proof: $\underline{D'_\mu \psi'} = \left[\partial_\mu - ig\left(\frac{-i}{g}(\partial_\mu U)U^{-1} + U\tilde{A}_\mu U^{-1}\right)\right] U\psi$

$$= \underline{(\partial_\mu U)\psi} + U(\partial_\mu \psi) - \underline{(\partial_\mu U)\psi} - igU\tilde{A}_\mu \psi$$

$$= U\left(\partial_\mu - ig\tilde{A}_\mu\right)\psi = \underline{U D_\mu \psi} \quad \checkmark$$

and from $D'_\mu \psi' = D'_\mu U\psi \overset{!}{=} U D_\mu \quad // U^{-1}\times$

$$\Rightarrow U^{-1} D'_\mu U = D_\mu \quad \checkmark$$

•) infinitesimal:

$$A_\mu^a t^a \to (1 - i\Theta^b t^b) A_\mu^a t^a (1 + i\Theta^b t^b)$$

$$- \frac{i}{g}\left[\partial_\mu (1 - i\Theta^a t^a)\right]\left[1 + i\Theta^b t^b\right]$$

$$= \ldots = A_\mu^a t^a - \frac{1}{g}\partial_\mu \Theta^a t^a + \underbrace{f^{abc}\Theta^b A_\mu^c t^a}_{\text{new!} \leftrightarrow \text{Non-Abelian!}}$$

•) kinetic terms: $F_{\mu\nu} = \frac{i}{g}\left[D_\mu, D_\nu\right] = F_{\mu\nu}^a t^a$

$$= \left(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f_{abc} A_\mu^b A_\nu^c\right) t^a$$

trafo: $F_{\mu\nu} \to F'_{\mu\nu} = U F_{\mu\nu} U^{-1}$ (insert $D'_\mu = U D_\mu U^{-1}$)

$\Rightarrow \mathrm{Tr}\{F_{\mu\nu} F^{\mu\nu}\}$ is gauge invariant!

Remarks: •) massive gauge bosons $m_W^2 A_\mu^a A^{\mu a}$

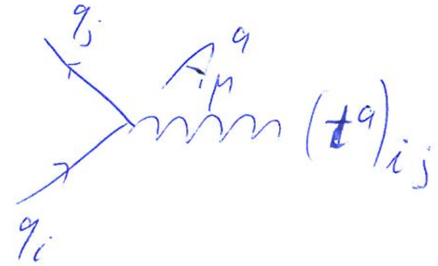
still forbidden...

•) $F_{\mu\nu} F^{\mu\nu} \sim (\partial A + g A^2)^2 \sim A^3 + A^4$

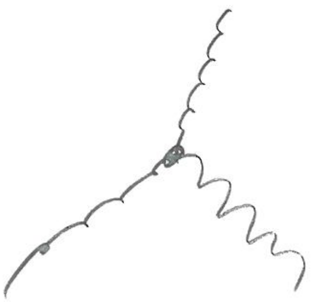
self-interactions

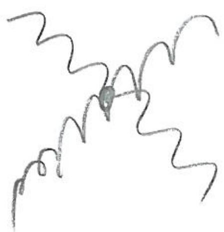
$\mapsto$ Non-Abelian

•) $i g \bar{\psi} \gamma^\mu t^a \psi A_\mu^a$



$g f_{abc} \partial_\mu A_\nu^a A^{b\mu} A^{c\nu} + g^2 f_{abc} f_{ade} A_\mu^b A_\nu^c A^{d\mu} A^{e\nu}$



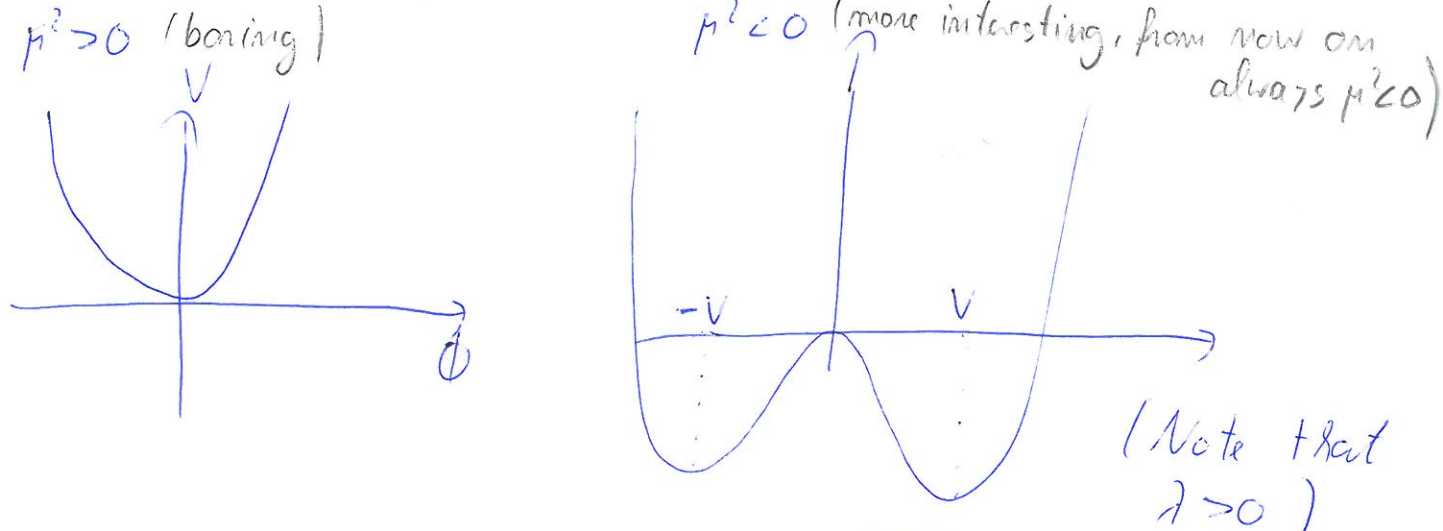


3) Spontaneous Symmetry Breaking + Higgs-Mechanism

$\mathcal{L}$ possesses symmetry which the ground state does <u>not</u> obey

a) $\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - V(\phi)$; $V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4}\phi^4$

Symmetry : $\phi \to -\phi$ „ Z_2 "



(Note that $\lambda > 0$)

$v = \sqrt{-\mu^2/\lambda}$

Choose $\langle \phi \rangle = +v$ and do physics around the minimum: $\phi = v + \eta(x)$

($\to$ we can only do perturbation theory)

$$\Rightarrow \mathcal{L} = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \lambda v^2 \eta^2 - \lambda v \eta^3 - \frac{1}{2}\lambda \eta^4$$

$$\Rightarrow \boxed{m_\eta^2 = 2\lambda v^2}$$

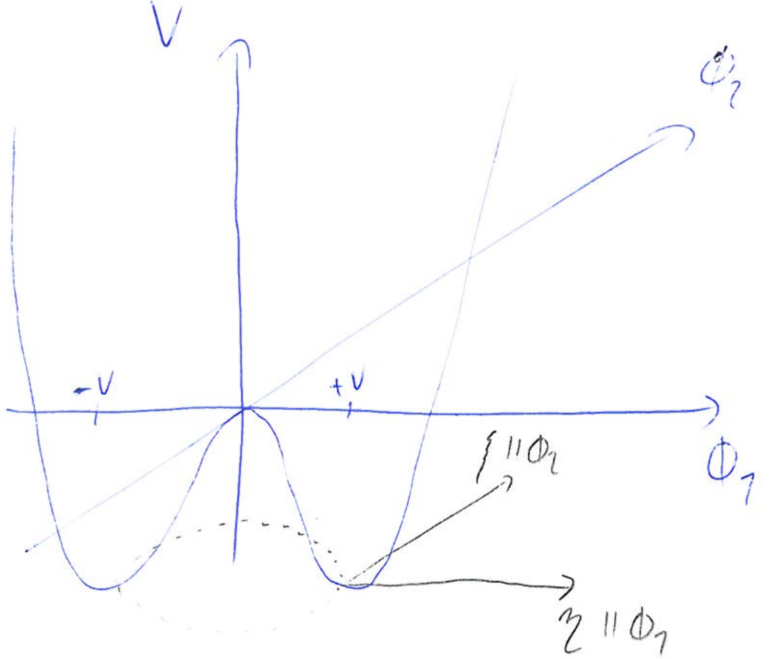
correct sign for mass!

this mass is generated by spontaneous symmetry breaking, the $\phi \to -\phi$ symmetry got lost in the ground state!

b) $\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi^*) - \mu^2 \phi^* \phi - \lambda(\phi^* \phi)^2$

with $\phi = \sqrt{\frac{1}{2}}(\phi_1 + i\phi_2)$

Symmetry: $\phi \to e^{i\alpha}\phi$ global $U(1)$



choose minimum

$\langle \phi \rangle = +v = \sqrt{-\mu^2/\lambda}$

physics around minimum:

$\phi = \sqrt{\frac{1}{2}}\left(v + \eta(x) + i\xi(x)\right)$

$\Rightarrow \mathcal{L} = \frac{1}{2}(\partial_\mu \xi)^2 + \frac{1}{2}(\partial_\mu \eta)^2 + \mu^2 \eta^2 \quad + \mathcal{O}(\text{cubic, quartic})$

$\rightarrow$ massive η

massless ξ . "flat direction", feels no resistance when moving

This massless boson ξ is called a <u>Goldstone boson</u> (occurs whenever a continuous symmetry is spontaneously broken)

c) $\mathcal{L} = [(\partial^\mu + ieA^\mu)\phi^*][(\partial_\mu - ieA_\mu)\phi] - \mu^2 \phi^* \phi - \lambda(\phi^*\phi)^2$

recall: $A_\mu \rightarrow A_\mu + \frac{1}{e}(\partial_\mu \alpha)$

Symmetry: $\phi \rightarrow e^{i\alpha(x)}\phi(x)$ <u>local</u> $U(1)$

choose $\phi = \sqrt{\frac{1}{2}}(v + \eta(x) + i\xi(x))$

$$\Rightarrow \mathcal{L} = \frac{1}{2}(\partial_\mu \xi)^2 + \frac{1}{2}(\partial_\mu \eta)^2 \underbrace{- v^2 \lambda \eta^2}_{m_\eta} \underbrace{+ \frac{1}{2} e^2 v^2 A_\mu A^\mu}_{m_\gamma} \underbrace{- evA_\mu \partial^\mu \xi}_{?}$$

count ~~dof~~ degrees of freedom:

before: (2 + 2) (Φ + massless A_μ)

after: (3+2) (ξ, η + massive A_μ)

WTF?

the $A_\mu \partial^\mu \xi$ term looks suspicious, it is the coupling of our Goldstone-boson with the gauge field (goes with derivative)

The hint how to proceed we get from noting:

$$\Phi = \sqrt{\tfrac{1}{2}}\,(v + \eta + i\xi) \simeq \sqrt{\tfrac{1}{2}}\,(v+\eta)\, e^{i\xi/v}$$

! looks like a gauge trafo !

Thus, we choose a particular gauge:

$$\Phi = \sqrt{\tfrac{1}{2}}\,(v + h(x))\, e^{i\,\Theta(x)/v}$$ such that $h(x)$ is real

and $$A_\mu \to A_\mu + \frac{1}{ev}\,\partial_\mu \Theta(x)$$

aim is to get rid of Θ

insert this in $\mathcal{L}$

$$\Rightarrow \mathcal{L} = \frac{1}{2}(\partial_\mu h)^2 - \lambda v^2 h^2 + \frac{1}{2} e^2 v^2 A_\mu^2 + \dots$$

$\rightarrow$ the GB has disappeared, it has been „eaten" to become the third degree of freedom of the now massive field A_μ.

d) global $O(N)$ with $\frac{1}{2}N(N-1)$ generators t^a

$$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_N \end{pmatrix} \qquad \mathcal{L} = \frac{1}{2}\partial_\mu \phi^i \partial^\mu \phi^i - \frac{1}{2} m^2 \phi^i \phi^i - \frac{\lambda}{4}(\phi^i \phi^i)^2$$

choose $\langle \phi \rangle = \begin{pmatrix} 0 \\ \vdots \\ v \end{pmatrix} \} N-1$ vacuum invariant under rotations of the $N-1$ zero entries $\Rightarrow O(N-1)$ with $\frac{1}{2}(N-1)(N-2)$ generators $\tilde{t}_a$

$$\Rightarrow \text{write } \phi = \exp\left(i\alpha^a \frac{\tilde{t}^a}{v}\right) \begin{pmatrix} 0 \\ \vdots \\ v+h(x) \end{pmatrix} \} N-1$$

$\Rightarrow$ I will find $\frac{1}{2}(N-1)(N-2) - \frac{1}{2}N(N-1) = N-1$ GB

~~$\Rightarrow$ there is one massless GB for each broken generator "Goldstone theorem"~~

3) The Standard Model

want to describe weak interactions and electromagnetism.

$\Rightarrow$ $U(1)$ should be there; call it $\boxed{U(1)_Y}$ "Hyper-charge"

in addition: weak interactions deal with

$p \to n$ transitions, $\Rightarrow u \to d$

$\leftrightarrow$ Heisenberg-theory $\begin{pmatrix} p \\ n \end{pmatrix} \leftrightarrow \begin{pmatrix} u \\ d \end{pmatrix}$

$\Rightarrow SU(2)$

Observation: parity violation, chiral theory, left-handed and right-handed treated differently, right-handed fermions seem to be "sterile" under $SU(2)$ $\Rightarrow$ $\boxed{SU(2)_L}$

we define $\quad \psi_1 = \begin{pmatrix} u \\ d \end{pmatrix}_L$ or $\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \sim 2_L \quad , \quad Y_1$

2_L: doublet of $SU(2)_L$ irreducible representation

Y_1: gets hypercharge Y_1

(recall: $\quad u_L = \frac{1}{2}(1-\gamma_5)u \quad ; \quad P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$ projections
$\quad \overline{u_L} = \bar{u} P_R \quad ; \quad P_L^2 = P_R^2 = 1 \quad ; \quad P_L P_R = 0$)

$$\psi_2 = u_R \quad \text{or} \quad \nu_R \qquad \sim 1_L \quad , \quad Y_2$$

$$\psi_3 = d_R \quad \text{or} \quad e_R \qquad \sim 1_L \quad , \quad Y_3$$

1_L: singlets of $SU(2)_L$ do not transform

(This is typical for model building: 1) choose group! 2) choose how particles transform!)

Note that we violate parity maximally from the very beginning!