How to construct the Standard Model

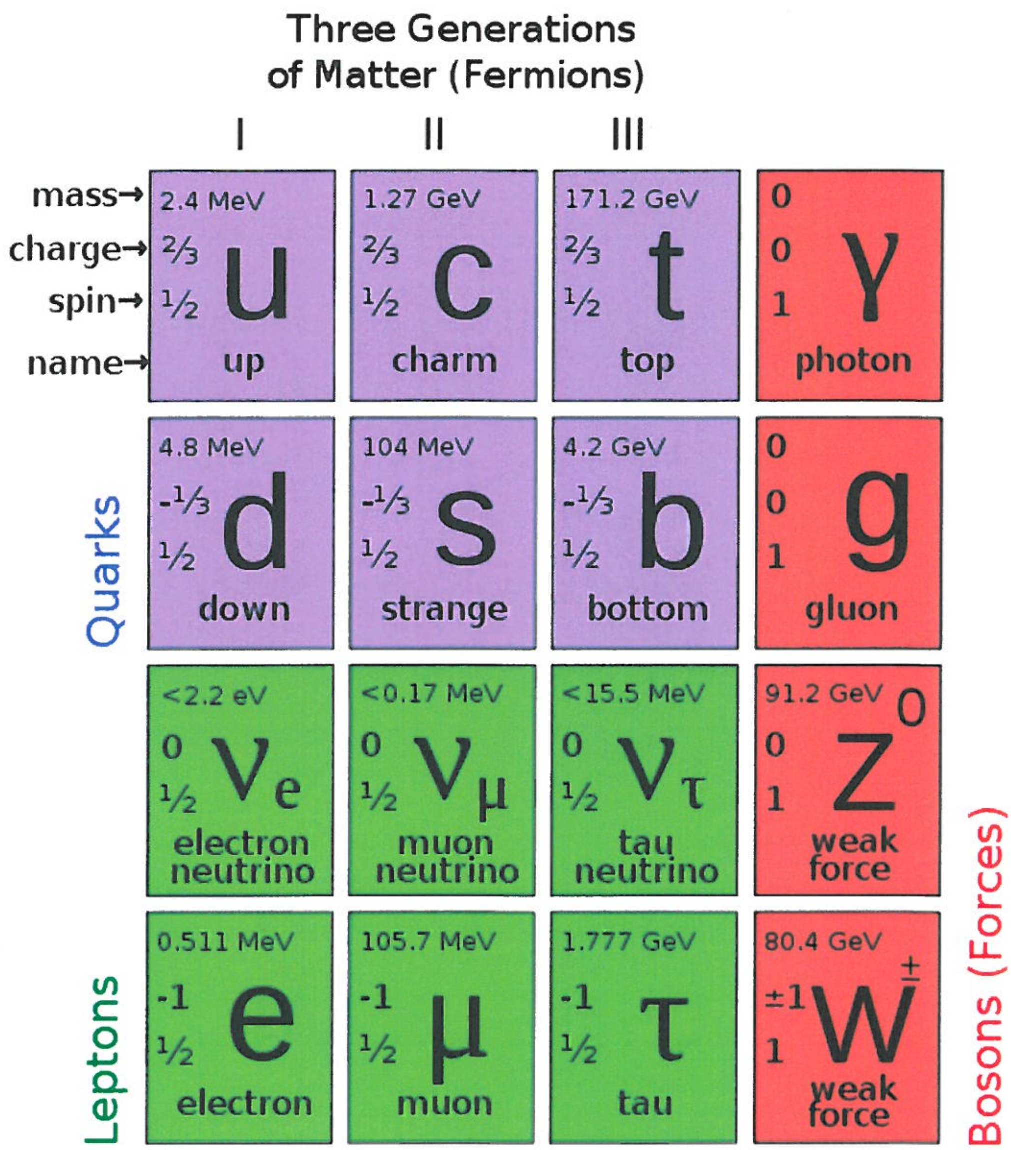
- Central part of the lecture

- Outline:

    - Gauge Symmetries: Abelian and Non-Abelian

    - $SU(3) \times \underbrace{SU(2) \times U(1)}_{\text{(focus on this part)}}$ for the SM

    - Electroweak symmetry breaking and the Higgs



(+ Higgs-boson)

- SM will describe interactions + the fact that some are massive, others are not massive

- <u>cannot</u> tell us: why I, II, III? why chiral?
  parameter values? Fermion mixing?
  Dark matter? Baryon asymmetry?
  Conservation of L and B?
  ....

1) The gauge principle

we deal with fermions, bosons, scalars (?)

$$(\not{p}-m)\psi = 0\,;\ \left(\partial_\mu \partial^\mu [+m^2]\right)A^\nu = 0\,;\ (\partial_\mu\partial^\mu + m^2)\phi = 0$$

described by Lagrange density (Lagrangian) $\mathcal{L}$

<u>Action</u>: $S = \int d^4x\, \mathcal{L}$ $\qquad [S] = [\hbar]$

$$\mathcal{L} = \mathcal{L}(\phi, \partial_\mu \phi)$$

from <u>principle of least action</u>, $\delta S = 0$,

it follows that $\boxed{\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} = \frac{\partial \mathcal{L}}{\partial \phi}}$

Euler - Lagrange equation

examples: $\mathcal{L} = \bar{\psi}(\not{p} - m)\psi$ $\qquad (*)$

$$\mathcal{L} = \frac{1}{2}\left[(\partial_\mu \phi)(\partial^\mu \phi) - m^2 \phi^2\right]$$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - j^\mu A_\mu$$

we note that (*) is invariant under

$$\psi \to e^{i\alpha}\psi \simeq (1+i\alpha)\psi \equiv \psi'$$

<u>global gauge transformation</u>

$\alpha = \text{const}$

· insert in $\mathcal{L}$ and use that $\delta\mathcal{L} = 0$

$$\Rightarrow 0 = \frac{\partial\mathcal{L}}{\partial\psi}\delta\psi + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\delta(\partial_\mu\psi)$$

$$= \frac{\partial\mathcal{L}}{\partial\psi}(\psi' - \psi) + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\left[\partial_\mu\psi' - \partial_\mu\psi\right]$$

$$= \frac{\partial\mathcal{L}}{\partial\psi}(i\alpha\psi) + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}(i\alpha\,\partial_\mu\psi)$$

$$= i\alpha\underbrace{\left[\frac{\partial\mathcal{L}}{\partial\psi} - \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\right)\right]}_{=0}\psi + i\alpha\,\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\psi\right)$$

$$\Rightarrow i\alpha\,\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\psi\right) = 0$$

we need $\frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)} = \frac{\partial}{\partial(\partial_\mu \psi)}\left[\bar{\psi}(+i\gamma_\mu \partial^\mu - m)\psi\right]$

$$= +i\bar{\psi}\gamma^\mu\psi$$

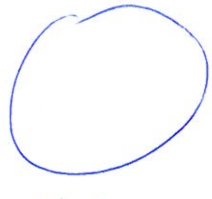
$$\Rightarrow \boxed{\partial_\mu j^\mu = 0 \quad \text{with } j^\mu = \bar{\psi}\gamma^\mu\psi}$$

$\Rightarrow$ conserved current due to invariance under global trafo „U(1) Abelian gauge transformation"

$\to$ this is of course the Noether theorem

there is also a conserved charge:

$$\frac{d}{dt} Q \equiv \frac{d}{dt}\int d^3x\, j^0 = -\int d^3x\, \vec{\nabla}\vec{j} \underset{\substack{\uparrow \\ \text{Gauss}}}{=} -\oint d\vec{S}\,\vec{j} = 0$$



Volume covered by surface S $\quad \psi(x\to\infty) = 0$

charge constant if no charge leaves volume, or: any charge leaving volume must be accounted for by flux $\vec{j}$ leaving volume now we generalize: $\alpha(x)$ local trafo

problem: $\bar\psi(i\partial\!\!\!/ - m)\psi \to \bar\psi e^{-i\alpha(x)}(+i\partial\!\!\!/ - m) e^{i\alpha(x)}\psi$

$$= -\bar\psi m \psi + \bar\psi e^{-i\alpha(x)}\left[i(\partial\!\!\!/ \psi) e^{i\alpha(x)} + i(\partial\!\!\!/ \alpha)\psi e^{i\alpha(x)}\right]$$

$$= \bar\psi(i\partial\!\!\!/ - m)\psi - \bar\psi\gamma^\mu\psi(\partial_\mu\alpha)$$

solution: $\mathcal{L} = \bar\psi(i D\!\!\!\!/ - m)\psi$

$$\to \bar\psi e^{-i\alpha(x)}(i D\!\!\!\!/' - m) e^{i\alpha(x)}\psi$$

such that $D'_\mu = e^{i\alpha(x)} D_\mu$

only possibility to achieve this:

$$\boxed{\begin{aligned} D_\mu &= \partial_\mu - ieA_\mu \\ A_\mu &\to A_\mu + \tfrac{1}{e}\partial_\mu\alpha \end{aligned}}$$

covariant derivative

gauge field

proof: $D'_\mu\psi' = (\partial_\mu - ieA_\mu - i\partial_\mu\alpha) e^{i\alpha(x)}\psi$

$$= \cancel{i(\partial_\mu\alpha) e^{i\alpha}\psi} - ieA_\mu e^{i\alpha}\psi - \cancel{i(\partial_\mu\alpha) e^{i\alpha}\psi} + e^{i\alpha}\partial_\mu\psi$$

$$= e^{i\alpha(x)}(\partial_\mu - ieA_\mu)\psi = e^{i\alpha(x)} D_\mu\psi$$

$$\Rightarrow \mathcal{L} = \underbrace{\bar{\psi}(\not{p}-m)\psi}_{\text{free}} + \underbrace{e\bar{\psi}\gamma^\mu\psi A_\mu}_{\text{interaction}} - \underbrace{\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\text{kinetic}}$$

<u>Note:</u> 1) $F_{\mu\nu}' = (\partial_\mu A_\nu' - \partial_\nu A_\mu') = \partial_\mu(A_\nu + \frac{1}{e}\partial_\nu\alpha) - \partial_\nu(A_\mu + \frac{1}{e}\partial_\mu\alpha)$

$$= \partial_\mu A_\nu - \partial_\nu A_\mu + \underbrace{\frac{1}{e}(\partial_\mu(\partial_\nu\alpha) - \partial_\nu(\partial_\mu\alpha))}_{=0}$$

$$= F_{\mu\nu} \quad (\text{gauge invariant on its own})$$

2) $\frac{i}{e}[D_\mu, D_\nu] = \frac{i}{e}[\partial_\mu - ieA_\mu, \partial_\nu - ieA_\nu]$

$$= \frac{i}{e}(-ie)\{[A_\mu, \partial_\nu] + [\partial_\mu, A_\nu]\}$$

$$= A_\mu\partial_\nu - \partial_\nu A_\mu + \partial_\mu A_\nu - A_\nu\partial_\mu$$

$$= \cancel{A_\mu\partial_\nu} - (\partial_\nu A_\mu) - \cancel{A_\mu\partial_\nu} + (\partial_\mu A_\nu) + \cancel{A_\nu\partial_\mu} - \cancel{A_\nu\partial_\mu}$$

$$= \partial_\mu A_\nu - \partial_\nu A_\mu = \underline{F_{\mu\nu}}$$

3) $$m_\gamma^2 A_\mu A^\mu \to m_\gamma^2 \left(A_\mu + \frac{1}{e}\partial_\mu\alpha\right)\left(A^\mu + \frac{1}{e}\partial^\mu\alpha\right)$$

$$= m_\gamma^2 \left(A_\mu A^\mu + A_\mu(\partial^\mu\alpha)/e + \frac{1}{e}(\partial_\mu\alpha)A^\mu + \frac{1}{e^2}(\partial_\mu\alpha)\partial^\mu\alpha\right)$$

not gauge invariant

$\Rightarrow m_\gamma = 0$

Lesson: local symmetry (gauge invariance)

needs existence of

(massless) gauge field

interactions

$\Rightarrow$ generalize this and apply to all interactions

2) Non-Abelian gauge symmetries

$e^{i\alpha(x)}$ is Abelian group: $e^{i\alpha(x)} e^{i\beta(x)} = e^{i\beta(x)} e^{i\alpha(x)}$

turns out, we have to use Non-Abelian groups, in particular $SU(N)$, the group of unitary $N \times N$ matrices with $\det = +1$.

→ an infinitesimal transformation has the form

$$U = 1 + i\,\alpha^a t^a = 1 + i\,\vec{\alpha}\,\vec{t} \simeq e^{i\vec{\alpha}\vec{t}}$$

with α_a infinitesimal parameters and t_a "generators" of the group $SU(N)$.

properties: $UU^\dagger = 1 + i\alpha^a(t^a - t^{a\dagger}) \Rightarrow \boxed{t^{a\dagger} = t^a}$

$(\det e^A = e^{\mathrm{Tr}\,A})$ $\quad \det U = 1 + i\alpha^a\,\mathrm{Tr}\{t^a\} \Rightarrow \boxed{\mathrm{Tr}\,t^a = 0}$

→ generators are hermitian and traceless

→ there are $N^2 - 1$ generators, which obey

this vector space is called Lie-Algebra of Lie group ← $[t^a, t^b] = i f^{abc}\, t^c$ (commutator is antihermitian and traceless) where f^{abc} are structure constants (antisymmetric)

Examples: 1) $SU(2)$: 3 Generators (Pauli-matrices)

$$\sigma_{1,2,3} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\left[\frac{\sigma_i}{2}, \frac{\sigma_j}{2}\right] = i\,\varepsilon_{ijk}\,\sigma_k/2$$

2) $SU(3)$: 8 Generators (Gell-Mann matrices)

$$\lambda_{1,\dots,8} = \dots$$

$$\left[\frac{\lambda_i}{2}, \frac{\lambda_j}{2}\right] = i f^{ijk} \frac{\lambda_k}{2}$$

$$f^{123} = 1$$

$$f^{147,246,257,345} = \frac{1}{2}$$

$$f^{156,367} = -\frac{1}{2}$$

$$f^{458,678} = \sqrt{3}/2$$

- Note that $\mathcal{U}$ is $N \times N$ matrix.
  $\Rightarrow$ to transform something, we need to find a „vector", e.g. $L = \begin{pmatrix} \nu \\ e \end{pmatrix}$, $Q = \begin{pmatrix} u \\ d \end{pmatrix}$ $\rightarrow SU(2)$

state Q forms an irreducible representation of the Lie group

$$q = \begin{pmatrix} q_r \\ q_g \\ q_b \end{pmatrix} \qquad \rightarrow SU(3)$$

E.g.: $\bar{Q} \rightarrow \bar{Q}\mathcal{U}^{+}$, $Q \rightarrow \mathcal{U}Q \Rightarrow \bar{Q}Q \rightarrow \bar{Q}\mathcal{U}^{+}\mathcal{U}Q = \bar{Q}Q$ ✓

Formalism and general aspects:

$$\boxed{\psi_i \rightarrow \mathcal{U}_{ij}\psi_j} \quad \text{with} \quad \mathcal{U}_{ij} = \exp\{-i\theta^a t^a\}_{ij}$$
$$= (1 - i\theta^a t^a)_{ij}$$

invariance of $\bar{\psi}(\not{p} - m)\psi$ achieved through

$$\partial_\mu \longrightarrow D_\mu = \partial_\mu - ig A_\mu^a t^a \equiv \partial_\mu - ig \vec{A}\vec{t} \equiv \partial_\mu - ig \tilde{A}_\mu$$

$$\tilde{A}_\mu \longrightarrow \frac{-i}{g}(\partial_\mu \mathcal{U})\mathcal{U}^{-1} + \mathcal{U}\tilde{A}_\mu \mathcal{U}^{-1}$$

such that $D_\mu' \psi' = \mathcal{U} D_\mu \psi$ or, equivalently,

$$\not{D}' = \mathcal{U}\not{D}\mathcal{U}^{-1}$$

(„A_μ^a transform as the adjoint representation")