

Lecture:

Standard Model of Particle Physics

Heidelberg SS 2012

Fermi Theory



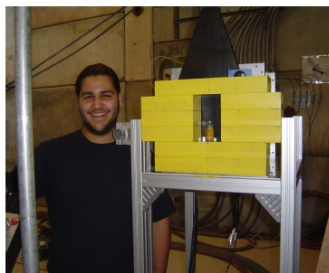
Particle Physics Practical Course at PSI (Switzerland) 2012

What's up ?

- During the semester break in summer we perform at the Paul Scherrer Institute (Switzerland) a real beam-line experiment to teach students in [experimental particle physics](#).
- About 10-12 students from the ETH Zurich and the Universities Zurich and Heidelberg spend three weeks at PSI to perform an experiment. The course includes lectures about several topics of experimental techniques. Main emphasis, however, is put on the practical work and “hands on”.
- Students plan and construct a small experiment from unused detector components. After commissioning the real fun starts: data taking all day and night (7/24) using one of the beamlines at PSI. During and after data taking a full analysis of the data is performed and summarised in a written document.

Examples from previous measurements:

- Branching Ratio: $B(\pi \rightarrow \mu \nu) / B(\pi \rightarrow e \nu)$
- Panofski Ratio: $B(\pi^+ \rightarrow \pi^0 \pi^+) / B(\pi^+ \rightarrow \pi^0 \gamma)$
- Lifetime and Muon-decay parameters



Date of next course:

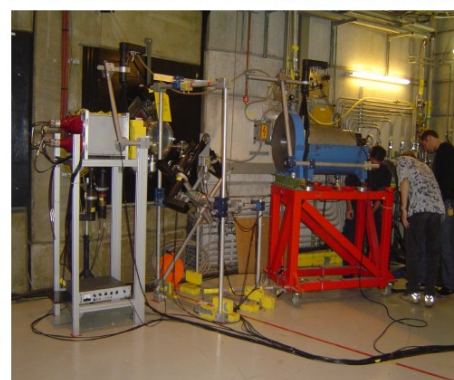
28. August - 15. September 2012

Contact Persons:

C.Grab : grab@phys.ethz.ch
A.Schöning: schoning@physi.uni-heidelberg.de
P.Robmann: peter@physik.uzh.ch

Limited number of places, please register!

This course (MVPSI) is part of the Master Programme at the Faculty of Physics and Astronomy in Heidelberg!



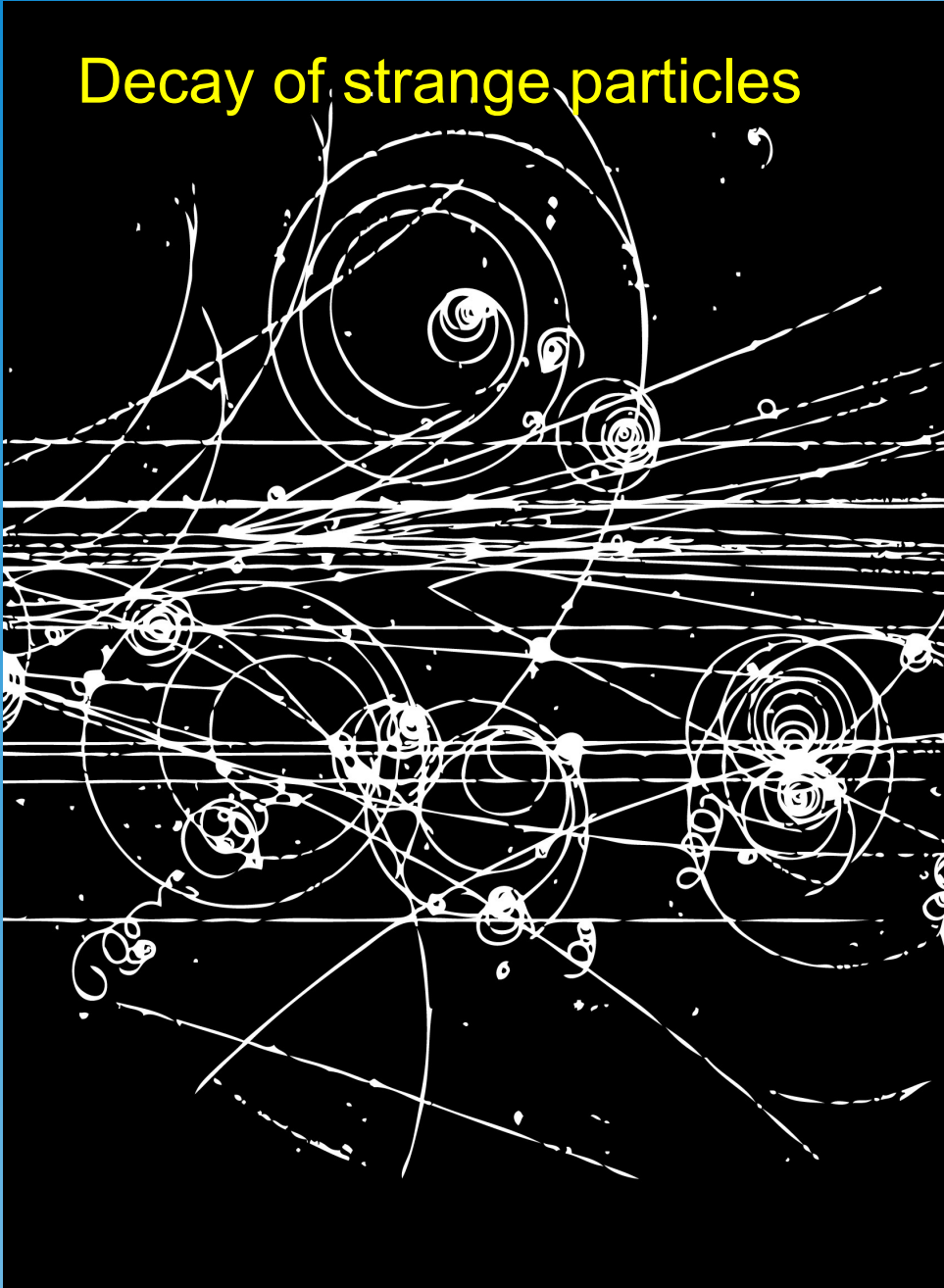
Fermi Theory

- **Unified description of all kind of Beta Decays?**
 - nuclear decays
 - muon and pion decay
 - decay of strange hadrons and heavy quarks
- **Description of weak scattering processes?**
 - at low energy processes
 - at high energy

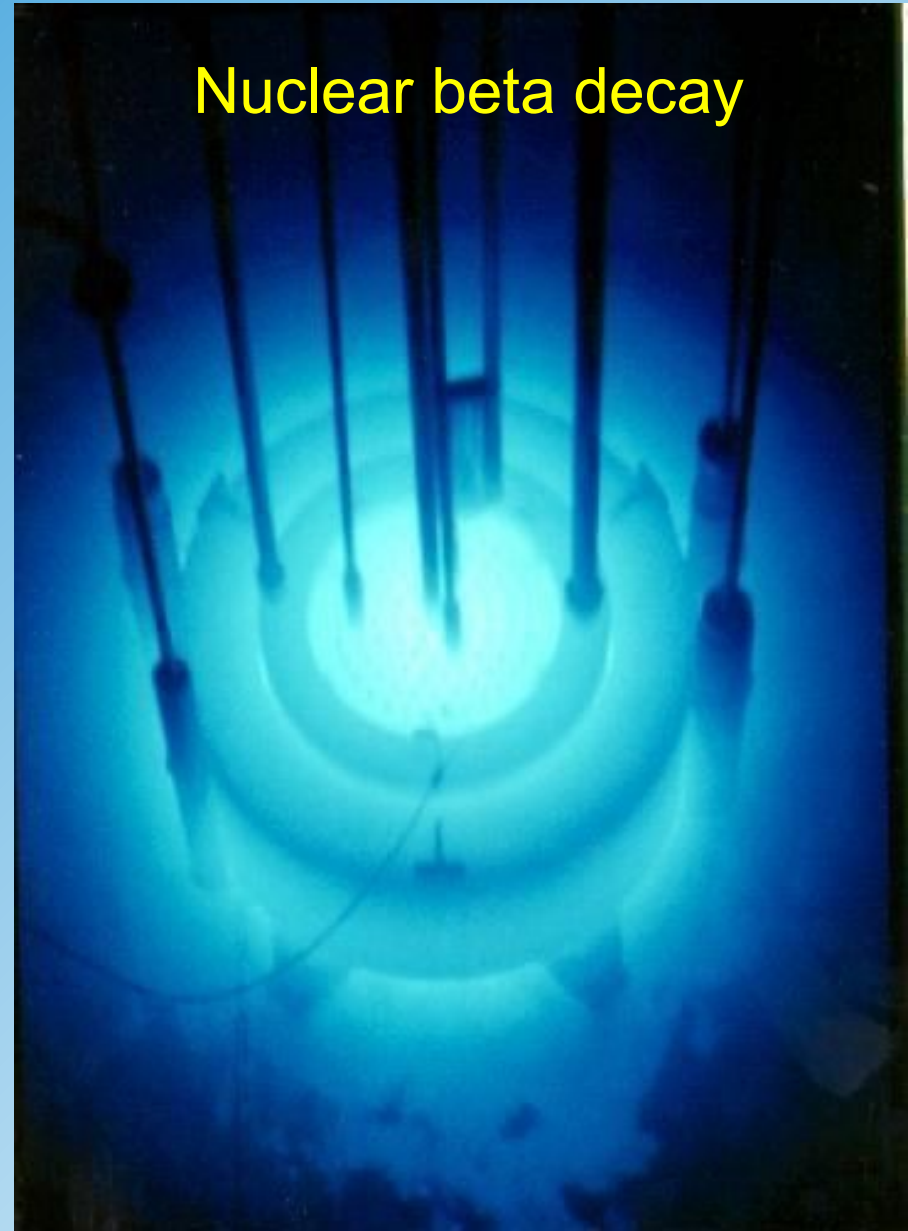
Isotop	Halbwertzeit
${}^1_0\text{n}$	11,7 m
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${}^{115}_{49}\text{In}$	6×10^{14} a

Weak Force

Decay of strange particles



Nuclear beta decay



Recap I

Lagrangian (independent of energy)

$$L = \frac{G}{\sqrt{2}} (\bar{f} \Gamma f') (\overline{f'''} \tilde{\Gamma} f''')$$

most general ansatz for operator Γ :

Vector Current:

$$j_V^\mu = \bar{\psi} \gamma^\mu \psi$$

Axial-vector Current:

$$j_A^\mu = \bar{\psi} \gamma^\mu \gamma^5 \psi$$

scalar coupling:

$$\lambda = \bar{\psi} \psi$$

pseudoscalar coupling:

$$\lambda = \bar{\psi} \gamma^5 \psi$$

Tensor Coupling

$$\sigma_A^{\mu\nu} = \bar{\psi} (\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu) \psi$$

Recap II

Vector Current conservation

$$\bar{u} \gamma^\mu u = \bar{u}_L \gamma^\mu u_L + \bar{u}_R \gamma^\mu u_R \quad (\text{no helicity flip})$$

Scalar Coupling

$$\bar{u} u = \bar{u}_R u_L + \bar{u}_L u_R \quad (\text{helicity flip!})$$

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What is the difference between helicity and chirality?

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What is the difference between helicity and chirality?

What is the difference between helicity and polarisation?

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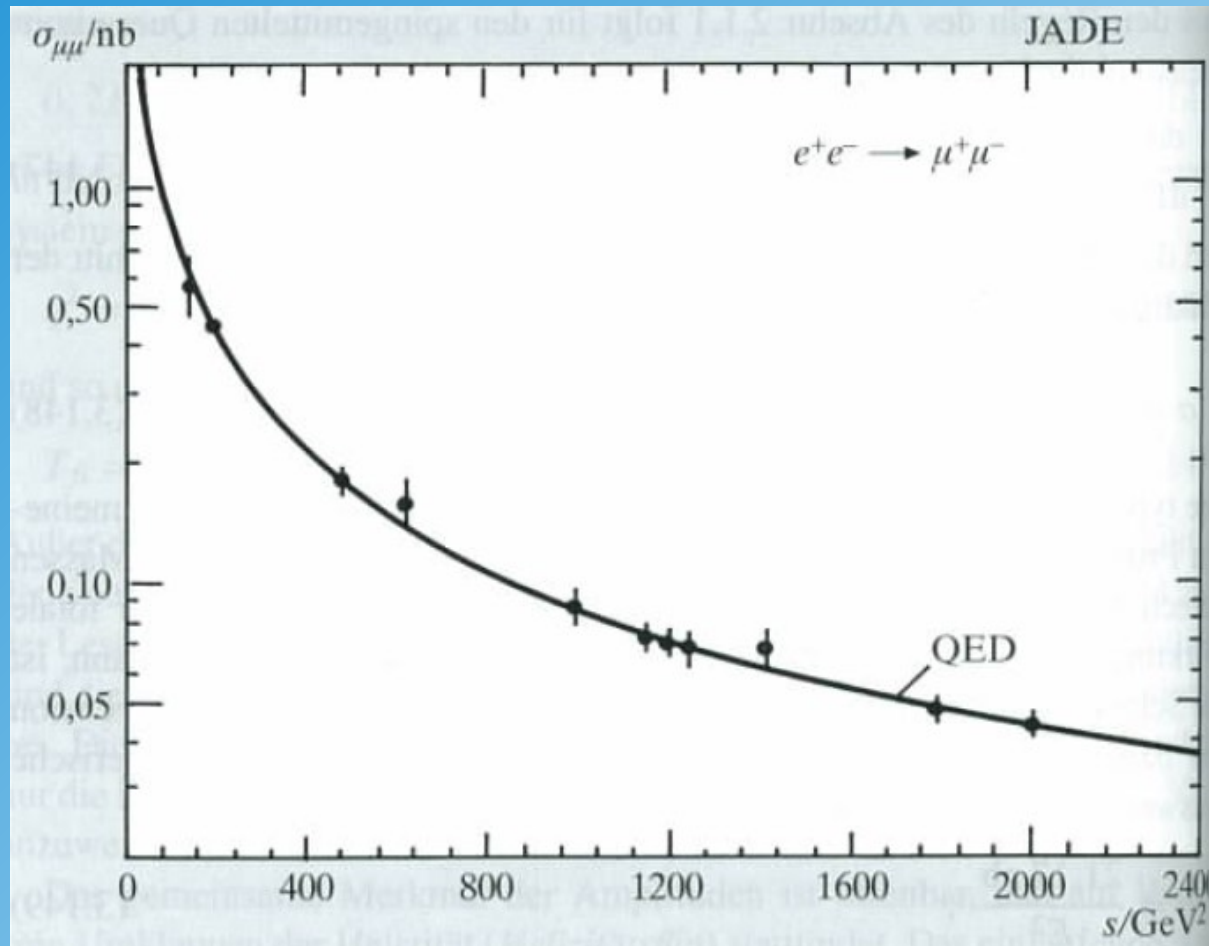
What is the difference between helicity and polarisation?

What is the difference between helicity flip and spin flip?

Recap III

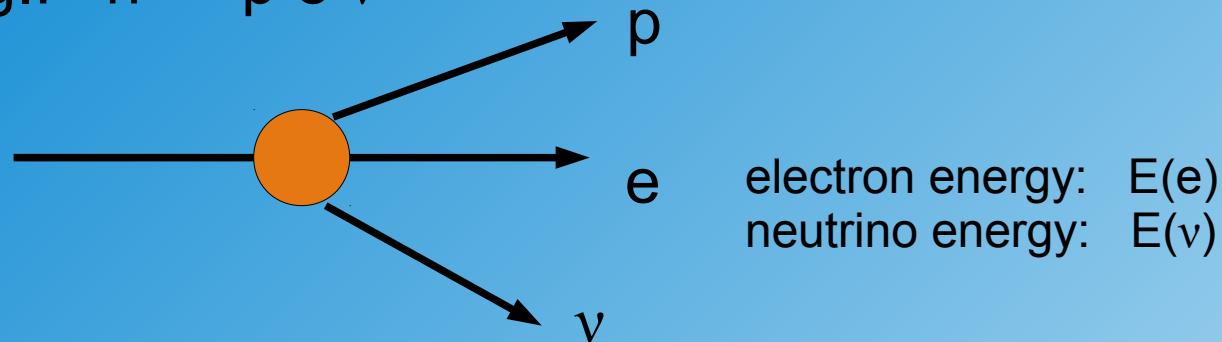
Energy dependence electromagnetic interaction:

$$\frac{1}{s}$$



3-Body Decay

e.g.: $n \rightarrow p e \nu$



Fermi's Golden rule:

$$dN(p) dp = 2 \frac{\pi}{\hbar} |\langle f | H | i \rangle|^2 \frac{dn}{dE_0} \quad \text{with:} \quad |H_{fi}|^2 = |\langle f | H | i \rangle|^2 = \text{const}$$

$$\text{Phase space:} \quad \frac{dn}{dE_0} = \frac{V^2}{4\pi^4 \hbar^4} p_e^2 dp_e p_\nu^2 dp_\nu \frac{1}{dE_0}$$

Beta Spectrum:

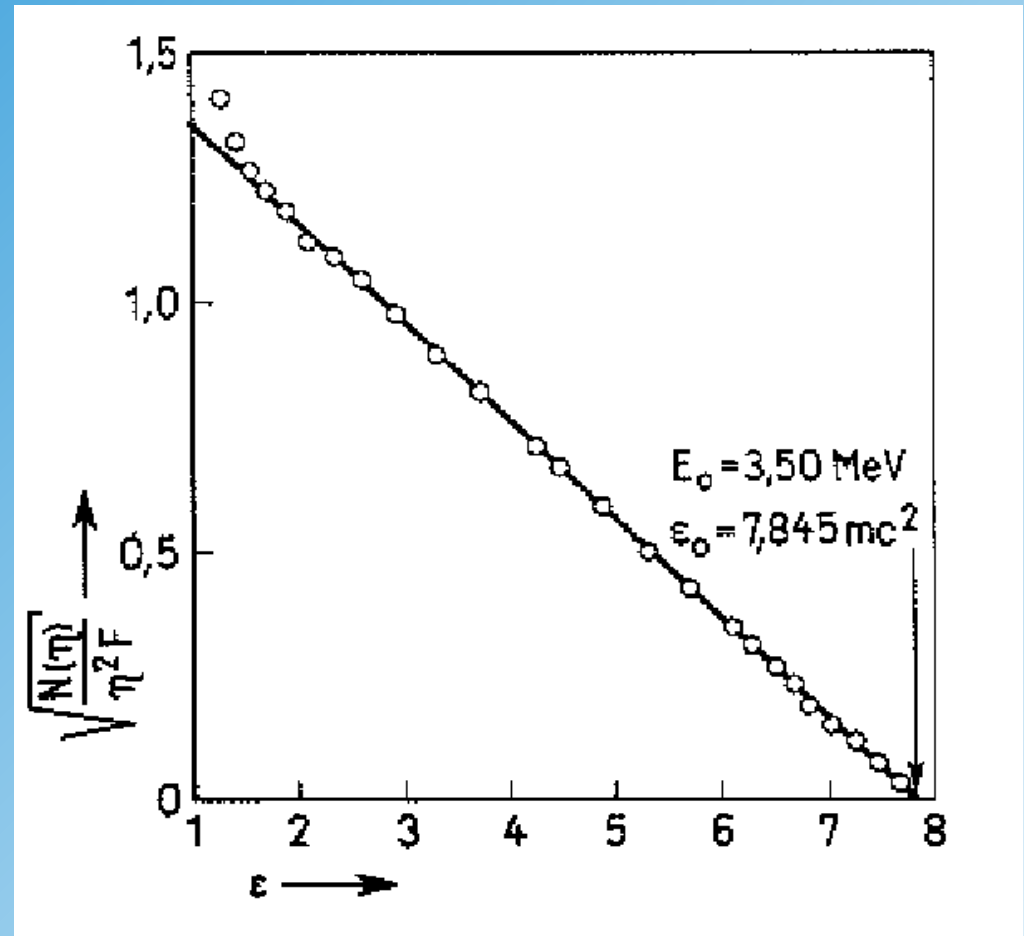
$$dN(\eta) d\eta = |H_{fi}|^2 \eta^2 (\epsilon_0 - \epsilon)^2 d\eta \quad \text{with:} \quad \begin{aligned} \eta &= p_e / m_e \\ \epsilon &= E_e / m_e \end{aligned}$$

Kurie-Plot (Fermi diagram)

$$\sqrt{\frac{dN(\eta)}{|H_{fi}|^2 \eta^2}} = \epsilon_0 - \epsilon$$

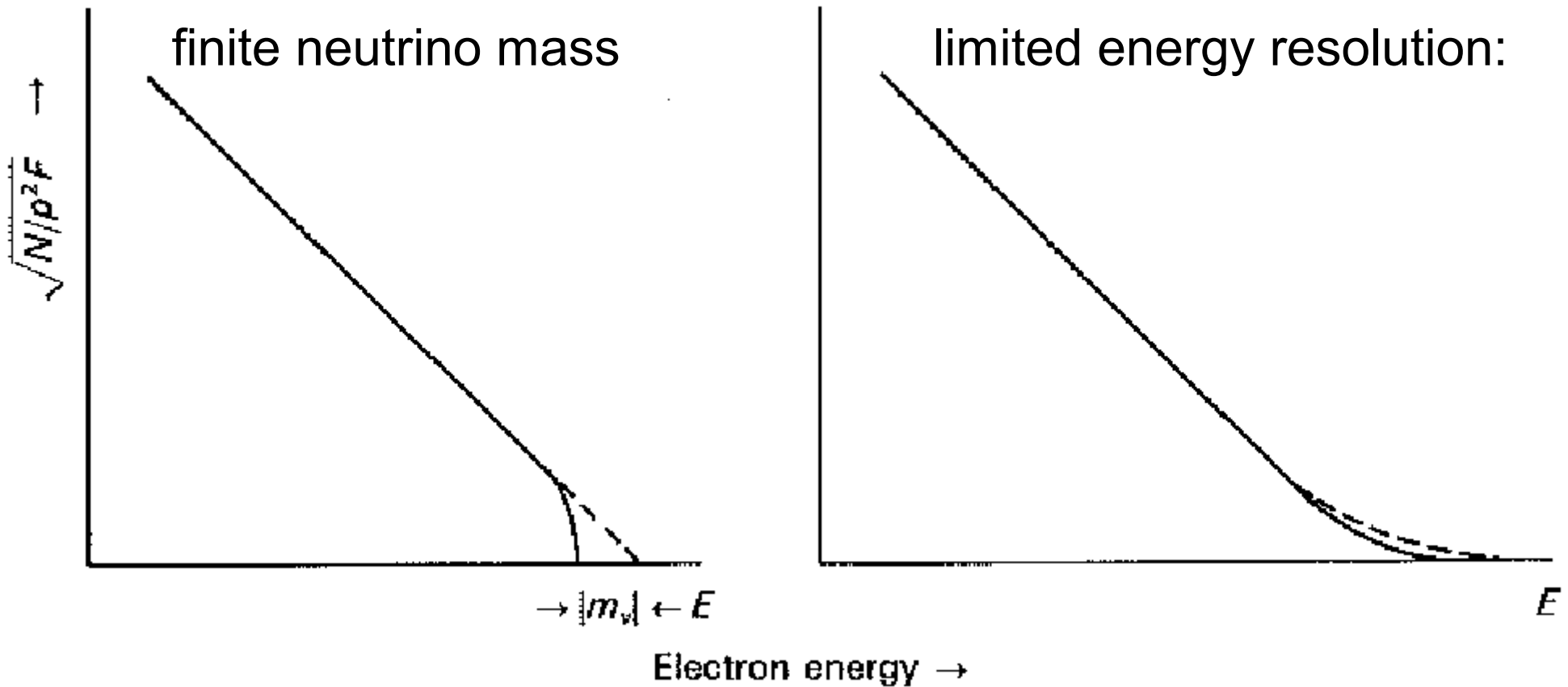
Beta decay of ${}^6\text{He} \rightarrow {}^6\text{Li} e^- \nu$

Note that the neutrino mass was set to zero here!

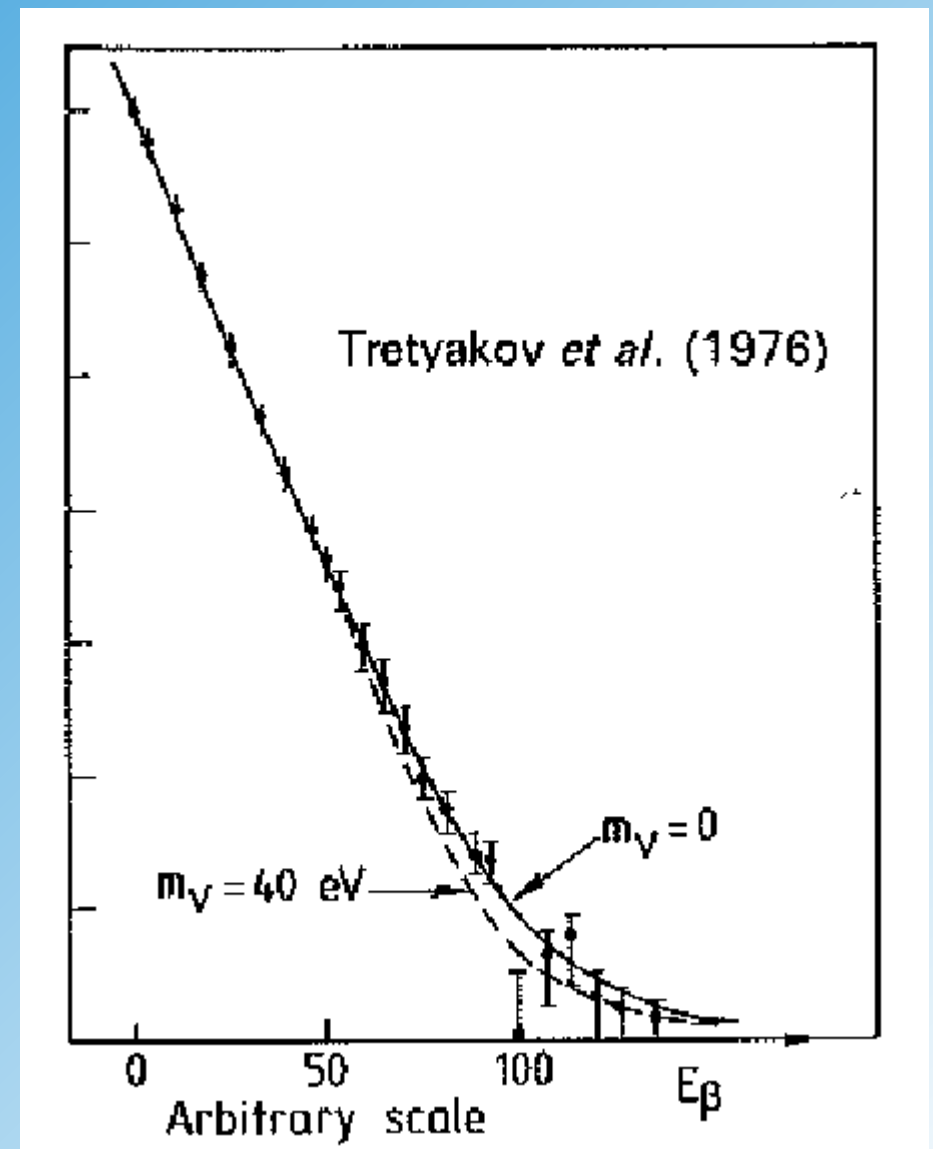
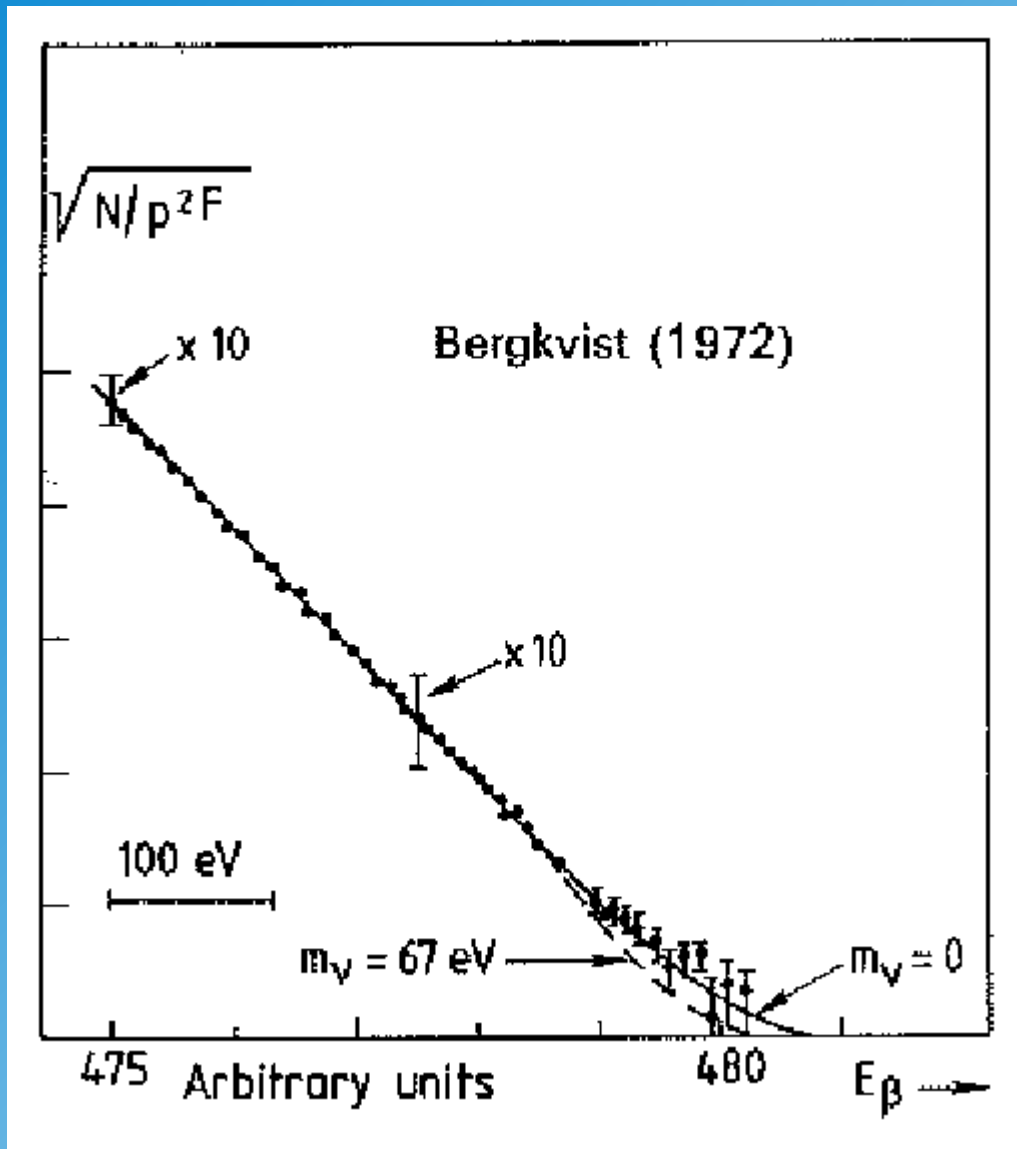


- linear function!
- matrix element independent of energy!

Mass and Resolution Effects



Neutrino-Mass Measurement



Katrin Experiment

Measure Beta-Spectrum in the tritium decay

current limit: $m_\nu < 1$ eV



expected limit:

$m_\nu < \sim 0.1-0.3$ eV

Lifetime in Beta Decay

Transition probability depends only on available decay energy E_0

from beta Spectrum:

$$dN(\eta) d\eta = |H_{fi}|^2 \eta^2 (\epsilon_0 - \epsilon)^2 d\eta$$

with $\eta \sim \epsilon$

$$\text{Decay width} \propto \epsilon_0^5 \propto M^5$$

Lifetime depends on the fifth power of the

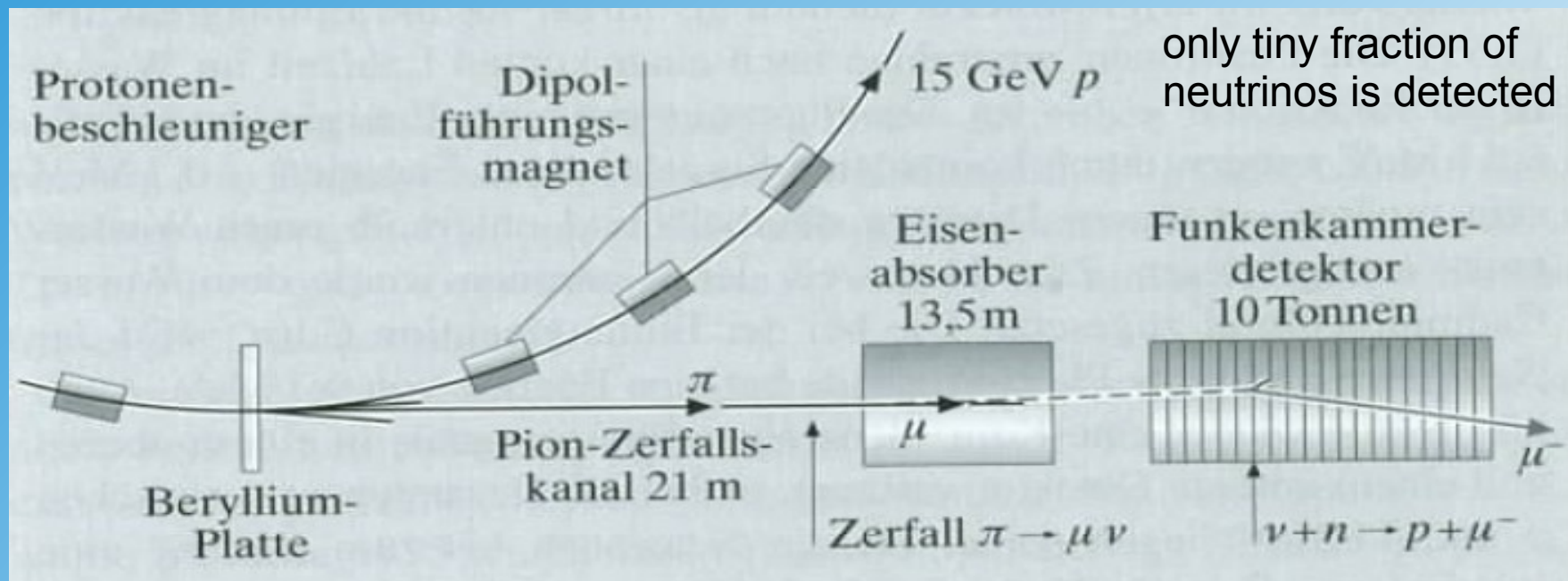
- particle mass (muon decay)
- Q value (nuclear decay $Q \sim E_e + E_\nu$)

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${}^1_0\text{n}$	11,7 m
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Weak Force

- Lifetimes in weak decays of order 10^{-10} seconds – 10^{10} years
- Interaction length:
 - Nuclear interaction (Fe): $\lambda_{\text{strong}} \sim O(10) \text{ cm}$
 - Weak interaction (Fe): $\gg 10^3 \text{ km}$ (neutrino energy dependent)

Discovery of muon neutrino (Lederman et al.):

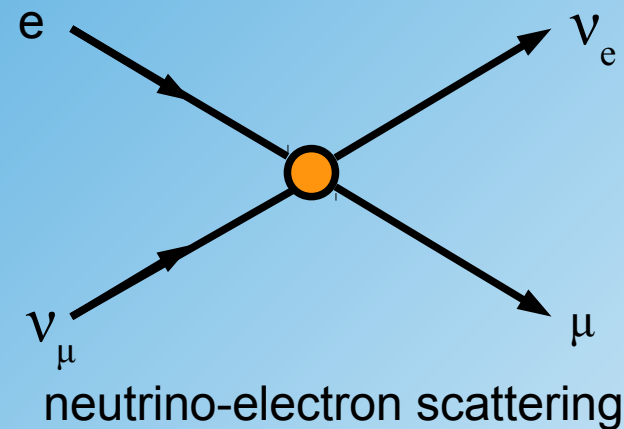


The weak force is really weak!

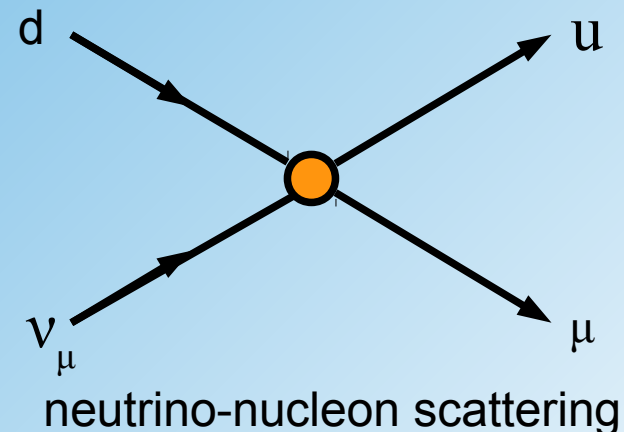
Weak scattering processes

- A constant matrix element (Fermi theory) give an energy dependent cross section of weak scattering processes
- Reason: phase space!

$$\sigma(\nu_{\mu} e \rightarrow \mu \nu_e) \propto G_F^2 s$$



$$\sigma(\nu_{\mu} d \rightarrow \mu u) \propto G_F^2 s$$

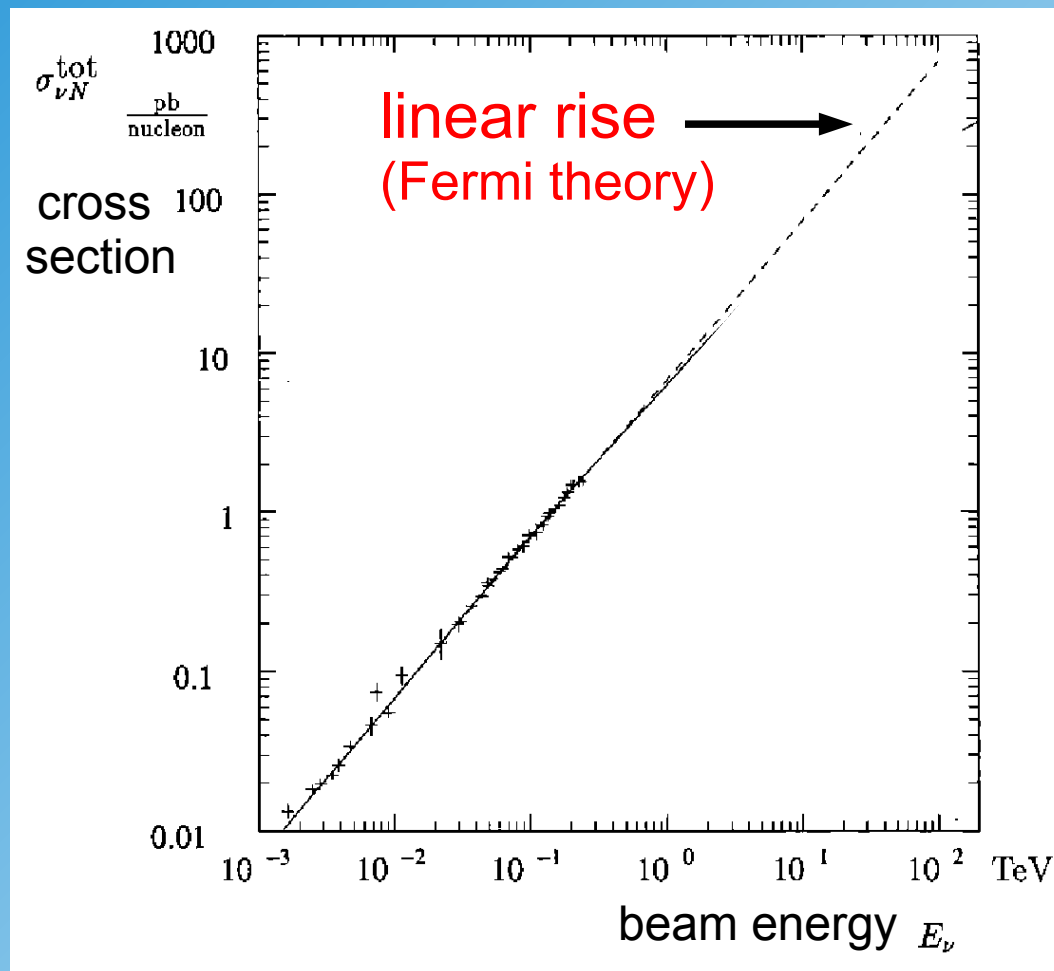


Weak scattering processes I

Kinematics:

- Fixed Target: $s = 2 E_\nu M_{target}$

Scattering cross section should rise linearly with neutrino beam energy



Weak scattering processes II

- Center of mass energies limited in Fixed Target Experiments
- Trick: invert reaction at colliders:

$$\nu_e d \rightarrow e^- u \quad \leftrightarrow \quad e^+ d \rightarrow \bar{\nu}_e u$$

Kinematics:

- Fixed Target: $s = 2 E_\nu^{target} M_{target}$
- Collider: $s = 4 E_e^{coll} E_p$ (HERA: electron-proton)

from comparison:

$$E_\nu^{target} \sim 2 E_e^{coll} \frac{E_p}{M_{target}} \approx 50 \text{ TeV}$$

Electron-Proton Collider HERA

$$E_e = 26.7 \text{ GeV} \quad E_p = 920 \text{ GeV}$$



Lorentz Invariant Kinematics of Deep Inelastic Scattering Process

The virtuality of the exchanged photon is given by:

$$Q^2 = -q^2 = -(p - p')^2$$

$$\propto \frac{1}{\sin^4 \theta / 2}$$

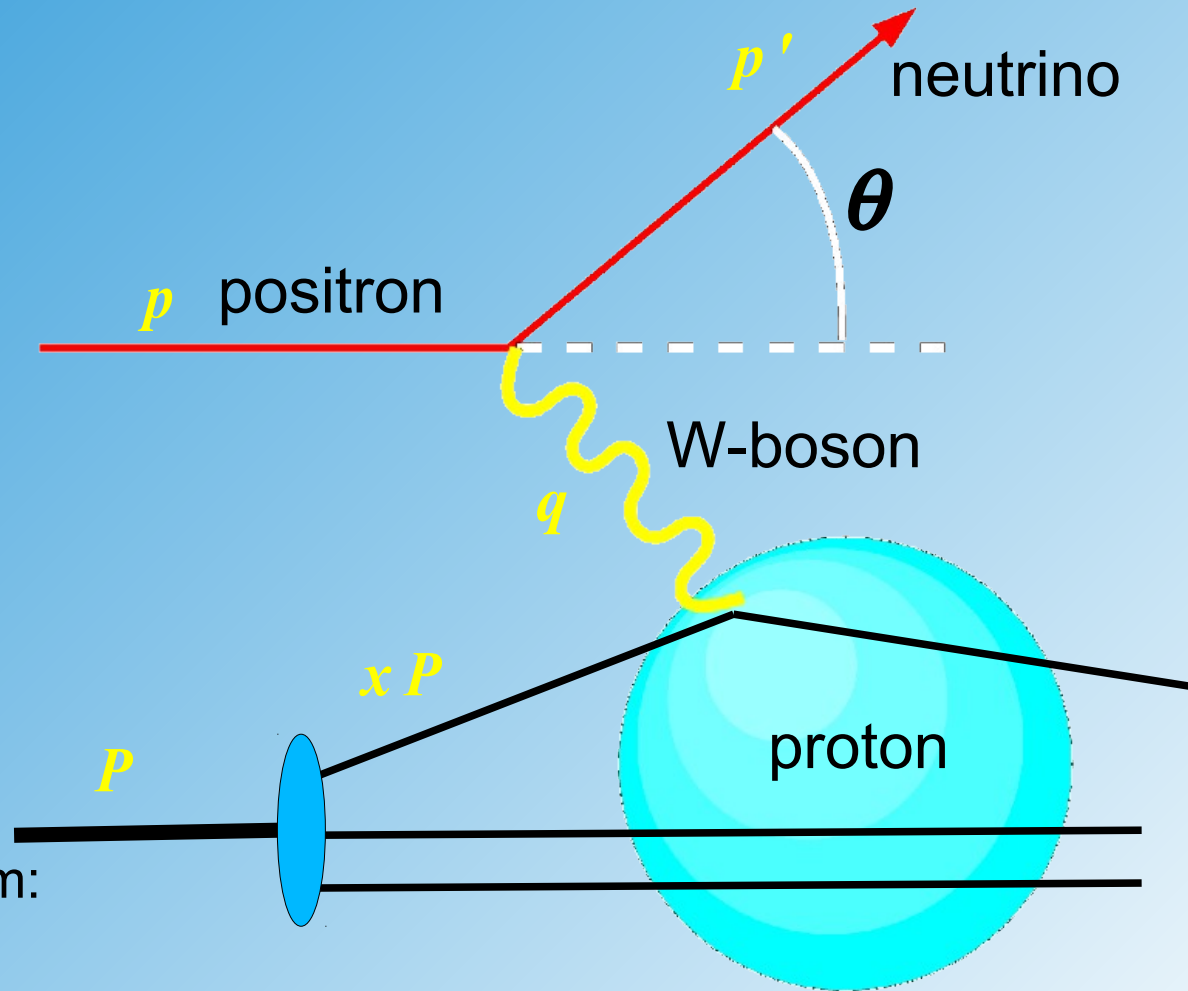
Relative energy loss (inelasticity):

$$y = \frac{\nu}{E_{\text{Elektron}}} = \frac{q P}{p P}$$

relative fraction of parton momentum:

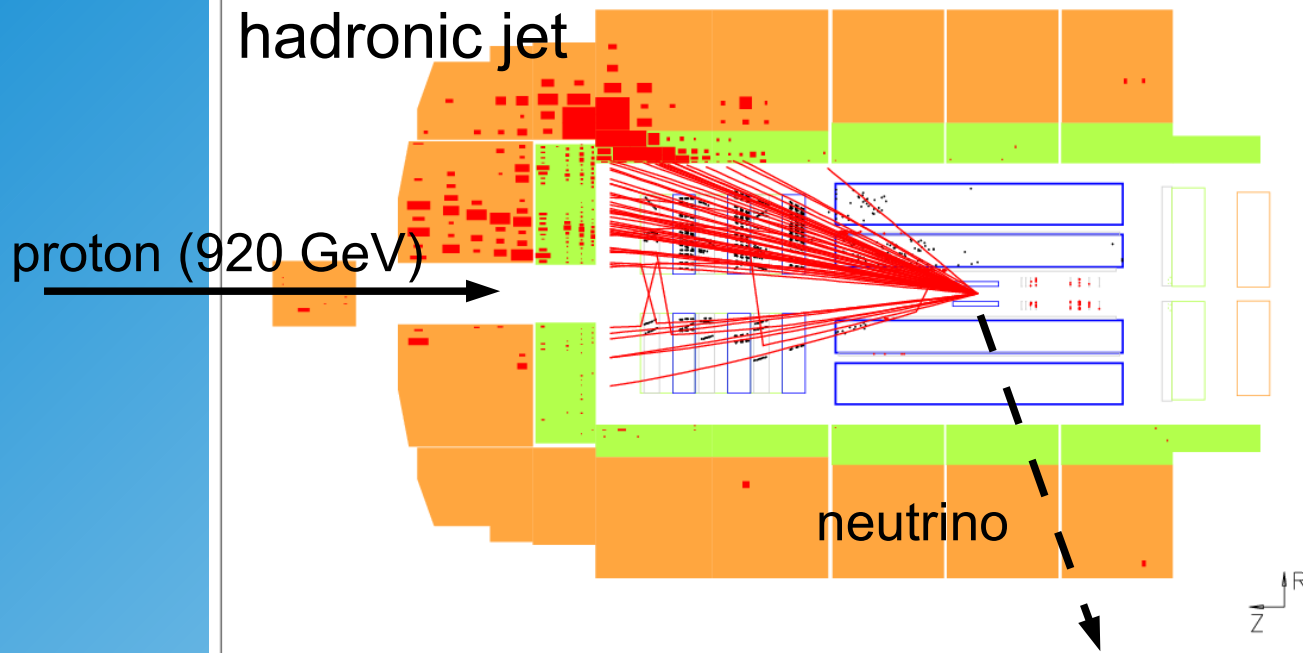
$$x = \frac{q^2}{2 q P} = \frac{Q^2}{s y}$$

with cms energy: $s = 2 p P$

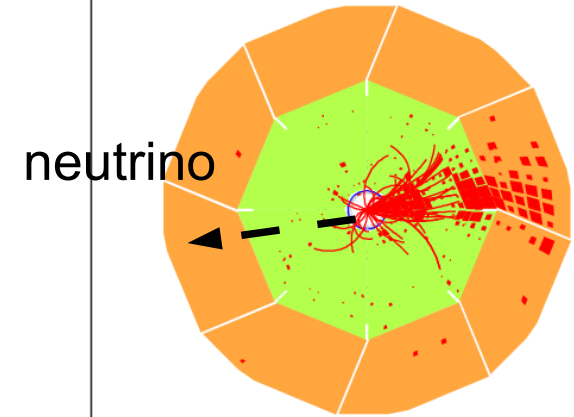


Charged Current Event at H1

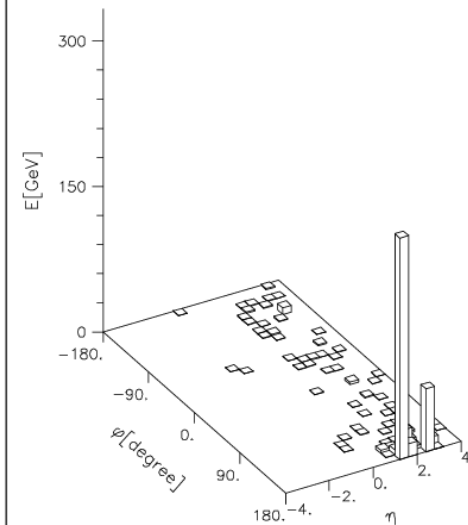
Pt=139 Q2=41067 x=0.77 y=0.53



$$e^+ d \rightarrow \bar{\nu}_e u$$

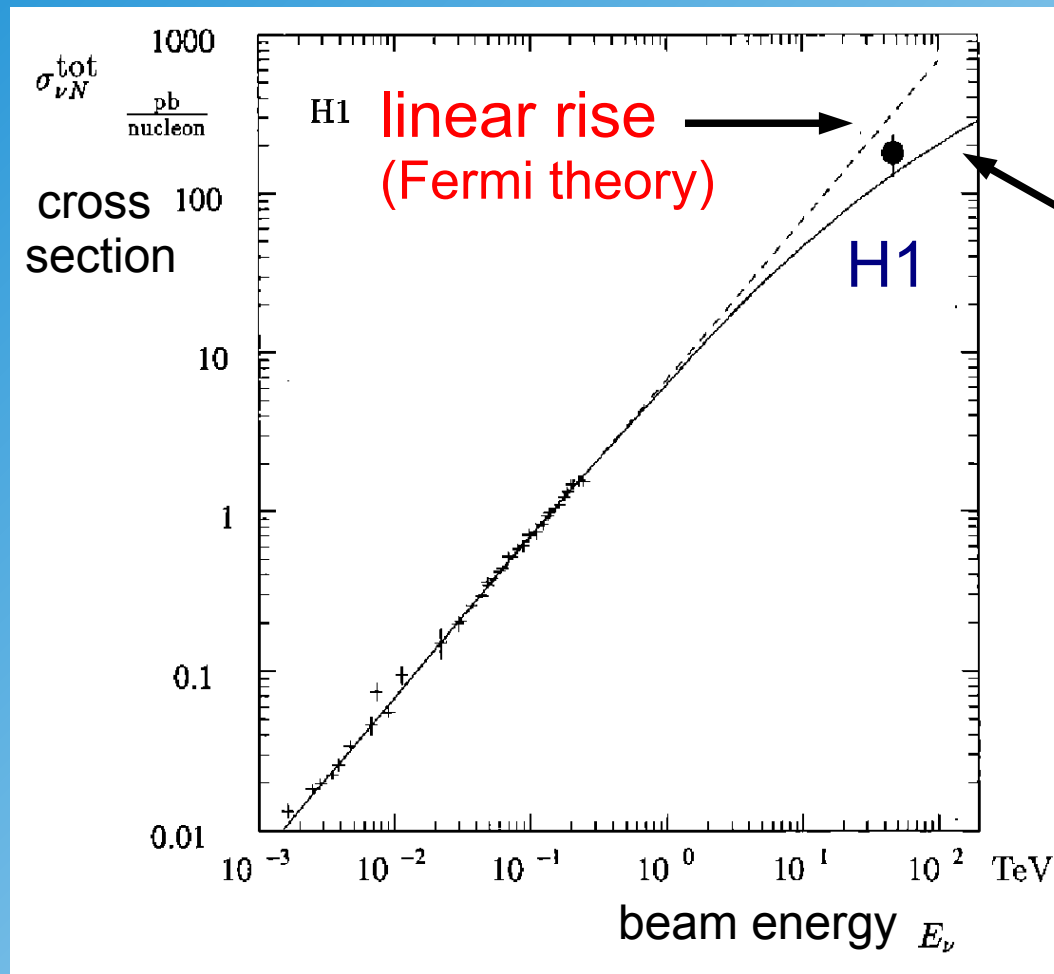


positron (26.7 GeV)



Weak scattering processes III

HERA beam energy translated into Fixed target



expected from
W-propagator

$$\frac{G_F}{\sqrt{2}} \rightarrow \frac{g}{M_W^2 + Q^2}$$

Breakdown of Fermi theory at high energies $s^{1/2} \sim 100 \text{ GeV}$

Lorentz Structure of Weak Process?

Lagrangian (independent of energy)

$$L = \frac{G}{\sqrt{2}} (\bar{f} \Gamma f') (\overline{f''} \tilde{\Gamma} f''')$$

most general ansatz for operator Γ :

Vector Current:

$$j_V^\mu = \bar{\psi} \gamma^\mu \psi \quad (\text{Fermi's proposal):}$$

Axial-vector Current:

$$j_A^\mu = \bar{\psi} \gamma^\mu \gamma^5 \psi$$

scalar coupling:

$$\lambda = \bar{\psi} \psi$$

pseudoscalar coupling:

$$\lambda = \bar{\psi} \gamma^5 \psi$$

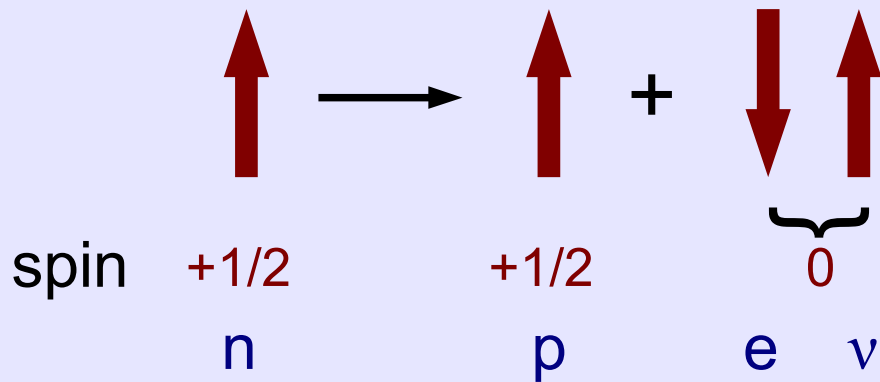
Tensor Coupling

$$\sigma_A^{\mu\nu} = \bar{\psi} (\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu) \psi$$

Test of Lorentz Structure?

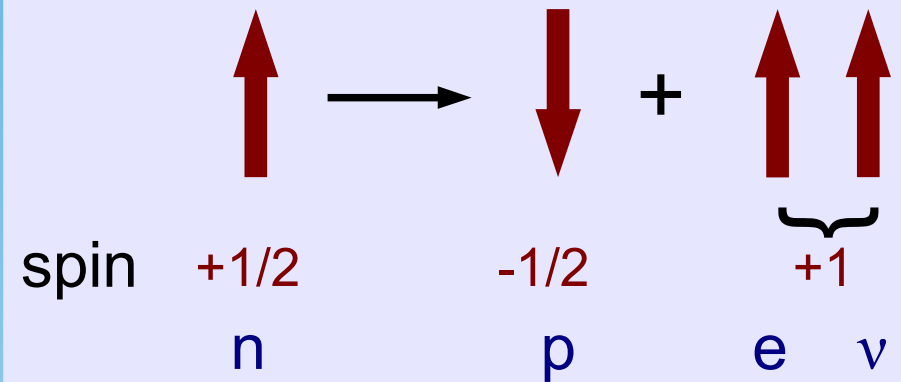
Different transitions in weak decays: $n \rightarrow p e \nu$

Fermi transition



- no spin flip ($\Delta J=0$)
- electron and neutrino in singlet state

Gamov Teller transition

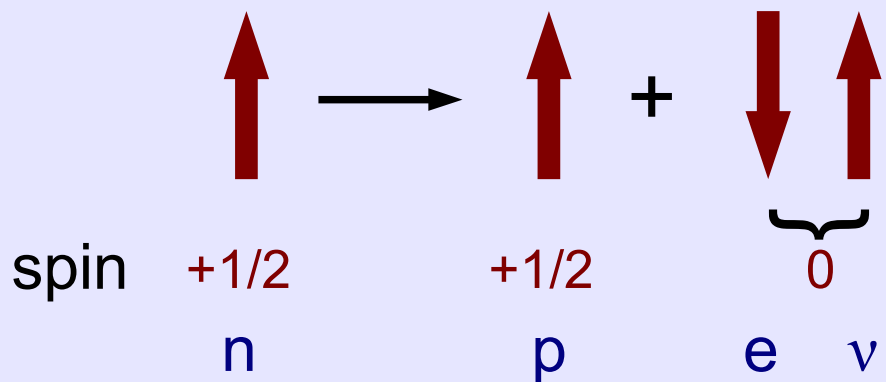


- spin flip ($\Delta J=0,1$)
- electron and neutrino in triplet state

Test of Lorentz Structure?

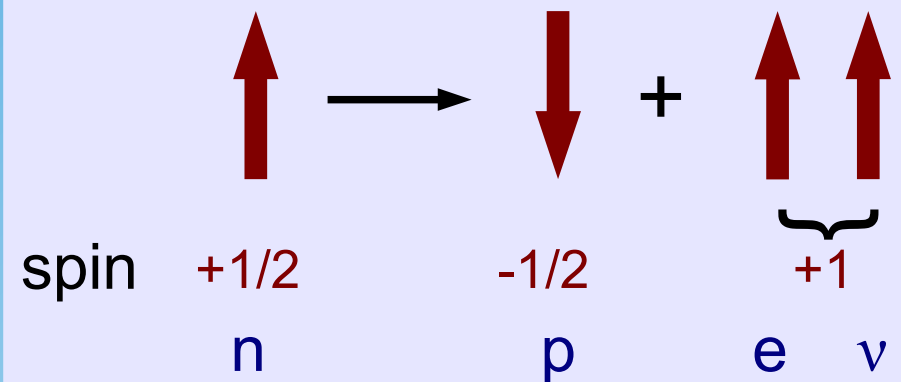
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Fermi transition



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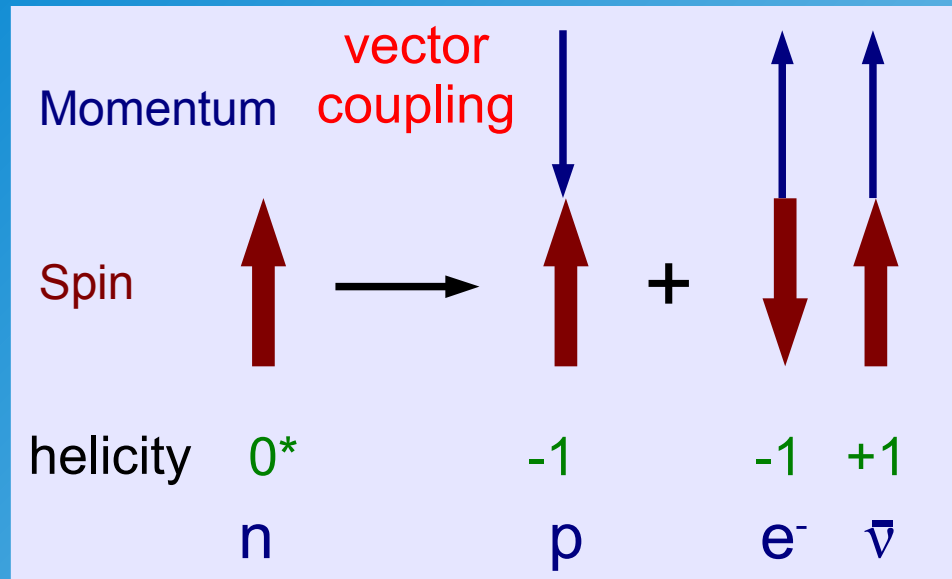


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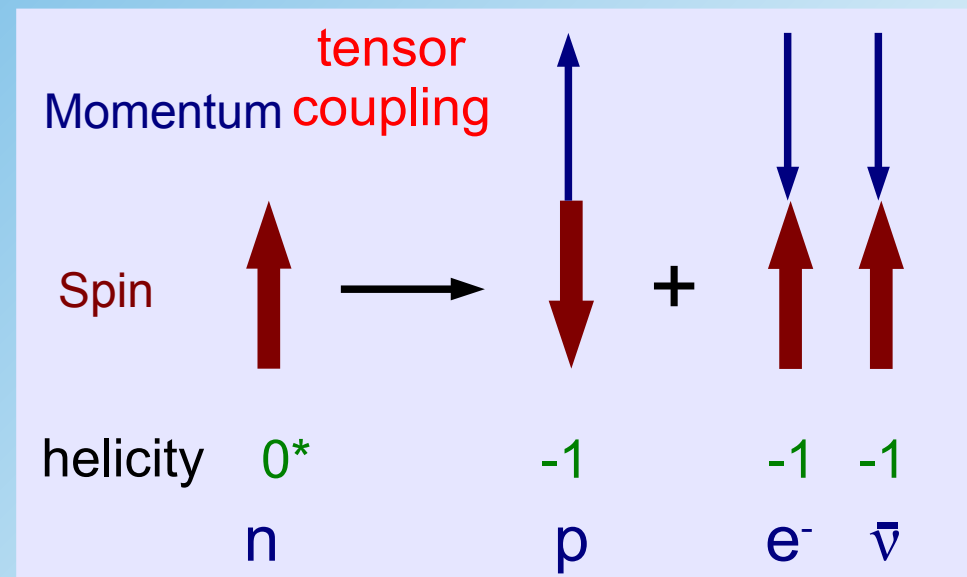
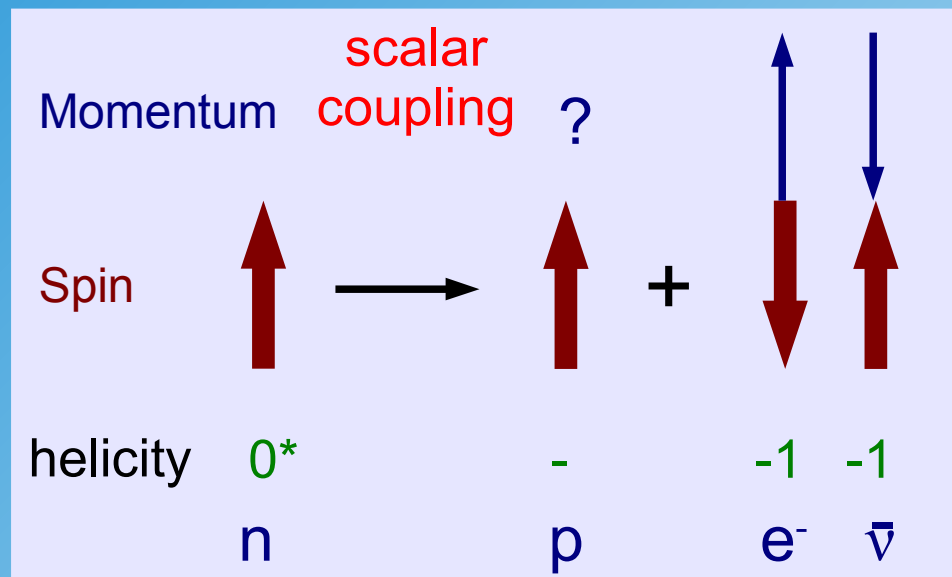
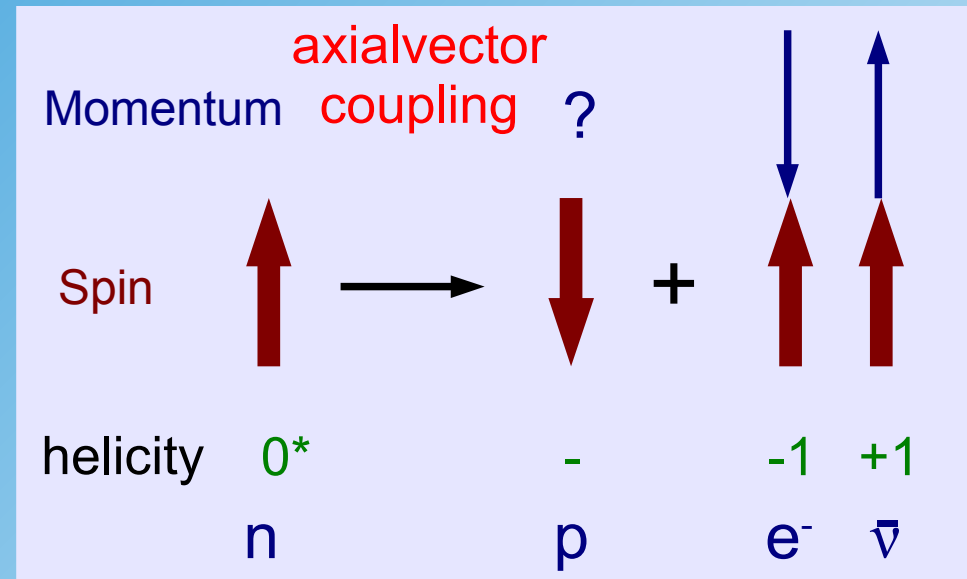
Note: spin flip \neq helicity flip

Test of Lorentz Structure?

Fermi transition

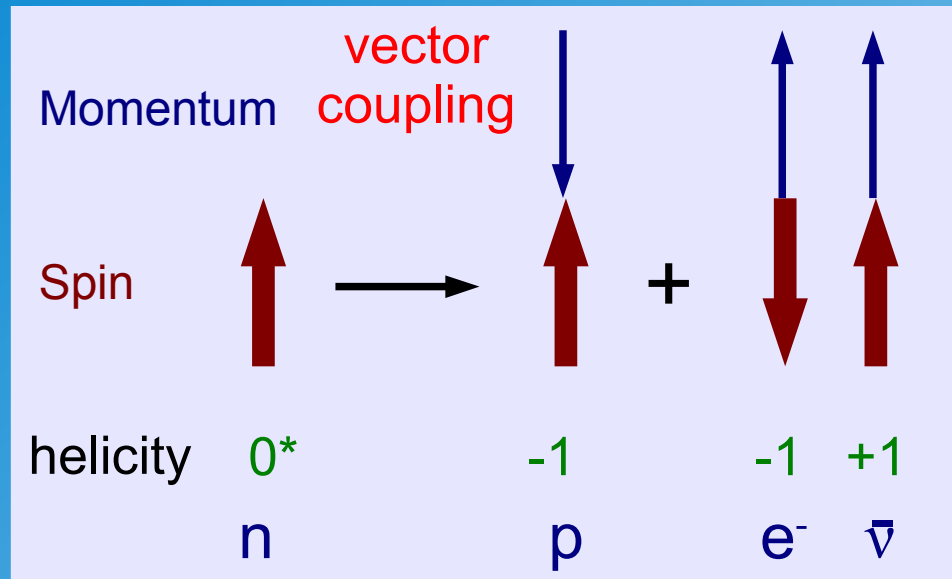


Gamov Teller transition

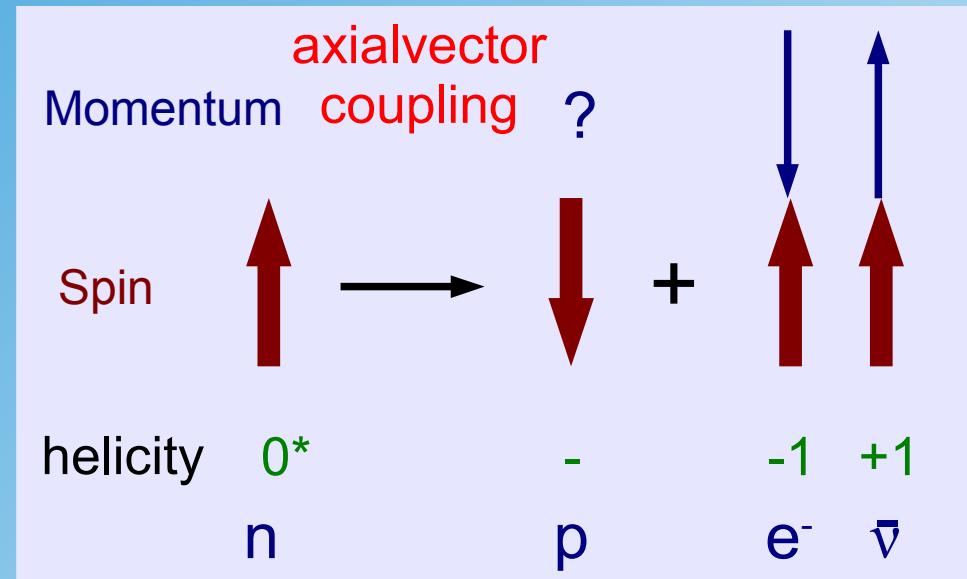


Test of Lorentz Structure?

Fermi transition



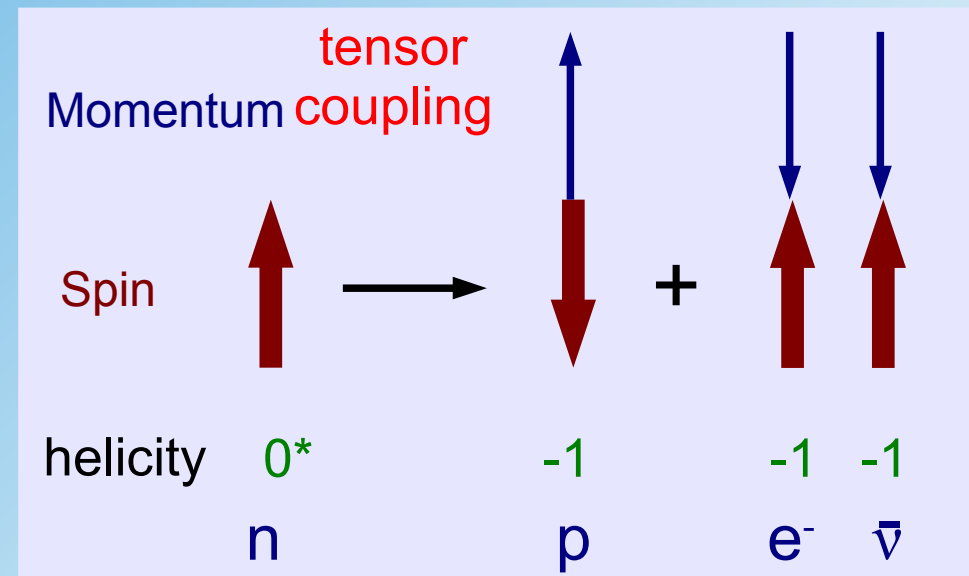
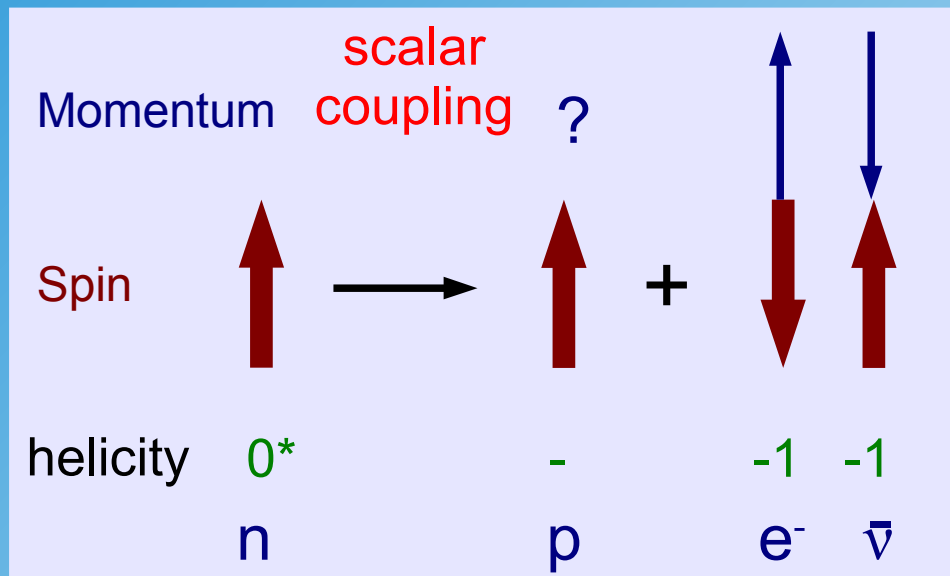
Gamov Teller transition



- Both, Fermi transitions and Gamov Teller transitions observed in weak interactions
- Vector and axialvector currents conserve helicities!

Test of Lorentz Structure?

- Scalar and tensor (pseudoscalar) couplings flip helicity!
- Have to measure spin-orientation of decay leptons!



The End

Gamma Matrices II

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\gamma^5 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

(in other representations
 \mathfrak{g} is diagonal)

Spinors of Helicity States

$$u_R = |\vec{p}, \lambda = +1/2\rangle \quad (u_1)$$

$$u_L = |\vec{p}, \lambda = -1/2\rangle \quad (u_2)$$

$$v_L = |\vec{p}, \lambda = -1/2\rangle \quad (v_1)$$

$$v_R = |\vec{p}, \lambda = +1/2\rangle \quad (v_2)$$

fermions:

$$u_R = \sqrt{E+m} \begin{pmatrix} 1 \\ 0 \\ \frac{|\vec{p}|}{E+m} \\ 0 \end{pmatrix}$$

$$u_L = \sqrt{E+m} \begin{pmatrix} 0 \\ 1 \\ 0 \\ \frac{-|\vec{p}|}{E+m} \end{pmatrix}$$

limit $p \rightarrow \infty$

anti-fermions:

$$v_L = \sqrt{E+m} \begin{pmatrix} \frac{|\vec{p}|}{E+m} \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$v_R = \sqrt{E+m} \begin{pmatrix} 0 \\ \frac{-|\vec{p}|}{E+m} \\ 0 \\ 1 \end{pmatrix}$$

$$u_R \rightarrow v_L$$

$$u_L \rightarrow v_R$$

Chirality Operator

limit $m \rightarrow 0$

$$u_R \sim \nu_L \sim \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad u_L \sim \nu_R \sim \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

operator:

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & \underline{1} \\ \underline{1} & 0 \end{pmatrix}$$

right chirality left chirality

$$\gamma_5 u_R = u_R \quad \gamma_5 u_L = -u_L$$

$$\gamma_5 \nu_L = \nu_L \quad \gamma_5 \nu_R = -\nu_R$$

eigenvalues ± 1

left-handed (chiral) particles: -1

right-handed (chiral) particles: +1

note: a right-handed chiral anti-particle has a left-handed helicity

Projection Operator

definition: $\Pi^\pm = \frac{1 \pm \gamma_5}{2}$ in the limit of $|E| \rightarrow \text{infinity}$

fermions

$$\begin{aligned} \Pi^+ u_R &= u_R & \Pi^+ u_L &= 0 \\ \Pi^- u_L &= u_L & \Pi^- u_R &= 0 \end{aligned}$$

anti-fermions

$$\begin{aligned} \Pi^+ \nu_L &= \nu_L & \Pi^+ \nu_R &= 0 \\ \Pi^- \nu_R &= \nu_R & \Pi^- \nu_L &= 0 \end{aligned}$$

projection:

$$\begin{aligned} \Pi^+ \psi &= R & (\text{right-handed (chiral) state}) \\ \Pi^- \psi &= L & (\text{left-handed (chiral) state}) \end{aligned}$$

reformulate Dirac Equation:

$$i \gamma^\mu \partial_\mu R = m L \quad i \gamma^\mu \partial_\mu L = m R$$

note: massive fermions must have left-handed and right handed components

Vector and Axial Currents

Vector Current:

$$j_V^\mu = \bar{\psi} \gamma^\mu \psi \quad (R \gamma^\mu R, L \gamma^\mu L) \quad \text{in QED: } \partial_\mu j_V^\mu = 0 \quad (\text{conservation of currents})$$

Axial-vector Current:

$$j_A^\mu = \bar{\psi} \gamma^\mu \gamma^5 \psi \quad \text{note: } \gamma^\mu \gamma^5 = -\gamma^5 \gamma^\mu$$

Left (right)-handed Current:

$$j_L^\mu = \bar{\psi} \gamma^\mu \Pi^- \psi \quad j_R^\mu = \bar{\psi} \gamma^\mu \Pi^+ \psi$$

relations:

$$j_L^\mu = 1/2 (j_V^\mu - j_A^\mu)$$

weak interaction (V-A theory):

$$j_R^\mu = 1/2 (j_V^\mu + j_A^\mu)$$

