# Fermi - Theory : Theory

$\pi^+ \to \mu^+ \nu_\mu$ vs. $\pi^0 \to \gamma\gamma$

$\mu \to e \nu \nu$

$10^{-(8..6)}s$ vs. $10^{-16}s$

? electromagnetism

Solution: weak interaction: Fermi-theory

Goal: describe fundamental equations, basic processes, problems

## 1) Fundamentals

at <u>low</u> energies, physics could be described by <u>point-like</u> 4-fermion interactions

$$\mathcal{L} = G(\bar{f}\,\Gamma\, f')(\bar{f}''\,\Gamma\, f''')$$

where $G$ has dimension $E^{-2}$

recall that $\Gamma$ could be one of the following „bilinear covariants"

$\bar{\Psi}\Psi \rightarrow \Gamma = \mathbb{1}$ Scalar

$\bar{\Psi}\gamma^\mu\Psi \rightarrow \Gamma = \gamma^\mu$ Vector

$\bar{\Psi}\sigma^{\mu\nu}\Psi \rightarrow \Gamma = \sigma^{\mu\nu}$ Tensor

$\bar{\Psi}\gamma^5\gamma^\mu\Psi \rightarrow \Gamma = \gamma^5\gamma^\mu$ Axial vector

$\bar{\Psi}\gamma^5\Psi \rightarrow \Gamma = \gamma_5$ Pseudoscalar

Careful analysis of parity violation, neutrino helicity, ... (→ André Schöning) lead to <u>V - A Theory</u>

$\boxed{\mathcal{L} = \frac{1}{\sqrt{2}} G_F J_\mu^+ J^\mu}$ with $\boxed{J_\mu = \sum_{\ell,\ell'} \bar{\Psi}_\ell \gamma_\mu (1-\gamma_5) \Psi_{\ell'}}$

for instance: $J^\mu = \bar{u}\,\gamma^\mu (1-\gamma_5)\, d$

$$J^\mu = \bar{e}\,\gamma^\mu (1-\gamma_5)\, \nu_e$$

where $u, d, e, \nu_e, \ldots$ are spinors for up-quark, down-quark, electron, electron-neutrino, ...

$\rightarrow$ $J^\mu$ are charge raising/lowering $\leftrightarrow$ take care that $\Delta Q_{tot} = 0$ ...

Note that $\frac{1}{2}(1 \pm \gamma_5)$ is projection operator

for <u>chirality</u>: $P_L = \frac{1}{2}(1-\gamma_5)$

$$P_R = \frac{1}{2}(1+\gamma_5)$$

identity: $P_L u = u_L$ ; $\overline{u_L} = \bar{u} P_R$ ;

$$\overline{u_L}\, u_L = \overline{u_R}\, u_R = 0 ;$$

$$\bar{u}\, u = (\overline{u_R} + \overline{u_L})(u_L + u_R) = \overline{u_R}\, u_L + \overline{u_L}\, u_R$$

$$= \overline{u_R}\, u_L + h.c. ;$$

$$\bar{u}\,\gamma^\mu d_L = \frac{1}{2}\bar{u}\,\gamma^\mu (1-\gamma_5)\, d = \frac{1}{2}\bar{u}(1+\gamma_5)\gamma^\mu d$$

$$= \overline{u_L}\,\gamma^\mu d = \overline{u_L}\,\gamma^\mu d_L ;$$

$\Rightarrow$ only left-handed particles are selected

$\rightarrow$ PARITY VIOLATION! $\boxed{P}$

$$\Rightarrow \boxed{\mathcal{L} = 4/\sqrt{2}\, G_F \left(\overline{e_L}\, \gamma^\mu (\nu_e)_L\right)\left(\overline{u_L}\, \gamma_\mu\, d_L\right)}$$

+ h.c. - terms: $\overline{e_L}\, \gamma^\mu \nu_L \xrightarrow{h.c.} (P_L \nu)^+ (\gamma^\mu)^+ (\overline{e})^+$

$$= \nu^+ P_L^+ \gamma^{\mu +} \gamma^0 e$$

$$= \underbrace{\nu^+ \gamma^0}_{\overline{\nu}}\, \underbrace{\gamma_0 P_L \gamma_0}_{P_R}\, \underbrace{\gamma_0 \gamma^{\mu +} \gamma^0}_{\gamma^\mu} e$$

$$= \overline{\nu_L}\, \gamma^\mu e = \overline{\nu_L}\, \gamma^\mu e_L$$

Remarks: 1) left-handed neutrinos are automatically selected

$P_L u_1 = 0 \quad ; \quad P_L u_2 = u_2$

$P_R u_1 = u_1 \quad ; \quad P_R u_2 = 0 \qquad$ (for $m=0$)

2) recall that chirality $\neq$ helicity for massive fermions; approx. equal for ultrarelativistic fermions.

Chirality is not a good quantum number, Helicity is a good quantum number

3) antiparticles:

$$P_L V_1 = V_1 \; ; \; P_L V_2 = 0 \qquad (m=0)$$
$$P_R V_1 = 0 \; ; \; P_R V_2 = V_2$$

$\Rightarrow$ right-handed anti-neutrinos are selected

$\Rightarrow$ since charge conjugation turns $\nu_L$ into $\bar{\nu}_L$:

CHARGE - VIOLATION $\boxed{C}$

4) at this stage: $CP$ is conserved! $(\nu_L \longrightarrow \bar{\nu}_R)$

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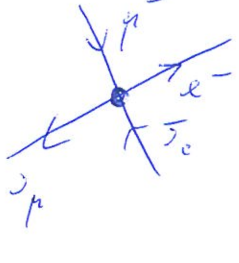
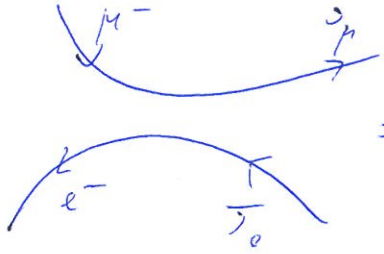
Size of $G_F$: from $\mathcal{L} = \frac{4}{\sqrt{2}} G_F \left(\overline{\mu_L} \gamma^\mu (\nu_\mu)_L\right)\left(\overline{(\nu_e)_L} \gamma_\mu e_L\right)$

obtain $\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu) = \frac{G_F^2 m_\mu^5}{192\pi^3} = 2.2 * 10^{-6} s$




or




$\Rightarrow G_F = 1.166 \cdot 10^{-5} \, GeV^{-2}$

$\simeq \left(\frac{1}{300 GeV}\right)^2$

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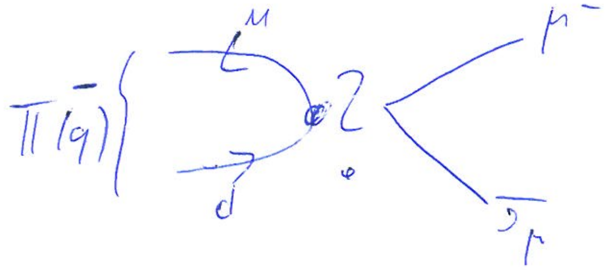
# 2) Basic Processes in Fermi-theory

## a) Pion decay




$$\mathcal{M} = \frac{2G_F}{\sqrt{2}}\, \bar{\mu}_L \gamma_\mu \nu_L \quad X^\mu$$

pionic physics

→ $\pi^-$ is spinless

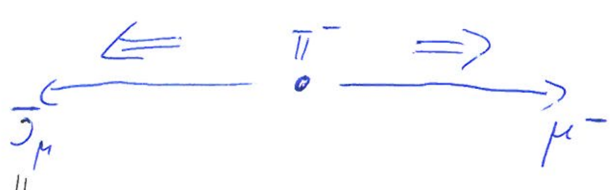
→ $\mathcal{M}$ should be Lorentz-invariant
⇒ vector or axialvector

$$\Rightarrow X^\mu = f_\pi(q^2)\, q^\mu = f_\pi\, q^\mu$$

decay constant

$$\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu) = \frac{G_F^2}{8\pi} f_\pi^2\, m_\pi\, m_\mu^2 \left(1 - m_\mu^2/m_\pi^2\right)$$

Helicity:




Total Spin $J=0$ because $\pi^-$ is scalar

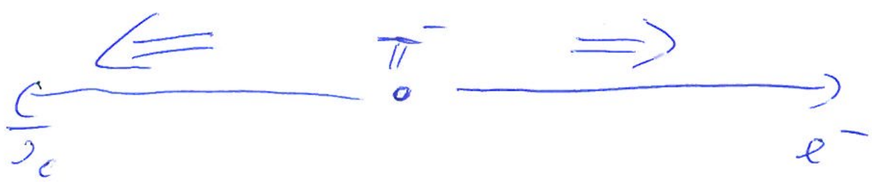
⇓

MUST HAVE
POSITIVE
HELICITY
(massless)

Now consider decay into $e^- \bar{\nu}_e$

$$\frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} = \left(\frac{m_e}{m_\mu}\right)^2 \left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right)^2 \sim 10^{-4}$$

prefers decay into <u>heavier</u> particle !?




the lighter the mass, the smaller the left-handed part of a spin 1/2 particle !

$$P_L u_\uparrow = \frac{1}{2}\sqrt{E+m}\left(1 - \frac{p}{E+m}\right)\begin{pmatrix}1\\0\\-1\\0\end{pmatrix}$$

moving in z-direction

$\Rightarrow P_L u_\uparrow \propto m/\sqrt{E}$ very small for $e^-$, larger for $\mu^-$

# b) Neutrino - lepton scattering

$\nu_e e^- \to \nu_e e^-$ : educated guess: $\sigma = G_F^2 s$

fixed target: $s = 2 m_e E_\nu$

$$\Rightarrow \sigma \simeq 5 \times 10^{-17}\, b \left(\frac{E_\nu}{GeV}\right)$$

$$\left[\text{recall: } \sigma(e^+e^- \to \mu^+\mu^-) \sim \frac{\alpha^2}{s} \sim 10^{-8} b \left(\frac{GeV^2}{s}\right)\right]$$

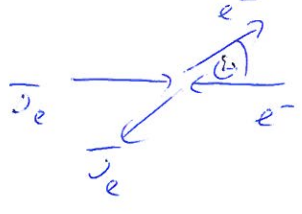
$\Rightarrow$ weak interactions are weak !

Correct result: $\frac{d\sigma}{d\Omega}(\nu_e e^- \to \nu_e e^-) = \frac{G_F^2 s}{4\pi^2}$

from $|\mathcal{M}|^2 = 16\, G_F^2 s^2$

for $\bar{\nu}_e e^- \to \bar{\nu}_e e^-$ : $|\mathcal{M}|^2 = 16\, G_F^2 t^2$

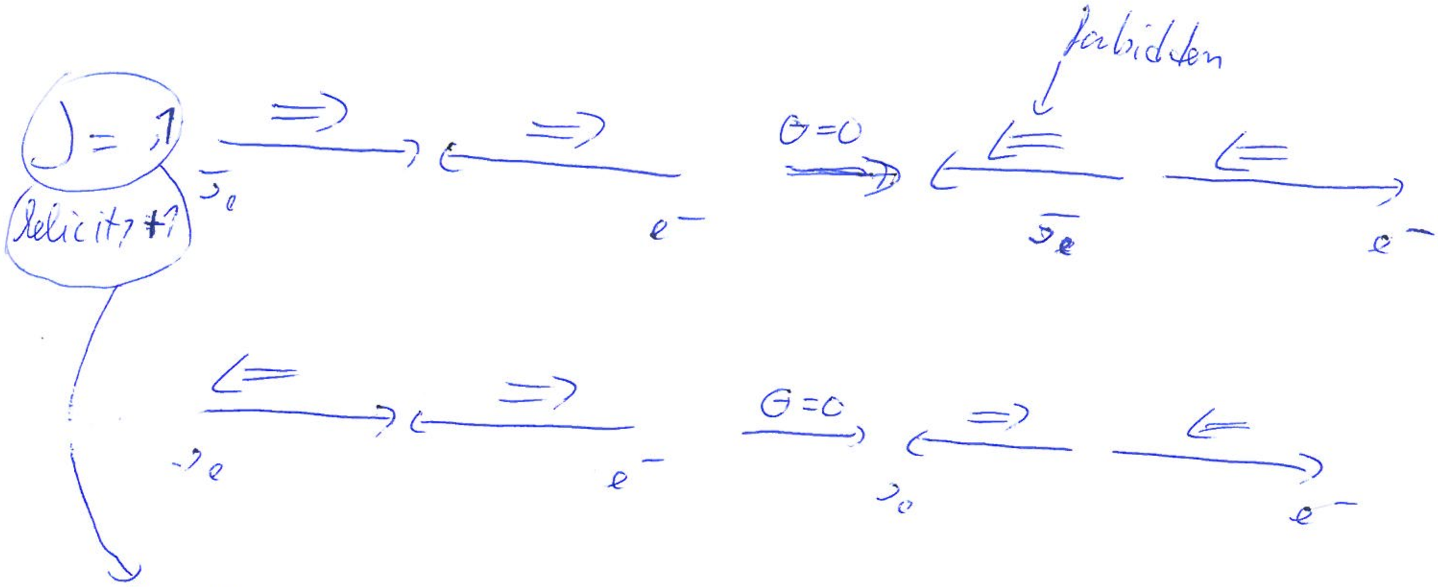
$$= 16\, G_F^2 s^2 (1 - \cos\Theta)^2$$




$$\Rightarrow \frac{\sigma(\nu_e e^-)}{\sigma(\bar{\nu}_e e^-)} = 3$$

Again, helicity arguments make these factors plausible!

$d\sigma(\bar{\nu}_e e) = 0$ for $\theta = 0$ (backward scattering)




factor 3! only one of the 3 states allowed

# 3) Problems and Interpretation of Fermi-Theory

## A) Problems:

### 1) Unitarity:

$\sigma \simeq G_F^2 s$ grows with s

From unitarity of S-matrix (conservation of probability)

it followed $\sigma_{tot} = \frac{1}{s} \operatorname{Im}\{\mathcal{M}(\theta=0)\}$

we also have: $\sigma = \int d\Omega \frac{|\overline{\mathcal{M}}|^2}{64\pi^2 s}$ for $2 \to 2$

Use partial wave decomposition:

$$\mathcal{M} = 16\pi \sum_{l=0}^{\infty} (2l+1) \underbrace{P_l(\cos\theta)}_{\text{Legendre}} \underbrace{a_l}_{\text{partial waves}}$$

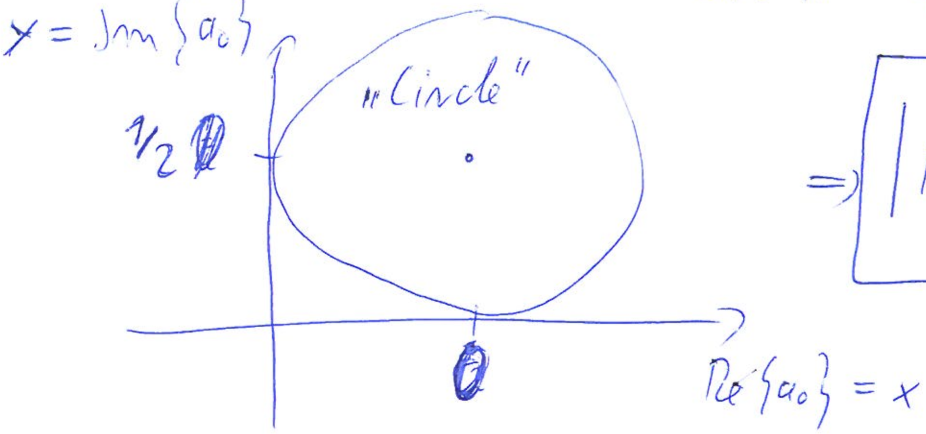
$$\Rightarrow \sigma = \int \underbrace{d\Omega}\ \frac{1}{64\pi^2 s}\,(16\pi)^2 \sum_{\ell,\ell'} (2\ell+1)(2\ell'+1)\,\underbrace{P_\ell P_{\ell'}}_{\frac{4\pi}{2\ell+1}\delta_{\ell\ell'}}\ a_\ell\, a_{\ell'}$$

$$= \frac{16\pi}{s} \sum_{\ell=0}^{\infty} (2\ell+1)\,|a_\ell|^2$$

$$\stackrel{!}{=} \frac{16\pi}{s} \sum_{\ell=0}^{\infty} \mathrm{Im}\{a_\ell\}(2\ell+1) \quad \left(P_\ell(\cos\theta = 1) = 1\right)$$

$$\Rightarrow |a_\ell|^2 = \mathrm{Im}\{a_\ell\} \Rightarrow x^2 + y^2 = y$$

$$\text{or: } (y - 1/2)^2 + x^2 = (1/2)^2$$




$$\Rightarrow \boxed{|\mathrm{Re}\{a_0\}| < \frac{1}{2}}$$

$\Rightarrow G_F$ is point-like $\Rightarrow \ell = 0$, s-wave

$\Rightarrow \sigma = \frac{1}{2}\frac{16\pi}{s}|a_0|^2$ : should go with $1/s$

conflict with $\sigma \sim G_F^2 s$ !

Problem occurs when $\frac{1}{2} \frac{16\pi}{s} \frac{1}{4} = G_F^2 \frac{s}{\pi}$

$$\Rightarrow \sqrt{s} \lesssim \sqrt[4]{\frac{2\pi^2}{G_F^2}} \simeq 617 \text{ GeV}$$

## 2) Renormalizability

Recall UV infinities in QED:  $-q^2 \gg m^2$

were absorbed in bare charges / masses.

There is an infinite number of infinite Feynman diagrams... if a finite number of redefinitions works, the theory is renormalizable (well defined)

One requirement: dimensionless coupling !

$$G_F \sim \left(\frac{1}{300 \text{ GeV}}\right)^2$$

(the more loops I include, the higher the degree of divergence, because each $G_F$ in a diagram must be compensated by a power $\Lambda^2$)

## B) Interpretation of Fermi-theory

Exchange of massive charged boson

$$\mathcal{L}_{fund} = -\frac{g}{2\sqrt{2}}\left[\bar{\nu}_\mu \gamma_\mu (1-\gamma_5)\mu + \bar{\nu}_e \gamma_\mu (1-\gamma_5) e\right] W^{+\mu}$$

$$+\frac{1}{2} m_W^2 W_\mu^+ (W^{+\mu})^+ + \mathcal{L}_{kin}$$

(origin: later)

@ low energy: $\partial_\mu W^{+\mu} = 0$ : kinetic term irrelevant

$\Rightarrow$ Euler Lagrange is simply: $\frac{\partial \mathcal{L}}{\partial W_\mu^+} = 0$

$$\Rightarrow W^\mu = \frac{g}{2\sqrt{2}}\left[\bar{\nu}_\mu \gamma_\mu (1-\gamma_5)\mu + \bar{\nu}_e \gamma_\mu (1-\gamma_5) e\right]\frac{1}{m_W^2}$$

insert in $\mathcal{L}_{fund}$:

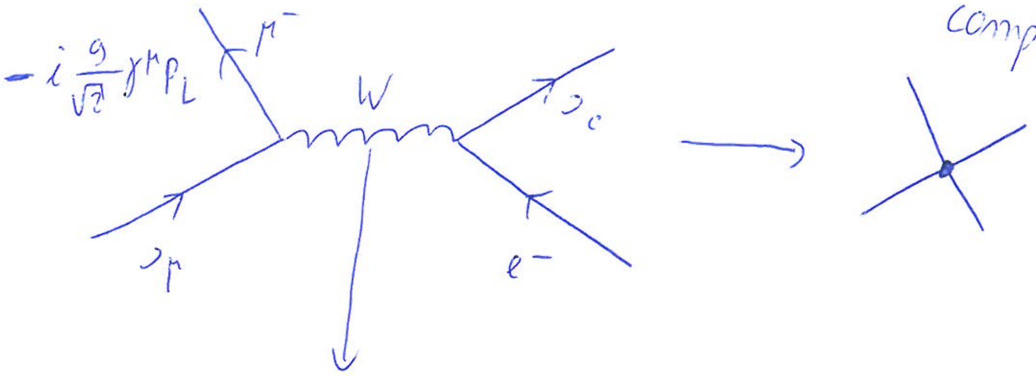
$$\mathcal{L}_{eff} = -\frac{g^2}{8m_W^2}\left[\bar{\nu}_\mu \gamma_\mu (1-\gamma_5)\mu\right]\left[\bar{e}\gamma^\mu (1-\gamma_5)\nu_e\right]$$

$$\Rightarrow \boxed{\frac{g^2}{8 m_W^2} = \frac{G_F}{\sqrt{2}}} \Rightarrow \frac{g}{m_W} \simeq \frac{1}{123\ \text{GeV}}$$

turns out: $m_W \simeq 80$ GeV

$\Rightarrow g \simeq 0.65$

compare with: $e = \sqrt{4\pi\alpha} \simeq 0.3$




propagator:

$$\frac{i\left(-g^{\mu\nu} + q^\mu q^\nu / m_W^2\right)}{q^2 - m_W^2} \xrightarrow{m_W^2 \gg q} \frac{+i g^{\mu\nu}}{+m_W^2}$$

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This is therefore an <u>effective theory</u>, i.e. the „ultraviolett completion" is not observable (and often not important) at energies that are much lower than the energy scale of the ~~experiments.~~ true theory. (here, e.g. $m_\mu \simeq 105$ MeV vs. $m_W \simeq 80$ GeV)