Lecture:

Standard Model of Particle Physics

Heidelberg SS 2012

Experimental Tests of QED
Part 1

Overview

PART I

- Cross Sections and QED tests
- Accelerator Facilities + Experimental Results

PART II

- Tests of QED in Partice Decays and Resonances
- QED Radiative Effects

Measurement of Cross Sections

$$e^+e^- \rightarrow X$$

$$(e^-e^- \rightarrow X)$$

- **test predictions of QED**

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➔ **test predictions of QED**

Why is α_{em}=1/137 so small?

Breakdown at higher energies?

Measurement of Cross Sections

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$$(e^- e^- \rightarrow X)$$

- **test predictions of QED**

Why is α_{em}=1/137 so small?
Breakdown at higher energies?

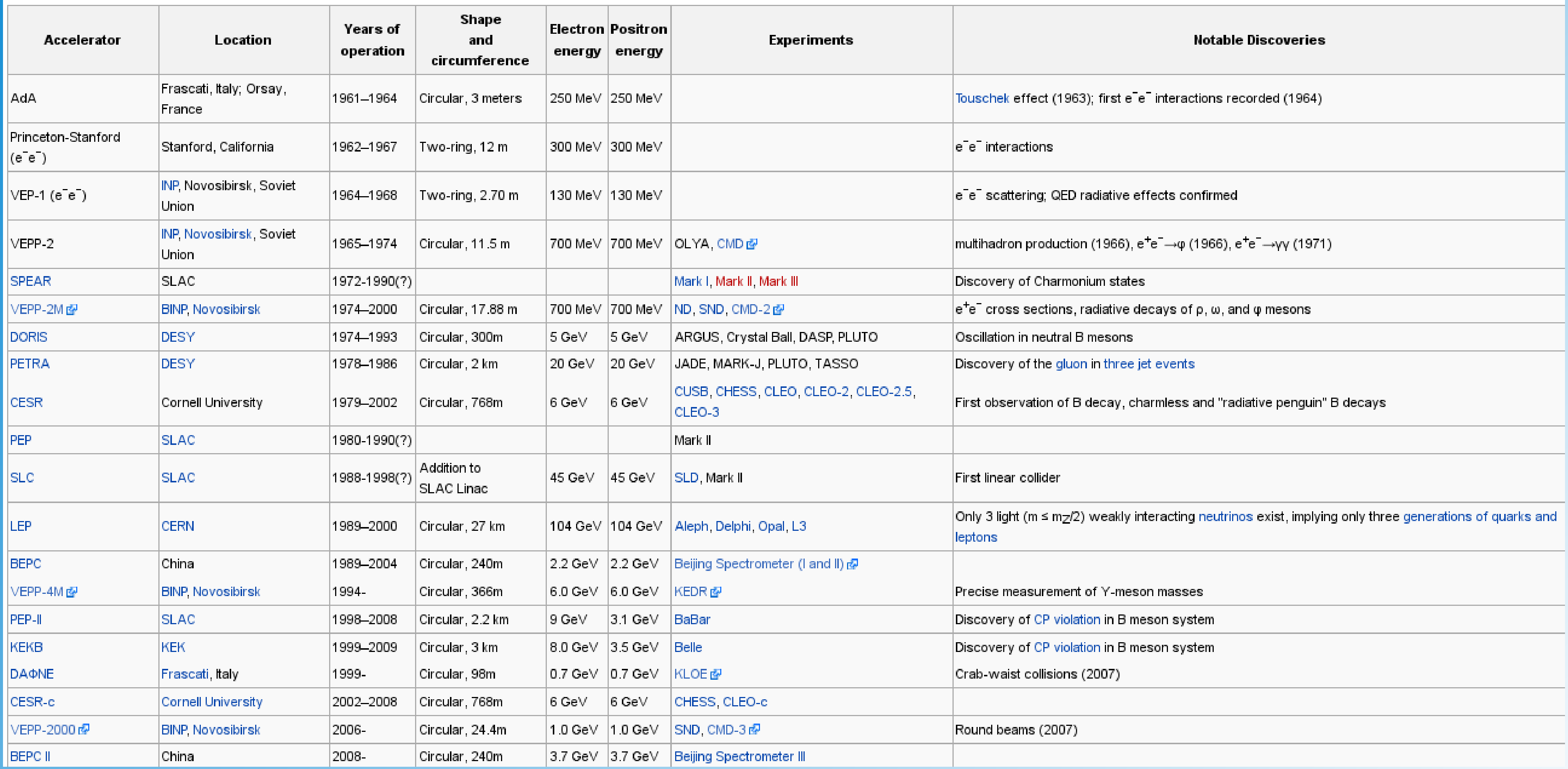
Reactions depend on center of mass → many different accelerators

Synchrotron Radiation Law::

$$P \propto \frac{E^4}{R^2}$$

- large accelerators required for high energies

List of ee-Accelerators



Accelerator	Location	Years of operation	Shape and circumference	Electron energy	Positron energy	Experiments	Notable Discoveries
AdA	Frascati, Italy; Orsay, France	1961–1964	Circular, 3 meters	250 MeV	250 MeV		Touschek effect (1963); first e^-e^- interactions recorded (1964)
Princeton-Stanford (e^-e^-)	Stanford, California	1962–1967	Two-ring, 12 m	300 MeV	300 MeV		e^-e^- interactions
VEP-1 (e^-e^-)	INP, Novosibirsk, Soviet Union	1964–1968	Two-ring, 2.70 m	130 MeV	130 MeV		e^-e^- scattering; QED radiative effects confirmed
VEPP-2	INP, Novosibirsk, Soviet Union	1965–1974	Circular, 11.5 m	700 MeV	700 MeV	OLYA, CMD	multihadron production (1966), $e^+e^- \to \varphi$ (1966), $e^+e^- \to \gamma\gamma$ (1971)
SPEAR	SLAC	1972-1990(?)				Mark I, Mark II, Mark III	Discovery of Charmonium states
VEPP-2M	BINP, Novosibirsk	1974–2000	Circular, 17.88 m	700 MeV	700 MeV	ND, SND, CMD-2	e^+e^- cross sections, radiative decays of ρ, ω, and φ mesons
DORIS	DESY	1974–1993	Circular, 300m	5 GeV	5 GeV	ARGUS, Crystal Ball, DASP, PLUTO	Oscillation in neutral B mesons
PETRA	DESY	1978–1986	Circular, 2 km	20 GeV	20 GeV	JADE, MARK-J, PLUTO, TASSO	Discovery of the gluon in three jet events
CESR	Cornell University	1979–2002	Circular, 768m	6 GeV	6 GeV	CUSB, CHESS, CLEO, CLEO-2, CLEO-2.5, CLEO-3	First observation of B decay, charmless and "radiative penguin" B decays
PEP	SLAC	1980-1990(?)				Mark II	
SLC	SLAC	1988-1998(?)	Addition to SLAC Linac	45 GeV	45 GeV	SLD, Mark II	First linear collider
LEP	CERN	1989–2000	Circular, 27 km	104 GeV	104 GeV	Aleph, Delphi, Opal, L3	Only 3 light ($m \leq m_Z/2$) weakly interacting neutrinos exist, implying only three generations of quarks and leptons
BEPC	China	1989–2004	Circular, 240m	2.2 GeV	2.2 GeV	Beijing Spectrometer (I and II)	
VEPP-4M	BINP, Novosibirsk	1994-	Circular, 366m	6.0 GeV	6.0 GeV	KEDR	Precise measurement of Υ-meson masses
PEP-II	SLAC	1998–2008	Circular, 2.2 km	9 GeV	3.1 GeV	BaBar	Discovery of CP violation in B meson system
KEKB	KEK	1999–2009	Circular, 3 km	8.0 GeV	3.5 GeV	Belle	Discovery of CP violation in B meson system
DAΦNE	Frascati, Italy	1999-	Circular, 98m	0.7 GeV	0.7 GeV	KLOE	Crab-waist collisions (2007)
CESR-c	Cornell University	2002–2008	Circular, 768m	6 GeV	6 GeV	CHESS, CLEO-c	
VEPP-2000	BINP, Novosibirsk	2006-	Circular, 24.4m	1.0 GeV	1.0 GeV	SND, CMD-3	Round beams (2007)
BEPC II	China	2008-	Circular, 240m	3.7 GeV	3.7 GeV	Beijing Spectrometer III	

AdA Accelerator

- First e^+ e^- collider ever
- AdA = Anello di Accumulazione (Frascati/Orsay, 1961-64)
- Energy = 250 MeV Electrons x 250 MeV Positrons

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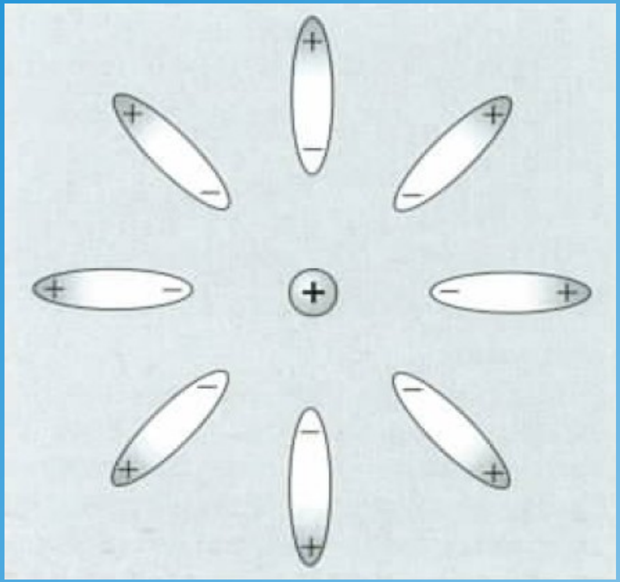
Motivation:

- Bruno Touschek: excite the dielectric vacuum to create vector mesons (e.g. rho meson predicted to be light!)

Note: at that time all new particles had been discovered in hadronic interactions (ie. proton beams)!

Dielectric Vacuum

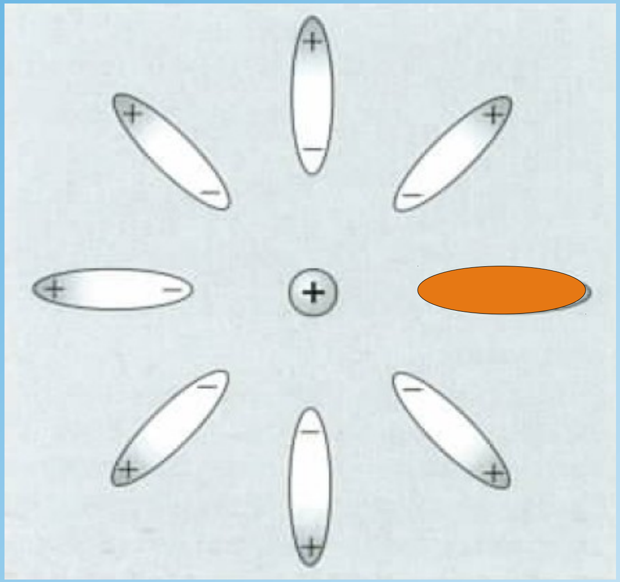
classical dielectric



bare electrical charge shielded
by induced dipoles

high energy →

“excited dielectric”



rho-meson

bare charge shielded
by vacuum polarisation

AdA Accelerator

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- Energy = 250 MeV Electrons x 250 MeV Positrons

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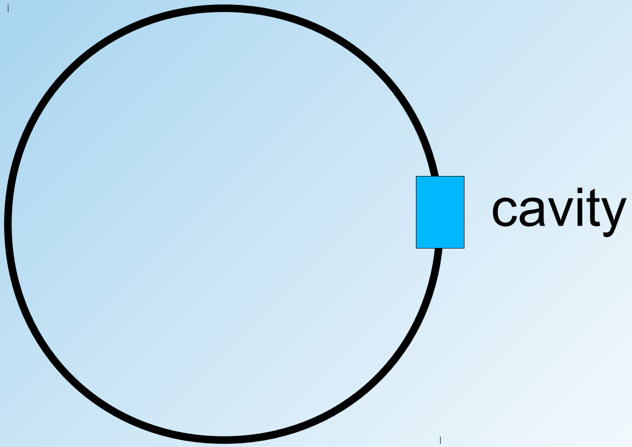
"Revolutionary" concept as the rho-meson is electrically neutral and was predicted to explain (as carrier) strong interactions

Remark: Indeed, Touschek was right. The strong force can be tested in $e^+ e^-$ collisions. But not in AdA (too low luminosity, too low energy)

AdA Challenges I

- How to store electrons and positrons?
  - magneto-optical storage ring (→ known at this time, synchroton radiation facilities)



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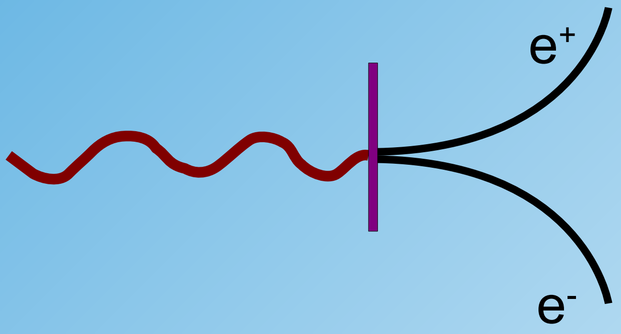

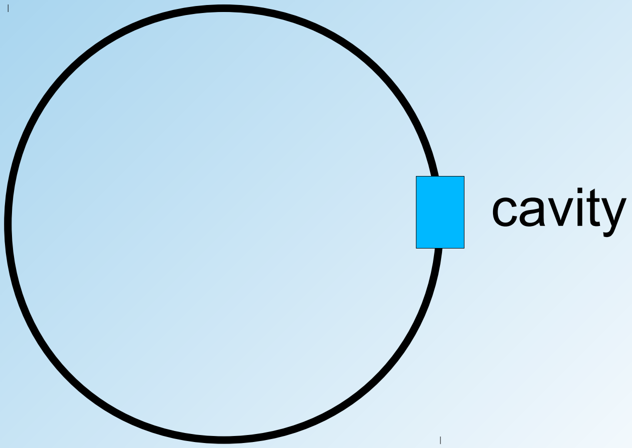

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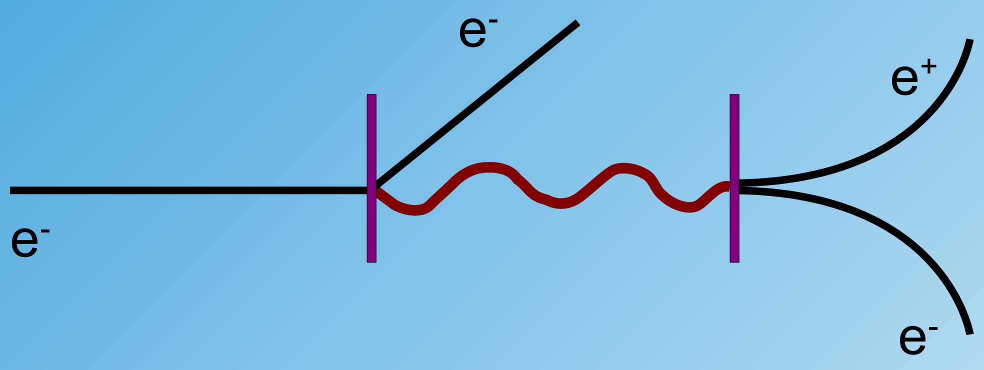
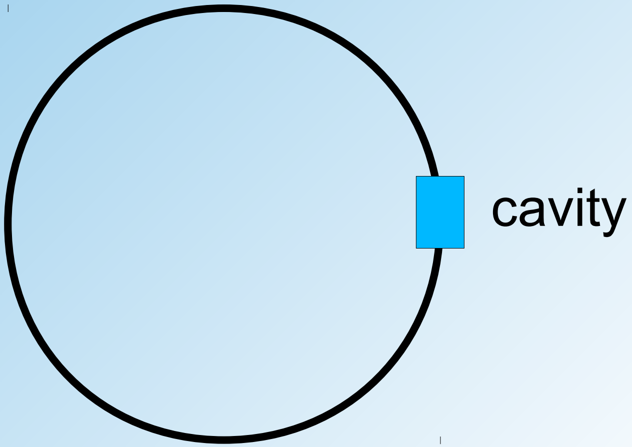
- How to produce the photons (E > 5-10 MeV)
  - Bremsstrahlung from high energetic electrons at target $e^- N \rightarrow \gamma e^- N$ using a linear electron accelerator (→ also known)





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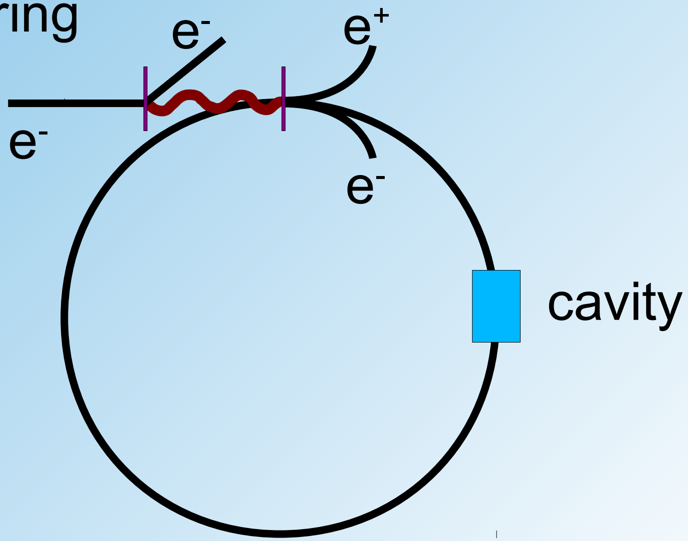
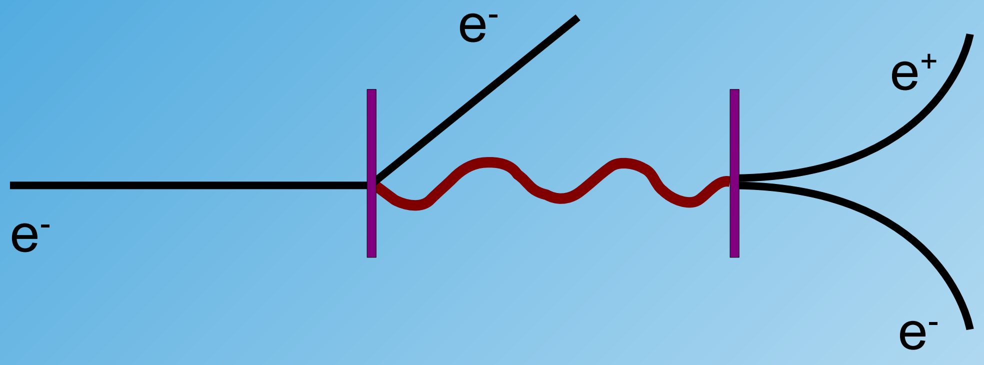
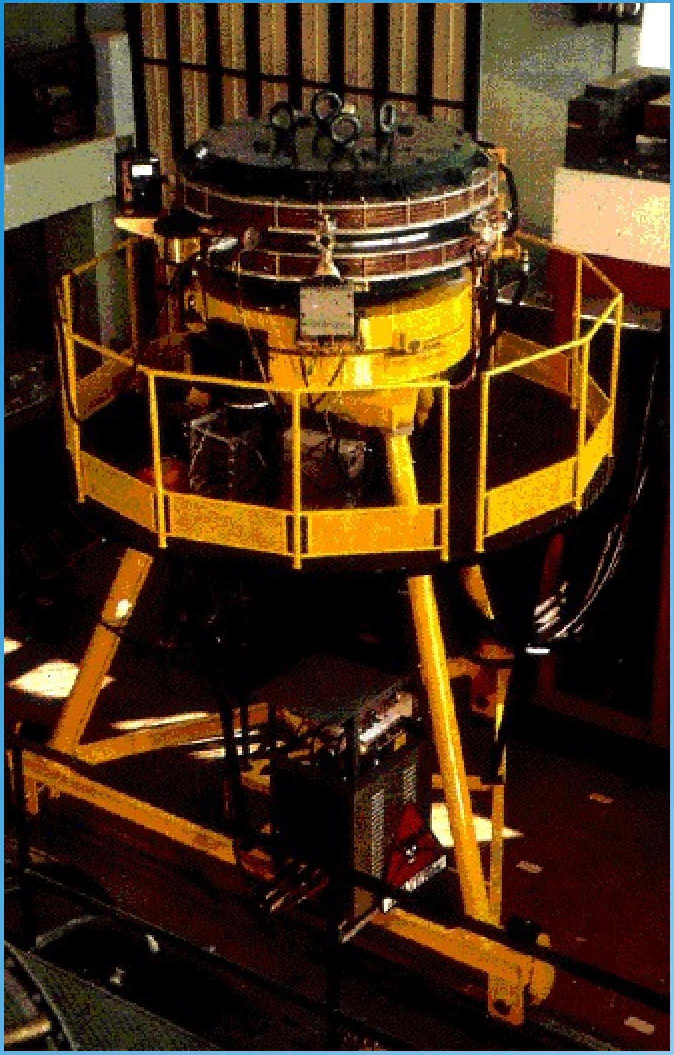
- How to produce the photons (E > 5-10 MeV)
  - Bremsstrahlung from high energetic electrons at target $e^- N \to \gamma e^- N$ using a linear electron accelerator (→ also known)
- How to fill the storage ring with electrons and positrons???
  - place the conversion target inside the storage ring

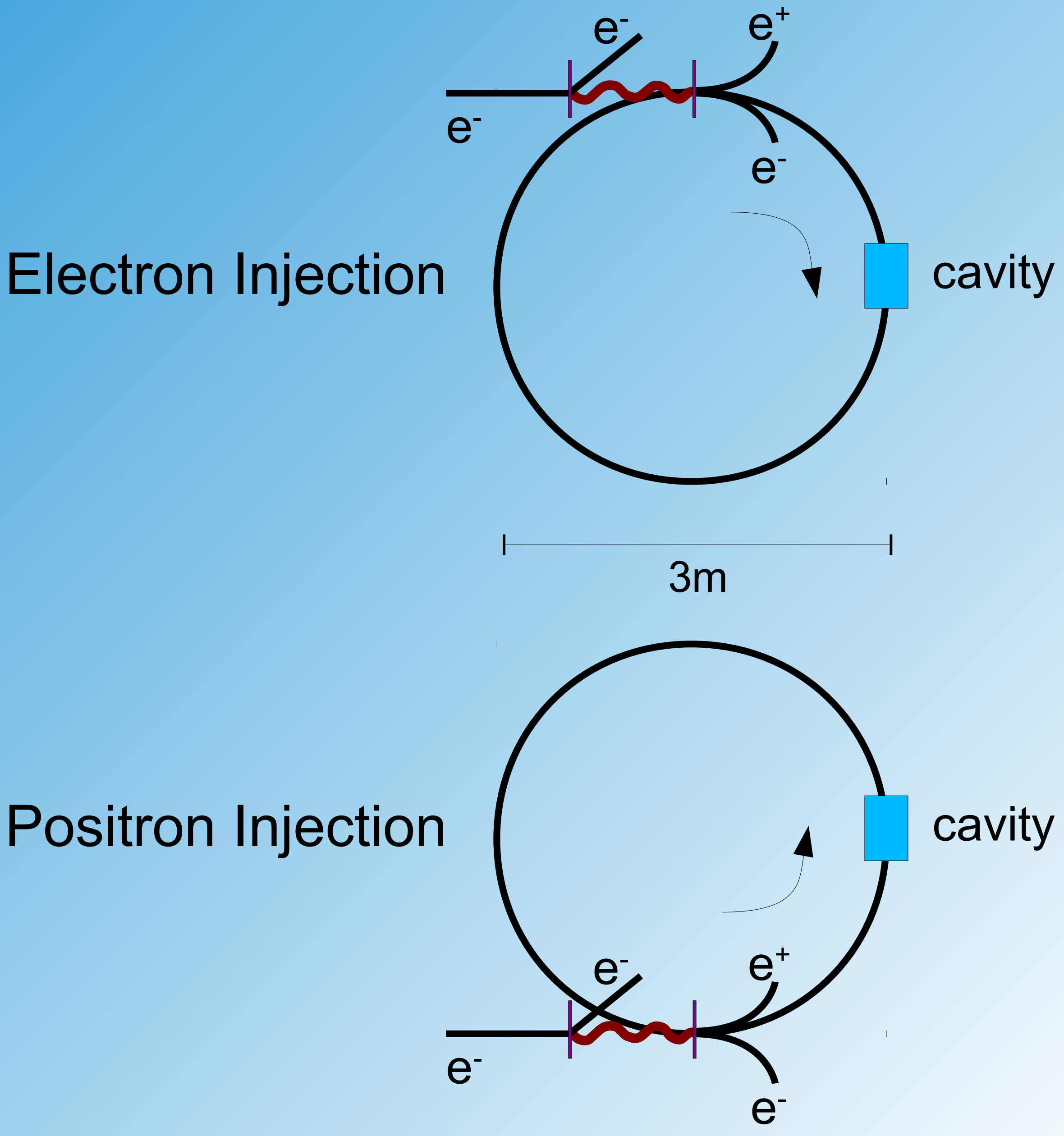




MAGNETIC DISCUSSION

AdA Concept




e-
e+
e-
e-
Electron Injection
cavity
3m
Positron Injection
cavity
e-
e+
e-
e-

AdA Challenges II

- How to make electrons and positrons collide?

Note: AdA is a single storage ring: electrons and positrons see same optics but in reverse direction

B.Touschek: *It is guaranteed that an electron and a positron necessarily meet in a single orbit because QED is CP (charge-parity)*

Bruno Touschek

AdA Challenges II

- How to make electrons and positrons collide?

Note: AdA is a single storage ring: electrons and positrons see same optics but in reverse direction

B.Touschek: *It is guaranteed that an electron and a positron necessarily meet in a single orbit because QED is CP (charge-parity)*

If a ring collider works, then CP(T) invariance of QED is confirmed!!!

Note: CP(T) invariance says that a positron can be regarded as an electron traveling in reverse time direction.

Touschek was right, in a very short time AdA was commissioned and electron-positron collisions were observed – much more than just a technical achievement!

AdA Challenges III

- How to measure that electron-positron collisions take place?
- How many collisions?

Definition of "Luminosity" Measurement (source factor)

$$R = L\,\sigma$$

Relation between rate of events and cross section of process

Luminosity Measurement in Ring

Accelerator Formula

$$L=\frac{N_1 N_2 f}{4\pi A}$$

N_1 and N_2 and beam cross section A are unknown and have to be precisely measured → difficult

More simple ansatz – use reference process(es):

$e^+ e^- \rightarrow e^+ e^-$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}(\cos\theta)} = \frac{\pi\alpha^2}{s}\left(u^2\left(\frac{1}{s}+\frac{1}{t}\right)^2+\left(\frac{t}{s}\right)^2+\left(\frac{s}{t}\right)^2\right)$$

ultrarelativistic approx. (Bhabha 1936)

$e^+ e^- \rightarrow \gamma\gamma$

$$\frac{d\sigma}{d\Omega}(e^+e^- \to \gamma\gamma) = \frac{\alpha^2}{2s}\frac{u^2+t^2}{tu}$$

annihilation process (Compton-like)

Both processes are forward peaked!

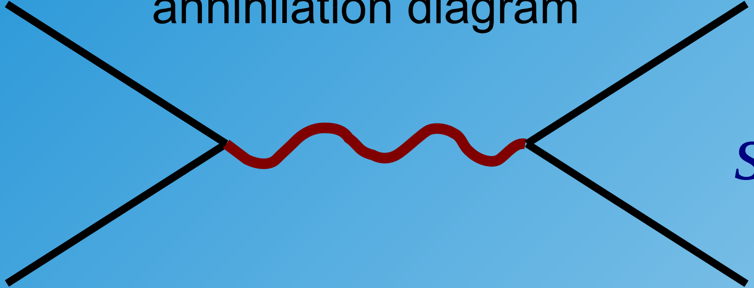
t-pole $t = -s\sin^2(\theta/2)$

Bhabha Scattering

$$\frac{d\sigma}{d\Omega}\left(e^{+}e^{-}\rightarrow e^{+}e^{-}\right)=\frac{\alpha^{2}}{2s}\left(\frac{u^{2}+t^{2}}{s^{2}}+\frac{s^{2}+u^{2}}{t^{2}}+\frac{2u^{2}}{st}\right)$$

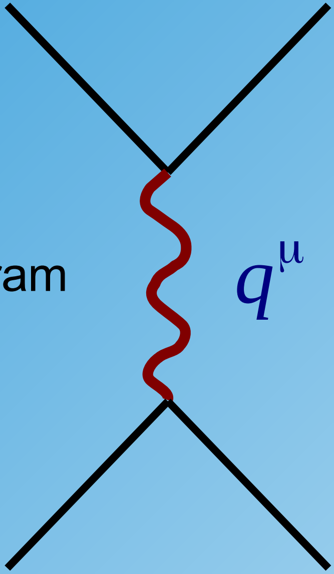
annihilation scattering interference

annihilation diagram



$s=q^{\mu}q_{\mu}$ s-pole from photon propagator

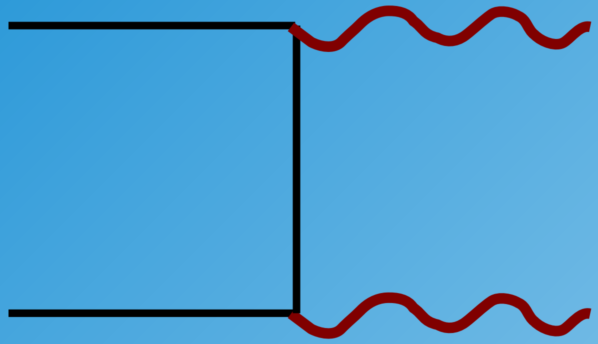
scattering diagram



$t=q^{\mu}q_{\mu}$ t-pole from photon propagator

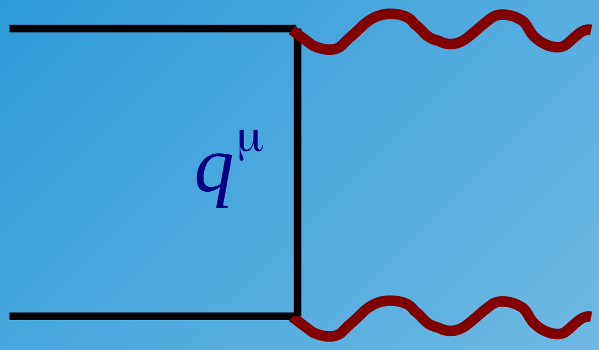
Photon Pair Production

$$\frac{d\sigma}{d\Omega}\left(e^+e^-\to\gamma\gamma\right)=\frac{\alpha^2}{2s}\left(\frac{u^2+t^2}{tu}\right)$$



Photon Pair Production

$$\frac{d\sigma}{d\Omega}(e^+e^- \to \gamma\gamma) = \frac{\alpha^2}{2s}\left(\frac{u}{t}+\frac{t}{u}\right)$$



$t = q^\mu q_\mu$

t-pole from electron propagator

Photon Pair Production

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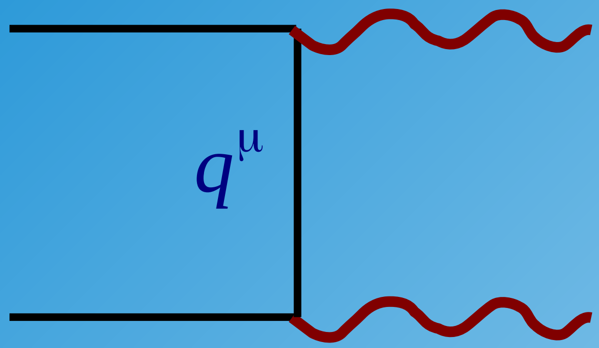
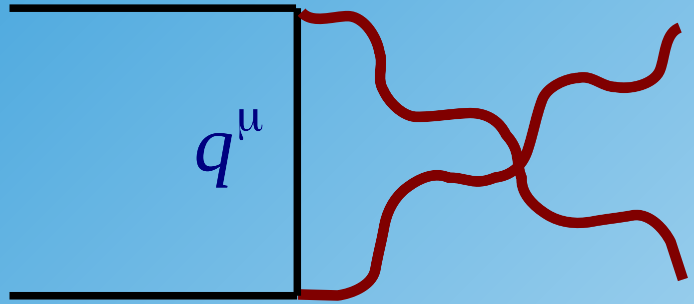
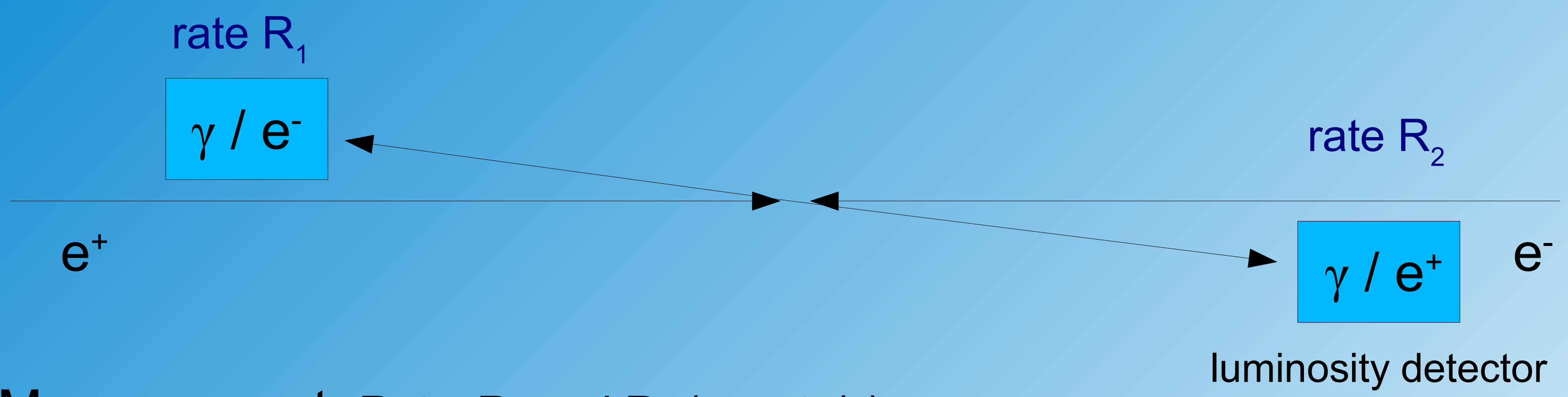


$t=q^{\mu}q_{\mu}$ t-pole from electron propagator



$u=q^{\mu}q_{\mu}$ u-pole from electron propagator

Sketch of Luminosity Measurement



Measurement: RatesR_1 and R_2 (counts/s)

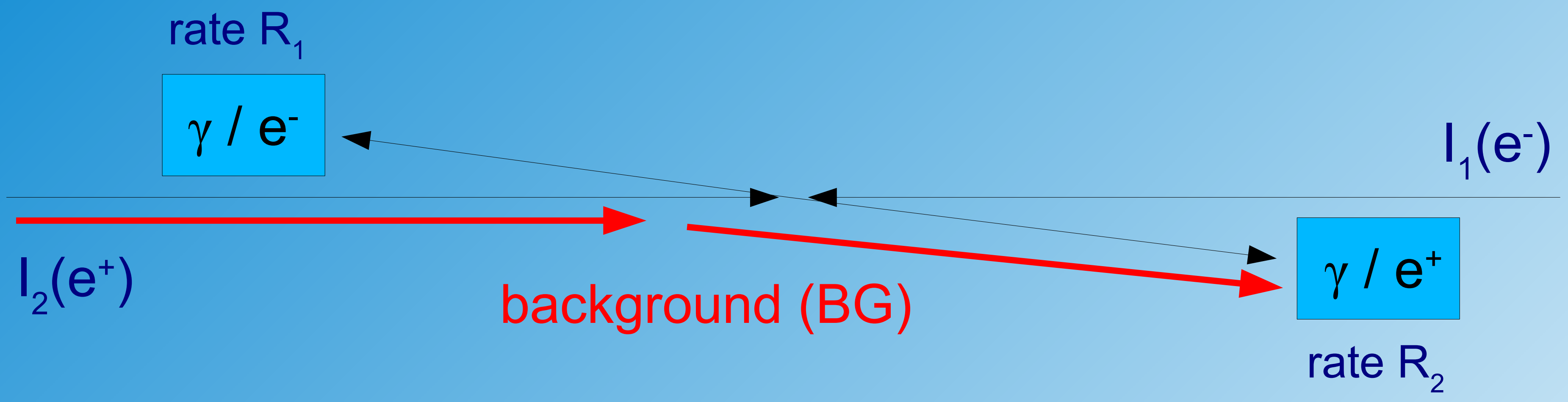
Use: **$R = L\,\sigma_{Detector}$** ↔ **$L = R/\,\sigma_{Detector}$**

$$\sigma_{Detector} = \int_{Detector} \frac{d\sigma}{d\Omega}\, d\Omega$$

acceptance calculation is an experimental task!

$\sigma_{Detector}$ is the **observable** cross section ≠ total cross cross section

Background for Luminosity Measurement



rate R_1

γ / e^-

$I_1(e^-)$

$I_2(e^+)$

background (BG)

γ / e^+

rate R_2

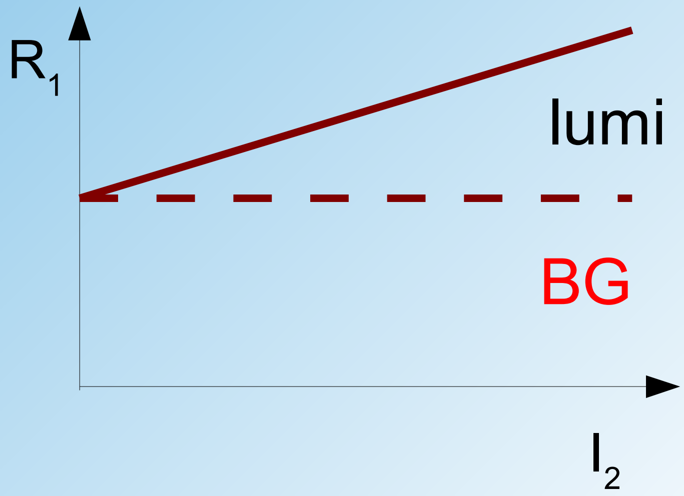
Problem: beam induced background, e.g. electron-rest gas scattering)

Ansatz:

$$R_1 = a_1 I_1 + b I_1 I_2 = I_1 (a_1 + b I_2)$$

$$R_2 = a_2 I_2 + b I_1 I_2 = I_2 (a_2 + b I_1)$$

BG lumi

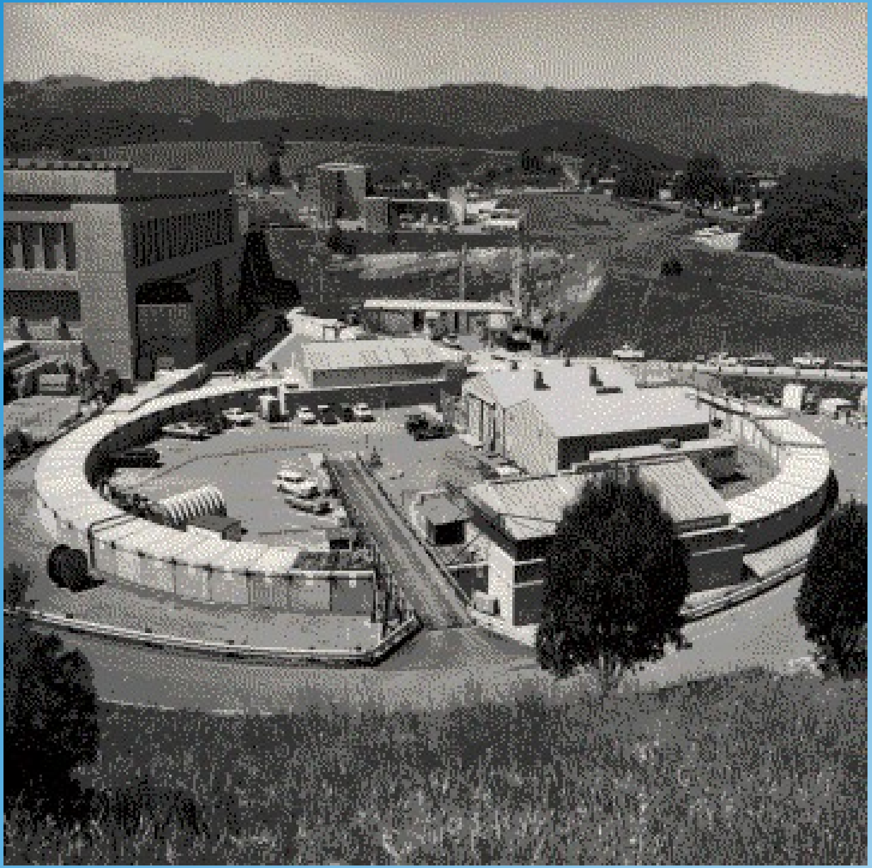


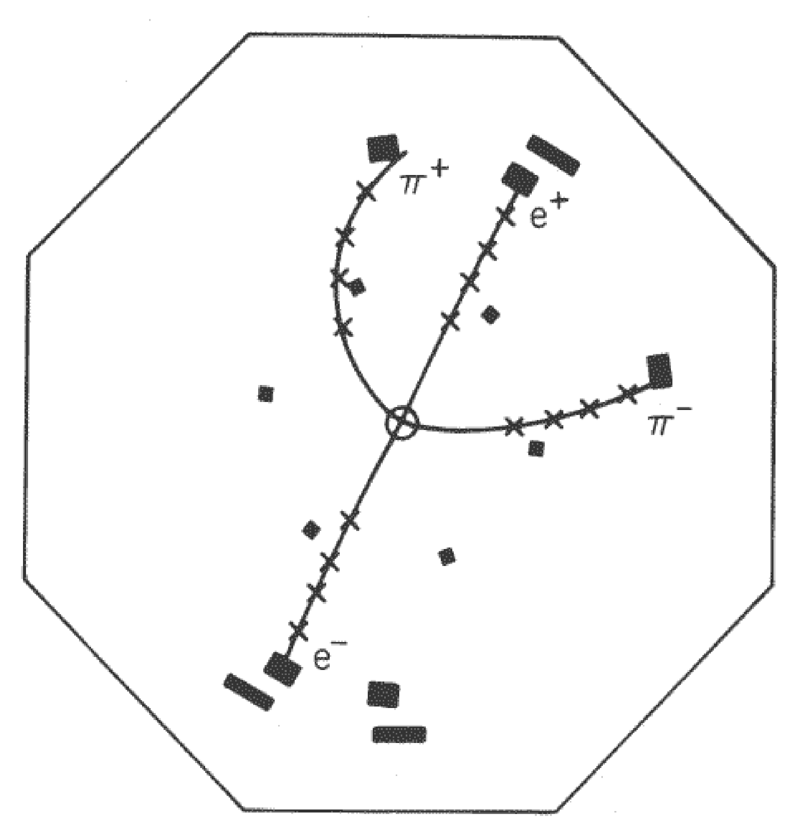
The Big e^+e^- Accelerators

- **SPEAR** (Stanford Positron Electron Accelerator Ring) at SLAC (1974-1990), $s^{1/2}$=3-8 GeV, Discovery of the Charm Quark
- **PETRA** (Positron Electron Tandem Ringanlage) at DESY (1978-1986), $s^{1/2}$=38 GeV, Discovery of Gluon-Jets
- **TRISTAN** at KEK, Japan (1986-1989) $s^{1/2}$=50-64 GeV (discovery of the “desert”)
- Large Electron-Positron Collider, Geneva (1988-2000): $s^{1/2}$=90 GeV (**LEP I**, Z-factory), $s^{1/2}$=200 GeV (**LEP II**, WW factory)
- Stanford Linear Accelerator at SLAC, Stanford (1991-1998) $s^{1/2}$=90 GeV (**SLC**, Z-factory)

SPEAR at SLAC

- Stanford Positron Electron Accelerator Ring (1974-1990), $s^{1/2}$=3-7 GeV, Discovery the J/Psi

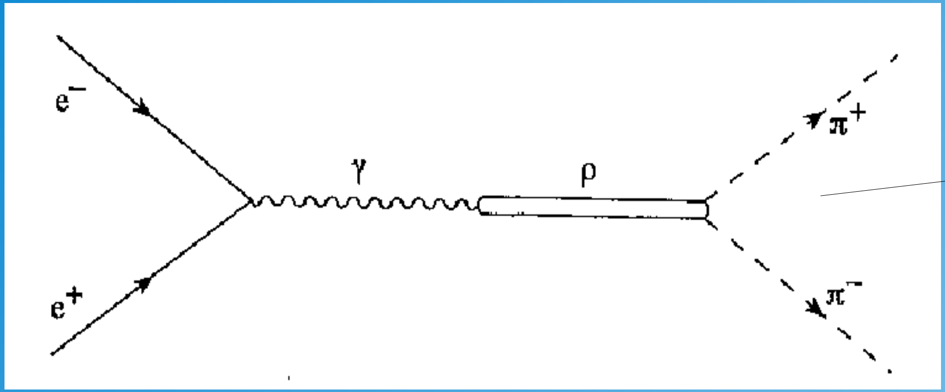




Discovery of the Charm Quark

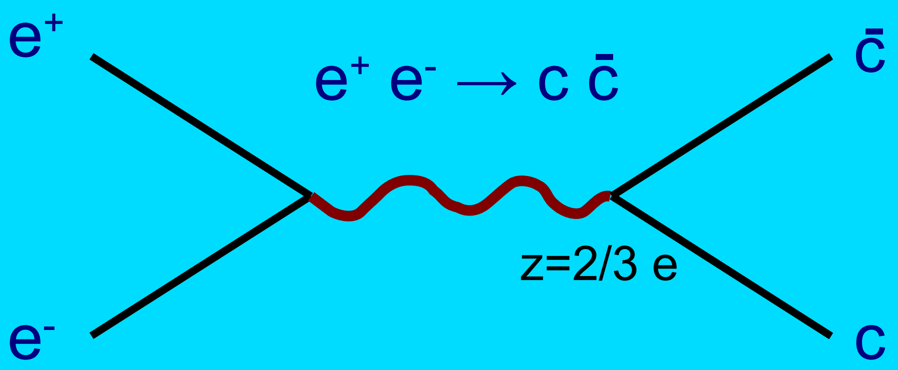
$$\Psi(2S) \rightarrow J/\Psi \pi^+ \pi^- \rightarrow e^+ e^- \pi^+ \pi^-$$

Quark-Pair Production



Resonant Rho production

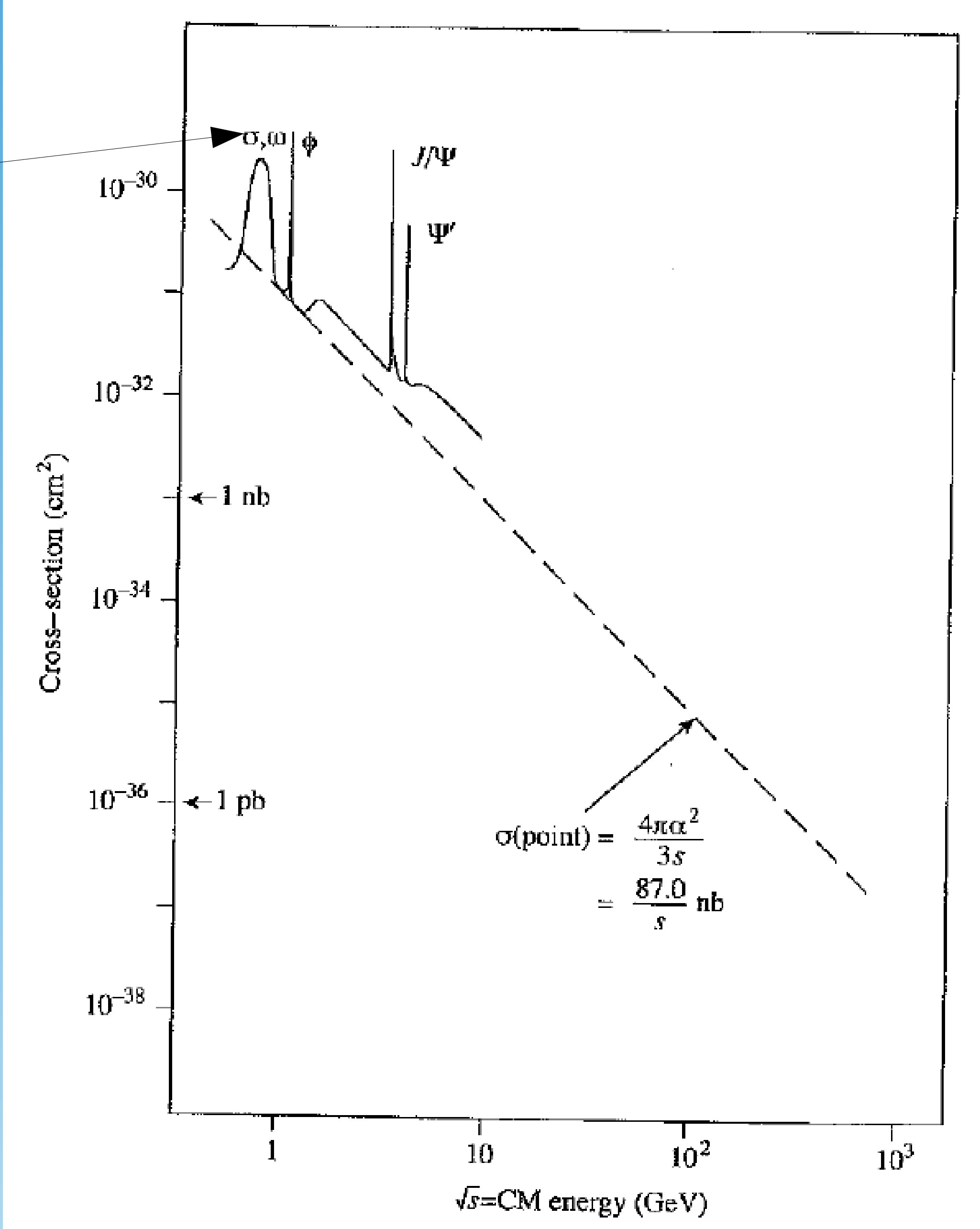
➔ later



$e^+ e^- \to c\,\bar{c}$

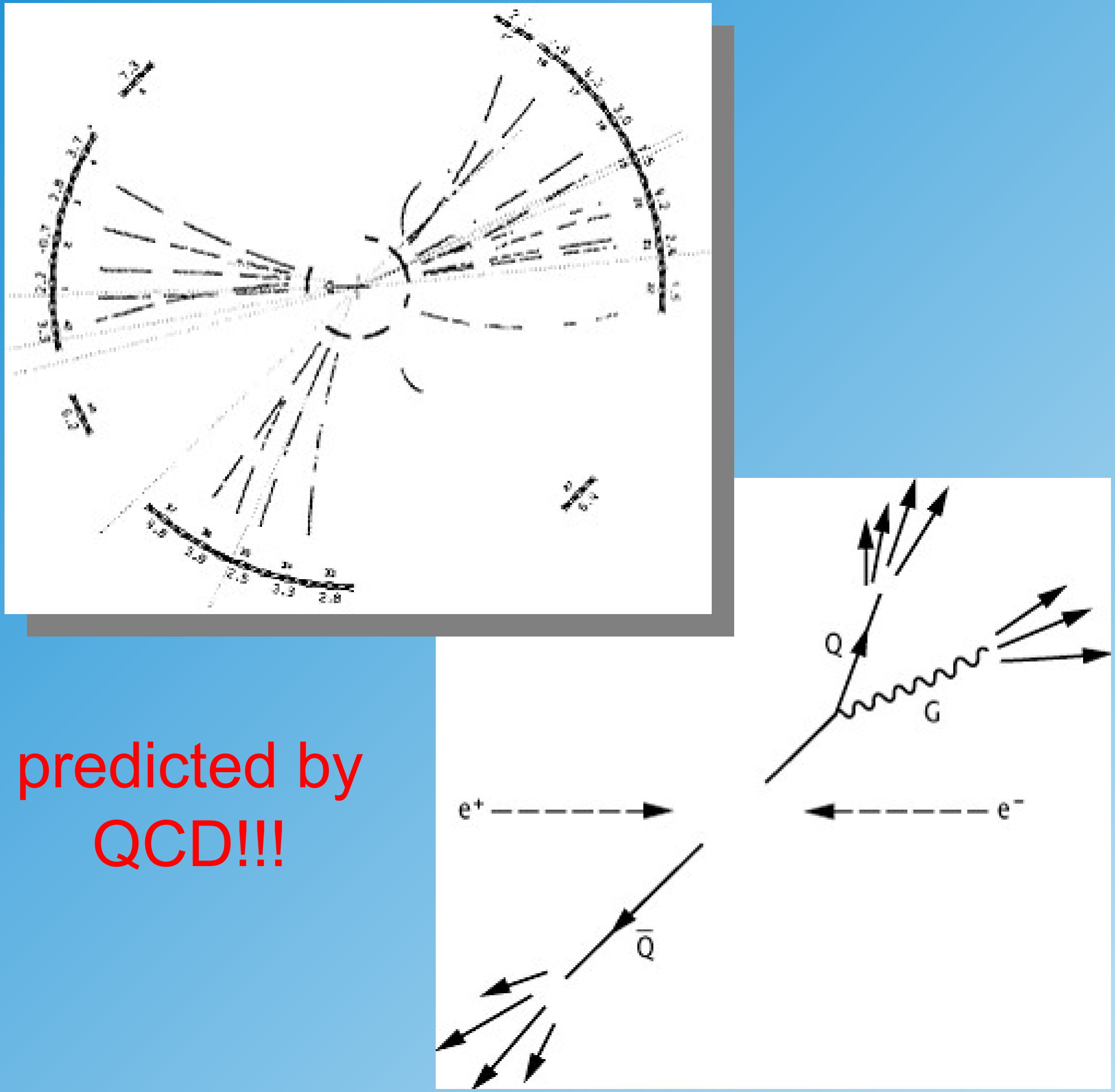
similar to muon-pair production

$$\frac{d\sigma}{d\Omega}(e^+e^- \to c\bar{c}) = \frac{\alpha^2 q^2}{2s}\left(\frac{u^2+t^2}{s^2}\right)$$

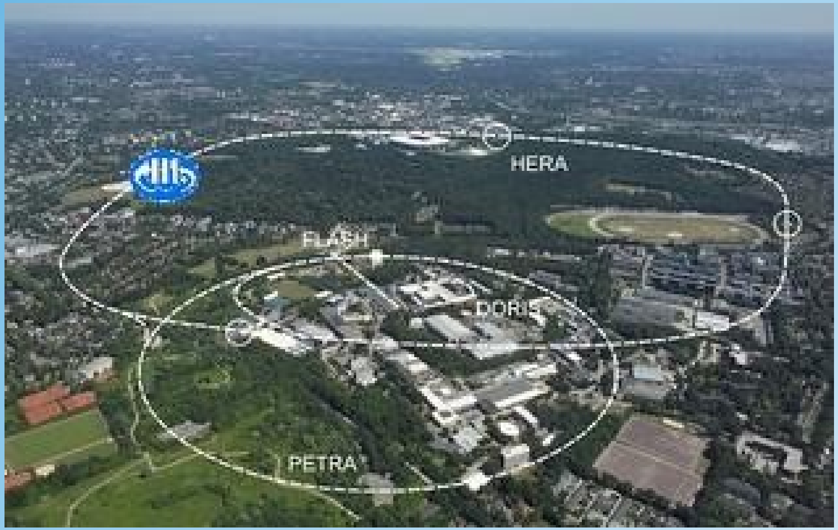


PETRA at DESY

- Positron Electron Tandem Ring Anlage (1978-1986), $s^{1/2}$=38 GeV, Discovery of Gluon Jets



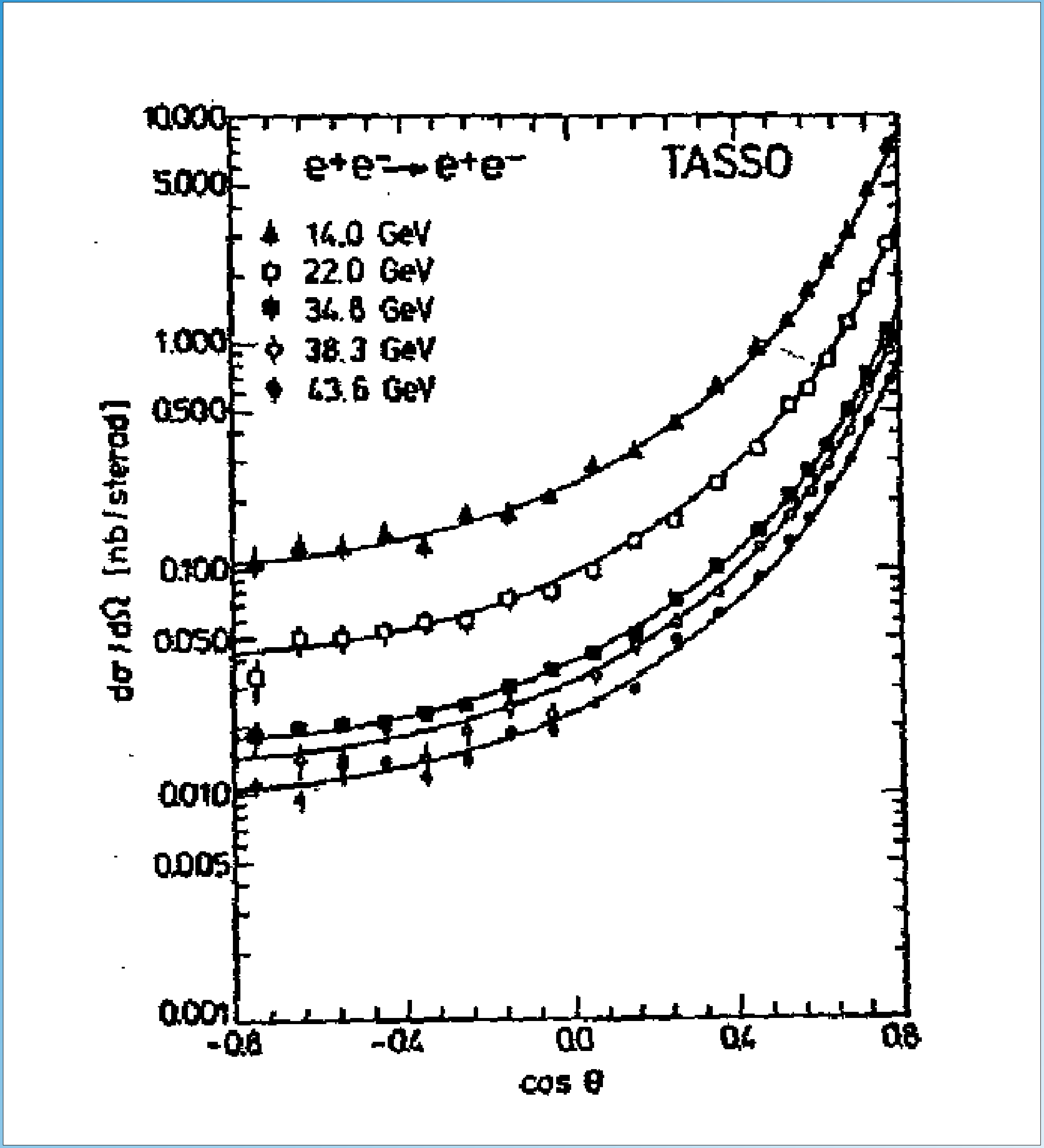
predicted by QCD!!!



Tasso at PETRA

QED Test:

Bhabha scattering



Total Muon Pair Production C.S.

$$\frac{d\sigma}{dt} = -\frac{2\pi\alpha^2}{s^2}\,\frac{t^2+u^2}{s^2}$$

$$s+t+u=\sum m_i^2 \approx 0$$

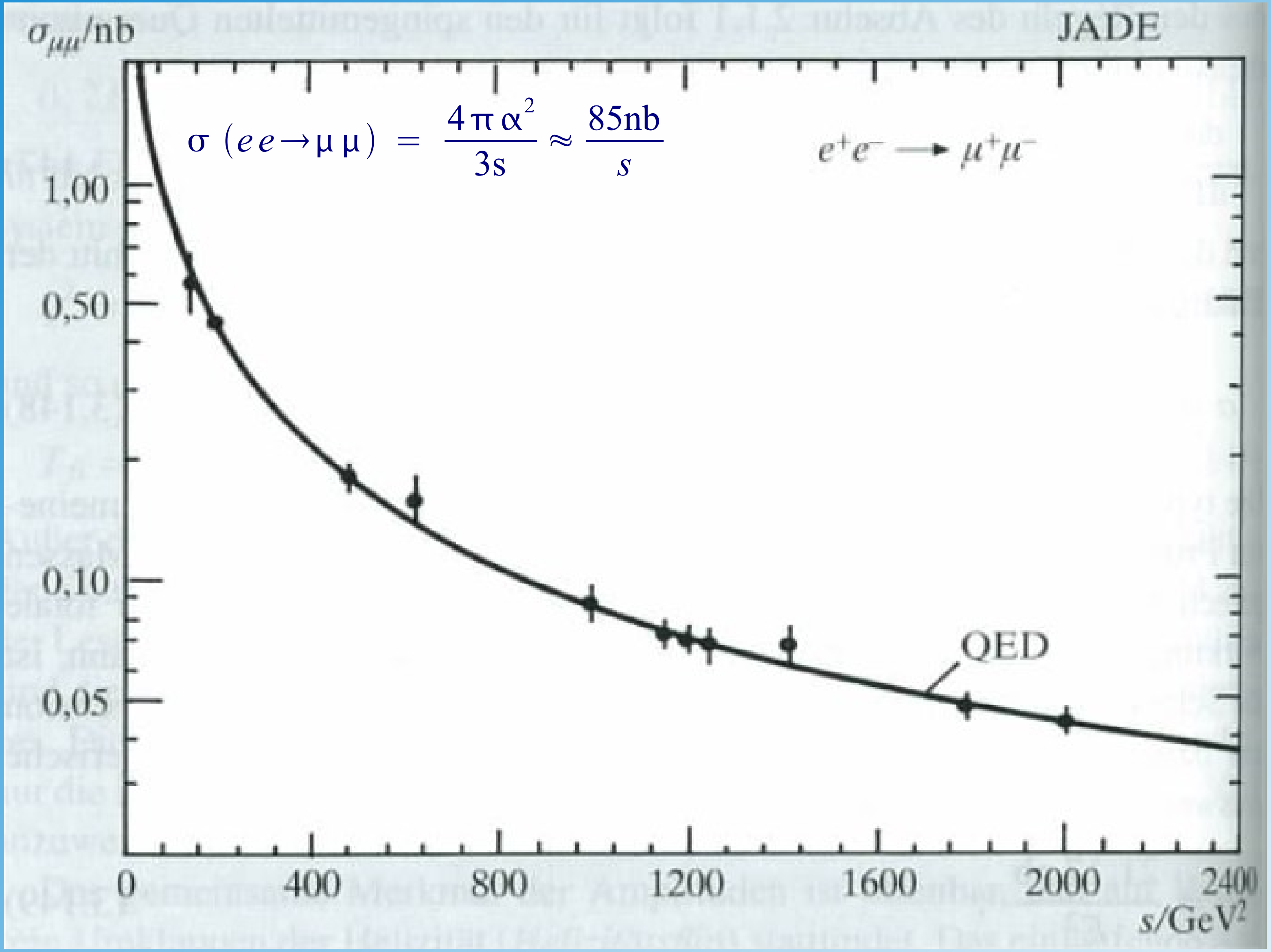
(only two independent)

$$\rightarrow\ u^2=t^2+s^2+2ts$$

$$\frac{d\sigma}{dt} = -2\pi\alpha^2\,\frac{2\mathrm{t}^2+s^2+2\mathrm{ts}}{s^4}$$

$$\sigma = -\int_{-s}^{0} 2\pi\alpha^2\,\frac{2\mathrm{t}^2+s^2+2\mathrm{ts}}{s^4} = -2\pi\alpha^2\,\frac{-2/3\mathrm{s}^3-s^3+s3}{s^4} = \frac{4\pi\alpha^2}{3\mathrm{s}}$$

Myon Pair Production



$$\sigma(ee \rightarrow \mu\mu) = \frac{4\pi\alpha^2}{3s} \approx \frac{85\text{nb}}{s}$$

PETRA accelerator (DESY)

Quark-Pair Production

Difficulty:

quarks and antiquarks are exp. difficult to distinguish

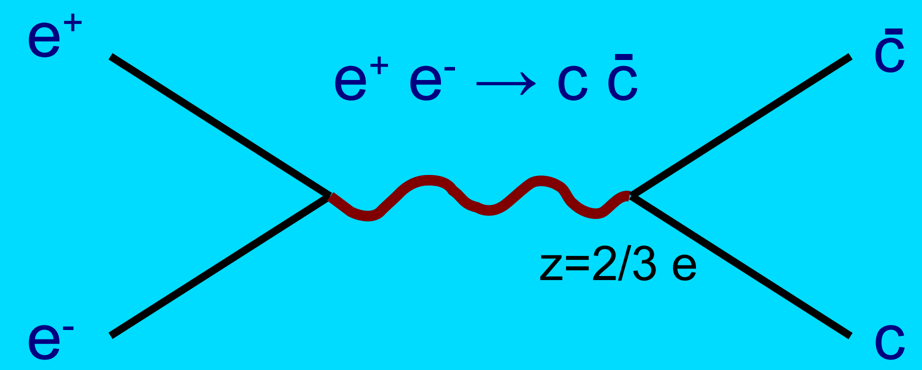
$$t = -\frac{s}{2}\left(1 \mp \cos\theta\right)$$

$$u = -\frac{s}{2}\left(1 \pm \cos\theta\right)$$

different signs for quarks and antiquarks

$$t^2 + u^2 = -\frac{s}{2}\left(1 + \cos^2\theta\right)$$

quarks and antiquarks averaged!



similar to muon-pair production

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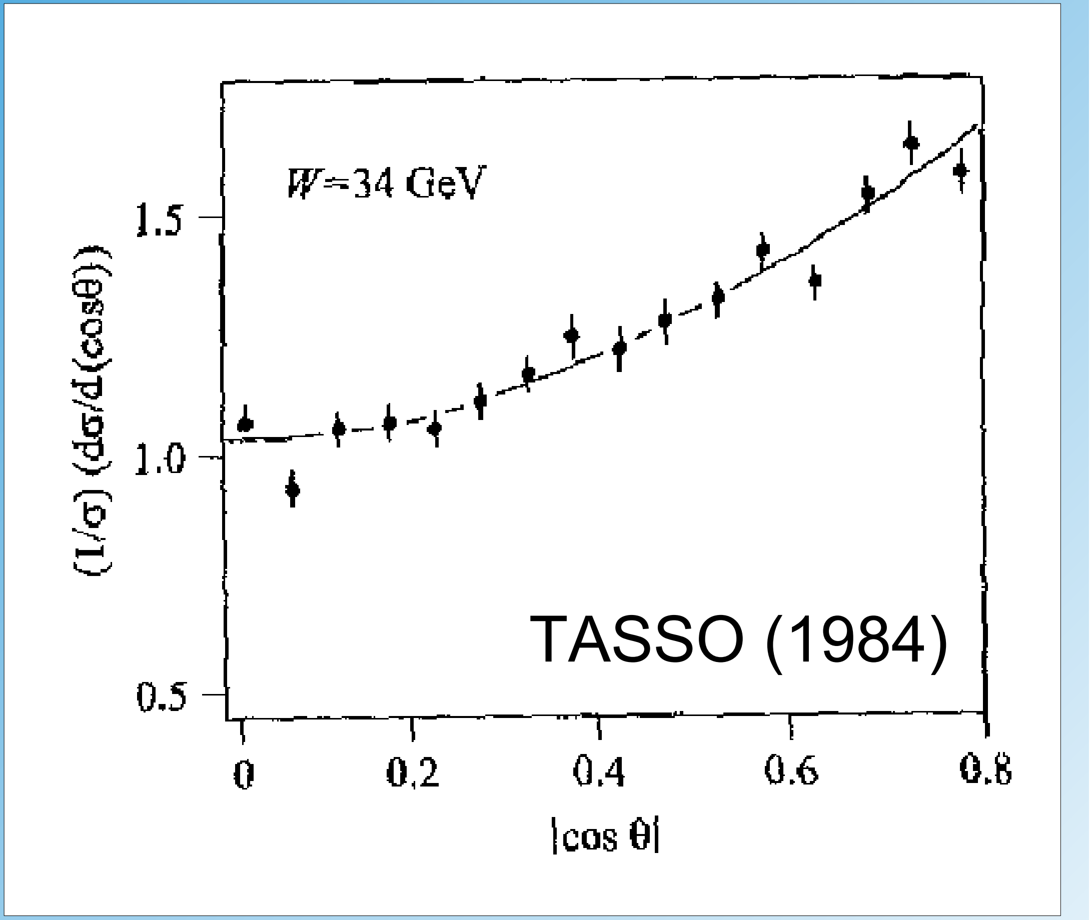
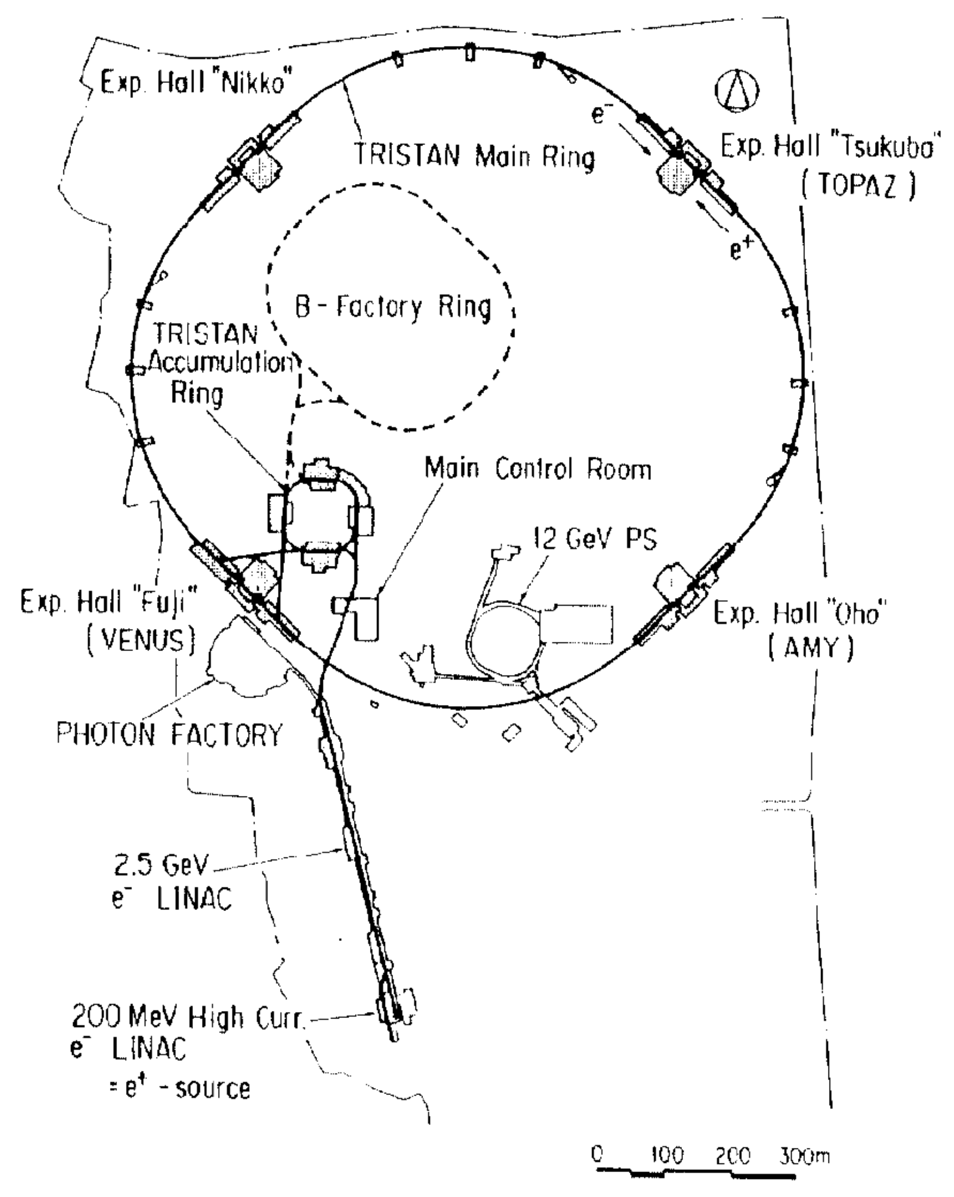


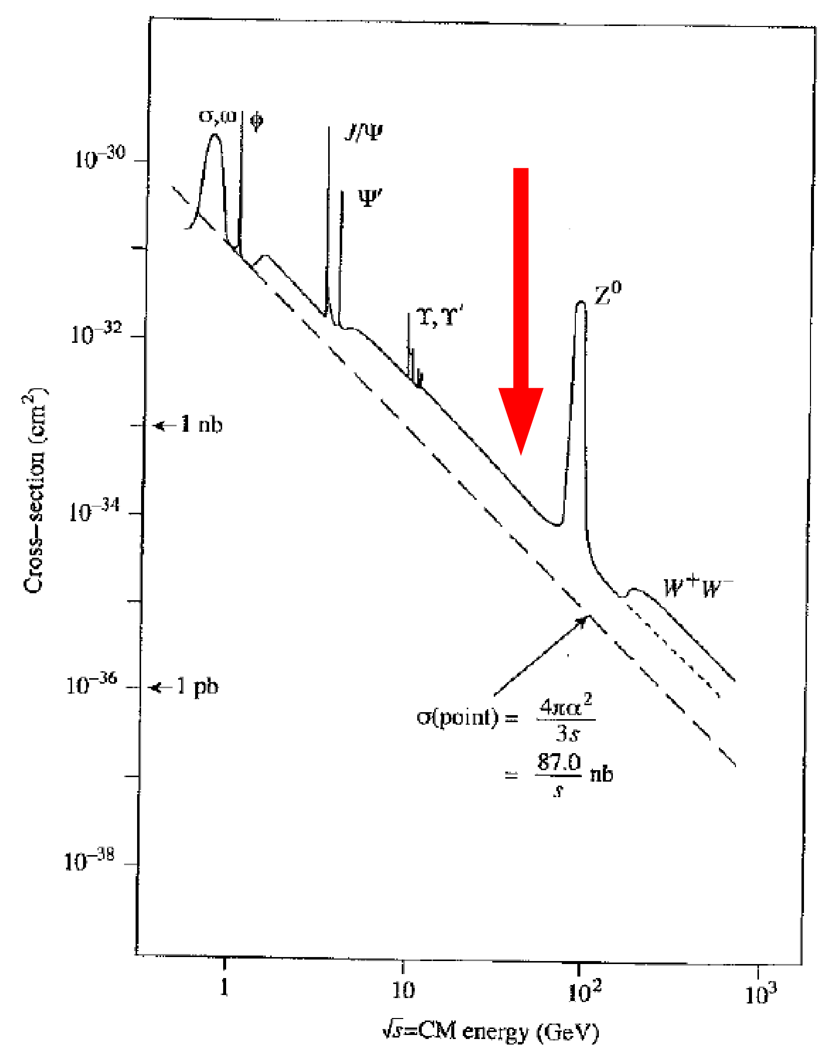
TASSO (1984)

Tristan Collider at KEK

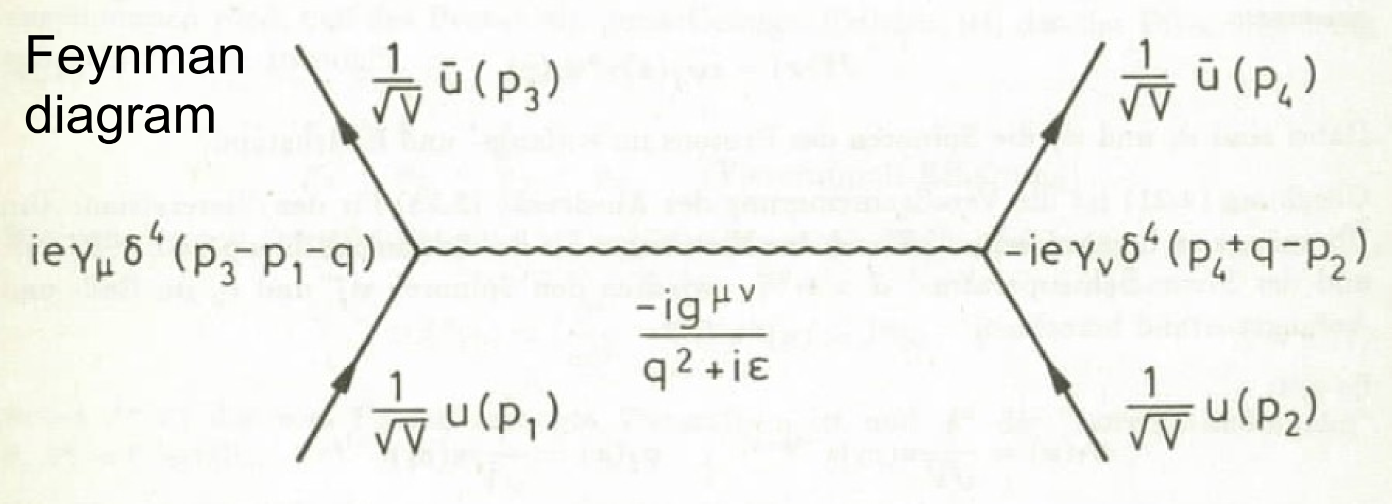
1986-1989: $s^{1/2}$=50-64 GeV

Search for the top in the “desert”





QED Tests in e^+e^- collisions



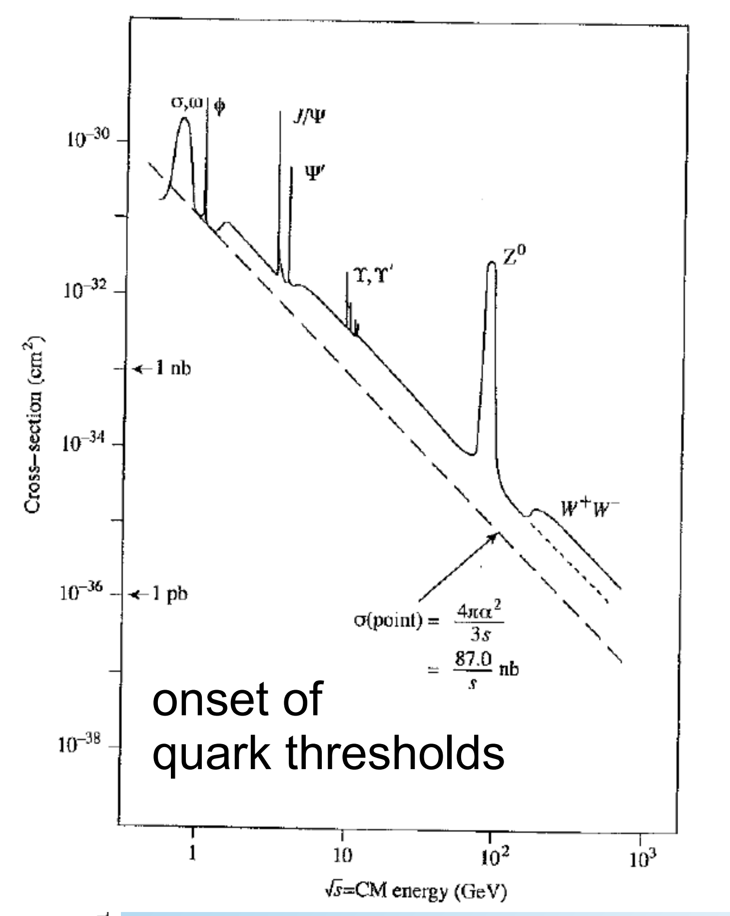
<u>Possible tests:</u>

- universality of charges (leptons, quarks, ...)
- energy dependence of coupling (“running”)
- test of perturbation theory
- Lorentz structure of coupling
- propagator effect → new physics
- test crossing symmetries (→ gauge invariance)

Measurement of R_{had}

Test of Quark Charges

$$R=\frac{e^{+}e^{-}\rightarrow hadrons}{e^{+}e^{-}\rightarrow\mu^{+}\mu^{-}}$$



onset of quark thresholds

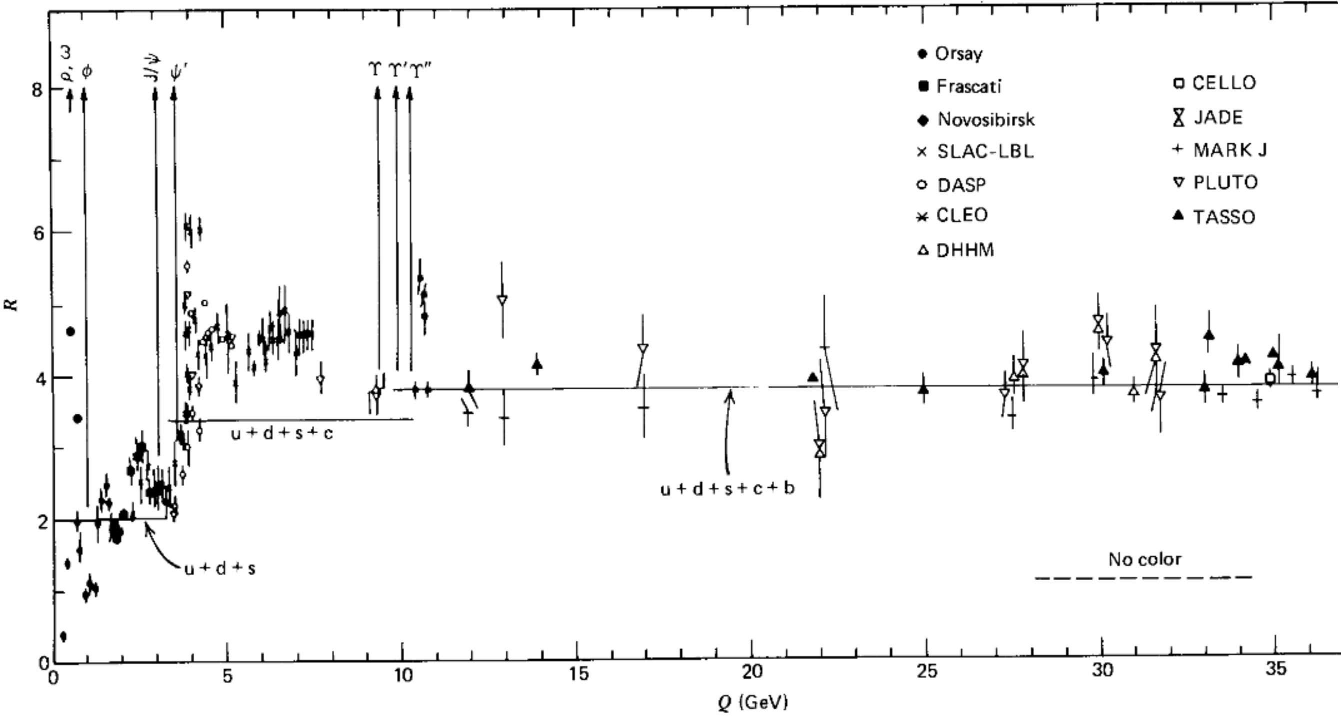
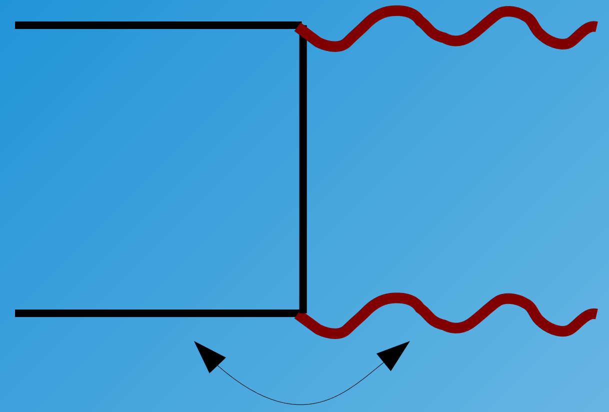


Fig. 11.3 Ratio R of (11.6) as a function of the total e^-e^+ center-of-mass energy. (The sharp peaks correspond to the production of narrow 1^- resonances just below or near the flavor thresholds.)

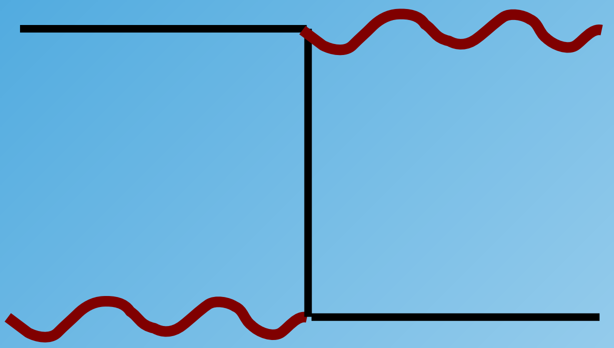
Crossing Symmetries

$$\frac{d\sigma}{d\Omega}(e^+ e^- \to \gamma\gamma) = \frac{\alpha^2}{2s}\left(\frac{u}{t}+\frac{t}{u}\right)$$



$$t \to -t$$
$$s \to u$$

$$\frac{d\sigma}{d\Omega}(\gamma e^- \to \gamma e^-) = \frac{\alpha^2}{2s}\left(\frac{-s}{u}+\frac{-u}{s}\right)$$



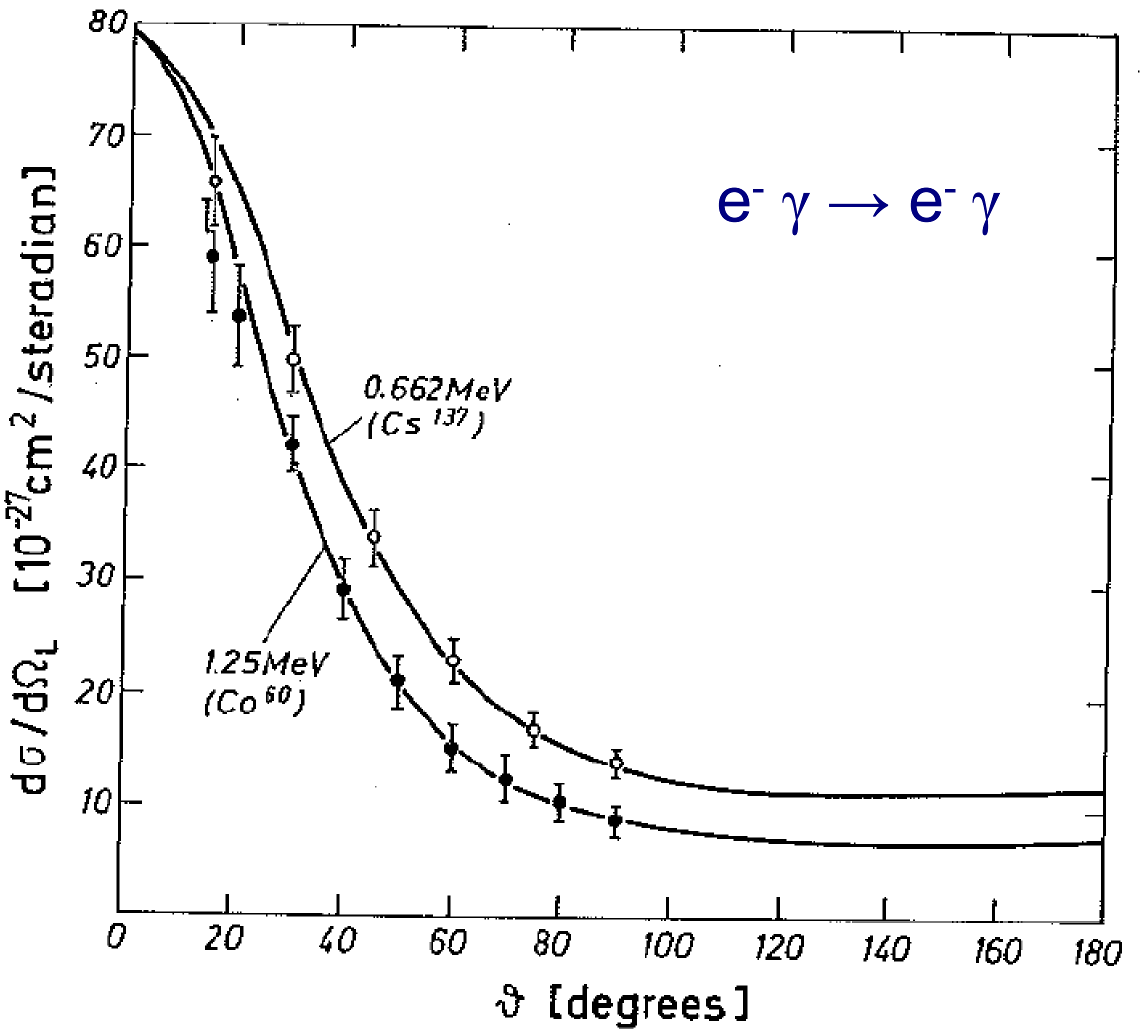
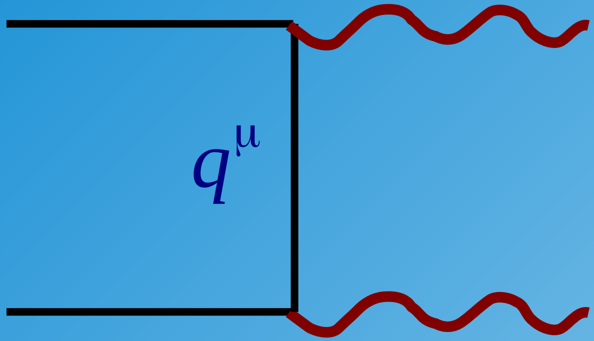


Figure 10.7 Experimental results for Compton scattering. The curves correspond to the Klein–Nishina formula (10.41) for photon energies $\omega = 0.662$ MeV and $\omega = 1.25$ MeV. The experimental data are from Hofstadter (1949) and Bernstein (1956) (after Evans 1958).

The Low Energy Limit

Electromagnetic coupling at low energy:



$$Q^2 = -t = -(q^\mu q_\mu) \rightarrow 0$$

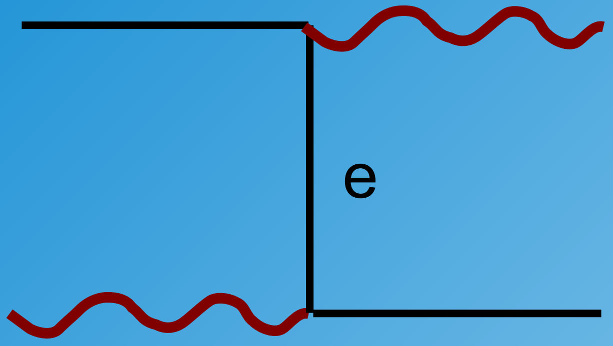
$Q^2 = m_e$ is kinematically forbidden

Thompson scattering cross section $\sigma_t = \frac{8\pi}{3r_e^2} = \frac{8\pi}{3}\left(\frac{\alpha\lambda_c}{2\pi}\right)^2$

used to determine α

The Low Energy Limit

Electromagnetic coupling at low energy:



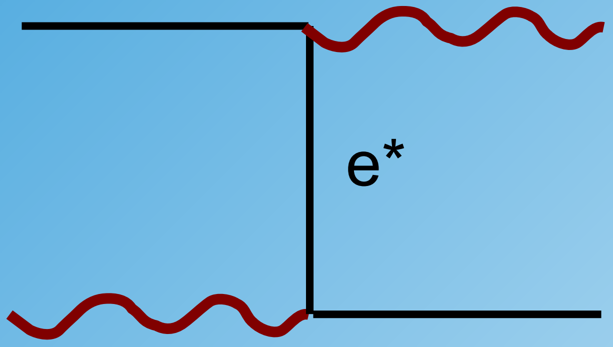
$$Q^2 = -t = -(q^\mu q_\mu) \rightarrow m_e^2$$

$Q^2 = m_e$ is kinematically forbidden

Thompson scattering cross section $\sigma_t = \frac{8\pi}{3 r_e^2} = \frac{8\pi}{3}\left(\frac{\alpha \lambda_c}{2\pi}\right)^2$

not dependent on energy! Used to determine α

General cross section:

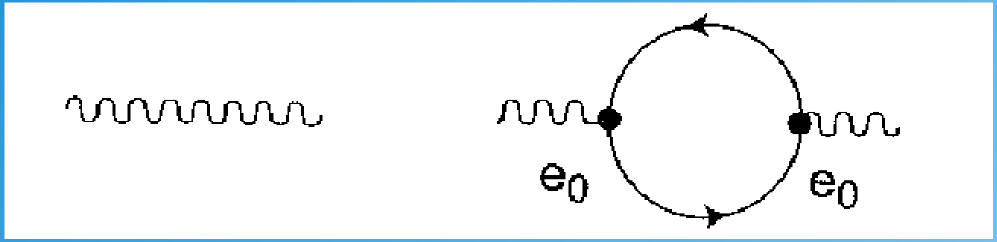


$$\frac{d\sigma}{d\Omega}(\gamma e^- \rightarrow \gamma e^-) = \frac{\alpha^2}{2s}\left(\frac{-s}{u} + \frac{-u}{s}\right)$$

Running of alpha$_{\text{em}}$

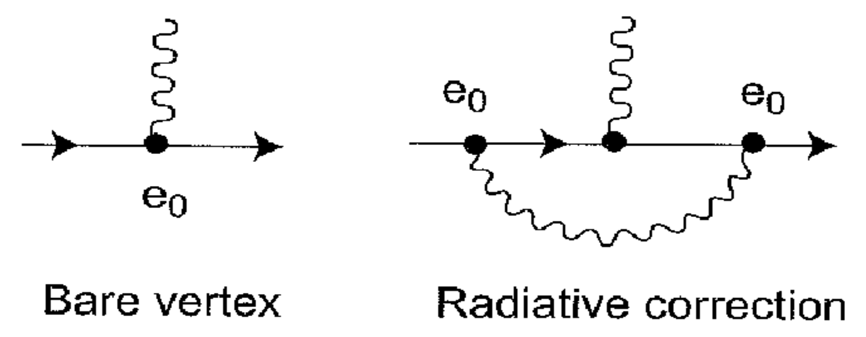
$$\alpha(Q=0)=1/137 \longrightarrow \alpha(Q=90\,GeV)=1/128$$

self-energy corrections



alpha is not a constant!

vertex corrections



dressed charge!

- measure em. coupling for different (high) energies
- search for new physics effects at mass scale Λ

$$\frac{d\sigma}{d\Omega}=\frac{d\sigma(QED)}{d\Omega}\left(1+\frac{s}{\Lambda^2}\right)$$

Lorentz-Structure of Electromagnetic Interaction

From Maxwell Equations:

$$\partial_\nu \partial^\nu A^\mu(x) = e\, J^\mu(x)$$

in QED: $\partial_\mu j_V^\mu = 0$ (conservation of currents)

electromagnetic interaction described by vector currents!

Also true at high energies?

Vector Current:

$$j_V^\mu = \bar{\psi}\gamma^\mu\psi$$

Axial-vector Current:

$$j_A^\mu = \bar{\psi}\gamma^\mu\gamma^5\psi$$

scalar coupling:

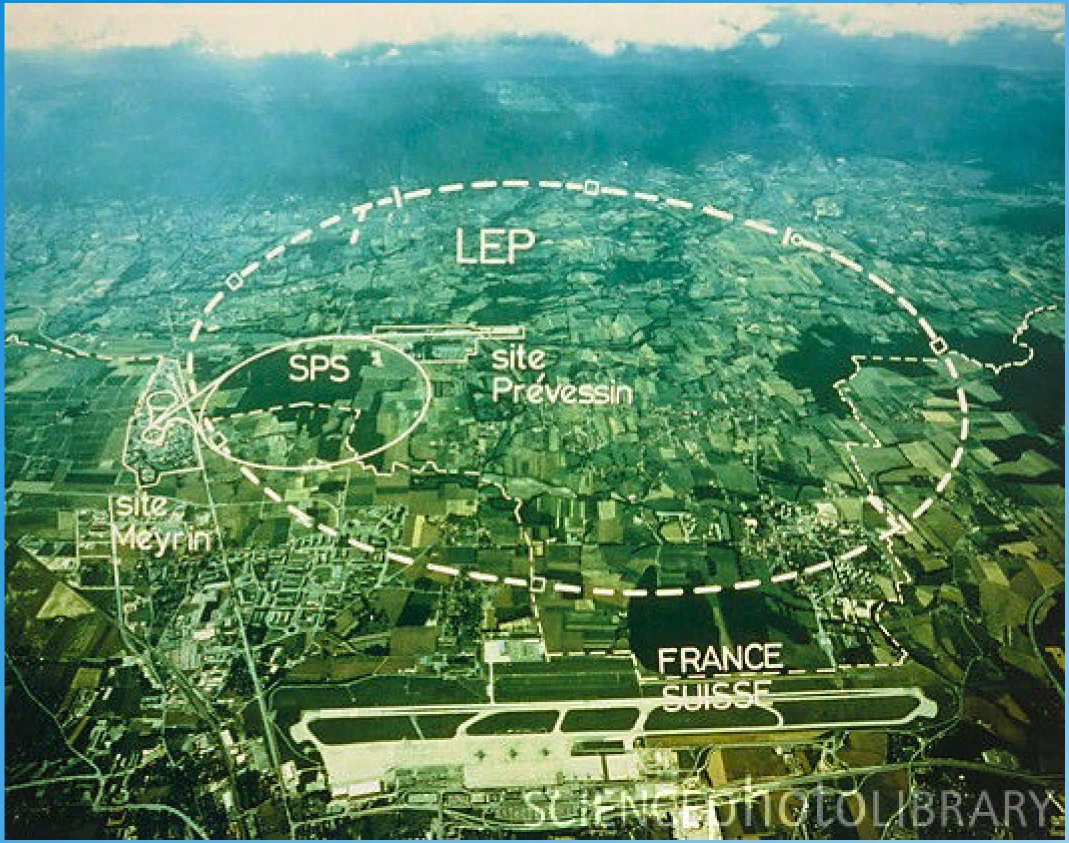
$$\lambda = \bar{\psi}\psi$$

pseudoscalar coupling:

$$\lambda = \bar{\psi}\gamma^5\psi$$

➜ lead in genreral to different angular distributions!

LEP Collider



biggest electron-positron collider with up to 200 GeV centre of mass

4 experiments:
ALEPH, DELPHI, L3, OPAL

LEP1: “Z-factory”

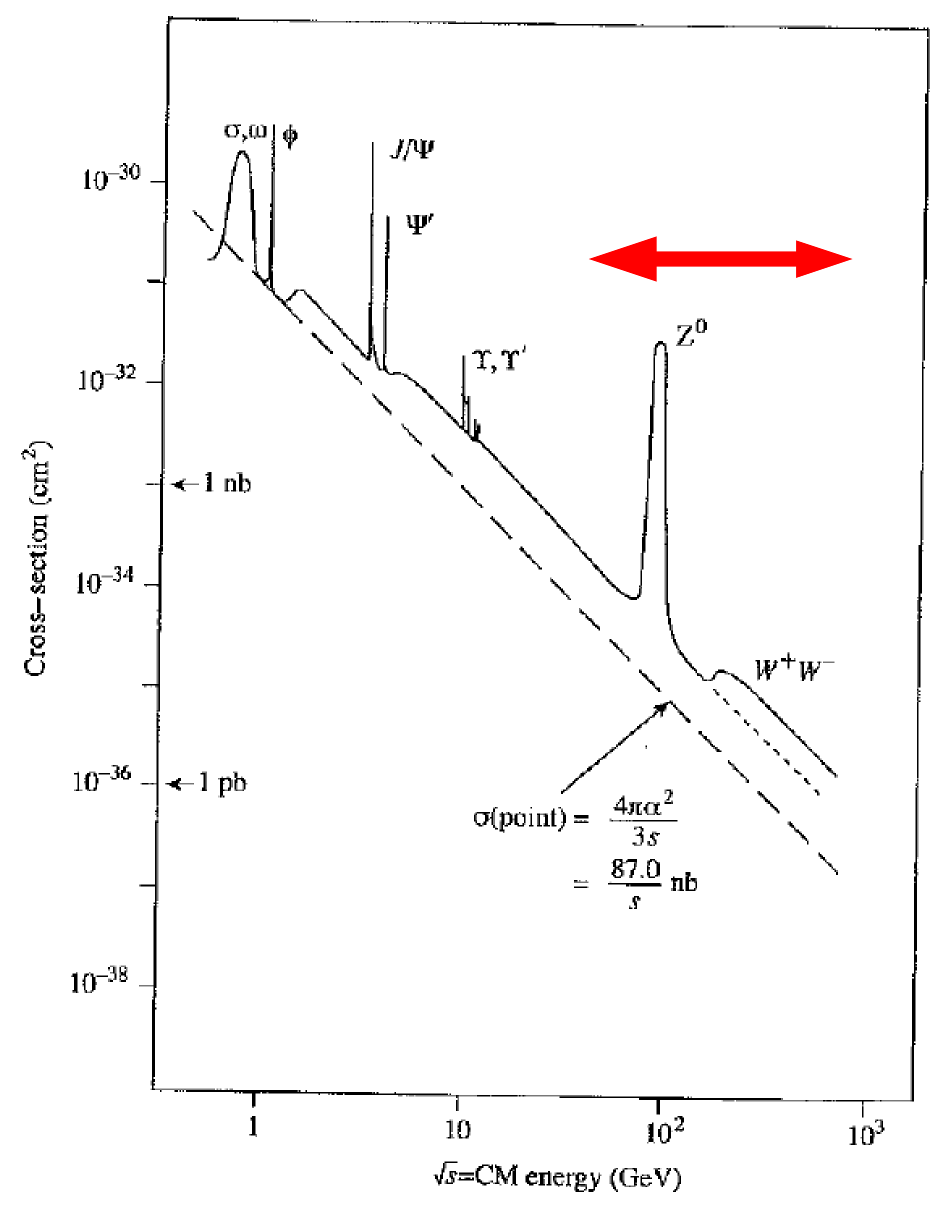
LEP2: “WW factory”

LEP Regime

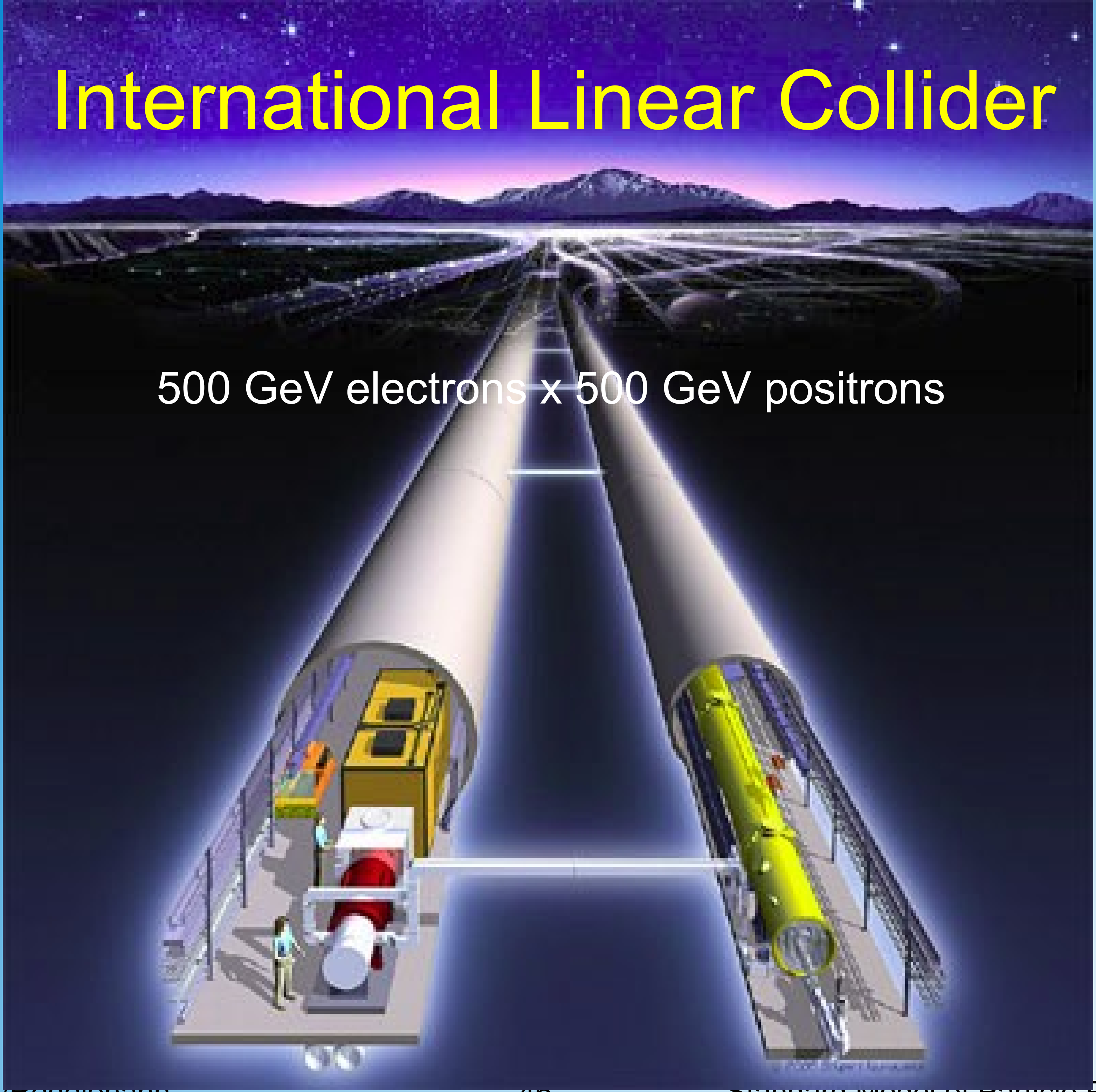
Processes:

- $e^+ e^- \rightarrow Z$
- $e^+ e^- \rightarrow W^+ W^-$

at LEP energies radiative effects play an important role!



International Linear Collider



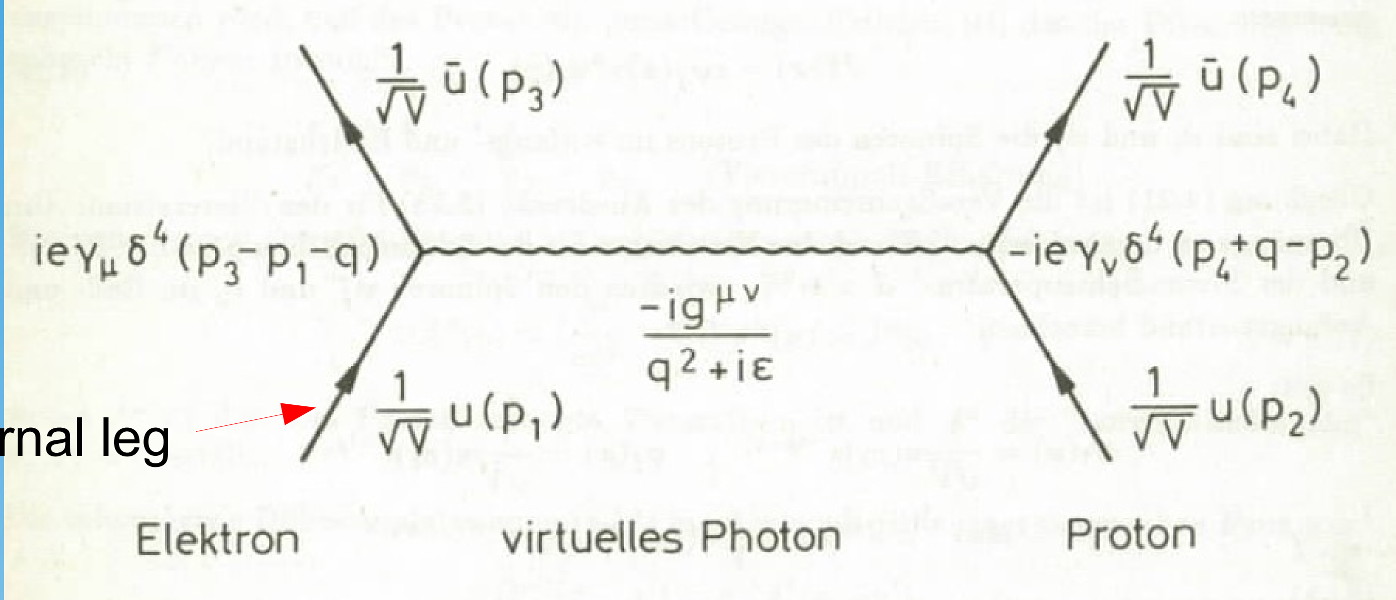
500 GeV electrons x 500 GeV positrons

Backup

Fermion-Fermion Scattering

$$S_{fi}^{(1)} = ie^2 \int \frac{d^4q}{(2\pi)^4}\,\delta^4(p_3 - p_1 - q)\,\delta^4(p_4 + q - p_2)\,(2\pi)^8 \\ \cdot \frac{1}{V^2}\,\overline{u}(p_3)\gamma_\mu u(p_1) \cdot \frac{-g^{\mu\nu}}{q^2 + i\epsilon} \cdot \overline{u}(p_4)\gamma_\nu u(p_2)$$

particle-particle (here electron-proton) scattering



external leg

lowest oder perturbation theory: leading order graph (Born)

Gamma Matrices I

Gamma matrices γ^μ are chosen such that γ^0 is hermitian while γ^k (k=1,2,3) are anti-hermitian

$$(\gamma^0)^\dagger = \gamma^0, \quad (\gamma^0)^\mu = 1,$$

$$(\gamma^k)^\dagger = -(\gamma^k), \quad (\gamma^k)^\mu = -1 \quad (k=1,2,3)$$

We define γ^5 as the hermitian matrix: $\gamma^5 = i\,\gamma^0\,\gamma^1\,\gamma^2\,\gamma^3$, $\qquad (\gamma^5)^2 = 1$,

The 4x4 gamma matrices can be represented by (representation where γ^0 is diagonal):

$$\gamma^k = \begin{pmatrix} 0 & \underline{\sigma}^k \\ -\underline{\sigma}^k & 0 \end{pmatrix}, \qquad \gamma^0 \overset{\text{def}}{=} \beta = \begin{pmatrix} \underline{1} & 0 \\ 0 & -\underline{1} \end{pmatrix}, \qquad \gamma^5 = \begin{pmatrix} 0 & \underline{1} \\ \underline{1} & 0 \end{pmatrix}$$

(note several representations exist)

With the 2x2 Pauli matrices:

$$\underline{\sigma}^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \underline{\sigma}^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \underline{\sigma}^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Gamma matrices anti-commute: $\gamma^i\,\gamma^k + \gamma^k\,\gamma^i = 0$ for $i \neq k$

Gamma Matrices II

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \qquad \gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \qquad \gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\gamma^5 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

(in other representations
g^5 is diagonal)