•) angular distribution

$$d\Omega = 2\pi \, d\cos\Theta \quad , \text{ use } t = (E_1 - E_3)^2 - (\vec{p}_1 - \vec{p}_3)^2 :$$

$$\frac{d\Omega}{dt} = \frac{d\Omega}{d\cos\Theta}\left|\frac{d\cos\Theta}{dt}\right| = \frac{4\pi s}{\sqrt{\lambda(s, m_1^2, m_2^2)}\sqrt{\lambda(s, m_3^2, m_4^2)}}$$

$$= \frac{\pi}{|\vec{p}|\,|\vec{p}\,'|}$$

•) relative velocity

$$V_{12} = |\vec{V}_1 - \vec{V}_2| = \left|\frac{\vec{p}_1}{E_1} - \frac{\vec{p}_2}{E_2}\right| = \frac{|\vec{p}_1|}{E_1 E_2}(E_1 + E_2)$$

$$\Rightarrow E_1 E_2 V_{12} = \sqrt{s}\sqrt{E_1^2 - m_1^2} = \sqrt{s}\left[\frac{1}{4s}(s + m_1^2 - m_2^2)^2 - m_1^2\right]^{1/2}$$

$$= \left[\frac{1}{4}(m_1^2 + \cancel{m_2^2} + 2(p_1 \cdot p_2) + m_1^2 - \cancel{m_2^2})^2 - s m_1^2\right]^{1/2}$$

$$\Rightarrow \boxed{E_1 E_2 V_{12} = \sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}}$$ Møller flux factor

(not really frame invariant...)

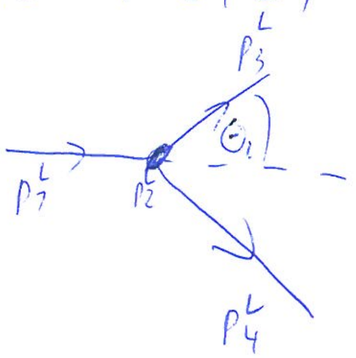
$$\simeq s/2 \text{ for } p_i^2 \gg m_i^2$$

(we need this later on when discussing cross sections)

[Note: $E_1 E_2 V_{12} = \sqrt{s}\,|\vec{p}_1|$ in CMS]

B) Labsystem

$$p_2^L = (E_2^L, \vec{0}) = (m_2, \vec{0}) \quad \leftrightarrow \text{ target experiment}$$



e.g. $\cancel{E_1} \; p_1^L \cdot p_2^L = E_1^L \cdot m_2 = \frac{1}{2}(p_1^L + p_2^L)^2 - \frac{1}{2} m_1^2 - \frac{1}{2} m_2^2$

$$\Rightarrow \quad E_1^L = \frac{1}{2m_2}\left(s - m_1^2 - m_2^2\right) \quad \left(\begin{array}{l} s \simeq 2 m_2 E_1^L \text{ for } s \to \infty \\ \simeq 2 m_2 |\vec{p}_1^L| \end{array} \right)$$

$$|\vec{p}_1^L| = \sqrt{E_1^{L2} - m_1^2} = \frac{1}{2m_2}\sqrt{\lambda(s, m_1^2, m_2^2)}$$

other momenta/energies, $\Theta_L \rightarrow$ exercises

•) Connection center-of-mass and labsystem

e.g. $$\frac{|\vec{p}\,|}{|\vec{p}_1^L|} = \frac{m_2}{\sqrt{s}} = \frac{m_2}{\sqrt{m_1^2 + m_2^2 + 2E_1^L m_2}} = \frac{E_1^2 - m_1^2}{E_1^{L2} - m_1^2}$$

$$\Rightarrow \quad E_1 = \frac{E_1^L m_2 + m_1^2}{\sqrt{m_1^2 + m_2^2 + 2 m_2 E_1^L}} \quad \text{etc.}$$

·) Examples

a) $pp \to ppp\bar{p}$ in LS : $E_1 \geq 7 m_p$

b) $pp \to ppp\bar{p}$ in cms : $E_1 \geq 4 m_p$

c) LHC : pp-collisions

each beam : $|\vec{p}| = 3.5$ TeV 2010/11

$\sqrt{s} = (E_1 + E_2) = 7$ TeV

since April 5th : $|\vec{p}| = 4$ TeV each

$\Rightarrow \sqrt{s} = 8$ TeV

d) T2K : protons on target : $E_p = 30$ GeV

$\Rightarrow \sqrt{s} = \sqrt{2 m_N E_p} \simeq 7.75$ GeV

e) HERA : $e^\pm p$-collisions : $\left.\begin{array}{l} E_e = 27.5 \text{ GeV} \\ E_p \simeq 920 \text{ GeV} \end{array}\right\} \sqrt{s} \simeq 2\sqrt{E_e E_p} \simeq 318$ GeV

f) eLHC (?) : $\left.\begin{array}{l} E_e \simeq 67 \text{ GeV} \\ E_p \simeq 7 \text{ TeV} \end{array}\right\} \sqrt{s} \simeq 1.37$ TeV

I 3) Crossing symmetry

so far $p_1 + p_2 \to p_3 + p_4$ assuming that all momenta are physical: $p = (E, \vec{p})$; $E = +\sqrt{\vec{p}^2 + m^2}$

BUT: is analytic relation valid for arbitrary (even timelike) $p = (E, \vec{p})$ with $E = -\sqrt{\vec{p}^2 + m^2}$

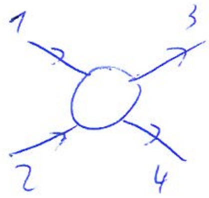
$\Rightarrow$ $p_1 + p_2 = p_3 + p_4$ (1)

$p_1 + (-p_3) = (-p_2) + p_4$ (2) (p_3, p_2 have negative E) [$E = p^0$]

$\boxed{\text{ANTIPARTICLES!}}$

$p_1 + (-p_4) = p_3 + (-p_2)$ (3) (p_4, p_2 have negative E) [$E = p^0$]

(1) corresponds to $1 + 2 \to 3 + 4$

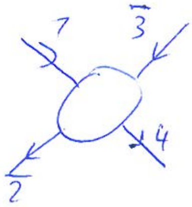


$p_1 + p_2 = p_3 + p_4$: $s = (p_1 + p_2)^2 > 0$

„s channel"

$p_{\bar{3}, \bar{2}}$ have positive E

(2) is interpreted as $1 + \bar{3} \to \bar{2} + 4$

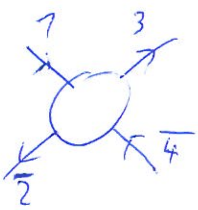


$p_1 + p_{\bar{3}} = p_{\bar{2}} + p_4$

„t channel"

$p_{\bar{2}, \bar{4}}$ have positive E

(3) is interpreted as $1 + \bar{4} \to \bar{2} + 3$



$p_1 + p_{\bar{4}} = p_3 + p_{\bar{2}}$

„u channel"

→ Here $p_{\bar{3}}$ denotes antiparticle with positive E

→ „s, t, u - channel" according to variable which is positive

e.g. $t = (p_1 - p_3)^2$

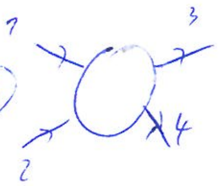
in ②: p_3 has negative E;

in CMS: $\vec{p}_1 - \vec{p}_3 = \vec{p}_1 + \vec{p}_{\bar{3}} = 0$

$\Rightarrow t = (E_1 - E_3)^2 = (E_1 + |E_3|)^2 \geq 0$

$s, u \leq 0$

(s)



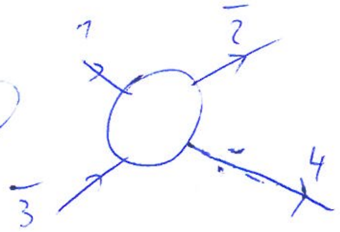
$e^+ e^- \to \mu^+ \mu^-$

In general: blob described by

$T_s(s, t, u) \equiv T(s, t, u)$ $\quad s \geq 0$, $\; t, u \leq 0$

comes from dynamics (QED, QCD, SM, BSM, ...) is analytic function of s, t, u;

(t)



$e^+ \mu^- \to \mu^+ e^-$

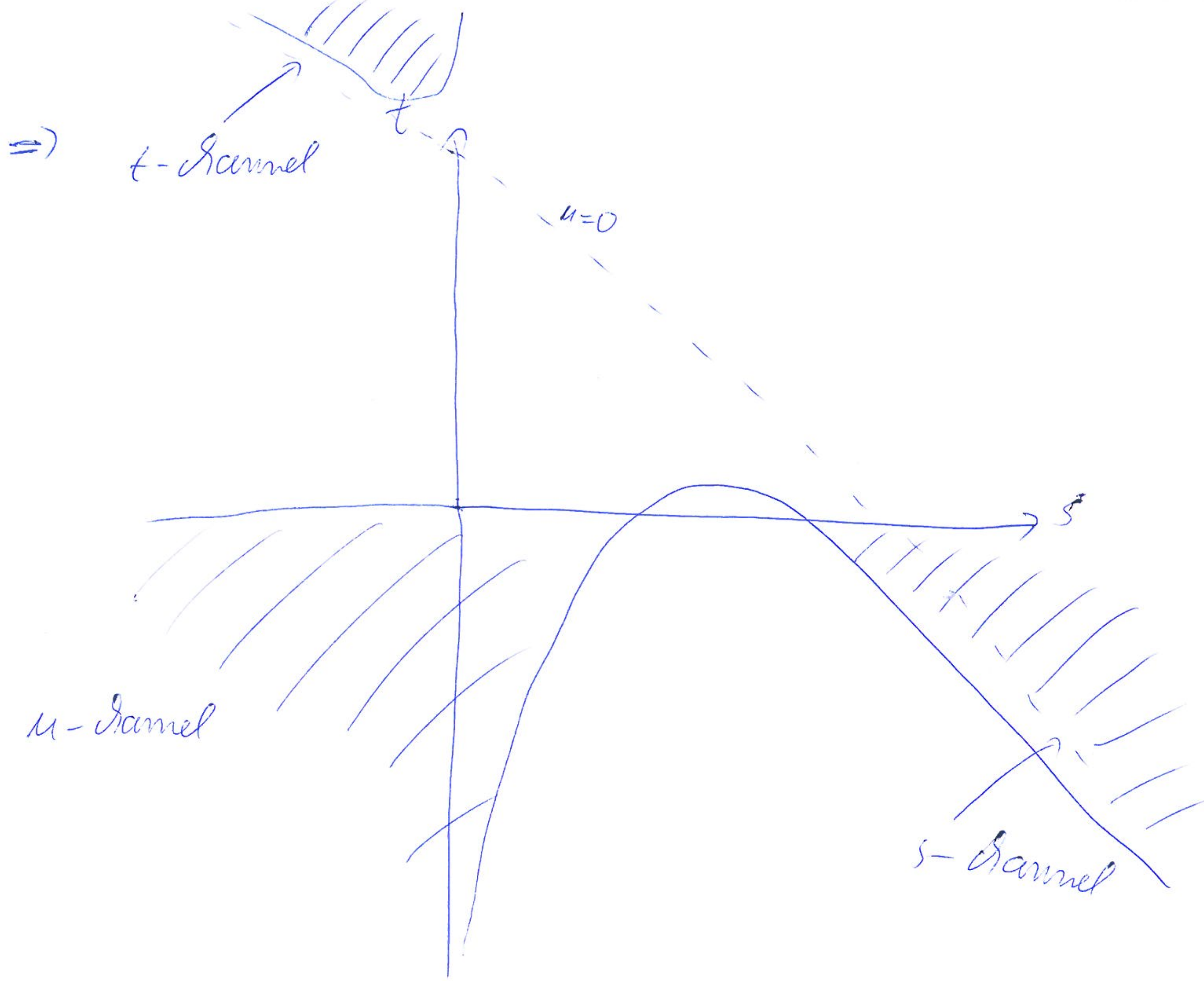
$t > 0$
$s, u \leq 0$

One can continue T analytically in a different regime of s, t, u, i.e. a regime in which $t \geq 0$; $s, u \leq 0$

For practical calculations: call the positive variable s

$$\Rightarrow T^{e^+e^- \mu^+\mu^-}(s, t, u)\Big|_{s \leftrightarrow t} = T^{e^+\mu^- \mu^+ e^-}$$

<u>CROSSING</u>



Remarks:

1) if (S): $\underset{1}{e^-}\,\underset{2}{e^-} \rightarrow \underset{3}{e^-}\,\underset{4}{e^-}$ (Møller-scattering)

$\Rightarrow$ $\underset{1}{e^-}\,\underset{\bar{3}}{e^+} \rightarrow \underset{4}{e^-}\,\underset{\bar{2}}{e^+}$ $\quad s \longleftrightarrow t$ (Bhabha-scattering)

and 3rd possibility

$\underset{1}{e^-}\,\underset{\bar{4}}{e^+} \rightarrow \underset{3}{e^-}\,\underset{\bar{2}}{e^+}$ $\quad s \longleftrightarrow u$

BUT: u-channel is same process as t-channel

$\Rightarrow$ expect $t \leftrightarrow u$ symmetry in T

Result: $|\bar{M}|^2 \propto \left(\frac{s^2+u^2}{t^2} + \frac{2s^2}{tu} + \frac{s^2+t^2}{u^2}\right)$

2) On Interpretation as antiparticles

recall: $j^\mu = -ie\,(\phi^* \partial^\mu \phi - \phi\, \partial^\mu \phi^*)$

from Klein-Gordon eq.

free particle: $\phi \propto N e^{-ipx}$

e^- with $+p$: $j^\mu(e^-) = -2e\,|N|^2 p^\mu = -2e|N|^2\,(E, \vec{p})$

e^+ with $+p$: $j^\mu(e^+) = +2e\,|N|^2 p^\mu = -2e|N|^2\,(-p^\mu)$

$= -2e|N|^2\,(-E, -\vec{p})$

$\Rightarrow \quad j^\mu(e^+) = j^\mu(e^-)\big|_{p^\mu < 0}$

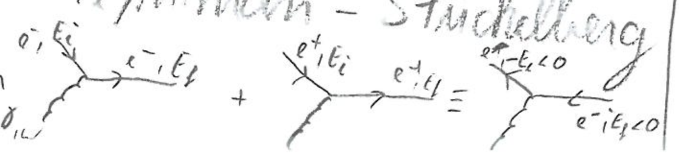
$\Rightarrow$ particle with $-p^\mu \equiv$ antiparticle with $+p^\mu$

emission of positron $+\vec{p}$, $+E$ — absorption of electron $-\vec{p}$, $-E$

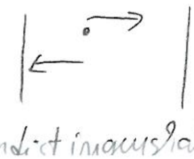
$\longleftrightarrow \quad e^{iEt} = e^{i(-E)(-t)}$

Feynman - Stückelberg

$(-)$ energy conservation in



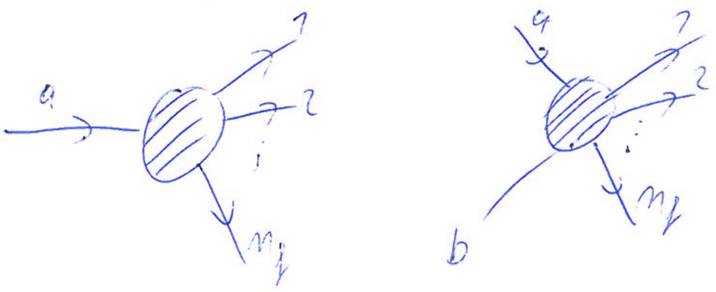
later: propagator with $E<0$ solutions going backwards in t.
[were eliminated in Maxwell theory by boundary conds.]
Causality?



indistinguishable

II) Cross section and phase space

generic problem:



decay scattering

<u>Remarks:</u>

1) $|i\rangle \xrightarrow{} |f\rangle$ Dynamics (later)

2) a) deal with plane waves in box of volume V
[or $\delta(\vec{p}_f - \vec{p}_i)$] [or Volume]

b) $\delta(E_f - E_i) \Rightarrow$ infinite time between $|f\rangle$ and $|i\rangle$

$\boxed{\text{„} V, T \to \infty \text{"}}$ at the end

wave packet approach with localized particles gives same result...

3) introduce $\boxed{\langle f | S | i \rangle \equiv S_{fi}}$ S-Matrix (unitary)

$$S_{fi} = \delta_{fi} - i\,(2\pi)^4\, \delta(P_f - P_i)\, T_{fi}$$

e.g. $=$ 

Transition matrix

II 1) Transition rate, cross section, decay rate

transition rate

$$\boxed{dW_{fi} = \frac{|S_{fi}|^2}{T}\, dN_f}$$

Sum over final states, integrate over all momenta

$$dN_f = \prod_{f=1}^{n_f} \frac{d^3 p_f}{(2\pi)^3}\,\frac{V}{2E_f}$$

number of states in intervall $\vec{p} \to \vec{p} + \delta\vec{p}$ in a box of volume V

normalize free particle wave functions to 2E in V

e.g. $\int d^3x\, \psi\psi^+ = 2E$

$$|S_{fi}|^2 = (2\pi)^8\, |T_{fi}|^2 \left[\delta(P_f - P_i)\right]^2$$

$$[\delta(P_f - P_i)]^2 = \delta(P_f - P_i)\int_{-T/2}^{T/2}\int_{-V/2}^{V/2} dt\, d^3x\, e^{-i(E_f - E_i)t}\, e^{-i(\vec{p}_f - \vec{p}_i)\vec{x}}$$

$$= \delta(P_f - P_i)\underbrace{\int_{-T/2,\,-V/2}^{T/2,\,V/2} dt\, d^3x}_{VT\, =\, \text{,,}\delta(0)\text{''}}$$

after integration over final states

$\longleftrightarrow$ Fermi's Golden Rule

$$(2\pi)^4\,\delta(0) = \int d^4x = VT$$

$$\Rightarrow\; dW_{fi} = (2\pi)^{4-3n_f}\, V^{1+n_f}\, \delta(P_f - P_i)\, |T_{fi}|^2 \prod_{f=1}^{n_f} \frac{d^3 p_f}{2E_f}$$

now define $\displaystyle T_{fi} = \prod_i \frac{1}{\sqrt{2E_i V}} \prod_{f=1}^{n_f} \frac{1}{\sqrt{2E_f V}}\, M_{fi}$

$$\Rightarrow \boxed{dW_{fi} = \frac{V^{1-n_i}}{(2\pi)^{3n_f-4}} \delta(P_f - P_i) |M_{fi}|^2 \prod_i \frac{1}{2E_i} \prod_f \frac{d^3 p_f}{2E_f}}$$

$$\tilde{\vec{p}} = \vec{p}^{\,2} + m^2$$

[use that $\int d^4p\, \delta(p^2 - m^2)\, \Theta(E) = \int d^3p \int dE\, \delta(E^2 - \tilde{\vec{p}}^{\,2})\, \Theta(E)$

$$= \int d^3p \int \frac{dE}{2|\tilde{\vec{p}}|} \left[\delta(E - |\tilde{\vec{p}}|) + \delta(E + |\tilde{\vec{p}}|) \right] \Theta(E)$$

$$= \int \frac{d^3p}{2E} \Rightarrow \text{Lorentz invariant}$$]

A) Decay

$$\boxed{\Gamma = \underbrace{\frac{1}{2E_a}}_{E_a = M} \frac{1}{(2\pi)^{3n_f-4}} \int \frac{d^3p_1}{2E_1} \cdots \frac{d^3p_{n_f}}{2E_{n_f}} |M_{fi}|^2 \delta(P_f - P_a)}$$

and life-time is defined as $\boxed{\tau = \frac{1}{\Gamma}}$

B) Cross section

$$\sigma = \frac{\#\text{ transitions per time}}{\#\text{ of incoming particles per area and time} \equiv \text{flux}}$$

$$\text{flux} = (\text{density of incoming particles}) \times (\text{velocity relative to target})$$

$$= \frac{1}{V} |\vec{v}_a - \vec{v}_b| = \frac{1}{V} \frac{1}{E_a E_b} \sqrt{(p_a \cdot p_b)^2 - m_a^2 m_b^2}$$

(page 12, I2)

$$\Rightarrow \boxed{\sigma = \frac{1}{4\sqrt{(p_a \cdot p_b)^2 - m_a^2 m_b^2}} \underbrace{\frac{1}{(2\pi)^{3n_f - 4}} \int \frac{d^3 p_1}{2E_1} \cdots \frac{d^3 p_{n_f}}{2E_{n_f}} \delta(p_f - p_i)}_{\text{Lorentz-invariant phase space (LIPS)}} |\mathcal{M}_{fi}|^2}$$

Labsystem: page 21: $\prod_i \frac{1}{2E_i} = \frac{1}{2E_1} \frac{1}{2M}$

$$\text{flux} = \frac{1}{V} |\vec{v}_1| = \frac{1}{V} \frac{|\vec{p}_1|}{E_1}$$

e.g. $d\sigma = \frac{1}{2E\,2M} \frac{1}{4\pi^2} |M|^2 \frac{d^3 k_1}{2E_1} \frac{d^3 k_2}{2E_2}$ for negligable incoming mass