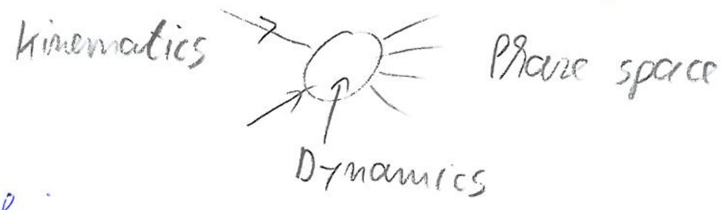
Theoretical Basics of Elementary Particle Physics

Block of 4 theory lectures (longest block...), followed by experimental tests, theoretical details, more exptl. tests. ⇒ 7 lectures

Goal: motivate and learn all tools to calculate elementary processes



Outline:

I) Relativistic Kinematics

I 1) Decay of a particle

I 2) 2-to-2 scattering

I 3) Crossing Symmetry

II] Cross section and phase space

II 1) Transition rate, cross section, decay rate

II 2) 2-to-2 cross section

III] QED, QFT, Feynman rules

III 1) How to describe fermions and photons

III 2) How to propagate fermions and photons

III 3) Feynman rules and elementary processes

I] Relativistic kinematics

1st Reminder: „natural units" $c = \hbar = 1$

$$\Rightarrow [c] = [L][T]^{-1} \overset{!}{=} 1 \Rightarrow [L] = [T]$$

$$[\hbar] = [E][T] \overset{!}{=} 1 \Rightarrow [E] = [T]^{-1} = [L]^{-1}$$

$$\Rightarrow \boxed{[M] = [L]^{-1} = [T]^{-1}}$$

numerics: $\hbar = 6.58 \cdot 10^{-25}$ GeV s

$$\Rightarrow \boxed{1\ \text{GeV}^{-1} \simeq 6.6 \cdot 10^{-25}\ \text{s}}$$

$\left(\tau = \frac{1}{\Gamma}\right)$

$c = 2.99 \cdot 10^{10}$ cm s^{-1}

$$\Rightarrow \boxed{1\ \text{fm} \simeq 5\ \text{GeV}^{-1}}$$

$(f \leftrightarrow 10^{-15})$

e.g. $\lambda_\pi \equiv \frac{\hbar}{m_\pi c} \simeq 1.4$ fm

$\alpha = \frac{e^2}{4\pi\hbar c} = \frac{e^2}{4\pi} \simeq \frac{1}{137}$ (@ low energies...)

cross section $[\sigma] = [L]^2 = [M]^{-2}$

$b = 10^{-24}$ cm^2 „barn"

$$\boxed{\left(\frac{1}{\text{GeV}}\right)^2 = 0.3894\ \text{mb}}$$

e.g. $\sigma_{had} \simeq \frac{1}{m_\pi^2} \simeq 20$ mb

2^{nd} reminder: notation $x^\mu = (x^0 = t, x^1, x^2, x^3)$

$$= (t, \vec{x})$$

$$x_\mu = g_{\mu\nu} x^\nu = (t, -\vec{x})$$

$$g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\tau^2 = t^2 - \vec{x}^2 = g_{\mu\nu} x^\mu x^\nu = x^\mu x_\mu \equiv x^2$$ invariant: $\tau^2 = \tau'^2$

measured by observer moving relative with v

"Eigenzeit" (proper time) $d\tau = dt \sqrt{1 - \left(\frac{d\vec{x}}{dt}\right)^2} \equiv dt/\gamma \quad ; \gamma \geq 1$

4-velocity: $u^\mu = \frac{dx^\mu}{d\tau} = \frac{dx^\mu}{dt} \frac{dt}{d\tau} = \gamma(1, \vec{v})$

$$u^2 = \gamma^2(1 - \vec{v}^2) = 1 > 0$$

always timelike

4-momentum: $p^\mu = m u^\mu = m\gamma(1, \vec{v}) \equiv (p^0, \vec{p})$

$$p^2 = m^2 u^2 = m^2 = E^2 - \vec{p}^2 \; (= p^{0\,2} - \vec{p}^2)$$

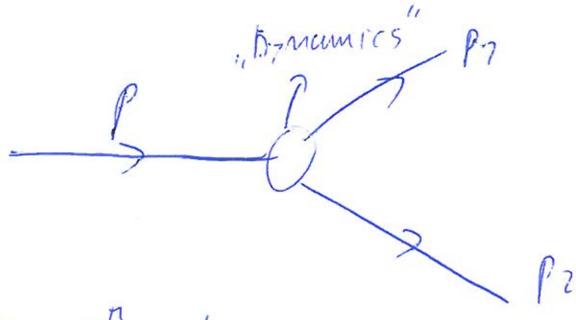
$$E = +\sqrt{\vec{p}^2 + m^2}$$

relativistic for $E \gg m$;

$$\Rightarrow |\vec{p}| \simeq E\left(1 - \frac{m^2}{2E^2}\right)$$

non-relativistic for $\cancel{m} \ll E$ ($p \gg m$); $|E| \simeq m\left(1 + \frac{\vec{p}^2}{2m^2}\right)$

I1) Decay of a particle



$p = p^\mu$ etc

rest system: $p = (M, \vec{0})$

$$d\tau^2 = d\tau'^2 = dt'^2/\gamma^2$$

$$\Rightarrow \boxed{dt' = \gamma\, d\tau \geq d\tau}$$

eg: $\tau(\pi^+ \to \mu + \nu) \simeq 2.6 \cdot 10^{-8}\,s$

@ $E_\pi = 20\,\text{GeV}$: $\gamma = \underbrace{\frac{E_\pi}{m_\pi}}_{\text{factor by which it „lives longer"}} \simeq 143$

$$E_{kin} = E - m = m(\gamma - 1)$$

$$\Rightarrow v^2 = 1 - \frac{1}{(1 + E_{kin}/m)^2}$$

$$\Rightarrow \gamma^2 = \frac{1}{1-v^2} = \left(1 + \frac{E_{kin}}{m}\right)^2$$

$$\Rightarrow \gamma = 1 + \frac{E-m}{m} = \frac{E}{m}$$

OR: $v^2 = \frac{\vec{p}^2}{m^2\gamma^2}$

$$\Rightarrow 1 - \frac{1}{\gamma^2} = \frac{\vec{p}^2}{m^2\gamma^2}$$

$$\Rightarrow \gamma^2 = \frac{\vec{p}^2}{m^2} + 1 = E^2/m^2$$

$p = p_1 + p_2$ energy-momentum conservation

$$p^2 = M^2 \; ; \; p_1^2 = m_1^2 \; ; \; p_2^2 = m_2^2$$

$$p = (M, \vec{0}) \; ; \; p_1 = (E_1, \vec{p}_1) \; ; \; p_2 = (E_2, \vec{p}_2)$$

$$p\, p_i = M E_i \Rightarrow E_i = \frac{p \cdot p_i}{M} = \frac{1}{M}(p_1 \cdot p_i + p_2 \cdot p_i)$$

use: $p_1 \cdot p_2 = \frac{1}{2}\left[(p_1+p_2)^2 - p_1^2 - p_2^2\right]$

$$= \frac{1}{2}(M^2 - m_1^2 - m_2^2)$$

$$\Rightarrow \boxed{E_1 = \frac{1}{2M}(M^2 + m_1^2 - m_2^2) \; ; \; E_2 = \frac{1}{2M}(M^2 + m_2^2 - m_1^2) \\ |\vec{p}_1| = \sqrt{E_1^2 - m_1^2} = |\vec{p}_2| = \frac{1}{2M}\sqrt{\lambda(M^2, m_1^2, m_2^2)}}$$

we have defined: $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$

$$= \left[a - (\sqrt{b} + \sqrt{c})^2\right]\left[a - (\sqrt{b} - \sqrt{c})^2\right]$$

$$= a^2 - 2a(b+c) + (b-c)$$

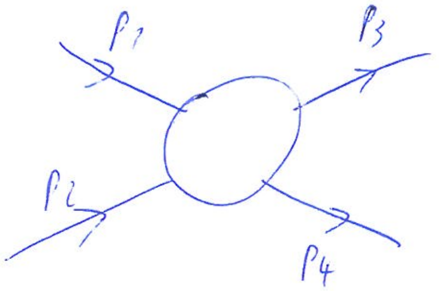
$$\longrightarrow a^2 \text{ for } a \gg b, c$$

$$\longrightarrow a^2 - 4ab \text{ for } b = c$$

$$= a(a - 4b)$$

$\lambda(a,b,c) = \lambda(b,a,c) = \lambda(c,b,a) = \dots$ symmetric

I2) 2-to-2 Scattering



$$p_i^2 = m_i^2$$

$$p_1 + p_2 = p_3 + p_4 \qquad (*)$$

if $m_1 = m_3$; $m_2 = m_4$: elastic scattering

Lorentz-invariant quantities:

$$p_i^2 \; ; \; \underbrace{p_1 \cdot p_2,\ p_1 \cdot p_3,\ p_1 \cdot p_4,\ p_2 \cdot p_3,\ p_2 \cdot p_4,\ p_3 \cdot p_4}$$

$6 - \underset{(*)}{4} \Rightarrow 2$ linearly independent choice of the 2 kinematic variables:

e.g. E_1, Θ_{13} better: Lorentz-invariant quantities

$$\Rightarrow \boxed{\begin{aligned} s &= (p_1+p_2)^2 = (p_3+p_4)^2 \\ t &= (p_1-p_3)^2 = (p_4-p_2)^2 \\ u &= (p_1-p_4)^2 = (p_3-p_2)^2 \end{aligned}}$$

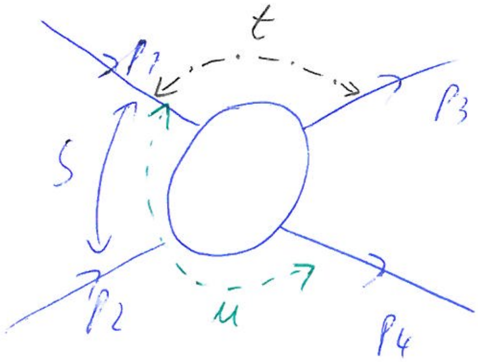
Mandelstam variables

(work with 3 ↔ later: crossing)

$$s + t + u = \sum_{i=1}^{4} m_i^2 \longleftrightarrow$$ 2 independent kinematic variables

s: square of center-of-mass energy

t: square of momentum transfer



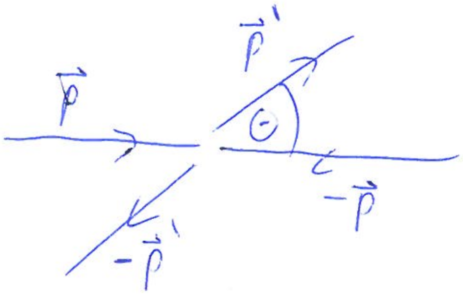
Important reference frames:

1) center-of-mass system: $\vec{p}_1 + \vec{p}_2 = 0 = \vec{p}_3 + \vec{p}_4$
(cms)

2) lab system : $\vec{p}_2 = 0$
(LS)

(3) Breit system : $\vec{p}_1 + \vec{p}_3 = 0$) rare

A) Center-of-mass system



$$p_1 = (E_1 = \sqrt{\vec{p}^{\,2} + m_1^2},\ \vec{p})$$
$$p_2 = (E_2 = \sqrt{\vec{p}^{\,2} + m_2^2},\ -\vec{p})$$
$$p_3 = (E_3 = \sqrt{\vec{p}^{\,\prime 2} + m_3^2},\ \vec{p}\,')$$
$$p_4 = (E_4 = \sqrt{\vec{p}^{\,\prime 2} + m_4^2},\ -\vec{p}\,')$$

$$s = (p_1 + p_2)^2 = (E_1 + E_2)^2 = (E_3 + E_4)^2 \ ;$$

$$p_1 (p_1 + p_2) = p_1 (E_1 + E_2, \vec{0}) = E_1 (E_1 + E_2) = E_1 \sqrt{s} \ ;$$

$$p_1^2 = m_1^2 \ ; \ p_1 \cdot p_2 = \frac{1}{2} \underbrace{(p_1 + p_2)^2}_{s} - \tfrac{1}{2} p_1^2 - \tfrac{1}{2} p_2^2$$

$$\Rightarrow \boxed{E_1 = \frac{1}{2\sqrt{s}}\left(s + m_1^2 - m_2^2\right)}$$

in the same way one finds:

$$\boxed{\begin{aligned} E_2 &= \frac{1}{2\sqrt{s}}\left(s + m_2^2 - m_1^2\right) \\ E_{3,4} &= \frac{1}{2\sqrt{s}}\left(s \pm m_3^2 \mp m_4^2\right) \end{aligned}}$$

$$\Rightarrow \boxed{\begin{aligned} |\vec{p}| &= \sqrt{E_{1,2}^2 - m_{1,2}^2} = \frac{1}{2\sqrt{s}}\sqrt{\lambda(s, m_1^2, m_2^2)} \\ |\vec{p}\,'| &= \sqrt{E_{3,4}^2 - m_{3,4}^2} = \frac{1}{2\sqrt{s}}\sqrt{\lambda(s, m_3^2, m_4^2)} \end{aligned}}$$

$$\boxed{|\vec{v}_i| = \frac{|\vec{p}_i|}{m_i \gamma} = \frac{|\vec{p}_i|}{m_i \, E_i/m_i} = \frac{\vec{p}_i}{E_i}}$$

also:

$$\boxed{\cos\Theta = \frac{s(t-u) + (m_1^2 - m_2^2)(m_3^2 - m_4^2)}{\sqrt{\lambda(s, m_1^2, m_2^2)\,\lambda(s, m_3^2, m_4^2)}}}$$ |Exercise|

(asymptotic behavior: $s >> m_i^2$: $E_i \simeq \frac{\sqrt{s}}{2} \simeq |\vec{p}| \simeq |\vec{p}\,'|$)

physical region: e.g. $t = (p_1 - p_3)^2 = f(\cos\Theta)$

$\Rightarrow t_{min,max}$ from $\cos\Theta = \pm 1$

e.g. $t \in [-s, 0]$ for $m_i = 0$

aus $|\vec{p}|, |\vec{p}'|^2 \geq 0$: $\quad s_{min} = \max\{(m_1+m_2)^2, (m_3+m_4)^2\}$

„threshold"

•) <u>Exampl elastic scattering</u> $\quad m_1 = m_3 ; m_2 = m_4$ (e.g. $\pi N \to \pi N$, $e\mu \to e\mu$)

$$\Rightarrow \quad E_1 = E_3 \quad ; E_2 = E_4$$

$$|\vec{p}| = |\vec{p}'| = \frac{1}{2\sqrt{s}}\sqrt{\lambda(s, m_1^2, m_2^2)}$$

$$t = (p_1 - p_3)^2 = -(\vec{p}_1 - \vec{p}_3)^2 = -2\vec{p}^{\,2}(1-\cos\Theta)$$

$$\Rightarrow \quad \boxed{-4\vec{p}^{\,2} \leq t \leq 0}$$

$\Theta = \pi$ backward $\qquad$ $\Theta = 0$ forward

$$u = (p_1 - p_4)^2 = (E_1 - E_4, \vec{p}_1 - \vec{p}_4)^2 =$$

$$= \underbrace{(E_1 - E_4)^2}_{\left[\frac{1}{\sqrt{s}}(m_1^2 - m_2^2)\right]^2} - \underbrace{(\vec{p}_1 + \vec{p}_4)^2}_{\left[2\vec{p}^{\,2}(1+\cos\Theta)\right]}$$

$\Rightarrow$

$\Rightarrow$ maximum of t when: $t = 0$ $(\theta = 0)$

$$u = 2m_1^2 + 2m_2^2 - s$$

minimum of t when: $t = -\dfrac{\lambda(s, m_1^2, m_2^2)}{s} = -4\vec{p}^{\,2}$ $(\theta = \pi)$

$$u = \frac{(m_1^2 - m_2^2)^2}{s}$$