

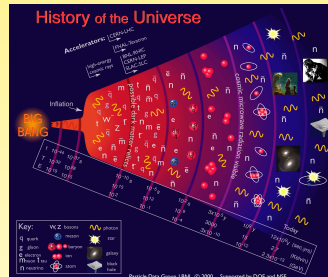
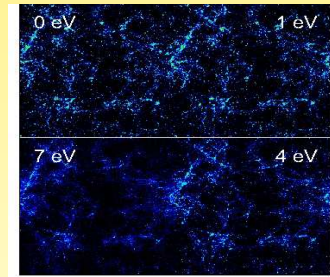
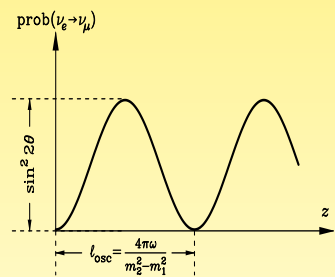
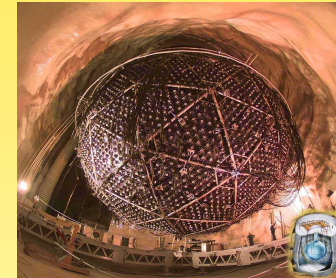
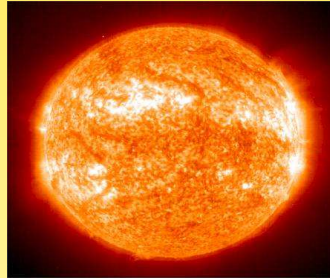
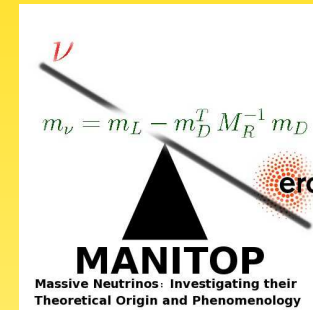
Introduction to Neutrino (Oscillation) Physics



MAX-PLANCK-INSTITUT
FÜR KERNPHYSIK

STANDARD MODEL
OF PARTICLE PHYSICS
RODEJOHANN/SCHOENING

17/07/13



Literature

- ArXiv:
 - Bilenky, Giunti, Grimus: *Phenomenology of Neutrino Oscillations*, hep-ph/9812360
 - Akhmedov: *Neutrino Physics*, hep-ph/0001264
 - Grimus: *Neutrino Physics – Theory*, hep-ph/0307149
- Textbooks:
 - Fukugita, Yanagida: *Physics of Neutrinos and Applications to Astrophysics*
 - Kayser: *The Physics of Massive Neutrinos*
 - Giunti, Kim: *Fundamentals of Neutrino Physics and Astrophysics*
 - Schmitz: *Neutrino Physik*

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I Basics

- 1) Introduction
- 12) History of the neutrino
- 13) Fermion mixing, neutrinos and the Standard Model

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I Basics

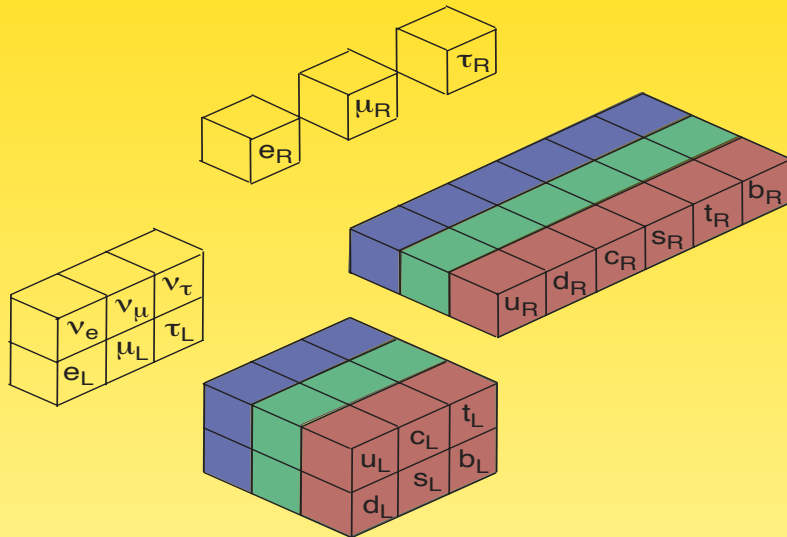
1) Introduction

12) History of the neutrino

13) Fermion mixing, neutrinos and the Standard Model

I1) Introduction

Standard Model of Elementary Particle Physics: $SU(3)_C \times SU(2)_L \times U(1)_Y$



Species	#	Σ
Quarks	10	10
Leptons	3	13
Charge	3	16
Higgs	2	18

18 free parameters...

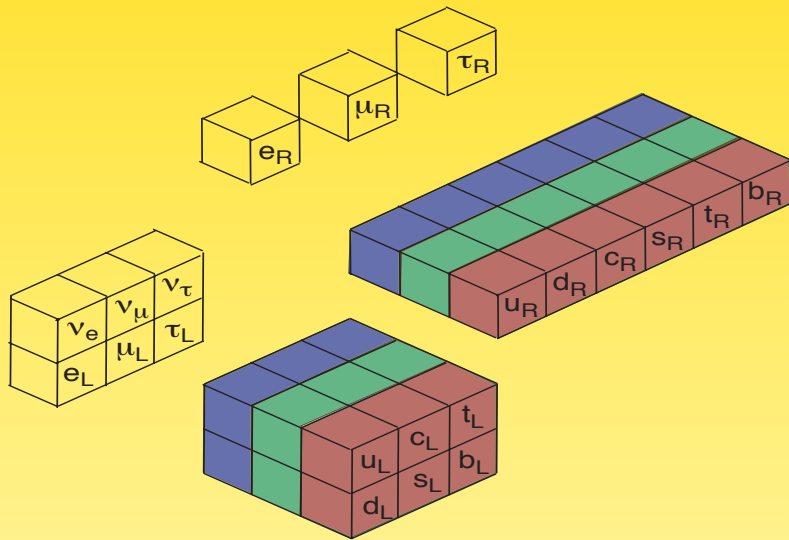
+ Dark Matter

+ Gravitation

+ Dark Energy

+ **Baryon Asymmetry**

Standard Model of Elementary Particle Physics: $SU(3)_C \times SU(2)_L \times U(1)_Y$



Species	#	Σ
Quarks	10	10
Leptons	3	13
Charge	3	16
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+ Neutrino Mass m_ν

Standard Model* of Particle Physics

add neutrino mass matrix m_ν (and a new energy scale?)

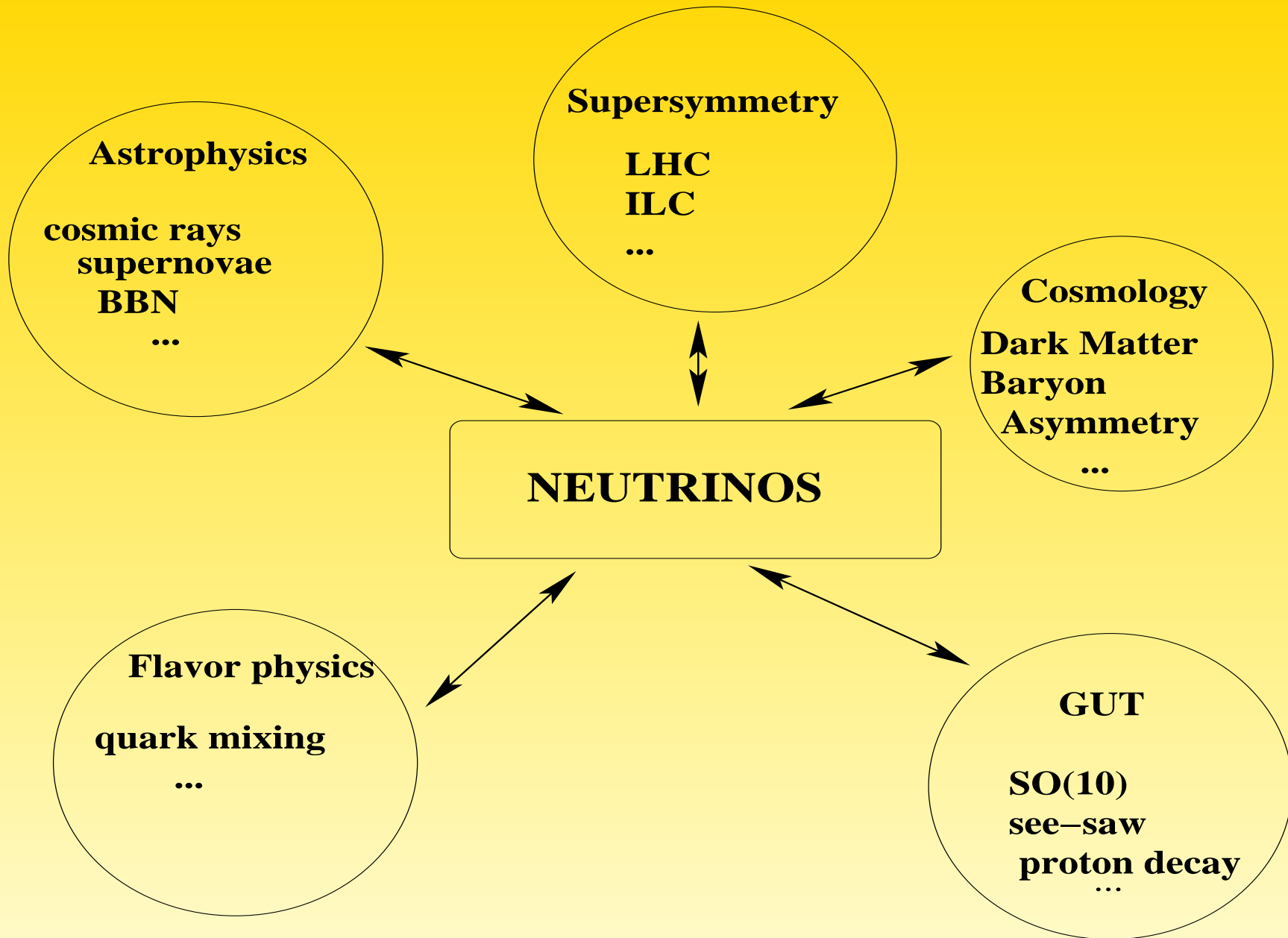
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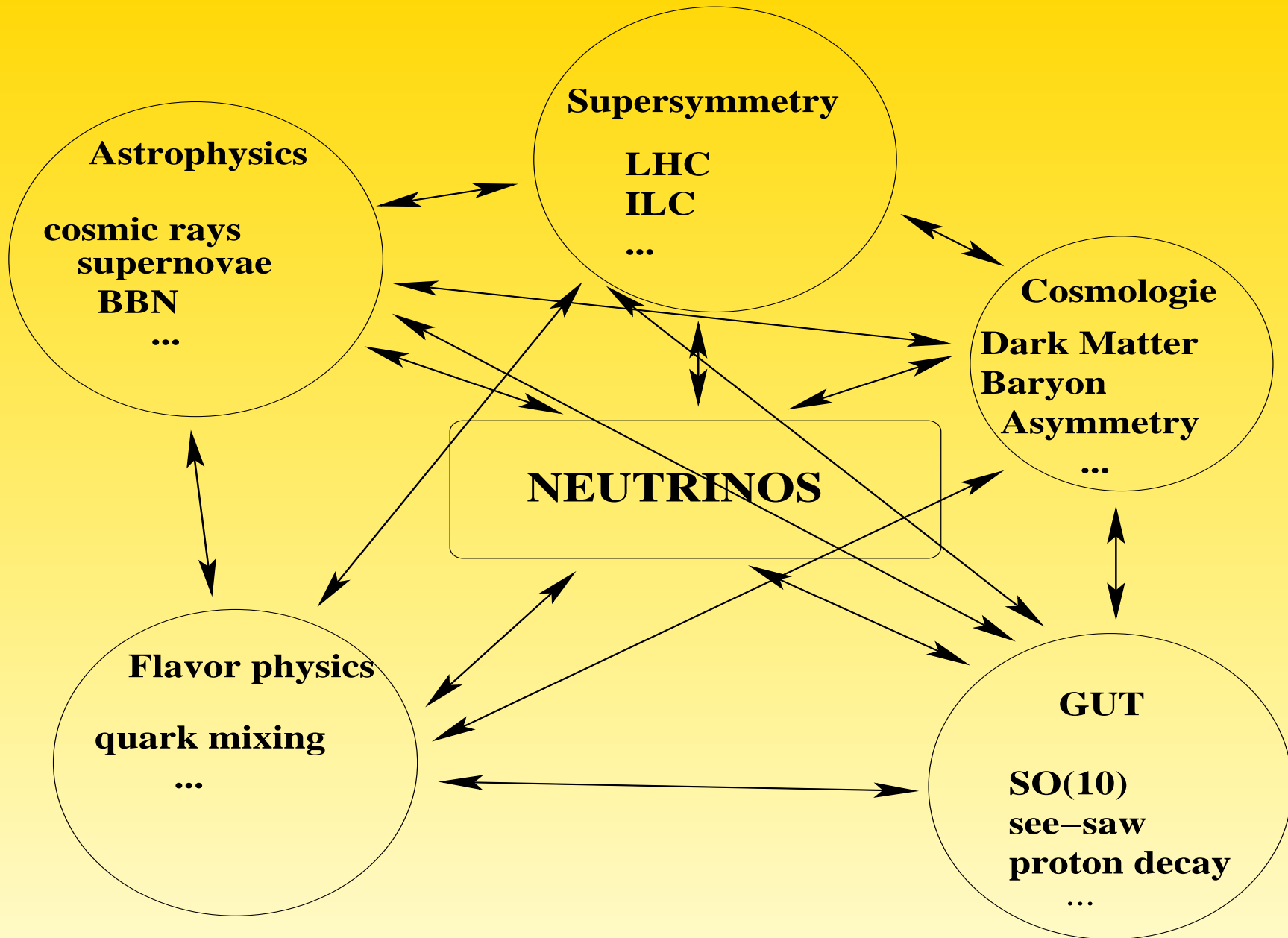
Standard Model* of Particle Physics

add neutrino mass matrix m_ν (and a new energy scale?)

Species	#	Σ		Species	#	Σ
Quarks	10	10		Quarks	10	10
Leptons	3	13	→	Leptons	12 (10)	22 (20)
Charge	3	16		Charge	3	25 (23)
Higgs	2	18		Higgs	2	27 (25)

Two roads towards more understanding: Higgs and **Flavor**





General Remarks

- Neutrinos interact weakly: can probe things not testable by other means
 - solar interior
 - geo-neutrinos
 - cosmic rays
- Neutrinos have no mass in SM
 - probe scales $m_\nu \propto 1/\Lambda$
 - happens in GUTs
 - connected to new concepts, e.g. Lepton Number Violation

⇒ particle and source physics

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12) History

1926 problem in spectrum of β -decay

1930 Pauli postulates "neutron"

Mythos. Photographie of Pauli 0393
Abschrift/15.12.30 PM

Offener Brief an die Gruppe der Radioaktiven bei der
Gesellschafts-Tagung zu Tübingen.

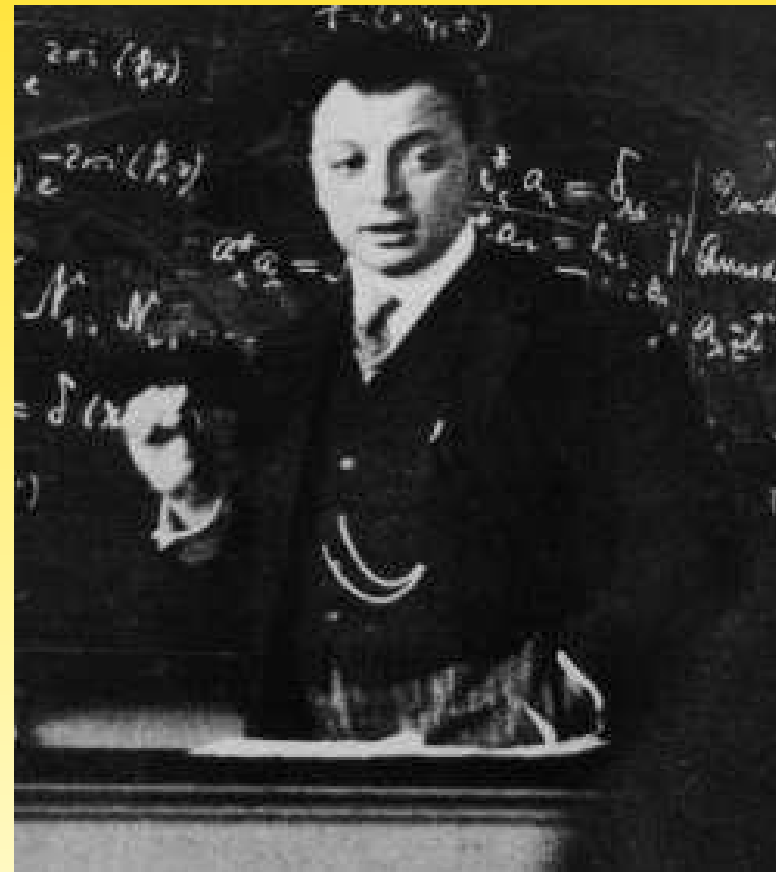
Abschrift

Physikalisches Institut
der Eidg. Technischen Hochschule
Zürich

Zürich, 4. Dec. 1930
Ulrichstrasse

Liebe Radioaktive Damen und Herren,

Wie der Ueberbringer dieser Zeilen, den ich huldvollst
anzuhören bitte, Ihnen des näheren auseinandersetzen wird, bin ich
angesichts der "falschen" Statistik der N - und $Li-6$ Kerne, sowie
des kontinuierlichen beta-Spektrums auf einen verzweifelten Ausweg
verfallen um den "Wechselatz" (1) der Statistik und den Energienatz
zu retten. Nämlich die Möglichkeit, es könnten elektrisch neutrale
Teilchen, die ich Neutronen nennen will, in den Kernen existieren,
welche den Spin $1/2$ haben und das Ausschliessungsprinzip befolgen und
sich von Lichtquanten ausserdem noch dadurch unterscheiden, dass sie
sich mit Lichtgeschwindigkeit laufen. Die Masse der Neutronen
würde von derselben Grössenordnung wie die Elektronenmasse sein und
jedenfalls nicht grösser als $0,01$ Protonenmasse. Das kontinuierliche
beta-Spektrum wäre dann verständlich unter der Annahme, dass beim
beta-Zerfall mit dem Elektron jeweils noch ein Neutron emittiert
wird, derart, dass die Summe der Energien von Neutron und Elektron
konstant ist.



- 1932 Fermi theory of β -decay
- 1956 discovery of $\bar{\nu}_e$ by Cowan and Reines (NP 1985)
- 1957 Pontecorvo suggests neutrino oscillations
- 1958 helicity $h(\nu_e) = -1$ by Goldhaber $\Rightarrow V - A$
- 1962 discovery of ν_μ by Lederman, Steinberger, Schwartz (NP 1988)
- 1970 first discovery of solar neutrinos by Ray Davis (NP 2002); solar neutrino problem
- 1987 discovery of neutrinos from SN 1987A (Koshiba, NP 2002)
- 1991 $N_\nu = 3$ from invisible Z width
- 1998 SuperKamiokande shows that atmospheric neutrinos oscillate
- 2000 discovery of ν_τ
- 2002 SNO solves solar neutrino problem
- 2010 the third mixing angle

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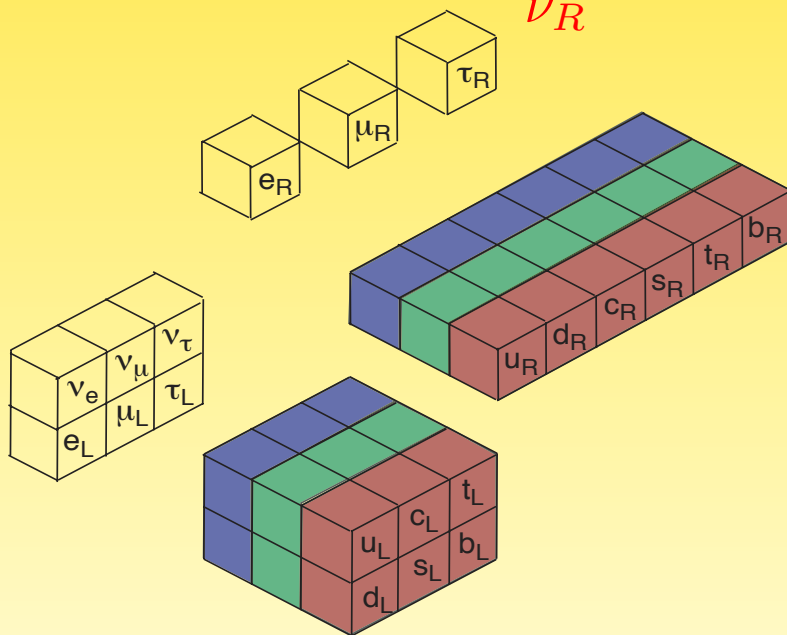
13) Neutrinos and the Standard Model

$$SU(3)_c \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_c \times U(1)_{em} \text{ with } Q = I_3 + \frac{1}{2} Y$$

$$L_e = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \sim (1, 2, -1)$$

$$e_R \sim (1, 1, -2)$$

$$\nu_R \sim (1, 1, 0) \quad \text{total SINGLET!!}$$



Mass Matrices

3 generations of quarks

$$L'_1 = \begin{pmatrix} u' \\ d' \end{pmatrix}_L, \quad L'_2 = \begin{pmatrix} c' \\ s' \end{pmatrix}_L, \quad L'_3 = \begin{pmatrix} t' \\ b' \end{pmatrix}_L$$

$$u'_R, c'_R, t'_R \equiv u'_{i,R} \quad \text{and} \quad d'_R, s'_R, b'_R \equiv d'_{i,R}$$

gives mass term

$$\begin{aligned} -\mathcal{L}_Y &= \sum_{i,j} \overline{L'_i} \left[g_{ij}^{(d)} \Phi d'_{j,R} + g_{ij}^{(u)} \tilde{\Phi} u'_{j,R} \right] \\ &\xrightarrow{\text{EWSB}} \sum_{i,j} \frac{v}{\sqrt{2}} g_{ij}^{(d)} \overline{d'_{i,L}} d'_{j,R} + \frac{v}{\sqrt{2}} g_{ij}^{(u)} \overline{u'_{i,L}} u'_{j,R} \\ &= \overline{d'_L} M^{(d)} d'_R + \overline{u'_L} M^{(u)} u'_R \end{aligned}$$

arbitrary complex 3×3 matrices in “flavor (interaction, weak) basis”

Diagonalization

$$U_d^\dagger M^{(d)} V_d = D^{(d)} = \text{diag}(m_d, m_s, m_b)$$

$$U_u^\dagger M^{(u)} V_u = D^{(u)} = \text{diag}(m_u, m_c, m_t)$$

with unitary matrices $U_{u,d} U_{u,d}^\dagger = U_{u,d}^\dagger U_{u,d} = V_{u,d} V_{u,d}^\dagger = V_{u,d}^\dagger V_{u,d} = \mathbb{1}$

in Lagrangian:

$$-\mathcal{L}_Y = \overline{d'_L} M^{(d)} d'_R + \overline{u'_L} M^{(u)} u'_R$$

$$= \underbrace{\overline{d'_L} U_d}_{\overline{d_L}} \underbrace{U_d^\dagger M^{(d)} V_d}_{D^{(d)}} \underbrace{V_d^\dagger d'_R}_{d_R} + \underbrace{\overline{u'_L} U_u}_{\overline{u_L}} \underbrace{U_u^\dagger M^{(u)} V_u}_{D^{(u)}} \underbrace{V_u^\dagger u'_R}_{u_R}$$

physical (mass, propagation) states $u_L = \begin{pmatrix} u \\ c \\ t \end{pmatrix}_L$

in interaction terms:

$$-\mathcal{L}_{\text{CC}} = \frac{g}{\sqrt{2}} W_{\mu}^{+} \overline{u'_L} \gamma^{\mu} d'_L$$

$$= \frac{g}{\sqrt{2}} W_{\mu}^{+} \underbrace{\overline{u'_L} U_u}_{\overline{u_L}} \gamma^{\mu} \underbrace{U_u^{\dagger} U_d}_V \underbrace{U_d^{\dagger} d'_L}_{d_L}$$

Cabibbo-Kobayashi-Maskawa (CKM) matrix survives:

$$V = U_u^{\dagger} U_d$$

Structure in Wolfenstein-parametrization:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A \lambda^3 (\rho - i \eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A \lambda^2 \\ A \lambda^3 (1 - \rho - i \eta) & -A \lambda^2 & 1 \end{pmatrix}$$

with $\lambda = \sin \theta_C = 0.22535 \pm 0.00065$, $A = 0.811_{-0.012}^{+0.022}$,

$\bar{\rho} = (1 - \frac{\lambda^2}{2}) \rho = 0.131_{-0.013}^{+0.026}$, $\bar{\eta} = 0.345_{-0.014}^{+0.013}$

Lesson to learn:

$$|V| = \begin{pmatrix} 0.97427 \pm 0.00015 & 0.22534 \pm 0.000065 & 0.00351^{+0.00015}_{-0.00014} \\ 0.22520 \pm 0.000065 & 0.97344 \pm 0.00016 & 0.0412^{+0.0011}_{-0.0005} \\ 0.00867^{+0.00029}_{-0.00031} & 0.0404^{+0.0011}_{-0.0005} & 0.999146^{+0.000021}_{-0.000046} \end{pmatrix}$$

small mixing in the quark sector
related to hierarchy of masses?

$$M = \begin{pmatrix} 0 & a \\ a & b \end{pmatrix} = U D U^T \quad \text{with } U = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$$

where $D = \text{diag}(m_1, m_2)$

from 11-entry one gets

$$\tan \theta = \sqrt{\frac{m_1}{m_2}}$$

compare with $\sqrt{m_d/m_s} \simeq 0.22$ and $\tan \theta_C \simeq 0.23$

Number of parameters in V for N families:

$$\begin{array}{l}
 \text{complex } N \times N \\
 \text{unitarity} \\
 \text{rephase } u_i, d_i
 \end{array}
 \begin{array}{c}
 2 N^2 \\
 -N^2 \\
 -(2 N - 1)
 \end{array}
 \left| \begin{array}{c}
 2 N^2 \\
 N^2 \\
 (N - 1)^2
 \end{array} \right.$$

a real matrix would have $\frac{1}{2} N (N - 1)$ rotations around ij -axes

in total:

families	angles	phases
2	1	0
3	3	1
4	6	3
N	$\frac{1}{2} N (N - 1)$	$\frac{1}{2} (N - 2) (N - 1)$

Masses in the SM:

$$-\mathcal{L}_Y = g_e \bar{L} \Phi e_R + g_\nu \bar{L} \tilde{\Phi} \nu_R + h.c.$$

with

$$L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \text{and} \quad \tilde{\Phi} = i\tau_2 \Phi^* = i\tau_2 \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}^* = \begin{pmatrix} \phi^0 \\ -\phi^+ \end{pmatrix}^*$$

after EWSB: $\langle \Phi \rangle \rightarrow (0, v/\sqrt{2})^T$ and $\langle \tilde{\Phi} \rangle \rightarrow (v/\sqrt{2}, 0)^T$

$$-\mathcal{L}_Y = g_e \frac{v}{\sqrt{2}} \bar{e}_L e_R + g_\nu \frac{v}{\sqrt{2}} \bar{\nu}_L \nu_R + h.c. \equiv m_e \bar{e}_L e_R + m_\nu \bar{\nu}_L \nu_R + h.c.$$

\Leftrightarrow in a renormalizable, lepton number conserving model with Higgs doublets the absence of ν_R means absence of m_ν

Lepton Masses

$$\begin{aligned}
 -\mathcal{L}_Y &= \overline{e'_L} M^{(\ell)} e'_R \\
 &= \underbrace{\overline{e'_L} U_\ell}_{\overline{e_L}} \underbrace{U_\ell^\dagger M^{(\ell)} V_\ell}_{D^{(\ell)}} \underbrace{V_\ell^\dagger e'_R}_{e_R}
 \end{aligned}$$

and in charged current term:

$$\begin{aligned}
 -\mathcal{L}_{CC} &= \frac{g}{\sqrt{2}} W_\mu^+ \overline{e'_L} \gamma^\mu \nu'_L \\
 &= \frac{g}{\sqrt{2}} W_\mu^+ \underbrace{\overline{e'_L} U_\ell}_{\overline{e_L}} \gamma^\mu \underbrace{U_\ell^\dagger U_\nu}_U \underbrace{U_\nu^\dagger \nu'_L}_{\nu_L}
 \end{aligned}$$

Rotation of ν_L is arbitrary in absence of m_ν : choose $U_\nu = U_\ell$

\Rightarrow Pontecorvo-Maki-Nakagawa-Saki (PMNS) matrix

$U = \mathbb{1}$ for massless neutrinos!!

\Rightarrow individual lepton numbers L_e, L_μ, L_τ are conserved

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II1) **The PMNS matrix**

II2) **Neutrino oscillations in vacuum**

II3) **Results and their interpretation – what have we learned?**

II4) **Prospects – what do we want to know?**

III1) The PMNS matrix

Neutrinos have mass, so:

$$-\mathcal{L}_{\text{CC}} = \frac{g}{\sqrt{2}} \bar{\ell}_L \gamma^\mu U \nu_L W_\mu^- \quad \text{with } U = U_\ell^\dagger U_\nu$$

Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix

$$\nu_\alpha = U_{\alpha i}^* \nu_i$$

connects flavor states ν_α ($\alpha = e, \mu, \tau$) to mass states ν_i ($i = 1, 2, 3$)

Number of parameters in U for N families:

complex $N \times N$	$2 N^2$	$2 N^2$
unitarity	$-N^2$	N^2
rephase ν_i, ℓ_i	$-(2 N - 1)$	$(N - 1)^2$

a real matrix would have $\frac{1}{2} N (N - 1)$ rotations around ij -axes

in total:

families	angles	phases
2	1	0
3	3	1
4	6	3
N	$\frac{1}{2} N (N - 1)$	$\frac{1}{2} (N - 2) (N - 1)$

this assumes $\bar{\nu}\nu$ mass term, what if $\nu^T \nu$?

Number of parameters in U for N families:

$$\begin{array}{r|l}
 \text{complex } N \times N & 2N^2 \\
 \text{unitarity} & -N^2 \\
 \text{rephase } \ell_\alpha & -N
 \end{array}
 \quad
 \begin{array}{l}
 2N^2 \\
 N^2 \\
 N(N-1)
 \end{array}$$

a real matrix would have $\frac{1}{2} N (N - 1)$ rotations around ij -axes

in total:

families	angles	phases	extra phases
2	1	1	1
3	3	3	2
4	6	6	3
N	$\frac{1}{2} N (N - 1)$	$\frac{1}{2} N (N - 1)$	$N - 1$

Extra $N - 1$ “Majorana phases” because of mass term $\nu^T \nu$

(absent for Dirac neutrinos)

Majorana Phases

- connected to Majorana nature, hence to Lepton Number Violation
- I can always write: $U = \tilde{U} P$, where all Majorana phases are in $P = \text{diag}(1, e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}, \dots)$:
- 2 families:

$$U = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\alpha} \end{pmatrix}$$

- 3 families: $U = R_{23} \tilde{R}_{13} R_{12} P$

$$\begin{aligned}
 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} P \\
 &= \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{i\delta} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{-i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{-i\delta} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{-i\delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{-i\delta} & c_{23} c_{13} \end{pmatrix} P
 \end{aligned}$$

with $P = \text{diag}(1, e^{i\alpha}, e^{i\beta})$

Dirac vs. Majorana neutrinos

why are neutrinos (probably) Majorana, why is their mass so small, how do I show that neutrinos are Majorana, what does this mean for physics beyond the Standard Model,...

See next term:-)

- MVSpec Standard Model II (Rodejohann + Lindner)
- MVSem Astroparticle Physics: Theory and Experiment (Rodejohann et al.)

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II Neutrino Oscillations

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II2) Neutrino Oscillations in Vacuum

Neutrino produced with charged lepton α is **flavor state**

$$|\nu(0)\rangle = |\nu_\alpha\rangle = U_{\alpha j}^* |\nu_j\rangle$$

evolves with time as

$$|\nu(t)\rangle = U_{\alpha j}^* e^{-i E_j t} |\nu_j\rangle$$

amplitude to find state $|\nu_\beta\rangle = U_{\beta i}^* |\nu_i\rangle$:

$$\begin{aligned} \mathcal{A}(\nu_\alpha \rightarrow \nu_\beta, t) &= \langle \nu_\beta | \nu(t) \rangle = U_{\beta i} U_{\alpha j}^* e^{-i E_j t} \underbrace{\langle \nu_i | \nu_j \rangle}_{\delta_{ij}} \\ &= U_{\alpha i}^* U_{\beta i} e^{-i E_i t} \end{aligned}$$

Probability:

$$\begin{aligned}
 P(\nu_\alpha \rightarrow \nu_\beta, t) &\equiv P_{\alpha\beta} = |\mathcal{A}(\nu_\alpha \rightarrow \nu_\beta, t)|^2 \\
 &= \sum_{ij} \underbrace{U_{\alpha i}^* U_{\beta i} U_{\beta j}^* U_{\alpha j}}_{\mathcal{J}_{ij}^{\alpha\beta}} \underbrace{e^{-i(E_i - E_j)t}}_{e^{-i\Delta_{ij}}} \\
 &= \dots =
 \end{aligned}$$

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4 \sum_{j>i} \text{Re}\{\mathcal{J}_{ij}^{\alpha\beta}\} \sin^2 \frac{\Delta_{ij}}{2} + 2 \sum_{j>i} \text{Im}\{\mathcal{J}_{ij}^{\alpha\beta}\} \sin \Delta_{ij}$$

with phase

$$\begin{aligned}
 \frac{1}{2}\Delta_{ij} &= \frac{1}{2} (E_i - E_j) t \simeq \frac{1}{2} \left(\sqrt{p_i^2 + m_i^2} - \sqrt{p_j^2 + m_j^2} \right) L \\
 &\simeq \frac{1}{2} \left(p_i \left(1 + \frac{m_i^2}{2p_i^2} \right) - p_j \left(1 + \frac{m_j^2}{2p_j^2} \right) \right) L \simeq \frac{m_i^2 - m_j^2}{4E} L
 \end{aligned}$$

$$\frac{1}{2}\Delta_{ij} = \frac{m_i^2 - m_j^2}{4E} L \simeq 1.27 \left(\frac{\Delta m_{ij}^2}{\text{eV}^2} \right) \left(\frac{L}{\text{km}} \right) \left(\frac{\text{GeV}}{E} \right)$$

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4 \sum_{j>i} \text{Re}\{\mathcal{J}_{ij}^{\alpha\beta}\} \sin^2 \frac{\Delta_{ij}}{2} + 2 \sum_{j>i} \text{Im}\{\mathcal{J}_{ij}^{\alpha\beta}\} \sin \Delta_{ij}$$

- $\alpha = \beta$: survival probability
- $\alpha \neq \beta$: transition probability
- requires $U \neq \mathbb{1}$ and $\Delta m_{ij}^2 \neq 0$
- $\sum_{\alpha} P_{\alpha\beta} = 1 \leftrightarrow$ conservation of probability
- $\mathcal{J}_{ij}^{\alpha\beta}$ invariant under $U_{\alpha j} \rightarrow e^{i\phi_{\alpha}} U_{\alpha j} e^{i\phi_j}$
 \Rightarrow Majorana phases drop out!

CP Violation

In oscillation probabilities: $U \rightarrow U^*$ for anti-neutrinos

Define asymmetries:

$$\begin{aligned}\Delta_{\alpha\beta} &= P(\nu_\alpha \rightarrow \nu_\beta) - P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) = P(\nu_\alpha \rightarrow \nu_\beta) - P(\nu_\beta \rightarrow \nu_\alpha) \\ &= 4 \sum_{j>i} \text{Im}\{\mathcal{J}_{ij}^{\alpha\beta}\} \sin \Delta_{ij}\end{aligned}$$

- 2 families: U is real and $\text{Im}\{\mathcal{J}_{ij}^{\alpha\beta}\} = 0 \forall \alpha, \beta, i, j$
- 3 families:

$$\Delta_{e\mu} = -\Delta_{e\tau} = \Delta_{\mu\tau} = \left(\sin \frac{\Delta m_{21}^2}{2E} L + \sin \frac{\Delta m_{32}^2}{2E} L + \sin \frac{\Delta m_{13}^2}{2E} L \right) J_{\text{CP}}$$

$$\text{where } J_{\text{CP}} = \text{Im} \{ U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^* \}$$

$$= \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta$$

vanishes for one $\Delta m_{ij}^2 = 0$ or one $\theta_{ij} = 0$ or $\delta = 0, \pi$

- CP violation in survival probabilities vanishes:

$$P(\nu_\alpha \rightarrow \nu_\alpha) - P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\alpha) \propto \sum_{j>i} \text{Im}\{\mathcal{J}_{ij}^{\alpha\alpha}\} = \sum_{j>i} \text{Im}\{U_{\alpha i}^* U_{\alpha i} U_{\alpha j}^* U_{\alpha j}\} = 0$$

- Recall that $U = U_\ell^\dagger U_\nu$

If charged lepton masses diagonal, then m_ν is diagonalized by PMNS matrix:

$$m_\nu = U \text{diag}(m_1, m_2, m_3) U^T$$

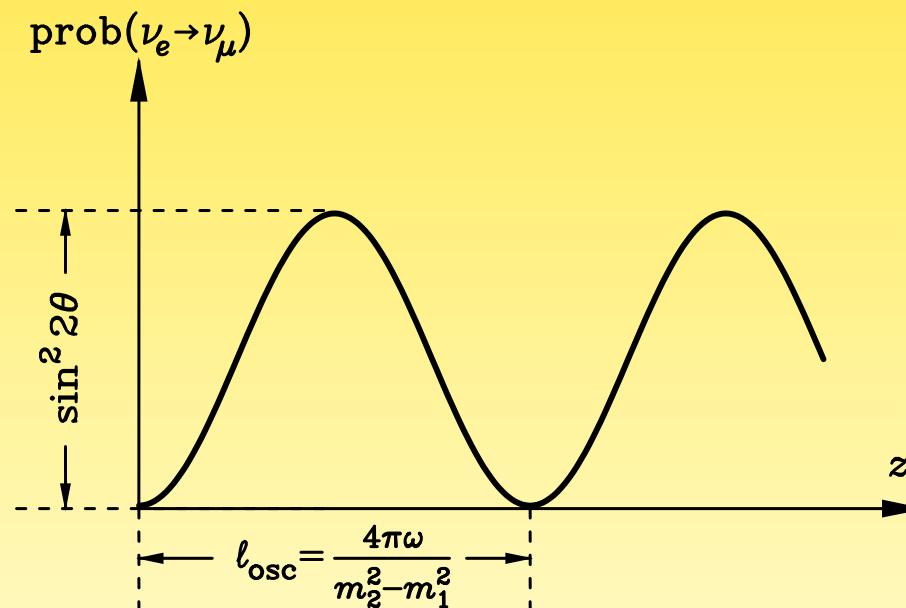
Define $h = m_\nu m_\nu^\dagger$ and find that

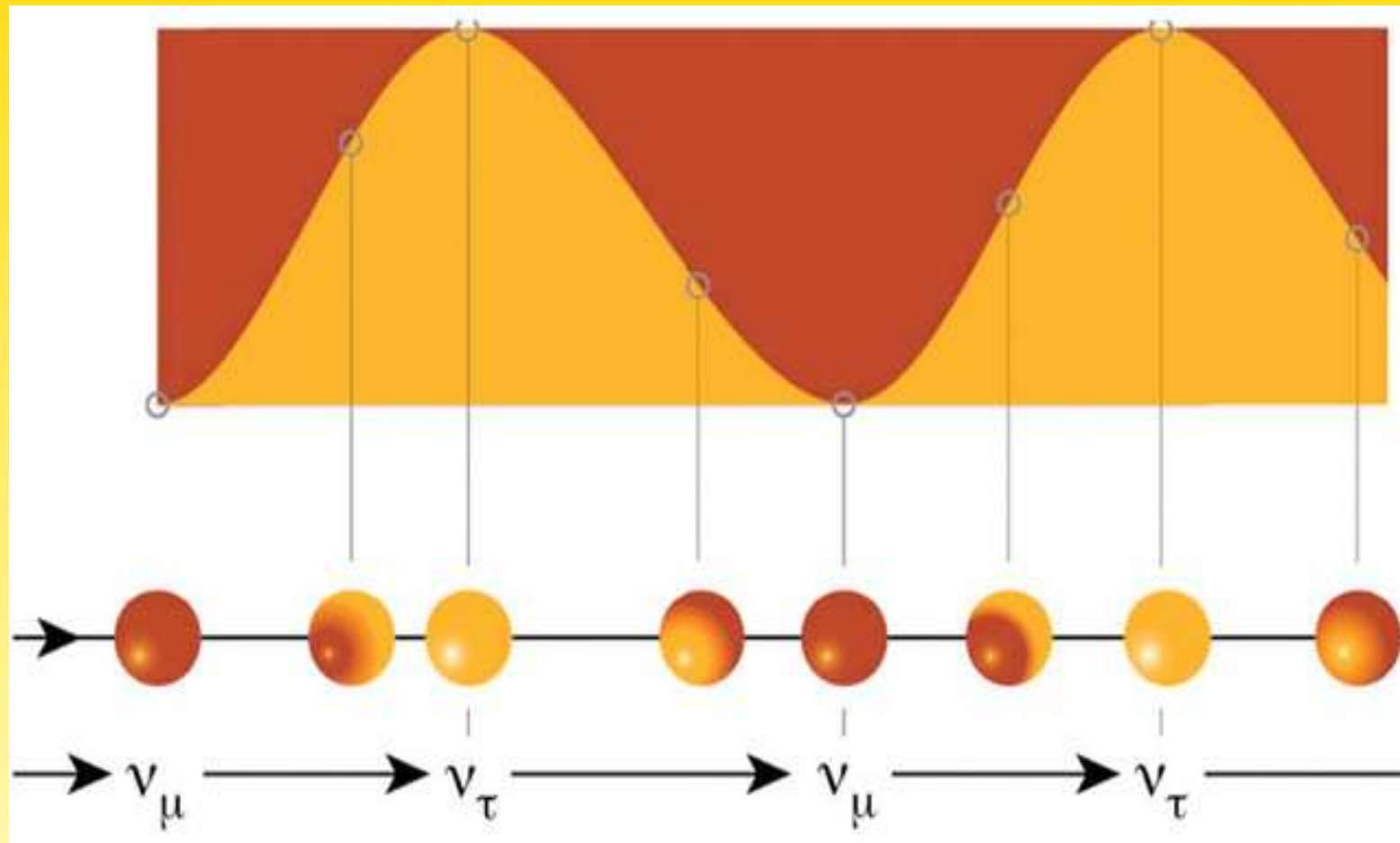
$$\text{Im}\{h_{12} h_{23} h_{31}\} = \Delta m_{21}^2 \Delta m_{31}^2 \Delta m_{32}^2 J_{\text{CP}}$$

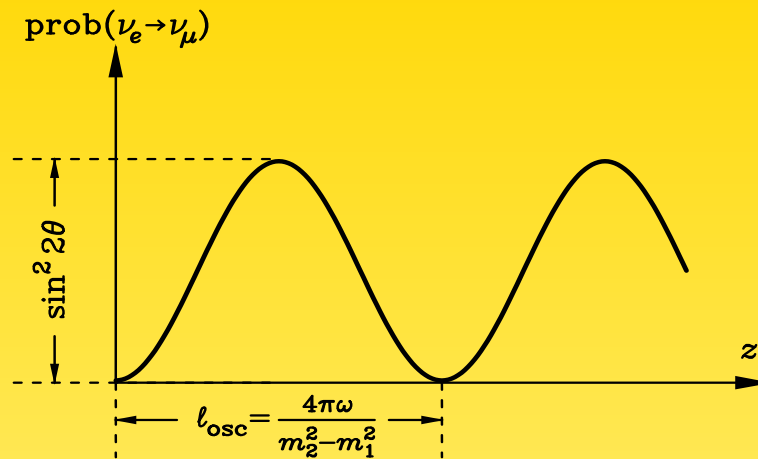
Two Flavor Case

$$U = \begin{pmatrix} U_{\alpha 1} & U_{\alpha 2} \\ U_{\beta 1} & U_{\beta 2} \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \Rightarrow \mathcal{J}_{12}^{\alpha\alpha} = |U_{\alpha 1}|^2 |U_{\alpha 2}|^2 = \frac{1}{4} \sin^2 2\theta$$

and transition probability is $P_{\alpha\beta} = \sin^2 2\theta \sin^2 \frac{\Delta m_{21}^2}{4E} L$





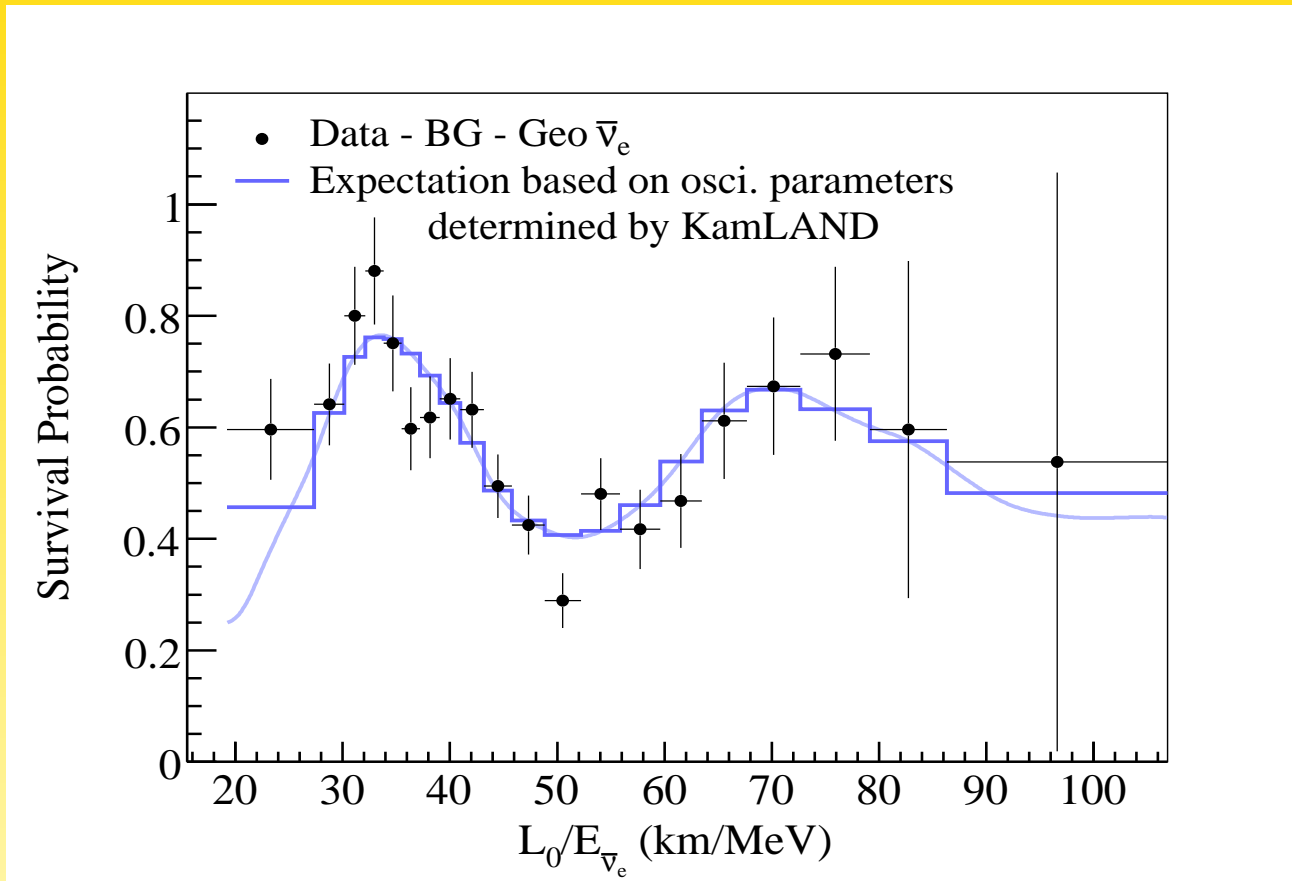


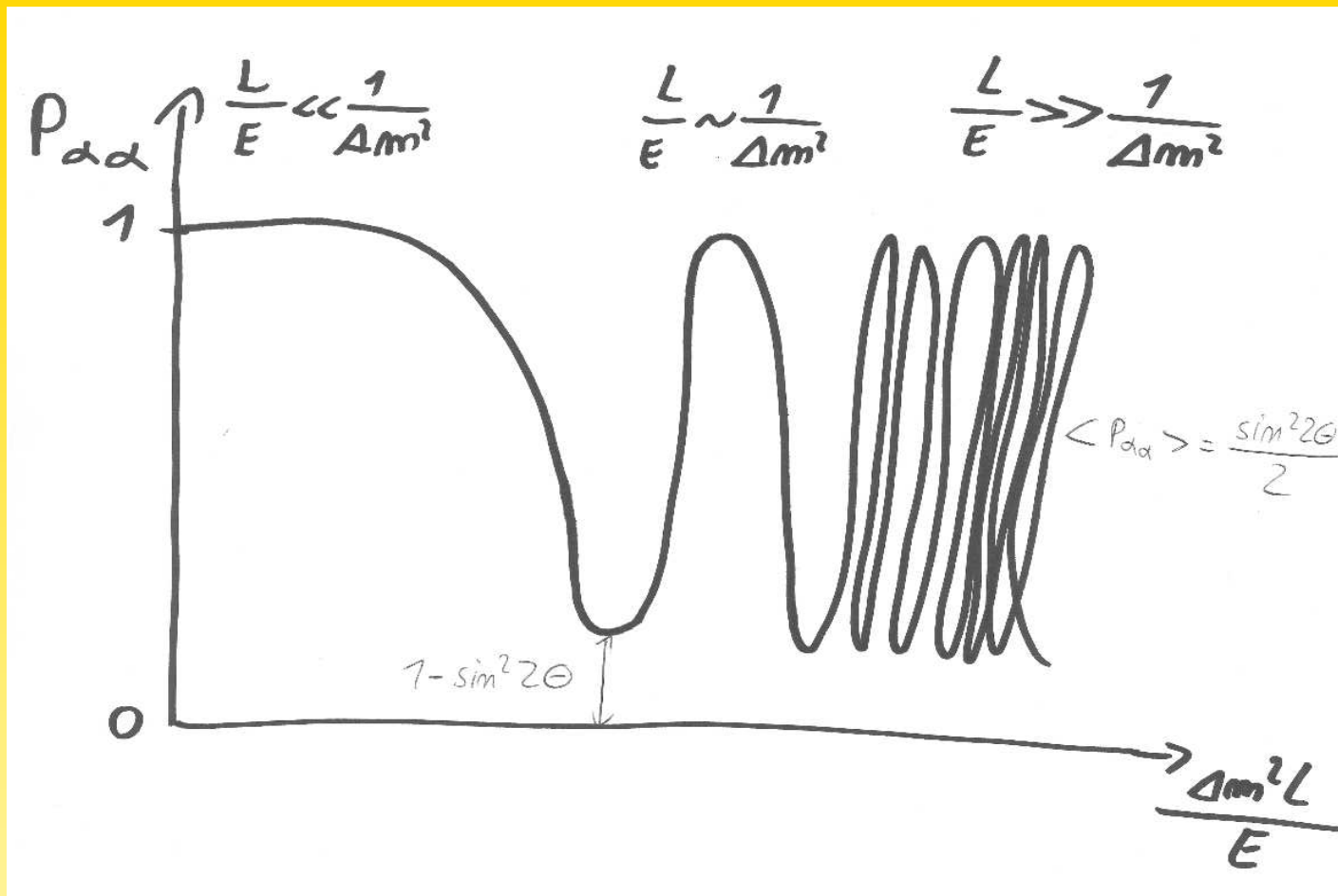
- amplitude $\sin^2 2\theta$
- maximal mixing for $\theta = \pi/4 \Rightarrow \nu_\alpha = \sqrt{\frac{1}{2}} (\nu_1 + \nu_2)$
- oscillation length $L_{\text{osc}} = 4\pi E / \Delta m_{21}^2 = 2.48 \frac{E}{\text{GeV}} \frac{\text{eV}^2}{\Delta m_{21}^2} \text{ km}$

$$\Rightarrow P_{\alpha\beta} = \sin^2 2\theta \sin^2 \pi \frac{L}{L_{\text{osc}}}$$

is distance between two maxima (minima)

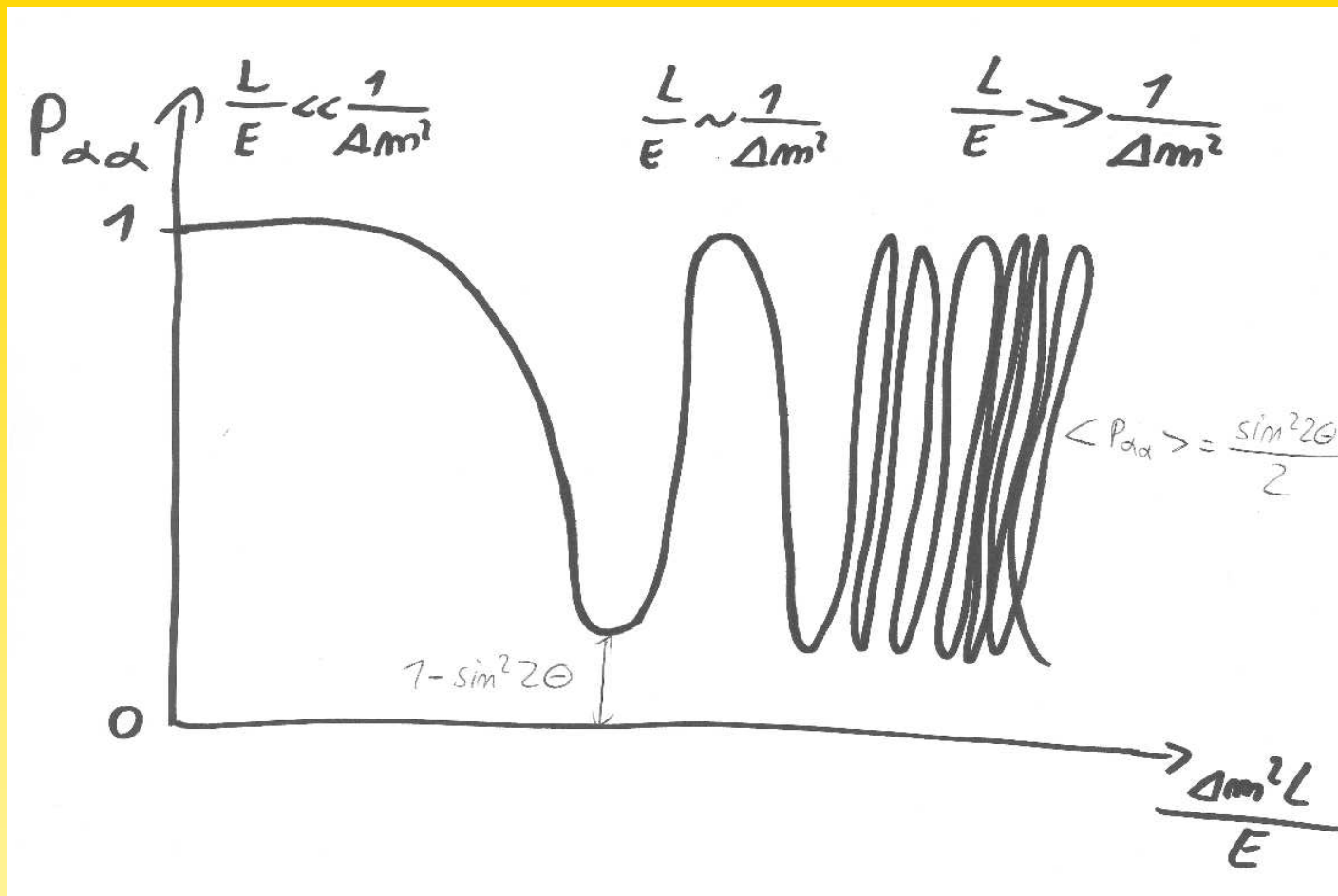
e.g.: $E = \text{GeV}$ and $\Delta m^2 = 10^{-3} \text{ eV}^2$: $L_{\text{osc}} \simeq 10^3 \text{ km}$





$L \gg L_{osc}$: fast oscillations $\langle \sin^2 \pi L / L_{osc} \rangle = \frac{1}{2}$
 and $P_{\alpha\alpha} = 1 - 2 |U_{\alpha 1}|^2 |U_{\alpha 2}|^2 = |U_{\alpha 1}|^4 + |U_{\alpha 2}|^4$

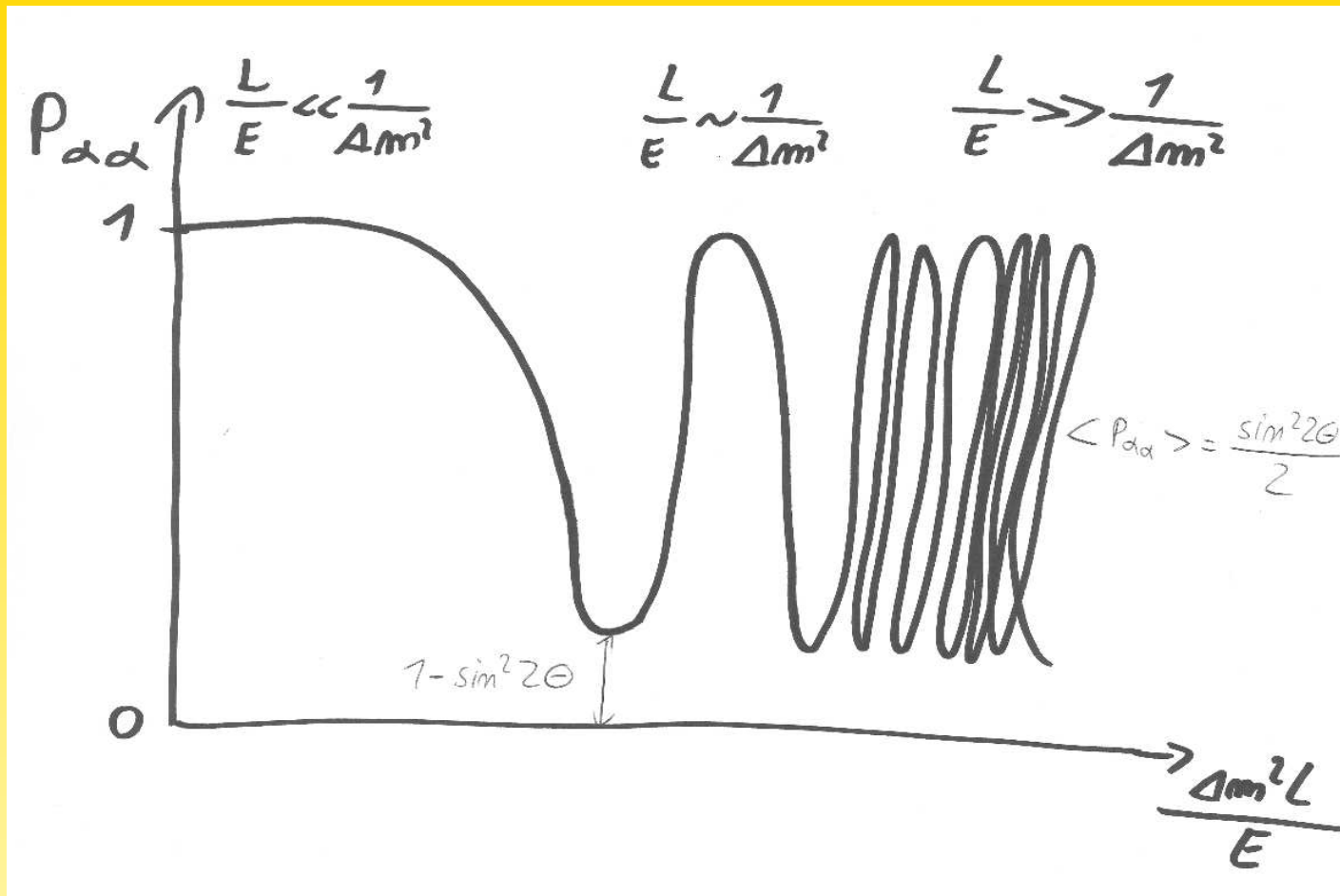
sensitivity to mixing



$L \gg L_{\text{osc}}$: fast oscillations $\langle \sin^2 \pi L / L_{\text{osc}} \rangle = \frac{1}{2}$

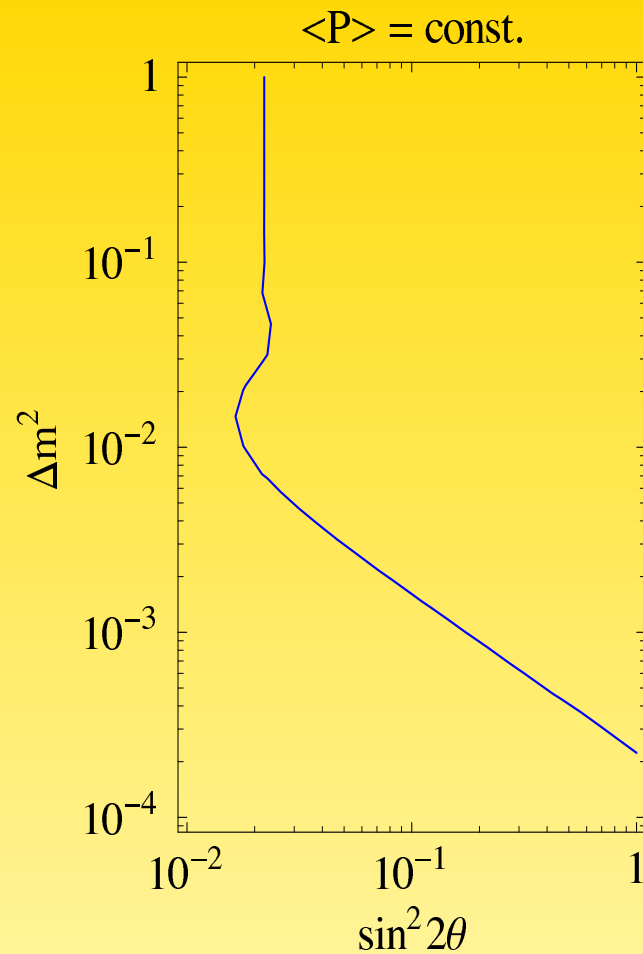
and $P_{\alpha\beta} = 2 |U_{\alpha 1}|^2 |U_{\alpha 2}|^2 = |U_{\alpha 1}|^2 |U_{\beta 1}|^2 + |U_{\alpha 2}|^2 |U_{\beta 2}|^2 = \frac{1}{2} \sin^2 2\theta$

sensitivity to mixing



$L \ll L_{\text{osc}}$: hardly oscillations and $P_{\alpha\beta} = \sin^2 2\theta (\Delta m^2 L / (4E))^2$

sensitivity to product $\sin^2 2\theta \Delta m^2$



large Δm^2 : sensitivity to mixing

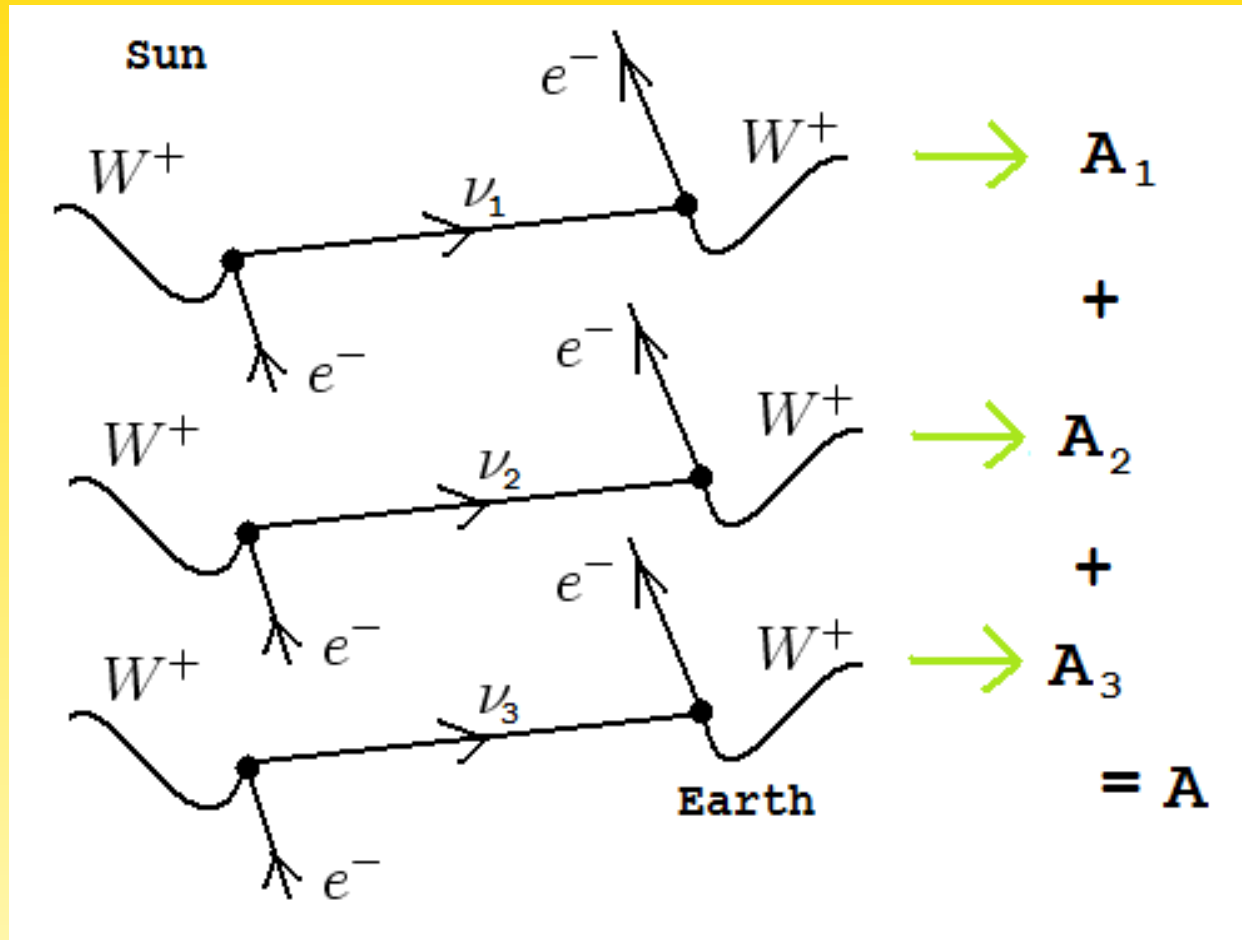
small Δm^2 : sensitivity to $\sin^2 2\theta$ Δm^2

maximal sensitivity when $\Delta m^2 L/E \simeq 2\pi$

Characteristics of typical oscillation experiments

Source	Flavor	E [GeV]	L [km]	$(\Delta m^2)_{\min}$ [eV ²]
Atmosphere	$\begin{pmatrix} - \\ \nu_e \end{pmatrix}, \begin{pmatrix} - \\ \nu_\mu \end{pmatrix}$	$10^{-1} \dots 10^2$	$10 \dots 10^4$	10^{-6}
Sun	ν_e	$10^{-3} \dots 10^{-2}$	10^8	10^{-11}
Reactor SBL	$\bar{\nu}_e$	$10^{-4} \dots 10^{-2}$	10^{-1}	10^{-3}
Reactor LBL	$\bar{\nu}_e$	$10^{-4} \dots 10^{-2}$	10^2	10^{-5}
Accelerator LBL	$\begin{pmatrix} - \\ \nu_e \end{pmatrix}, \begin{pmatrix} - \\ \nu_\mu \end{pmatrix}$	$10^{-1} \dots 1$	10^2	10^{-1}
Accelerator SBL	$\begin{pmatrix} - \\ \nu_e \end{pmatrix}, \begin{pmatrix} - \\ \nu_\mu \end{pmatrix}$	$10^{-1} \dots 1$	1	1

Quantum Mechanics



Can't distinguish the individual m_i : coherent sum of amplitudes and interference

Quantum Mechanics

Textbook calculation is completely wrong!!

- $E_i - E_j$ is not Lorentz invariant
- massive particles with different p_i and same E violates energy and/or momentum conservation
- definite p : in space this is e^{ipx} , thus no localization

Quantum Mechanics

consider E_j and $p_j = \sqrt{E_j^2 - m_j^2}$:

$$p_j \simeq E + m_j^2 \left. \frac{\partial p_j}{\partial m_j^2} \right|_{m_j=0} \equiv E - \xi \frac{m_j^2}{2E}, \quad \text{with } \xi = -2E \left. \frac{\partial p_j}{\partial m_j^2} \right|_{m_j=0}$$

$$E_j \simeq p_j + m_j^2 \left. \frac{\partial E_j}{\partial m_j^2} \right|_{m_j=0} = p_j + \frac{m_j^2}{2p_j} = E + \frac{m_j^2}{2E} (1 - \xi)$$

in pion decay $\pi \rightarrow \mu\nu$:

$$E_j = \frac{m_\pi^2}{2} \left(1 - \frac{m_\mu^2}{m_\pi^2} \right) + \frac{m_j^2}{2m_\pi^2}$$

thus,

$$\xi = \frac{1}{2} \left(1 + \frac{m_\mu^2}{m_\pi^2} \right) \simeq 0.8 \quad \text{in } E_i - E_j \simeq (1 - \xi) \frac{\Delta m_{ij}^2}{2E}$$

wave packet with size $\sigma_x (\gtrsim 1/\sigma_p)$ and group velocity $v_i = \partial E_i / \partial p_i = p_i / E_i$:

$$\psi_i \propto \exp \left\{ -i(E_i t - p_i x) - \frac{(x - v_i t)^2}{4\sigma_x^2} \right\}$$

1) wave packet separation should be smaller than σ_x !

$$L \Delta v < \sigma_x \Rightarrow \frac{L}{L_{\text{osc}}} < \frac{p}{\sigma_p}$$

(loss of coherence: interference impossible)

2) m_ν^2 should NOT be known too precisely!

$$\text{if known too well: } \Delta m^2 \gg \delta m_\nu^2 = \frac{\partial m_\nu^2}{\partial p_\nu} \delta p_\nu \Rightarrow \delta x_\nu \gg \frac{2 p_\nu}{\Delta m^2} = \frac{L_{\text{osc}}}{2\pi}$$

(I know which state ν_i is exchanged, localization)

In both cases: $P_{\alpha\alpha} = |U_{\alpha 1}|^4 + |U_{\alpha 2}|^4$ (same as for $L \gg L_{\text{osc}}$)

Quantum Mechanics

total amplitude for $\alpha \rightarrow \beta$ should be given by

$$A \propto \sum_j \int \frac{d^3p}{2E_j} \mathcal{A}_{\beta j}^* \mathcal{A}_{\alpha j} \exp \{ -i(E_j t - px) \}$$

with production and detection amplitudes

$$\mathcal{A}_{\alpha j} \mathcal{A}_{\beta j}^* \propto \exp \left\{ -\frac{(p - \tilde{p}_j)^2}{4\sigma_p^2} \right\}$$

we expand around \tilde{p}_j :

$$E_j(p) \simeq E_j(\tilde{p}_j) + \left. \frac{\partial E_j(p)}{\partial p} \right|_{p=\tilde{p}_j} (p - \tilde{p}_j) = \tilde{E}_j + v_j (p - \tilde{p}_j)$$

and perform the integral over p :

$$A \propto \sum_j \exp \left\{ -i(\tilde{E}_j t - \tilde{p}_j x) - \frac{(x - v_j t)^2}{4\sigma_x^2} \right\}$$

the probability is the integral of $|A|^2$ over t :

$$P = \int dt |A|^2 \propto \exp \left\{ -i \left[(\tilde{E}_j - \tilde{E}_k) \frac{v_j + v_k}{v_j^2 + v_k^2} - (\tilde{p}_j - \tilde{p}_k) \right] x \right\} \\ \times \exp \left\{ -\frac{(v_j - v_k)^2 x^2}{4\sigma_x^2 (v_j^2 + v_k^2)} - \frac{(\tilde{E}_j - \tilde{E}_k)^2}{4\sigma_p^2 (v_j^2 + v_k^2)} \right\}$$

now express average momenta, energy and velocity as

$$\tilde{p}_j \simeq E - \xi \frac{m_j^2}{2E}$$

$$\tilde{E}_j \simeq E + (1 - \xi) \frac{m_j^2}{2E}, \quad v_j = \frac{\tilde{p}_j}{\tilde{E}_j} \simeq 1 - \frac{m_j^2}{2E^2}$$

this we insert in first exponential of P :

$$\left[(\tilde{E}_j - \tilde{E}_k) \frac{v_j + v_k}{v_j^2 + v_k^2} - (\tilde{p}_j - \tilde{p}_k) \right] = \frac{\Delta m_{jk}^2 L}{2E}$$

the second exponential (damping term) can also be rewritten and the final probability is

$$P \propto \exp \left\{ -i \frac{\Delta m_{ij}^2}{2E} L - \left(\frac{L}{L_{jk}^{\text{coh}}} \right)^2 - 2\pi^2 (1 - \xi)^2 \left(\frac{\sigma_x}{L_{jk}^{\text{osc}}} \right)^2 \right\}$$

with

$$L_{jk}^{\text{coh}} = \frac{4\sqrt{2}E^2}{|\Delta m_{jk}^2|} \sigma_x \quad \text{and} \quad L_{jk}^{\text{osc}} = \frac{4\pi E}{|\Delta m_{jk}^2|}$$

expressing the two conditions (coherence and localization) for oscillation discussed before

Contents

II Neutrino Oscillations

II1) The PMNS matrix

II2) Neutrino oscillations in vacuum

II3) Results and their interpretation – what have we learned?

II4) Prospects – what do we want to know?

II3) Results and their interpretation – what have we learned?

- Main results as by-products:
 - check solar fusion in Sun → solar neutrino problem
 - look for nucleon decay → atmospheric neutrino oscillations
- almost all current data described by 2-flavor formalism
- sophisticated: confirm genuine 3-flavor effects:
 - third mixing angle
 - mass ordering
 - CP violation
- have entered precision era

Interpretation in 3 Neutrino Framework

assume $\Delta m_{21}^2 \ll \Delta m_{31}^2 \simeq \Delta m_{32}^2$ and small θ_{13} :

- atmospheric and accelerator neutrinos: $\Delta m_{21}^2 L/E \ll 1$

$$P(\nu_\mu \rightarrow \nu_\tau) \simeq \sin^2 2\theta_{23} \sin^2 \frac{\Delta m_{31}^2}{4E} L$$

- solar and KamLAND neutrinos: $\Delta m_{31}^2 L/E \gg 1$

$$P(\nu_e \rightarrow \nu_e) \simeq 1 - \sin^2 2\theta_{12} \sin^2 \frac{\Delta m_{21}^2}{4E} L$$

- short baseline reactor neutrinos: $\Delta m_{21}^2 L/E \ll 1$

$$P(\nu_e \rightarrow \nu_e) \simeq 1 - \sin^2 2\theta_{13} \sin^2 \frac{\Delta m_{31}^2}{4E} L$$

Solar Neutrinos

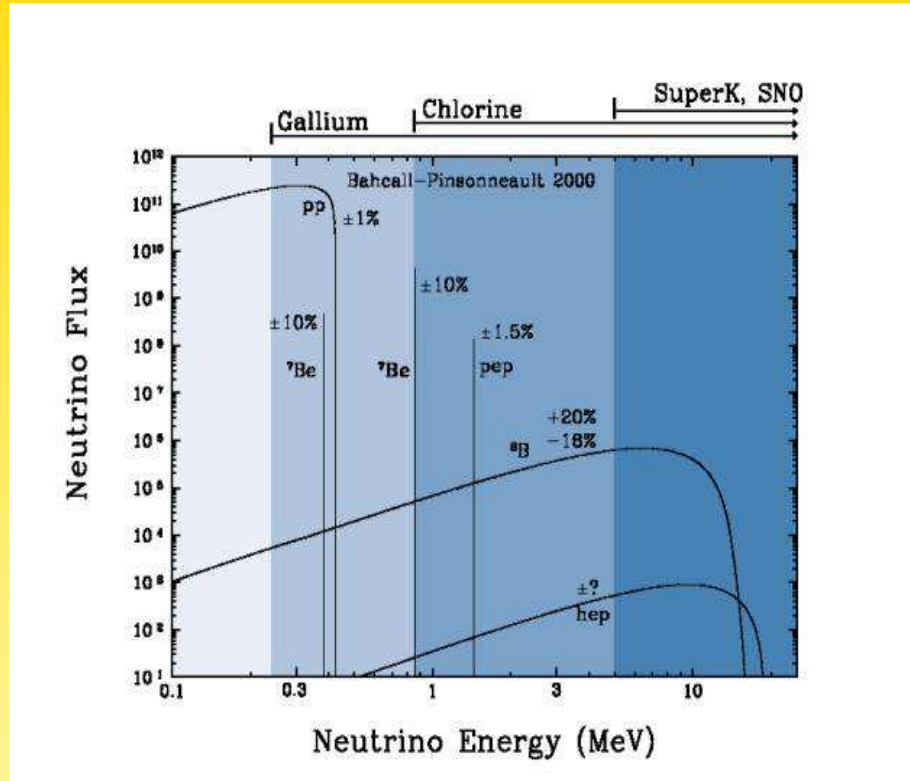
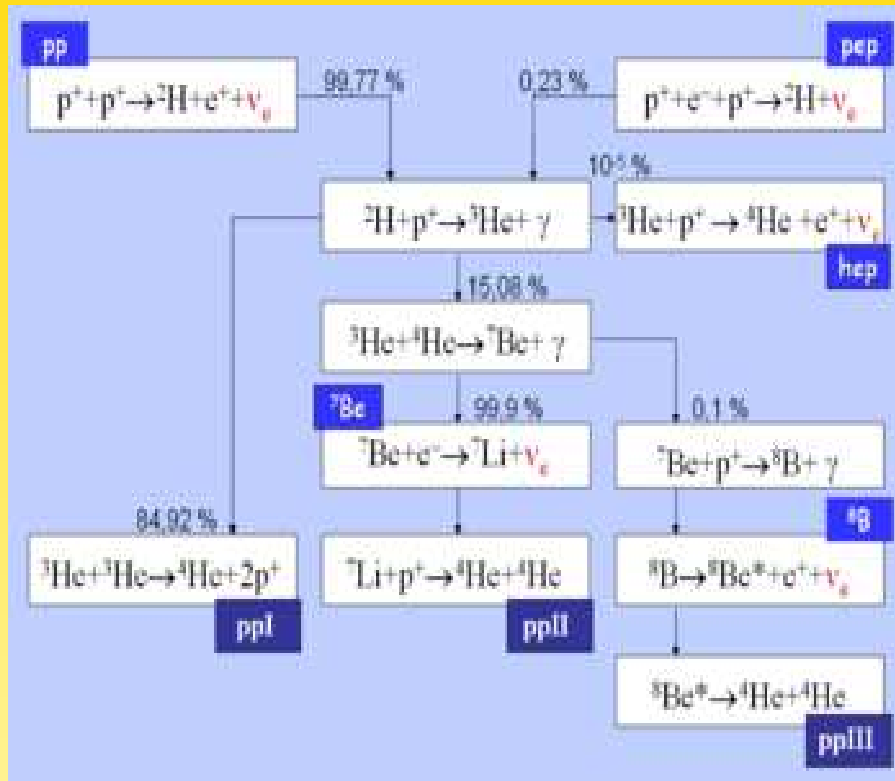
98% of energy production in fusion of net reaction



26 MeV of the energy go in photons, i.e., 13 MeV per ν_e ;

get neutrino flux from solar constant

$$S = 8.5 \times 10^{11} \text{ MeV cm}^{-2} \text{ s}^{-1} \Rightarrow \Phi_\nu = \frac{S}{13 \text{ MeV}} = 6.5 \times 10^{10} \text{ cm}^{-2} \text{ s}^{-1}$$



Solar Standard Model (SSM) predicts 5 sources of neutrinos from *pp*-chain

Bahcall *et al.*

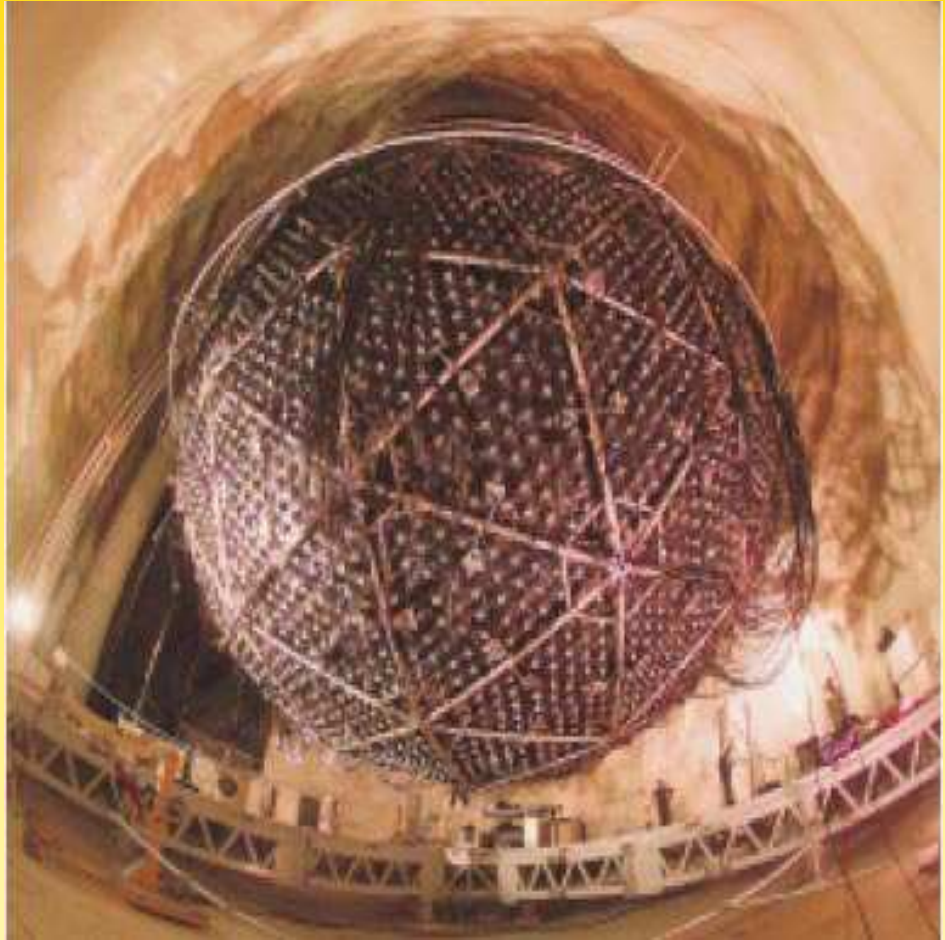
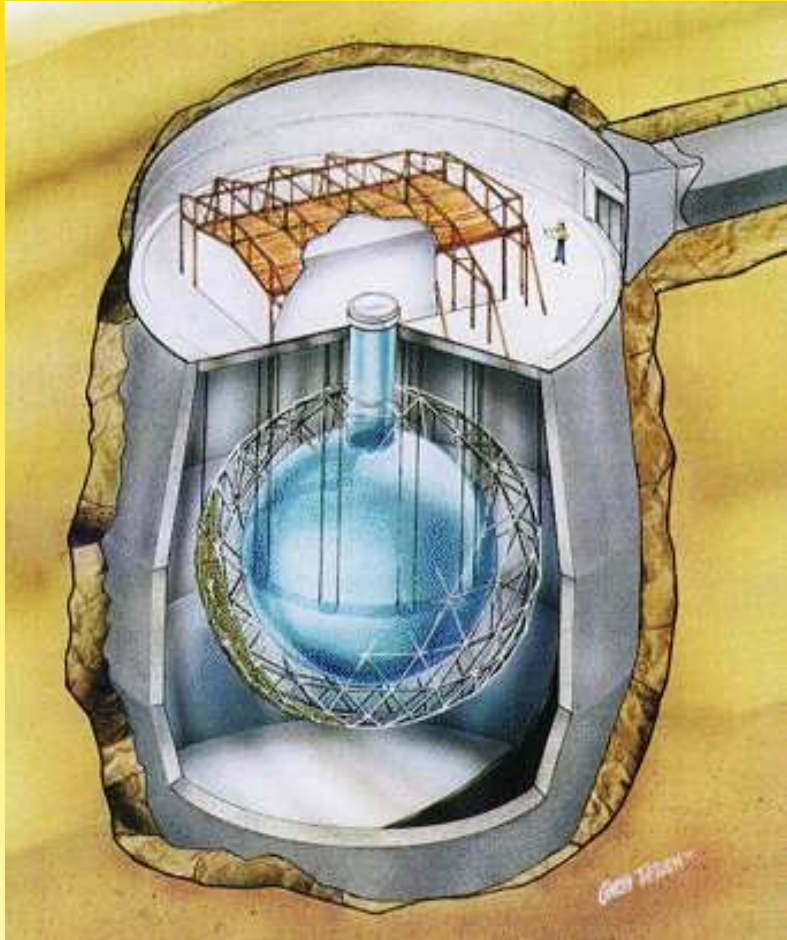
Different experiments sensitive to different energy, hence different neutrinos

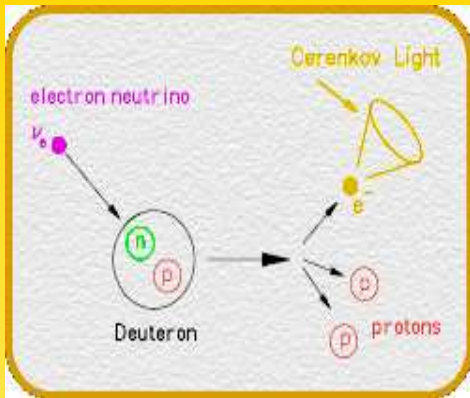
- Homestake: $\nu_e + {}^{37}\text{Cl} \rightarrow {}^{37}\text{Ar} + e^-$
- Gallex, GNO, SAGE: $\nu_e + {}^{71}\text{Ga} \rightarrow {}^{71}\text{Ge} + e^-$
- (Super)Kamiokande: $\nu_e + e^- \rightarrow \nu_e + e^-$

All find less neutrinos than predicted by SSM, deficit is energy dependent:

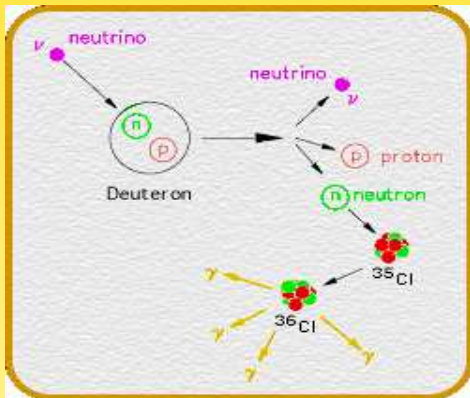
“solar neutrino problem”

Breakthrough came with SNO experiment, using heavy water

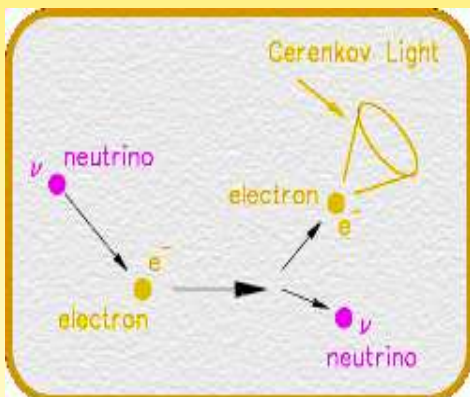




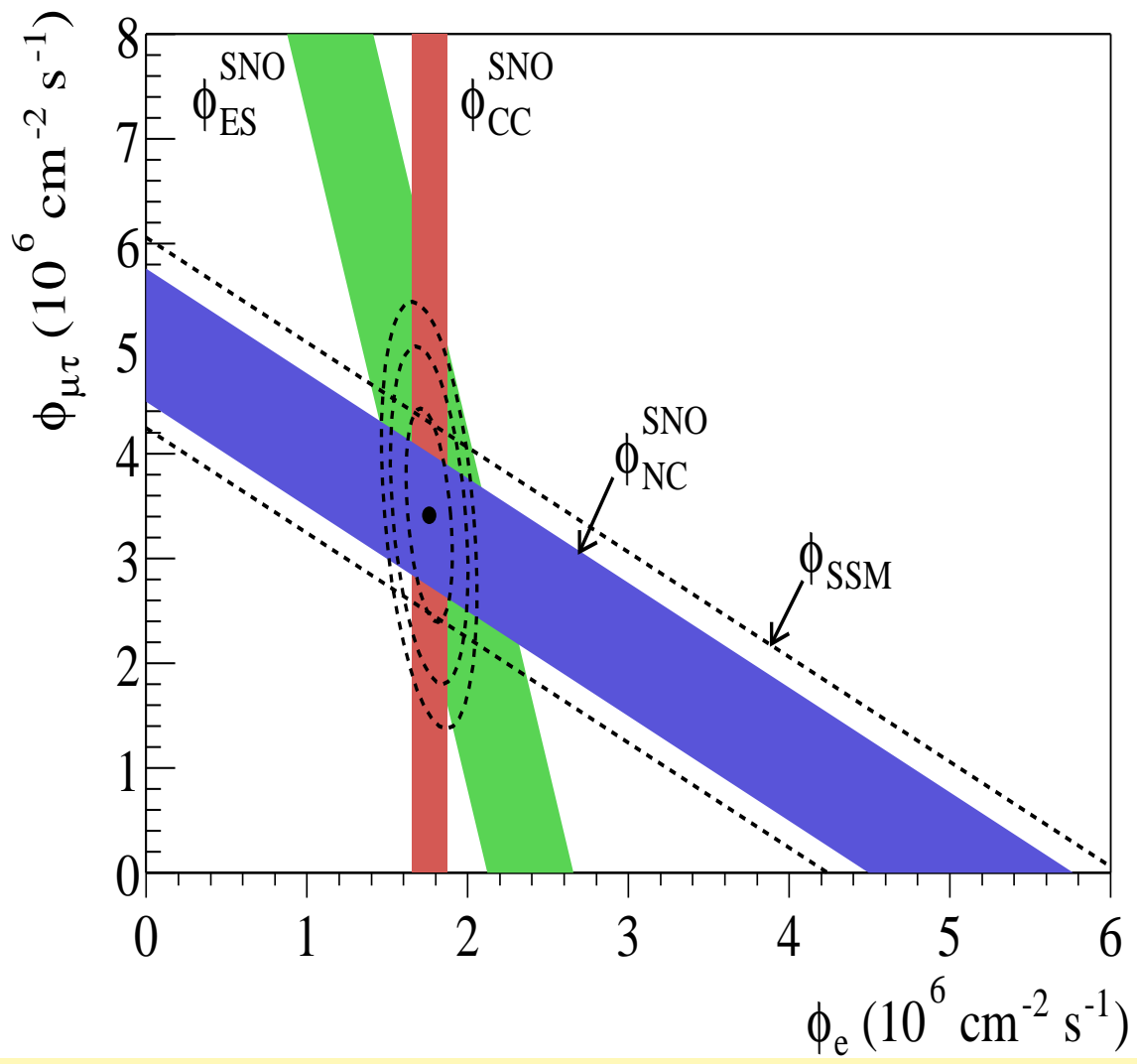
charged current: $\Phi(\nu_e)$



neutral current: $\Phi(\nu_e) + \Phi(\nu_{\mu\tau})$



elastic scattering: $\Phi(\nu_e) + 0.15 \Phi(\nu_{\mu\tau})$



Results of fits give

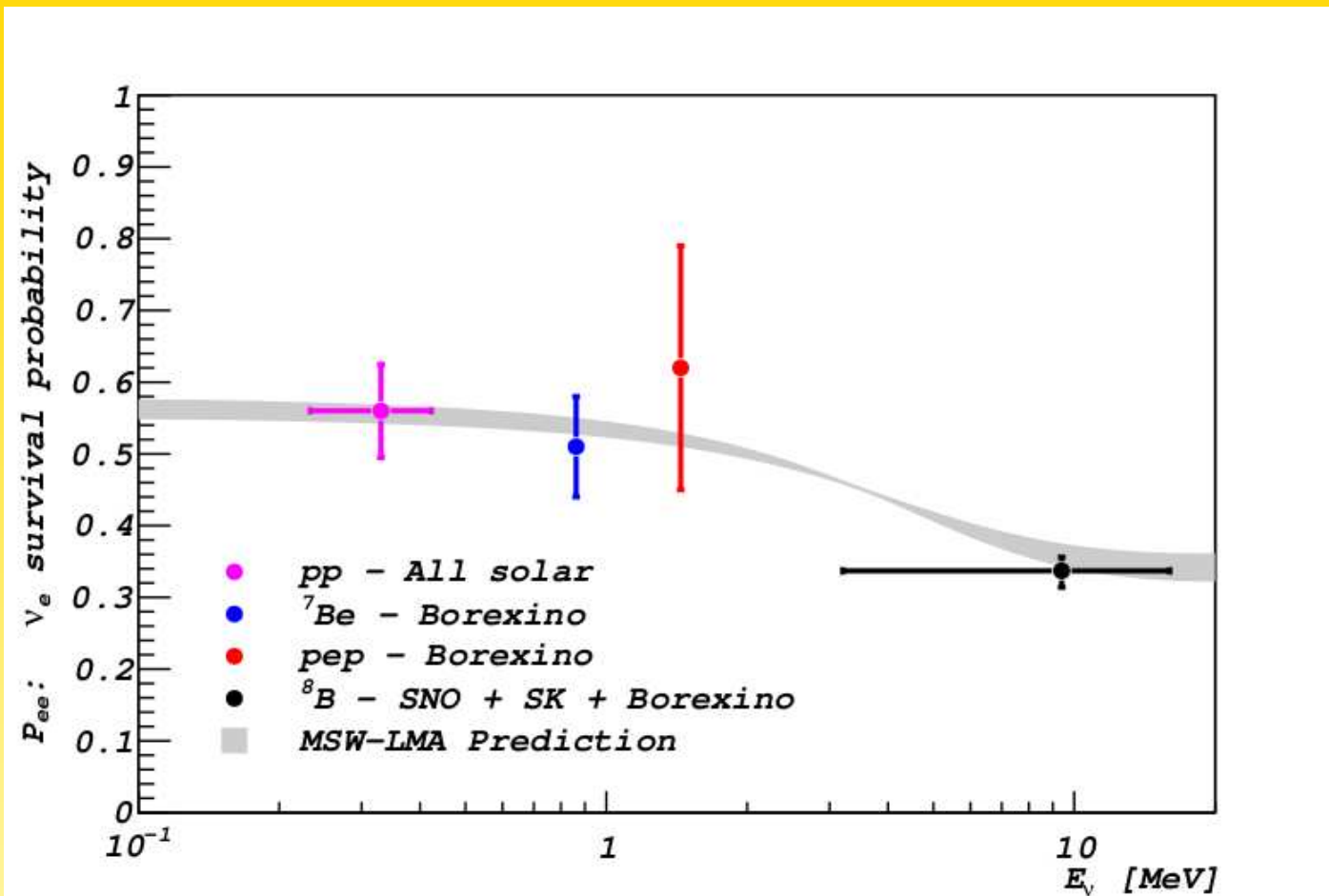
$$\sin^2 \theta_{12} \simeq 0.30$$

$$\Delta m_{21}^2 \equiv \Delta m_{\odot}^2 \simeq 8 \times 10^{-5} \text{ eV}^2$$

only works with matter effects and resonance in Sun

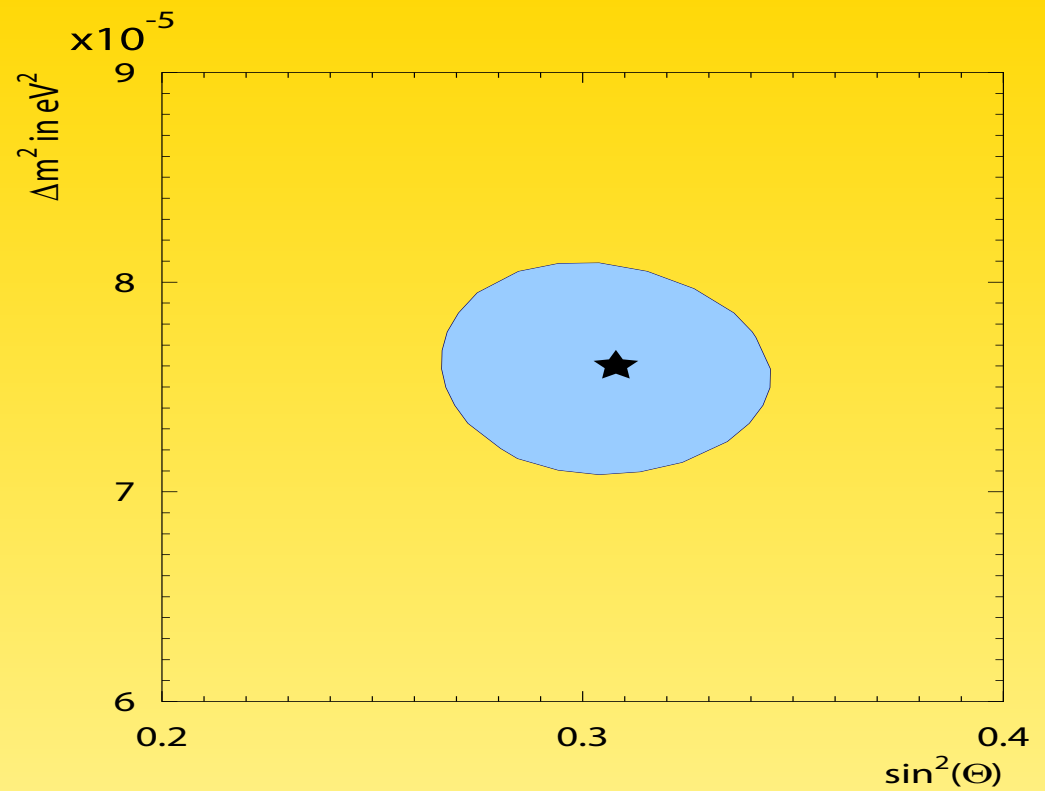
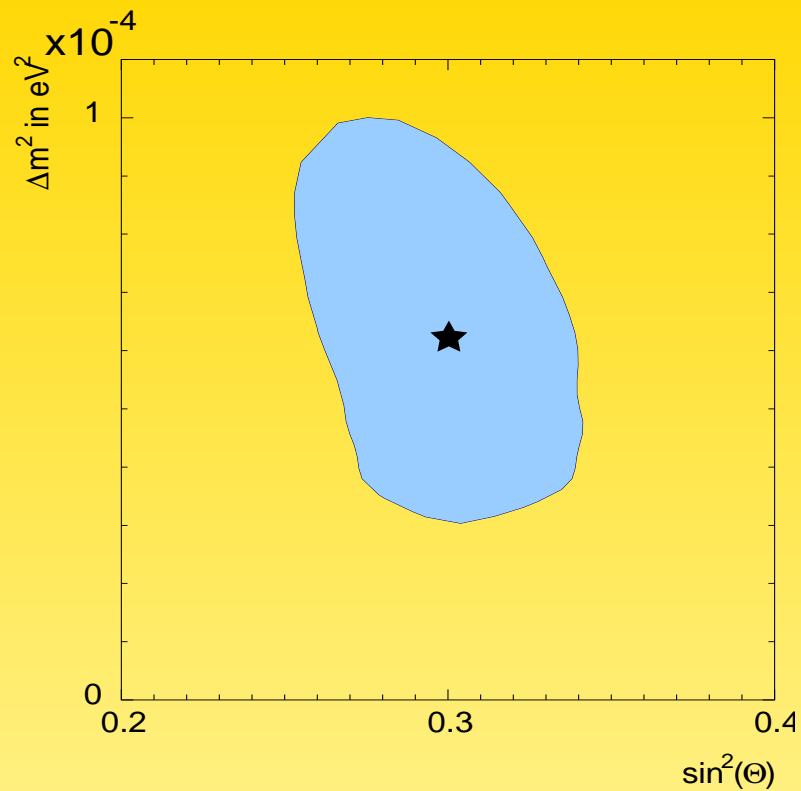
$$\Rightarrow \Delta m_{\odot}^2 \cos 2\theta_{12} = (m_2^2 - m_1^2) (\cos^2 \theta_{12} - \sin^2 \theta_{12}) > 0$$

choosing $\cos 2\theta_{12} > 0$ fixes $\Delta m_{\odot}^2 > 0$



$$\text{low } E: P_{ee} = 1 - \frac{1}{2} \sin^2 2\theta_{12} \simeq \frac{5}{9}$$

$$\text{large } E: P_{ee} = \sin^2 \theta_{12} = \frac{1}{3}$$

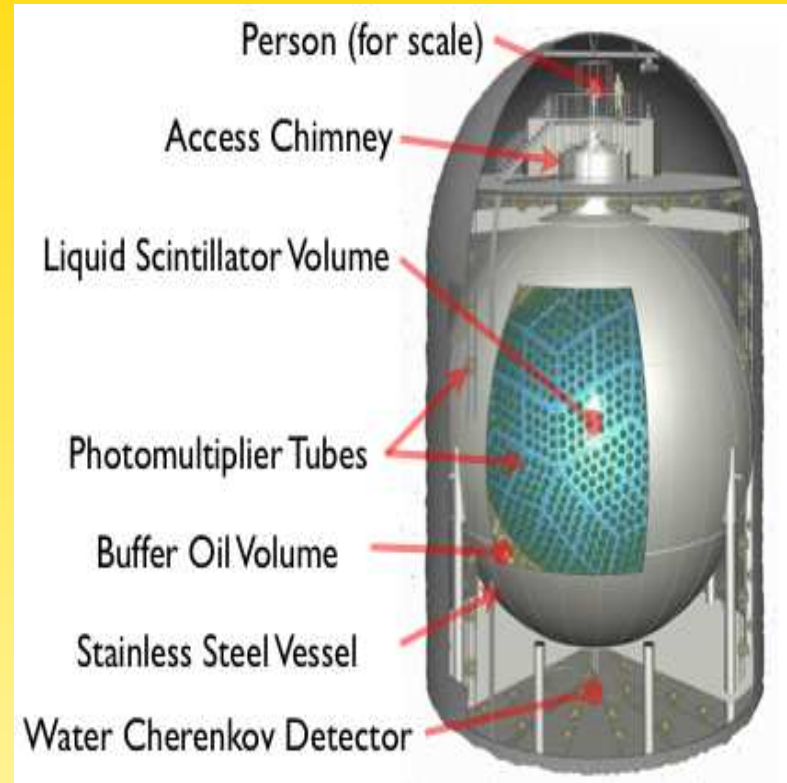
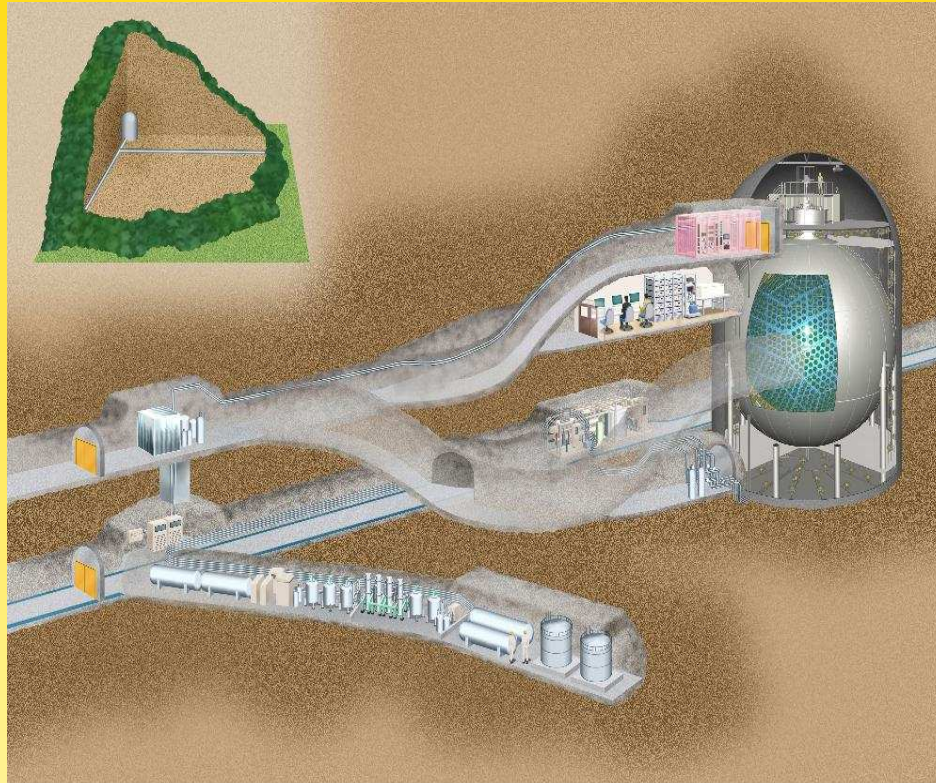


KamLAND: reactor neutrinos

$n \rightarrow p + e^- + \bar{\nu}_e$ with $E \simeq$ few MeV

If $L \simeq 100$ km:

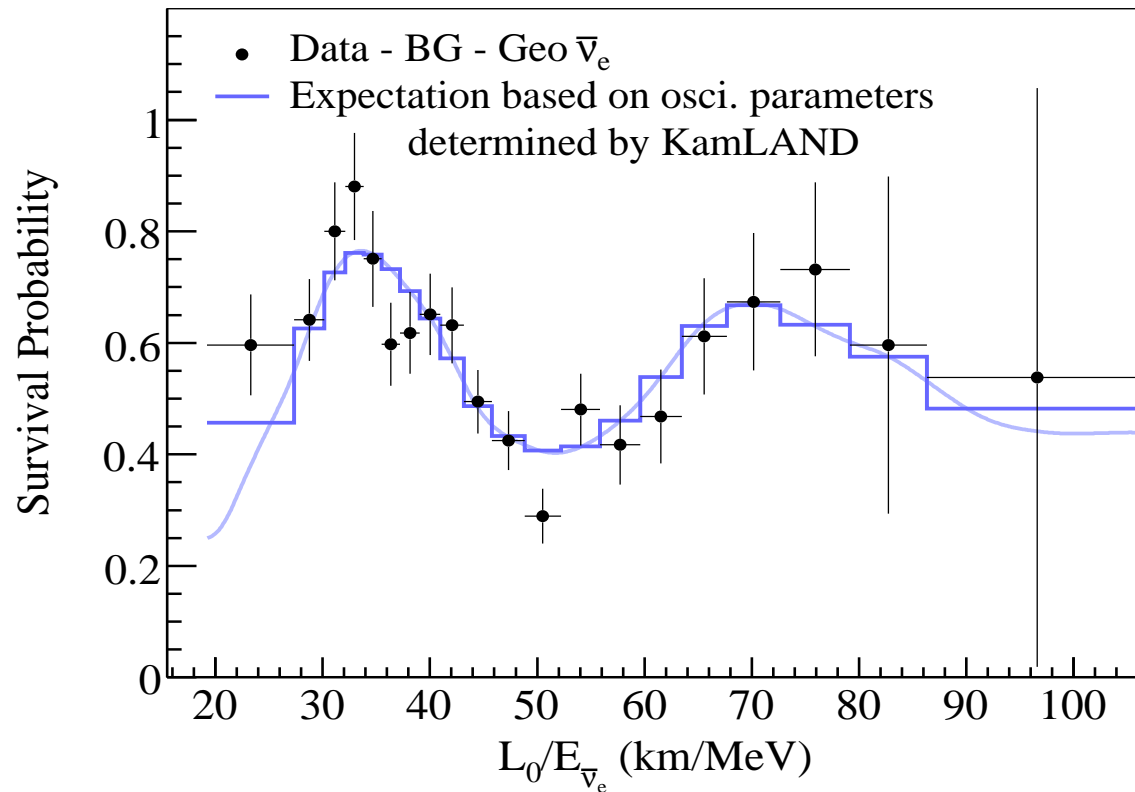
$$\frac{\Delta m_{\odot}^2}{E} L \sim 1 \Rightarrow \text{solar } \nu \text{ parameters!!}$$



$$\bar{\nu}_e + p \rightarrow n + e^+ \text{ with } E_\nu \simeq E_{\text{prompt}} + E_n^{\text{recoil}} + 0.8 \text{ MeV}$$

$$200 \mu\text{s later: } n + p \rightarrow d + \gamma$$

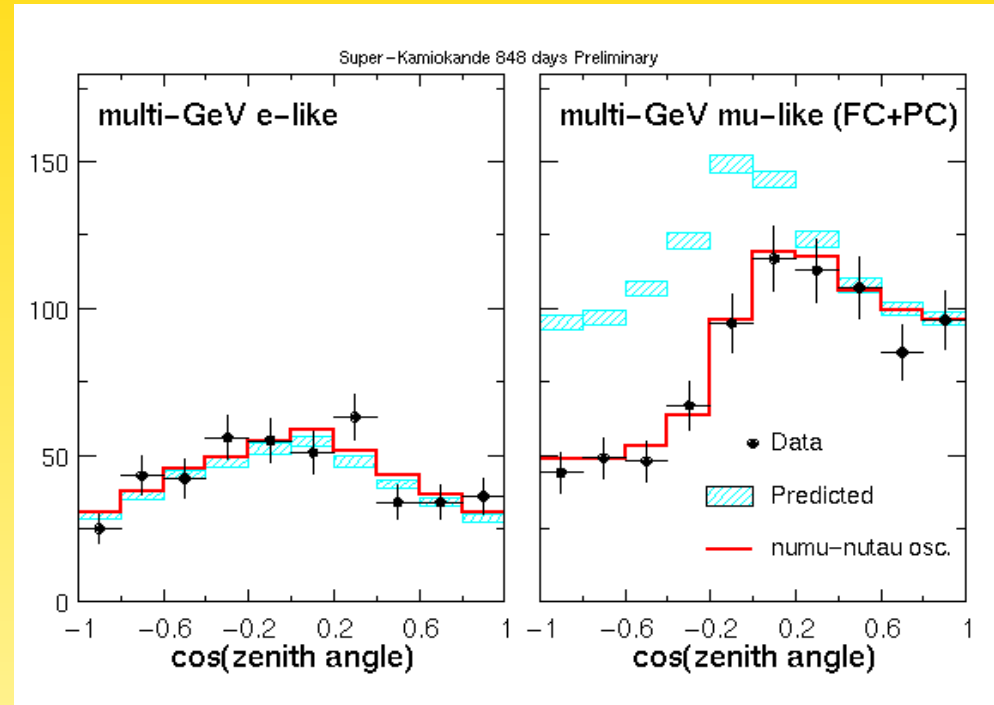
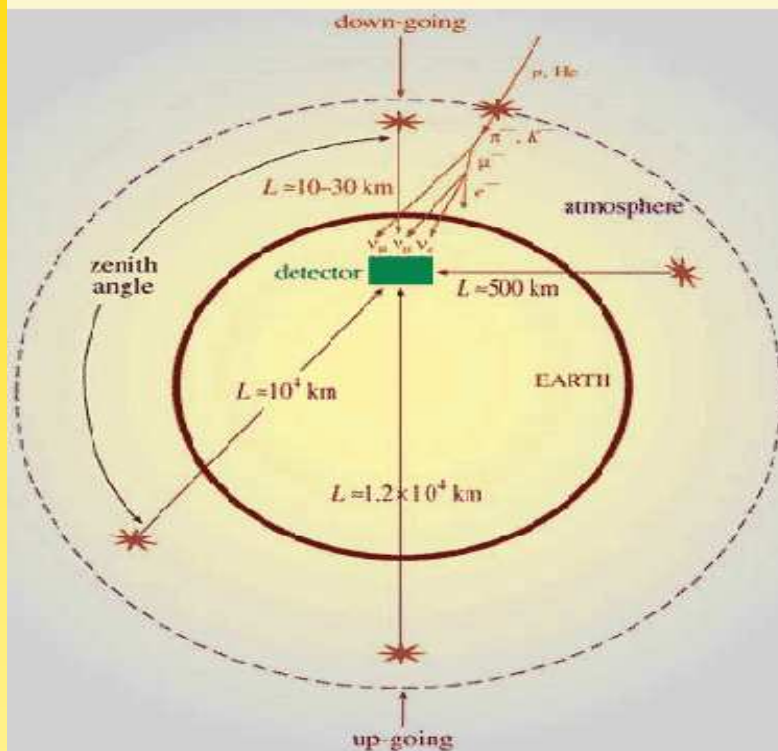
Neutrinos do oscillate



KamLAND

Atmospheric Neutrinos

Figure 4

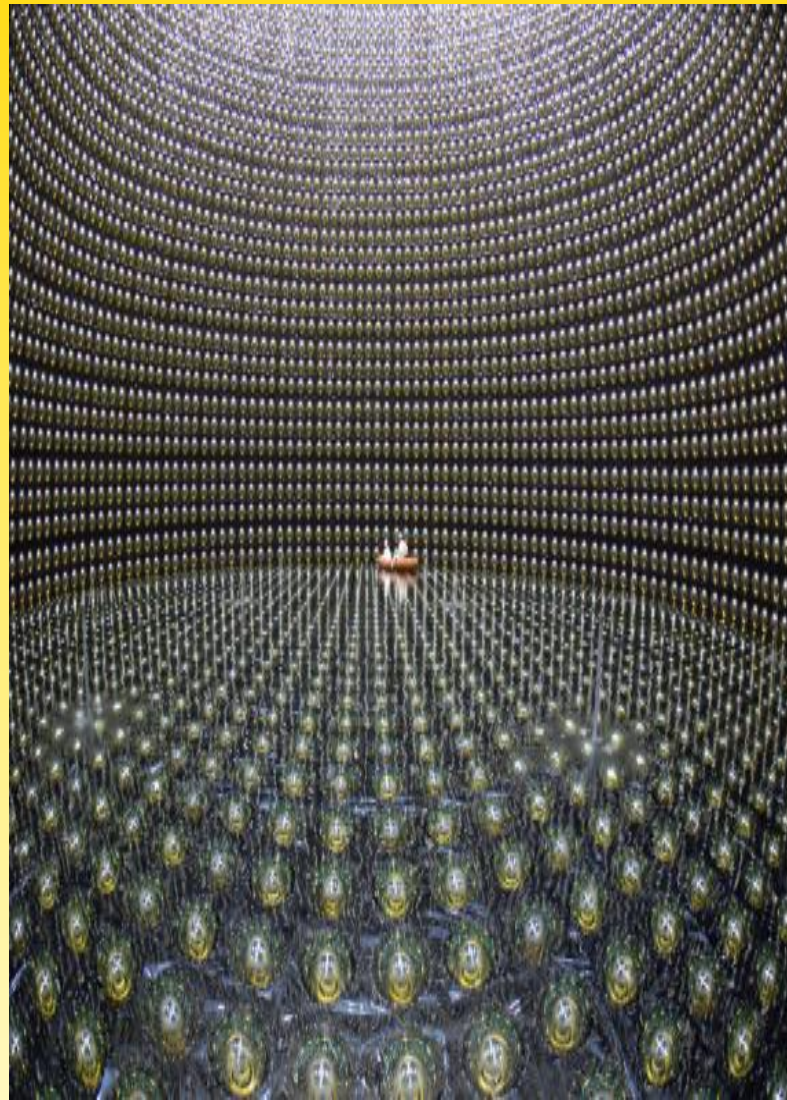
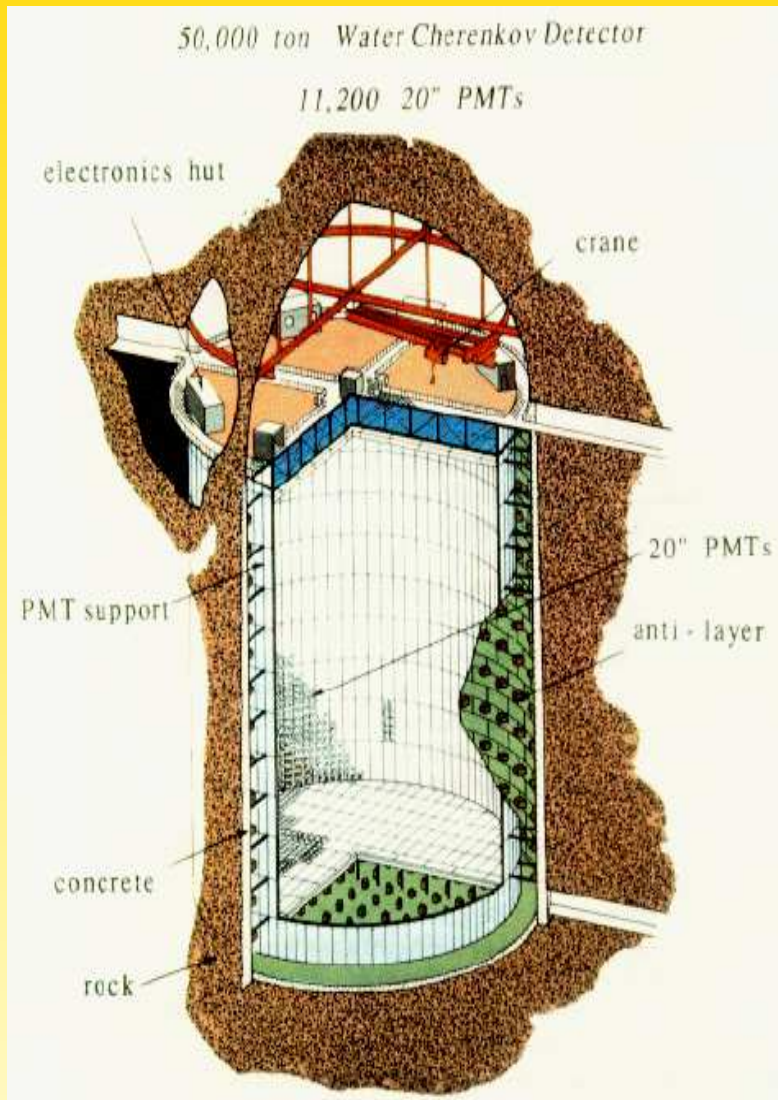


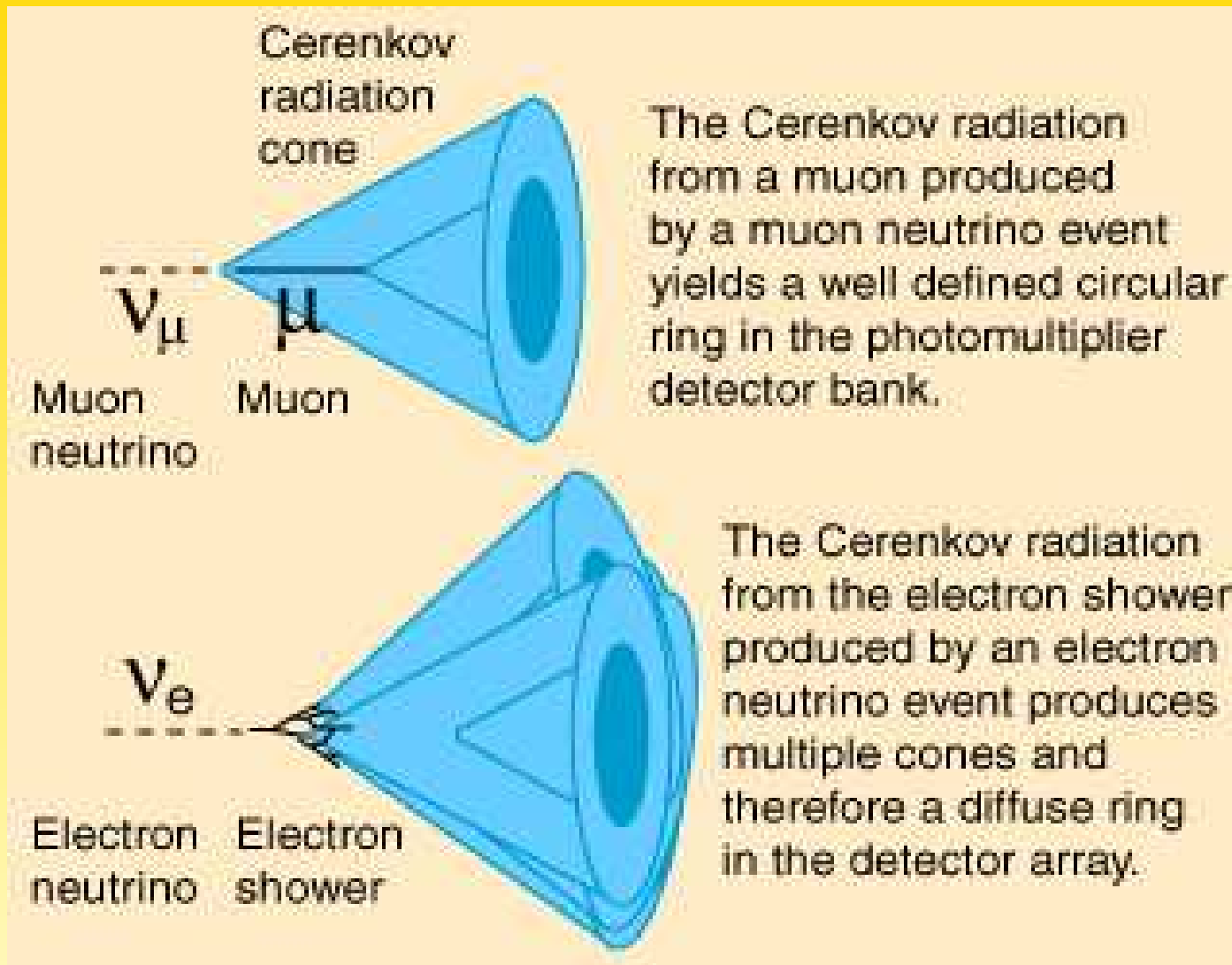
zenith angle $\cos \theta = 1$ $L \simeq 500$ km

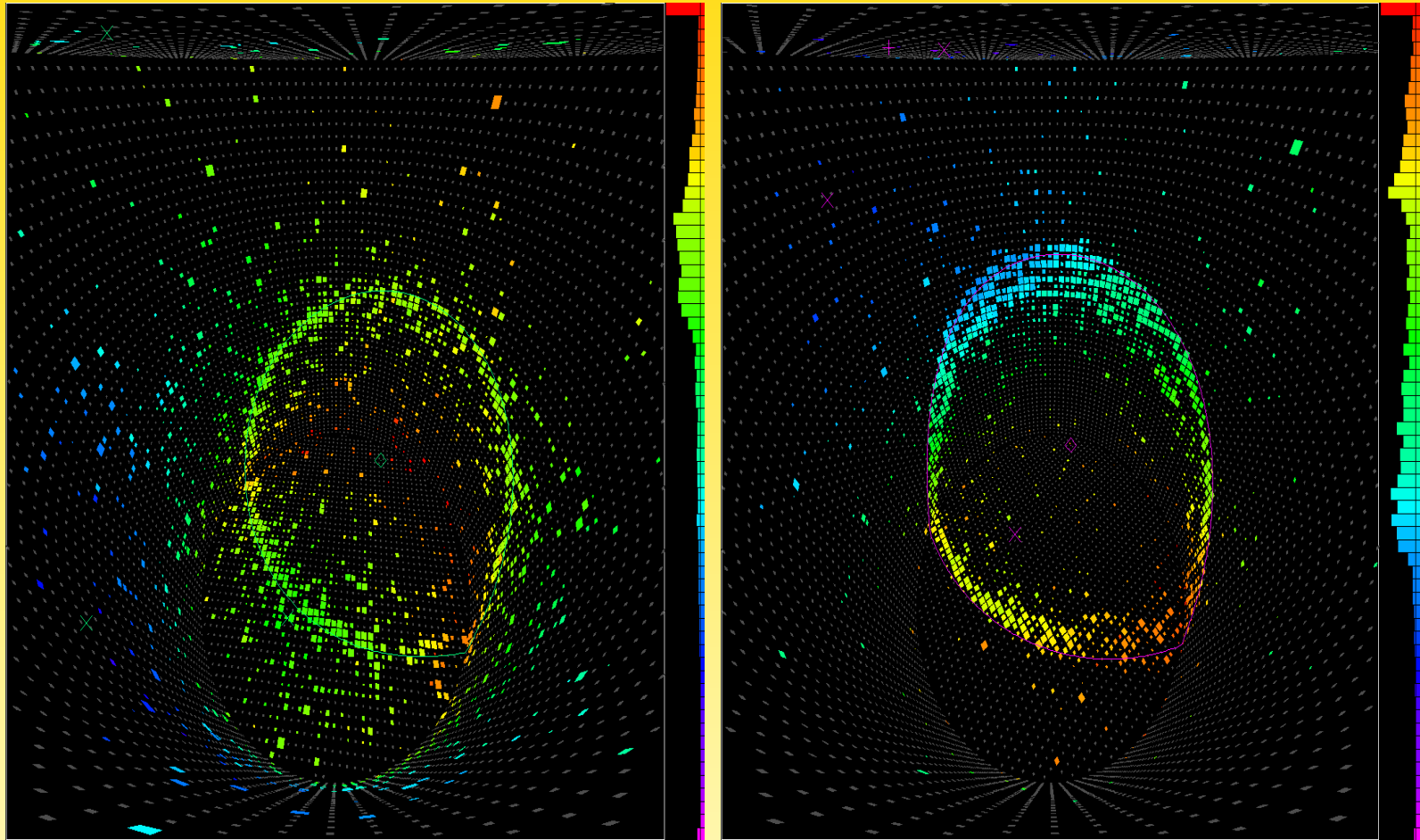
zenith angle $\cos \theta = 0$ $L \simeq 10$ km down-going

zenith angle $\cos \theta = -1$ $L \simeq 10^4$ km up-going

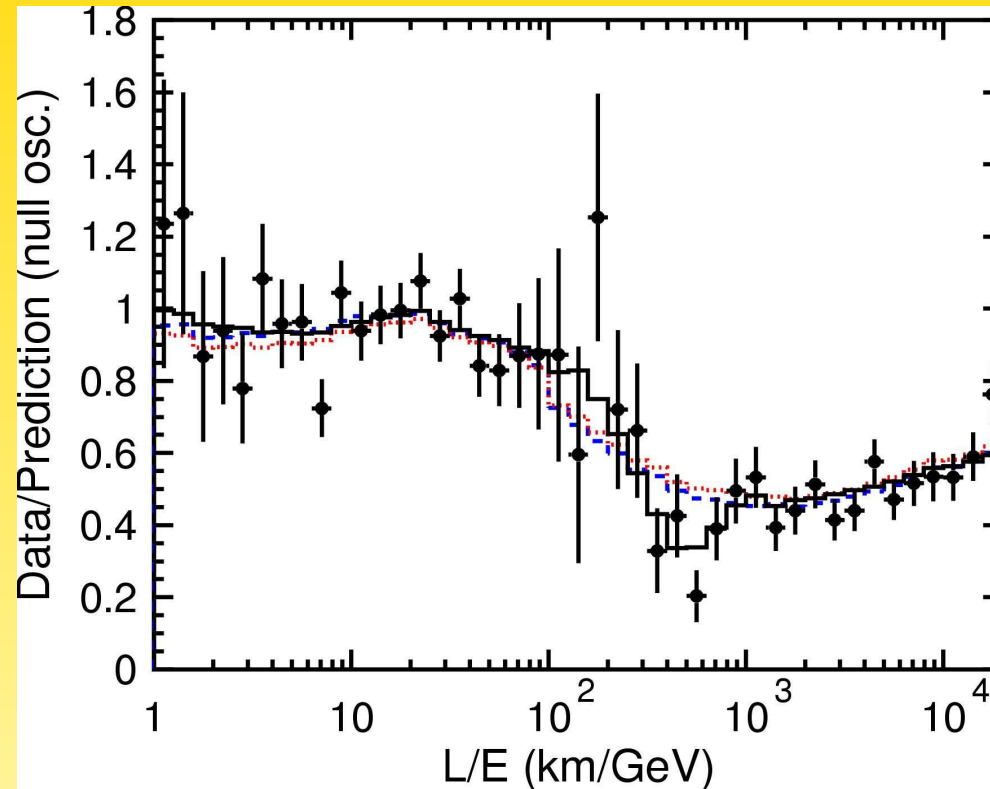
SuperKamiokande







Atmospheric Neutrinos



Dip at $L/E \simeq 500$ km/GeV \Rightarrow **Oscillatory Behavior!!**
(3.8 σ evidence for ν_τ appearance)

Testing Atmospheric Neutrinos with Accelerators: K2K, MINOS, T2K, OPERA, No ν A

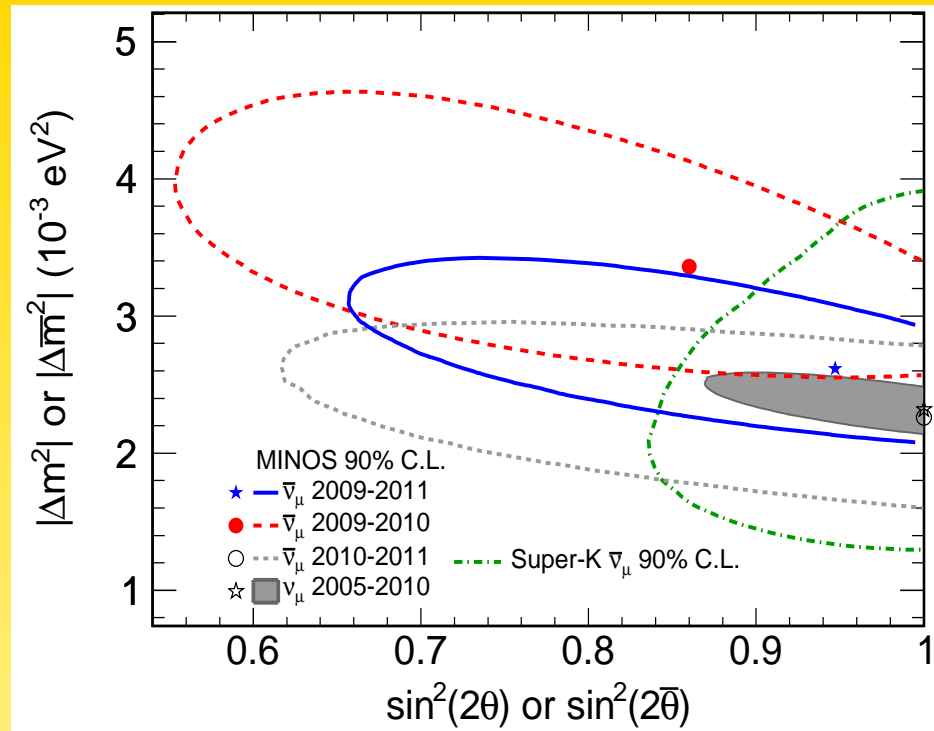
Proton beam

$$p + X \rightarrow \pi^\pm, K^\pm \rightarrow \pi^\pm \rightarrow \nu_\mu^{(-)} \quad \text{with } E \simeq \text{GeV}$$

If $L \simeq 100$ km:

$$\frac{\Delta m_A^2}{E} L \sim 1 \Rightarrow \text{atmospheric } \nu \text{ parameters!!}$$

$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2 2\theta_{23} \sin^2 \frac{\Delta m_{31}^2}{4E} L$$



Results of fits give

$$\sin^2 \theta_{23} \simeq 0.50 \quad \text{maximal mixing?!}$$

$$|\Delta m_{31}^2| \equiv \Delta m_A^2 \simeq 2.5 \times 10^{-3} \text{ eV}^2 \simeq 30 \Delta m_{\odot}^2$$

NEW TREND (2012): less-than-maximal θ_{23} ?

Non-maximal θ_{23} ?

LBL accelerator experiments have octant-asymmetric amplitude (plus higher order terms with sensitivity to δ and $\text{sgn}(\Delta m_A^2)$)

$$P(\nu_\mu \rightarrow \nu_\mu) \propto \cos^2 \theta_{13} \sin^2 \theta_{23} (1 - \cos^2 \theta_{13} \sin^2 \theta_{23})$$

$$P(\nu_\mu \rightarrow \nu_e) \propto \cos^2 \theta_{13} \sin^2 \theta_{13} \sin^2 \theta_{23}$$

MINOS and T2K disappearance data most important, preference for $\theta_{23} \neq \pi/4$

atmospheric data:

$$N_e - N_e^0 \propto (R \sin^2 \theta_{23} - 1) f(\Delta m_A^2, \theta_{13}) + (R \cos^2 \theta_{23} - 1) g(\Delta m_\odot^2, \theta_{12}) \\ - C \sin \theta_{13} \sin \theta_{23} \cos \theta_{23} \cos \delta$$

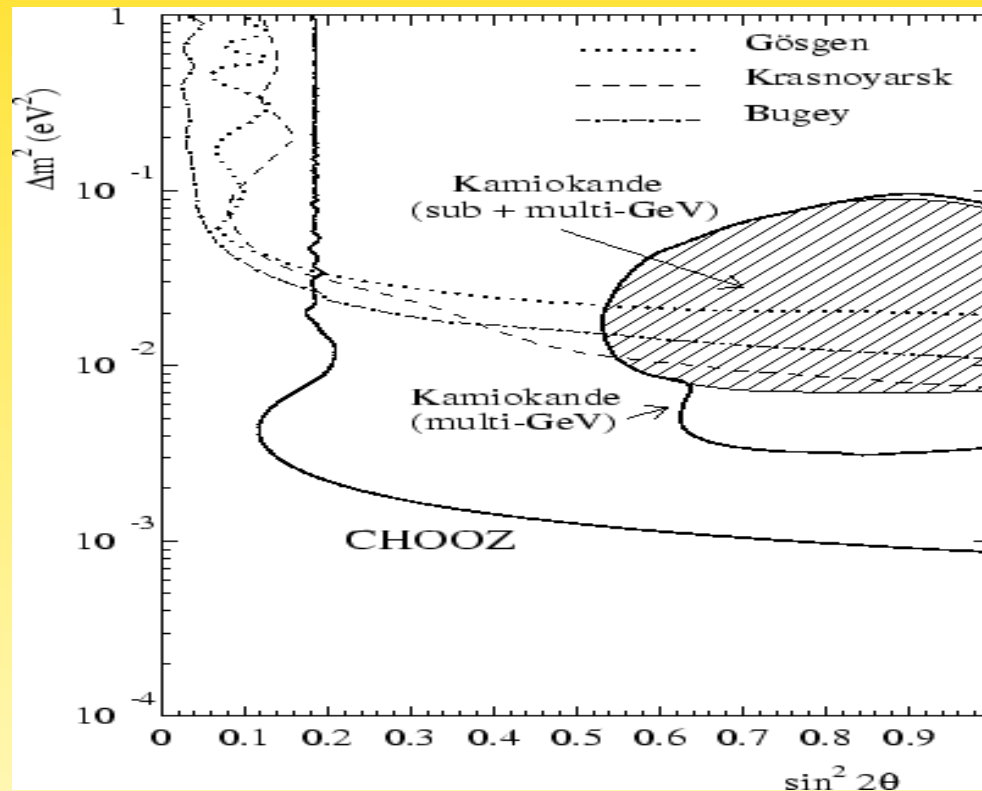
slight electron excess in sub-GeV atmospheric data sets easier explained by

$$\cos \delta = -1 \text{ and } \theta_{23} < \pi/4$$

The third mixing: Short-Baseline Reactor Neutrinos

$E_\nu \simeq \text{MeV}$ and $L \simeq 0.1 \text{ km}$:

$\frac{\Delta m_A^2}{E} L \sim 1 \Rightarrow$ atmospheric ν parameters!!



$$P_{ee} = 1 - \sin^2 2\theta_{13} \sin^2 \frac{\Delta m_A^2}{4E} L$$

3 families: $U = R_{23} \tilde{R}_{13} R_{12} P$

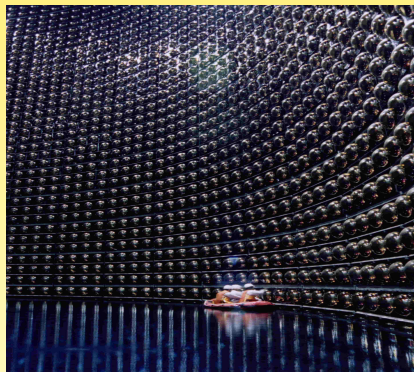
$$\begin{aligned}
 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} P \\
 &= \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{i\delta} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{-i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{-i\delta} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{-i\delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{-i\delta} & c_{23} c_{13} \end{pmatrix} P
 \end{aligned}$$

with $P = \text{diag}(1, e^{i\alpha}, e^{i\beta})$

$$U = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{i\delta} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{-i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{-i\delta} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{-i\delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{-i\delta} & c_{23} c_{13} \end{pmatrix} =$$

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{atmospheric and LBL accelerator}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{\text{SBL reactor}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{solar and LBL reactor}}$$

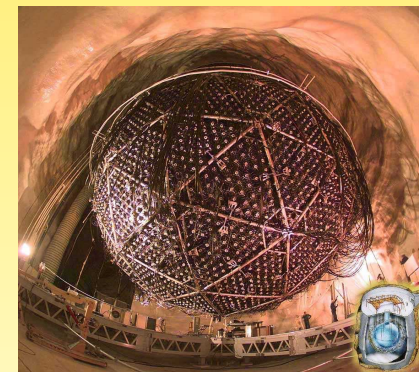
atmospheric and
LBL accelerator



SBL reactor



solar and
LBL reactor



$$U = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{i\delta} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{-i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{-i\delta} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{-i\delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{-i\delta} & c_{23} c_{13} \end{pmatrix} =$$

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{atmospheric and LBL accelerator}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{\text{SBL reactor}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{solar and LBL reactor}}$$

atmospheric and
LBL accelerator

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} \\ 0 & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{pmatrix}$$

$$(\sin^2 \theta_{23} = \frac{1}{2})$$

$$\Delta m_{\text{A}}^2$$

SBL reactor

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$(\sin^2 \theta_{13} = 0)$$

$$\Delta m_{\text{A}}^2$$

solar and

LBL reactor

$$\begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{3}} & \sqrt{\frac{2}{3}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$(\sin^2 \theta_{12} = \frac{1}{3})$$

$$\Delta m_{\odot}^2$$

Tri-bimaximal Mixing

approximation to PMNS matrix:

$$U_{\text{TBM}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix} \quad \text{Harrison, Perkins, Scott (2002)}$$

with mass matrix

$$(m_\nu)_{\text{TBM}} = U_{\text{TBM}}^* m_\nu^{\text{diag}} U_{\text{TBM}}^\dagger = \begin{pmatrix} A & B & B \\ \cdot & \frac{1}{2}(A+B+D) & \frac{1}{2}(A+B-D) \\ \cdot & \cdot & \frac{1}{2}(A+B+D) \end{pmatrix}$$

$$A = \frac{1}{3} (2m_1 + m_2 e^{-2i\alpha}) , \quad B = \frac{1}{3} (m_2 e^{-2i\alpha} - m_1) , \quad D = m_3 e^{-2i\beta}$$

\Rightarrow Flavor symmetries...

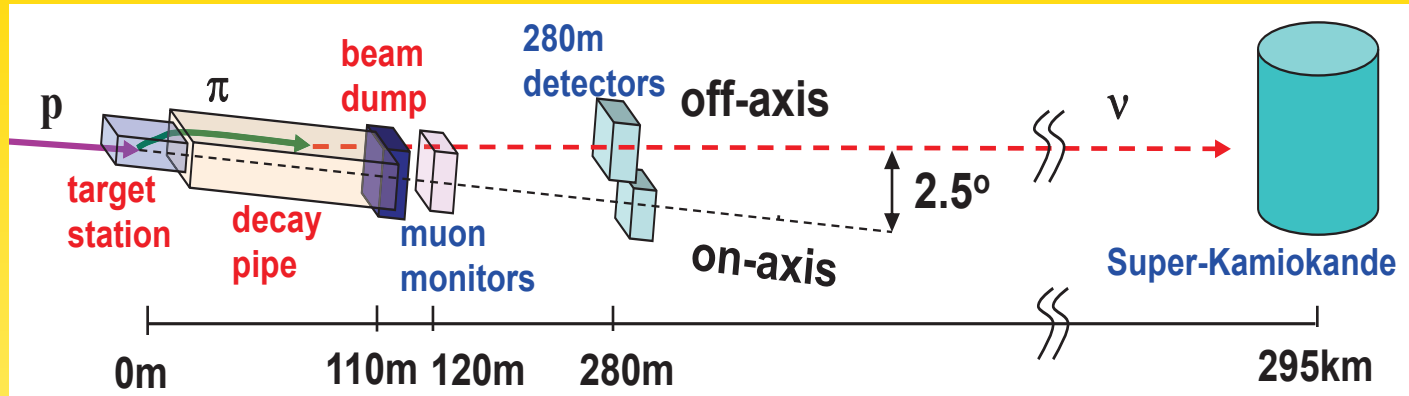
Tri-bimaximal Mixing

$$U_{\text{TBM}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}$$

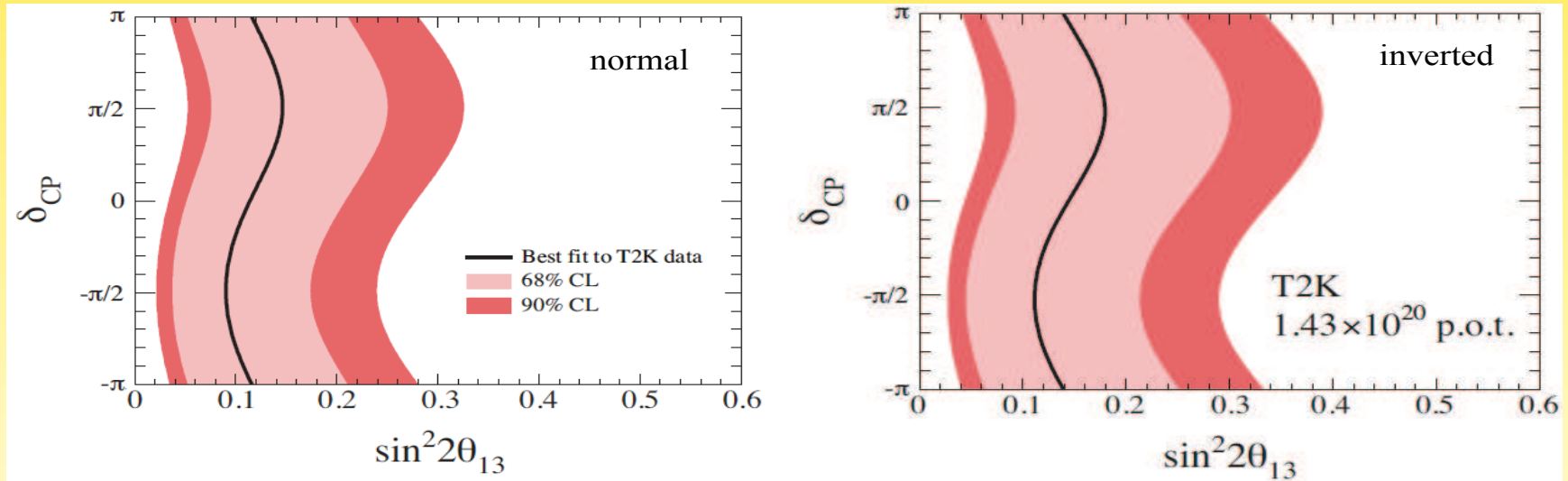
Harrison, Perkins, Scott (2002)

This was still okay till end of 2010...

T2K: 2.5σ



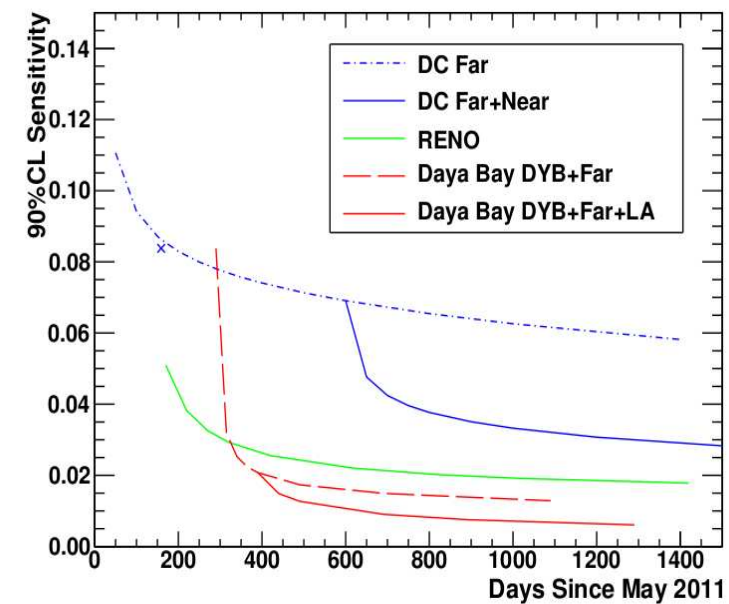
$$P(\nu_\mu \rightarrow \nu_e) \simeq \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 \frac{\Delta m_A^2 L}{4E}$$



More data

- MINOS: 1.7σ
- Double Chooz: $0.017 < \sin^2 2\theta_{13} < 0.16$ at 90 % C.L.

$$P_{ee} = 1 - \sin^2 2\theta_{13} \sin^2 \frac{\Delta m_A^2 L}{4E}$$



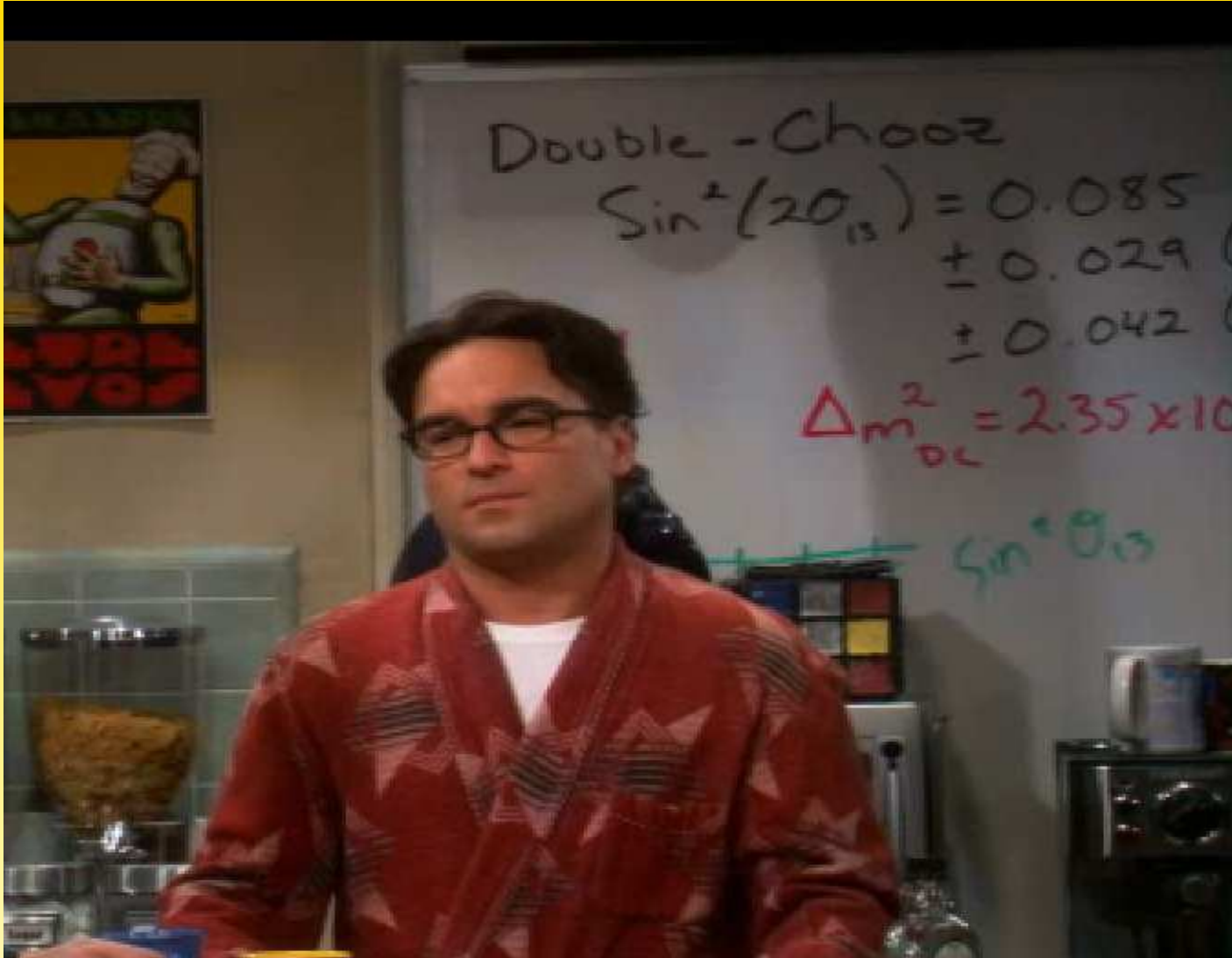
θ_{13} : status

dominated by reactor experiments:

Double Chooz: $\sin^2 2\theta_{13} = 0.109 \pm 0.030 \pm 0.025 \neq 0$ at 2.9σ

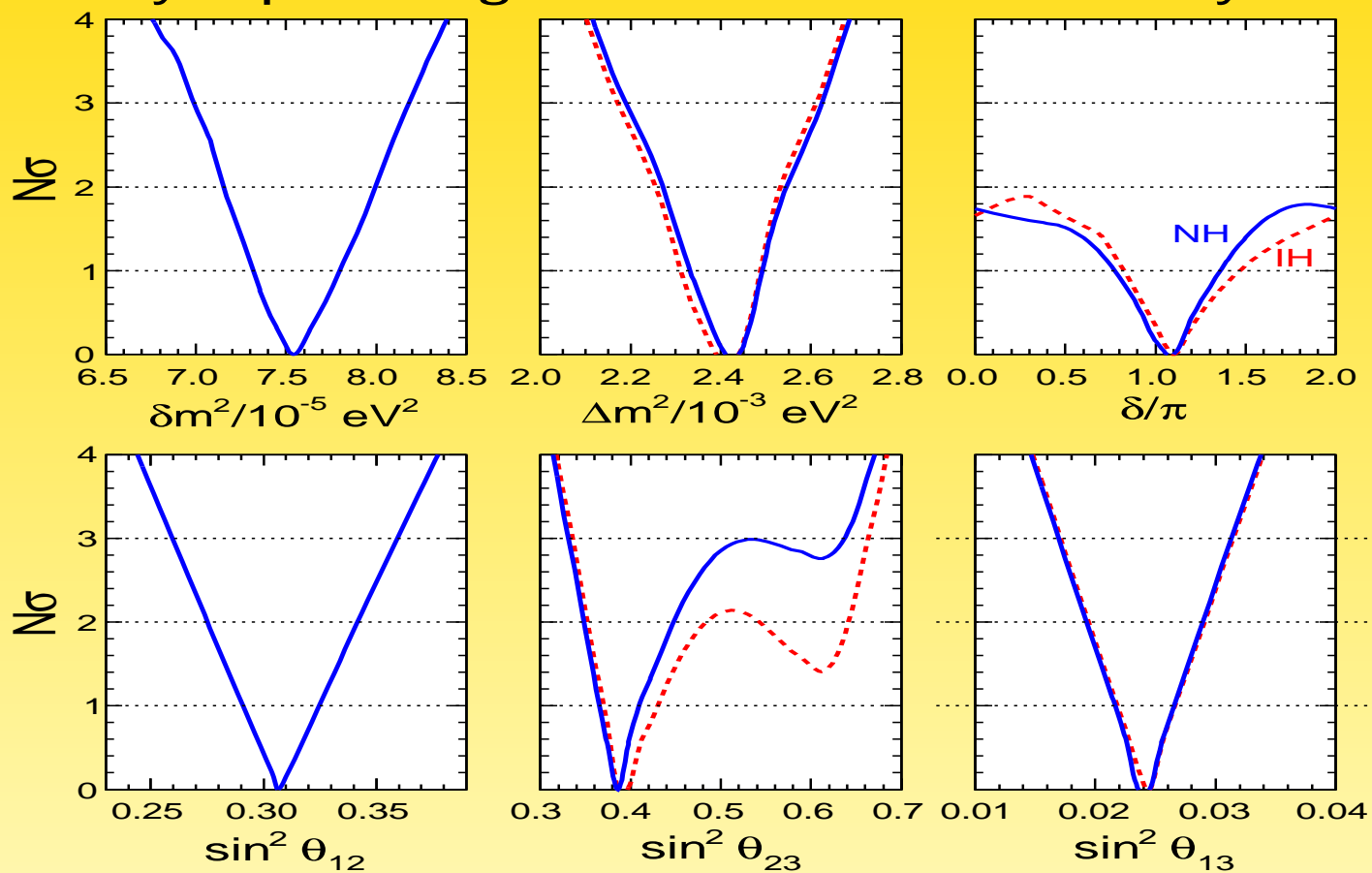
Daya Bay: $\sin^2 2\theta_{13} = 0.089 \pm 0.010 \pm 0.005 \neq 0$ at 7.7σ

RENO: $\sin^2 2\theta_{13} = 0.113 \pm 0.013 \pm 0.019 \neq 0$ at 4.9σ



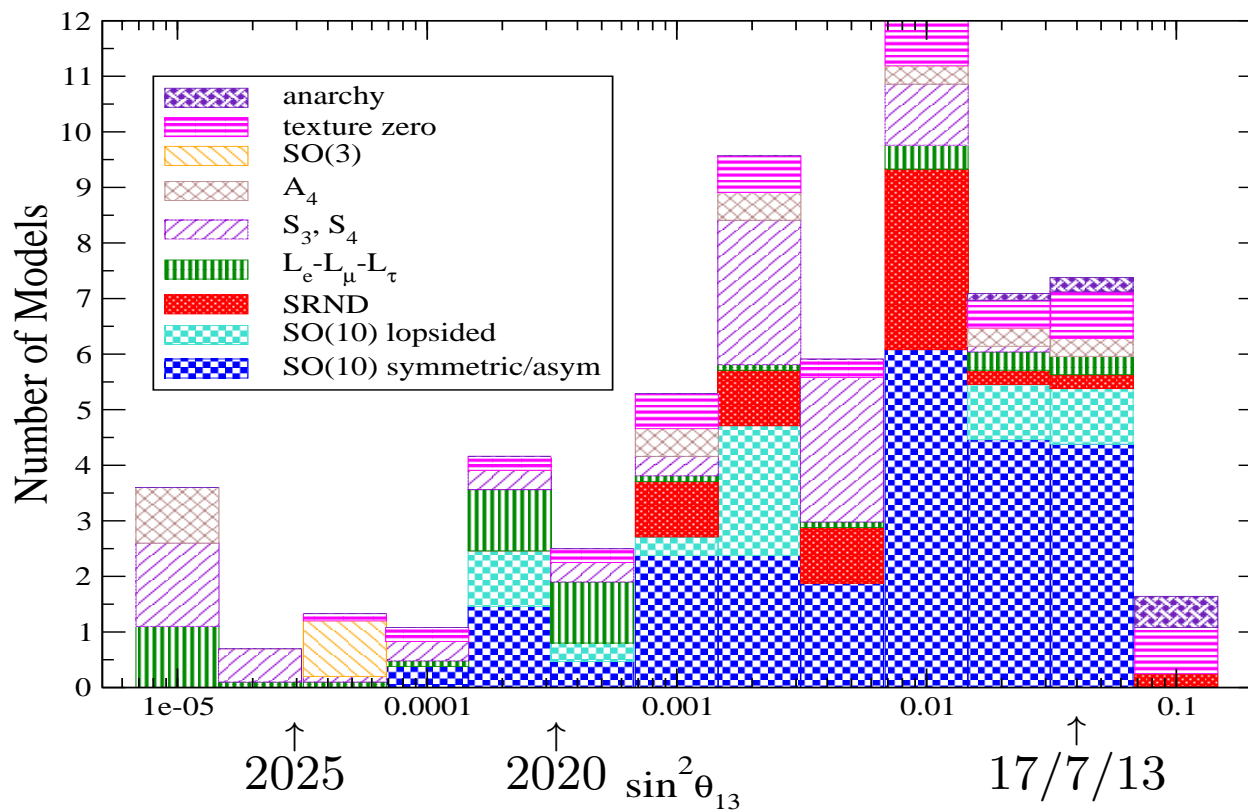
at least, Double Chooz are the only ones who made it to Big Bang Theory...

Synopsis of global 3ν oscillation analysis



What's that good for?

Predictions of All 63 Models

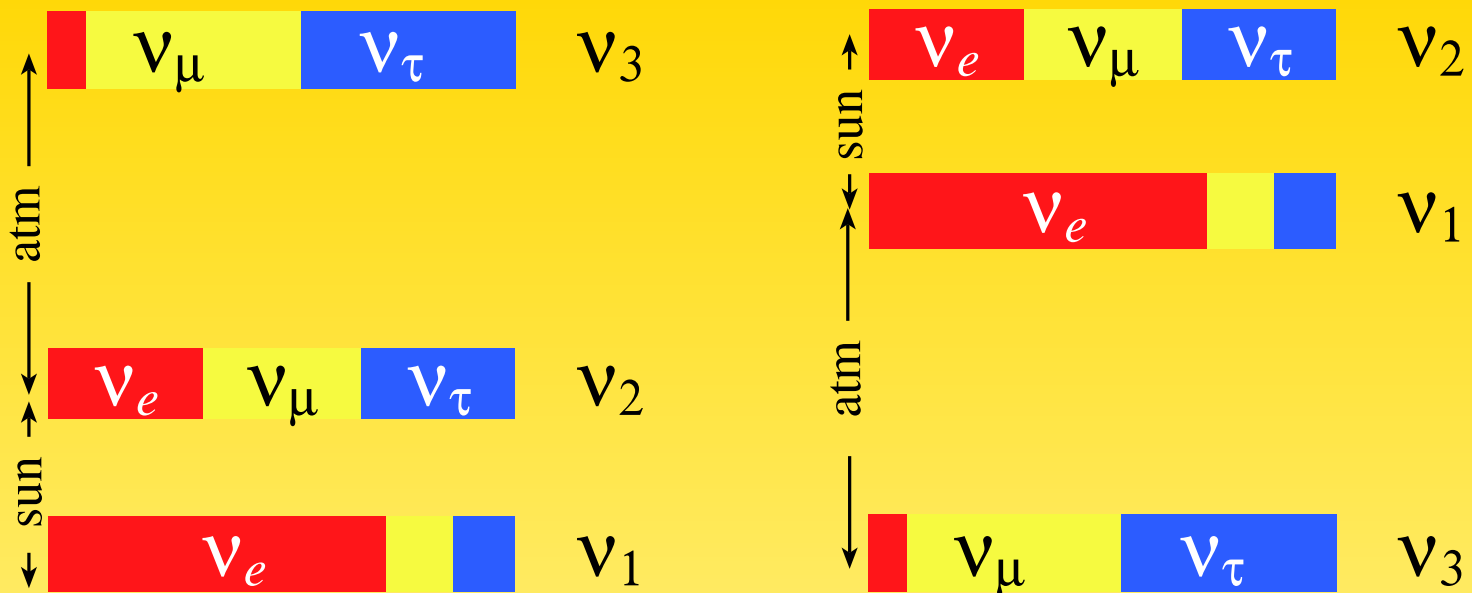


Albright, Chen

CKM vs. PMNS

$$|V_{\text{CKM}}| \simeq \begin{pmatrix} 0.974 & 0.225 & 0.004 \\ 0.225 & 0.973 & 0.041 \\ 0.009 & 0.040 & 0.999 \end{pmatrix}$$

$$|U_{\text{PMNS}}| \simeq \begin{pmatrix} 0.82 & 0.55 & 0.16 \\ 0.38 & 0.57 & 0.69 \\ 0.38 & 0.57 & 0.68 \end{pmatrix}$$



$$|U|^2 \simeq \begin{pmatrix} 0.795 \dots 0.846 & 0.513 \dots 0.585 & 0.126 \dots 0.178 \\ 0.205 \dots 0.543 & 0.416 \dots 0.730 & 0.579 \dots 0.808 \\ 0.215 \dots 0.548 & 0.409 \dots 0.725 & 0.567 \dots 0.800 \end{pmatrix}$$

- normal ordering: $\Delta m_{31}^2 > 0$
- inverted ordering: $\Delta m_{31}^2 < 0$

Contents

II Neutrino Oscillations

II1) The PMNS matrix

II2) Neutrino oscillations in vacuum and matter

II3) Results and their interpretation – what have we learned?

II4) Prospects – what do we want to know?

II4) Prospects – what do we want to know?

9 physical parameters in m_ν

- θ_{12} and $m_2^2 - m_1^2$ (or θ_\odot and Δm_\odot^2)
- θ_{23} and $|m_3^2 - m_2^2|$ (or θ_A and Δm_A^2)
- θ_{13} (or $|U_{e3}|$)
- m_1, m_2, m_3
- $\text{sgn}(m_3^2 - m_2^2)$
- Dirac phase δ
- Majorana phases α and β (or α_1 and α_2 , or ϕ_1 and ϕ_2 , or...)

The future: open issues for neutrinos oscillations

Look for *three flavor effects*:

- precision measurements
 - how maximal is θ_{23} ? how small/large is U_{e3} ?

- sign of Δm_{32}^2 ?

$$\tan 2\theta_m = f(\text{sgn}(\Delta m^2))$$

- is there CP violation?

- Problems:

- two *small* parameters: $\Delta m_{\odot}^2 / \Delta m_{\text{A}}^2 \simeq 1/30$ and $|U_{e3}| \lesssim 0.2$
- 8-fold degeneracy for fixed L/E and $\nu_e \rightarrow \nu_\mu$ channels

Degeneracies

Expand 3 flavor oscillation probabilities in terms of $R = \Delta m_{\odot}^2 / \Delta m_{\text{A}}^2$ and $|U_{e3}|$:

$$\begin{aligned}
 P(\nu_e \rightarrow \nu_\mu) \simeq & \sin^2 2\theta_{13} \sin^2 \theta_{23} \frac{\sin^2 (1-\hat{A})\Delta}{(1-\hat{A})^2} + R^2 \sin^2 2\theta_{12} \cos^2 \theta_{23} \frac{\sin^2 \hat{A}\Delta}{\hat{A}^2} \\
 & + \sin \delta \sin 2\theta_{13} R \sin 2\theta_{12} \cos \theta_{13} \sin 2\theta_{23} \sin \Delta \frac{\sin \hat{A}\Delta \sin (1-\hat{A})\Delta}{\hat{A}(1-\hat{A})} \\
 & + \cos \delta \sin 2\theta_{13} R \sin 2\theta_{12} \cos \theta_{13} \sin 2\theta_{23} \cos \Delta \frac{\sin \hat{A}\Delta \sin (1-\hat{A})\Delta}{\hat{A}(1-\hat{A})}
 \end{aligned}$$

with $\hat{A} = 2\sqrt{2} G_F n_e E / \Delta m_{\text{A}}^2$ and $\Delta = \frac{\Delta m_{\text{A}}^2}{4E} L$

- $\theta_{23} \leftrightarrow \pi/2 - \theta_{23}$ degeneracy
- θ_{13} - δ degeneracy
- δ - $\text{sgn}(\Delta m_{\text{A}}^2)$ degeneracy

Solutions: more channels, different L/E , high precision,...

Degeneracies

Expand 3 flavor oscillation probabilities in terms of $R = \Delta m_{\odot}^2 / \Delta m_A^2$ and $|U_{e3}|$:

$$P(\nu_e \rightarrow \nu_\mu) \simeq \sin^2 2\theta_{13} \sin^2 \theta_{23} \frac{\sin^2 (1-\hat{A})\Delta}{(1-\hat{A})^2} + R^2 \sin^2 2\theta_{12} \cos^2 \theta_{23} \frac{\sin^2 \hat{A}\Delta}{\hat{A}^2}$$

$$+ \sin \delta \sin 2\theta_{13} R \sin 2\theta_{12} \cos \theta_{13} \sin 2\theta_{23} \sin \Delta \frac{\sin \hat{A}\Delta \sin (1-\hat{A})\Delta}{\hat{A}(1-\hat{A})}$$

$$+ \cos \delta \sin 2\theta_{13} R \sin 2\theta_{12} \cos \theta_{13} \sin 2\theta_{23} \cos \Delta \frac{\sin \hat{A}\Delta \sin (1-\hat{A})\Delta}{\hat{A}(1-\hat{A})}$$

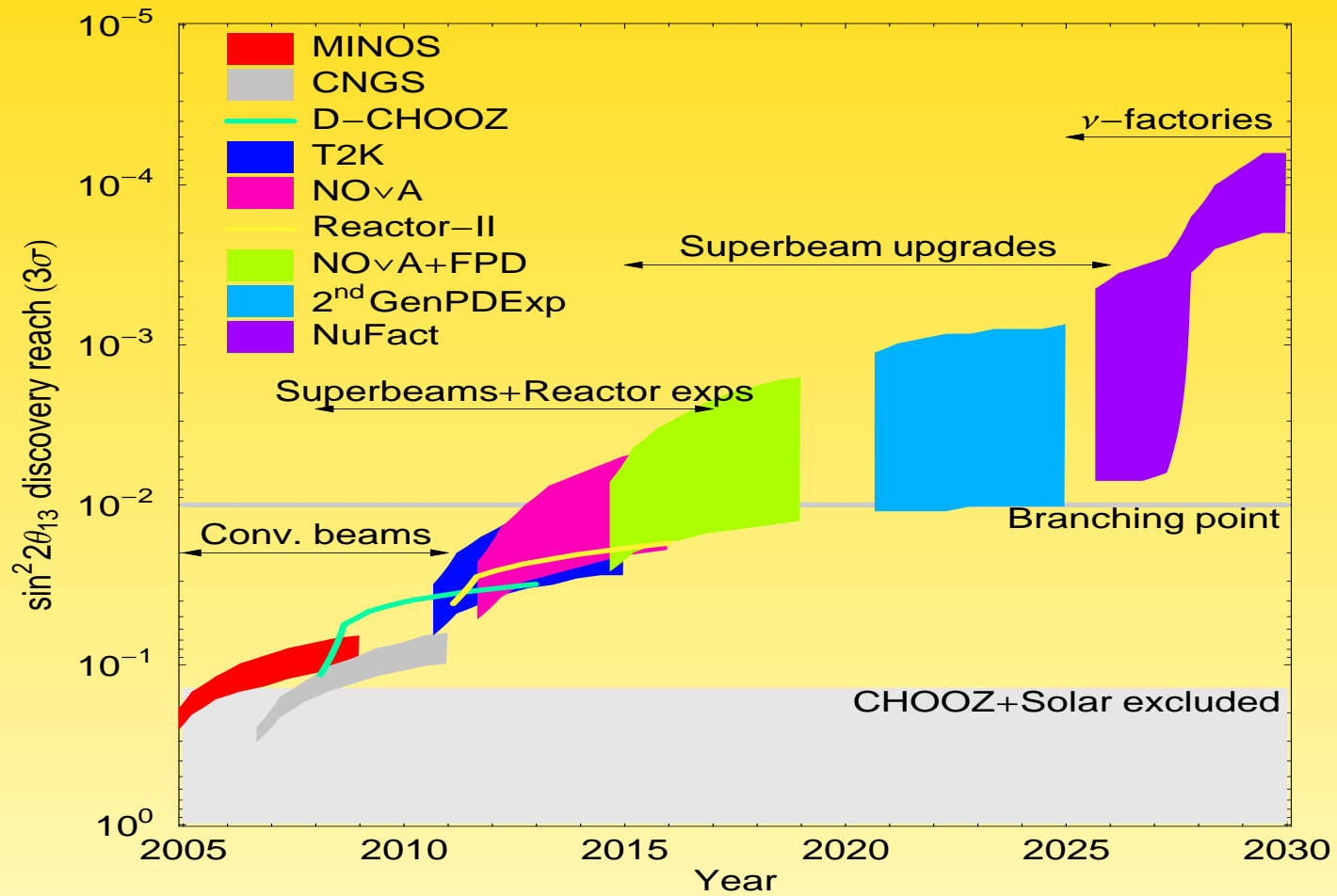
$$\text{with } \hat{A} = 2\sqrt{2} G_F n_e E / \Delta m_A^2 \text{ and } \Delta = \frac{\Delta m_A^2}{4E} L$$

If $\hat{A} \Delta = \pi$:

$$P(\nu_e \rightarrow \nu_\mu) \simeq \sin^2 2\theta_{13} \sin^2 \theta_{23} \frac{\sin^2 (1-\hat{A})\Delta}{(1-\hat{A})^2}$$

This is the “magic baseline” of $L = \frac{\sqrt{2}\pi}{G_F n_e} \simeq 7500$ km

Typical time scale

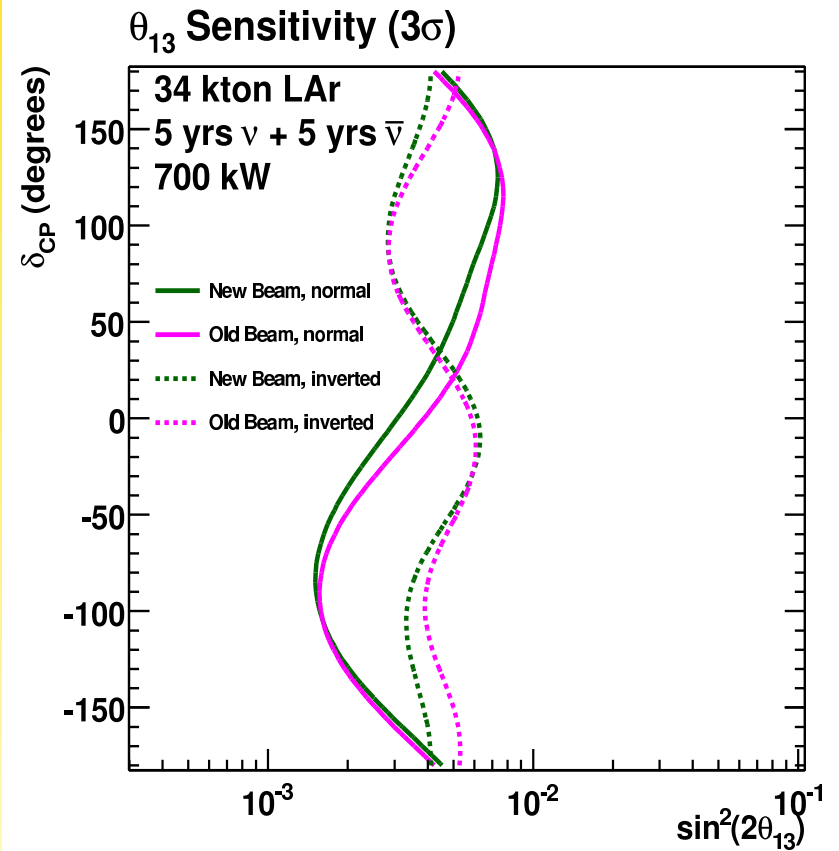
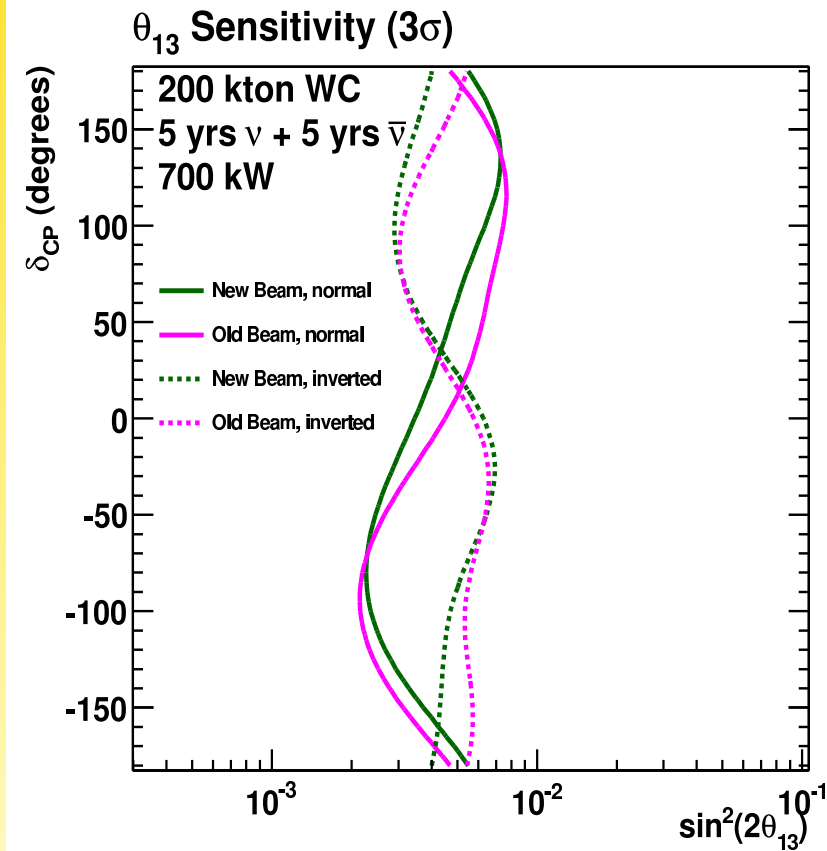


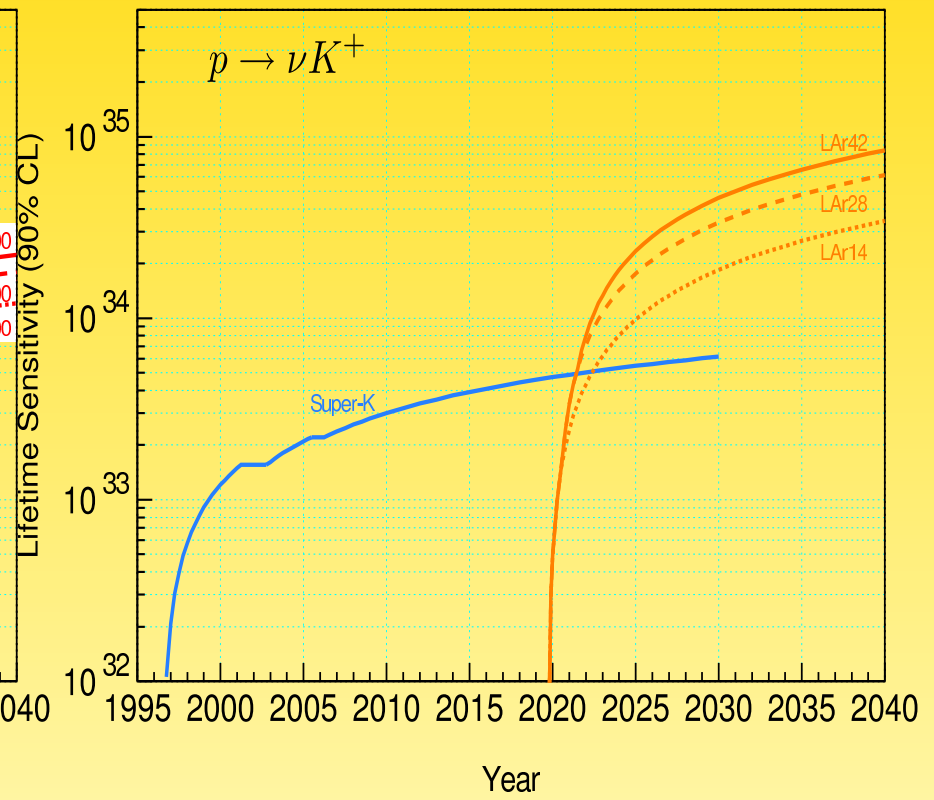
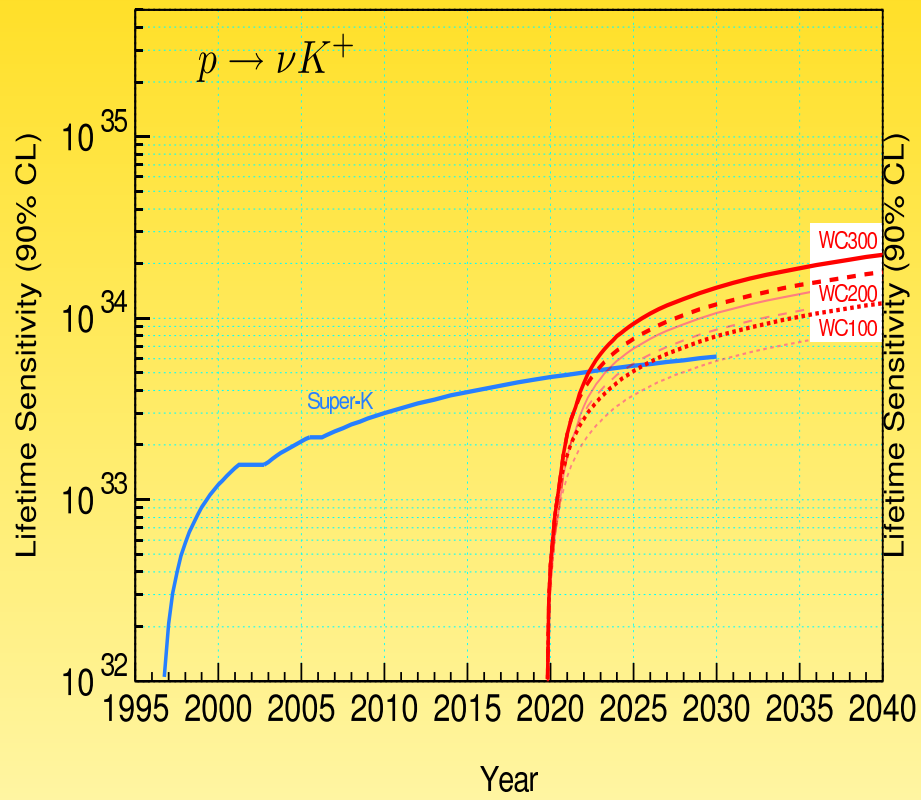
Future experiments

- what detector?
 - Water Cerenkov?
 - liquid scintillator?
 - liquid argon?
- Neutrino Physics
 - oscillations (hierarchy, CP, precision)
 - non-standard physics (NSIs, unitarity violation, steriles, extra forces, . . .)
- other physics
 - SN (burst and relic)
 - geo-neutrinos
 - p -decay

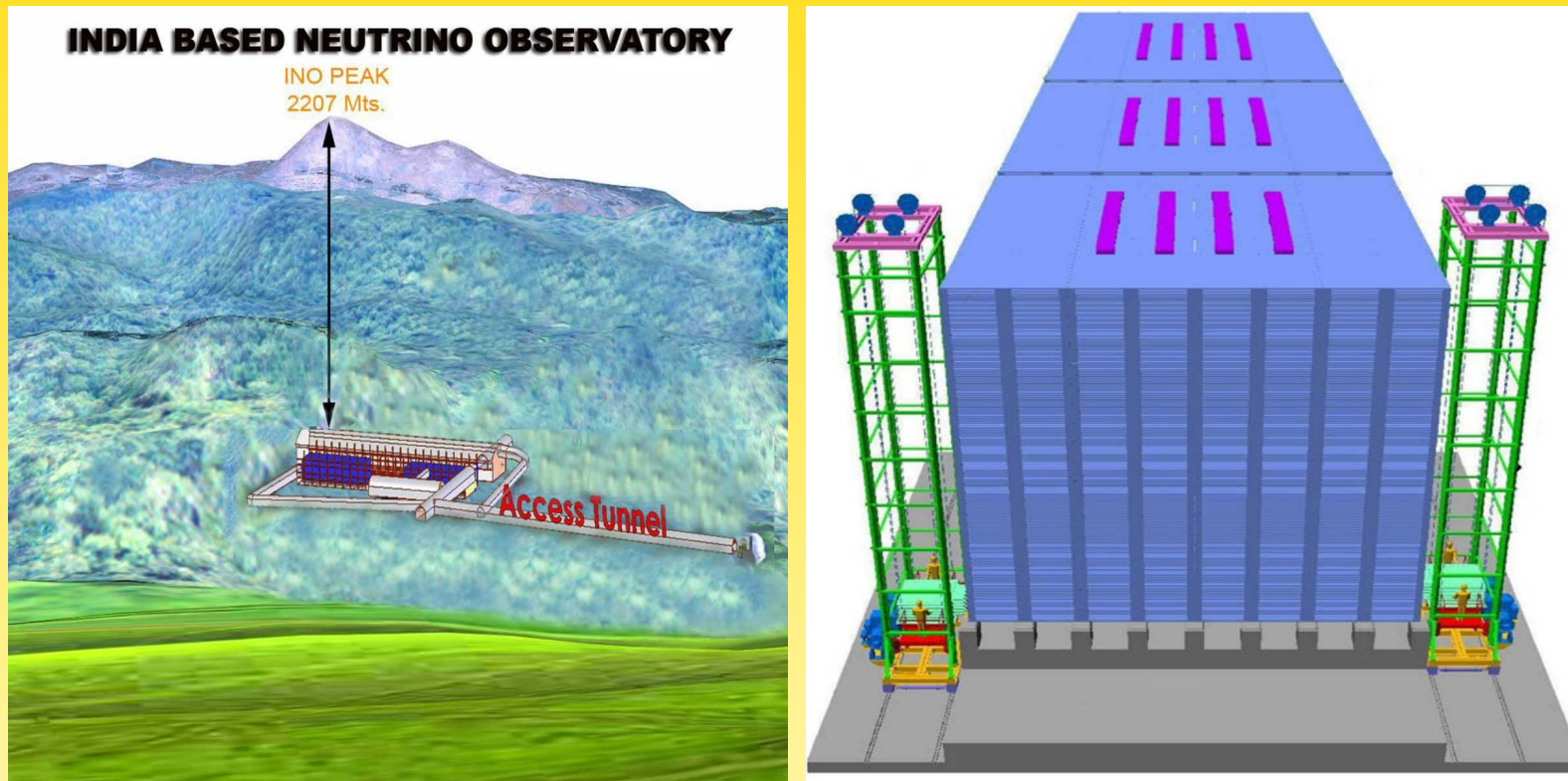
Example LBNE

FNAL \rightarrow Homestake, $L = 1300$ km



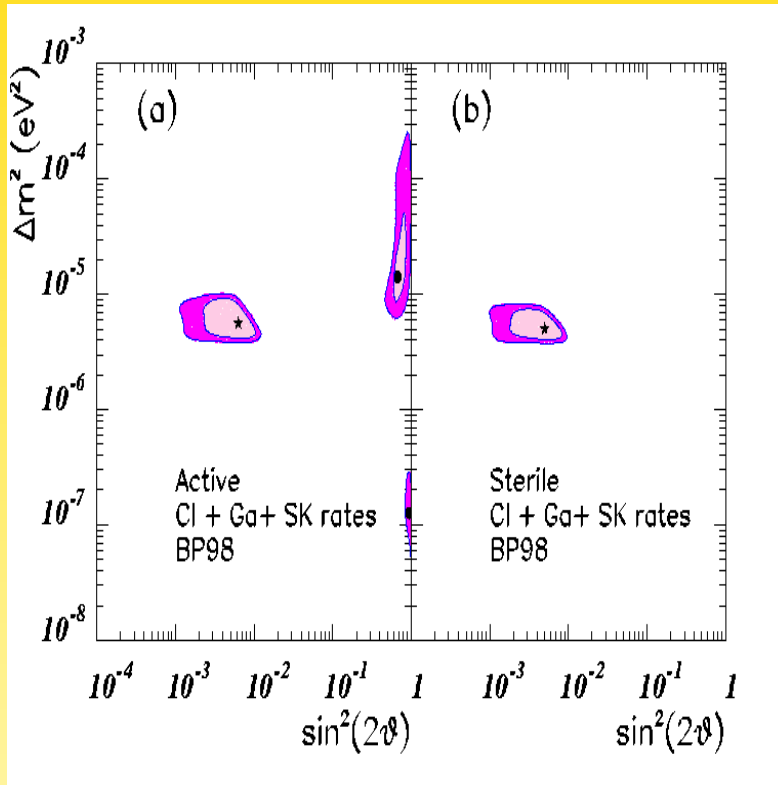


Example ICAL at INO

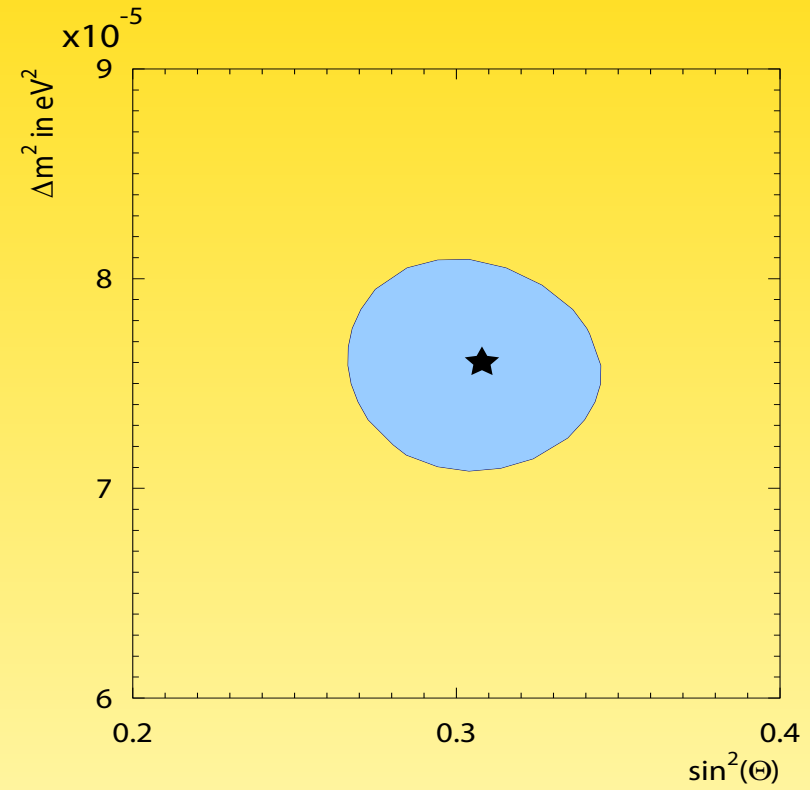


7300 km from CERN, 6600 km from JHF at Tokai

Precision era!



1998



today

Summary

