Recap:

$$-\mathcal{L}_Y = \left[\overline{d'_L} \; M^{(d)} \; d'_R + \overline{u'_L} \; M^{(u)} \; u'_R \right]$$

with $d'_{L,R} = \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}_{L,R}$ $\quad u'_{L,R} = \begin{pmatrix} u' \\ c' \\ t' \end{pmatrix}_{L,R}$

Flavor states with mass matrices

diagonalize (= basis transformation)

$$\Rightarrow \boxed{\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} W_\mu^+ \overline{u_L} \; V \gamma^\mu d_L}$$

$d'_L = \mathcal{U}_d \, d_L$
$u'_L = \mathcal{U}_u \, u_L$ $\quad ; V = \mathcal{U}_u^+ \mathcal{U}_d$

with $\mathcal{U}_d^+ M^{(d)} M^{(d)+} \mathcal{U}_d = \mathrm{diag}(m_d^2, \dots)$

Flavor $\quad$ mass

CKM-matrix

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

elements have to be determined by experiment

V is complex for $N \geq 3$ families

$N=3$: $\underset{\text{„angles"}}{3} + \underset{\text{„phase"}}{1}$ parameters

Explicit example: 2 generations

V would have $2^2 = 4$ parameters (complex + unitary)

can be written:

$$V = \begin{pmatrix} e^{i\omega_1} & 0 \\ 0 & e^{i\omega_2} \end{pmatrix} \begin{pmatrix} e^{i\frac{\eta}{2}} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} c_1 & s_1 \\ -s_1 & c_1 \end{pmatrix} \begin{pmatrix} e^{-i\frac{\eta}{2}} & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_1 e^{i\omega_1} & s_1 e^{i(\omega_1+\frac{\eta}{2})} \\ -s_1 e^{i(\omega_2-\frac{\eta}{2})} & c_1 e^{i\omega_2} \end{pmatrix}$$

with $c_1 = \cos\Theta_1$
$s_1 = \sin\Theta_1$

$$\mathcal{L}_{CC} \sim (\bar{u}, \bar{c}) \begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

$$= \bar{u} d\, c_1 e^{i\omega_1} + \bar{u} s\, s_1 e^{i(\omega_1+\frac{\eta}{2})} + \bar{c} d\, (-s_1) e^{i(\omega_2-\frac{\eta}{2})}$$
$$+ \bar{c} s\, c_1 e^{i\omega_2}$$

phases can be removed by „rephasing"

$$u \rightarrow e^{i(\omega_1+\eta)} u$$

$$d \rightarrow e^{i\eta} d$$

$$c \rightarrow e^{i\omega_2} c$$

and V is simply $V = \begin{pmatrix} \cos\Theta & \sin\Theta \\ -\sin\Theta & \cos\Theta \end{pmatrix}$

Cabibbo angle: e.g. $\mu\text{-decay} \propto G_F^2$

$\beta\text{-decay} \propto G_F^2 V_{ud}^2 = G_F^2 \cos^2\Theta$

$\sin\Theta = 0.23$

$\sin^2\Theta \simeq 0.05$

another explicit example:

suppose $M^{(u)}$ diagonal and

$$M^{(d)} = \begin{pmatrix} 0 & a \\ a & b \end{pmatrix}$$ with a, b real and positive

$$\Rightarrow V^{+} M^{(d)} V^{*} = D = \mathrm{diag}(m_d, m_s) \Rightarrow V D V^{T} = M^{(d)}$$

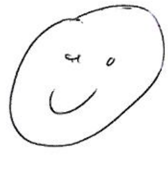
$$\Rightarrow \boxed{(V D V^{T})_{11} = 0}$$ with $V = \begin{pmatrix} c & s \\ -s & c \end{pmatrix}$, or, better:

$$V = \begin{pmatrix} c & s e^{-i\delta} \\ -s e^{i\delta} & c \end{pmatrix} \Rightarrow V^{T} = \begin{pmatrix} c & -s e^{i\delta} \\ s e^{-i\delta} & c \end{pmatrix}$$

$$\Rightarrow \dots \; m_d c^2 + m_s s^2 e^{-2i\delta} = 0$$ choose $\delta = \frac{\pi}{2}$

$$\Rightarrow m_d c^2 - m_s s^2 = 0 \Rightarrow \frac{m_d}{m_s} \simeq \sin^2\theta_c$$

or $$\boxed{\sin\theta_c \simeq \sqrt{\frac{m_d}{m_s}}}$$



$$\downarrow \qquad\qquad \downarrow$$

$$0.23 \qquad\qquad \sqrt{\frac{5}{100}} \simeq 0.22$$

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other extreme case ($\leftrightarrow$ neutrinos)

$$M = \begin{pmatrix} 0 & a \\ a & 0 \end{pmatrix} \Rightarrow \sqrt{\tfrac{1}{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & a \\ a & 0 \end{pmatrix} \sqrt{\tfrac{1}{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} a & 0 \\ 0 & -a \end{pmatrix}$$

mixing <u>not</u> related to masses

Parametrization of V

is arbitrary and unphysical

one possible choice: $V = R_{23}(\theta_{23})\, R_{13}(\theta_{13}, \delta)\, R_{12}(\theta_{12})$

with $R_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$

$$R_{12} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_{13} = \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & 1 \end{pmatrix}$$ where I put the phase is arbitrary

$c_{ij} = \cos\theta_{ij} \qquad s_{ij} = \sin\theta_{ij}$

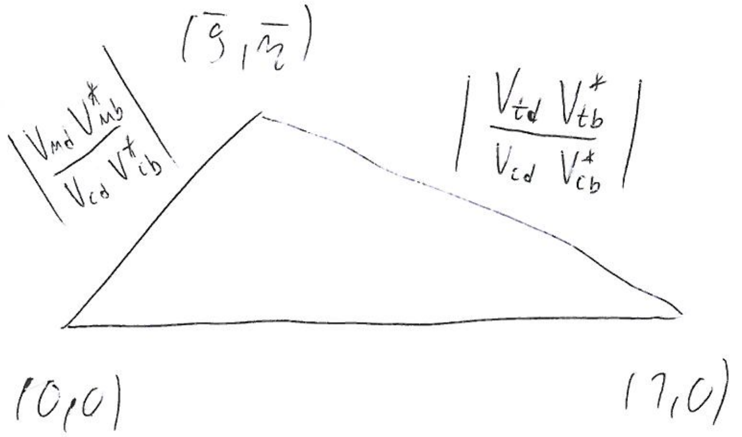
more intuitive (Wolfenstein)

$$V = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^3)$$

individual elements measured by various methods.

test consistency be reconstructing <u>unitary triangles</u> from $VV^\dagger = \mathbb{1}$ or $V^\dagger V = \mathbb{1}$

most convenient one: 1st and third row
(all sides of same length $\sim \lambda^3$)



Insert: what about neutrinos?

$-\mathcal{L} = \overline{e'_L} \; M^{(\ell)} \; e'_R$ with $e'_{L/R} = \begin{pmatrix} e' \\ \mu' \\ \tau' \end{pmatrix}_{L/R}$, $\nu'_L = \begin{pmatrix} \nu_e' \\ \nu_\mu' \\ \nu_\tau' \end{pmatrix}_L$

$$= \underbrace{\overline{e'_L}\, U_\ell}_{\overline{e_L}} \underbrace{U_\ell^\dagger M^{(\ell)} V_\ell}_{\text{diag}} \underbrace{V_\ell^\dagger e'_R}_{\equiv e_R}$$

$$\to -\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} W_\mu^+ \underbrace{\overline{e'_L}\, U_\ell}_{\overline{e_L}} U_\ell^\dagger \gamma^\mu U_\nu \underbrace{U_\nu^\dagger \nu'_L}_{\nu_L}$$

Rotation with U_ν has no effect if there is no mass term for neutrinos

$\Rightarrow$ choose $U_\nu = U_\ell \Rightarrow V = \mathbb{1}$ there is no analogue of CKM matrix for leptons!

[If $m_\nu \neq 0$: there will be a mixing matrix:

Pontecorvo
Maki
Nakagawa
Sakata

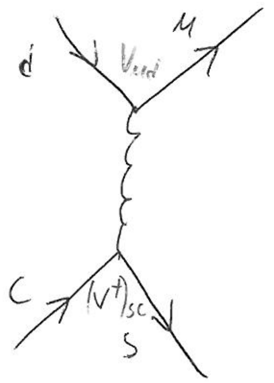
} Matrix ($\rightarrow$ Wednesday)]

CP Violation

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} \Big[W_\mu^+ \, \underset{u,c}{\bar{u}_L} \, \gamma^\mu (1-\gamma_5) \, V \, \underset{d,s}{d_L}$$

$$+ W_\mu^- \, \underset{d,s}{\bar{d}_L} \, \gamma^\mu \, V^+ \, \underset{u,c}{u_L} \Big] \qquad \text{is h.c.}$$

u: incoming up-quark or outgoing up-antiquark
$\bar{u}$ outgoing " incoming " — Stückelberg

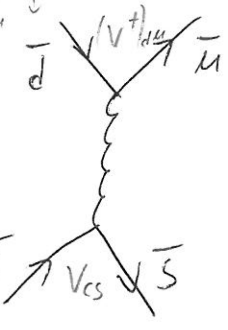


$$\mathcal{M}(dc \to us) \equiv \mathcal{M} \sim V_{ud} V_{cs}^* \, (\bar{u}_u \gamma^\mu (1-\gamma_5) u_d) \times$$
$$(\bar{u}_s \gamma_\mu (1-\gamma_5) u_c)$$

$$\left[\Rightarrow \mathcal{M}(\bar{u}\bar{s} \to \bar{d}\bar{c}) = \mathcal{M} \qquad (\text{Stückelberg}) \right]$$

now consider anti-particle process $\bar{d}\bar{c} \to \bar{u}\bar{s}$: (CPT!)

incoming down antiquark
$\Rightarrow \bar{d} V^+ u$



incoming up-like antiquark
$\Downarrow$
$\bar{u} V d$

$$\mathcal{M}(\bar{d}\bar{c} \to \bar{u}\bar{s}) \equiv \mathcal{M}' \sim V_{ud}^* V_{cs} \, (\bar{u}_d \gamma^\mu (1-\gamma_5) u_u) \times$$
$$(\bar{u}_c \gamma_\mu (1-\gamma_5) u_s)$$
$$\left[= \mathcal{M}(us \to dc) \quad (\text{Stückelberg}) \right]$$

$$\Rightarrow \mathcal{M}' = \mathcal{M}^+$$

underlying reason: total Hamiltonian $\underbrace{\mathcal{H}_{tot}}_{\substack{\text{contains process} \\ \text{plus CPT-transformed} \\ \text{process}}} \sim \mathcal{M} + \mathcal{M}^+$ must be hermitian

$\mathcal{M} \sim \langle f | \mathcal{H} | i \rangle$; $\mathcal{M}' \sim \langle \bar{f} | \mathcal{H}^+ | \bar{i} \rangle \sim \mathcal{M}^+$

now consider the CP- transformed process:

so far: weak IA seems to be invariant: $\nu_L \xrightarrow{P} \nu_R$ ↯

C: flips sign of all charges, e.g. lepton number „charge conjugation". not really particle → anti-particle

$\nu_L \xrightarrow{C} \bar{\nu}_L$ ↯

$\nu_L \xrightarrow{CP} \bar{\nu}_R$ ✓

for a spinor: $u_i \xrightarrow{CP} P u_i^c$ with $P = \gamma^0$

$$u^c = C\bar{u}^T \quad ; \; C = i\gamma_2\gamma_0$$

$$\overline{u^c} = -u^T C^{-1}$$

a) C

$$\bar{u}_u \gamma^\mu(1-\gamma_5)u_d \longrightarrow \overline{u_u^c}\gamma^\mu(1-\gamma_5)u_d^c = -u_u^T C^{-1}(\gamma^\mu - \gamma^\mu\gamma^5)C\bar{u}_d^T$$

$$= u_u^T\left(\gamma^{\mu T} + (\gamma^\mu\gamma^5)^T\right)\bar{u}_d^T \qquad \Big/ \text{ with } \begin{array}{l} C^{-1}\gamma^\mu C = -\gamma^{\mu T} \\ C^{-1}\gamma^\mu\gamma^5 C = (\gamma^\mu\gamma^5)^T \end{array}$$

$$= u_u^T\left(\gamma^\mu(1+\gamma^5)\right)^T\bar{u}_d^T$$

$$= -\bar{u}_d\gamma^\mu(1-\gamma_5)u_u$$

b) P

with $\gamma^{0-1}\gamma^\mu(1-\gamma_5)\gamma_0 = \gamma^{\mu+}(1-\gamma_5)$, it follows

CP $\quad -\bar{u}_d\gamma^{\mu+}(1-\gamma_5)u_u$

$$\Rightarrow \mathcal{M}_{CP} = V_{ud}V_{cs}^*\left(\bar{u}_d\gamma^{\mu+}(1-\gamma_5)u_u\right)\left(\bar{u}_c\gamma_\mu^+(1-\gamma_5)u_s\right) \qquad \Big/ \begin{array}{l}\gamma^{0+} = \gamma^0 \\ \gamma^{i+} = -\gamma^i\end{array}$$

$$= V_{ud}V_{cs}^*\left(\bar{u}_d\gamma^\mu(1-\gamma_5)u_u\right)\left(\bar{u}_c\gamma_\mu(1-\gamma_5)u_s\right)$$

$\Rightarrow$ if V are complex : $\mathcal{M}_{CP} \neq \mathcal{M}^T$

cf. $\mathcal{M} \sim \langle f | \mathcal{H} | i \rangle \to \langle f | CP^{-1} \underbrace{CP \mathcal{H} CP^{-1}} CP | i \rangle \approx \mathcal{M}_{CP}$

not the antiparticle process

$\Rightarrow$ CP violation for $N \geq 3$ generations

Final remark: To generate a measurable difference between a process and its CP-transf. process, one needs a strong + weak phase (plus more than one channel $\leftrightarrow$ interference)

$\mathcal{A} = A_1 e^{i(\delta+\eta)} + A_2$ for $a \to b$

$\mathcal{A}^{CP} = A_1 e^{i(-\delta+\eta)} + A_2$ for $(a \to b)^{CP}$

δ weak $\xrightarrow{CP} -\delta$

η strong $\xrightarrow{CP} +\eta$

Asymmetry: $\Delta = |\mathcal{A}|^2 - |\mathcal{A}^{CP}|^2 = |A_1|^2 + |A_2|^2 + 2A_1A_2\cos(\delta+\eta)$
$- |A_1|^2 - |A_2|^2 - 2A_1A_2\cos(\eta-\delta)$

$= 2A_1A_2\left(\cos(\delta+\eta) - \cos(\delta-\eta)\right)$

$= 0$ if $\eta = 0$
if $\delta = 0$