Flavor Theory

2 lectures on (some) aspects of flavor
+1 on neutrino oscillations

Flavor: $\longleftrightarrow$ SM fermions come in 3 copies

matter we are made of...

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L \quad \begin{pmatrix} t \\ b \end{pmatrix}_L \quad + u_R, d_R, c_R, s_R, b_R, t_R$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \quad + e_R, \mu_R, \tau_R$$

identical up to mass $\Rightarrow$ can mix $\Rightarrow$ interesting effects!

related to ~~...~~ <u>Flavor Violation</u> (processes that change flavor e.g. $u \to s$)

and <u>CP violation</u>

and <u>problems of the Standard Model</u>

+ <u>Neutrino masses and Oscillations!</u>

How to describe flavor

recall: $\mathcal{L}_Y = -g_d \bar{L} \Phi d_R - g_u \bar{L} \tilde{\Phi} u_R$

with $L = \begin{pmatrix} u \\ d \end{pmatrix}_L$, $\Phi = \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} / \sqrt{2}$, $\tilde{\Phi} = i \tau_2 \Phi^*$

and $m_{u,d} = \frac{g_{u,d} v}{\sqrt{2}}$ mass given by Yukawa coupling times vev

Now: 3 generations $L_1' = \begin{pmatrix} u' \\ d' \end{pmatrix}_L$, $L_2' = \begin{pmatrix} c' \\ s' \end{pmatrix}_L$, $L_3' = \begin{pmatrix} t' \\ b' \end{pmatrix}_L$

$u_R', c_R', t_R' \equiv u_{iR}'$; $d_R', s_R', b_R' \equiv d_{iR}'$

„FLAVOR STATES"

leads to most general Yukawa terms:

$$\mathcal{L}_Y = -\sum_{i,j} \bar{L}_i' \left[g_{ij}^{(d)} \Phi d_{jR}' + g_{ij}^{(u)} \tilde{\Phi} u_{jR}' \right]$$

$$\rightarrow -\sum \left[\bar{d}_{iL}' \underbrace{\frac{g_{ij}^{(d)} v}{\sqrt{2}}}_{M_{ij}^{(d)}} d_{jR}' + \bar{u}_{iL}' \underbrace{\frac{g_{ij}^{(u)} v}{\sqrt{2}}}_{M_{ij}^{(u)}} u_{jR}' \right]$$

$$\Rightarrow \mathcal{L} \equiv -\left[\overline{d'_L}\, M^{(d)}\, d'_R + \overline{u'_L}\, M^{(u)}\, u_R\right]$$

with $d'_{L,R} = \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}_{L,R}$ and $u'_{L,R} = \begin{pmatrix} u' \\ c' \\ t' \end{pmatrix}_{L,R}$

Mass matrices $M^{(d)}, M^{(u)}$ (remember: similar for W_3^μ, B^μ masses)

Diagonalization to find the diagonal states, e.g. $\overline{u_L}\, m_u\, u_R$. "physical states", "mass states":

$$\left.\begin{aligned} \mathcal{U}_d^+ M^{(d)} V_d &= D^d \equiv \text{diag}(m_d, m_s, m_b) \\ \mathcal{U}_u^+ M^{(u)} V_u &= D^u \equiv \text{diag}(m_u, m_c, m_t) \end{aligned}\right\} \text{"bi-unitary transformation"}$$

$$\mathcal{U}_u \mathcal{U}_u^+ = V_u V_u^+ = \ldots = \mathbb{1}$$

Proof: consider $H = M' M'^+$ (hermitian)

diagonalize: $\mathcal{U}_L^+ M' M'^+ \mathcal{U}_L = D^2 = \text{diag}(m_1^2, m_2^2, m_3^2) = m_i^2\, \delta_{ij}$

the m_i^2 are real and positive:

$$m_i^2 = \sum_j (\mathcal{U}_L^+ M')_{ij} (M'^+ \mathcal{U}_L)_{ji} = \sum (\mathcal{U}_L^+ M')_{ij} (\mathcal{U}_L^+ M')^+_{ji}$$

$$= \sum (\mathcal{U}_L^+ M')_{ij} (\mathcal{U}_L^+ M')^*_{ij} = \sum |(\mathcal{U}_L^+ M')_{ij}|^2$$

now write $M' = \mathcal{U}_L D V_R^+$ with $D_{ij} = +\sqrt{m_i^2}\,\delta_{ij}$

V_R is given by: $V_R = (M')^{-1}\mathcal{U}_L D$, because when

inserting one gets: $M' = \mathcal{U}_L D D \mathcal{U}_L^+ ((M')^+)^{-1}$

V_R is unitary, since:

$$V_R^+ V_R = \underbrace{D^{-1}\mathcal{U}_L^+ M'}_{\text{from } M' = \mathcal{U}_L D V_R^+} \underbrace{(M')^{-1}\mathcal{U}_L D}_{\text{from def. } V_R = M'^{-1}\mathcal{U}_L D} = \mathbb{1}$$ (similar for $V_R V_R^+$)

Number of parameters: $\underset{2N^2}{M'} = \underset{N^2}{\mathcal{U}_L}\ \underset{N}{D}\ \underset{N^2}{V_R^+}$ for $N \times N$ matrices

$\Rightarrow$ bi-unitary diagonalization possible, with

$\mathcal{U}_L^+ M' V_R = D$; where the m_i can be chosen positive! (or zero)

---

if M' is real: orthogonal instead of unitary $\mathcal{U}_L$, V_R

---

back to Lagrangian

$$-\mathcal{L} = \underbrace{\overline{d'_L} \mathcal{U}_d}_{\overline{d}_L} \underbrace{\mathcal{U}_d^\dagger M^{(d)} V_d}_{D^d} \underbrace{V_d^\dagger d'_R}_{\equiv d_R} + \underbrace{\overline{u'_L} \mathcal{U}_u}_{\equiv \overline{u}_L} \underbrace{\mathcal{U}_u^\dagger M^{(u)} V_u}_{D^u} \underbrace{V_u^\dagger u'_R}_{\equiv u_R}$$

the new states in d_L, u_L, u_R, d_R are mass states, their mass matrix is diagonal!

not done yet, consider charged current term:

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} W_\mu^+ \overline{u'_L} \gamma^\mu d'_L$$

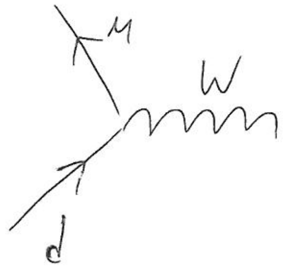
$$= \frac{g}{\sqrt{2}} W_\mu^+ \underbrace{\overline{u'_L} \mathcal{U}_u}_{\overline{u}_L} \mathcal{U}_u^\dagger \gamma^\mu \mathcal{U}_d \underbrace{\mathcal{U}_d^\dagger d'_L}_{d_L}$$

$$\Longrightarrow \boxed{V = \mathcal{U}_u^\dagger \mathcal{U}_d}$$

Cabibbo
Kobayashi
Maskawa

! CKM-Matrix in charged ~~lepton~~ current term!

⇒ Feynman-rules:



$ig\,\gamma_\mu(1-\gamma_5)\frac{1}{2\sqrt{2}}\,V_{ud}$

different strength for different transitions

What about neutral currents?

$\mathcal{L} \sim Z_\mu\,\bar{u}'\,(c_V - c_A\gamma_5)\,\gamma_\mu\,u' \;\to\; \bar{u}\,(\ \ )\,u$

since $U_{u,d}$ is unitary

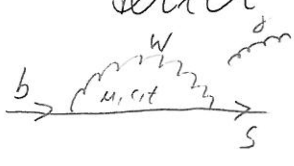
⇒ no Flavor Changing Neutral Currents

Glashow
Illiopolus GIM mechanism
Maiani

at loop level: there are FCNC

⇒ search for FCNC promising to detect new physics beyond the SM

e.g.



$b \to s\gamma$

How many parameters in V?

N families $\Rightarrow$ V is unitary $N \times N$ matrix

	#	$\#_{tot}$
complex $N \times N$	$2N^2$	$2N^2$
$VV^\dagger = \mathbb{1}$	$-N^2$	N^2
rephase u_i, d_i (mod one unphysical total phase)	$-(2N-1)$	$(N-1)^2$

a real matrix would have $\frac{1}{2}N(N-1)$ Euler angles
$\Rightarrow$ the remaining parameters are phases

families	angles	phases
2	1	0
3	3	1
4	6	3
N	$\frac{1}{2}N(N-1)$	$\frac{1}{2}(N-2)(N-1)$