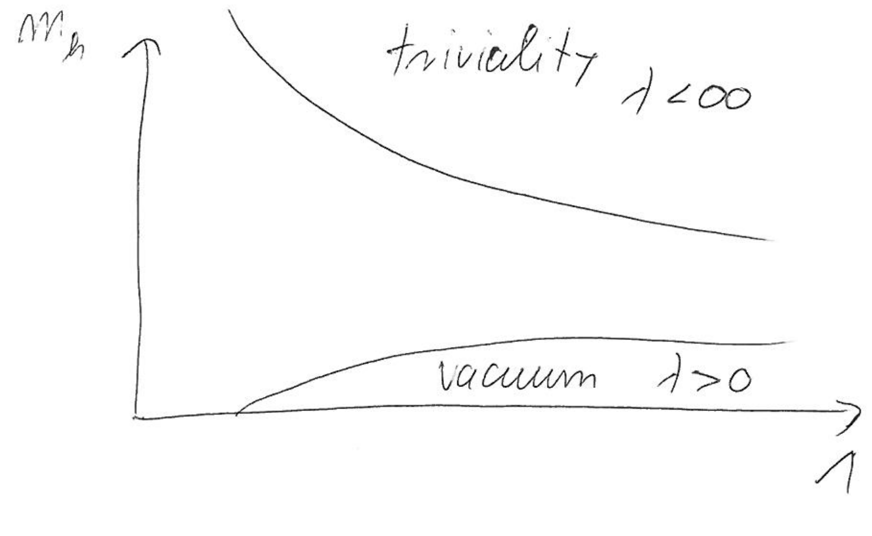
Recap.:

Couplings depend on energy scale!

apply to λ, Higgs self-coupling, in order to get feeling for „sensible" values of $m_h^2 = 2\lambda v^2$, or to have interpretation of measured value



Higgs-mass seems to lie in an interesting region...

today: more aspects of Higgs
more constraints, and a nice „bonus" feature of the Higgs

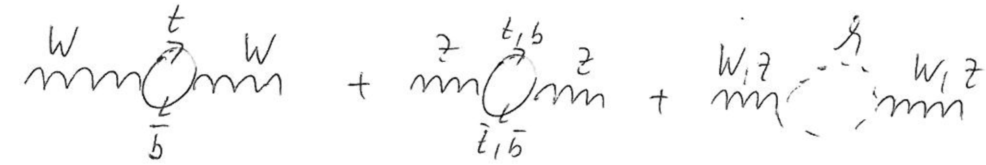
Indirect constraints on the Higgs

we have direct constraints (colliders → André Schöning)

+ theoretical constraints (triviality, vacuum stability)

+ <u>indirect constraints</u> (m$_h$ in loop processes)

For instance:



shifts <u>pole</u> of propagator ⇒ shifts <u>mass</u> of W, Z

$$\Rightarrow \Delta\rho = \Delta\left(\frac{m_W^2}{m_Z^2 \cos^2\Theta_W}\right) = \frac{3G_F}{8\pi^2 2\sqrt{2}}\left[m_t^2 + m_b^2 - 2\frac{m_t^2 m_b^2}{m_t^2 - m_b^2}\log\frac{m_t^2}{m_b^2} - \frac{11}{9} m_Z^2 s_W^2 \log\frac{m_h^2}{m_Z^2}\right]$$

- $m_{t,b}$ part $\to 0$ for $m_t = m_b$ and $m_f = 0$
- $g' = 0$ ($U(1)_Y$ - charge) : $\Delta\rho = 0$

"custodial symmetry" extra $SU(2)$ in $V(\Phi\Phi^\dagger)$

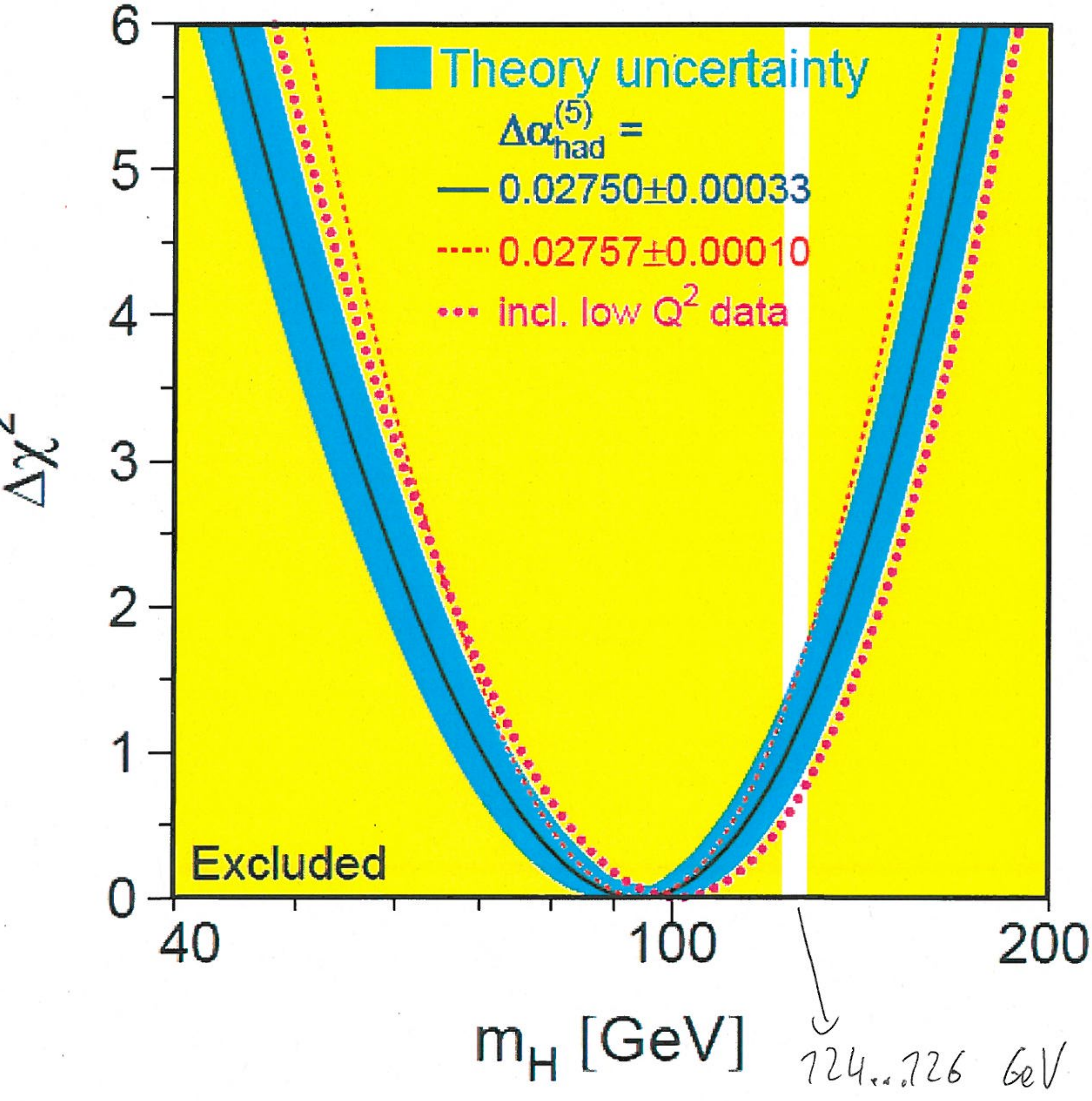
- observation: $\Delta\rho \simeq 0$
- m_h : logarithmically ... ⇐ weak

Fit to 9 and many many many more observables ...

$$m_H = 94^{+29}_{-24}\ \mathrm{GeV} \qquad (68\%\ \mathrm{C.L.})$$

$$\Rightarrow m_H \leq 125\ \mathrm{GeV} \qquad (95\%\ \mathrm{C.L.})$$

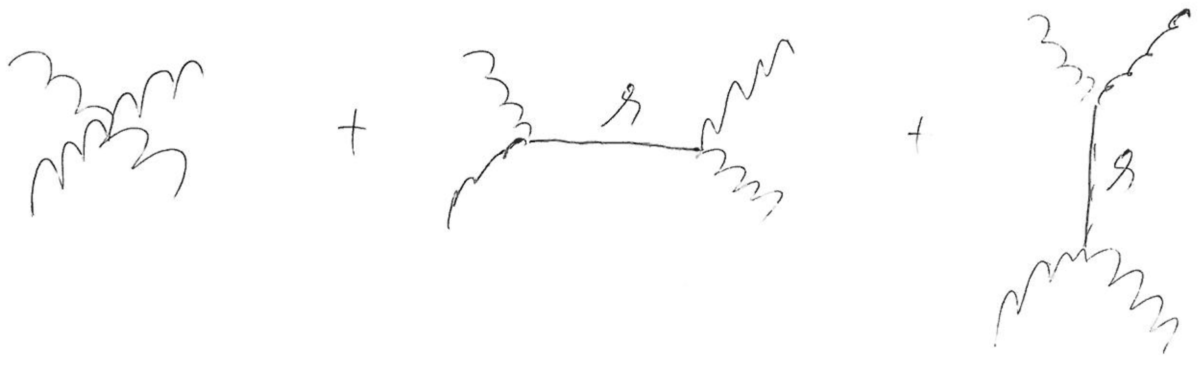
arXiv:1302.3415



124...126 GeV

another indirect limit: Unitarity

$W^+ W^- \to W^+ W^-$ scattering



can be lengthy... useful property:

equivalence theorem

Consider polarization vectors (sheet 5)

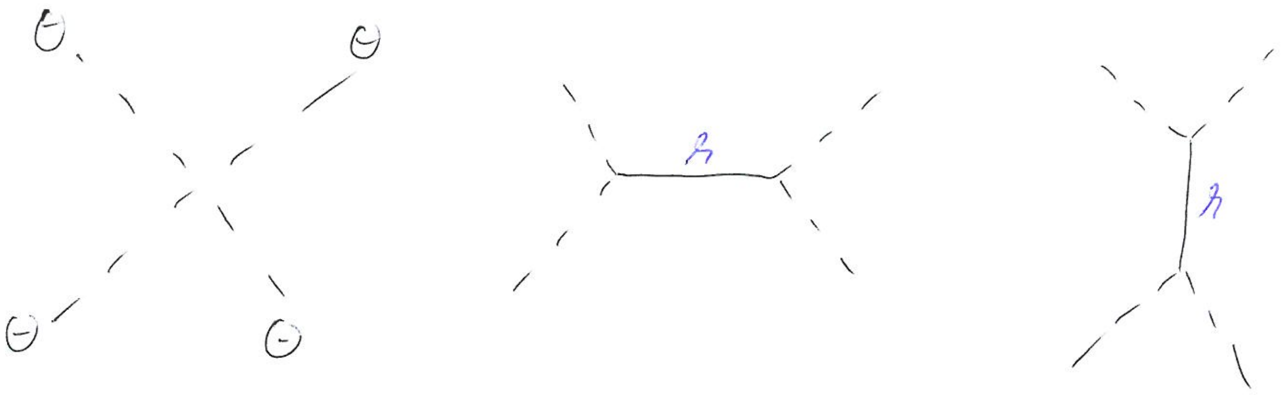
$$\varepsilon_\mu^{(\lambda=0)} = \frac{1}{m_W}(|\vec{p}|, 0, 0, E)$$

$$\varepsilon_\mu^{(\lambda=\pm 1)} = \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0)$$

@ high energy: dominated by longitudinal polarization

$\Rightarrow$ enough to consider Goldstone bosons

$$\left(\Phi = \exp\left\{ i \frac{\vec{\sigma}\vec{\Theta}}{v} \right\} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \frac{1}{\sqrt{2}} \right)$$



Feynman rules: insert $\Phi \simeq \frac{1}{\sqrt{2}}\begin{pmatrix}\Theta_1 + i\Theta_2 \\ v + h - i\Theta_3\end{pmatrix}$ in

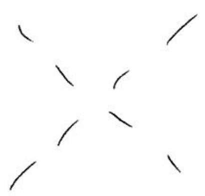
potential $V = \mu(\Phi^\dagger\Phi) + \lambda(\Phi^\dagger\Phi)^2$

$$\Rightarrow V = \frac{m_h^2}{8v^2}\left(\sum \Theta_i^2\right)^2 + \frac{m_h^2}{2v}\, h \sum \Theta_i^2 + \dots$$

charged Goldstone bosons: $\Theta_\pm = \sqrt{\tfrac{1}{2}}\,(\Theta_1 \pm i\Theta_2)$

$$\Rightarrow V = \frac{m_h^2}{2v^2}\,\Theta_+\Theta_-\,\Theta_+\Theta_- + \frac{m_h^2}{v}\, h\,\Theta_+\Theta_- + \dots$$

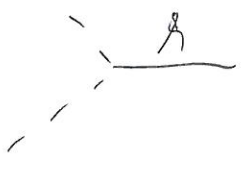
$\Rightarrow$



$$-2i\,\frac{m_h^2}{v^2}$$

factor 4: $\left.\begin{matrix} 2\,\Theta_+ \\ 2\,\Theta_- \end{matrix}\right\}$ in V

for each Θ_+: 2 possible ways to identify them in $\mathcal{L}$



$$-i\, m_h^2/v$$

$$\Rightarrow \boxed{i\mathcal{M} = \frac{-2i m_h^2}{v^2} + \left(\frac{-i m_h^2}{v}\right)^2 \left(\frac{i}{s-m_h^2} + \frac{i}{t-m_h^2}\right)}$$

matrix element for longitudinal $W_L W_L \to W_L W_L$ scattering

now remember: $\mathcal{M} = 16\pi \sum_\ell (2\ell+1) P_\ell(\cos\theta)\, a_\ell$

partial wave decomposition. With $\mathrm{Re}\{a_\ell\} < 1/2$.

Our matrix element in the limit of large s:

$|\mathcal{M}| \simeq 2\frac{m_h^2}{v^2}$ should be smaller than 8π

$$\Rightarrow \boxed{m_h < \sqrt{4\pi v^2} \simeq 870 \text{ GeV}} \quad (\text{or } \lambda < 2\pi)$$

(of course, could be systematic cancellations of $a_{1,2,3,\dots}$
$\Rightarrow$ „perturbative unitarity")

Lesson: at around TeV energies, something should regularize gauge boson scattering $\Rightarrow$ LHC

Higgs and Fermion masses

not mentioned so far: $\boxed{\text{mass terms for fermions are forbidden by gauge invariance}}$

e.g. up-quarks:

$$m_u \bar{u} u = m_u \underbrace{\bar{u}_L}_{\text{part of } SU(2)_L \text{ doublet}} \underbrace{u_R}_{SU(2)_L \text{ singlet}} + h.c. \qquad \psi_1 = \begin{pmatrix} u \\ d \end{pmatrix}_L$$

<u>BUT:</u> I can write, with $\tilde{\phi} = i\sigma_2 \phi^*$:

$$\mathcal{L} = - g_d \bar{\psi}_1 \phi d_R - g_u \bar{\psi}_1 \tilde{\phi} u_R$$

g_d, g_u are called <u>Yukawa couplings</u>

these terms are allowed:

1) Hypercharge: $\psi_1 : 1/3$; $\phi : 1$; $d_R : -2/3$; $u_R = 4/3$

2) $\psi_1 \to U_L \psi_1$; $\phi \to U_L \phi$; $d_R \to d_R$; $u_R \to u_R$

$\Rightarrow \bar{\psi}_1 \phi$ is invariant

3) what about $\overline{\Psi}_1 \tilde{\Phi}$?

$$\overline{\Psi}_1 \tilde{\Phi} = \overline{\Psi}_1 i\sigma_2 \Phi^* \longrightarrow \overline{\Psi}_1 U_L^+ i\sigma_2 U_L^* \Phi^*$$

useful property: $-i\sigma_i \, i\sigma_2 (-i\sigma_i^*) = i\sigma_2$

(recall $U_L = \exp\{ i \frac{\sigma_i}{2} \alpha_i \} \simeq 1 + i\alpha_i\sigma_i/$

$\Rightarrow$ $\overline{\Psi}_1 \tilde{\Phi}$ is invariant!

With $\Phi = \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}$ it follows $\tilde{\Phi} = \begin{pmatrix} \Phi_2^* \\ -\Phi_1^* \end{pmatrix} \longrightarrow \begin{pmatrix} v + h(x) \\ 0 \end{pmatrix} \frac{1}{\sqrt{2}}$

$\Rightarrow$ after SSB:

$$\mathcal{L} = \underbrace{-g_d \frac{v}{\sqrt{2}}}_{m_d} \overline{d}_L d_R \underbrace{-g_u \frac{v}{\sqrt{2}}}_{m_u} \overline{u}_L u_R \underbrace{- g_d \overline{d}_L d_R h(x)/\sqrt{2}}_{\text{Coupling Higgs to fermion pair}}$$

$\Rightarrow$ •) Symmetry breaking + fermion masses!

•) Coupling to Higgs proportional to <u>mass</u> of the fermion: $g_d = m_d / v$

•) one doublet enough for up- and down quarks

•) no ν_R $\Rightarrow$ no mass of neutrino ...

Note: also the coupling to W, Z is proportional to their masses!

from $(D_\mu \Phi)^\dagger (D^\mu \Phi)$: $h W^+ W^- \propto g\, m_W$ with $m_W \propto v g$

$hhWW \propto g^2$

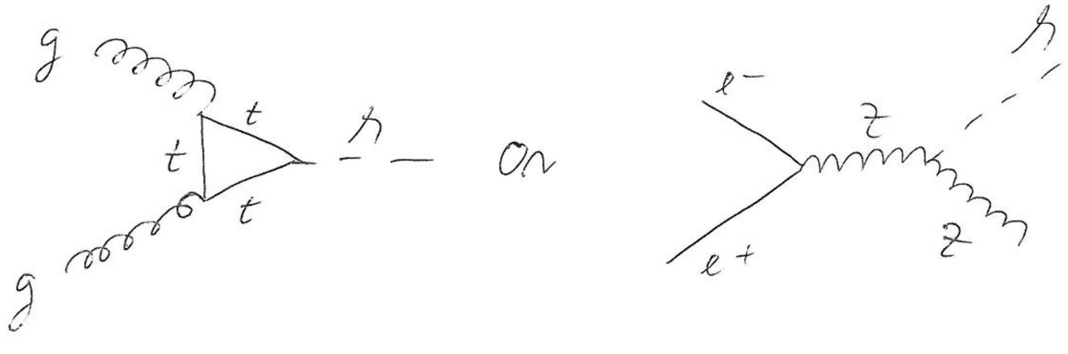
also Higgs-coupling to Higgs prop. to mass:

$\mathcal{L} \subset \lambda h^4 \rightarrow m_h^2 = 2 \lambda v^2$

$\mathcal{L} \subset \lambda v h^3$

$\Leftarrow$ production of Higgs in e^+e^- or pp, $p\bar{p}$-colliders very difficult (tiny couplings)

$\Rightarrow$ need other processes, e.g.



$\rightarrow$ see André Schöning's lectures