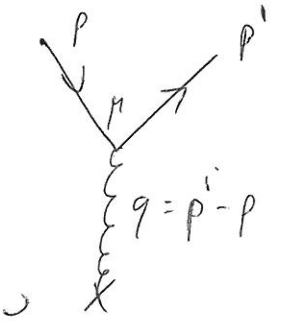
Higgs-Theory

aim: theoretical aspects/constraints on the Higgs-Boson

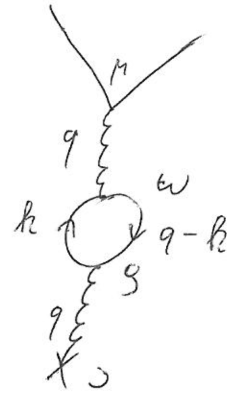
[experimental searches (André Schöning)]

1) Running Couplings

(partly addition to QED-part, feeling for it necessary for Higgs constraints)



„tree-level"



„loop-correction"
vacuum polarization
Correction to scattering of electrons off a potential A_μ $\left[= \left(\frac{Ze}{4\pi x}, \vec{0} \right) \right]$ static source, example, not really important

<u>tree:</u> $i\mathcal{M}_{\text{tree}} \propto e^2 \; \bar{u}(p')\gamma^\mu u(p) \; \frac{g_{\mu\nu}}{q^2} \; j^\nu(q)$

$(Ze\,\delta(\vec{x}), \vec{0})$ and then Fourier-trafo for fermion loops, never mind...

$$\mathcal{M}_{loop} \cancel{\propto} (-1)\; e\, \bar{u}(p') \gamma^\mu u(p) \frac{-i g_{\mu\nu}}{q^2} \int \frac{d^4k}{(2\pi)^4} \times$$

$$e\, (\gamma^\nu)_{\alpha\beta} \frac{(\not{k} + m)_{\beta\gamma}}{k^2 - m^2}\, e\, (\gamma^\rho)_{\gamma\delta} \frac{(\not{k} - \not{q} + m)_{\delta\alpha}}{(k-q)^2 - m^2} \quad \frac{g_{\rho\sigma}}{q^2}\, j^\sigma(q)$$

$\Rightarrow \mathrm{Tr}\{\ldots\}$

$\Rightarrow \int \frac{d^4k}{k^2} \rightarrow$ quadratic divergence?

gauge invariance: „only" logarithmic divergence

introduce <u>cut-off M^2</u>

<u>effect:</u> propagator receives modification:

$$-i\frac{g_{\mu\nu}}{q^2} + \left(-i\frac{g_{\mu\nu}}{q^2}\right) I^{\nu\rho} \left(-i\frac{g_{\rho\sigma}}{q^2}\right)$$

with $I^{\nu\rho} = -i g^{\nu\rho} q^2 I(q^2)$

with $I(q^2) = \frac{\alpha}{3\pi} \log M^2/_{-q^2} \equiv \frac{\alpha}{3\pi} \log M^2/_{Q^2}$

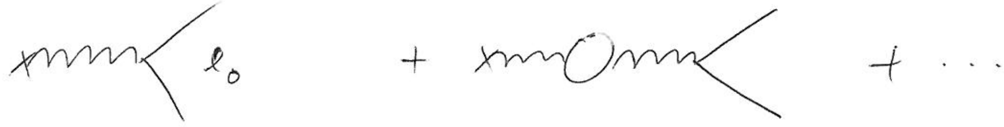
(for $-q^2 \gg m^2$)

(Tricks for calculation + theory background: QED, QFT) effect of the loop correction:

$\mathcal{M} \propto e^2 \to e(1-I)e$ with I log. divergent

<u>BUT:</u> consider what experimentalist measures:

$\mathcal{L} = \bar{\Psi} e_0 A \Psi$ with e_0 the <u>bare charge</u>



e_0^2 $\qquad\qquad$ $e_0(1-I)e_0$

$$\Rightarrow \boxed{\underbrace{e_0^2}_{\text{formally } \infty}(\underbrace{1-I}_{\infty}) \equiv \underbrace{e^2}_{\text{finite!}}} \text{ is finite!}$$

e.g. $\frac{4\pi}{137}$ order measured @ low energy

$$\Rightarrow e = e_0(1-I)^{1/2} \simeq e_0(1-I/2)\Big|_{Q^2=\mu^2}$$

depends on μ: energy at which expt. is performed

! this implies $e = e(Q^2)$!

luckily, the same infinity occurs for all orders in perturbation theory: "renormalizable theory" we can also do higher loop corrections

$e_0 \qquad e_0(1-I) \qquad e_0(1-I+I^2)$

$$\Rightarrow e^2 = e_0^2\left(1 - I + I^2 - I^3 + \dots\right)\Big|_{Q^2=\mu^2} \qquad \left(\text{geometric series}\right)$$

$$\Rightarrow \boxed{\alpha(Q^2) = \frac{\alpha_0}{1 + I(Q^2)}} = \frac{\alpha_0}{1 - \frac{\alpha_0}{3\pi}\log Q^2/\mu^2}$$

measure @ reference scale μ^2, at which $\alpha = \alpha_0(1 - I(\mu^2))$

$$\Rightarrow \alpha_0 = \alpha(\mu^2)(1 + I(\mu^2))$$

$$= \alpha(\mu^2)\left(1 + \frac{\alpha(\mu^2)}{3\pi}\log M^2/\mu^2\right)$$

correct to the order of α that we are working with

$$\Rightarrow \alpha(Q^2) = \frac{\alpha(\mu^2)\left(1 + \frac{\alpha(\mu^2)}{3\pi}\log M^2/\mu^2\right)}{1 - \frac{\alpha(\mu^2)}{3\pi}\log Q^2/M^2}$$

$$= \alpha(\mu^2)\left(1 + \frac{\alpha(\mu^2)}{3\pi}\log M^2/\mu^2\right)\left(1 + \frac{\alpha(\mu^2)}{3\pi}\log Q^2/M^2\right)$$

$$= \alpha(\mu^2)\left(1 + \frac{\alpha(\mu^2)}{3\pi}\underbrace{\left(\log M^2/\mu^2 + \log Q^2/M^2\right)}_{\log Q^2/\mu^2}\right)$$

$$\Rightarrow \boxed{\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi} \log Q^2/\mu^2}}$$

$\alpha \nearrow$ for $Q^2 \nearrow$

generic for QED and any $U(1)$

Note: $\alpha(Q_L^2) = \infty$ for some finite Q_L^2

"Landau pole"

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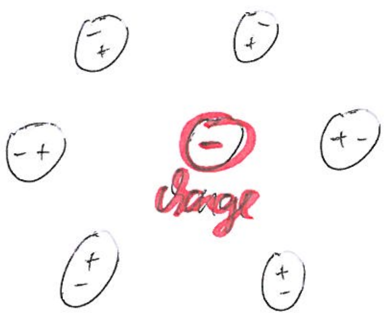
procedure confirmed by observation:

$$\alpha(Q^2 \simeq 0) = \frac{1}{137} \longrightarrow \alpha(Q^2 = M_Z^2) = \frac{1}{128}$$



$Q^2 \nearrow$: one sees more of $\ominus$

$Q^2 \searrow$: $\ominus$ is screened by e^+e^- pairs

the situation in QCD is different!

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \frac{\alpha_s(\mu^2)}{4\pi}\left(\frac{11}{3}N - \frac{2}{3}N_f\right)\log Q^2/\mu^2}$$

with $N = 3$ $(SU(N))$

$N_f = 6$ (u, c, t, d, s, b)

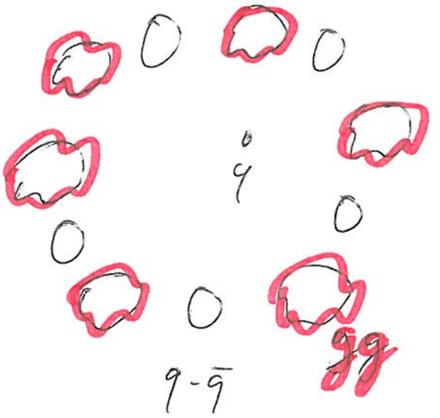
$\Rightarrow$ $\alpha_s \searrow$ for $Q^2 \nearrow$ (as long as $N_f \leq 16$)

„asymptotic freedom"

$\alpha_s \nearrow$ for $Q^2 \searrow$ „confinement"

$$\left(\alpha_s = \infty \quad \text{for} \quad Q^2 = \mu^2 \exp\left\{\frac{-12\pi}{(33-2N_f)\,\alpha_s(\mu^2)}\right\} \simeq (100\ \text{MeV})^2 \right)$$

reason for different behavior with respect to QED:



Gluon contribution leads to „anti-screening"

$\alpha_s(M_Z) = 0.12$

$\alpha_s(M_Z) = 0.33$

back to QED: $\alpha(Q^2) = \dfrac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi} \log Q^2/\mu^2}$

$\downarrow$

actually $N_f \frac{\alpha(\mu^2)}{3\pi}$ + quarks

define the β-function: $\beta_\alpha = \dfrac{\alpha^2}{3\pi}$

$$\boxed{\frac{d\alpha}{d\log Q^2} = \beta_\alpha}$$

$\beta_\alpha > 0 \Rightarrow \alpha \nearrow$ for $Q^2 \nearrow$

to see that indeed the running is reproduced: $g \equiv \alpha^{-1}$

$$\Rightarrow \frac{d\alpha}{d\log Q^2} = \frac{d}{d\log Q^2}\frac{1}{g} = -\frac{1}{g^2}\frac{dg}{d\log Q^2} = \frac{1}{3\pi}\frac{1}{g^2}$$

$$\Rightarrow \frac{dg}{d\log Q^2} = -\frac{1}{3\pi} \Rightarrow g(Q^2) = -\frac{1}{3\pi}\log Q^2 + C$$

C from boundary condition = reference measurement

$$g(\mu^2) = -\frac{1}{3\pi}\log\mu^2 + C \text{ ; insert in } g(Q^2):$$

$$\Rightarrow g(Q^2) = g(\mu^2) - \frac{1}{3\pi}\log Q^2/\mu^2$$

and thus: $\alpha(Q^2) = \dfrac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi}\log Q^2/\mu^2}$ ☺

2) Constraints on the Higgs-Boson

recall: $V = \mu^2 \phi^+ \phi + \lambda (\phi^+ \phi)^2$

leads to $v^2 = \frac{-\mu^2}{2\lambda} = (246 \text{ GeV})^2$

and $m_h^2 = 2\lambda v^2 = \left((125 \ldots 126) \text{ GeV}\right)^2$

described by 2 parameters (μ^2, λ), (v, m_h), ..

$$\phi = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} = \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \exp\left\{ -\frac{i \vec{\Theta} \vec{\tau}}{v} \right\} 1/\sqrt{2}$$

„unitary gauge"

$\Theta_i \rightarrow$ polarization degrees of gauge bosons $\Rightarrow$ mass

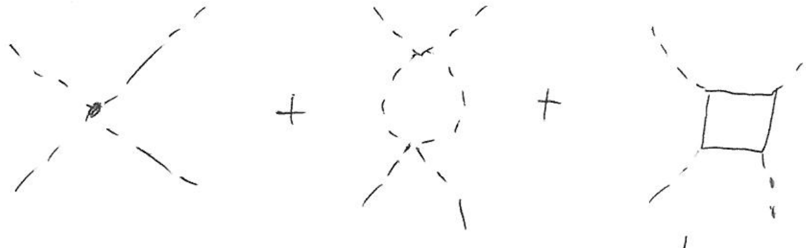
~~3 ways to constrain those 2 parameters~~
~~(colliders, data~~

since last summer: also m_h (or λ) known

3 methods:
1) direct (collider)
2) indirect (next Monday)
3) theoretical (today)

Running Higgs-self coupling

$\mathcal{L}$ contains term $\lambda \phi^4$



+ gauge-bosons

$$\lambda = \lambda(Q^2) = f(\lambda, \frac{m_t}{v}, g, g')$$

(next week: coupling Higgs-fermion $\propto \frac{m_f}{v} \equiv Y_f$)

Yukawa coupling

a) large λ

$$\frac{d\lambda}{d \log Q^2} \simeq \frac{3}{4\pi^2} \lambda \quad \Rightarrow \quad \lambda \nearrow \text{ for } Q^2 \nearrow$$

Landau pole

λ should be smaller than, say, 4π, otherwise the theory becomes non-perturbative

$$\Rightarrow \quad \lambda(\text{low energy}) \overset{!}{<} \lambda_{max} \Rightarrow m_h < m_h^{max}$$

„TRIVIALITY-BOUND"

only trivial theory ($\lambda = 0$) avoids Landau pole

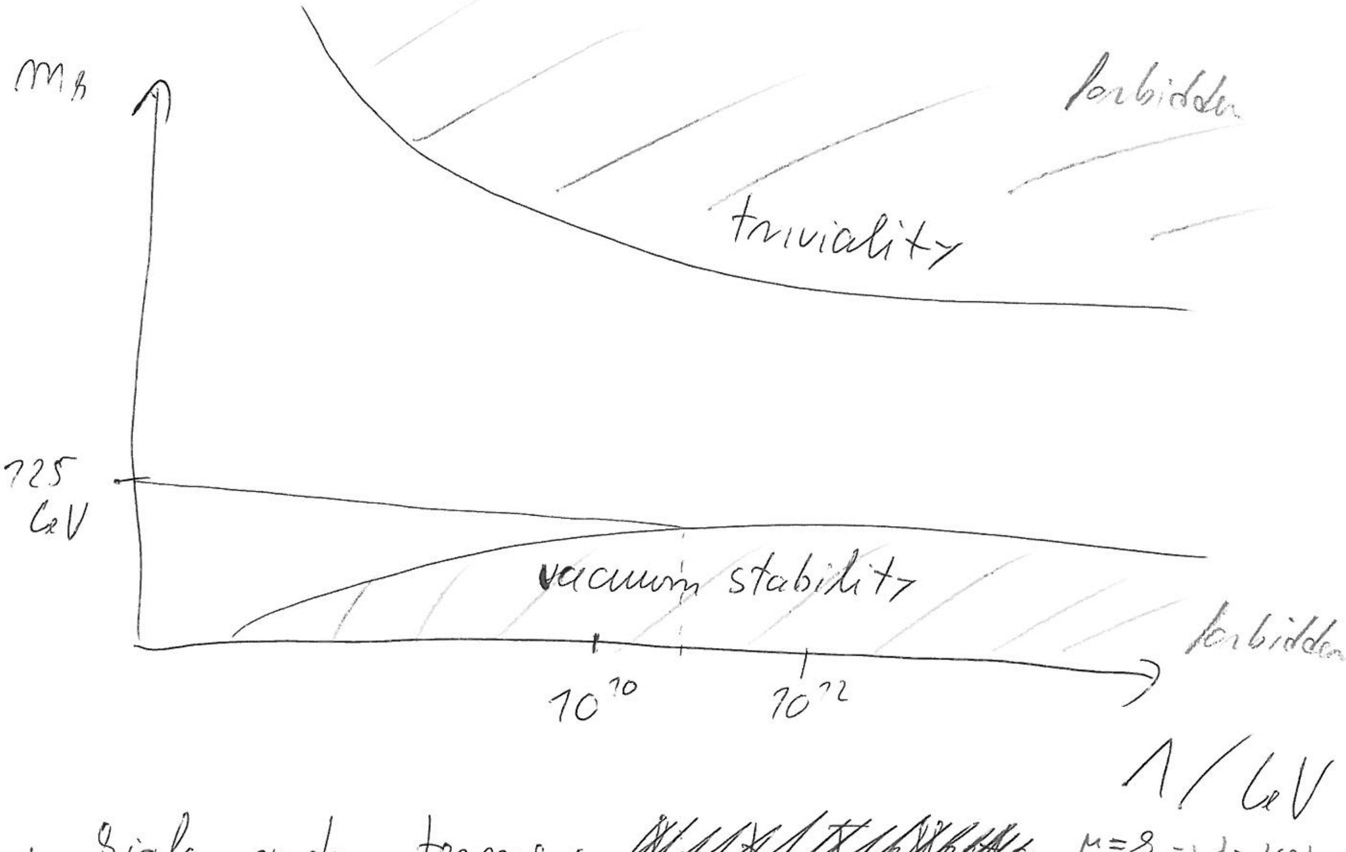
b) small λ

$$\frac{d\lambda}{d\log Q^2} \simeq -\frac{3}{16\pi^2} y_t^4 \Rightarrow \lambda \downarrow \text{ for } Q^2 \nearrow$$

λ should not become negative, otherwise the potential V is not bounded from below, no Higgs-mechanism, no ground state of theory (this would be BAD!)

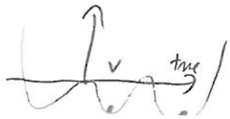
$$\Rightarrow \lambda(\text{low energy}) \overset{!}{>} \lambda_{min} \Rightarrow m_h > m_h^{min}$$

„vacuum stability bound"



+ higher order terms + ~~[illegible]~~ $\mu = \varrho \Rightarrow \lambda = \lambda(\varrho)$ if minimum is classical minimum all the time $\Rightarrow$ stable! if not, other minima will be developed

$\Rightarrow$ Higgs vacuum probably metastable . . .



tunneling...