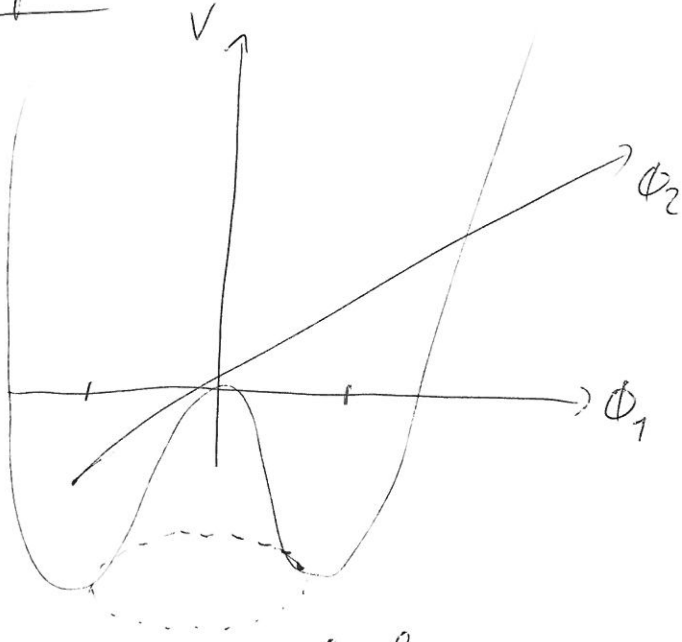
Recap.:



$V(\Phi) = \mu^2(\Phi\Phi^\dagger) + \lambda(\Phi\Phi^\dagger)^2$

spontaneous breaking of continuous symmetry:

$\Rightarrow$ massless Goldstone boson

local

in a gauged symmetry: GB $\rightarrow$ 3rd degree of freedom of gauge boson

$\begin{pmatrix} \Phi = (v+h)\, e^{-i\theta/v} \\ A_\mu = A_\mu - \frac{1}{ev}\partial_\mu \theta \end{pmatrix} \Rightarrow$ from now on: simply write $\Phi = v+h$!

Generalization: Goldstone theorem (sheet 7)

- gauge theory generated by N generators

  e.g. $\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}$ $O(3)$ symmetry $\leftrightarrow$ 3 generators

- vacuum invariant under $M < N$ generators

  e.g. $\langle \Phi \rangle = \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix}$ $O(2)$ symmetry $\leftrightarrow$ 1 generator

$\Rightarrow$ $N-M$ Goldstone bosons (can be eaten in local gauge th.

$\leftrightarrow$ M gauge bosons associated with unbroken generators remain massless

Construction of the Standard Model

focus on em. + weak IA (QCD $SU(3)_c$ is not broken!)

$\rightarrow$ $U(1)$; direct $U(1)_{QED}$ does not work

$\Rightarrow$ $\boxed{U(1)_Y}$ „hypercharge"

$\rightarrow$ β-decays, Fermi-theory, $W^\pm$ $\leftrightarrow$ Isospin $\Rightarrow$ $SU(2)$

$\rightarrow$ maximal parity violation $\Rightarrow$ $\boxed{SU(2)_L}$

$\Rightarrow$ expect $1+3$ gauge bosons, call them B_μ, W_μ^i

photon should be massless

$\Rightarrow$ vacuum should be invariant under electromagnetism

now, we face the typical problem of model building

$\rightarrow$ choose the group

$\rightarrow$ choose the particle content (i.e. representation)

$\rightarrow$ choose the breaking (scalar)

$$\Rightarrow \quad \psi_1 = \begin{pmatrix} u \\ d \end{pmatrix}_L \quad \text{or} \quad \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \sim 2_L \,,\; Y_1 \text{ or } \tilde{Y}_1$$

$$\psi_2 = u_R \quad \text{or} \quad \nu_R \quad \sim 1_L \,,\; Y_2 \text{ or } \tilde{Y}_2$$

$$\psi_3 = d_R \quad \text{or} \quad e_R \quad \sim 1_L \,,\; Y_3 \text{ or } \tilde{Y}_3$$

we need to work with different charges of our particles $\Rightarrow$ $U(1)_Y$ generated by $\exp\left\{ \frac{i}{2} g' \hat{Y} \beta(x) \right\}$

$\hat{Y}$ is generator of hypercharge: $\hat{Y} u_R = Y_2 u_R$

$\Rightarrow$ $SU(2)_L$ generated by $\exp\left\{ i \frac{\sigma_i}{2} \alpha_i(x) g \right\} \equiv U_L$

weak isospin is $\hat{I}_3$ $\cancel{= \exp\left\{ i \frac{\sigma_3}{2} \alpha_3(x) \right\}}$

$\cancel{\neq \exp\left\{ i \hat{I}_3 \alpha_3(x) \right\}}$

$$\hat{I}_3 u_L = \frac{1}{2} u_L$$
$$\hat{I}_3 d_L = -\frac{1}{2} d_L$$
$$\hat{I}_3 u_R = 0$$

$\Rightarrow$ transformation rules

$$\Psi_1 \to \Psi_1' = \exp\left\{g'\tfrac{1}{2} i Y_1 \beta(x)\right\} U_L \Psi_1 = \exp\left\{g'\tfrac{1}{2} \hat{Y} \beta(x)\right\} U_L \Psi_1$$

$$\Psi_{2,3} \to \Psi_{2,3}' = \exp\left\{g'\tfrac{1}{2} i Y_{2,3} \beta(x)\right\} \Psi_{2,3}$$

with $U_L = \exp\left\{ i g \frac{\sigma_i}{2} \alpha_i(x) \right\}$

There should be a scalar multiplet to allow for massive gauge bosons: „Higgs Doublet"

$$\Phi = \sqrt{\tfrac{1}{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \sim 2_L, \; Y_\Phi$$

potential $V = \mu^2 \Phi \Phi^\dagger + \lambda (\Phi^\dagger \Phi)^2$

$$\Rightarrow \Phi^\dagger \Phi = -\mu^2/2\lambda \equiv \tfrac{1}{2} v^2$$

choose $\phi_3 = v$ as minimum

we note that $\bar{\psi}_1 \gamma^\mu \psi_1 \to I_3^{u_L} \cdot \bar{u}_L \gamma^\mu u_L + I_3^{d_L} \bar{d}_L \gamma^\mu d_L$

$$+ \left(\bar{u}_L \gamma^\mu u_L + \bar{d}_L \gamma^\mu d_L\right) Y_1 \frac{1}{2}$$

and $I_3^{u_R} \bar{u}_R \gamma^\mu u_R + \frac{1}{2} Y_2 \bar{u}_R \gamma^\mu u_R$

$\Rightarrow$ electromagnetic charge should be linear combination of weak isospin and hypercharge (which we normalized arbitrarily)

$$\Rightarrow \boxed{\hat{Q} = \hat{I}_3 + \frac{\hat{Y}}{2}} \quad \text{(by convention)}$$

if the vacuum state of the Higgs is invariant under this $\hat{Q}$, the gauge boson associated to this operator will be massless $\leftrightarrow$ photon

$$\Rightarrow e^{i\delta(x)\hat{Q}} \begin{pmatrix} 0 \\ v \end{pmatrix} \stackrel{!}{=} \begin{pmatrix} 0 \\ v \end{pmatrix} \quad \text{with } \hat{Q}_\phi = \frac{1}{2}\begin{pmatrix} 1+Y_\phi & 0 \\ 0 & Y_\phi - 1 \end{pmatrix}$$

$$\Rightarrow Q_\phi \begin{pmatrix} 0 \\ v \end{pmatrix} \stackrel{!}{=} 0 \Rightarrow \boxed{Y_\phi = +1}$$

•) the remaining 3 gauge bosons will be massive ..

In analogy to our $U(1)$ example from last lecture, note that I can write:

$$\boxed{\Phi(x) = \sqrt{\tfrac{1}{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \exp\left\{ i \frac{\vec{\Theta}\,\vec{\sigma}}{v} \right\}}$$

associated gauge trafo of W_μ^i and B_μ^i will make the Θ_i eaten, to make the gauge bosons massive. (don't need to explicitly do this)

$$\Rightarrow \Phi = \sqrt{\tfrac{1}{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix} \left(1 + i\sigma_1 \tfrac{1}{v}\Theta_1 + i\sigma_2 \tfrac{1}{v}\Theta_2 + i\sigma_3 \tfrac{1}{v}\Theta_3 \right) = \ldots =$$

$$= \sqrt{\tfrac{1}{2}} \begin{pmatrix} \Theta_2 + i\Theta_1 \\ v + h - i\Theta_3 \end{pmatrix}$$

•) next step: masses of the bosons:

$(D_\mu \Phi)^\dagger (D^\mu \Phi)$ with $D_\mu = \partial_\mu + \frac{1}{2}(igW_\mu^i \sigma_i + ig' Y_\Phi B_\mu)$

define $W_\mu^\pm = \sqrt{\frac{1}{2}}(W_\mu^1 \mp i W_\mu^2)$ charged bosons

- focus on terms with v
- terms with h are gauge-boson-Higgs couplings

$$\Rightarrow \mathcal{L} = \frac{1}{2} v^2 g^2 W_\mu^+ W^{\mu -}$$

$$+ \frac{v^2}{8} \left(W_\mu^3 , B_\mu \right) \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix} \begin{pmatrix} W^{3,\mu} \\ B^\mu \end{pmatrix}$$

$$\Rightarrow \boxed{m_{W^\pm}^2 = \frac{1}{4} v^2 g^2}$$

2nd part: non-diagonal $\Rightarrow$ diagonalize (note: rank $1 \Rightarrow m_\gamma = 0$)

$$\boxed{M_A = 0} \qquad A_\mu = \frac{g' W_\mu^3 + g\, B_\mu}{\sqrt{g^2 + g'^2}}$$

$$\boxed{M_Z^2 = \frac{v^2}{4} (g^2 + g'^2)} \qquad Z_\mu = \frac{g W_\mu^3 - g' B_\mu}{\sqrt{g^2 + g'^2}}$$

with $\tan \Theta_W = g'/g$ Weinberg angle

$$\boxed{\begin{aligned} A_\mu &= c_W B_\mu + s_W W_\mu^3 \\ Z_\mu &= c_W W_\mu^3 - s_W B_\mu \end{aligned}}$$

„physical fields“ with diagonal mass terms

Note: $\frac{M_W^2}{M_Z^2} = \cos^2\Theta_W$ or:

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2\Theta_W} = 1$$

very sensitive to presence of new physics

Coupling of fermions to gauge bosons:

$\bar{\Psi}_{\frac{1}{2}} D_\mu \Psi_{\frac{1}{2}}$ has terms $\propto W_\mu^\pm, Z_\mu, A_\mu$

a) $A_\mu \Big[g \hat{I}_3 s_W \bar{u}_L \gamma^\mu u_L + g' \frac{\hat{Y}_L}{2} c_W \bar{u}_L \gamma^\mu u_L + g' c_W \frac{\hat{Y}_R}{2} \bar{u}_R \gamma^\mu u_R \Big]$

since $\hat{Q} = I_3 + \frac{\hat{Y}}{2} \Longrightarrow \boxed{g s_W = g' c_W = e}$

e.g. $Y_{u_L} = Y_{d_L} = 1/3$

$Y_{d_R} = -2/3$; $Y_{u_R} = 4/3$

$Y_{\nu_L} = -1$; $Y_{\nu_R} = 0$ (total singlet, not included in SM)

$\circ$) $Z_\mu [\dots] \propto \frac{e}{2 c_W s_W} Z_\mu \bar{f} \gamma^\mu (v_f - a_f \gamma_5) f$

	μ	d	ν	e
$2 v_f$	$1 - \frac{8}{3} s_W^2$	$-1 + \frac{4}{3} s_W^2$	1	$-1 + 4 s_W^2$
$2 a_f$	1	-1	1	-1

$\Rightarrow$ not purely $V-A$

$\circ$) $\mathcal{L}_{fund} = - \frac{g}{2 \sqrt{2}} \left[\bar{\nu}_\mu \gamma_\mu (1 - \gamma_5) \mu + \bar{\nu}_e \gamma_\mu (1 - \gamma_5) e \right] W^{\mu +}$

$P_L = \frac{1}{2}(1 - \gamma_5)$ def. of W_μ^+

$+ \frac{1}{2} m_W^2 W_\mu^+ W^{\mu -} \quad + \mathcal{L}_{kin}$

@ low energy : $\partial_\mu W^{\mu +} = 0$ (kinetic term irrelevant)

$\Rightarrow$ Euler Lagrange: $\frac{\partial \mathcal{L}}{\partial W_\mu^+} = 0$

$$\Rightarrow W^{\mu -} = \frac{g}{2\sqrt{2}\, m_W^2} \left[\bar{\nu}_\mu \gamma^\mu (1-\gamma_5) \mu + \bar{\nu}_e \gamma^\mu (1-\gamma_5) e \right]$$

insert back in $\mathcal{L}_{\text{kind}}$

$$\Rightarrow \mathcal{L}_{\text{eff}} = - \frac{g^2}{8 m_W^2} \left[\bar{\nu}_\mu \gamma_\mu (1-\gamma_5) \mu \right] \left[\bar{e} \gamma^\mu (1-\gamma_5) \nu_e \right]$$

μ-decay,

$$\Rightarrow \boxed{\frac{g^2}{8 m_W^2} = \frac{G_F}{\sqrt{2}}} \Rightarrow \frac{g}{m_W} \simeq \frac{1}{123 \text{ GeV}}$$

measure via production in collider

$$\Rightarrow \boxed{v = 246 \text{ GeV}}$$

$$m_W = 80 \text{ GeV} \Rightarrow g \simeq 0.65 \leftrightarrow e = \sqrt{4\pi\alpha} \simeq 0.3$$

$\Rightarrow$ Fermi-theory is „effective theory", i.e. ultraviolett completion not observable

$\Leftrightarrow$ fundamental energy scale $\gg$ energy scale of observation

$$m_W \gg m_\mu$$