Lecture:

Standard Model of Particle Physics

Heidelberg SS 2013

Fermi Theory

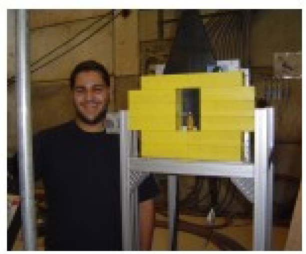
ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Particle Physics

Particle Physics Practical Course at the Paul Scherrer Institute (PSI, Switzerland) in Summer 2013

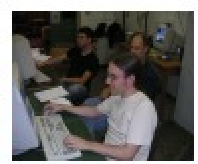
What's up ?

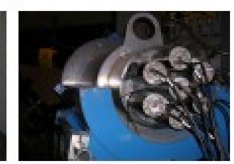
- During the semester break in summer we perform at the Paul Scherrer Institute (Switzerland) a real beam-line experiment to teach students in experimental particle physics.
- About 10-12 students from the ETH Zurich and the Universities Zurich and Heidelberg spend two to three weeks at PSI to perform an experiment. The course includes lectures about several topics of experimental techniques. Main emphasis, however, is put on the practical work and "hands on".
- Students plan and construct a small experiment from unused detector components. After commissioning the real fun starts: data taking all day and night (7/24) using one of the beamlines at PSI. During and after data taking a full analysis of the data is performed and summarised in a written document.



Examples from previous measurements:

- Branching Ratio: $B(\pi\to\mu\upsilon)/B(\pi\to e\upsilon)$
- Panofski Ratio: $B(\pi p\to n\pi)/B(\pi p\to n\gamma)$
- Lifetimes and decay parameters of muons and pions





Date of next course:

26. August - 13. September 2013

Contact Persons:

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C.Grab : grab@phys.ethz.ch
P.Robmann: peter@physik.uzh.ch

Limited number of places, please register!

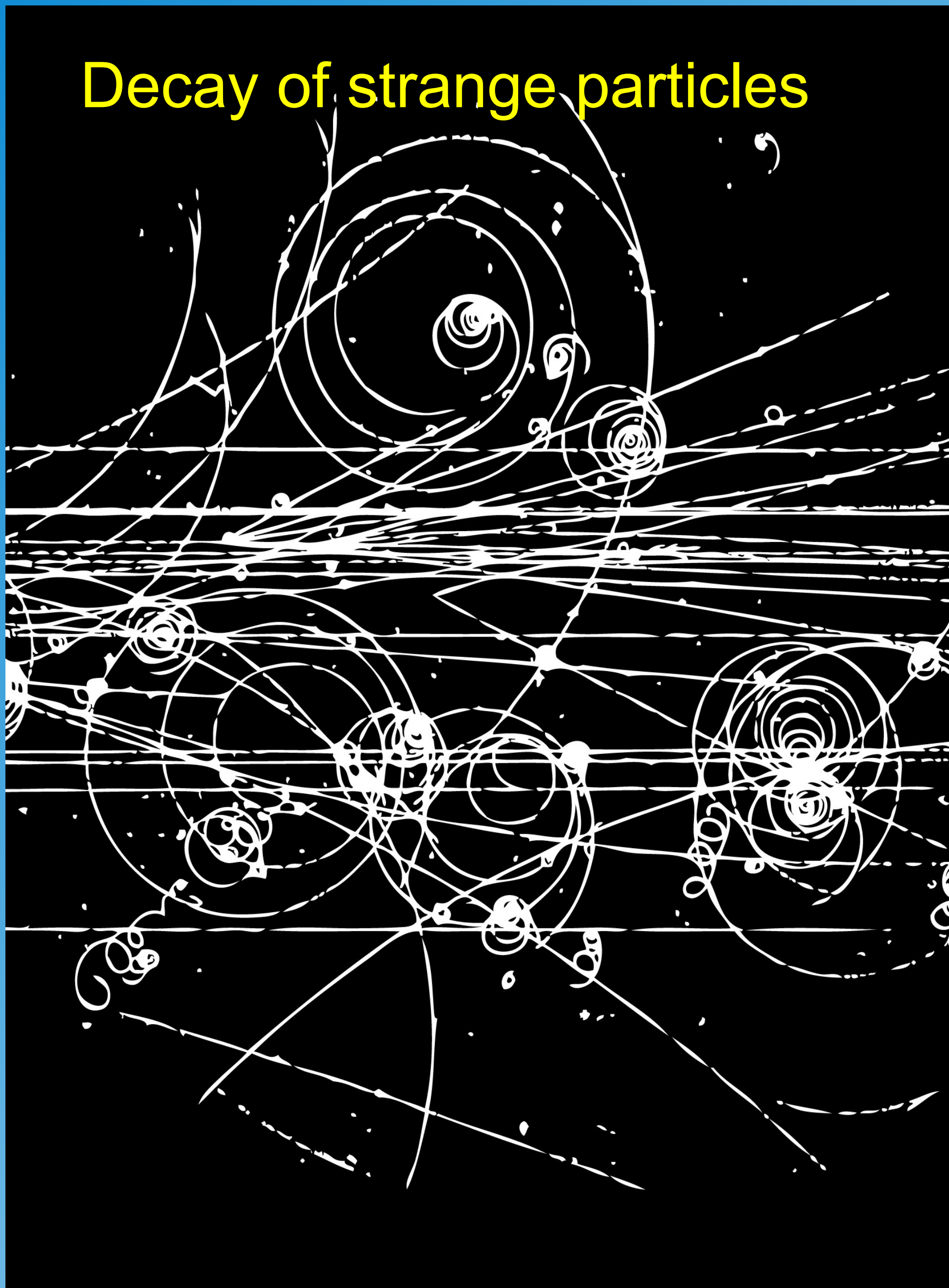
This course (MVPSI) is part of the Master Programme at the Faculty of Physics and Astronomy in Heidelberg!

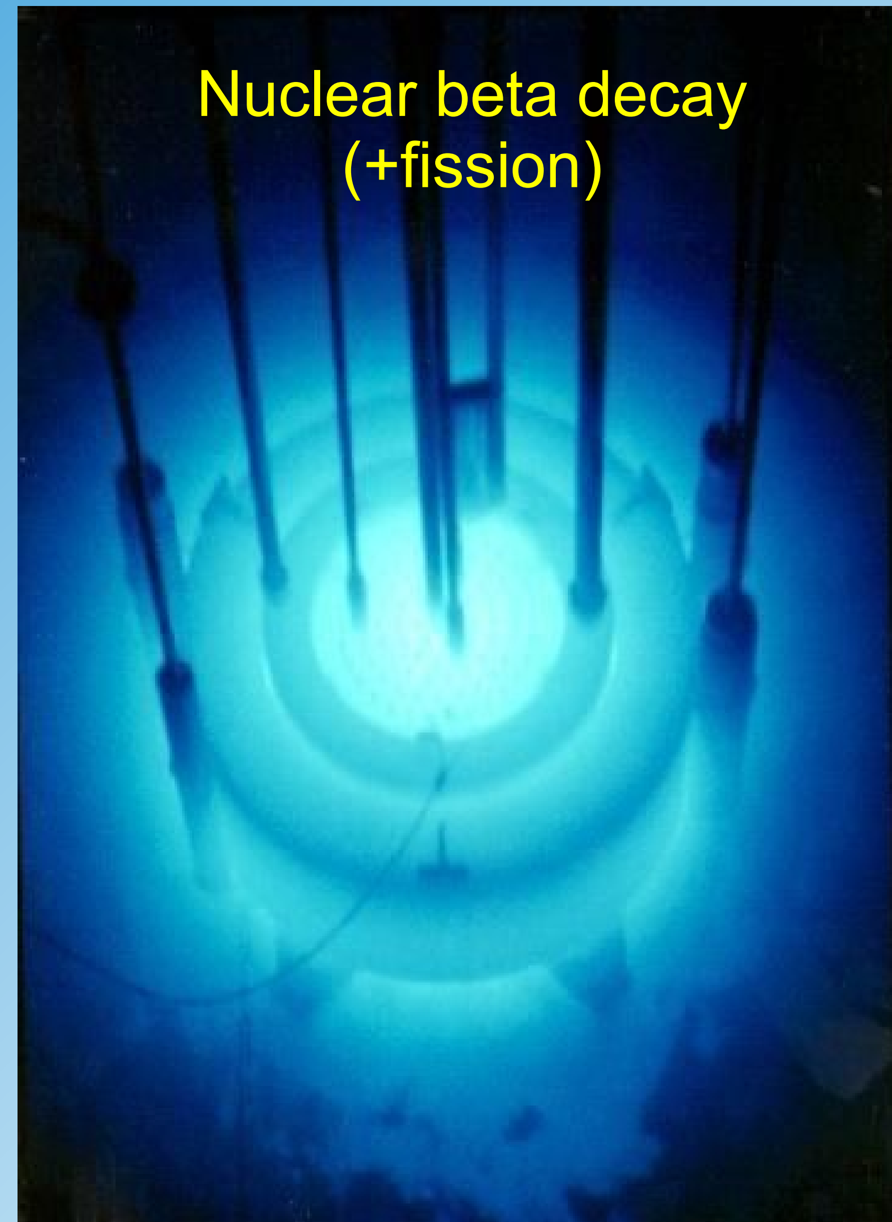
06/02/13 / A.Schöning

PSI

ETH Institute for Particle Physics

Weak Force





Fermi Theory

- **Unified description of all kind of beta decays?**
  - nuclear decays
  - muon and pion decay
  - decay of strange hadrons and heavy quarks

- **Description of weak scattering processes?**
  - at low energy
  - at high energy

Isotop	Halbwert-zeit
^{1}n	11,7 m
^{35}S	87 d
^{198}Au	2,7 d
^{91}Y	61 d
^{137}Cs	30 a
^{87}Rb	6×10^{10} a
^{115}In	6×10^{14} a

Recap I

Lagrangian (independent of energy)

$$L = \frac{G}{\sqrt{2}} \left(\bar{f} \; \Gamma \; f\,' \right) \left(\overline{f\,''} \; \tilde{\Gamma} \; f\,''' \right)$$

¿

most general ansatz for operator Γ:

Vector Current:

$$j_V^\mu = \bar{\psi} \gamma^\mu \psi$$

Axial-vector Current:

$$j_A^\mu = \bar{\psi} \gamma^\mu \gamma^5 \psi$$

scalar coupling:

$$\lambda = \bar{\psi} \psi$$

pseudoscalar coupling:

$$\lambda = \bar{\psi} \gamma^5 \psi$$

Tensor Coupling

$$\sigma_A^{\mu\nu} = \bar{\psi} \left(\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu \right) \psi$$

Recap II

Vector Current conservation

$\bar{u}\gamma^{\mu}u = \bar{u}_L\gamma^{\mu}u_L + \bar{u}_R\gamma^{\mu}u_R$ (no helicity flip)

Scalar Coupling

$\bar{u}u = \bar{u}_R u_L + \bar{u}_L u_R$ (helicity flip!)

Recap II

Vector Current conservation

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$$\bar{u}u = \bar{u}_R u_L + \bar{u}_L u_R$$ (helicity flip!)

What is the difference between helicity and chirality?

Recap II

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What is the difference between helicity and chirality?

What is the difference between helicity and polarisation?

Recap II

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Scalar Coupling

$$\bar{u}u = \bar{u}_R u_L + \bar{u}_L u_R$$ (helicity flip!)

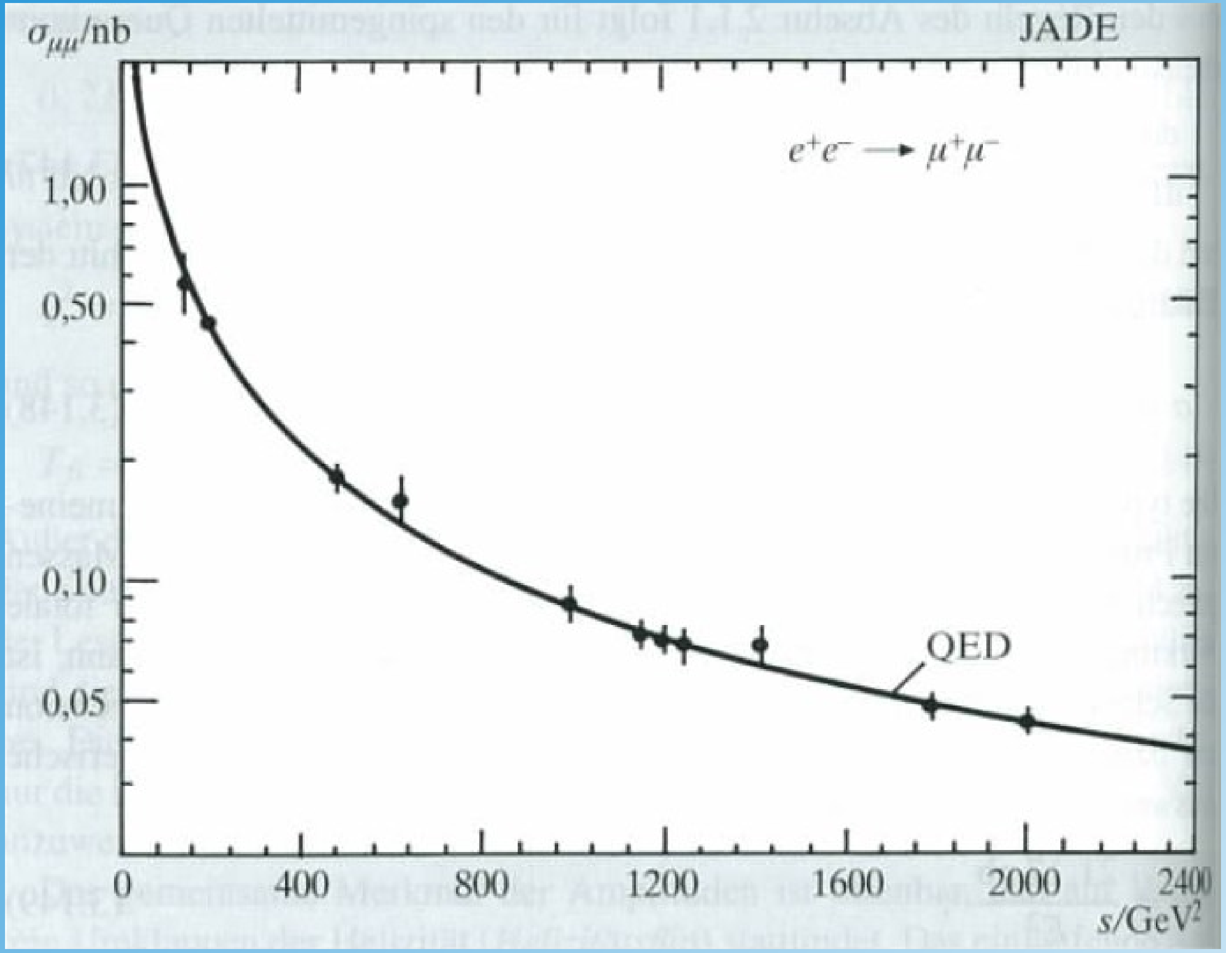
What is the difference between helicity and chirality?

What is the difference between helicity and polarisation?

What is the difference between helicity flip and spin flip?

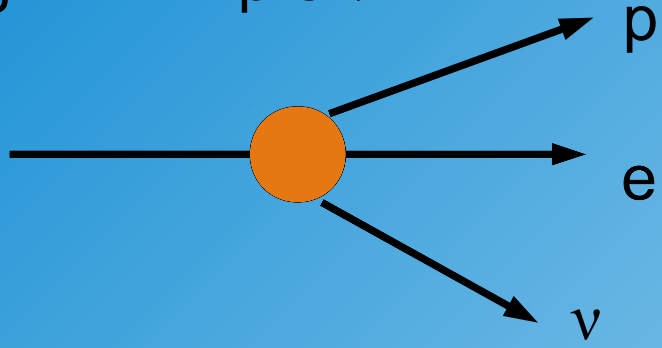
Recap III

Energy dependence of electromagnetic interaction: $\frac{1}{s}$



3-Body Decay

e.g.: n $\rightarrow$ p e ν



electron energy: E(e)
neutrino energy: E(ν)

Fermi's Golden rule:

$$dN(p)\,dp = 2\frac{\pi}{\hbar}\left|\langle f|H|i\rangle\right|^2 \frac{dn}{dE_0}$$ with: $$\left|H_{fi}\right|^2 = \left|\langle f|H|i\rangle\right|^2 = const$$

Phase space: $$\frac{dn}{dE_0} = \frac{V^2}{4\pi^4\hbar^4}\, p_e^2\, dp_e\, p_\nu^2\, dp_\nu \frac{1}{dE_0}$$

Beta Spectrum:

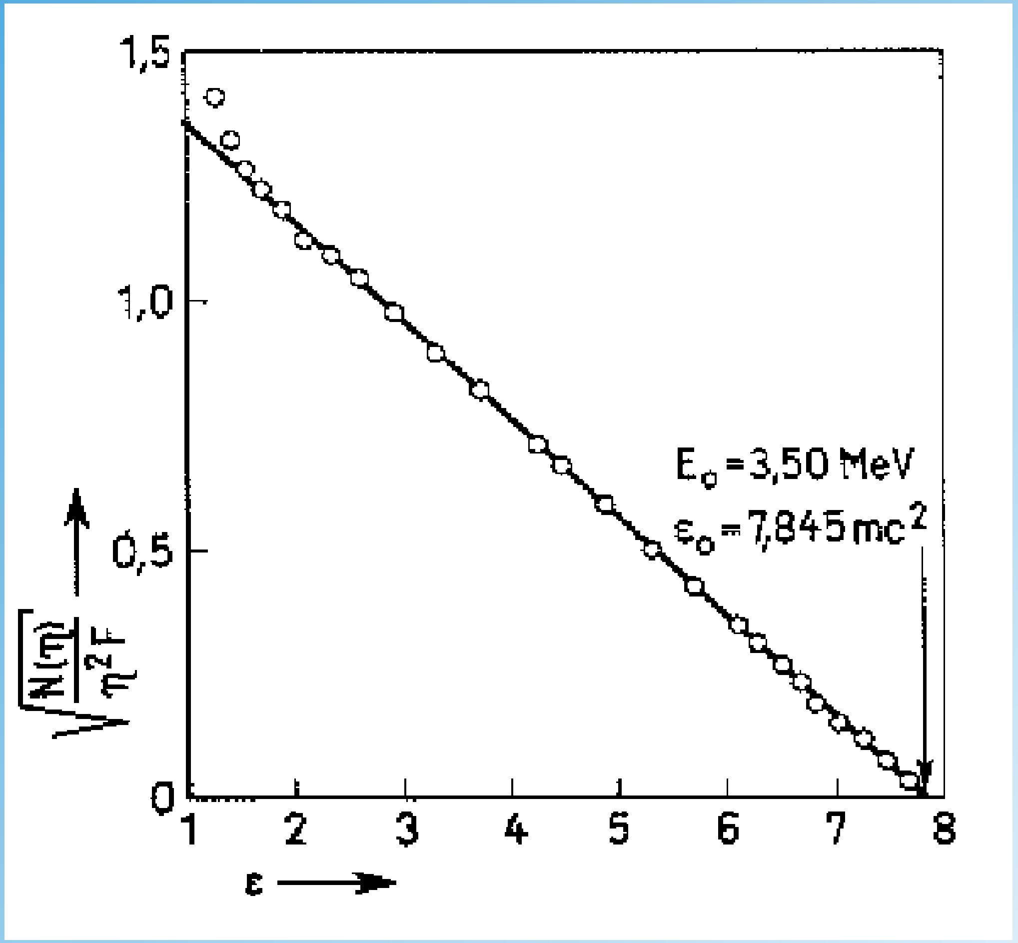
$$dN(\eta)\,d\eta = \left|H_{fi}\right|^2 \eta^2(\epsilon_0 - \epsilon)^2\, d\eta$$ with: $$\eta = p_e/m_e$$ $$\epsilon = E_e/m_e$$

Kurie-Plot (Fermi diagram)

$$\sqrt{\frac{dN(\eta)}{|H_{fi}|^2 \eta^2}} = \epsilon_0 - \epsilon$$

Beta decay of $^6He \rightarrow {}^6Li\, e^- \nu$

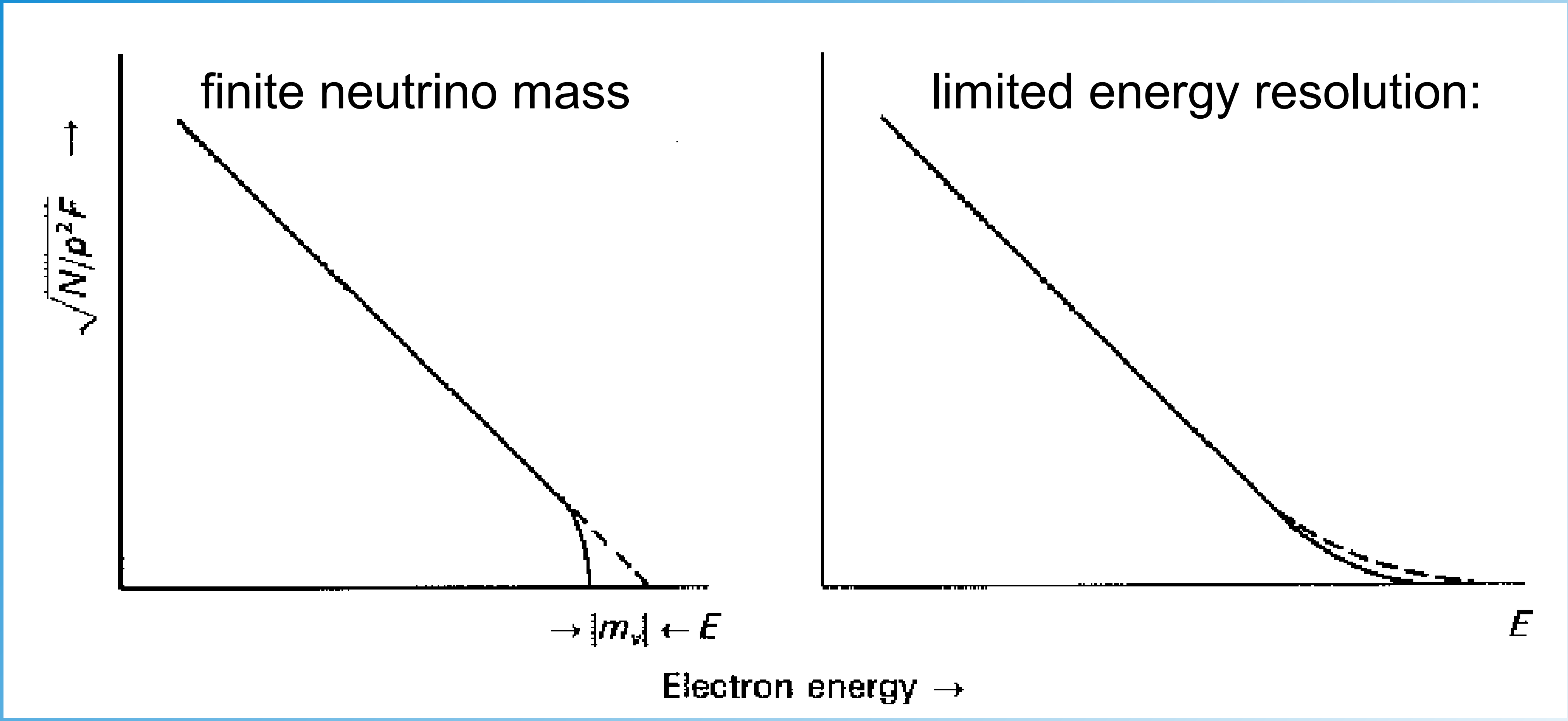
Note that the neutrino mass was set to zero here!



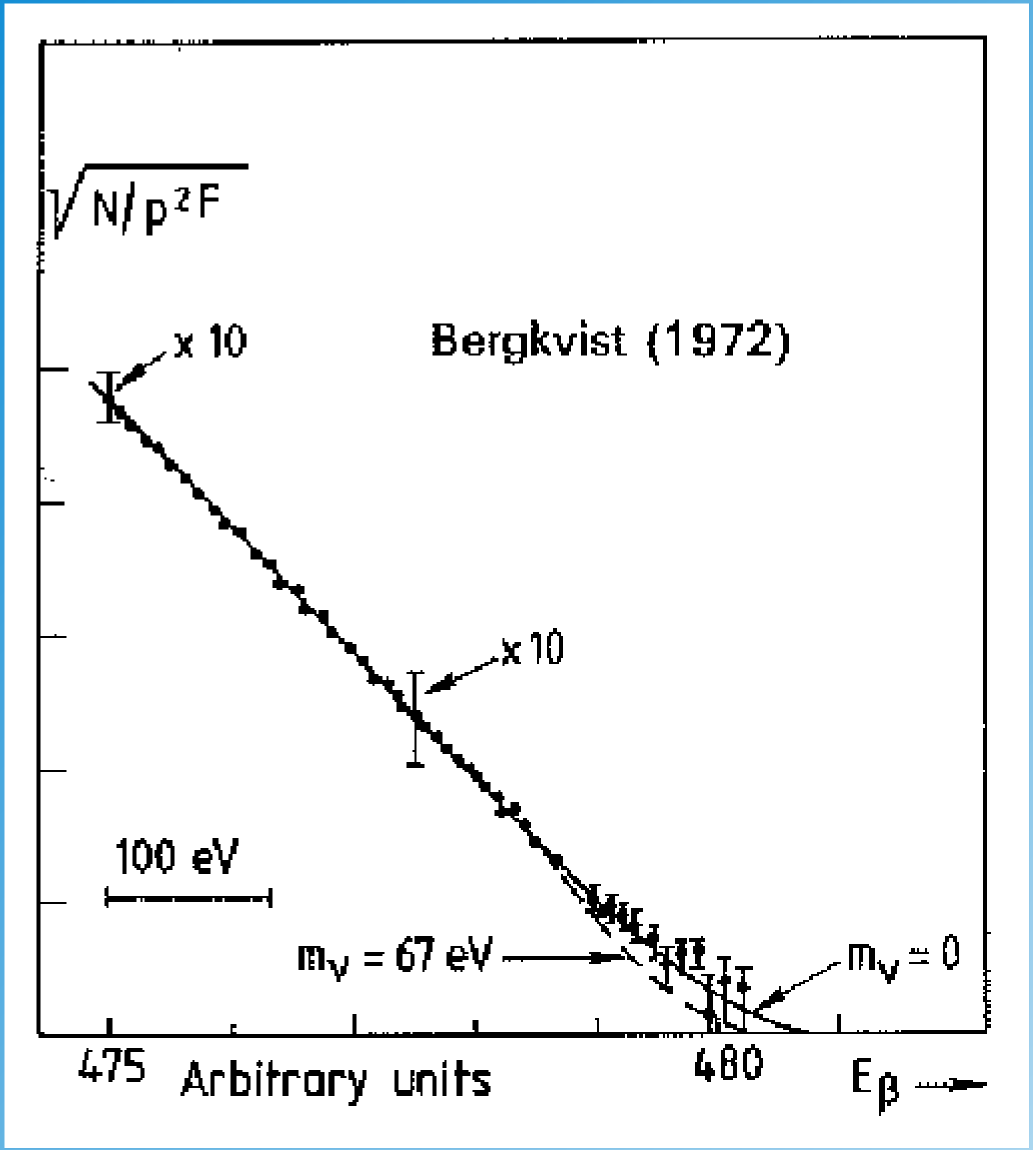
- **linear function!**
- **matrix element independent of energy!**
  - **Fermi Theory**

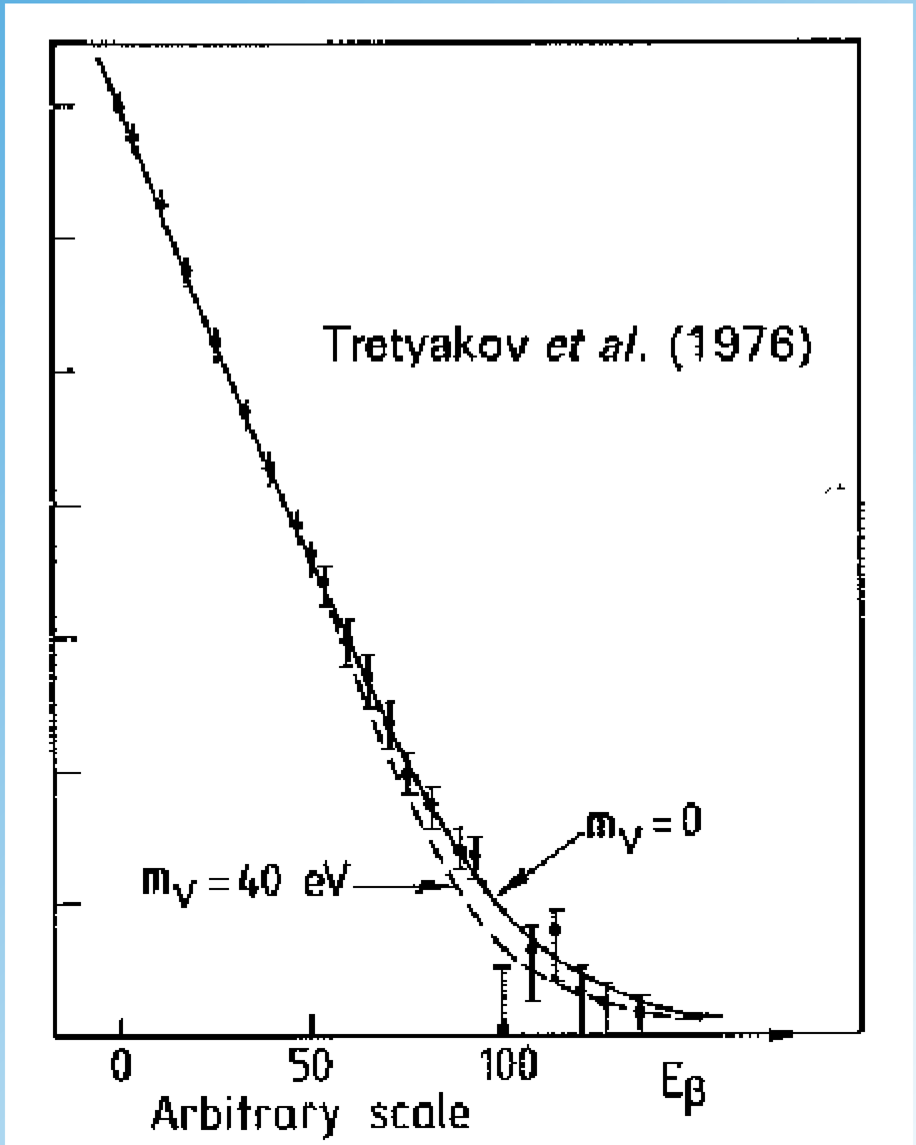
$$\sqrt{\frac{1}{|H_{fi}|^2}} = constant$$

Mass and Resolution Effects



Neutrino-Mass Measurement





Katrin Experiment

Measure Beta-Spectrum in the tritium decay

current limit: m_ν<1 eV

expected limit:

m_ν<~ 0.1-0.3 eV

Lifetime in Beta Decay

Transition probability depends only on available decay energy E_0

from beta Spectrum:

$$dN(\eta)\, d\eta = |H_{fi}|^2\, \eta^2 (\epsilon_0 - \epsilon)^2\, d\eta$$

with $\eta \sim \epsilon$

Decay width $\propto \epsilon_0^5 \propto M^5$

Isotop	Halbwertzeit
^{1}n	11,7 m
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Lifetime depends on the fifth power of the

- particle mass (muon decay)
- Q value (nuclear decay $Q \sim E_e + E_\nu$)

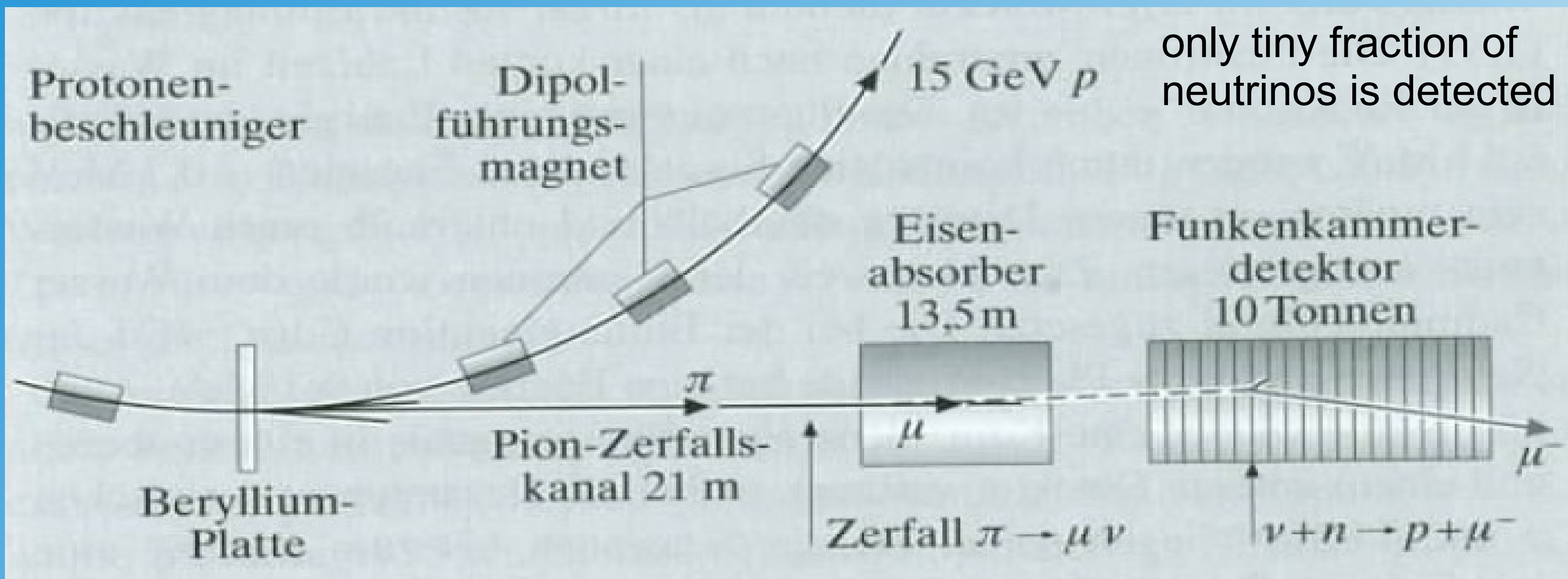
(only little recoil)

Weak Force

- Lifetimes in weak decays of order 10^{-10} seconds – 10^{10} years
- Interaction length:
  - Nuclear interaction (Fe): λ_{strong} ~ O(10) cm
  - Weak interaction (Fe): >> 10^3 km (neutrino energy dependent)

Weak Force

Discovery of muon neutrino (Lederman et al.):

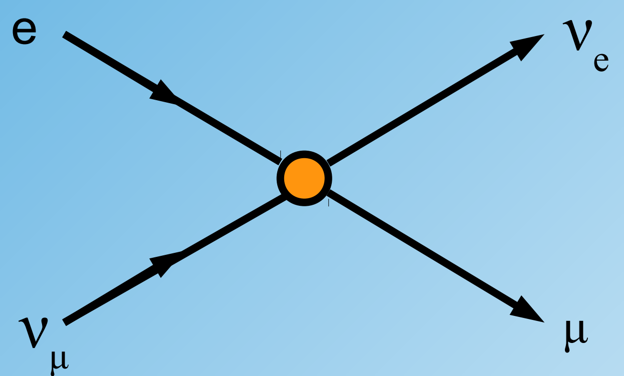


The weak force is really weak!

Weak scattering processes

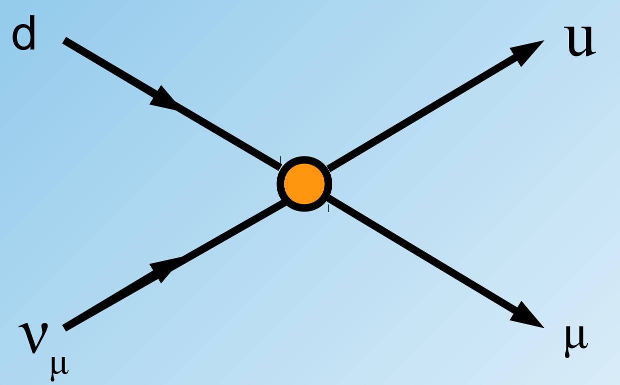
- A constant matrix element (Fermi theory) gives an energy dependent cross section in weak scattering processes
- Reason: phase space!

$$\sigma(\nu_\mu e \rightarrow \mu \nu_e) \propto G_F^2 s$$



neutrino-electron scattering

$$\sigma(\nu_\mu d \rightarrow \mu u) \propto G_F^2 s$$



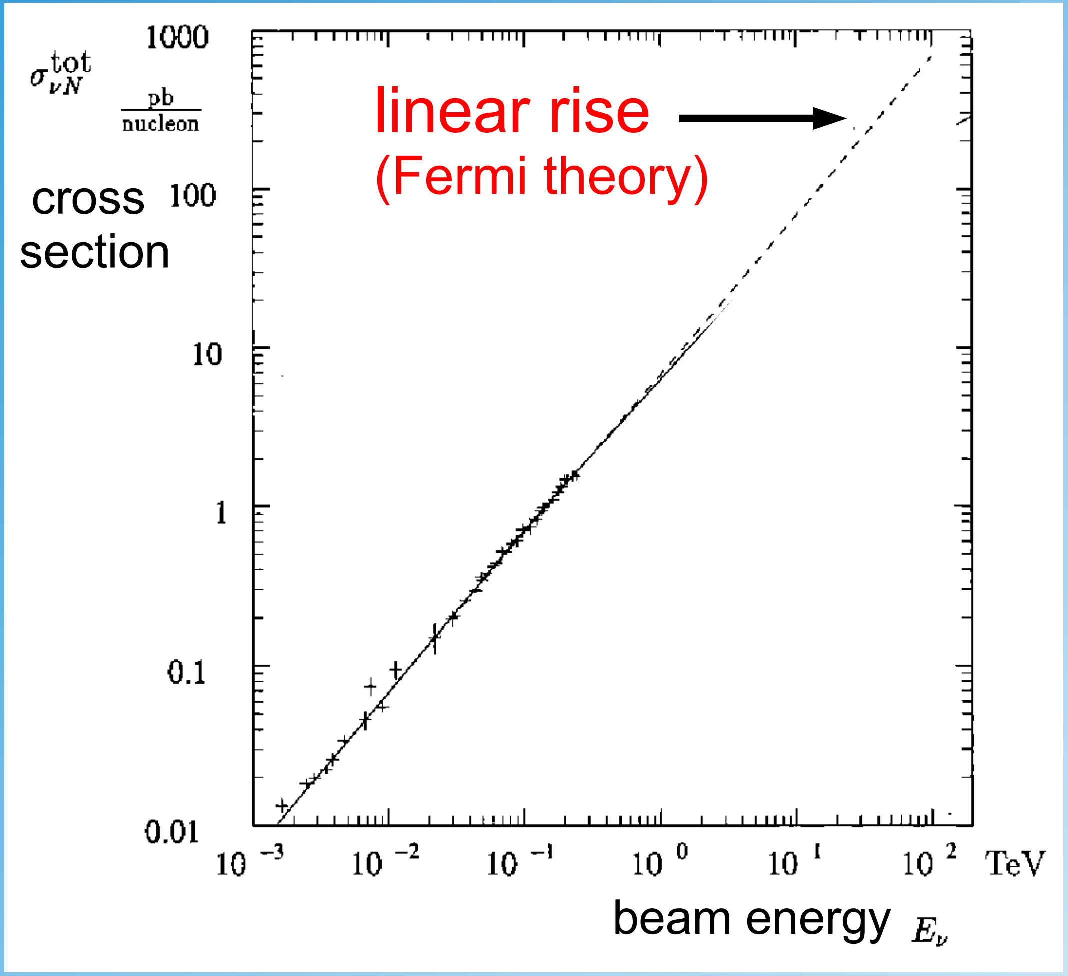
neutrino-nucleon scattering

Weak scattering processes I

Kinematics:

- Fixed Target: $s = 2 E_\nu M_{target}$

Scattering cross section should rise linearly with neutrino beam energy



Weak scattering processes II

- Center of mass energies limited in Fixed Target Experiments
- Trick: invert reaction at colliders:

$$\nu_e d \rightarrow e^- u \qquad \leftrightarrow \qquad e^+ d \rightarrow \bar{\nu}_e u$$

<u>Kinematics:</u>

- Fixed Target: $s = 2 E_\nu^{target} M_{target}$
- Collider: $s = 4 E_e^{coll} E_p$ (HERA: electron-proton)

from comparison:

$$E_\nu^{target} \sim 2 E_e^{coll} \frac{E_p}{M_{target}} \approx 50\ TeV$$

Electron-Proton Collider HERA

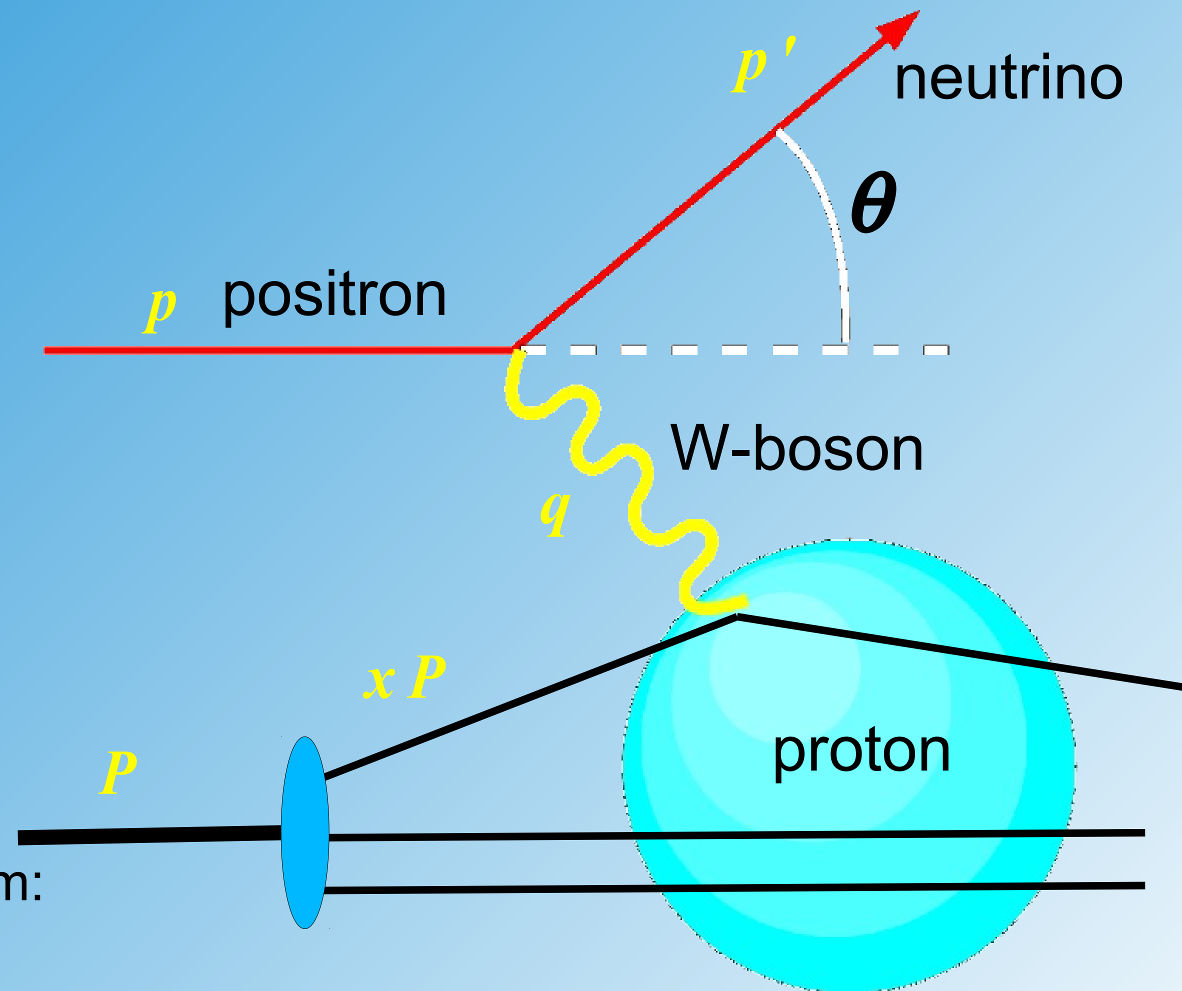
E_e = 26.7 GeV E_p= 920 GeV

Lorentz Invariant Kinematics of Deep Inelastic Scattering Process

The virtuality of the exchanged photon is given by:

$$Q^2 = -q^2 = -(p - p')^2$$

$$\propto \frac{1}{\sin^4 \theta/2}$$



Relative energy loss (inelasticity):

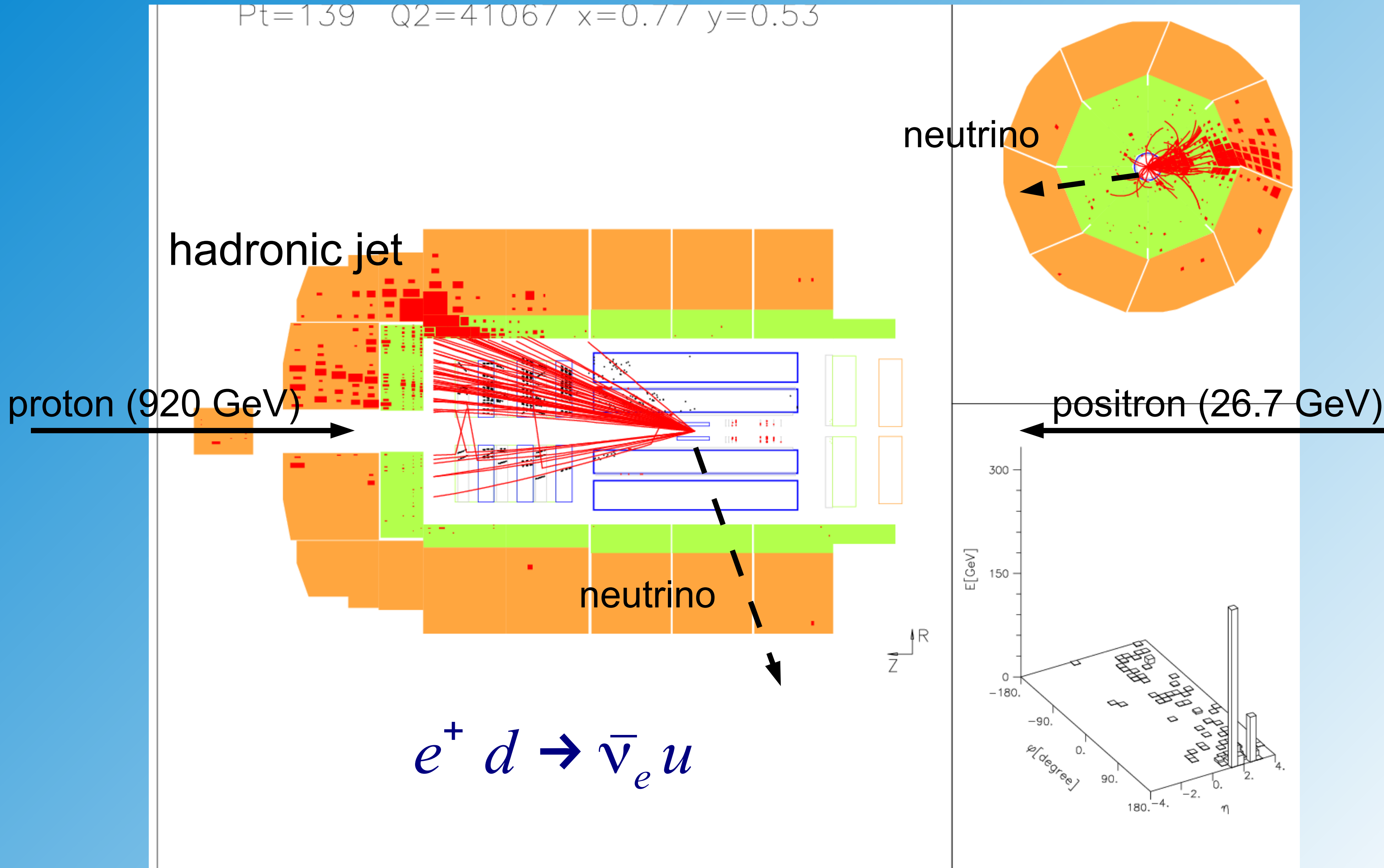
$$y = \frac{\nu}{E_{Elektron}} = \frac{q\,P}{p\,P}$$

relative fraction of parton momentum:

$$x = \frac{q^2}{2\,q\,P} = \frac{Q^2}{s\,y}$$

with cms energy: $s = 2\,p\,P$

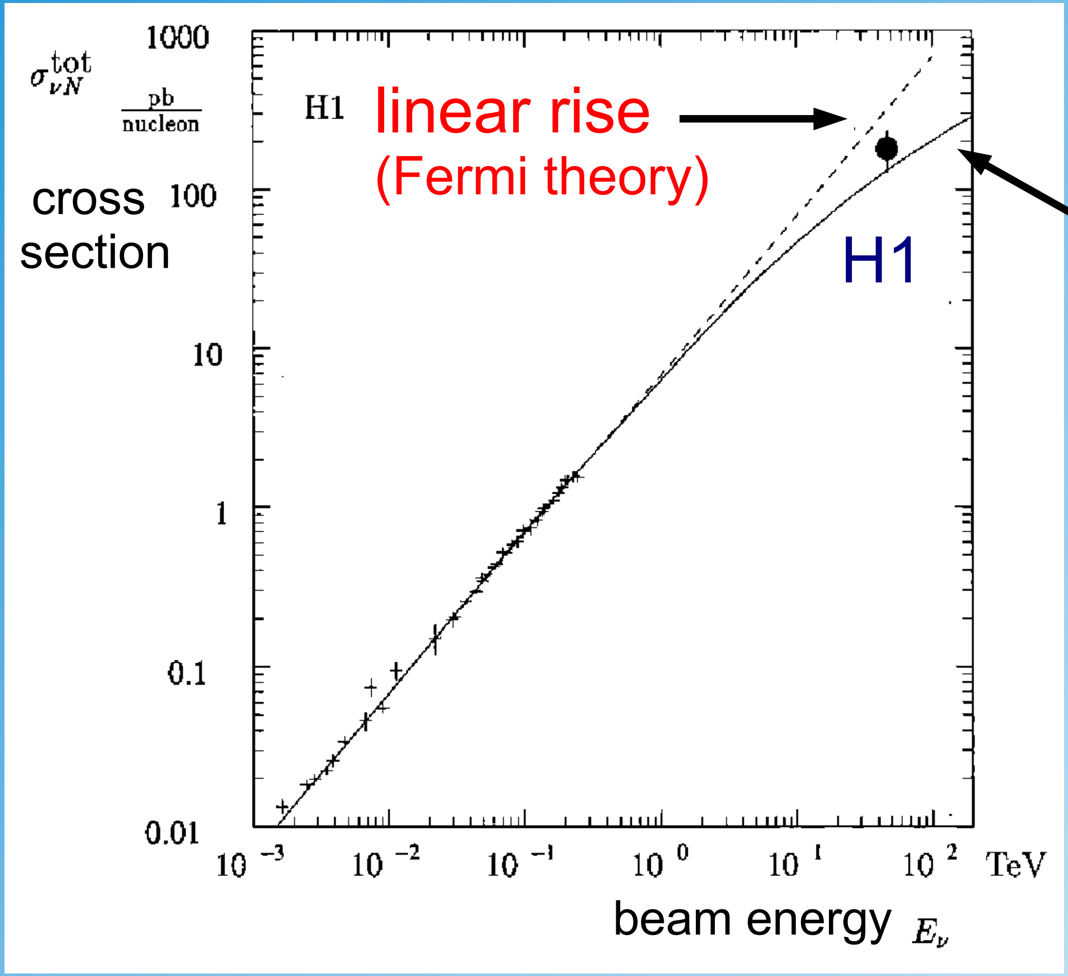
Charged Current Event at H1



$$e^{+}\, d \rightarrow \bar{\nu}_e\, u$$

Weak scattering processes III

HERA beam energy translated into Fixed target



1992 (13 events)

effect of W-propagator

$$\frac{G_F}{\sqrt{2}} \rightarrow \frac{g}{M_W^2 + Q^2}$$

Breakdown of Fermi theory at high energies $s^{1/2} \sim 100$ GeV

Lorentz Structure of Weak Process?

Lagrangian (independent of energy)

$$L = \frac{G}{\sqrt{2}} \left(\bar{f}\, \Gamma\, f\,' \right) \left(\overline{f\,''}\, \tilde{\Gamma}\, f\,''' \right)$$

¿

most general ansatz for operator Γ:

Vector Current:

$j_V^{\mu} = \bar{\psi}\, \gamma^{\mu}\, \psi$ (Fermi´s proposal):

Axial-vector Current:

$j_A^{\mu} = \bar{\psi}\, \gamma^{\mu}\, \gamma^{5}\, \psi$

scalar coupling:

$\lambda = \bar{\psi}\, \psi$

pseudoscalar coupling:

$\lambda = \bar{\psi}\, \gamma^{5}\, \psi$

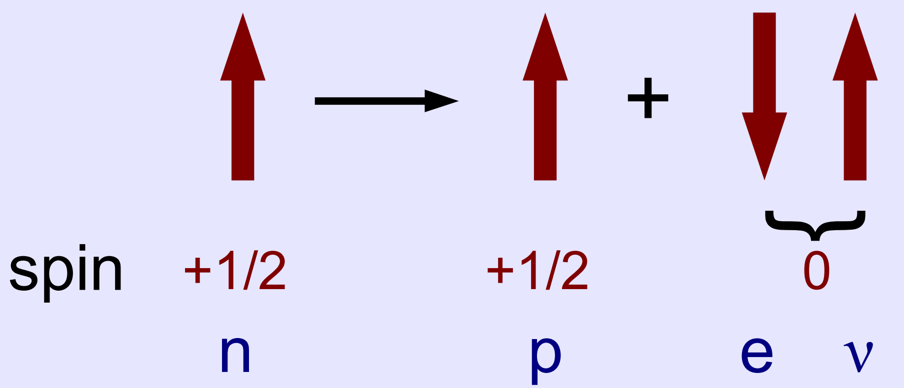
Tensor Coupling

$\sigma_A^{\mu\nu} = \bar{\psi} \left(\gamma^{\mu} \gamma^{\nu} - \gamma^{\nu} \gamma^{\mu} \right) \psi$

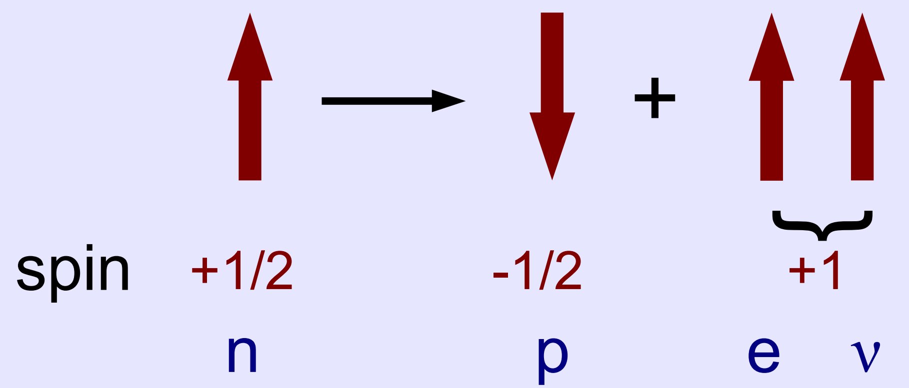
Test of Lorentz Structure?

Different transitions in weak decays: $n \rightarrow p\, e\, \nu$

<u>Fermi transition</u>



<u>Gamov Teller transition</u>



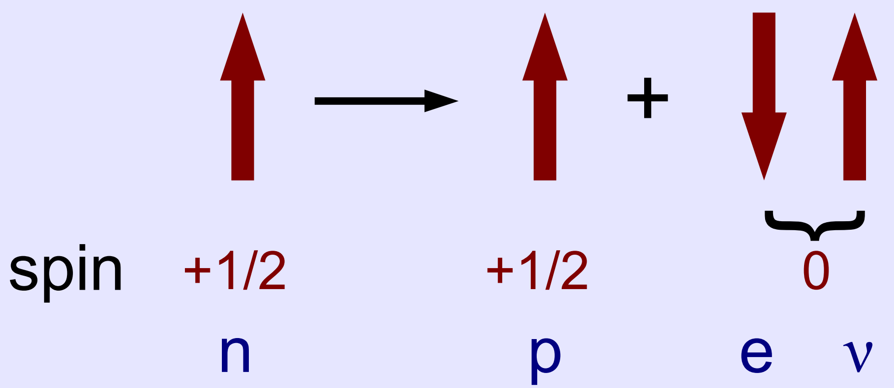
- no spin flip (ΔJ=0)
- electron and neutrino in singlet state

- spin flip (ΔJ=0,1)
- electron and neutrino in triplet state

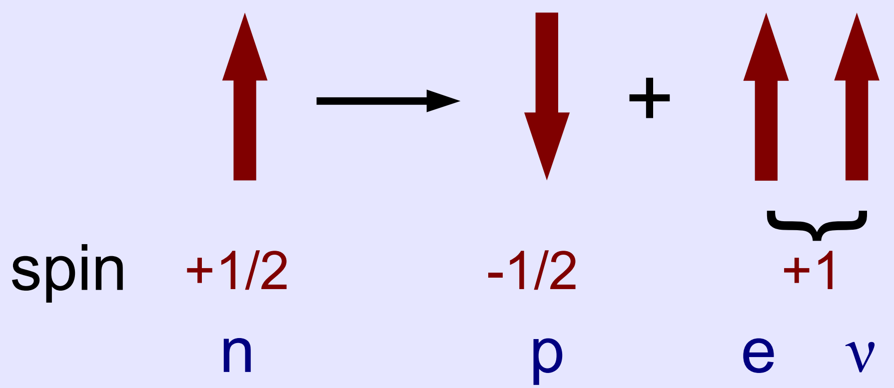
Test of Lorentz Structure?

Different transitions in weak decays: $n \rightarrow p\, e\, \nu$

Fermi transition



Gamov Teller transition



- no spin flip (ΔJ=0)
- electron and neutrino in singlet state

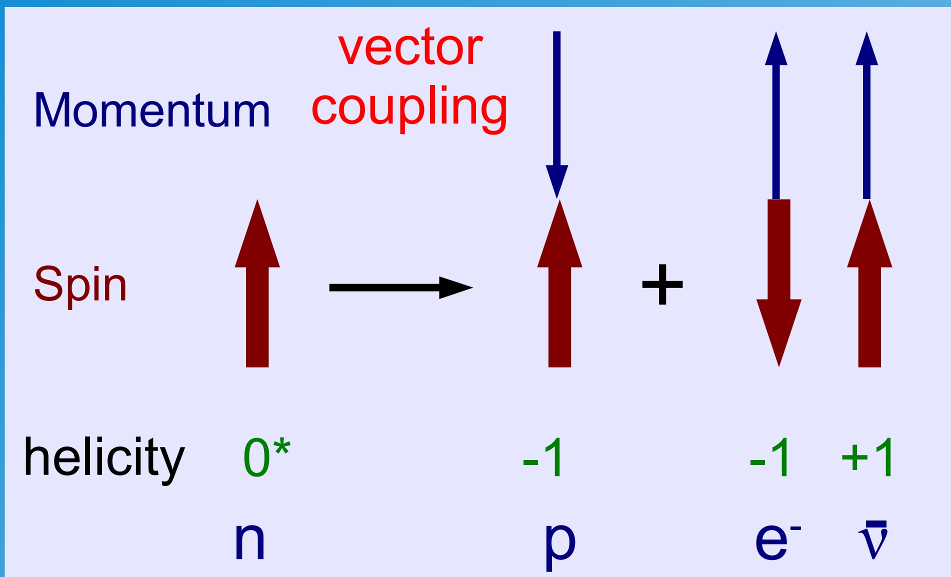
- spin flip (ΔJ=0,1)
- electron and neutrino in triplet state

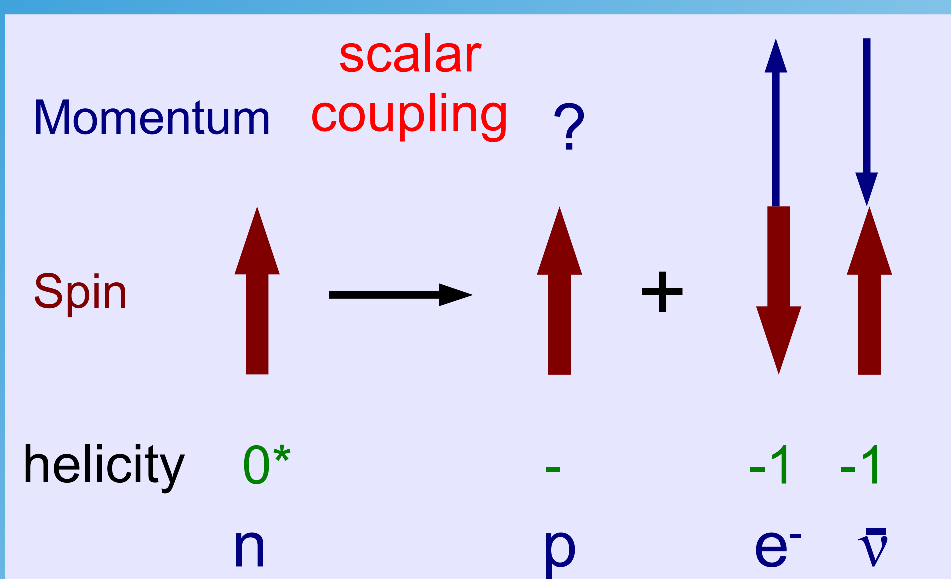
Note: **spin flip ≠ helicity flip**

Test of Lorentz Structure?

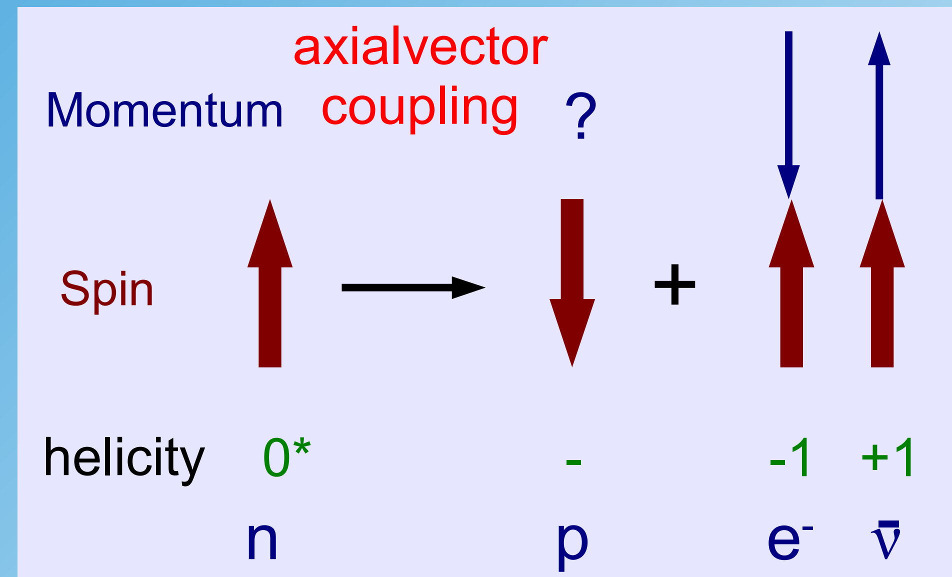
Fermi transition

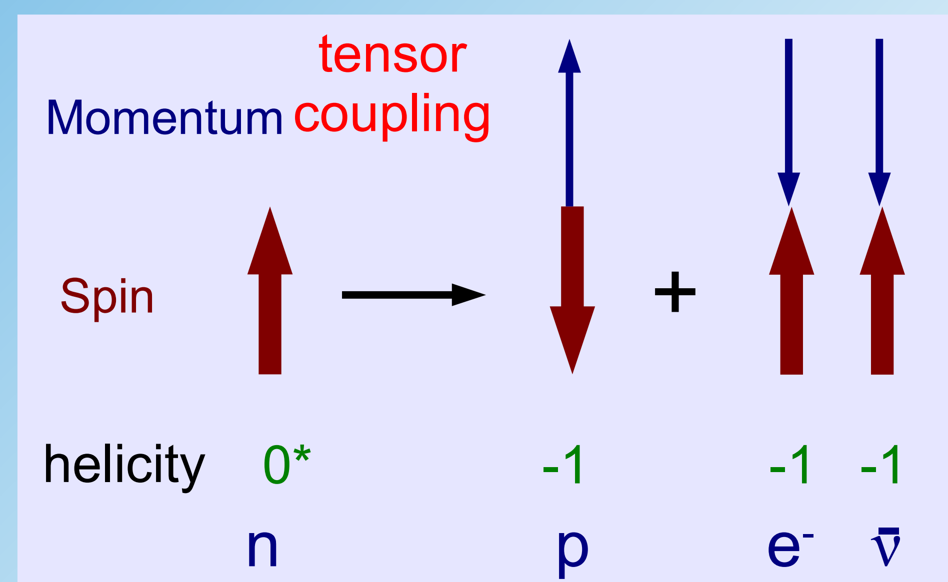
fix electron helicity= -1





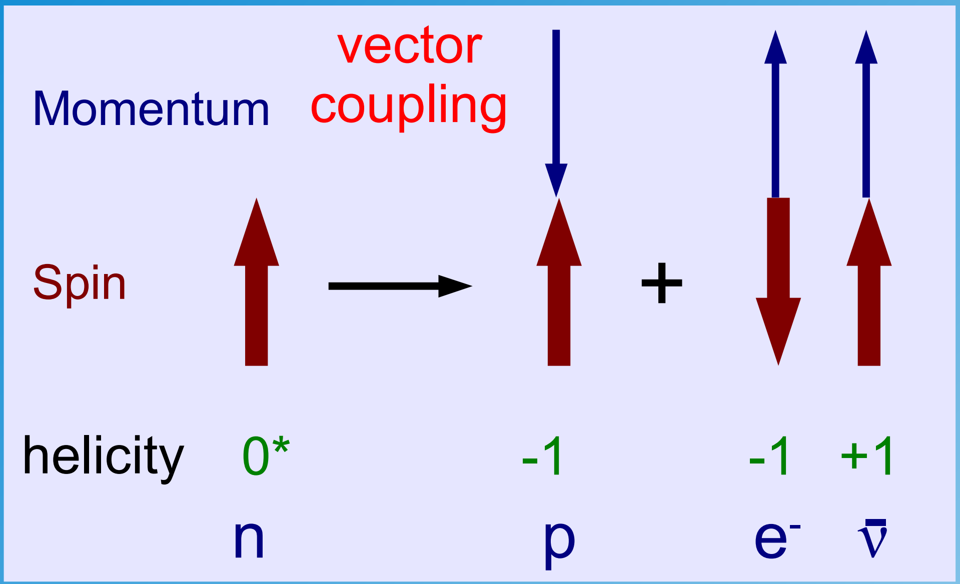
Gamov Teller transition

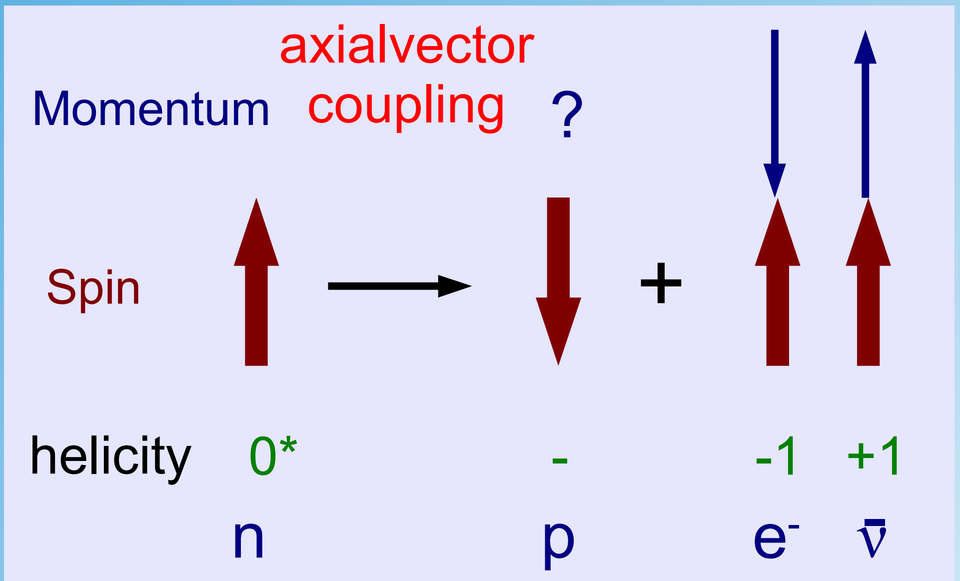


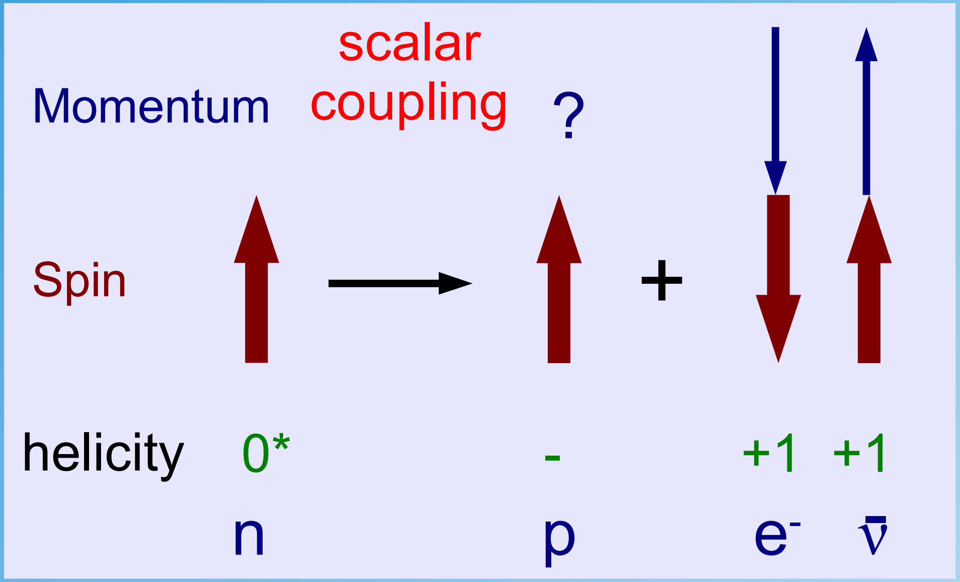


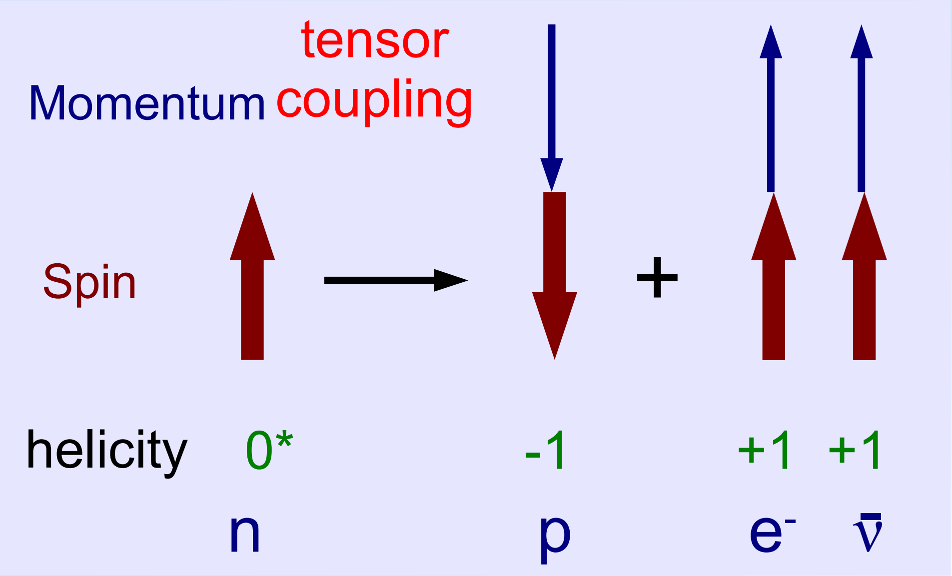
Test of Lorentz Structure?

Fermi transition RH anti-neutrino helicity Gamov Teller transition



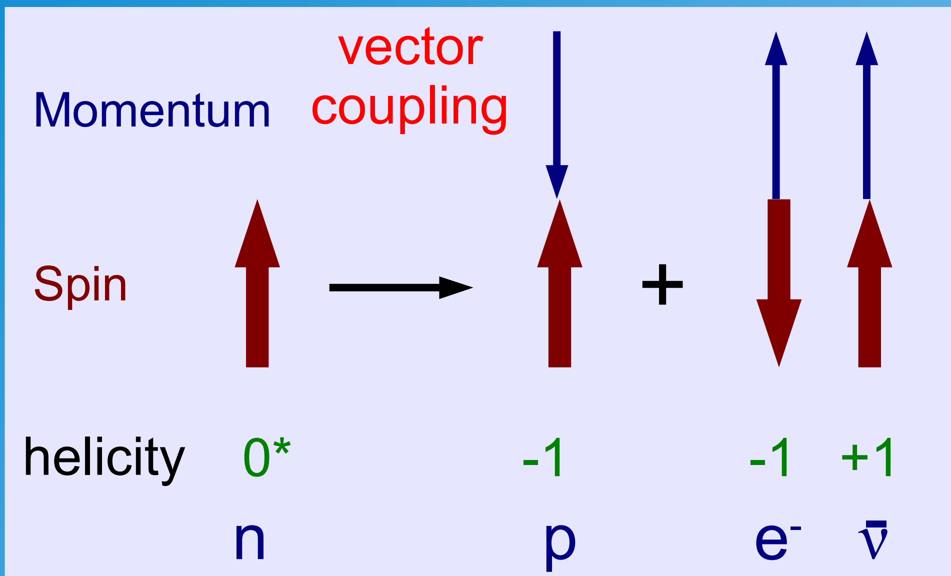


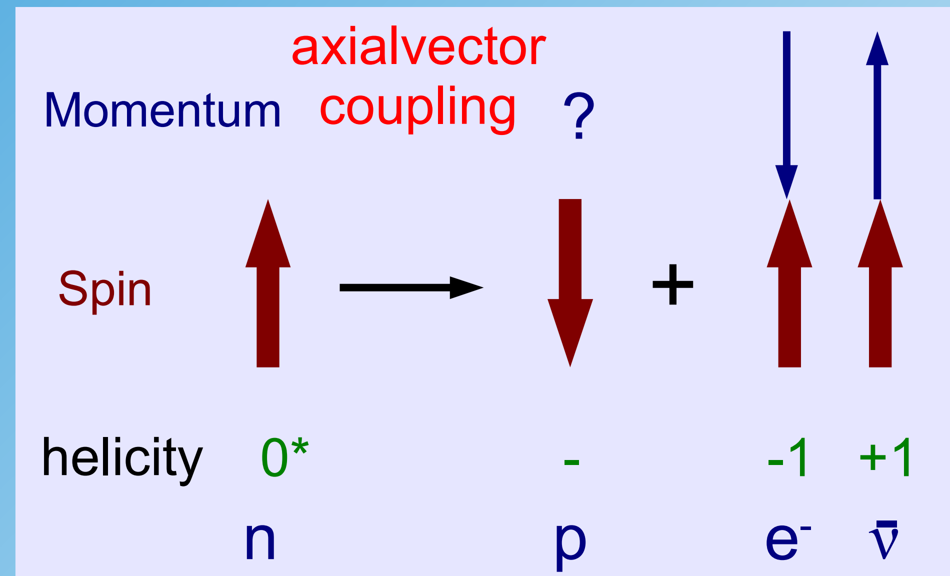




Test of Lorentz Structure?

Fermi transition   fix electron helicity= -1   Gamov Teller transition

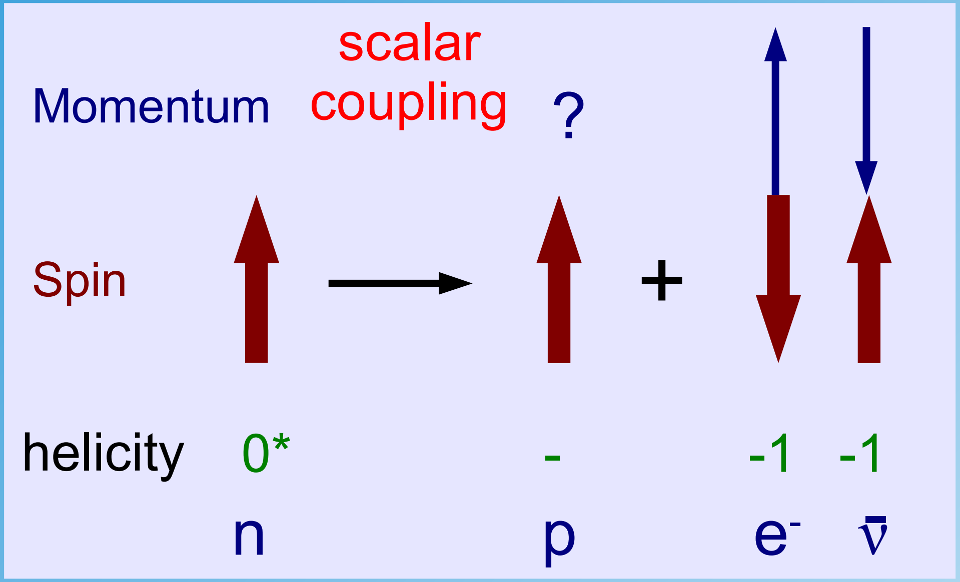


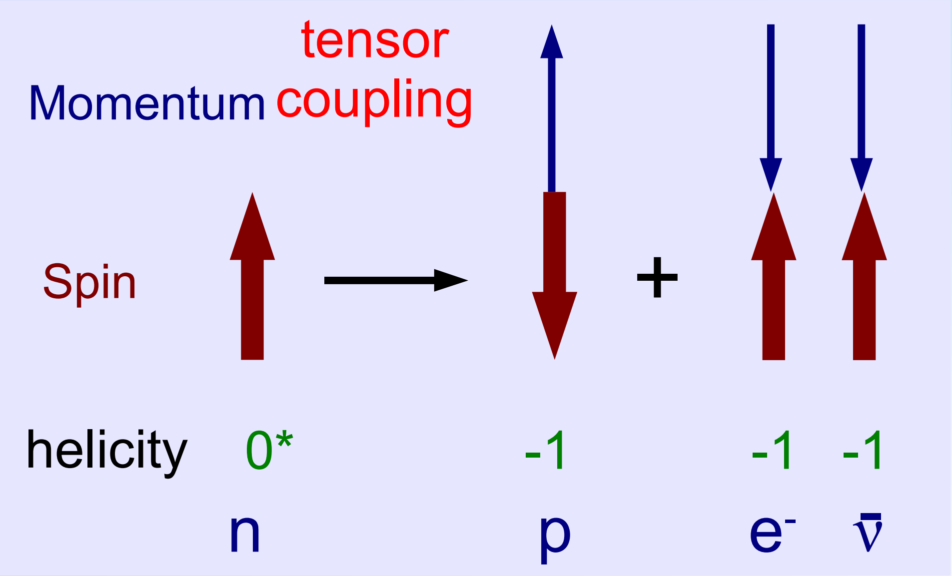


- Both, Fermi and Gamov Teller transitions observed in weak interactions
- Vector and Axialvector currents conserve helicities and realised in nature

Test of Lorentz Structure?

- Scalar and tensor (pseudoscalar) couplings flip helicity!
- Have to measure spin-orientation of decay leptons!





The End

Gamma Matrices II

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \quad \gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \quad \gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\gamma^5 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

(in other representations
g5 is diagonal)

Spinors of Helicity States

$$u_R = |\vec{p},\, \lambda = +1/2\rangle \qquad (u_1)$$
$$u_L = |\vec{p},\, \lambda = -1/2\rangle \qquad (u_2)$$
$$v_L = |\vec{p},\, \lambda = -1/2\rangle \qquad (v_1)$$
$$v_R = |\vec{p},\, \lambda = +1/2\rangle \qquad (v_2)$$

fermions:

$$u_R = \sqrt{E+m}\begin{pmatrix} 1 \\ 0 \\ \frac{|\vec{p}|}{E+m} \\ 0 \end{pmatrix} \qquad u_L = \sqrt{E+m}\begin{pmatrix} 0 \\ 1 \\ 0 \\ \frac{-|\vec{p}|}{E+m} \end{pmatrix}$$

anti-fermions:

$$v_L = \sqrt{E+m}\begin{pmatrix} \frac{|\vec{p}|}{E+m} \\ 0 \\ 1 \\ 0 \end{pmatrix} \qquad v_R = \sqrt{E+m}\begin{pmatrix} 0 \\ \frac{-|\vec{p}|}{E+m} \\ 0 \\ 1 \end{pmatrix}$$

limit $p \longrightarrow \infty$

$u_R \longrightarrow v_L$

$u_L \longrightarrow v_R$

Chirality Operator

limit m → 0

$$u_R \sim v_L \sim \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \qquad u_L \sim v_R \sim \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

operator:

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & \underline{1} \\ \underline{1} & 0 \end{pmatrix}$$

right chirality

$$\gamma_5 u_R = u_R$$
$$\gamma_5 v_L = v_L$$

left chirality

$$\gamma_5 u_L = -u_L$$
$$\gamma_5 v_R = -v_R$$

eigenvalues ± 1

left-handed (chiral) particles: -1

right-handed (chiral) particles: +1

note: a right-handed chiral anti-particle has a left-handed helicity

Projection Operator

definition: $\Pi^{\pm} = \frac{1 \pm \gamma_5}{2}$ in the limit of |E| → infinity

fermions

$\Pi^{+} u_R = u_R \qquad \Pi^{+} u_L = 0$

$\Pi^{-} u_L = u_L \qquad \Pi^{-} u_R = 0$

anti-fermions

$\Pi^{+} v_L = v_L \qquad \Pi^{+} v_R = 0$

$\Pi^{-} v_R = v_R \qquad \Pi^{-} v_L = 0$

projection:

$\Pi^{+} \psi = R$ (right-handed (chiral) state)

$\Pi^{-} \psi = L$ (left-handed (chiral) state)

reformulate Dirac Equation:

$$i \gamma^{\mu} \partial_{\mu} R = m L \qquad i \gamma^{\mu} \partial_{\mu} L = m R$$

note: massive fermions must have left-handed and right handed components

Vector and Axial Currents

Vector Current:

$j_V^\mu = \bar{\psi}\gamma^\mu\psi \quad (R\gamma^\mu R,\ \ L\gamma^\mu L)$ in QED: $\partial_\mu j_V^\mu = 0$ (conservation of currents)

Axial-vector Current:

$j_A^\mu = \bar{\psi}\gamma^\mu\gamma^5\psi$ note: $\gamma^\mu\gamma^5 = -\gamma^5\gamma^\mu$

Left (right)-handed Current:

$j_L^\mu = \bar{\psi}\gamma^\mu\Pi^-\psi \qquad j_R^\mu = \bar{\psi}\gamma^\mu\Pi^+\psi$

relations:

$j_L^\mu = 1/2\,(j_V^\mu - j_A^\mu)$

$j_R^\mu = 1/2\,(j_V^\mu + j_A^\mu)$

weak interaction (V-A theory):