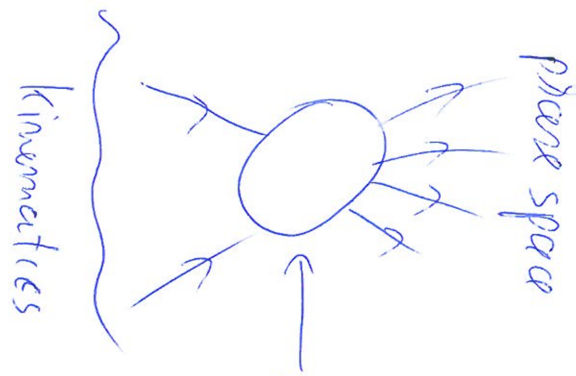
Basics of Elementary Particle Physics

6 lectures (4T, 2E) to motivate / learn / recall tools to calculate elementary processes.
+ measure

Classical exercise / task in particle physics:



Dynamics ↔ Feynman rules

Outline

I Kinematics (mostly repetition)

I 1) Decay

I 2) 2-2 Scattering

I 3) Crossing

II Cross section and phase space

II 1) Transition rate, cross section, decay rate

II 2) 2-2 cross section

III QED, QFT, Feynman rules

III 1) How to describe fermions and photons

III 2) How to propagate fermions and photons

III 3) Feynman rules and elementary processes

IV Theoretical QED Aspects

IV 1) External photons

IV 2) Running coupling

IV 3) Bremsstrahlung

+ Exptl. Aspects by André Schöning

I KINEMATICS

Reminder: •) Natural units: $\hbar = c = 1$

$$\Rightarrow [L] = [T]$$

$$[E] = [T]^{-1} = [M]$$

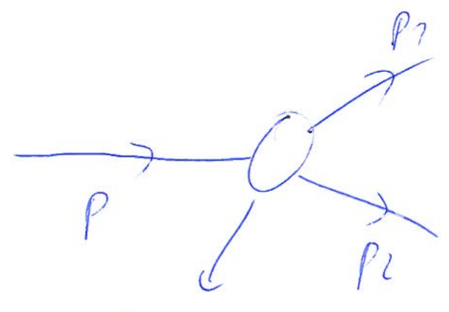
various useful conversions... ($\Rightarrow$ Exercise 1)

•) $p^\mu = (E, \vec{p})$; $p_\mu = (E, -\vec{p})$

$$p^2 = m^2 = E^2 - \vec{p}^{\,2}$$

$$E \simeq \begin{cases} |\vec{p}|\left(1 + \frac{m^2}{2\vec{p}^{\,2}}\right) & \text{highly relat. } E \gg m \\ m\left(1 + \frac{\vec{p}^{\,2}}{2m^2}\right) & \text{non-relat. } E \ll m \end{cases}$$

I1) Decay



„Dynamics"

$p^\mu = (M, \vec{0})$ rest system of particle

$$E_1 = \frac{1}{2M}\left(M^2 + m_1^2 - m_2^2\right)$$

$$E_2 = \frac{1}{2M}\left(M^2 + m_2^2 - m_1^2\right)$$

$$|\vec{p}_1| = |\vec{p}_2| = \frac{1}{2M}\sqrt{\lambda(M^2, m_1^2, m_2^2)}$$

(Exercise 2)

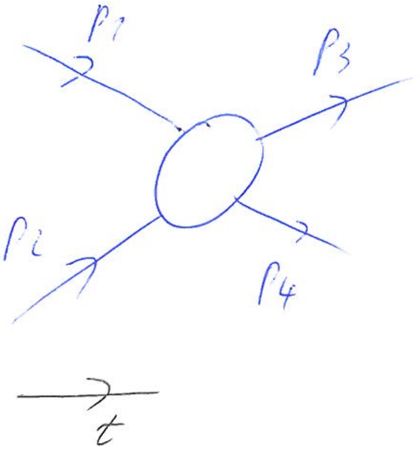
define important $\lambda(a,b,c) = a^2+b^2+c^2-2(ab+ac+bc)$

$$= \left[a-(\sqrt{b}+\sqrt{c})^2\right]\left[a-(\sqrt{b}-\sqrt{c})^2\right]$$

$$\simeq \begin{cases} a^2 & ; \ a \gg b,c \\ a(a-4b) & ; \ b=c \end{cases}$$

symmetric in a,b,c

I 2) 2-2 scattering



Lorentz-invariant:

$$\underbrace{p_i^2 \ ; \ p_1\cdot p_2,\ p_1\cdot p_3,\ p_1\cdot p_4,\ p_2\cdot p_3,\ p_2\cdot p_4,\ p_3\cdot p_4}$$

$6-4=2$ linearly independent!

$p_1+p_2=p_3+p_4$

Choice of those 2 is arbitrary, e.g. E_1, Θ_{13}

Lorentz-invariant choice:

$$\boxed{\begin{aligned} s &= (p_1+p_2)^2 = (p_1+p_3)^2 \\ t &= (p_1-p_3)^2 = (p_4-p_2)^2 \\ u &= (p_1-p_4)^2 = (p_3-p_2)^2 \end{aligned}}$$

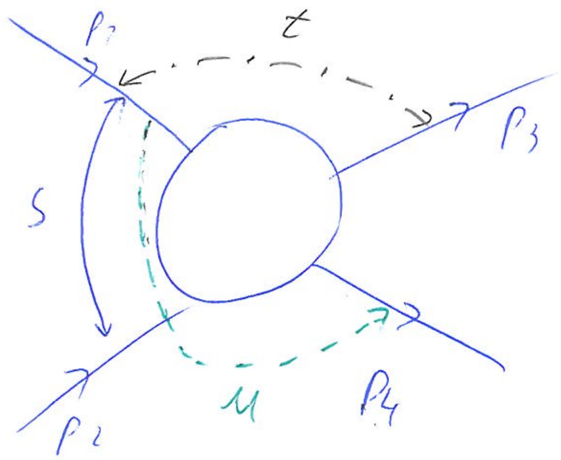
MANDELSTAM VARIABLES

should not be independent.. indeed:

$$s + t + u = \sum_{i=1}^{4} m_i^2$$

s: square of center-of-mass energy

t: square of momentum transfer



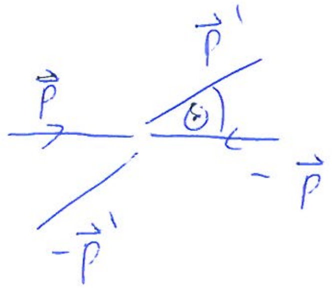
Important reference frames:

1) Center-of-mass: $\vec{p}_1 + \vec{p}_2 = 0 = \vec{p}_3 + \vec{p}_4$

2) Lab : $\vec{p}_2 = 0$

(3) Breit : $\vec{p}_1 + \vec{p}_3 = 0$) rare

a) CMS



$$E_{1,2} = \frac{1}{2\sqrt{s}}\left(s \pm m_1^2 \mp m_2^2\right)$$

$$E_{3,4} = \frac{1}{2\sqrt{s}}\left(s \pm m_3^2 \mp m_4^2\right)$$

$$|\vec{p}| = \frac{1}{2\sqrt{s}}\sqrt{\lambda(s, m_1^2, m_2^2)}$$

$$|\vec{p}\,'| = \frac{1}{2\sqrt{s}}\sqrt{\lambda(s, m_3^2, m_4^2)}$$

$$\cos\Theta = \frac{s(t-u) + (m_1^2 - m_2^2)(m_3^2 - m_4^2)}{\sqrt{\lambda(s, m_1^2, m_2^2)\,\lambda(s, m_3^2, m_4^2)}}$$

(Exercise 2)

-) asymptotic behavior: $s \gg m_i^2$ $\quad E_i = |\vec{p}| = |\vec{p}\,'| = \sqrt{s}/2$

-) physical region: e.g. $t = f(\cos\Theta)$

  $\Rightarrow$ $t_{min;max}$ from $\cos\Theta = \pm 1$

  e.g. $t \in [-s, 0]$ for $m_i = 0$

-) $|\vec{p}|, |\vec{p}\,'| \geq 0$: $s_{min} = \max\{(m_1+m_2)^2, (m_3+m_4)^2\}$

  "threshold"

#) example elastic scattering $(m_1 = m_3,\ m_2 = m_4)$

$$t = -2\vec{p}^{\,2}(1-\cos\Theta) \Rightarrow \boxed{-4\vec{p}^{\,2} \le t \le 0}$$

$E_1 = E_3$
$|\vec{p}| = |\vec{p}\,'|$

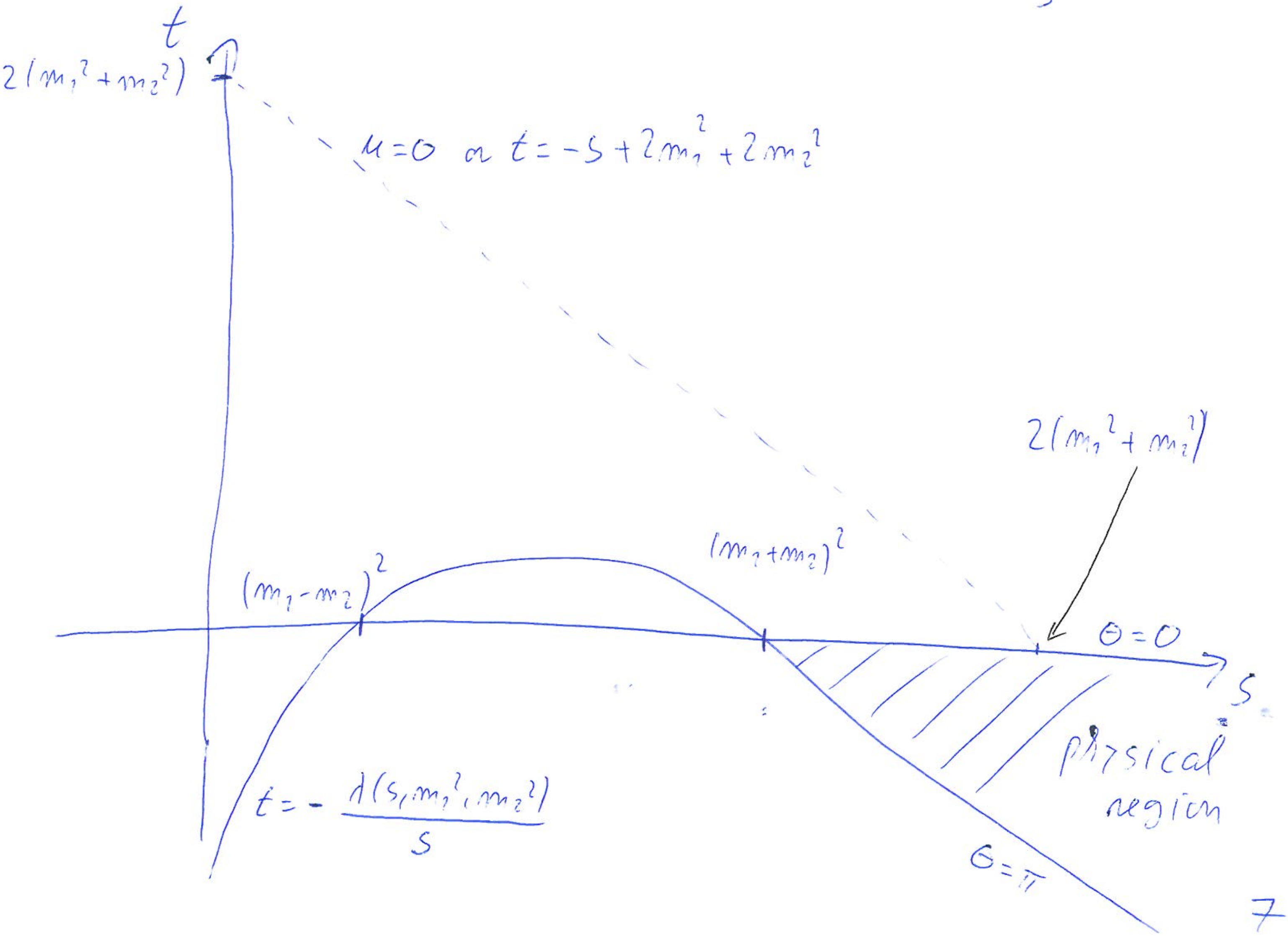
$\Theta = \pi$ backward

$\Theta = 0$ forward

$$u = \left[\frac{1}{\sqrt{s}}(m_1^2 - m_2^2)\right]^2 - 2\vec{p}^{\,2}(1+\cos\Theta)$$

$$\Rightarrow \quad t_{max} = 0 \ ; \ u = 2m_1^2 + 2m_2^2 - s$$

$$t_{min} = -4\vec{p}^{\,2} = -\frac{\lambda(s, m_1^2, m_2^2)}{s} \ ; \ u = \frac{(m_1^2 - m_2^2)^2}{s}$$



#) angular distribution

$$d\Omega = 2\pi \, d\cos\Theta$$

$$\Rightarrow \frac{d\Omega}{dt} = \frac{d\Omega}{d\cos\Theta}\left|\frac{d\cos\Theta}{dt}\right| = \frac{4\pi s}{\left[\lambda(s,m_1^2,m_2^2)\,\lambda(s,m_3^2,m_4^2)\right]^{1/2}}$$

$$= \frac{\pi}{|\vec{p}|\,|\vec{p}\,'|}$$

#) relative velocity

$$v_{12} = |\vec{v}_1 - \vec{v}_2| = \left|\frac{\vec{p}_1}{E_1} - \frac{\vec{p}_2}{E_2}\right| = \frac{|\vec{p}_1|}{E_1E_2}(E_1+E_2) = \frac{|\vec{p}_1|\sqrt{s}}{E_1E_2}$$

$$\Rightarrow E_1E_2v_{12} = \sqrt{s}\left(E_1^2-m_1^2\right)^{1/2} = \sqrt{s}\left[\frac{1}{4s}(s+m_1^2-m_2^2)^2 - \frac{4s}{4s}m_1^2\right]$$

$$= \left[\frac{1}{4}\left(s^2+m_1^4+m_2^4-2sm_1^2-2sm_2^2-2m_1^2m_2^2\right)\right]^{1/2}$$

$$= \left[\frac{1}{4}\left\{\underbrace{(s-m_1^2-m_2^2)^2}_{(2p_1\cdot p_2)^2} - 4m_1^2m_2^2\right\}\right]^{1/2}$$

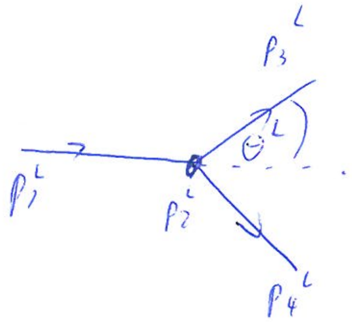
$$\Rightarrow \boxed{E_1E_2v_{12} = \sqrt{(p_1\cdot p_2)^2 - m_1^2m_2^2}}$$

Møller flux factor

$$\simeq s/2 \quad \text{for } p_i^2 \gg m_i^2$$

b) Labsystem

$p_2^L = (m_2, \vec{0})$ target experiment



$$|\vec{p}_1^{\,L}| = \frac{1}{2m_2}\sqrt{\lambda(s, m_1^2, m_2^2)}$$

$$|\vec{p}_3^{\,L}| = \frac{1}{2m_2}\sqrt{\lambda(u, m_2^2, m_3^2)}$$

$$|\vec{p}_4^{\,L}| = \frac{1}{2m_2}\sqrt{\lambda(t, m_2^2, m_4^2)}$$

$$E_1^L = \frac{1}{2m_2}\left(s - m_1^2 - m_2^2\right)$$

$$E_3^L = \frac{1}{2m_2}\left(m_2^2 + m_3^2 - u\right)$$

$$E_4^L = \frac{1}{2m_2}\left(m_2^2 + m_4^2 - t\right)$$

$$\cos\theta^L = \frac{m_2^2(-2m_1^2 - 2m_3^2 + m_4^2 + t) + (m_1^2 - s)(u - m_3^2)}{\left[\lambda(u, m_2^2, m_3^2)\;\lambda(s, m_1^2, m_2^2)\right]^{1/2}}$$

(Exercise 2)

I3) Crossing

- Remember Feynman – Stückelberg:

$$j^\mu = i\,(\phi^* \partial^\mu \phi - \phi\, \partial^\mu \phi^*)_e \quad \text{(from Klein-Gordon)}$$

$$\left[kG \cdot i\phi^* - (kG)^*(-i\phi)\right]$$

free particle $\phi \propto e^{-ipx}$:

e^- with p : $j^\mu(e^-) \propto -2e\,p^\mu$

e^+ with p : $j^\mu(e^+) \propto +2e\,p^\mu = -2e\,(-p^\mu)$

$$\Rightarrow \quad j^\mu(e^+) = j^\mu(e^-)\Big|_{p^\mu<0}$$

particle with $-p^\mu$ = antiparticle with $+p^\mu > 0$

$E>0$  =  $E<0$  t

emission of positron $\vec{p}$, $E>0$

absorption of electron $-\vec{p}$, $E<0$