Supernova Bound on keV-mass Sterile Neutrinos

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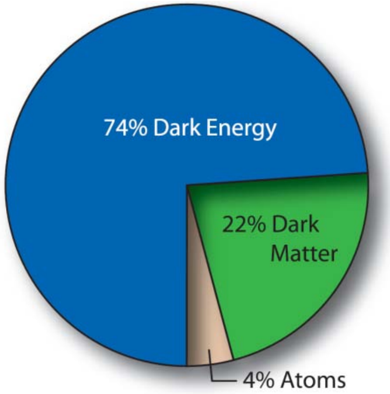
Outline

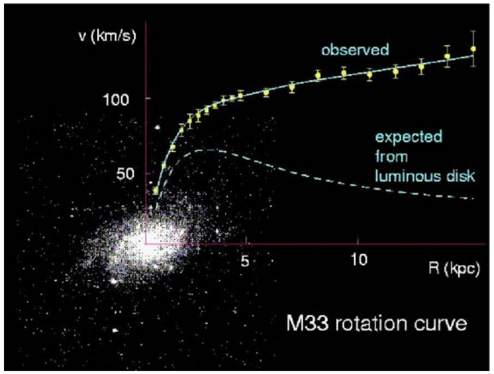
1. Motivation: WDM, Pulsar Kicks

2. Sterile Neutrinos in a SN Core

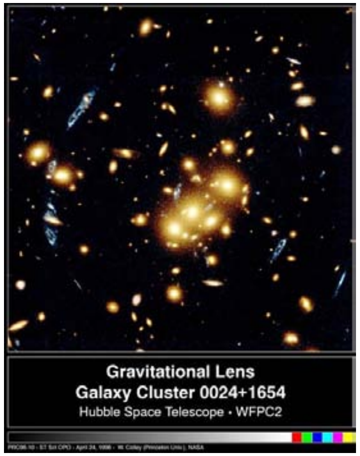
3. Energy losses and SN bounds

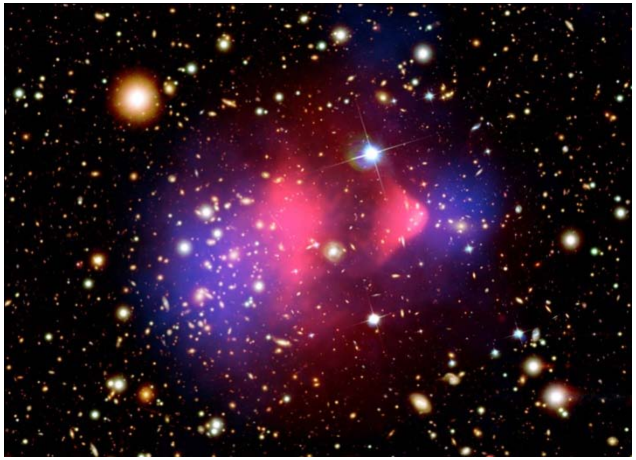
Robust evidence for Dark Matter



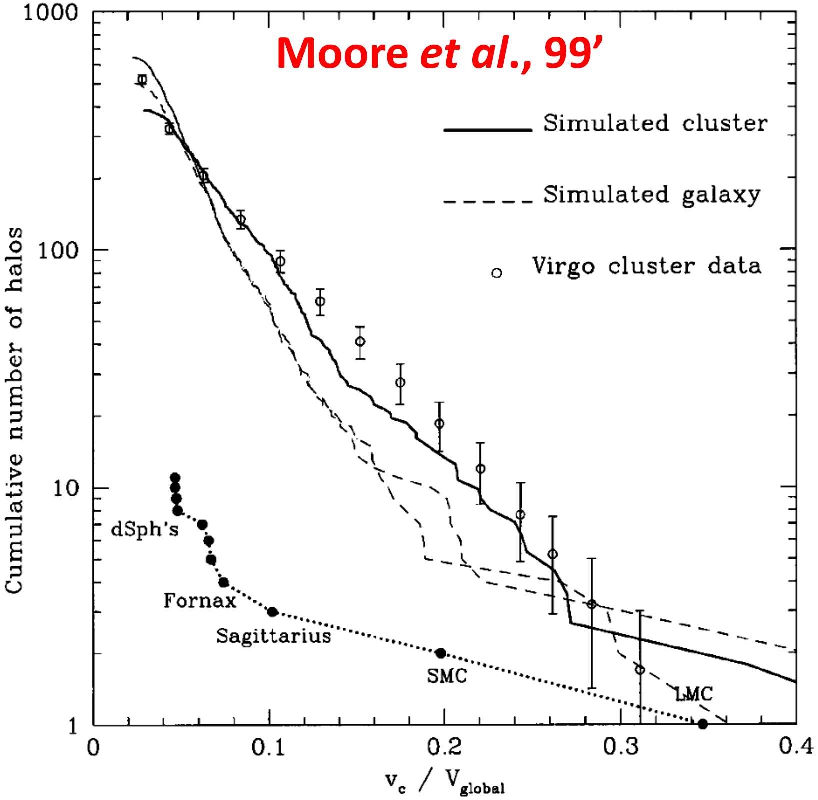


Cold or Warm DM ?

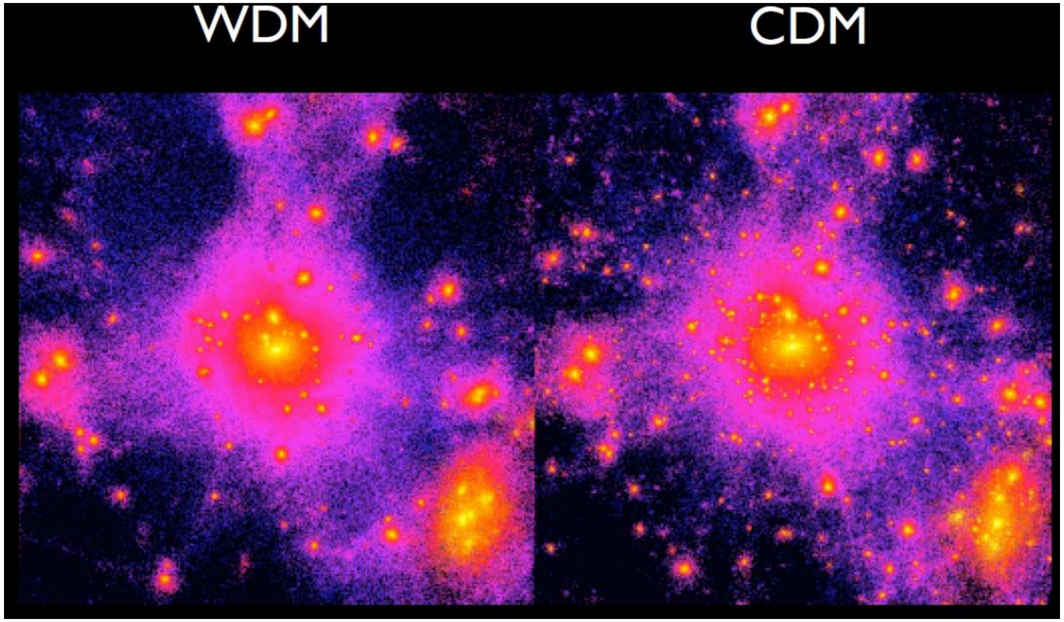




Abundance of Cosmic Substructure



Moore *et al.*, 99'



WDM candidate: keV-mass sterile neutrinos

In the early Universe: *production via neutrino oscillations*

$$\left|\nu_\alpha\right\rangle = +\cos\vartheta\left|\nu_1\right\rangle + \sin\vartheta\left|\nu_2\right\rangle$$
$$\left|\nu_s\right\rangle = -\sin\vartheta\left|\nu_1\right\rangle + \cos\vartheta\left|\nu_2\right\rangle$$

Described by two parameters: m_s & ϑ

Boltzmann equation: *distribution function of sterile neutrinos*

$$\frac{\partial}{\partial t} f_s(p,t) - H\, p\, \frac{\partial}{\partial p} f_s(p,t) \approx \Gamma(\nu_\alpha \to \nu_s; p, t)\, [f_\alpha(p,t) - f_s(p,t)]$$

Matter effects: *primordial lepton number asymmetry*

$$\sin^2 2\theta_m \approx \frac{(\Delta m^2/2p)^2 \sin^2 2\theta}{(\Delta m^2/2p)^2 \sin^2 2\theta + (\Delta m^2/2p \cos 2\theta - V_m - V_T)^2}$$

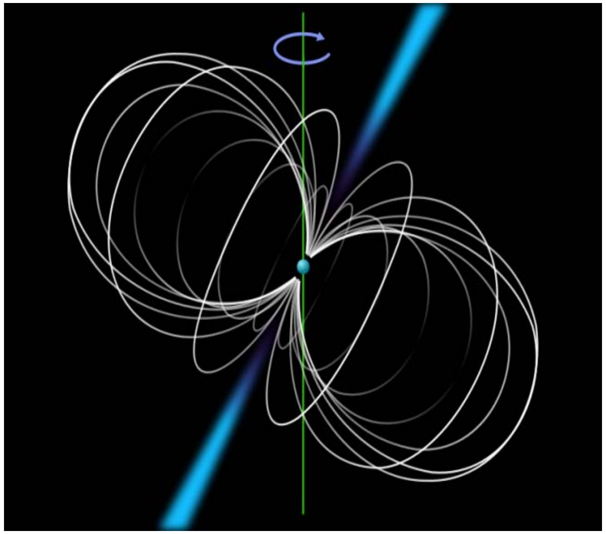
Non-resonant case: **Dodelson, Widrow, 94'**

$$\Omega_s h^2 \approx 0.1 \left(\frac{\sin^2\vartheta}{3\times 10^{-9}}\right)\left(\frac{m_s}{3\mathrm{keV}}\right)^{1.8}$$

Resonant case: **Shi, Fuller, 99'**

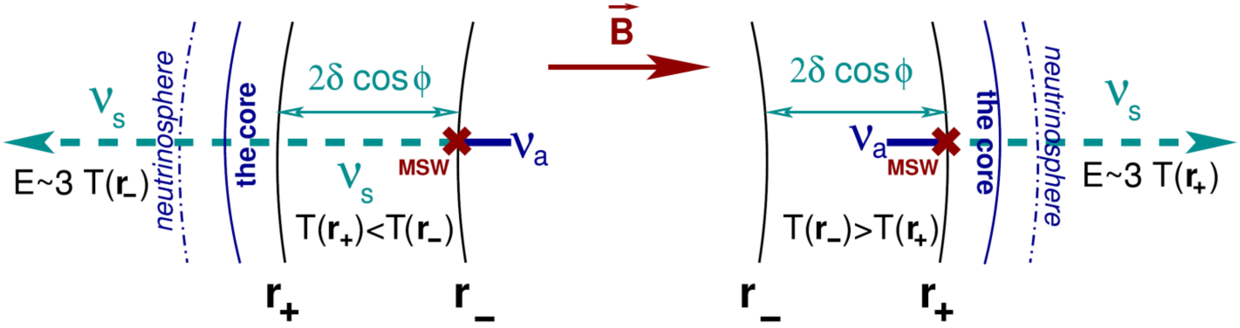
$$\Omega_s h^2 \approx 0.1 \left(\frac{m_s}{1\,\mathrm{keV}}\right)\left(\frac{L}{3\times 10^{-3}}\right)$$

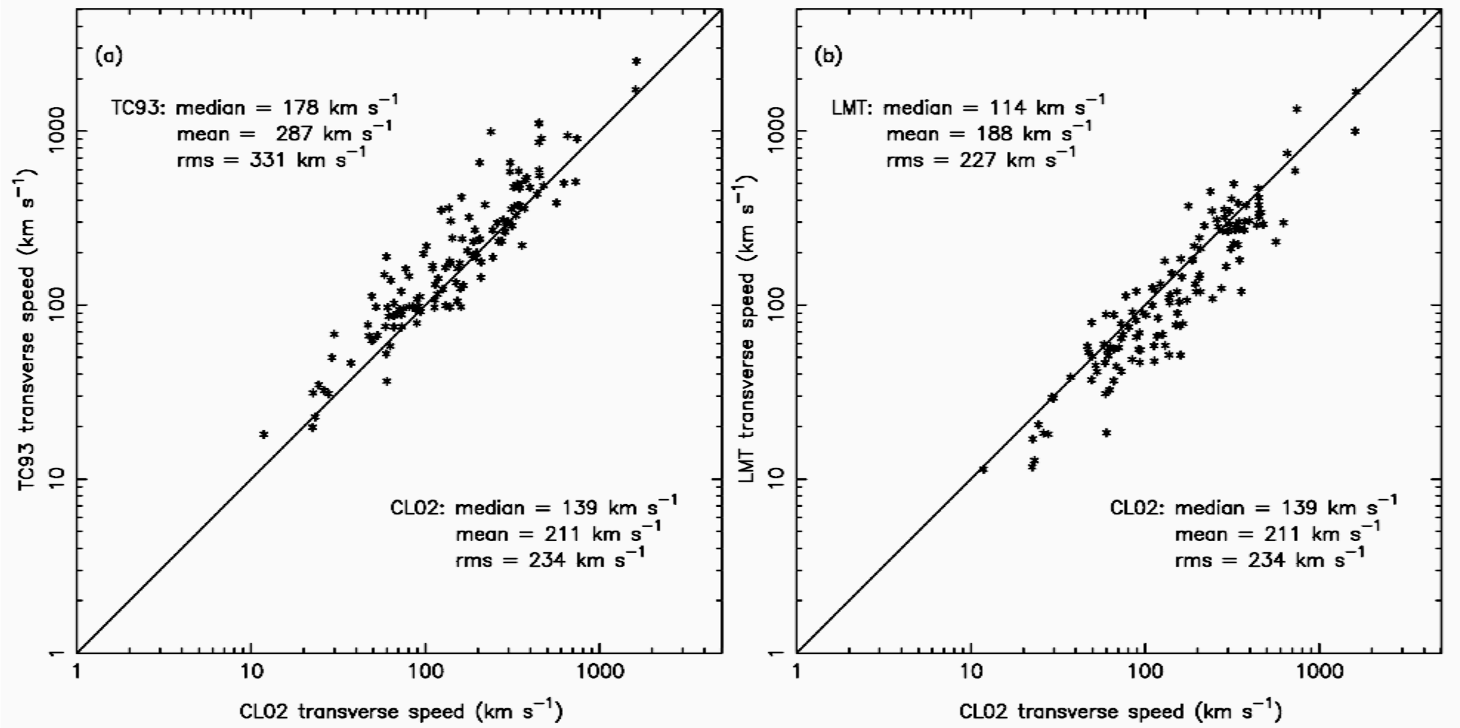
Pulsar velocities: asymmetric emission of sterile neutrinos



Neutrino oscillations
in magnetized media

Kusenko, 09’

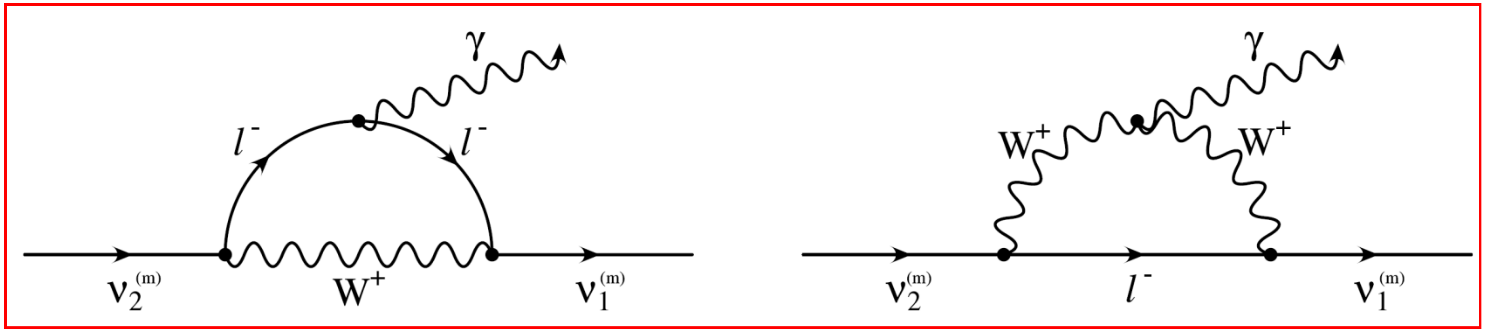


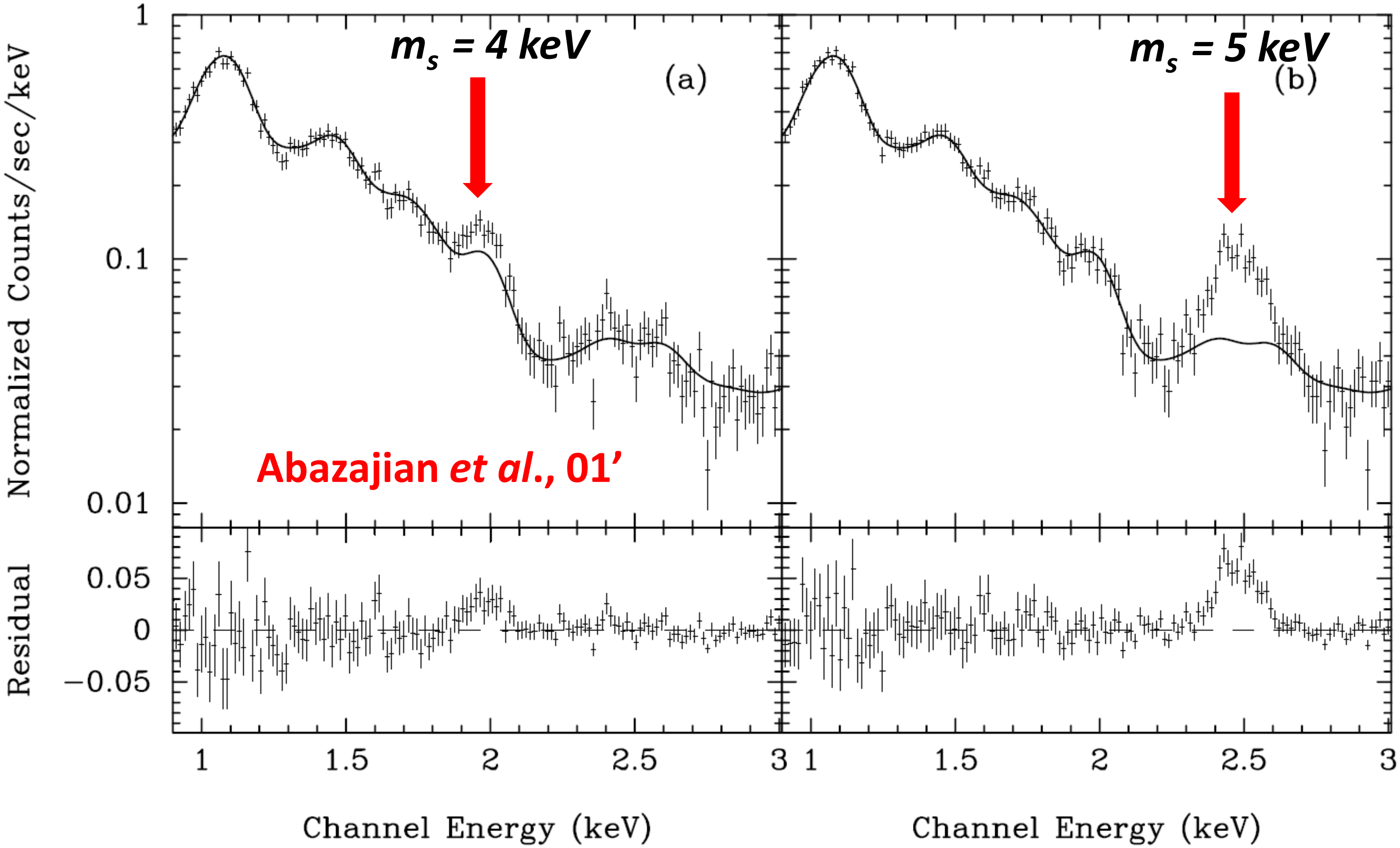


Hobbs *et al.*, 05’

A statistical study of 233 pulsar proper motions

X-ray observations: to discover keV-mass sterile neutrinos

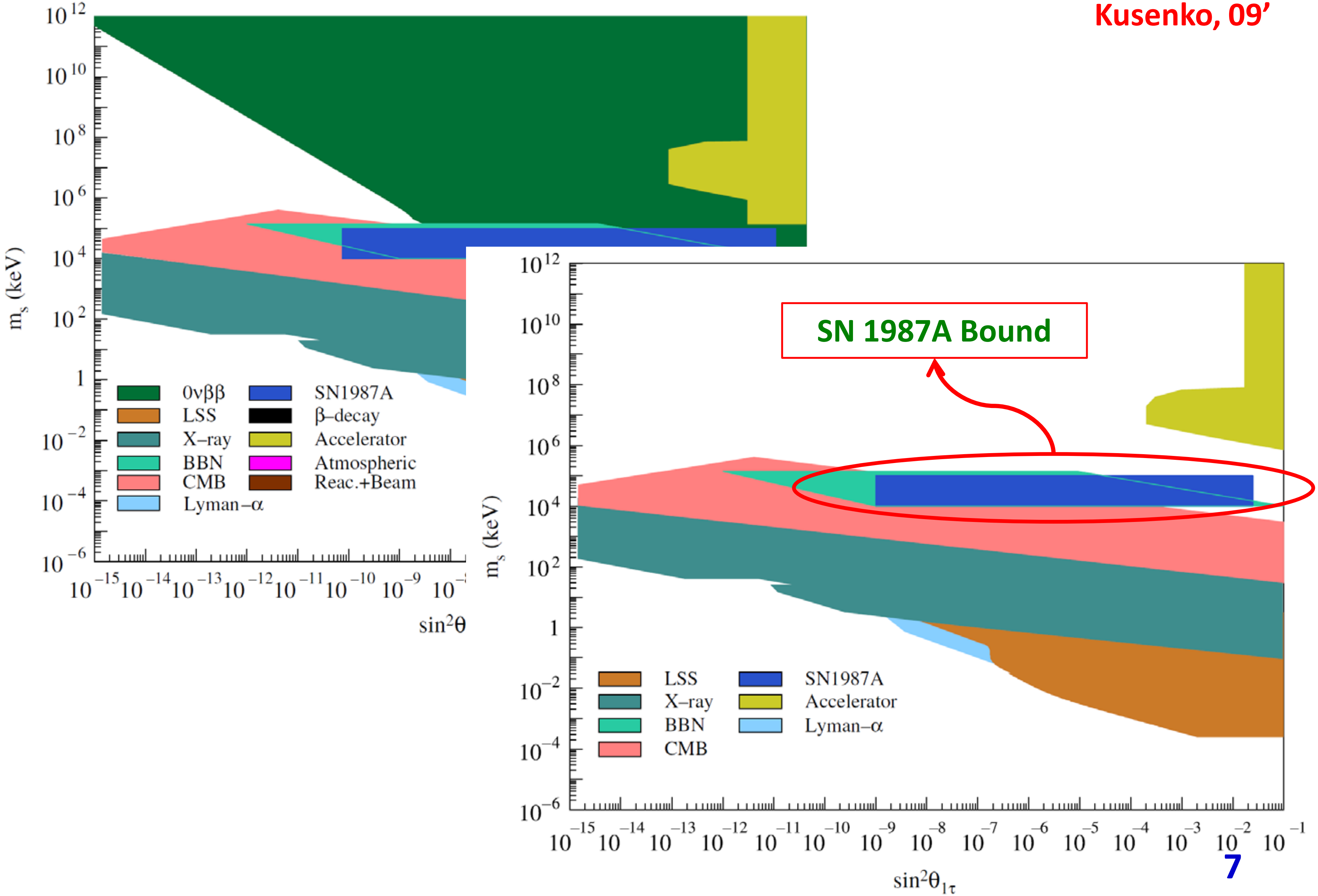




Abazajian *et al.*, 01’

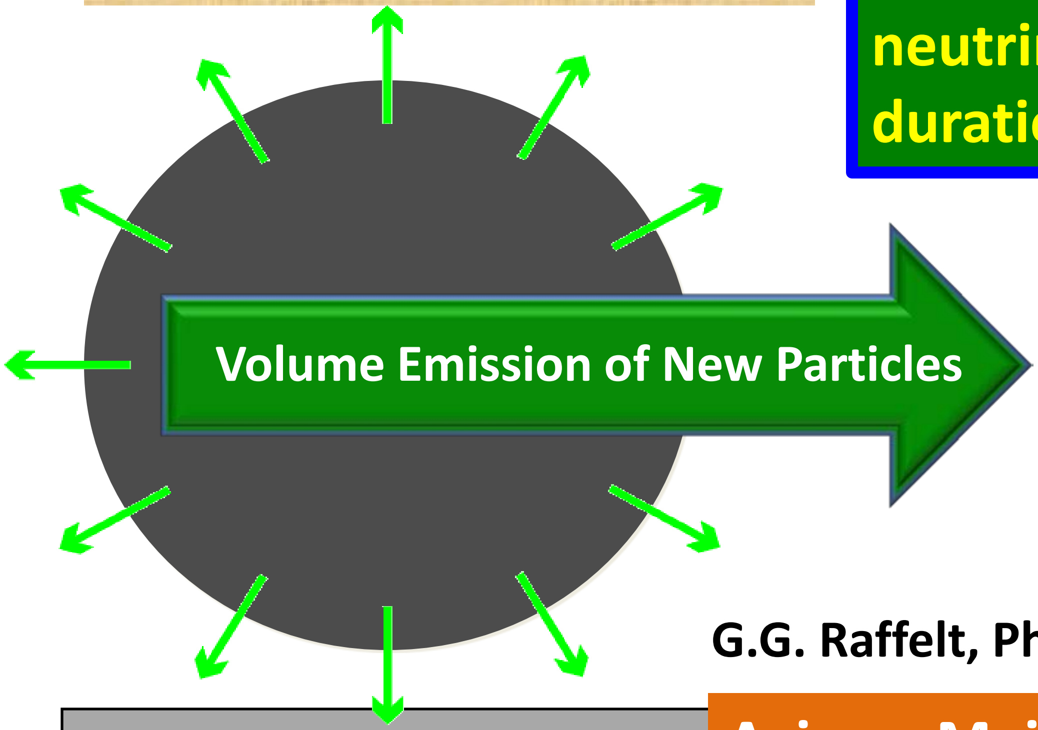
Various constraints on keV-mass sterile neutrinos

Kusenko, 09'



SN 1987A Bound

Standard energy-loss arguments



Neutrino Cooling

$$\varepsilon_\nu = 3.0\times10^{33}\ \mathrm{erg\,cm^{-3}s^{-1}}$$

Volume Emission of New Particles

Proto-Neutron Star

$\rho \approx \rho_{\mathrm{nuc}} = 3\times10^{14}\ \mathrm{g\,cm^{-3}}$

$T \approx 30$ MeV

If new weakly interacting particles can be produced in the supernova core, they will steal energies from neutrino bursts, which reduces the duration of neutrino signals.

$$\varepsilon_{\mathrm{new}} < 3.0\times10^{33}\ \mathrm{erg\,cm^{-3}s^{-1}}$$

G.G. Raffelt, Phys. Rept. 198 (1990) 1

Axions, Majorons, *Sterile Neutrinos, …*

Sterile Neutrinos in a SN Core

Production of sterile neutrinos:

Low matter density: neutrino flavor oscillations with matter effects

High matter density: production & absorption via scattering processes

Occupation-number formalism:

Sigl, Raffelt, 93’

$$\rho_{ij} = \left\langle b_i^+ b_j \right\rangle$$

$$\overline{\rho}_{ij} = \left\langle d_j^+ d_i \right\rangle$$

Diagonal terms: just the usual occupation numbers

Non-diagonal terms: encode the phase information

Equations of motion:

$$\dot{\rho}_{\mathbf{p}} = i[\rho_{\mathbf{p}}, \Omega_{\mathbf{p}}] + \sum_{i=1}^{n} \left[\left(I_i - \frac{1}{2} \left\{ I_i, \rho_{\mathbf{p}} \right\} \right) \mathcal{P}_{\mathbf{p}}^i - \frac{1}{2} \left\{ I_i, \rho_{\mathbf{p}} \right\} \mathcal{A}_{\mathbf{p}}^i \right]$$

Neutral-current interaction

$$+ \frac{1}{2} \sum_a \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left[\mathcal{W}_{\mathbf{p}'\mathbf{p}}^a \left(G^a \rho_{\mathbf{p}'} G^a (1 - \rho_{\mathbf{p}}) + \text{h.c.} \right) - \mathcal{W}_{\mathbf{p}\mathbf{p}'}^a \left(\rho_{\mathbf{p}} G^a (1 - \rho_{\mathbf{p}'}) G^a + \text{h.c.} \right) \right]$$

Sterile Neutrinos in a SN Core

Two-flavor mixing case

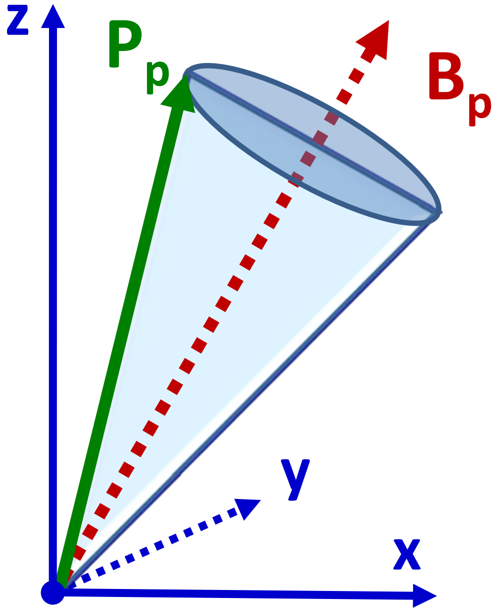
$$\rho_{\mathbf{p}} = \frac{1}{2}\left(n_{\mathbf{p}} + \mathbf{P}_{\mathbf{p}} \cdot \boldsymbol{\tau}\right)$$

$$\Omega_{\mathbf{p}} = \frac{1}{2}\left(E_{\mathbf{p}} + \mathbf{B}_{\mathbf{p}} \cdot \boldsymbol{\tau}\right)$$

$$\dot{\rho}_{\mathbf{p}} = i[\rho_{\mathbf{p}}, \Omega_{\mathbf{p}}] \longrightarrow \dot{\mathbf{P}}_{\mathbf{p}} = \mathbf{B}_{\mathbf{p}} \times \mathbf{P}_{\mathbf{p}}$$

Flavor polarization vectors rotate around magnetic fields

$$\mathbf{B}_{\mathbf{p}} = \left(\frac{\Delta m^2}{2E}\sin 2\vartheta, \quad 0, \quad \frac{\Delta m^2}{2E}\cos 2\vartheta - V_{\text{eff}}\right)$$



Matter Effects

Wolfenstein, 78'; Mikheyev, Smirnov, 85'

$$\sin^2 2\vartheta_{\nu,\bar{\nu}} = \frac{\sin^2 2\vartheta}{\sin^2 2\vartheta + \left(\cos 2\vartheta \mp (\pm) E / E_r\right)^2}$$

where the resonant energy is

$$E_r = \frac{\Delta m^2}{2|V_{\text{eff}}|}$$

maximal mixing if $E \sim E_r \cos 2\vartheta$

Sterile Neutrinos in a SN Core

Weak-damping limit

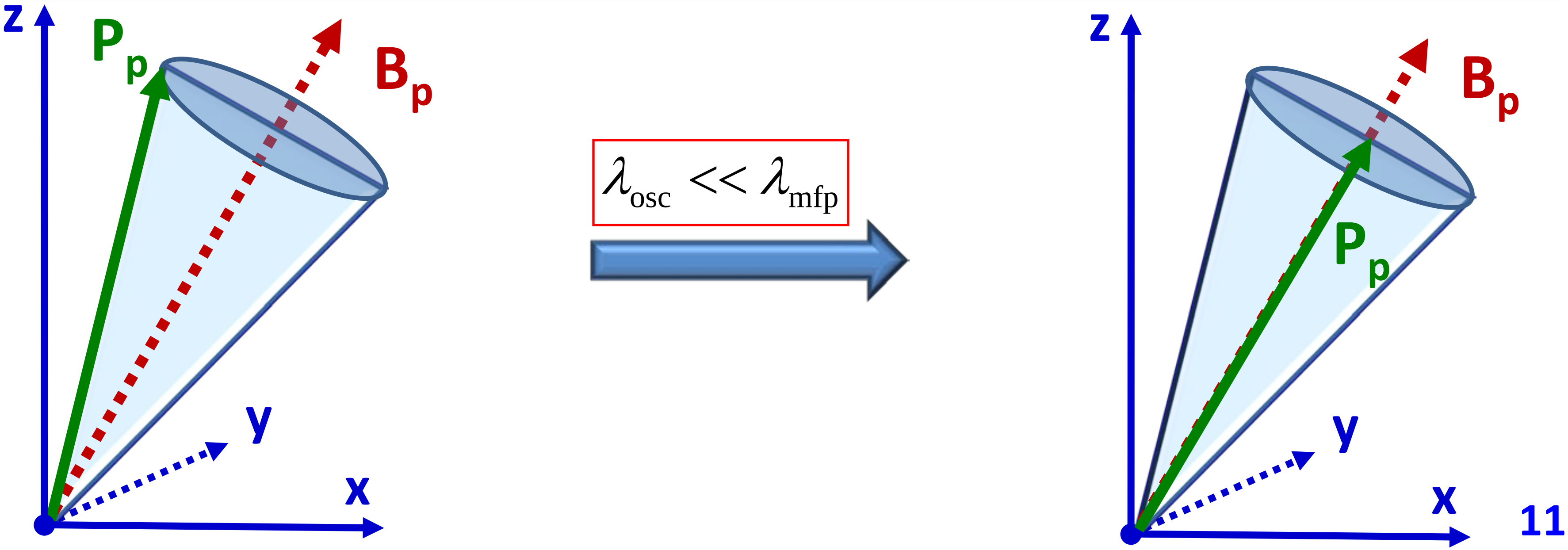
Oscillation length

$$\lambda_{\rm osc} = \frac{4\pi E}{\Delta\tilde{m}^2} < 0.7\,{\rm cm}\left(\frac{E}{30\ {\rm MeV}}\right)\left(\frac{10^{-4}}{\sin 2\vartheta}\right)\left(\frac{10{\rm keV}}{m_s}\right)^2$$

Mean free path

$$\lambda_{\rm mfp} = \frac{1}{N_{\rm B}\sigma_{\nu N}} \approx 10^3\,{\rm cm}\left(\frac{30\ {\rm MeV}}{E}\right)^2\left(\frac{10^{14}\,{\rm g\ cm}^{-3}}{\rho}\right)$$

Neutrinos oscillate many times before a subsequent collision with nucleons



Sterile Neutrinos in a SN Core

In the weak-damping limit

$$\boxed{\tilde{\rho}_{\mathbf{p}} = \frac{1}{2}\left[n_{\mathbf{p}} + \left(\mathbf{P}_{\mathbf{p}} \cdot \hat{\mathbf{B}}_{\mathbf{p}}\right)\left(\hat{\mathbf{B}}_{\mathbf{p}} \cdot \boldsymbol{\tau}\right)\right]}$$

averaged over a period of oscillation

Two independent parameters:
occupation numbers $f^{\alpha}_{\mathbf{p}}$ & $f^{s}_{\mathbf{p}}$

$$\boxed{\tilde{\rho}_{\mathbf{p}} = \begin{pmatrix} f^{\alpha}_{\mathbf{p}} & 0 \\ 0 & f^{s}_{\mathbf{p}} \end{pmatrix} + \frac{1}{2}\left(f^{\alpha}_{\mathbf{p}} - f^{s}_{\mathbf{p}}\right) t_{\mathbf{p}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}$$

Simplified equations of motion:

$$\dot{f}^{s}_{\mathbf{p}} = \frac{1}{4} s^{2}_{\mathbf{p}} \left\{ \left[(1 - f^{s}_{\mathbf{p}}) \mathscr{P}^{\alpha}_{\mathbf{p}} - f^{s}_{\mathbf{p}} \mathscr{A}^{\alpha}_{\mathbf{p}} \right] + \sum_{a} \left(g^{a}_{\alpha}\right)^{2} \int \frac{d^{3}\mathbf{p}}{(2\pi)^{3}} \left[\mathscr{W}^{a}_{\mathbf{p}'\mathbf{p}} f^{\alpha}_{\mathbf{p}'} (1 - f^{s}_{\mathbf{p}}) - \mathscr{W}^{a}_{\mathbf{p}\mathbf{p}'} f^{s}_{\mathbf{p}} (1 - f^{\alpha}_{\mathbf{p}'}) \right] \right\}$$

further simplification if sterile neutrinos escape from the SN core

$$\dot{f}^{s}_{\mathbf{p}} = \frac{1}{4} s^{2}_{\mathbf{p}} \left[\mathscr{P}^{\alpha}_{\mathbf{p}} + \sum_{a} \left(g^{a}_{\alpha}\right)^{2} \int \frac{d^{3}\mathbf{p}}{(2\pi)^{3}} \mathscr{W}^{a}_{\mathbf{p}'\mathbf{p}} f^{\alpha}_{\mathbf{p}'} \right]$$

set $f^{s}_{\mathbf{p}} = 0$

Lepton-number-loss rate $\dot{\mathscr{N}}_{\mathrm{L}} = \int \frac{d^{3}\mathbf{p}}{(2\pi)^{3}} \dot{f}^{s}_{\mathbf{p}}$ **Energy-loss rate** $\mathscr{E}_{s} = \int \frac{d^{3}\mathbf{p}}{(2\pi)^{3}} E \dot{f}^{s}_{\mathbf{p}}$

Sterile Neutrinos in a SN Core

Neutrino matter potentials

$$V_{\nu_e} = \sqrt{2} G_{\mathrm{F}} N_{\mathrm{B}} \left[Y_e - \frac{1}{2} Y_n + 2 Y_{\nu_e} + Y_{\nu_\mu} + Y_{\nu_\tau} \right]$$

$$V_{\nu_\mu} = \sqrt{2} G_{\mathrm{F}} N_{\mathrm{B}} \left[-\frac{1}{2} Y_n + Y_{\nu_e} + 2 Y_{\nu_\mu} + Y_{\nu_\tau} \right]$$

$$V_{\nu_\tau} = \sqrt{2} G_{\mathrm{F}} N_{\mathrm{B}} \left[-\frac{1}{2} Y_n + Y_{\nu_e} + Y_{\nu_\mu} + 2 Y_{\nu_\tau} \right]$$

Remarks:

1. degenerate electron neutrinos; the equation of state involved; charged current interactions; so we consider tau neutrinos for simplicity;
2. we assume the SN core to be homogeneous and isotropic.

Tau-sterile neutrino mixing

$$V_{\nu_\tau} = -\frac{G_{\mathrm{F}}}{\sqrt{2}} N_{\mathrm{B}} \left(1 - Y_e - 2 Y_{\nu_e} - 4 Y_{\nu_\tau} \right) < 0$$

Initial conditions:

$Y_e = 0.3$, $Y_{\nu_e} = 0.07$, $Y_{\nu_\mu} = Y_{\nu_\tau} = 0$

$$\sin^2 2\vartheta_{\nu,\bar{\nu}} = \frac{\sin^2 2\vartheta}{\sin^2 2\vartheta + \left(\cos 2\vartheta \pm E / E_r \right)^2}$$

1. the MSW resonance occurs in the antineutrino channel;
2. asymmetry between tau neutrinos and antineutrinos.

Energy-loss Rates and SN Bounds

Simple bounds in the 'vacuum limit': $E_r >> E$

$$\boxed{\sin^2 2\vartheta_{\nu,\bar{\nu}} = \frac{\sin^2 2\vartheta}{\sin^2 2\vartheta + \left(\cos 2\vartheta \pm E/E_r\right)^2}} \Longrightarrow \boxed{\vartheta_\nu \approx \vartheta_{\bar{\nu}} \approx \vartheta}$$

Energy-loss rates

$$\mathcal{E}_s = \underset{\nu+\bar{\nu}}{\boxed{2}}\int_0^\infty \frac{E^2}{2\pi^2}\frac{E}{\exp\left(E/T\right)+1}\left(\frac{1}{4}\sin^2 2\vartheta\right)\underset{\nu-N}{\boxed{\frac{N_\mathrm{B}G_\mathrm{F}^2E^2}{\pi}}}dE = 4N_\mathrm{B}G_\mathrm{F}^2T^6\vartheta^2$$

Supernova Bound

$$\boxed{\mathcal{E}_s = 4N_\mathrm{B}G_\mathrm{F}^2T^6\vartheta^2 < \mathcal{E}_\nu = 3.0\times10^{33}\,\mathrm{erg\,cm^{-3}s^{-1}}} \Longrightarrow \boxed{\vartheta^2 \le 10^{-8}}$$

$\rho \approx \rho_{\mathrm{nuc}} = 3\times10^{14}\ \mathrm{g\ cm^{-3}}$

$T \approx 30$ MeV

Such a simple bound is valid and mass-independent only in the 'vacuum limit'.

Energy-loss Rates and SN Bounds

Conditions for the vacuum limit

$$E_r = 3.25\,\mathrm{MeV}\left(\frac{m_s}{10\,\mathrm{keV}}\right)^2\left(\frac{10^{14}\,\mathrm{g\,cm^{-3}}}{\rho}\right)\left|Y_0 - Y_{\nu_\tau}\right|^{-1}$$

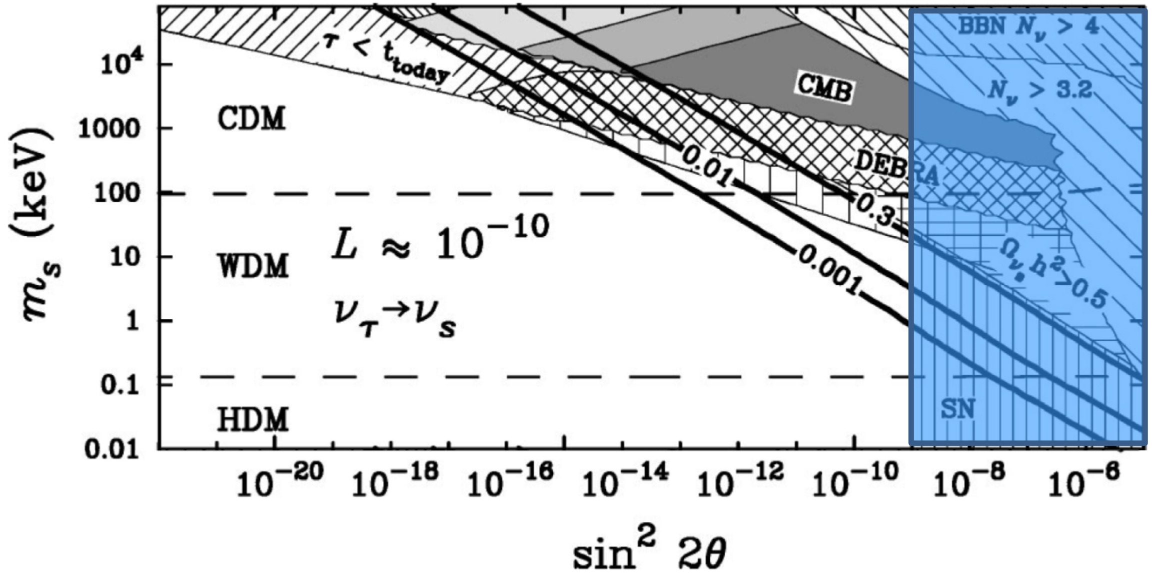
$Y_0 = (1 - Y_e - 2Y_{\nu_e})/4 = 0.14$

$$\boxed{m_s > 100\,\mathrm{keV}}$$

$$\boxed{Y_{\nu_\tau} \to Y_0}$$

Is the simple bound also valid for the small-mass range?

Abazajian, Fuller, Patel, 01'



A Stationary State?

$$\boxed{Y_{\nu_\tau} \to Y_0} \Rightarrow \boxed{\vartheta_\nu \approx \vartheta_{\bar\nu} \approx \vartheta}$$

Including the degen. param.

$$f_E^{\nu_\tau} = \frac{1}{\exp[E/T - \eta] + 1}$$

$$f_E^{\bar\nu_\tau} = \frac{1}{\exp[E/T + \eta] + 1}$$

Energy-loss Rates and SN Bounds

Criterion for a stationary state

Emission Rate = **Mixing angle in matter** × **Population of active neutrinos** × **Neutral-current interactions**

⇒ **Neutrino Emission Rate** = **Antineutrino Emission Rate**

Evolution of the degeneracy parameter

$$\dot{N}_{\nu_\tau} = -\frac{1}{4}\sum_a \int \frac{E^2 dE}{2\pi^2} \sin^2 2\vartheta_\nu \int \frac{E'^2 dE'}{2\pi^2} w^a_{E'E}\, f^{\nu_\tau}_{E'}$$

$$\dot{N}_{\bar{\nu}_\tau} = -\frac{1}{4}\sum_a \int \frac{E^2 dE}{2\pi^2} \sin^2 2\vartheta_{\bar{\nu}} \int \frac{E'^2 dE'}{2\pi^2} \overline{w}^a_{E'E}\, f^{\bar{\nu}_\tau}_{E'}$$

$$f^{\nu_\tau}_E = \frac{1}{\exp[E/T-\eta]+1}$$

$$f^{\bar{\nu}_\tau}_E = \frac{1}{\exp[E/T+\eta]+1}$$

Sterile neutrinos with mixing angles $\vartheta_\nu < \vartheta_c \approx 10^{-2}$ can escape from the core.

Energy-loss Rates and SN Bounds

Evolution of the degeneracy parameter

$$\frac{d}{dt}\eta(t)=\frac{N_{\mathrm{B}}G_{\mathrm{F}}^{2}s_{2\vartheta}^{2}T^{2}}{4\pi}\left[\mathcal{F}_{\bar{\nu}}(\eta)-\mathcal{F}_{\nu}(\eta)\right]\mathcal{G}^{-1}(\eta)$$

Feedback effects

Initial condition: *t = 0, η = 0*

$\boxed{\mathcal{F}_{\bar{\nu}}(0)-\mathcal{F}_{\nu}(0)>0}$ **Antineutrino Emission Rate > Neutrino Emission Rate**

↓

η increases (η >0)

Y_{ν_τ} **(τ number asymmetry)** ↑

$E_r \propto \left|Y_0 - Y_{\nu_\tau}\right|^{-1}$ ↑

$\vartheta_{\bar{\nu}}$ ↓ ϑ_{ν} ↑

↑ $f_E^{\nu_\tau}=\dfrac{1}{\exp\left[E/T-\eta\right]+1}$

↓ $f_E^{\bar{\nu}_\tau}=\dfrac{1}{\exp\left[E/T+\eta\right]+1}$

↓

Antineutrino Emission Rate ↓ **Neutrino Emission Rate** ↑

stable point η*

$\boxed{\mathcal{F}_{\bar{\nu}}(\eta^{*})=\mathcal{F}_{\nu}(\eta^{*})}$

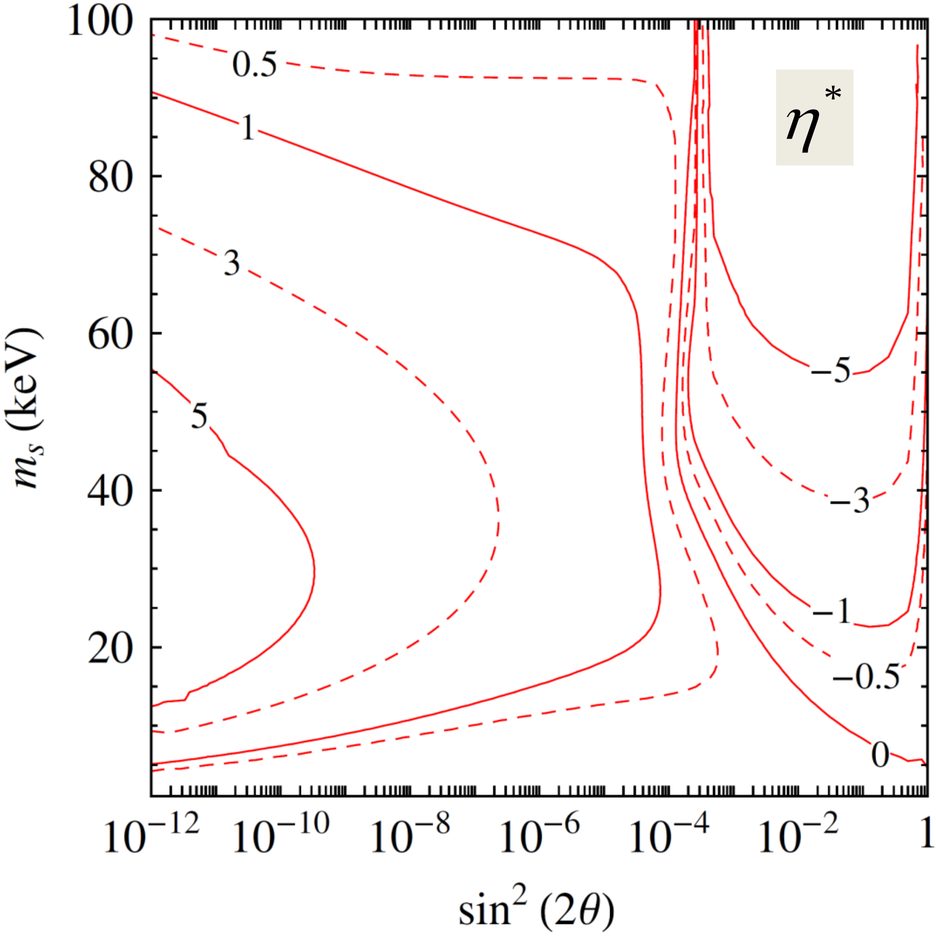
Neutrino Emission Rate = Antineutrino Emission Rate

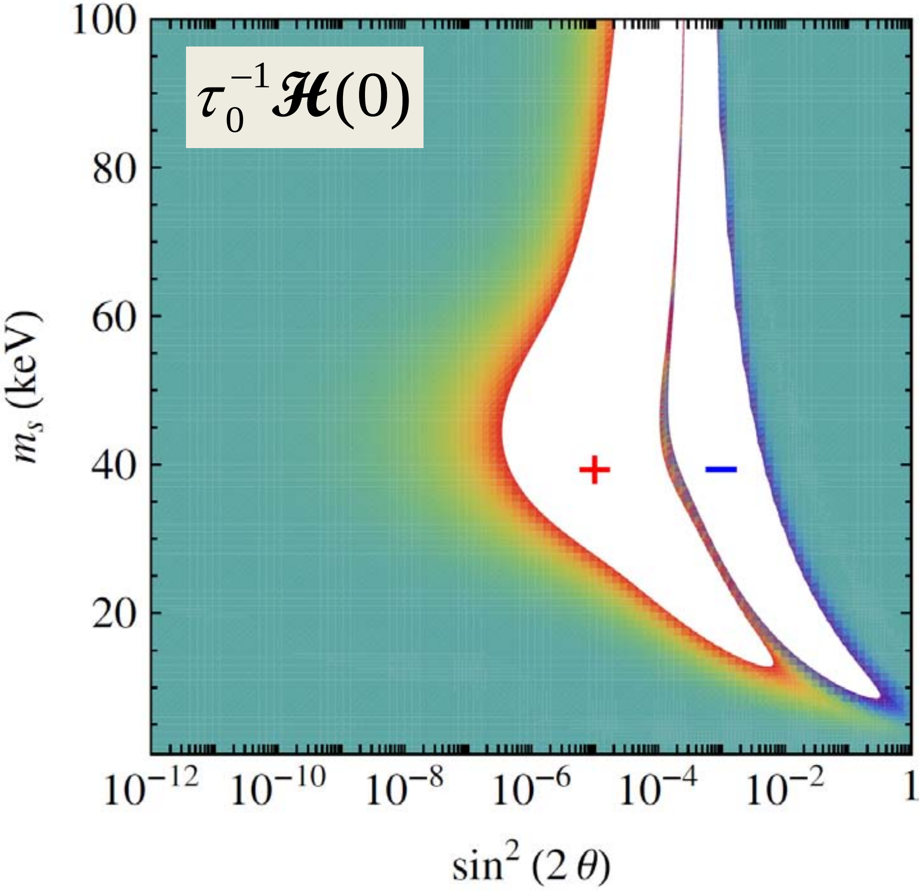
Energy-loss Rates and SN Bounds

Evolution of the degeneracy parameter

Raffelt, Zhou, 11'

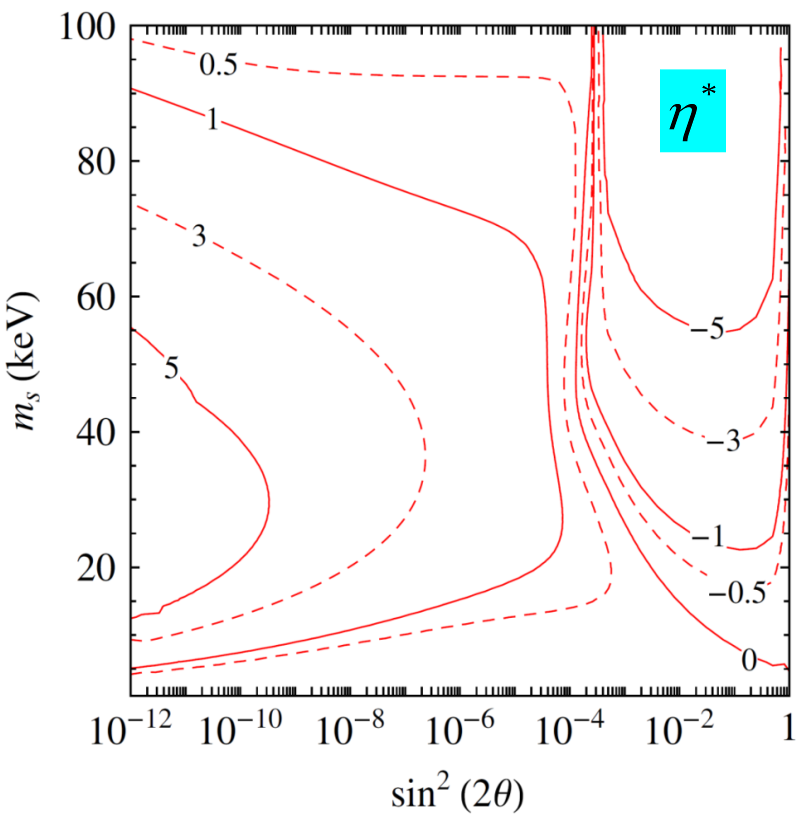
$$\frac{d}{dt}\eta(t)=\boxed{\frac{N_{\mathrm{B}}G_{\mathrm{F}}^{2}s_{2\vartheta}^{2}T^{2}}{4\pi}}\boxed{\left[\mathcal{F}_{\bar{\nu}}(\eta)-\mathcal{F}_{\nu}(\eta)\right]\mathcal{G}^{-1}(\eta)}$$

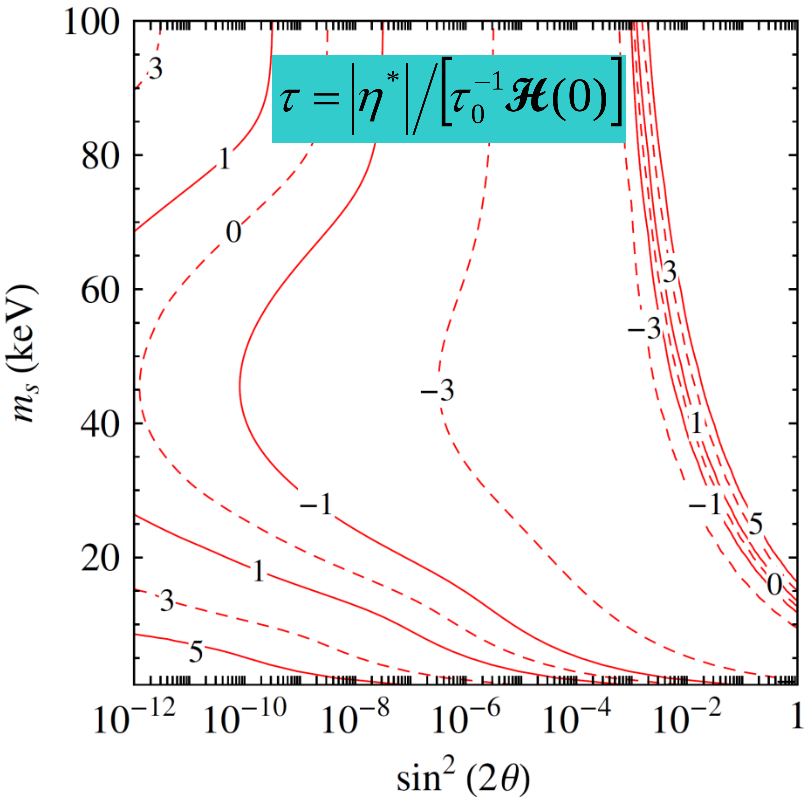




Energy-loss Rates and SN Bounds

1. **The stable point *η** can be either negative or positive, depending on the sterile neutrino mass and vacuum mixing angle;**
2. **The values of *η** are negative for large vacuum mixing angles, because more antineutrinos than neutrinos are trapped in the SN core;**
3. **We temporarily ignore the trapped sterile neutrinos, which may actually transfer energies rapidly due to their larger mean free paths.**




$\tau = \left|\eta^*\right| / \left[\tau_0^{-1}\mathcal{H}(0)\right]$

Energy-loss Rates and SN Bounds

Raffelt, Zhou, 11'

Energy-loss rate

$$\mathcal{E}_s(t) = \frac{N_\mathrm{B} G_\mathrm{F}^2 s_{2\vartheta}^2 T^6}{8\pi^3}\left[\mathcal{R}_{\bar{\nu}}(\eta) + \mathcal{R}_{\nu}(\eta)\right]$$

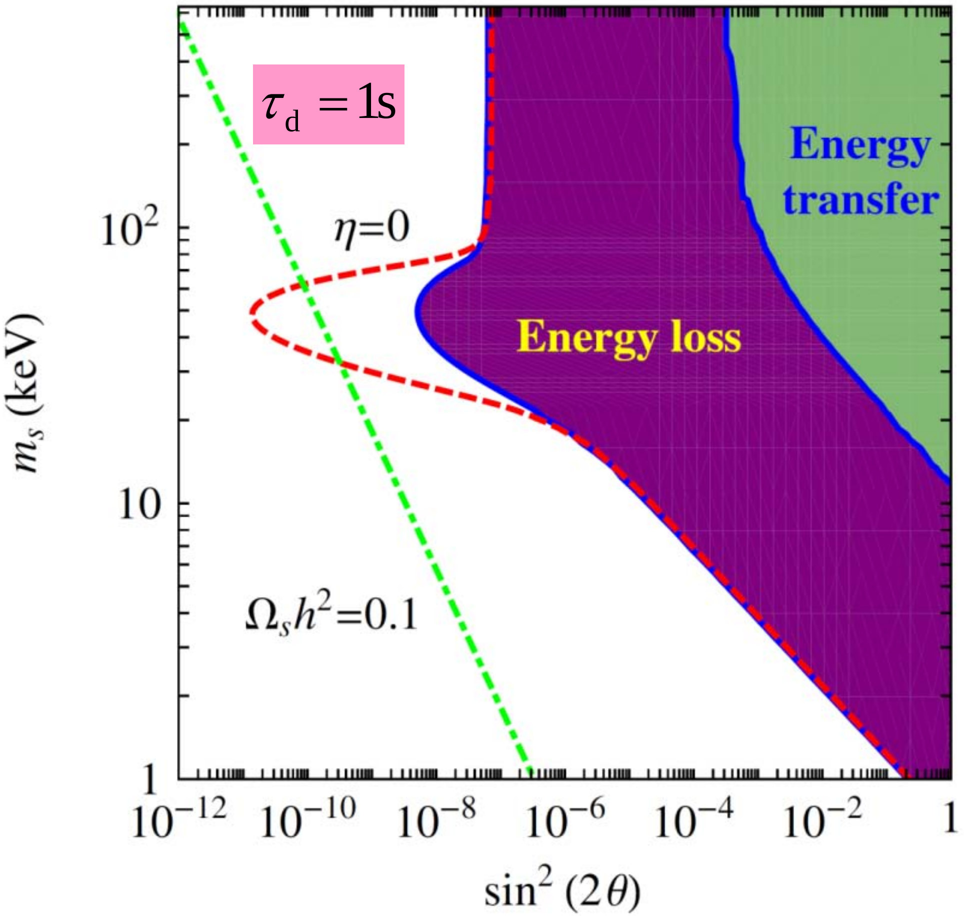
$$\mathcal{R}_{\bar{\nu}}(\eta) = \int_0^\infty \frac{x^5}{e^{x+\eta}+1}\,\frac{1-\mathcal{B}(x, x_r\varepsilon^-, x_r\varepsilon^+)}{s_{2\vartheta}^2 + \left(c_{2\vartheta} - x/x_r\right)^2}\,dx$$

Averaged energy-loss rate

$$\boxed{\left\langle \mathcal{E}_s \right\rangle = \tau_\mathrm{d}^{-1}\int_0^{\tau_\mathrm{d}} \mathcal{E}_s(t)\,dt}$$

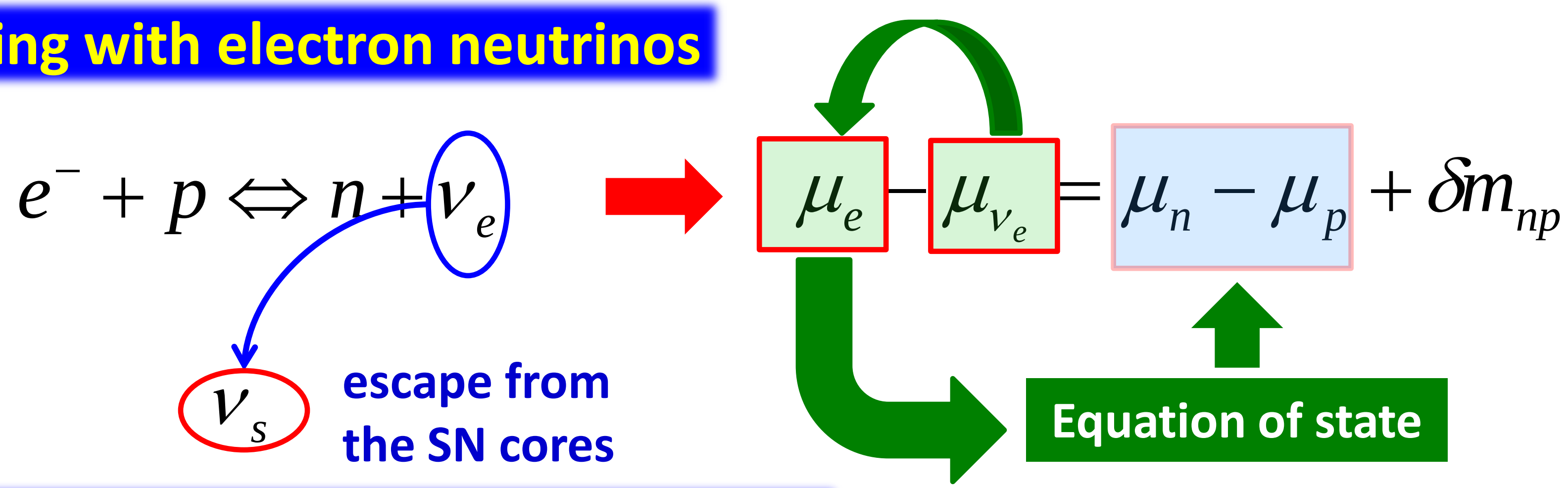
Supernova bound

$$\boxed{\left\langle \mathcal{E}_s \right\rangle < 3.0\times 10^{33}\,\mathrm{erg\,cm^{-3}s^{-1}}}$$



Sterile Neutrinos and SN Explosions

Mixing with electron neutrinos



$$e^- + p \Leftrightarrow n + \nu_e$$

ν_s escape from the SN cores

$$\boxed{\mu_e} - \boxed{\mu_{\nu_e}} = \boxed{\mu_n - \mu_p} + \delta m_{np}$$

Equation of state

How to constrain sterile neutrinos?

Remarks:

1. If the lepton-number loss is not significant, one can simply apply the standard energy-loss argument to the ν_e-ν_s mixing case;
2. For the warm-dark-matter mass range (*1* keV to *10* keV), the MSW resonance may be present and amplify the lepton-number-loss rate;
3. Sterile neutrinos have already done something important during the collapsing phase, such as reducing the electron number fraction Y_e and thus the size of the homologous core, and the energy of the shock wave.

Sterile Neutrinos and SN Explosions

Sterile neutrino assisted SN explosions?

Hidaka, Fuller, 06'

One-zone model of the collapsing core: the EoS & resonant ν_e-ν_s conversion,…

To include the neutrino trapping and diffusion, shock-wave propagation, …