New Process for Charged Lepton Flavor Violation Searches: $\mu^- e^- \to e^- e^-$ in a muonic atom

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Work in progress
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Introduction

Evidence of new physics

charged Lepton Flavor Violation (cLFV)

- Sensitive to high energy scale (> LHC energy)
- Reaction ratio of each cLFV depends on model structure

→ One of the leading prove for fundamental physics and model discrimination

Should study many types of cLFV processes to understand new physics!!

Introduction

New idea for cLFV search

$$\mu^- e^- \longrightarrow e^- e^-$$ in a muonic atom

What is target ?

- Flavor violation between μ and e

What is advantage ?

- Sensitive to both photonic dipole interaction and 4-Fermi contact interaction

- Clean signal [back-to-back dielectron]

Contents

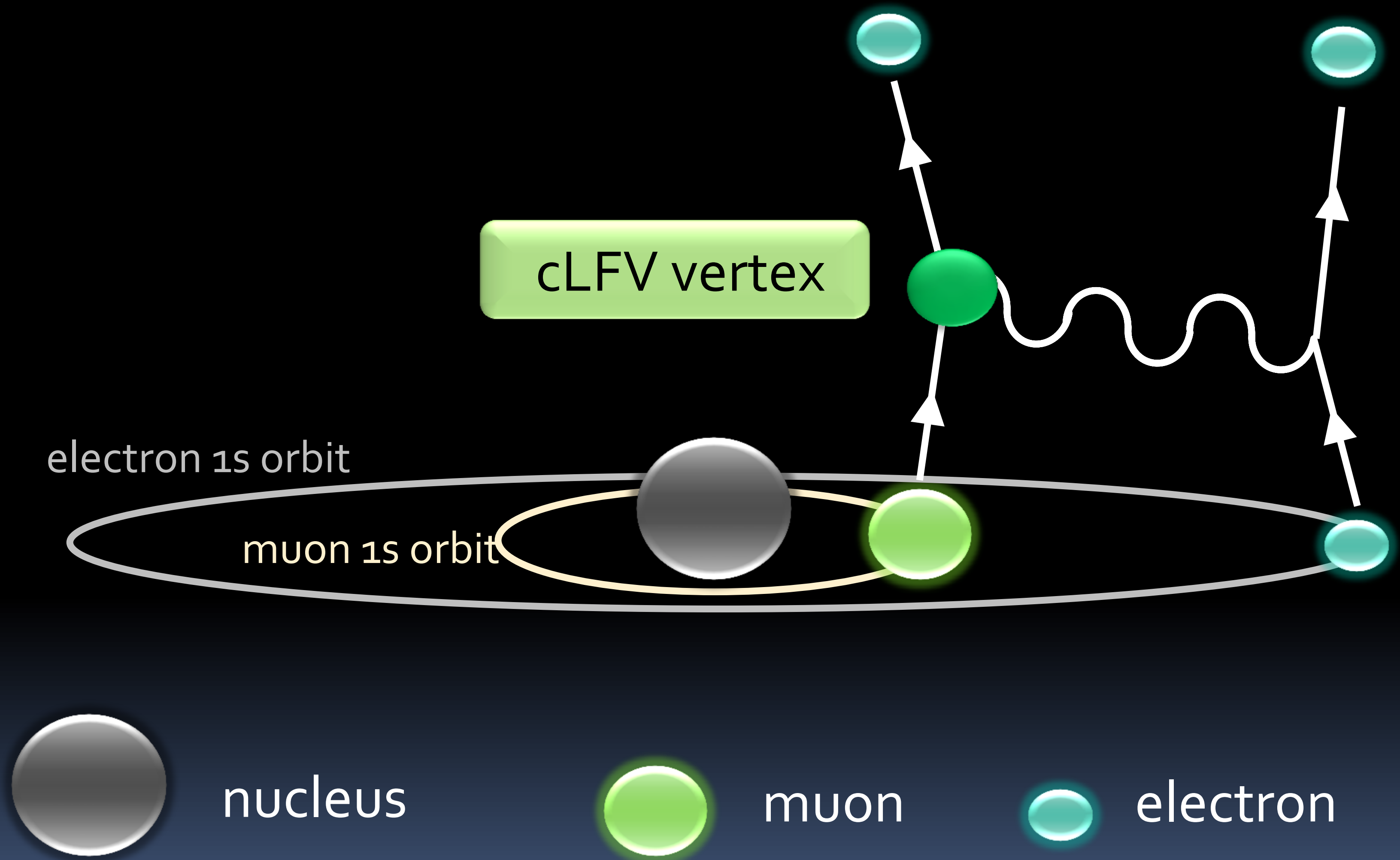
Reaction rate of $\mu^{-} e^{-} \longrightarrow e^{-} e^{-}$

Muonic atom

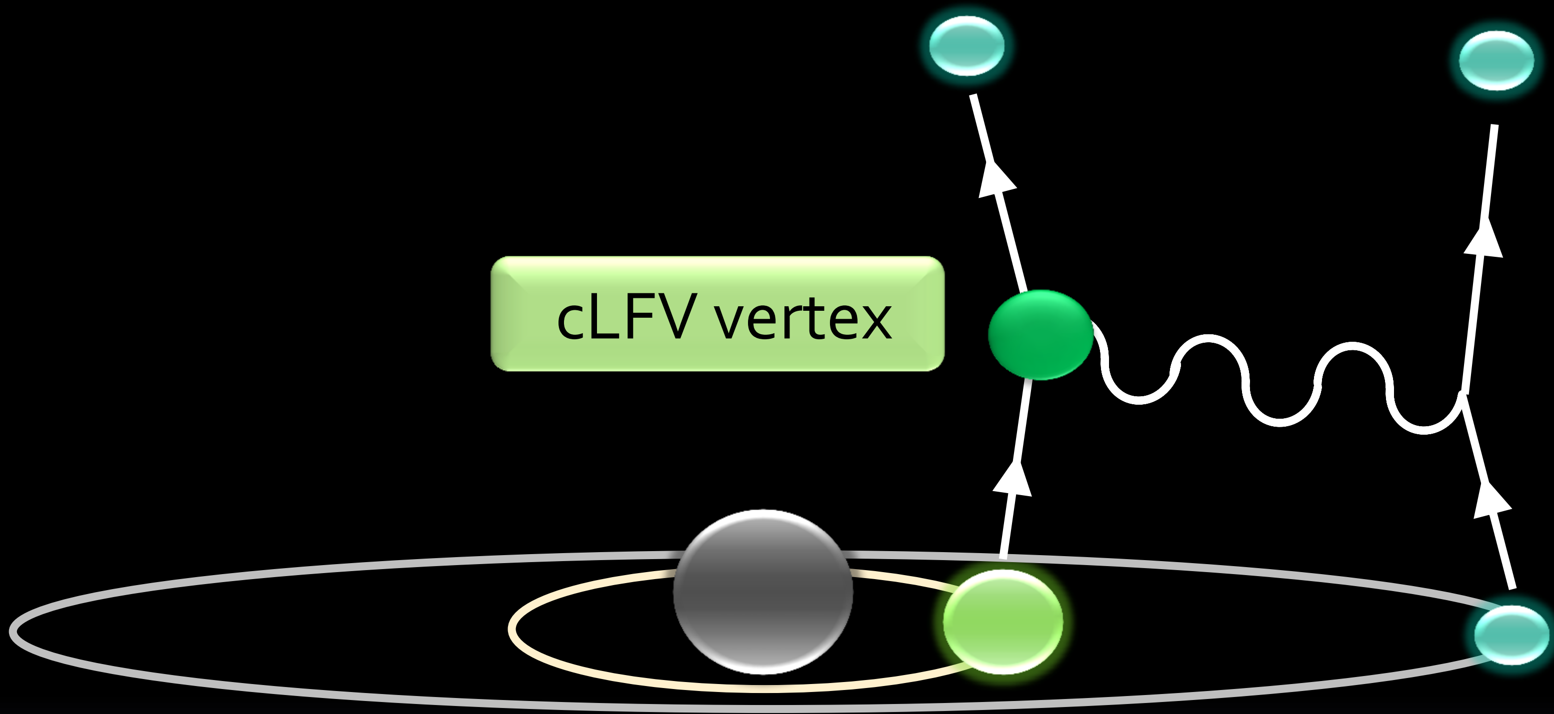
Applications

- Precision test of QED
- μ-e conversion reaction
- Muon catalyzed fusion

$\mu^- e^- \to e^- e^-$ in muonic atom

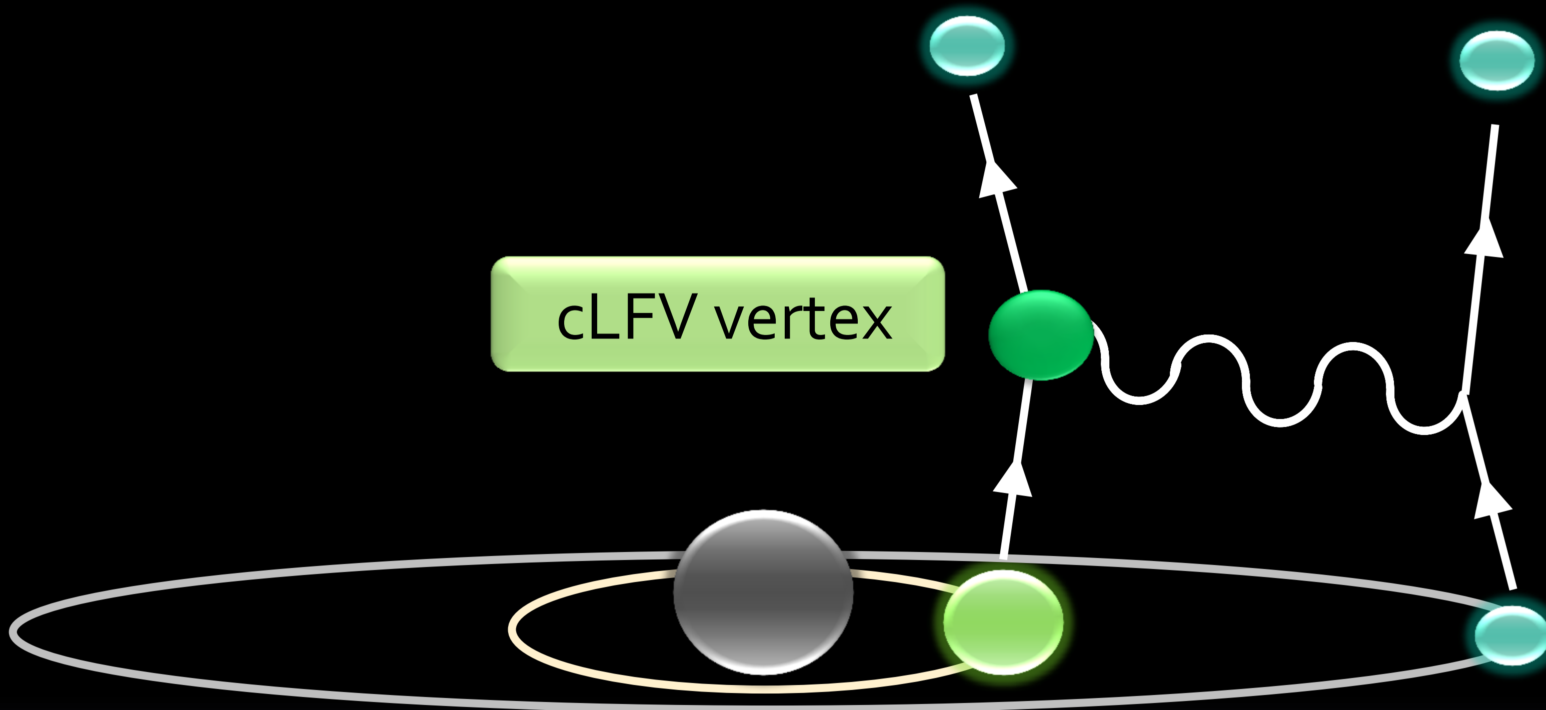


$\mu^- e^- \rightarrow e^- e^-$ in muonic atom



Reaction rate $\Gamma(\mu^- \mathrm{e}^- \rightarrow \mathrm{e}^- \mathrm{e}^-; Z) = 2\sigma v_{\mathrm{rel}} \left| \psi_{1\mathrm{S}}^{(\mathrm{e})}(0; Z-1) \right|^2$

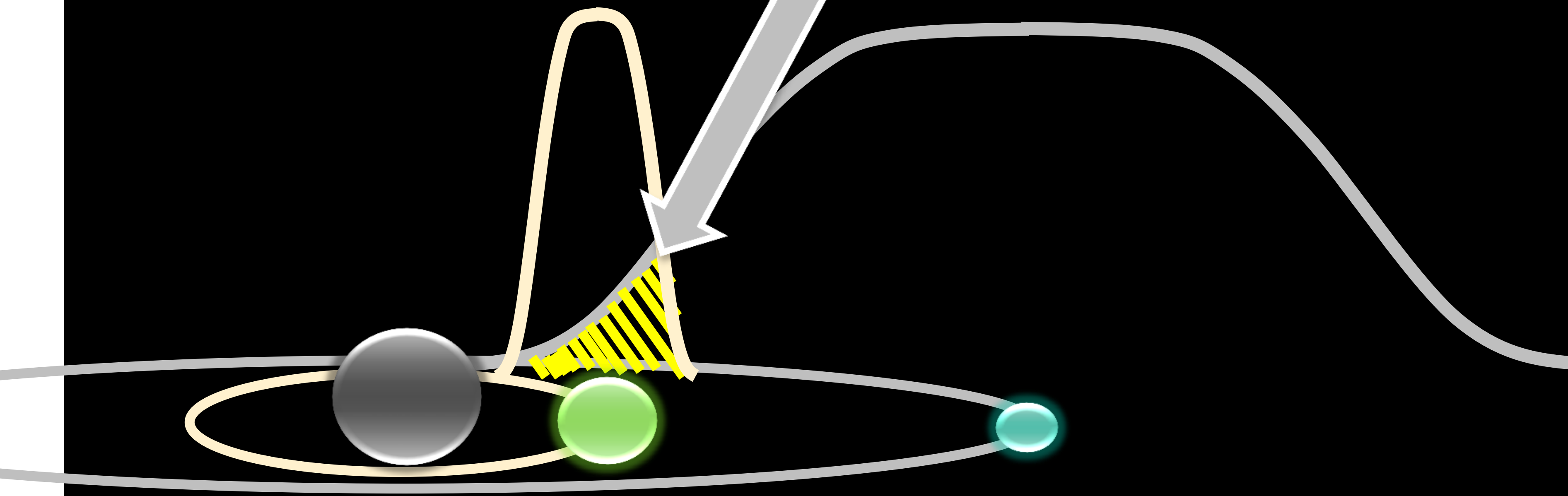
$\mu^- e^- \to e^- e^-$ in muonic atom



Reaction rate $\Gamma(\mu^- e^- \to e^- e^-; Z) = 2\sigma v_{rel} \left|\psi_{1S}^{(e)}(0; Z-1)\right|^2$

Overlap of wave functions

Overlap of wave functions

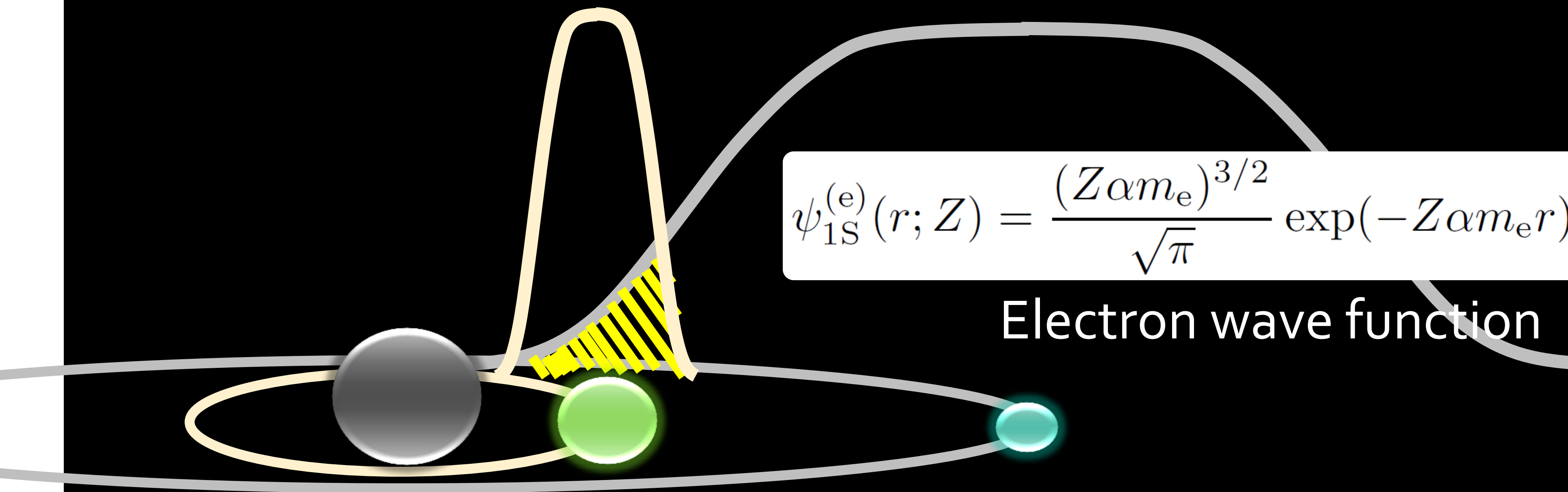


<u>Approximation</u>

Muon localization at nucleus position $\left(\because\ m_\mu >> m_e \right)$

Overlap = electron wave function at nucleus

Overlap of wave functions



$$\psi^{(e)}_{1S}(r;Z) = \frac{(Z\alpha m_e)^{3/2}}{\sqrt{\pi}} \exp(-Z\alpha m_e r)$$

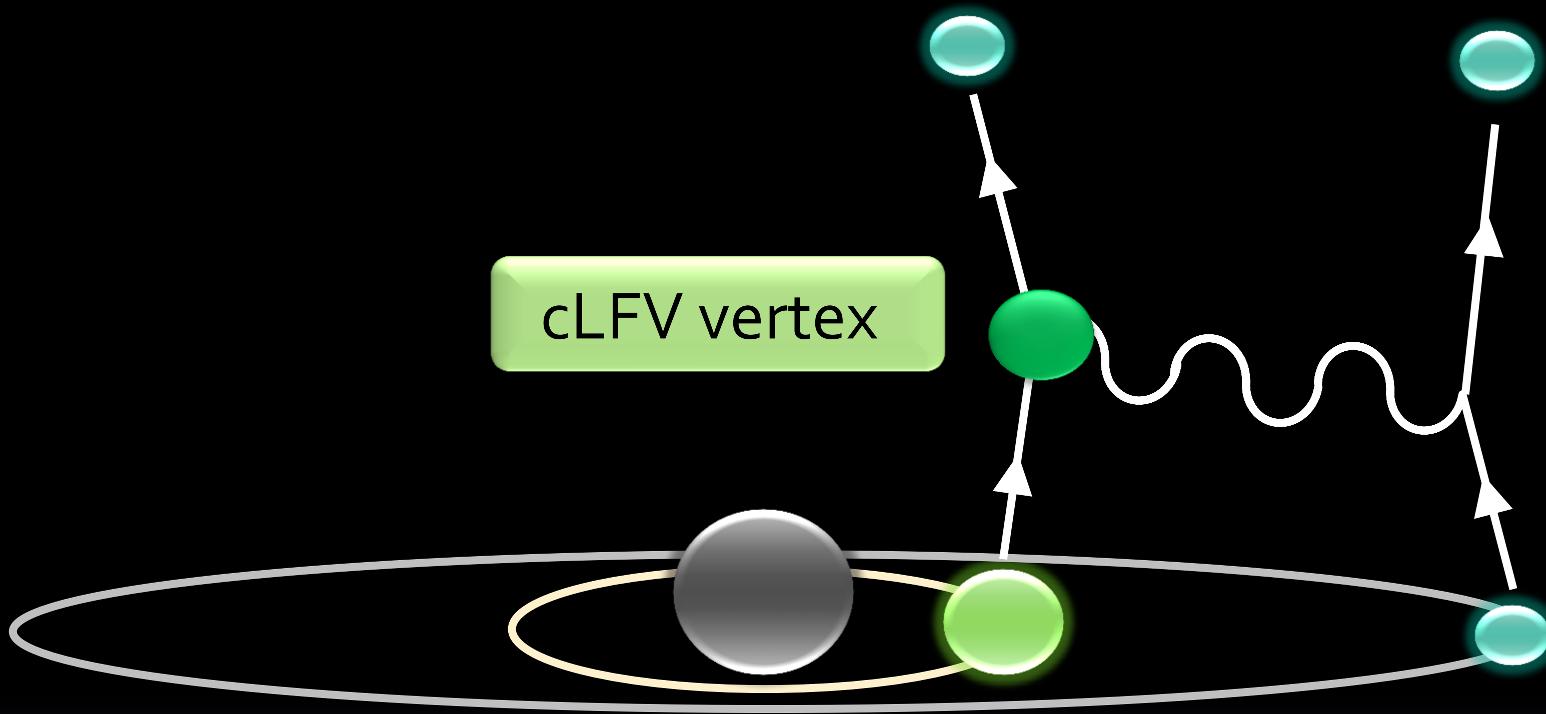
Electron wave function

r : distance from nucleus

Z : atomic number of nucleus

Overlap of wave functions $\left|\psi^{(e)}_{1S}(0;Z-1)\right|^2$

$\mu^- e^- \to e^- e^-$ in muonic atom



Reaction rate $\Gamma(\mu^- \mathrm{e}^- \to \mathrm{e}^- \mathrm{e}^-; Z) = 2\sigma v_{\mathrm{rel}} \left| \psi_{1S}^{(\mathrm{e})}(0; Z-1) \right|^2$

Cross section for elemental interaction

Effective Lagrangian

$$\mathcal{L}_{\mu^- e^- \to e^- e^-} = -\frac{4G_F}{\sqrt{2}} \Big[m_\mu A_R \, \overline{\mu_R} \sigma^{\mu\nu} e_L F_{\mu\nu} + m_\mu A_L \, \overline{\mu_L} \sigma^{\mu\nu} e_R F_{\mu\nu}$$

$$+ g_1 \left(\overline{\mu_R} e_L\right) \left(\overline{e_R} e_L\right) + g_2 \left(\overline{\mu_L} e_R\right) \left(\overline{e_L} e_R\right)$$

$$+ g_3 \left(\overline{\mu_R} \gamma^\mu e_R\right) \left(\overline{e_R} \gamma_\mu e_R\right) + g_4 \left(\overline{\mu_L} \gamma^\mu e_L\right) \left(\overline{e_L} \gamma_\mu e_L\right)$$

$$+ g_5 \left(\overline{\mu_R} \gamma^\mu e_R\right) \left(\overline{e_L} \gamma_\mu e_L\right) + g_6 \left(\overline{\mu_L} \gamma^\mu e_L\right) \left(\overline{e_R} \gamma_\mu e_R\right) + (\mathrm{H.c.}) \Big]$$

[Y. Kuno and Y. Okada Rev. Mod. Phys. **73** (2001)]

cLFV effective coupling constant

A_R A_L g_1 g_2 g_3 g_4 g_5 g_6

⇒ Sensitive to the structure of new physics

Effective Lagrangian

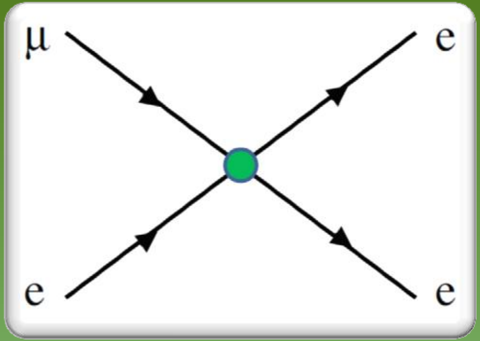
$$\mathcal{L}_{\mu^- e^- \to e^- e^-} = -\frac{4G_F}{\sqrt{2}}\Big[m_\mu A_R\, \overline{\mu_R}\sigma^{\mu\nu} e_L F_{\mu\nu} + m_\mu A_L\, \overline{\mu_L}\sigma^{\mu\nu} e_R F_{\mu\nu}$$

$$+ g_1 (\overline{\mu_R} e_L)(\overline{e_R} e_L) + g_2 (\overline{\mu_L} e_R)(\overline{e_L} e_R)$$

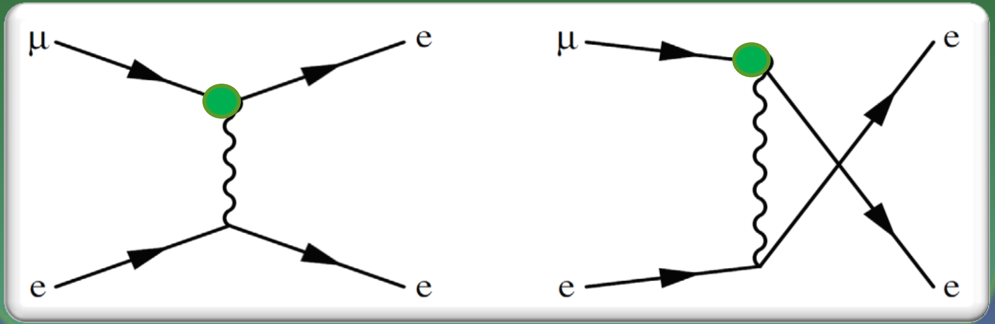
$$+ g_3 (\overline{\mu_R}\gamma^\mu e_R)(\overline{e_R}\gamma_\mu e_R) + g_4 (\overline{\mu_L}\gamma^\mu e_L)(\overline{e_L}\gamma_\mu e_L)$$

$$+ g_5 (\overline{\mu_R}\gamma^\mu e_R)(\overline{e_L}\gamma_\mu e_L) + g_6 (\overline{\mu_L}\gamma^\mu e_L)(\overline{e_R}\gamma_\mu e_R) + (\mathrm{H.c.})\Big]$$

4-Fermi interaction mediated by heavy particle



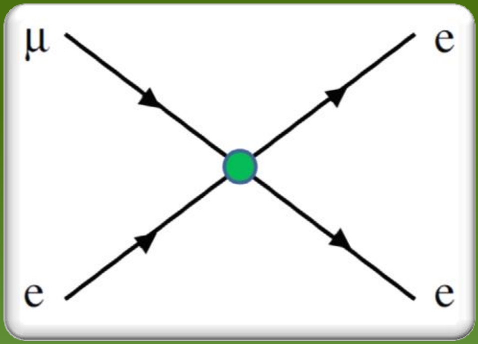
Dipole interaction mediated by photon



(1) 4-Fermi interaction dominant case

(No contributions from dipole interaction)

4-Fermi interaction
mediated by heavy particle



Branching ratio

$$\mathrm{Br}(\mu^- \mathrm{e}^- \to \mathrm{e}^- \mathrm{e}^-) \equiv \tilde{\tau}_\mu \Gamma(\mu^- \mathrm{e}^- \to \mathrm{e}^- \mathrm{e}^-)$$
$$= 24\pi (Z-1)^3 \alpha^3 \left(\frac{m_\mathrm{e}}{m_\mu}\right)^3 \frac{\tilde{\tau}_\mu}{\tau_\mu} G$$
$$= (3.31 \times 10^{-12})(Z-1)^3 (\tilde{\tau}_\mu / \tau_\mu) G \,.$$

Definition of branching ratio

$$\frac{\text{Event number of } \mu^- e^- \longrightarrow e^- e^-}{\text{Number of muonic atom}}$$

Branching ratio

$$\mathrm{Br}(\mu^- e^- \to e^- e^-) \equiv \tilde{\tau}_\mu \Gamma(\mu^- e^- \to e^- e^-)$$
$$= 24\pi (Z-1)^3 \alpha^3 \left(\frac{m_e}{m_\mu}\right)^3 \frac{\tilde{\tau}_\mu}{\tau_\mu} G$$
$$= (3.31 \times 10^{-12})(Z-1)^3 (\tilde{\tau}_\mu / \tau_\mu) G \, .$$

τ_μ Lifetime of free muon $(2.197 \times 10^{-6}\,\mathrm{s})$

$\tilde{\tau}_\mu$ Lifetime of bound muon

$2.19 \times 10^{-6}\,\mathrm{s}$ for $^{1}\mathrm{H}$

$(7 - 8) \times 10^{-8}\,\mathrm{s}$ for $^{238}\mathrm{U}$

Branching ratio

$$\mathrm{Br}(\mu^- \mathrm{e}^- \to \mathrm{e}^- \mathrm{e}^-) \equiv \tilde{\tau}_\mu \Gamma(\mu^- \mathrm{e}^- \to \mathrm{e}^- \mathrm{e}^-)$$
$$= 24\pi (Z-1)^3 \alpha^3 \left(\frac{m_\mathrm{e}}{m_\mu}\right)^3 \frac{\tilde{\tau}_\mu}{\tau_\mu} G$$
$$= (3.31 \times 10^{-12})(Z-1)^3 (\tilde{\tau}_\mu / \tau_\mu) G \,.$$

$$G \equiv G_{12} + 16 G_{34} + 4 G_{56} + 8 G'_{14} + 8 G'_{23} - 8 G'_{56}$$

$$G_{ij} \equiv |g_i|^2 + |g_j|^2 \qquad G'_{ij} \equiv \mathrm{Re}\,(g_i^* g_j)$$

g_i g_j Coupling constants in effective Lagrangian

Branching ratio

$$\mathrm{Br}(\mu^- e^- \to e^- e^-) \equiv \tilde{\tau}_\mu \Gamma(\mu^- e^- \to e^- e^-)$$
$$= 24\pi (Z-1)^3 \alpha^3 \left(\frac{m_e}{m_\mu}\right)^3 \frac{\tilde{\tau}_\mu}{\tau_\mu} G$$
$$= (3.31 \times 10^{-12})(Z-1)^3 (\tilde{\tau}_\mu / \tau_\mu) G\,.$$

Enhancement factor from overlap of wave functions

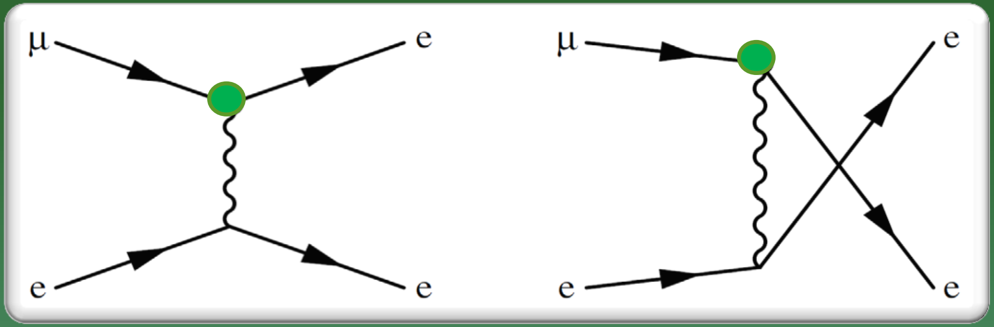
(∵ Positive charge attracts electron toward the nucleus position)

→ Heavy nuclei provides great advantage

(2) Dipole interaction dominant case

(No contributions from 4-Fermi interaction)

Dipole interaction mediated by photon



Branching ratio

$$\mathrm{Br}(\mu^- e^- \to e^- e^-)$$
$$= 1536\pi^2 (Z-1)^3 \alpha^4 (|A_\mathrm{R}|^2 + |A_\mathrm{L}|^2) \frac{m_\mathrm{e}}{m_\mu} \frac{\tilde{\tau}_\mu}{\tau_\mu}$$
$$= 2.08 \times 10^{-9} (Z-1)^3 (|A_\mathrm{R}|^2 + |A_\mathrm{L}|^2)(\tilde{\tau}_\mu / \tau_\mu)$$

Enhancement factor from overlap of wave functions

A_R A_L Coupling constants in effective Lagrangian

Branching ratio

$$\mathrm{Br}(\mu^- e^- \to e^- e^-)$$
$$= 1536\pi^2 (Z-1)^3 \alpha^4 \left(|A_\mathrm{R}|^2 + |A_\mathrm{L}|^2\right) \frac{m_\mathrm{e}}{m_\mu} \frac{\tilde{\tau}_\mu}{\tau_\mu}$$
$$= 2.08 \times 10^{-9} (Z-1)^3 \left(|A_\mathrm{R}|^2 + |A_\mathrm{L}|^2\right) (\tilde{\tau}_\mu / \tau_\mu)$$

Photon propagator
in non-relativistic limit

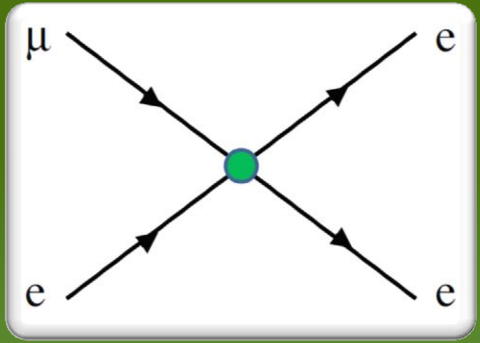
$$\frac{1}{q^2} \simeq \frac{1}{m_\mu m_e}$$

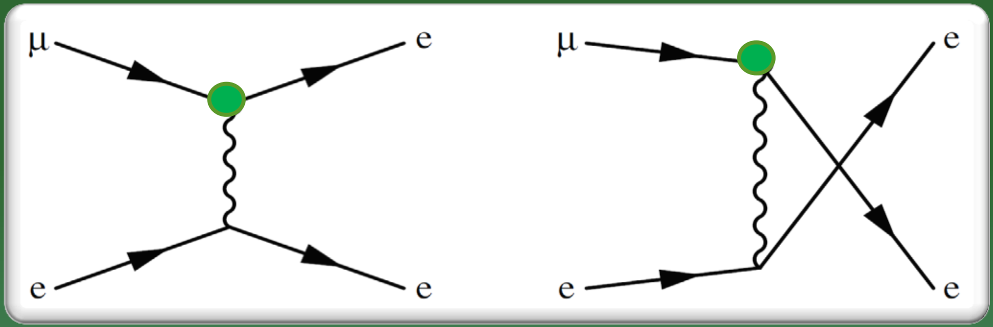
Enhancement factor
compared with 4-Fermi case

$$\frac{m_\mu^2}{m_e^2}$$

(3) Both type interactions comparable case

$$A_{L(R)} \simeq g_i \quad \left(i = 1, 2, \ldots 6 \right)$$





Ratio of 4-Fermi type BR and dipole type BR

$$\sigma v_{\mathrm{photonic}} \sim \alpha \frac{m_\mu^2}{m_e^2} \times \sigma v_{\mathrm{4Fermi}} \sim 10^3 \times \sigma v_{\mathrm{4Fermi}}$$

- One of the distinct features of $\mu^- e^- \longrightarrow e^- e^-$
- It is available for the discrimination of models

Numerical results

How to estimate required muon number

How many muons are required to break current cLFV limit and to discovery $\mu^- e^- \longrightarrow e^- e^-$ process ?

Available inputs

- BR($\mu^- e^- \longrightarrow e^- e^-$) as a function of effective coupling constants
- BR of other cLFV processes as a function of same effective coupling constants
- Current limit of BR of other cLFV processes

How to estimate required muon number

Effective couplings are canceled in ratio of them

⇒ Required muon number is estimated from the limit

Available inputs

- BR($\mu^- e^- \longrightarrow e^- e^-$) as a function of effective coupling constants
- BR of other cLFV processes as a function of same effective coupling constants
- Current limit of BR of other cLFV processes

Ratio of branching ratios of each cLFV process

(1) 4-Fermi interaction dominant case

$$\frac{\mathrm{Br}(\mu^- \mathrm{e}^- \to \mathrm{e}^- \mathrm{e}^-)}{\mathrm{Br}(\mu^+ \to \mathrm{e}^+ \mathrm{e}^+ \mathrm{e}^-)} \lesssim 192\pi (Z-1)^3 \alpha^3 \left(\frac{m_\mathrm{e}}{m_\mu}\right)^3 \frac{\tilde{\tau}_\mu}{\tau_\mu}$$

Limit from SINDRUM experiment

Br($\mu^- \longrightarrow e^- e^+ e^-$) < 1.0×10^{-12}

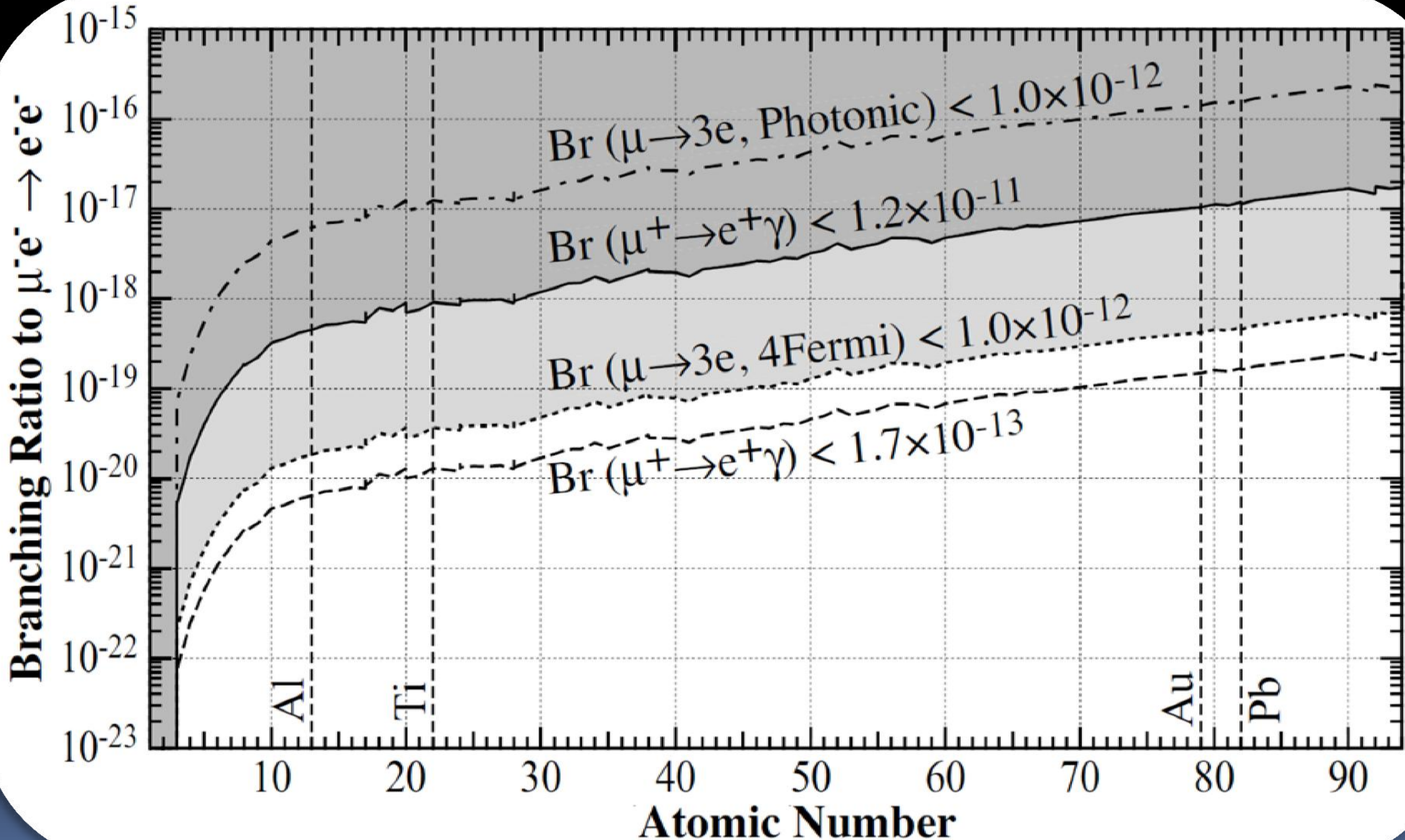
(2) Dipole interaction dominant case

$$\frac{\mathrm{Br}(\mu^- e^- \to e^- e^-)}{\mathrm{Br}(\mu^+ \to e^+ \gamma)} \lesssim 4 (Z-1)^3 \alpha^4 \frac{m_e}{m_\mu} \frac{\tilde{\tau}_\mu}{\tau_\mu}$$

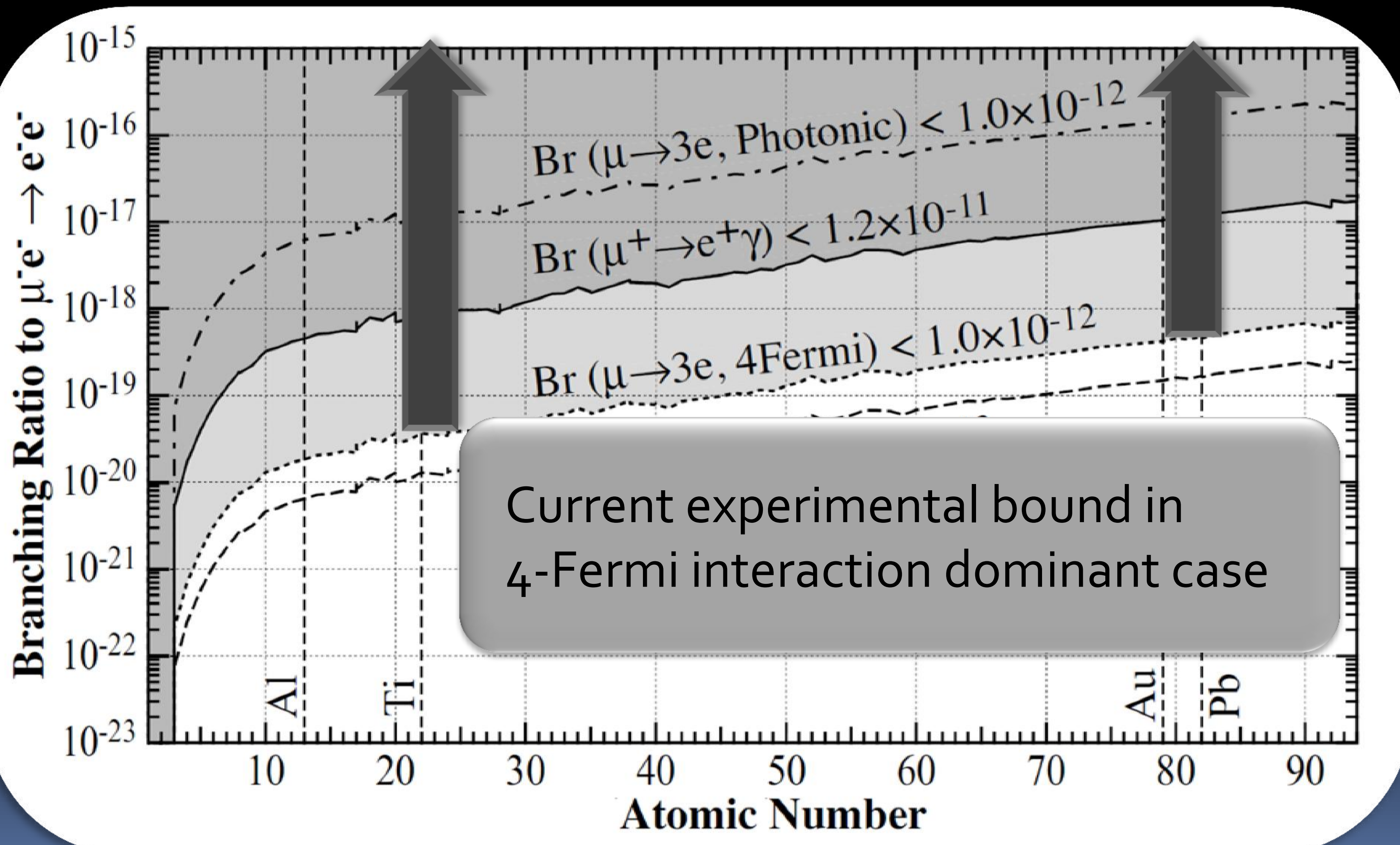
Limit from MEGA experiment

Br($\mu^+ \longrightarrow e^+ \gamma$) < 1.2×10^{-11}

Numerical result



Numerical result



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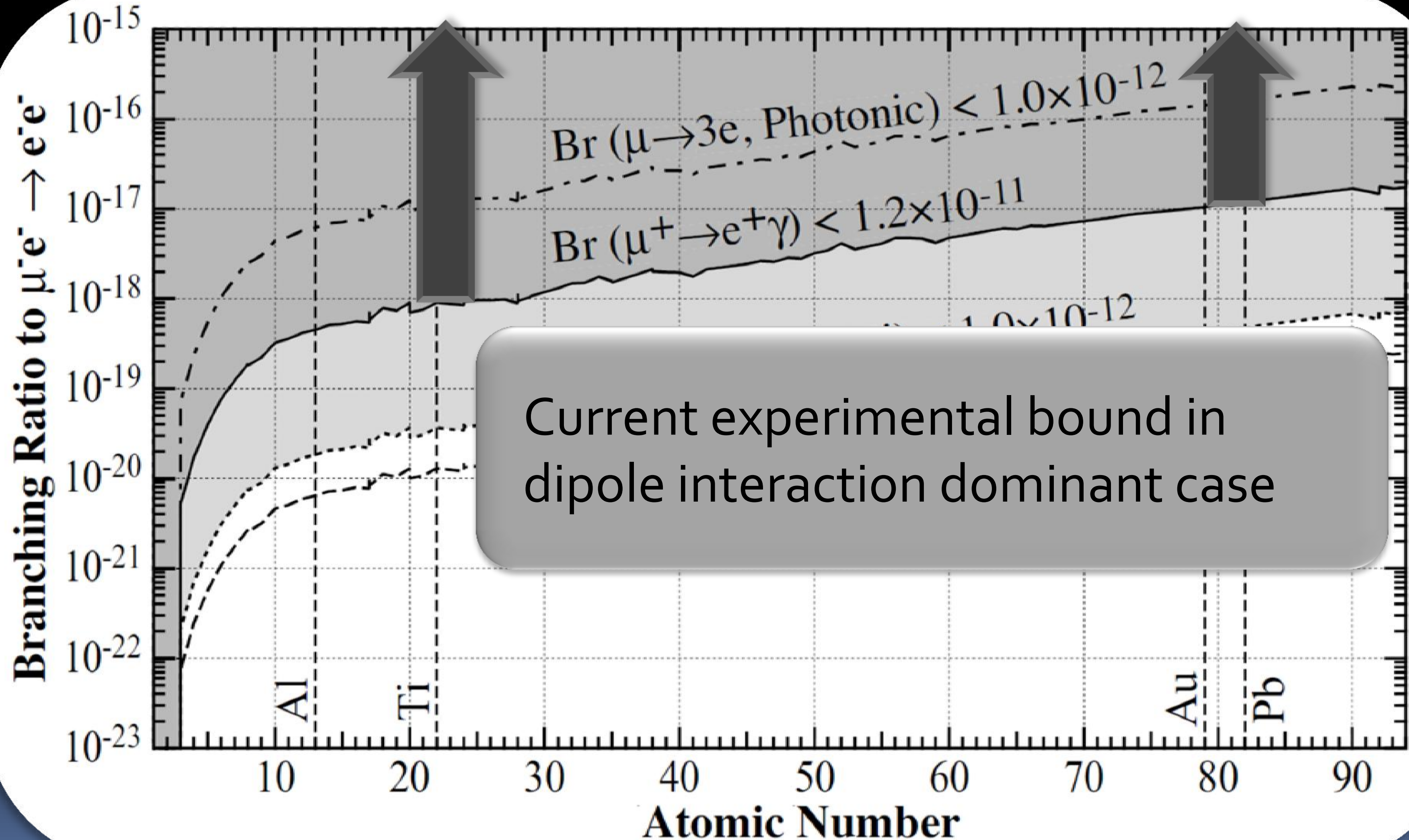
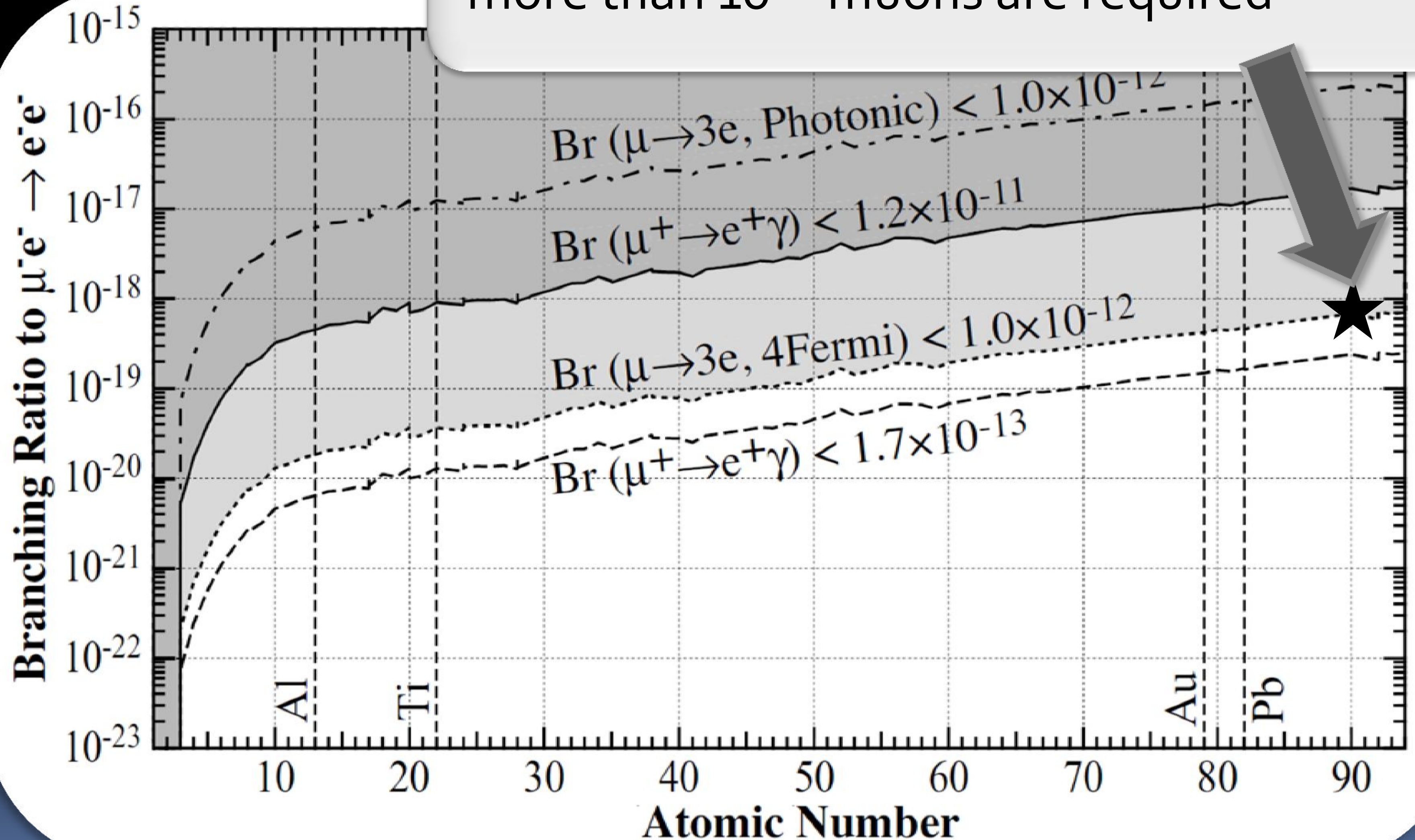


Current experimental bound in dipole interaction dominant case

Numerical r

To break world record and discovery cLFV, more than 10^{18} muons are required



Conclusion of previous work

Muon intensity in working and future experiments

Collaboration	Searching for	Intensity
MEG	$\mu \to e\gamma$	$10^{7.5}\ \mu/\mathrm{s}$
MUSIC	$\mu \to 3e$	$10^{8}\ \mu/\mathrm{s}$
COMET	$\mu^- N \to e^- N$	$10^{11}\ \mu/\mathrm{s}$
Mu2E (E973)	$\mu^- N \to e^- N$	$10^{11}\ \mu/\mathrm{s}$
PRISM	$\mu^- N \to e^- N$	$10^{12}\ \mu/\mathrm{s}$

For COMET experiment

More than 10^{18} muons per year $\sim 3 \times 10^{7}$ (s)

$\mu^- e^- \longrightarrow e^- e^-$ must serve complemental information to shed light on the nature of cLFV

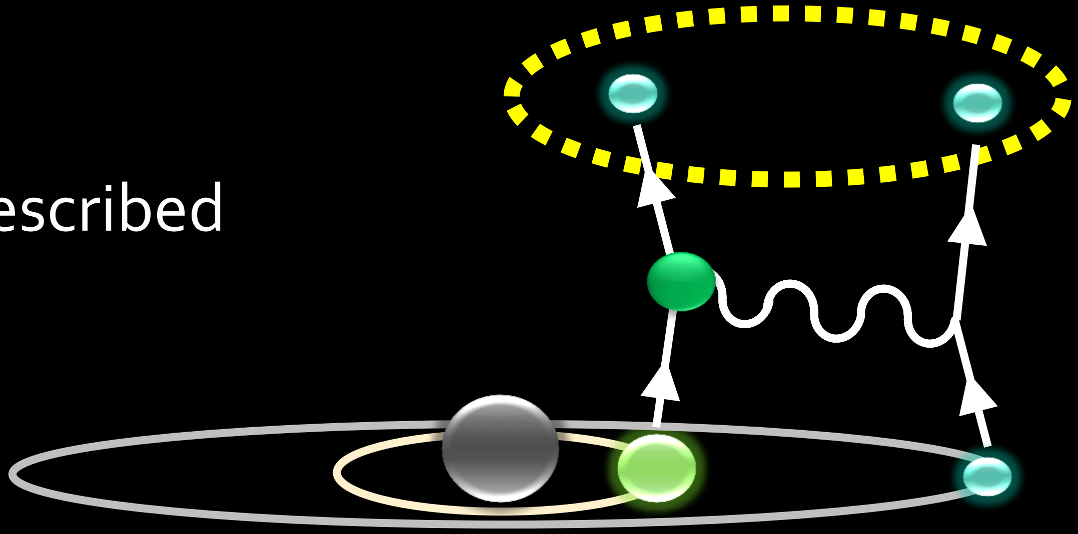
However it could not be a first cLFV signal ...???

Improvement and enhancement

Shortcoming of previous calculation and improvement

Shortcoming(1):

Final electrons are described by plane wave



Improvement

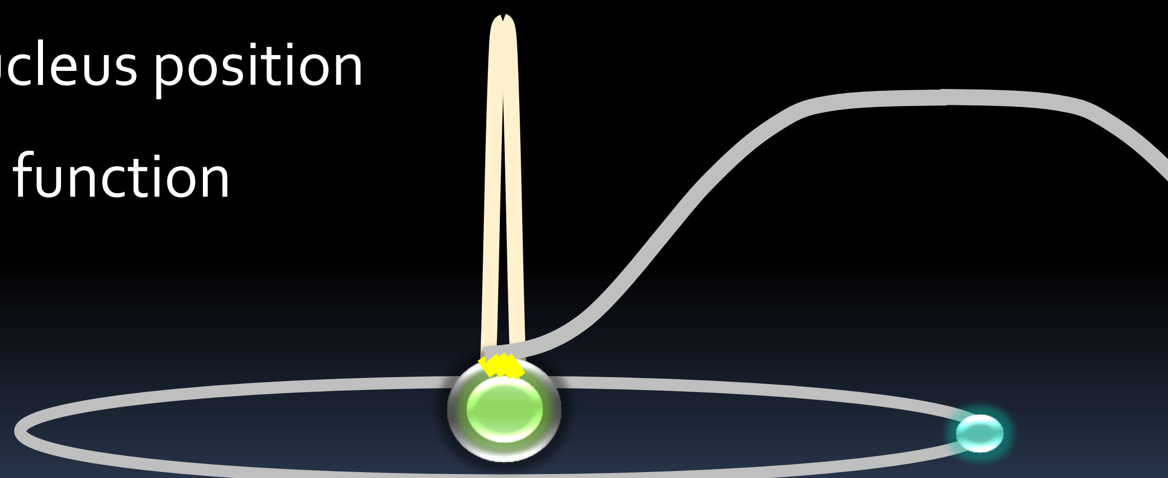
Distorted wave function by nucleus potential

Shortcoming of previous calculation and improvement

$$\Gamma(\mu^- \mathrm{e}^- \to \mathrm{e}^- \mathrm{e}^-; Z) = 2\sigma v_{\mathrm{rel}} \left| \psi_{1\mathrm{S}}^{(\mathrm{e})}(0; Z-1) \right|^2$$

Approximation in previous work

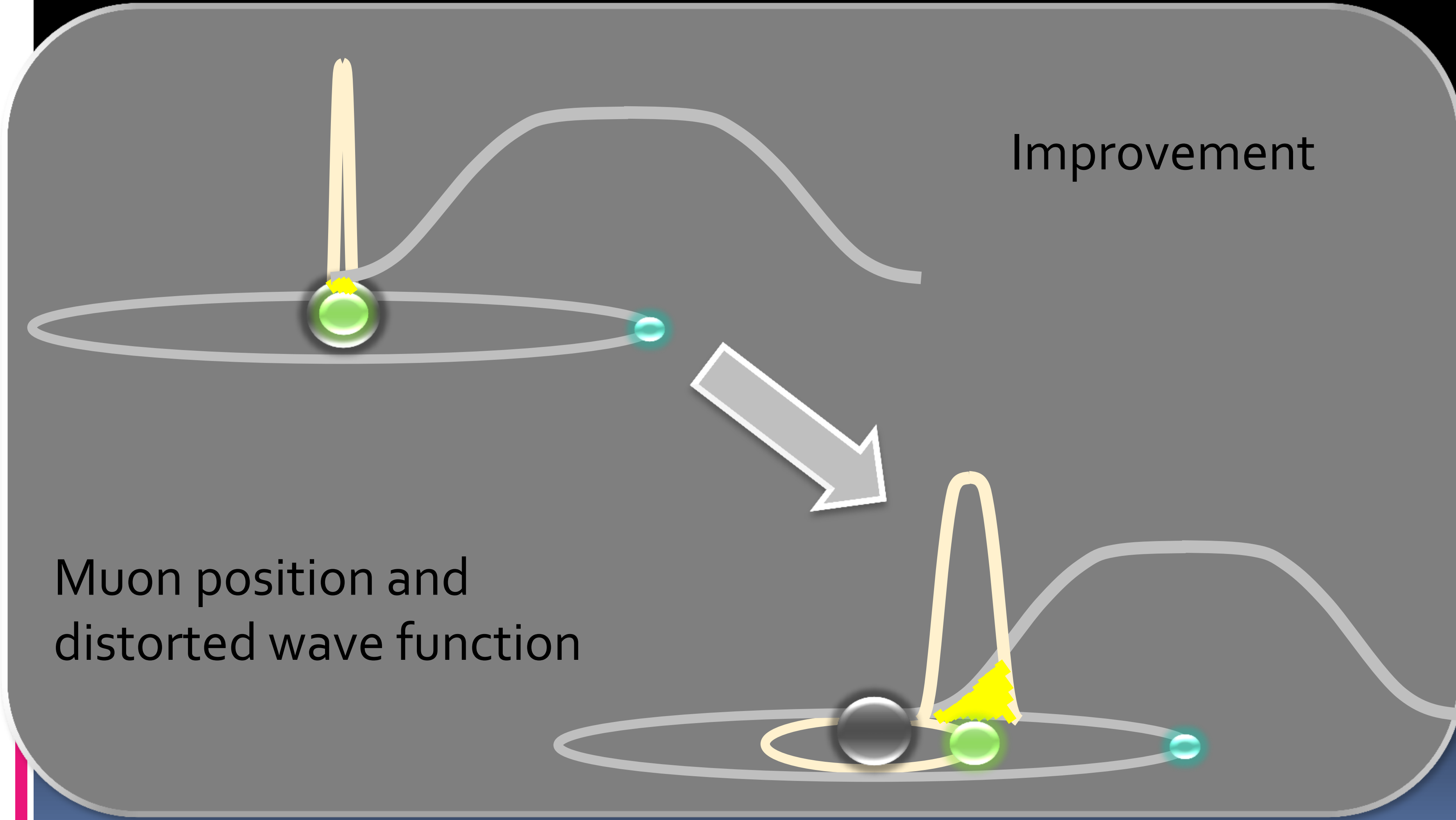
- Localized muon at nucleus position
- Non-relativistic wave function of electron



Shortcoming(2):

No information of muon position and nucleus potential

Shortcoming of previous calculation and improvement



New result (preliminary)

$$\text{Enhancement factor} = \frac{\text{Reaction rate with distorted wave}}{\text{Reaction rate with plane wave}}$$

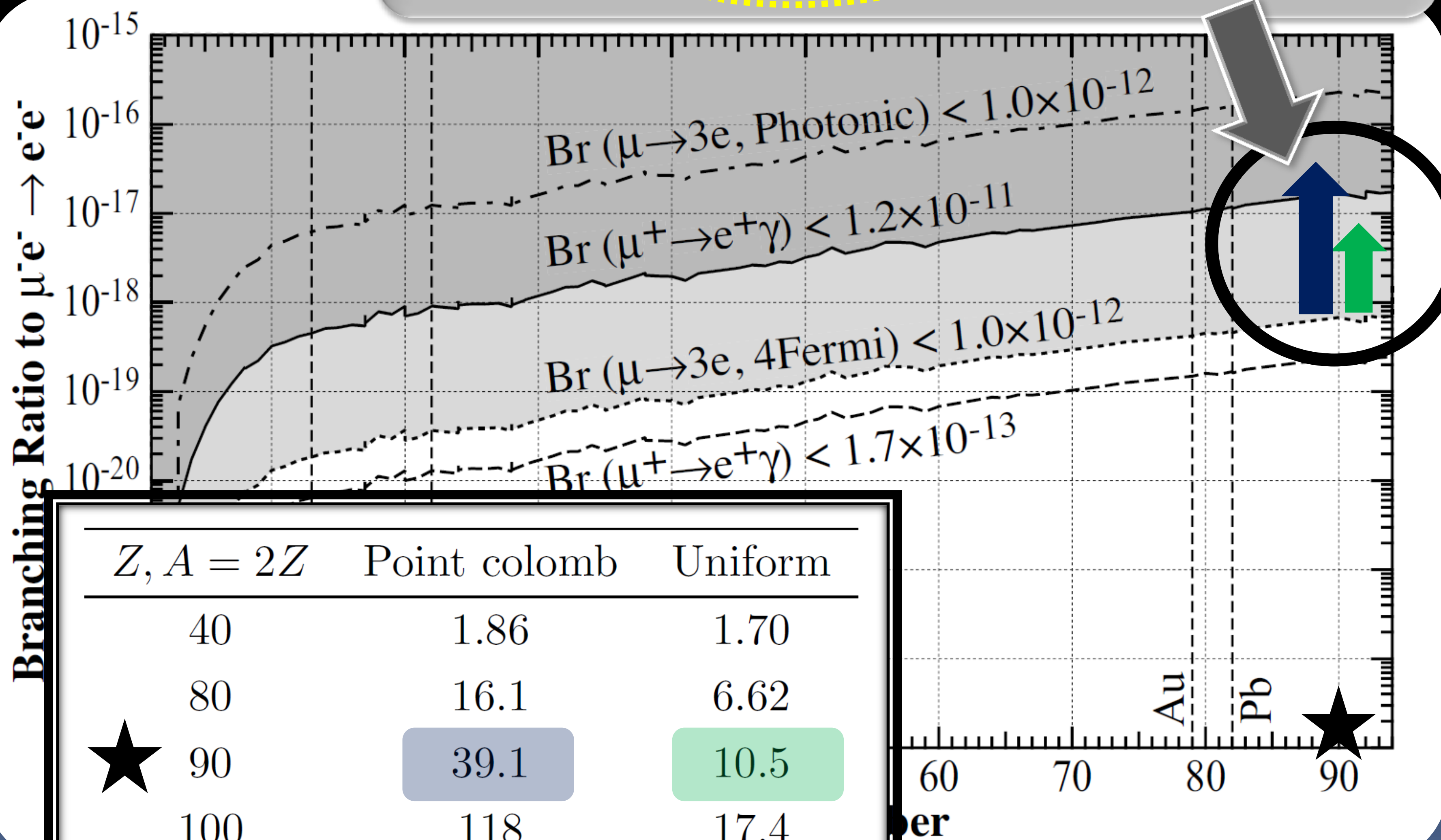
$Z, A = 2Z$	Point colomb	Uniform
40	1.86	1.70
80	16.1	6.62
90	39.1	10.5
100	118	17.4

New result

Enhancement factor

Preliminary!!

To break world record and discovery cLFV, more than 10^{16} - 10^{17} muons are required



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PRISM	$\mu^- N \to e^- N$	$10^{12}\ \mu/s$

For COMET experiment

More than 10^{18} muons per year $\sim 3\times 10^{7}$ (s)

⇨ Within the reach of next generation experiments

First discovery of cLFV comes from $\mu^- e^- \longrightarrow e^- e^-$ in a muonic atom !?

Summary

- $\mu^- e^- \longrightarrow e^- e^-$ in a muonic atom is one of the promising reactions to search for cLFV
  - Clean signal [back-to-back energetic dielectron]

    Cleaner experimental signature comparison with $\mu^- \longrightarrow e^- e^+ e^-$ and $\mu^+ \longrightarrow e^+ \gamma$
  - Sensitive to not only 4-Fermi int. but also photonic int.
  - Reaction rate: proportional to $(Z-1)^3$

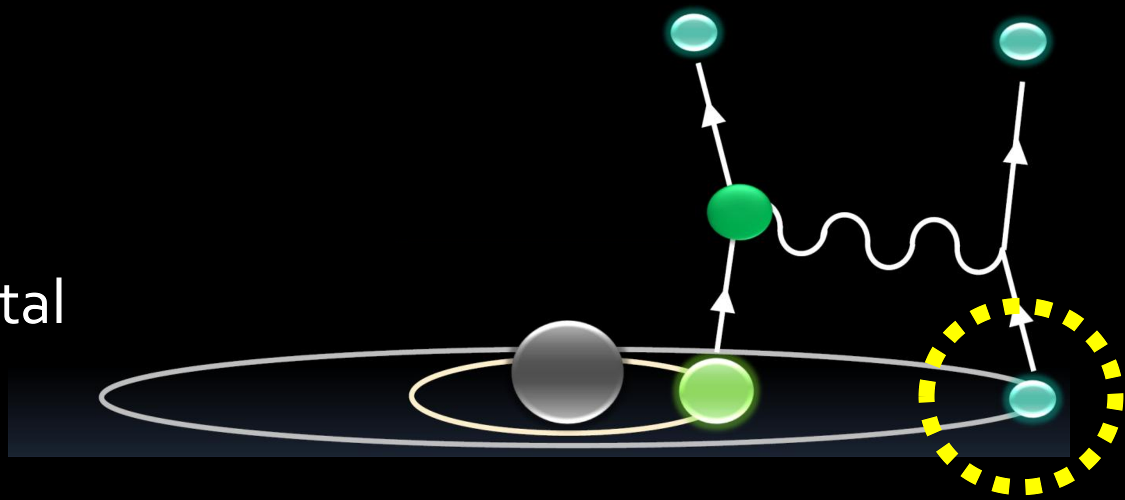
- Realistic wave functions lead large enhancement of reaction rate

- First signal of cLFV may be discovered from $\mu^- e^- \longrightarrow e^- e^-$ in next generation experiments !!

Back-up slides

Electron wave function (initial state)

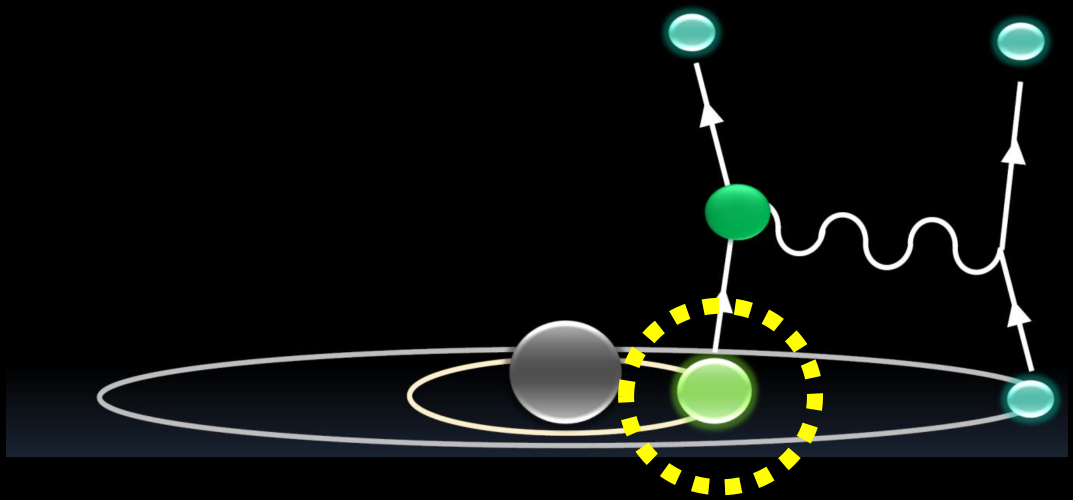
Initial state electron
= electron in atomic orbital



Electron orbit radius >> Nucleus size

Electron wave function in initial state = Wave function of Dirac particle in point Coulomb potential

Muon wave function



Muon Bohr radius $\simeq$ Nucleus size

⇨ Muon wave function penetrates into nucleus

Wave function has to be constructed taking into account nucleus size and Coulomb potential

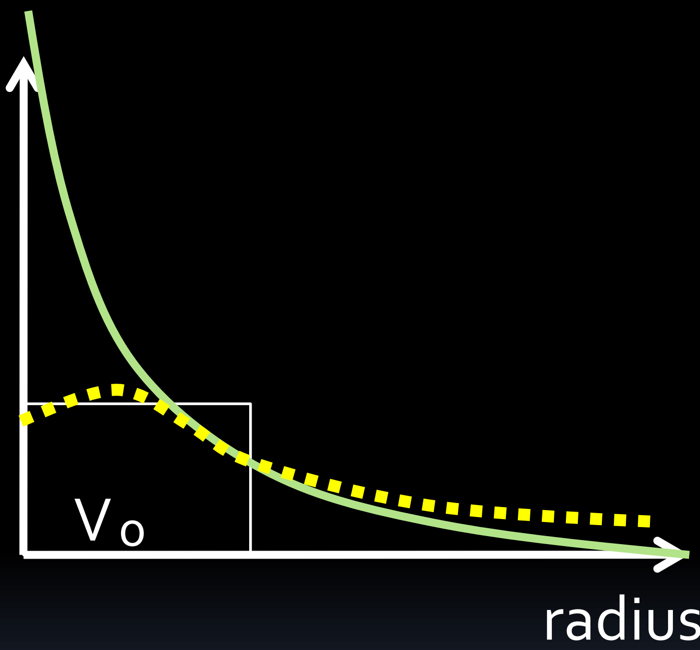
Muon wave function

Construction step of the muon wave function

(1) Deriving muon wave function with constant potential V_0

(2) Deriving muon wave function to be one in point Coulomb potential for radius $\longrightarrow \infty$

(3) Connecting these waves at an appropriate point

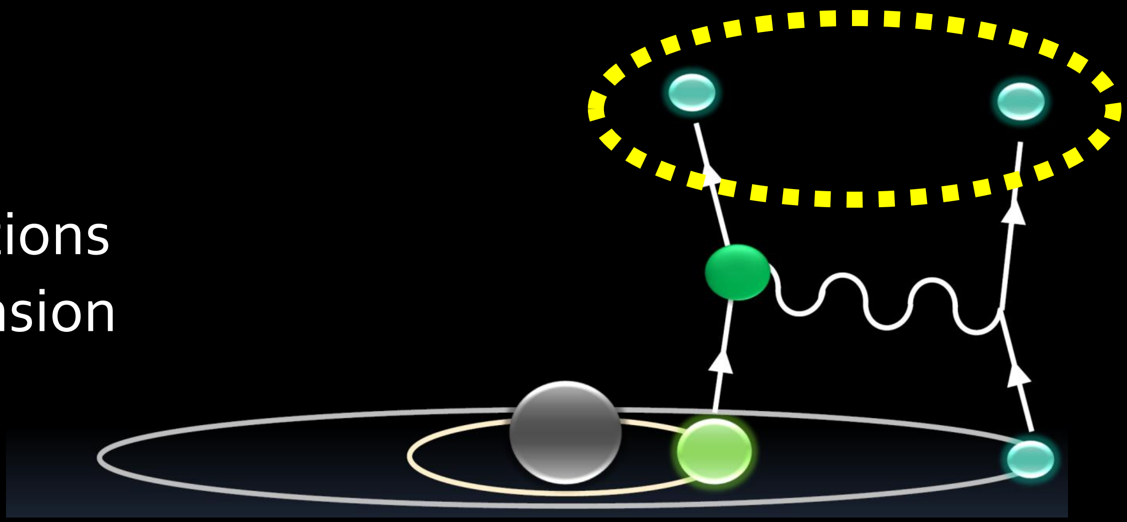


Requirements

$$\left.\frac{d\Psi_{(1)}/dr}{\Psi_{(1)}}\right|_{r=r_0} = \left.\frac{d\Psi_{(2)}/dr}{\Psi_{(2)}}\right|_{r=r_0}$$

Electron wave function (final state)

Constructing wave functions
using partial wave expansion

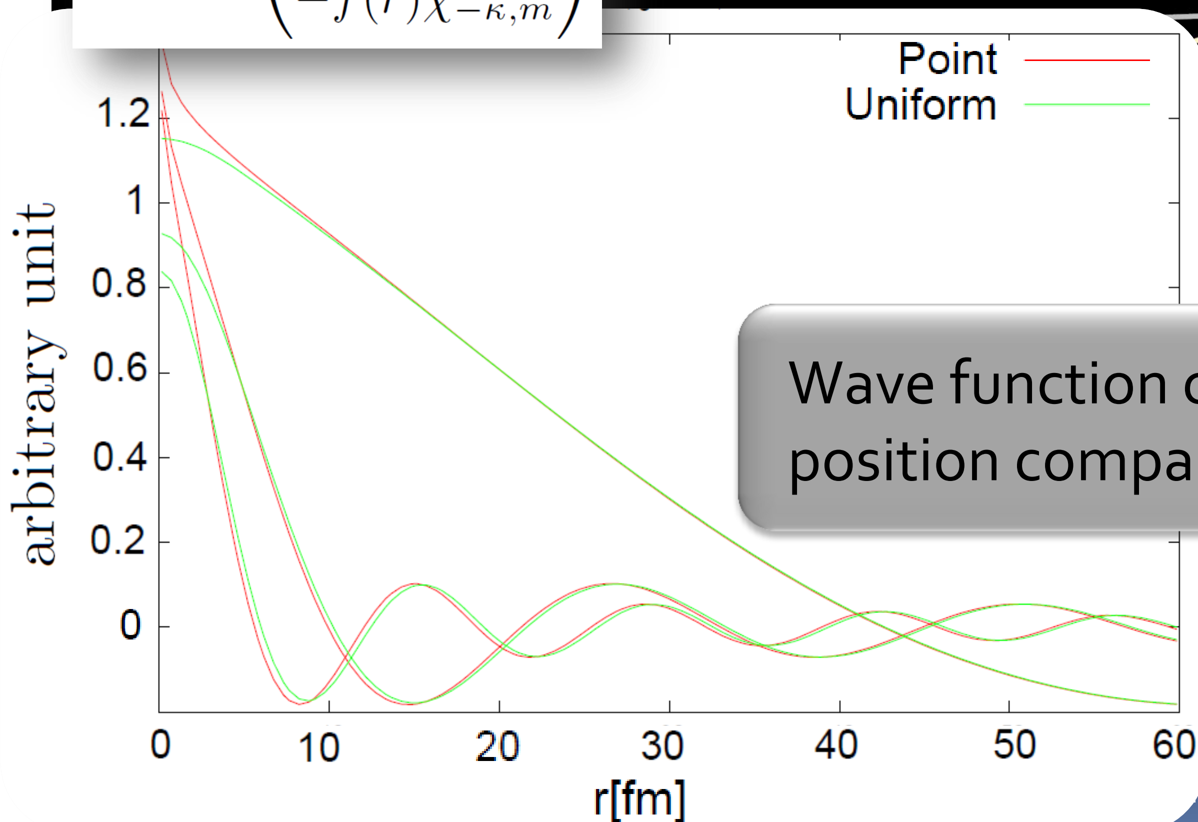


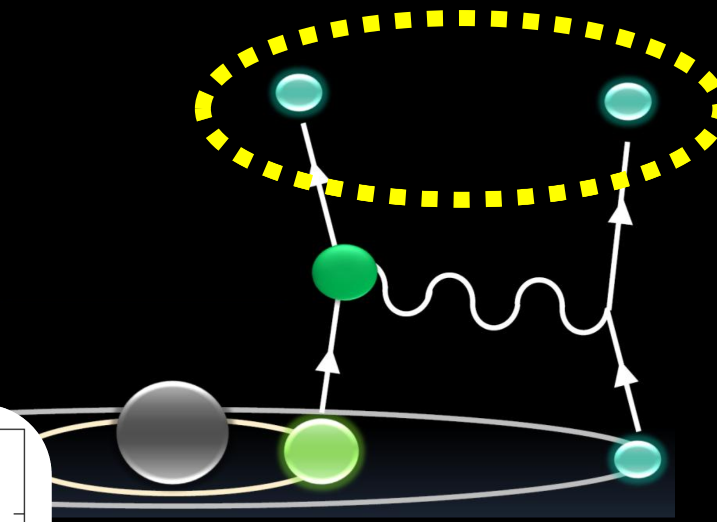
Requirements for constructing final state electrons

- To be wave functions in point Coulomb potential at points being quite far from nucleus
- To be consistent with total angular momentum of initial state

Electron wave function (final state)

$$\psi_{jlm} = \begin{pmatrix} ig(r)\chi_{\kappa,m} \\ -f(r)\chi_{-\kappa,m} \end{pmatrix}$$





Wave function concentrating on muon position compared with plane wave

Reaction rate

Simplest case:
scalar type 4-Fermi interaction

$$\mathcal{L} = -\frac{4G_F}{\sqrt{2}} g_{\mathrm{cLFV}} (\psi_1^\dagger \gamma^0 P_R \psi_\mu)(\psi_2^\dagger \gamma^0 P_R \psi_e)$$

Amplitude

$$\mathcal{M} = -\frac{4G_F}{\sqrt{2}} g_{\mathrm{cLFV}} \int d^3\boldsymbol{r} (\psi_1^\dagger \gamma^0 P_R \psi_\mu)(\psi_2^\dagger \gamma^0 P_R \psi_e) e^{-i(\boldsymbol{p_1}+\boldsymbol{p_2})\cdot \boldsymbol{r}}$$

ψ_μ Distorted muon wave function

ψ_e Distorted initial electron wave function

ψ_1 ψ_2 Distorted final electron wave functions

Reaction rate

Simplest case:
scalar type 4-Fermi interaction

Amplitude

$$\mathcal{M} = -\frac{4G_F}{\sqrt{2}} g_{\mathrm{cLFV}} \int d^3\boldsymbol{r} (\psi_1^\dagger \gamma^0 P_R \psi_\mu)(\psi_2^\dagger \gamma^0 P_R \psi_e) e^{-i(\boldsymbol{p_1}+\boldsymbol{p_2})\cdot \boldsymbol{r}}$$

Cross section with
distorted wave functions

$$\sigma v = \frac{1}{2!} \int \frac{d^3\boldsymbol{p}_1}{(2\pi)^3} \frac{d^3\boldsymbol{p}_2}{(2\pi)^3} (2\pi)\delta(E_i - E_f) \left|\mathcal{M}\right|^2$$

Integrate over spatial coordinate of vertex and
final state momentum by brute force!!

cLFV processes and experimental limit

process	present limit	future	
$\mu \rightarrow e\gamma$	$<1.2 \times 10^{-11}$	$<10^{-13}$	MEG at PSI
$\mu \rightarrow eee$	$<1.0 \times 10^{-12}$	$<10^{-14}$ - 10^{-16}	PSI or MuSIC
$\mu N \rightarrow eN$ (in Al)	none	$<10^{-16}$	Mu2e / COMET
$\mu N \rightarrow eN$ (in Ti)	$<4.3 \times 10^{-12}$	$<10^{-18}$	PRISM
$\tau \rightarrow e\gamma$	$<1.1 \times 10^{-7}$	$<10^{-9}$ - 10^{-10}	super (KEK)B factory
$\tau \rightarrow eee$	$<3.6 \times 10^{-8}$	$<10^{-9}$ - 10^{-10}	super (KEK)B factory
$\tau \rightarrow \mu\gamma$	$<4.5 \times 10^{-8}$	$<10^{-9}$ - 10^{-10}	super (KEK)B factory
$\tau \rightarrow \mu\mu\mu$	$<3.2 \times 10^{-8}$	$<10^{-9}$ - 10^{-10}	super (KEK)B factory