Seesaw in the bulk

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Based on the work with

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Contents

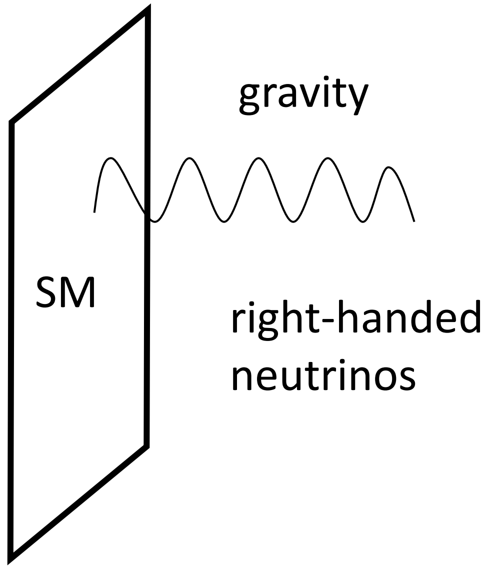
The interplay between neutrino physics and the extra dimensions

- The bulk right-handed neutrinos
- KK expansion and the propagator
- Flavor symmetry breaking on orbifolds

The bulk right-handed neutrinos

[Dienes,Dudas,Gherghetta,1999]

[Arkani-Hamed,Dimopoulos,Dvali,March-Russell,1999]



Standard Model + ν_R (bulk)

Dirac Yukawa

$$\int d^4x \, \frac{m}{\sqrt{\Lambda L}} \, \overline{\nu_R^k(x)} \nu_L(x)$$

Neutrino mass

$$\underbrace{\frac{m}{\sqrt{\Lambda L}} = \frac{\Lambda m}{M_{pl}}}_{\Lambda^3 L = M_{pl}^2} \approx \frac{10^4 \text{ GeV } 10^2 \text{ GeV}}{10^{18} \text{ GeV}} = 10^{-3} \text{ eV}$$

An interesting range of the neutrino mass shows up

Localization of the wave function

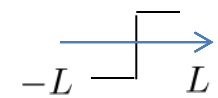
[Grossman, Neubert, 99]

[Chang, Hisano, Nakano, Okada, Yamaguchi, 99]

Dirac (kink) mass in the S1/Z2 bulk

$$m_d \theta(y) \overline{\Psi} \Psi$$

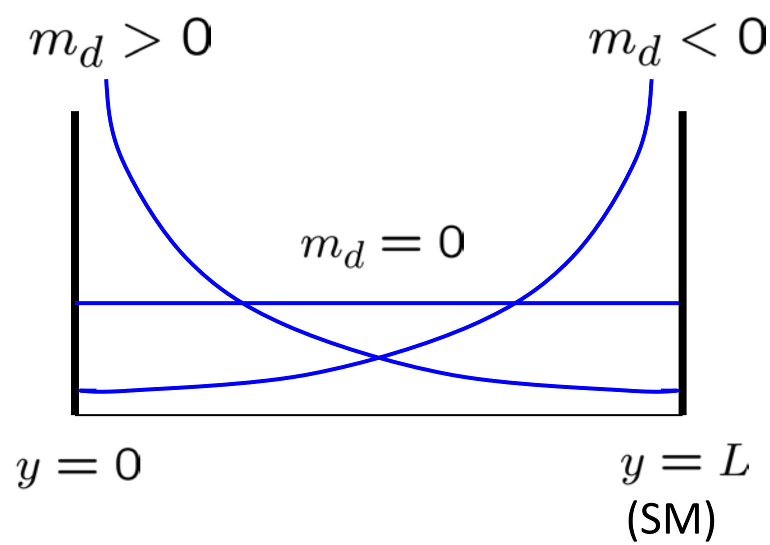
$\theta(y)$: "step function"



Zero mode function

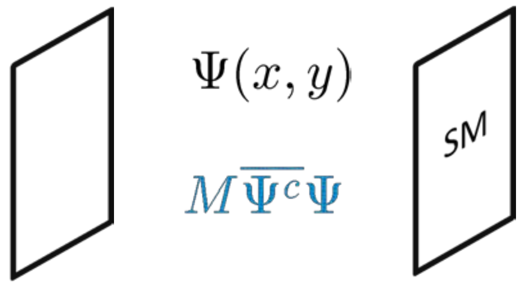
$$\chi^0(y) = \sqrt{\frac{m_d}{1 - e^{-2m_d L}}} e^{-m_d y}$$

$$m_d L \approx 28 \to 10^{-12} \text{ suppression}$$



Seesaw in five dimensions

[Dienes,Dudas,Gherghetta,99]
[Lukas, Ramond, Romanino, Ross, 00]



Lagrangian

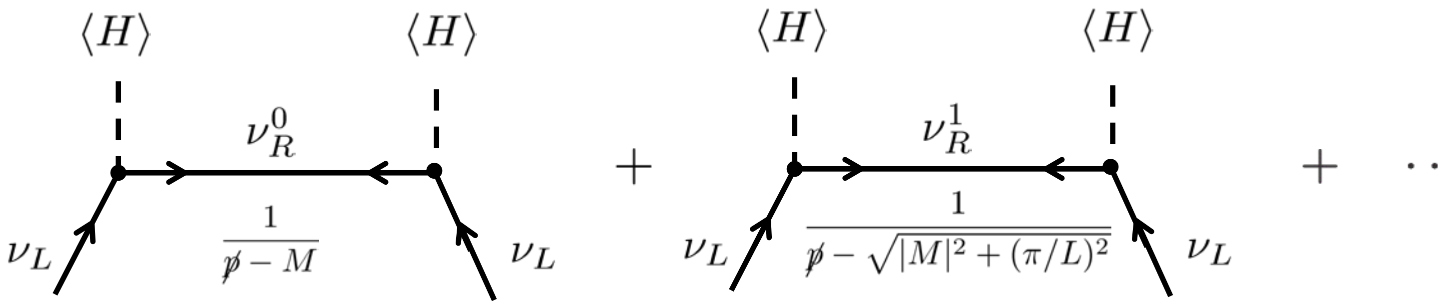
$$\mathcal{L} = i\overline{\Psi}\Gamma^M\partial_M\Psi - \frac{1}{2}(M\overline{\Psi^c}\Psi + \text{h.c.}) - \left(\frac{m}{\sqrt{\Lambda}}\overline{\Psi}\,\nu_L + \text{h.c.}\right)\delta(y-L)$$

Bulk Majorana

left-handed neutrinos

$$\Psi = \begin{pmatrix}\nu_R \\ \nu\end{pmatrix}, \qquad \Psi^c = i\Gamma^2\Gamma^0\gamma_5\overline{\Psi}^{\mathrm{T}}$$

Seesaw in five dimensions



Neutrino mass after the seesaw

$$M_\nu = \frac{1}{\tanh(ML)} \frac{m^2}{\Lambda}$$

$\Lambda > 10^{14}$ GeV for $\mathcal{O}(1)$ Yukawa couplings

Seesaw with the bulk Dirac mass

$$\mathcal{L} = i\overline{\Psi}\Gamma^M\partial_M\Psi - m_d\theta(y)\overline{\Psi}\Psi - \frac{1}{2}(M\overline{\Psi^c}\Psi + \text{h.c.})$$
$$- \left(\frac{m}{\sqrt{\Lambda}}\overline{\Psi}\nu_L + \text{h.c.}\right)\delta(y-L)$$

Bulk Dirac

The neutrino mass becomes

$$M_\nu = \frac{1}{\Lambda L}\frac{\widetilde{M}L\cosh(\widetilde{M}L) - m_d L\sinh(\widetilde{M}L)}{\sinh(\widetilde{M}L)}\frac{m^{\rm T}m}{M^*}, \quad \widetilde{M} = \sqrt{m_d^2 + |M|^2}$$

If $M \ll m_d$ and $\widetilde{M}L \gg 1$,

then $M_\nu \simeq \frac{M}{m_d}\frac{m^{\rm T}m}{\Lambda}$.

$\Rightarrow$ $\Lambda \approx \text{TeV} \rightarrow M/m_d \sim 10^{-10}$

for eV neutrino mass

[AW, Yoshioka, 2009]

Inverse seesaw

Whole neutrino mass matrix

$$\mathcal{M} = \begin{array}{c} \\ \nu_L^{\mathrm{T}} \\ \psi_R^{0\,\dagger} \\ \psi_R^{1\,\dagger} \\ \psi_L^{1\,\mathrm{T}} \\ \vdots \end{array} \begin{array}{c} \begin{array}{ccccc} \nu_L & \psi_R^{0\,*} & \psi_R^{1\,*} & \psi_L^{1} & \end{array} \\ \left(\begin{array}{c|cccc} & m_0 & m_1 & & \cdots \\ \hline m_0 & M & & & \cdots \\ m_1 & & M & M_{KK} & \cdots \\ & & M_{KK} & M & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{array}\right) \end{array}$$

small

The zero mode function $\chi^0(L) \sim e^{-m_d L}$ ⇒ Zero mode is negligible in seesaw

$$M \ll M_{KK}$$

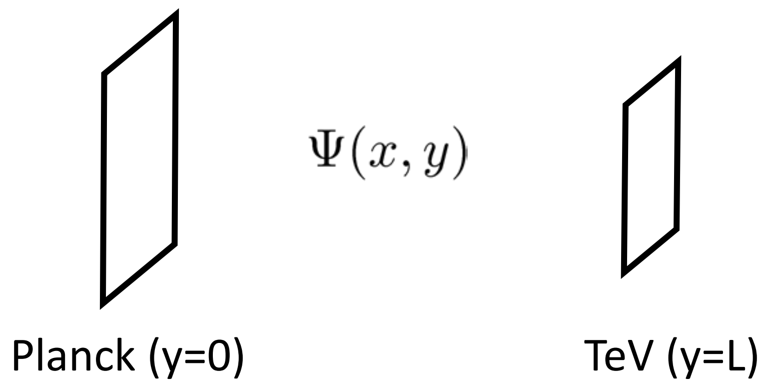
$$M_\nu = \frac{m^2}{\frac{M_{KK}^2}{M}} = \frac{M}{M_{KK}} \frac{m^2}{M_{KK}}$$

$$\mathrm{Det}\mathcal{M} = m_1^2 M \cdots = M_\nu M_{KK}^2 \cdots$$

On the warped geometry

Randall-Sumdrum background

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$$



$$\mathcal{L} = \sqrt{g}\left[i\overline{\Psi}\not{D}\Psi - m_d\theta(y)\overline{\Psi}\Psi - \left(\frac{1}{2}M\overline{\Psi^c}\Psi + \frac{m}{\sqrt{\Lambda}}\overline{\Psi}\nu_L\delta(y-L) + \text{h.c.}\right)\right]$$

How does the background change the neutrino mass ?

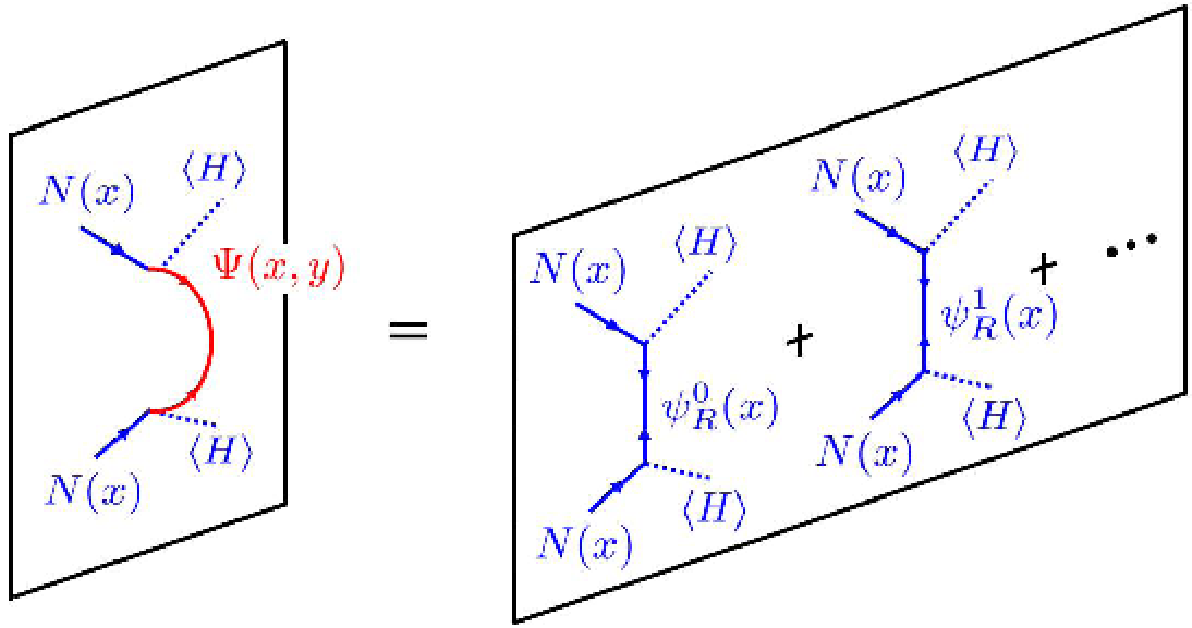
By the usual KK expansion, ...

$$\int_0^L dy\, e^{ky} \chi(y)^{n\,\mathrm{T}} M \chi(y)^m \;\approx\; \delta_{nm}$$

$$\mathcal{M} = \begin{array}{c} \\ \nu_L^{\mathrm{T}} \\ \psi_R^{0\,\dagger} \\ \psi_R^{1\,\dagger} \\ \psi_L^{1\,\mathrm{T}} \\ \psi_R^{2\,\dagger} \\ \psi_L^{2\,\mathrm{T}} \\ \vdots \end{array} \begin{array}{c} \begin{array}{ccccccc} \nu_L & \psi_R^{0\,*} & \psi_R^{1\,*} & \psi_L^{1} & \psi_R^{2\,*} & \psi_L^{2} & \end{array} \\ \left(\begin{array}{c|c|cc|cc|c} & m_0^{\mathrm{T}} & m_1^{\mathrm{T}} & & m_2^{\mathrm{T}} & & \cdots \\ \hline m_0 & -M_{R_{00}}^* & -M_{R_{01}}^* & & -M_{R_{02}}^* & & \cdots \\ \hline m_1 & -M_{R_{01}}^* & -M_{R_{11}}^* & M_{K_1} & -M_{R_{12}}^* & & \cdots \\ & & M_{K_1} & M_{L_{11}} & & M_{L_{12}} & \cdots \\ \hline m_2 & -M_{R_{02}}^* & -M_{R_{21}}^* & & -M_{R_{22}}^* & M_{K_2} & \cdots \\ & & & M_{L_{21}} & M_{K_2} & M_{L_{22}} & \cdots \\ \hline \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{array}\right) \end{array}$$

It is hard to perform seesaw diagonalization

Alternative to the KK expansion ⇒ 5D propagator



The equations for the bulk propagator

$$\left[e^{2k|y|}p^2 - m_d^2 - |M|^2 + \partial_y^2\right] \langle \Psi_L^c(p,y)\overline{\Psi_L}(p,y')\rangle$$
$$-ke^{k|y|}p_\mu\sigma^\mu \langle \Psi_R^c(p,y)\overline{\Psi_R}(p,y')\rangle = iM\delta(y-y')$$

➡ the warped effect is vanishing away at the low-energy limit

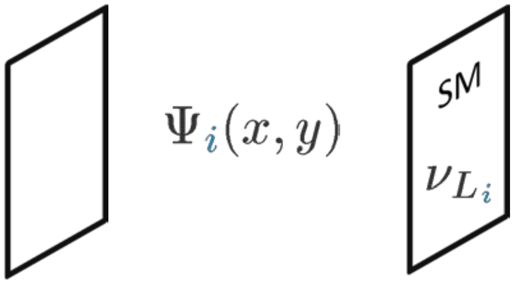
The result is

$$M_\nu = \frac{1}{\Lambda' L}\frac{\widetilde{M}L\cosh(\widetilde{M}L) - m_d L\sinh(\widetilde{M}L)}{\sinh(\widetilde{M}L)}\frac{m^{\mathrm{T}}m}{M^*}, \qquad \Lambda' = \Lambda e^{-kL}$$

- The neutrino mass is not much affected by the background
- The result is the same for more general metric $ds^2 = \rho(y)\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$
- The seesaw mass can be calculated without knowing the KK expansion

Even with a complicated backgroud where a suitable KK expansion cannot be find, the seesaw neutrino mass is calculable.

Three generation phenomenology



Flavor symmetry breaking

Symmetry breaking by the boundary conditions for the bulk fields [Scherk, Schwarz,79]

⇨ An application to symmetry among generation [Haba, AW, Yoshioka, 2006]

S_3 group → tribimaximal mixing

What if the S_4 group is used ? [Ishimori, Shimizu, Tanimoto, AW, 2010]

- a few free parameters
- variety of irreducible representations

Symmetry breaking on S1/Z2

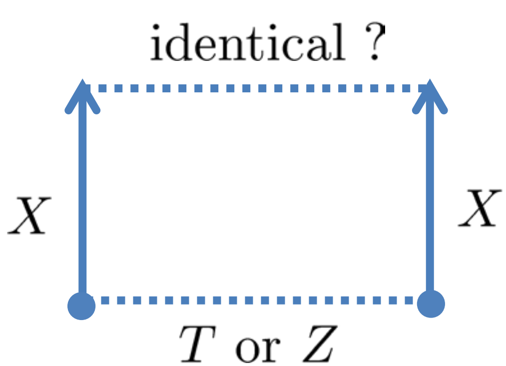
Two operations on S1/Z2

$$\begin{array}{ll} \text{translation} & \hat{T} : y \rightarrow y + 2\pi R \\ \text{reflection} & \hat{Z} : y \rightarrow -y \end{array}$$

Boundary conditions for the bulk fermions

$$\Psi(x, y + 2\pi R) = T\,\Psi(x, y)$$
$$\Psi(x, -y) = Z \otimes \gamma_5 \Psi(x, y)$$

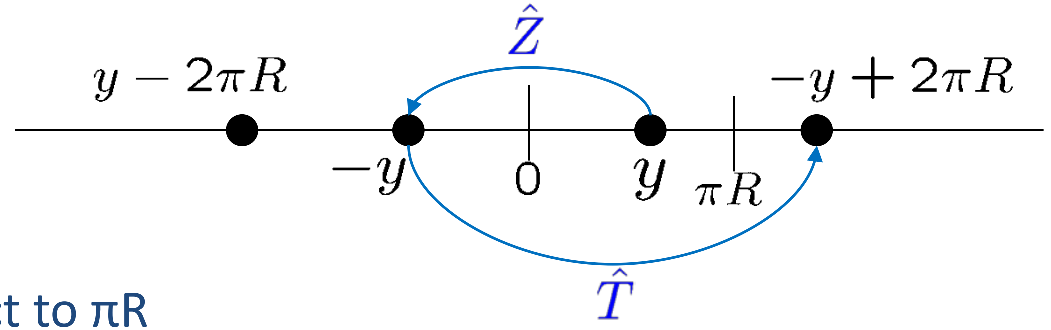
elements of the symmetry group



Symmetry is broken by the elements X which satisfiy $[T, X] \neq 0$ or $[Z, X] \neq 0$.

Another parity Z'

$$Z' = TZ$$



⇒ the reflection with respect to πR

$$(Z, T) \leftrightarrow (Z, Z')$$

Boundary conditions in terms of the two parities

$$\Psi_i(x, -y) = Z_{ij} \otimes \gamma_5 \Psi_j(x, y)$$
$$\Psi_i(x, -y + 2\pi R) = Z'_{ij} \otimes \gamma_5 \Psi_j(x, y)$$

Application to S_4 flavor symmetry

S_4 group

24 elements: $1,\ Q,\ P,\ Q^2,\ PQP^2,\ \cdots,\ QP.$

Irreducible representations: $\underline{1}, \underline{1}', \underline{2}, \underline{3}, \underline{3}'$

Exmaple of the triplet $\quad Q = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \quad P = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

Boundary conditions for the bulk right-handed neutrinos

$$\Psi_i(x,-y) \ = \ Z_{ij} \otimes \gamma_5 \Psi_j(x,y)$$
$$\Psi_i(x,-y+2\pi R) \ = \ Z'_{ij} \otimes \gamma_5 \Psi_j(x,y)$$

Z_{ij} and Z'_{ij} are identified as some S_4 elements.

Lagrangian of neutrino sector

$$\mathcal{L} = i\overline{\Psi}_j \Gamma^M \partial_M \Psi_j - \frac{1}{2}(\overline{\Psi^c_i}(M)_{ij}\Psi_j + \text{h.c.}) - \frac{1}{\sqrt{\Lambda}}\left(\overline{\Psi}_i(m)_{ij}\nu_{L_j} + \overline{\Psi^c_i}(m^c)_{ij}\nu_{L_j} + \text{h.c.}\right)\delta(y-\pi R)$$

Ψ_i and ν_{L_i} are assumed to be $\underline{3}$.

$$m_{ij} = m\delta_{ij}, \quad m^c_{ij} = m^c\delta_{ij}, \quad M_{ij} = M\delta_{ij}$$

4D Lagrangian is obtained by

- Fixing the boundary conditions (T and Z)
- Substituting the KK expansions into 5D Lagrangian
- Integrating it over 5D coordinate y

Example I: $Z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad Z' = 1$

KK expansion

$$\Psi_i(x,y) = \begin{pmatrix} \sum_n \chi^n_{R_{ij}}(y)\psi^n_{R_j}(x) \\ \sum_n \chi^n_{L_{ij}}(y)\psi^n_{L_j}(x) \end{pmatrix},$$

$$\chi^n_R(y) = \frac{1}{\sqrt{\pi R}} \begin{pmatrix} \cos\left(n\frac{y}{R}\right) & 0 & 0 \\ 0 & \frac{-1}{\sqrt{2}}\cos\left[\left(n-\frac{1}{2}\right)\frac{y}{R}\right] & \frac{1}{\sqrt{2}}\cos\left(n\frac{y}{R}\right) \\ 0 & \frac{1}{\sqrt{2}}\cos\left[\left(n-\frac{1}{2}\right)\frac{y}{R}\right] & \frac{1}{\sqrt{2}}\cos\left(n\frac{y}{R}\right) \end{pmatrix}$$

$$\chi^n_L(y) = \frac{1}{\sqrt{\pi R}} \begin{pmatrix} \sin\left(n\frac{y}{R}\right) & 0 & 0 \\ 0 & \frac{-1}{\sqrt{2}}\sin\left[\left(n-\frac{1}{2}\right)\frac{y}{R}\right] & \frac{1}{\sqrt{2}}\sin\left(n\frac{y}{R}\right) \\ 0 & \frac{1}{\sqrt{2}}\sin\left[\left(n-\frac{1}{2}\right)\frac{y}{R}\right] & \frac{1}{\sqrt{2}}\sin\left(n\frac{y}{R}\right) \end{pmatrix}$$

Majorana mass matrix after the seesaw

$$M_\nu = \begin{pmatrix} A & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}}(A+B) & \frac{1}{\sqrt{2}}(A-B) \\ 0 & \frac{1}{\sqrt{2}}(A-B) & \frac{1}{\sqrt{2}}(A+B) \end{pmatrix}, \quad \begin{cases} A = \frac{1}{\Lambda R}\frac{|M|R}{\tanh(\pi|M|R)}\frac{m^2}{M^*} \\ \\ B = \frac{1}{\Lambda R}|M|R\tanh(\pi|M|R)\frac{(m^c)^2}{M} \end{cases}$$

- only one mixing angle
- degenerate masses

Example II: $Z = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad Z' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

The result is

$$M_\nu = \frac{1}{\Lambda R}\Bigg[\overbrace{\frac{s|M|R}{c+1/2}\frac{m^2}{M^*}}^{m_1} \begin{pmatrix} \frac{4}{6} & \frac{-2}{6} & \frac{-2}{6} \\ \frac{-2}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{-2}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix} + \overbrace{\frac{|M|R}{\tanh(\pi|M|R)}\frac{m^2}{M^*}}^{m_2} \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$$

$$- \underbrace{\frac{s|M|R}{c+1/2}\frac{(m^c)^2}{M}}_{m_3} \begin{pmatrix} & & \\ & \frac{1}{2} & \frac{-1}{2} \\ & \frac{-1}{2} & \frac{1}{2} \end{pmatrix} - \frac{|M|R}{c+1/2}\frac{m m^c}{|M|} \begin{pmatrix} & \frac{1}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{-1}{2} & \\ \frac{-1}{2} & & \frac{1}{2} \end{pmatrix} \Bigg]$$

$c \equiv \cosh(2\pi|M|R)$, $s \equiv \sinh(2\pi|M|R)$

- S_4 is completely broken
- $m^c = 0 \rightarrow$ tri-bimaximal mixing with inverted hierarchy
- $MR \gg 1 \rightarrow$ tri-bimaximal

Deviation from the tri-bimaximal mixing

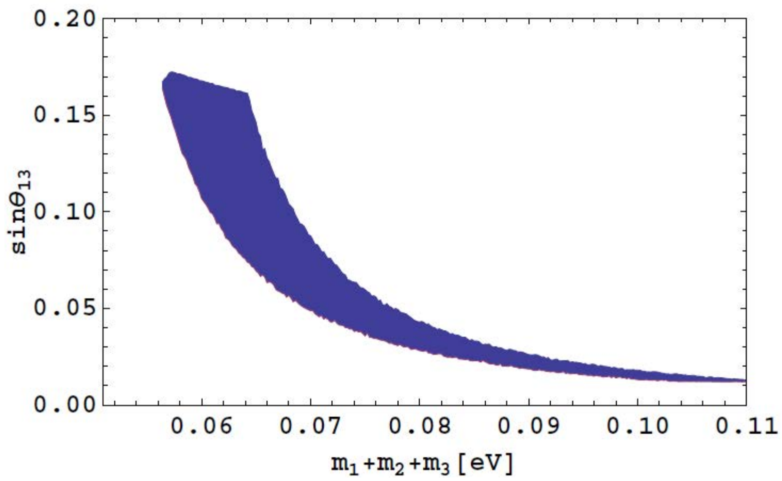
$$M_\nu = \frac{-|M|}{\Lambda} V_{\text{tri-bi}} \begin{pmatrix} \frac{-2s}{2c+1}\frac{m^2}{M^*} & 0 & \frac{\sqrt{3}}{2c+1}\frac{mm^c}{|M|} \\ 0 & \frac{-1}{\tanh(\pi|M|R)}\frac{m^2}{M^*} & 0 \\ \frac{\sqrt{3}}{2c+1}\frac{mm^c}{|M|} & 0 & \frac{2s}{2c+1}\frac{(m^c)^2}{M} \end{pmatrix} V_{\text{tri-bi}}^{\mathrm{T}},$$

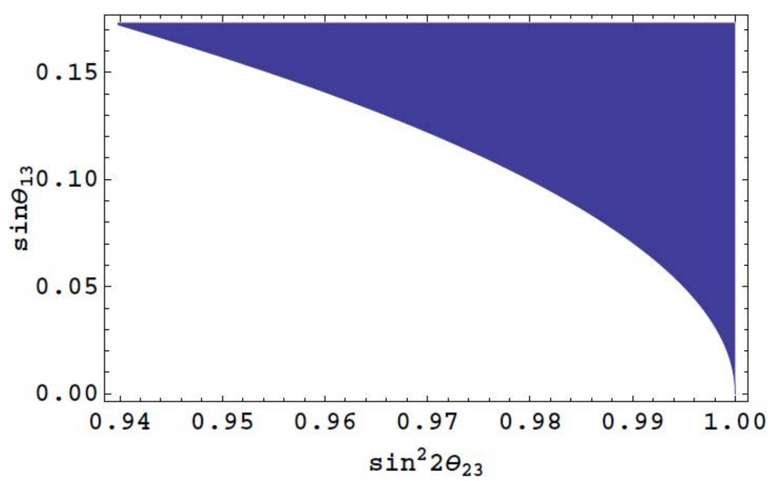
$$U_{e2} = \frac{1}{\sqrt{3}} e^{i\rho}$$

$$U_{e3} = \frac{2i}{\sqrt{6}} \sin\theta e^{i\rho}$$

$$U_{\mu 3} = -i\left(\frac{1}{\sqrt{2}}\cos\theta e^{i\sigma} + \frac{1}{\sqrt{6}}\sin\theta e^{i\rho}\right)$$

3 effective parameters: $|m|^2/\Lambda,\ |M|R,\ |m^c|/|m|$





Charged-lepton sector

For Example

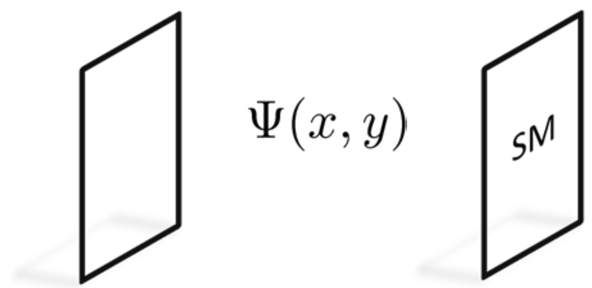
	e_R	(μ_R, τ_R)	(L_e, L_μ, L_τ)	H	(ϕ_1, ϕ_2, ϕ_3)
S_4	$\underline{1}$	$\underline{2}$	$\underline{3}$	$\underline{1}$	$\underline{3}$

$$M_\ell = vY_s \begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + vY_d \begin{pmatrix} 0 & 0 & 0 \\ \alpha_1 & \omega^2\alpha_2 & \omega\alpha_3 \\ \alpha_1 & \omega\alpha_2 & \omega^2\alpha_3 \end{pmatrix}, \qquad \alpha_i \equiv \langle\phi_i\rangle/\Lambda$$

$$\alpha_1 v \sim m_e,\ \alpha_2 v \sim m_\mu,\ \alpha_3 v \sim m_\tau$$

$\Rightarrow$ small mixing for the left-handed direction

Summary



We have explored ``bulk seesaw”

- Inverse seesaw (bulk Dirac mass)
- Geometry free nature of the neutrino mass
- Flavor symmetry breaking without scalar fields