Minimal neutrino texture in neutrino mass matrix

Yusuke Shimizu

MPIK, Heidelberg

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based on:

M. Fukugita, Y. S., M. Tanimoto and T. T. Yanagida, Phys. Lett. B **716** (2012) 294.

Y. S., R. Takahashi, M. Tanimoto, arXiv:1212.5913 (accepted in Phys. Lett. B).

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Plan of my talk

1. Introduction

Standard Model (SM): $SU(3)_C \times SU(2)_L \times U(1)_Y$

Particle	First	Second	Third	Mixing matrix
Quark	$\begin{pmatrix} u \\ d \end{pmatrix}_L$ u_R^c d_R^c	$\begin{pmatrix} c \\ s \end{pmatrix}_L$ c_R^c s_R^c	$\begin{pmatrix} t \\ b \end{pmatrix}_L$ t_R^c b_R^c	CKM matrix (Cabibbo-Kobayashi-Maskawa)
Lepton	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$ e_R^c	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$ μ_R^c	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$ τ_R^c	PMNS matrix (Pontecorvo-Maki-Nakagawa-Sakata)

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Generation (Flavor) Mysteries (Problems)

- Masses of elementary particles are different each generation.
- Lepton flavor mixing is quite different from quark one.

Neutrino Oscillation

- Neutrino mass hierarchies:
  Normal $\rightarrow m_1 < m_2 < m_3$.
  Inverted $\rightarrow m_3 < m_1 < m_2$.
  (Quasi-degenerated $\rightarrow m_1 \sim m_2 \sim m_3$)
- Lepton flavor mixing matrix:
  $|\nu_\alpha\rangle = U_{\alpha i}|\nu_i\rangle \quad (\alpha = e, \mu, \tau, \quad i = 1, 2, 3)$,

$$U \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\rho} & 0 \\ 0 & 0 & e^{i\sigma} \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\rho} & 0 \\ 0 & 0 & e^{i\sigma} \end{pmatrix}.$$

Experiments indicate large θ_{13}!!

- Experimental result by Daya Bay @Neutrino 2012

$$\sin^2 2\theta_{13} = 0.089 \pm 0.010\ (\text{stat}) \pm 0.005\ (\text{syst}).$$

- Consistent with RENO, Double Chooz, and T2K experiments.

- Global fit of the neutrino oscillation:
  D. V. Forero, M. Tortola and J. W. F. Valle, arXiv:1205.4018 [hep-ph].

parameter	best fit	2σ	3σ
$\sin^2\theta_{12}$	0.320	0.29-0.35	0.27-0.37
$\sin^2\theta_{23}$	0.613 (0.427) 0.600	0.38-0.66 0.39-0.65	0.36-0.68 0.37-0.67
$\sin^2\theta_{13}$	0.0246 0.0250	0.019-0.030 0.020-0.030	0.017-0.033
$\Delta m^2_{\rm sol}$ $[10^{-5}{\rm eV}^2]$	7.62	7.27-8.01	7.12-8.20
$\left\|\Delta m^2_{\rm atm}\right\|$ $[10^{-3}{\rm eV}^2]$	2.55 2.43	2.38-2.68 2.29-2.58	2.31-2.74 2.21-2.64

- $\sin\theta_{13}$ is nearly Cabibbo angle:

$$\sin\theta_{13} = 0.15 \pm 0.01, \quad \sin\theta_C = 0.225 \equiv \lambda.$$

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- The relation between masses and flavor mixing angles:

$$\sqrt{\frac{m_d}{m_s}} \simeq 0.225 \simeq \sin\theta_C.$$

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- The relation between masses and flavor mixing angles:

$$\sqrt{\frac{m_d}{m_s}} \simeq 0.225 \simeq \sin\theta_C.$$

- Neutrino masses and flavor mixing angles are related each other!!

$$\sqrt[4]{\frac{\Delta m^2_{\text{sol}}}{\Delta m^2_{\text{atm}}}} = 0.416 \simeq \mathcal{O}(\sqrt{\lambda}) \quad \Rightarrow \quad \sin^2\theta_{23},$$

$$\sqrt{\frac{\Delta m^2_{\text{sol}}}{\Delta m^2_{\text{atm}}}} = 0.173 \simeq \frac{\lambda}{\sqrt{2}} \quad \Rightarrow \quad \sin\theta_{13}.$$

Our purpose

- Building the neutrino mass texture with large θ_{13}.

We present two textures.

- 3×3 Dirac neutrino mass texture.
  M. Fukugita, Y. S., M. Tanimoto and T. T. Yanagida,
  Phys. Lett. B **716** (2012) 294.
- 3×2 Dirac neutrino mass texture.
  Y. S., R. Takahashi, M. Tanimoto,
  arXiv:1212.5913 (accepted in Phys. Lett. B).

2. Large θ_{13} and neutrino mass matrix

- Texture with $\sin\theta_C = \sqrt{m_d/m_s}$: 2 parameters

  S. Weinberg, HUTP-77-A057, Trans.New York Acad.Sci.38:185-201, 1977.

$$M_d = \begin{pmatrix} 0 & A \\ A & B \end{pmatrix} = \begin{pmatrix} 0 & \sqrt{m_d m_s} \\ \sqrt{m_d m_s} & m_s \end{pmatrix}.$$

- Extend to 3 generations: 3 parameters

  H. Fritzsch, Phys. Lett. B73 (1978) 317; Nucl. Phys. B115 (1979) 189.

$$M_d = \begin{pmatrix} 0 & A_d & 0 \\ A_d & 0 & B_d \\ 0 & B_d & C_d \end{pmatrix}, \quad M_u = \begin{pmatrix} 0 & A_u & 0 \\ A_u & 0 & B_u \\ 0 & B_u & C_u \end{pmatrix},$$

$$V_{us} \simeq \sqrt{\frac{m_d}{m_s}} - \sqrt{\frac{m_u}{m_c}} \simeq 0.185, \quad V_{cb} \simeq \sqrt{\frac{m_s}{m_b}} - \sqrt{\frac{m_c}{m_t}} \simeq 0.065.$$

so good   too large

- Fritzsch texture does not work in quark sector...

Apply to lepton sector

M. Fukugita, M. Tanimoto, T. Yanagida, Prog. Theor. Phys. 89 (1993) 263.

- The charged lepton and Dirac neutrino mass matrices:

$$m_E = \begin{pmatrix} 0 & A_\ell & 0 \\ A_\ell & 0 & B_\ell \\ 0 & B_\ell & C_\ell \end{pmatrix}, \quad m_{\nu D} = \begin{pmatrix} 0 & A_\nu & 0 \\ A_\nu & 0 & B_\nu \\ 0 & B_\nu & C_\nu \end{pmatrix}.$$

- We assume that the right-handed Majorana neutrino mass matrix is proportional to unit one: $M_R = M_0 \mathbf{1}$.
- The left-handed Majorana neutrino mass eigenvalues:

$$m_i = \left(U_\nu^T m_{\nu D}^T M_R^{-1} m_{\nu D} U_\nu \right)_i .$$

- Pontecorvo-Maki-Nakagawa-Sakata(PMNS) mixing matrix:

$$U_{\text{PMNS}} = U_\ell^\dagger Q U_\nu, \quad Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & e^{i\tau} \end{pmatrix}.$$

The neutrino mixing matrix elements

$$U_\nu(1,1) = \sqrt{\frac{m_{2D} m_{3D}(m_{3D} - m_{2D})}{(m_{2D} + m_{1D})(m_{3D} - m_{2D} + m_{1D})(m_{3D} - m_{1D})}},$$

$$U_\nu(1,2) = -\sqrt{\frac{m_{1D} m_{3D}(m_{3D} + m_{1D})}{(m_{2D} + m_{1D})(m_{3D} - m_{2D} + m_{1D})(m_{3D} + m_{2D})}} \simeq \sqrt{\frac{m_{1D}}{m_{2D}}} = \sqrt[4]{\frac{m_1}{m_2}},$$

$$U_\nu(1,3) = \sqrt{\frac{m_{1D} m_{2D}(m_{2D} - m_{1D})}{(m_{3D} - m_{1D})(m_{3D} - m_{2D} + m_{1D})(m_{3D} + m_{2D})}} \simeq \frac{m_{2D}}{m_{3D}}\sqrt{\frac{m_{1D}}{m_{3D}}} = \sqrt{\frac{m_2}{m_3}}\sqrt[4]{\frac{m_1}{m_3}},$$

$$U_\nu(2,1) = \sqrt{\frac{m_{1D}(m_{3D} - m_{2D})}{(m_{2D} + m_{1D})(m_{3D} - m_{1D})}},$$

$\Uparrow$

$$U_\nu(2,2) = \sqrt{\frac{m_{2D}(m_{3D} + m_{1D})}{(m_{2D} + m_{1D})(m_{3D} + m_{2D})}},$$

Because of seesaw mechanism.

$$U_\nu(2,3) = \sqrt{\frac{m_{3D}(m_{2D} - m_{1D})}{(m_{3D} + m_{2D})(m_{3D} - m_{1D})}} \simeq \sqrt{\frac{m_{2D}}{m_{3D}}} = \sqrt[4]{\frac{m_2}{m_3}},$$

$$U_\nu(3,1) = -\sqrt{\frac{m_{1D}(m_{2D} - m_{1D})(m_{3D} + m_{1D})}{(m_{3D} - m_{1D})(m_{3D} - m_{2D} + m_{1D})(m_{2D} + m_{1D})}},$$

$$U_\nu(3,2) = -\sqrt{\frac{m_{2D}(m_{2D} - m_{1D})(m_{3D} - m_{2D})}{(m_{3D} + m_{2D})(m_{3D} - m_{2D} + m_{1D})(m_{2D} + m_{1D})}},$$

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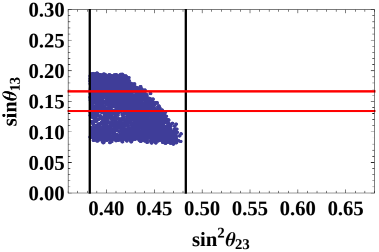
Lepton mixing matrix elements

M. Fukugita, Y. S., M. Tanimoto and T. T. Yanagida, Phys. Lett. B **716** (2012) 294.

- Free parameter: $m_1,\ \sigma,\ \tau$

$$U_{e2} \simeq -\left(\frac{m_1}{m_2}\right)^{1/4} + \left(\frac{m_e}{m_\mu}\right)^{1/2} e^{i\sigma},$$
$$U_{\mu 3} \simeq \left(\frac{m_2}{m_3}\right)^{1/4} e^{i\sigma} - \left(\frac{m_\mu}{m_\tau}\right)^{1/2} e^{i\tau},$$
$$U_{e3} \simeq \left(\frac{m_e}{m_\mu}\right)^{1/2} U_{\mu 3} + \left(\frac{m_2}{m_3}\right)^{1/2} \left(\frac{m_1}{m_3}\right)^{1/4}.$$



- Charged lepton contribution:

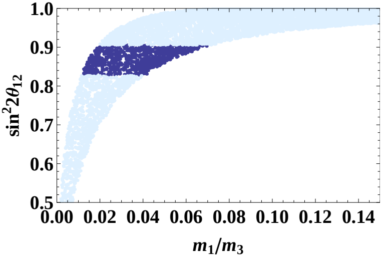
$$\left(\frac{m_e}{m_\mu}\right)^{1/2} \simeq 0.0695,$$
$$\left(\frac{m_\mu}{m_\tau}\right)^{1/2} \simeq 0.244.$$

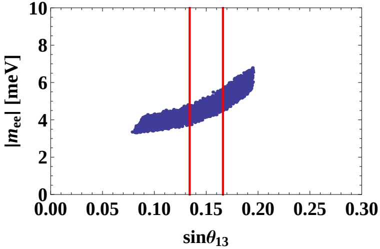
$$\sin^2\theta_{23} \simeq \sqrt[4]{\frac{\Delta m^2_{\text{sol}}}{\Delta m^2_{\text{atm}}}} = 0.416 \simeq \mathcal{O}(\sqrt{\lambda}),$$
$$\sin\theta_{13} \simeq (\sin\theta_{23})^3 \sin\theta_{12} \simeq 0.158.$$

- Neutrino mass hierarchy: Normal hierarchy

Neutrino less double beta decay

- m_1 is restricted by θ_{12}:

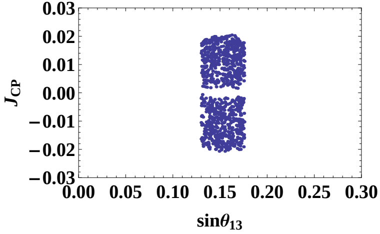




$$|m_{ee}| = 4 - 5.5 \text{ meV}.$$

CP violation in lepton sector

$$J_{CP} = \text{Im}\left[U_{\mu 3} U^*_{\tau 3} U^*_{\mu 2} U_{\tau 2}\right].$$



If θ_{12}, θ_{23}, θ_{13} are more precise, σ and τ are more restricted and J_{CP} is more predictive.

3. Toward minimal texture

- Before reactor experiments were reported θ_{13} ($|U_{e3}| \equiv \sin\theta_{13}$), the tri-bimaximal mixing (TBM) $V_{\text{tri-bi}}$ was good scheme.

$$U_{\text{PMNS}} = V_{\text{tri-bi}} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix},$$

$$|U_{e2}| = \frac{1}{\sqrt{3}}, \quad |U_{e3}| = 0, \quad |U_{\mu 3}| = \frac{1}{\sqrt{2}}.$$

- The left-handed Majorana neutrino mass matrix:

$$M_\nu^{\text{TBM}} = \frac{m_1+m_3}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{m_2-m_1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_1-m_3}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

- TBM is realized by non-Abelian discrete group.
  H. Ishimori, T. Kobayashi, H. Okada, Y. S., and M. Tanimoto, JHEP **0904** (2009) 011; Lect. Notes Phys. **858** (2012) 1.

We consider deviation from tri-bimaximal mixing

We discuss three cases as 1-2, 1-3, 2-3 mixing deviation from tri-bimaximal one.

W. Rodejohann and H. Zhang, Phys. Rev. D **86** (2012) 093008.
A. Damanik, arXiv:1206.0987 [hep-ph].

- Case I: 1-2 mixing deviation from tri-bimaximal one.

$$U_{\text{PMNS}} = V_{\text{tri-bi}} \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$|U_{e2}| = \left| \frac{2\sin\phi}{\sqrt{6}} + \frac{\cos\phi}{\sqrt{3}} \right|, \quad |U_{e3}| = 0, \quad |U_{\mu 3}| = \frac{1}{\sqrt{2}}.$$

$|U_{e3}| = 0$, then unfavored.

We consider deviation from tri-bimaximal mixing

- Case II: 1-3 mixing deviation from tri-bimaximal one.

$$U_{\text{PMNS}} = V_{\text{tri-bi}} \begin{pmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{pmatrix},$$

$$|U_{e2}| = \frac{1}{\sqrt{3}}\,, \quad |U_{e3}| = \left|\frac{2\sin\phi}{\sqrt{6}}\right|, \quad |U_{\mu 3}| = \left|-\frac{\sin\phi}{\sqrt{6}} + \frac{\cos\phi}{\sqrt{2}}\right| .$$

- Adding the $(1,3)$ or $(1,2)$ off diagonal matrix:

$$M_\nu = M_\nu^{\text{TBM}} + \left[\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right].$$

- Realized by A_4 symmetry $(\mathbf{1}', \mathbf{1}'')$ and S_4 symmetry $(\mathbf{2})$ and ...

  Y. S., M. Tanimoto and A. Watanabe, Prog. Theor. Phys. **126** (2011) 81.

- In this case, $\sin^2\theta_{12} > \frac{1}{3}$, which is unfavored.

We consider deviation from tri-bimaximal mixing

- Case Ⅲ: 2-3 mixing deviation from tri-bimaximal one.

$$U_{\text{PMNS}} = V_{\text{tri-bi}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{pmatrix},$$

$$|U_{e2}| = \left|\frac{\cos\phi}{\sqrt{3}}\right|, \quad |U_{e3}| = \left|\frac{\sin\phi}{\sqrt{3}}\right|, \quad |U_{\mu 3}| = \left|\frac{\cos\phi}{\sqrt{2}} + \frac{\sin\phi}{\sqrt{3}}\right| .$$

- In this case, $\sin^2\theta_{12} < \frac{1}{3}$.
- We consider the framework of Split Seesaw:
  A. Kusenko, F. Takahashi and T. T. Yanagida, Phys. Lett. B **693** (2010) 144.

$$M_{R1} \sim \mathcal{O}(\text{keV}) \ll M_{R2},\ M_{R3} \sim \mathcal{O}(10^{12}\ \text{GeV}),$$
$$Y^D_{1i} \ll Y^D_{2i},\ Y^D_{3i}.$$

- M_{R1} is the sterile neutrino: Dark matter candidate.
- Realized in 5D theory compactified on S^1/Z_2.

- We can separate the neutrino mass matrix:

$$M_R^{3\times3} = \begin{pmatrix} M_{R1}^{1\times1} & 0 \\ 0 & M_R^{2\times2} \end{pmatrix}, \quad M_D = \begin{pmatrix} Y_{3\times1}^D & Y_{3\times2}^D \end{pmatrix} v.$$

- By using seesaw mechanism:

$$M_\nu = Y_{3\times2}^D \left(M_R^{2\times2}\right)^{-1} (Y^D)_{2\times3}^T v^2 + \sum_i Y_{1i}^D M_{R1}^{-1} (Y^D)_{i1}^T v^2.$$

$\Downarrow$ Flavor mixing

$\Downarrow$ No effect on flavor mixing

- We can consider "Minimal Texture".

- The right-handed Majorana and Dirac neutrino mass matrices:

$$M_R^{2\times 2} = \begin{pmatrix} M_{R2} & 0 \\ 0 & M_{R3} \end{pmatrix}, \quad M_D = Y^D_{3\times 2} v = \begin{pmatrix} a & d \\ b & e \\ c & f \end{pmatrix} v.$$

- The condition of 2-3 mixing deviation (normal hierarchy):

$$2a - b - c = 0, \quad \text{and} \quad 2d - e - f = 0.$$

- In this condition, left-handed Majorana neutrino mass matrix:

$$M_\nu = V_{\text{TBM}}^T \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{3}{4}\left(\frac{(b+c)^2}{M_{R2}} + \frac{(e+f)^2}{M_{R3}}\right) & \frac{\sqrt{\frac{3}{2}}((b-c)(b+c)M_{R3}+(e-f)(e+f)M_{R2})}{2M_{R2}M_{R3}} \\ 0 & \frac{\sqrt{\frac{3}{2}}((b-c)(b+c)M_{R3}+(e-f)(e+f)M_{R2})}{2M_{R2}M_{R3}} & \frac{(b-c)^2 M_{R3}+(e-f)^2 M_{R2}}{2M_{R2}M_{R3}} \end{pmatrix} v^2 V_{\text{TBM}}.$$

- In this case, Majorana mass can rescale, then the right-handed Majorana and Dirac neutrino mass matrices:

$$M_R^{2\times 2} = M_R \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad M_D = \begin{pmatrix} \frac{b+c}{2} & \frac{e+f}{2} \\ b & e \\ c & f \end{pmatrix} v.$$

Two zero texture in normal hierarchy

- The general texture:

$$M_D = \begin{pmatrix} \frac{b+c}{2} & \frac{e+f}{2} \\ b & e \\ c & f \end{pmatrix} v.$$

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- (i) $b + c = 0$ and $f = 0$:

$$M_D = \begin{pmatrix} 0 & \frac{e}{2} \\ b & e \\ -b & 0 \end{pmatrix} v \to \begin{pmatrix} 0 & \frac{e}{2} \\ \frac{1}{\sqrt{2}} & e \\ -\frac{1}{\sqrt{2}} & 0 \end{pmatrix} v.$$

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- (ii) $b + c = 0$ and $e = 0$:

$$M_D = \begin{pmatrix} 0 & \frac{f}{2} \\ b & 0 \\ -b & f \end{pmatrix} v \to \begin{pmatrix} 0 & \frac{f}{2} \\ \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & f \end{pmatrix} v.$$

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- (iii) $c = 0$ and $e = 0$:

$$M_D = \begin{pmatrix} \frac{b}{2} & \frac{f}{2} \\ b & 0 \\ 0 & f \end{pmatrix} v \to \begin{pmatrix} \frac{1}{2} & \frac{f}{2} \\ 1 & 0 \\ 0 & f \end{pmatrix} v.$$

Two zero texture in normal hierarchy

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$$M_D = \begin{pmatrix} \frac{b+c}{2} & \frac{e+f}{2} \\ b & e \\ c & f \end{pmatrix} v.$$

- (i) $b + c = 0$ and $f = 0$:

$$M_D = \begin{pmatrix} 0 & \frac{e}{2} \\ b & e \\ -b & 0 \end{pmatrix} v \to \begin{pmatrix} 0 & \frac{e}{2} \\ \frac{1}{\sqrt{2}} & e \\ -\frac{1}{\sqrt{2}} & 0 \end{pmatrix} v.$$

- (ii) $b + c = 0$ and $e = 0$:

$$M_D = \begin{pmatrix} 0 & \frac{f}{2} \\ b & 0 \\ -b & f \end{pmatrix} v \to \begin{pmatrix} 0 & \frac{f}{2} \\ \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & f \end{pmatrix} v.$$

- (iii) $c = 0$ and $e = 0$:

$$M_D = \begin{pmatrix} \frac{b}{2} & \frac{f}{2} \\ b & 0 \\ 0 & f \end{pmatrix} v \to \begin{pmatrix} \frac{1}{2} & \frac{f}{2} \\ 1 & 0 \\ 0 & f \end{pmatrix} v.$$

- In the phenomenology, (i) is marginal but (ii) and (iii) are unfavored.

- In case (i), the left-handed Majorana neutrino mass matrix:

$$M_\nu = \frac{v^2}{M_R} V_{\text{TBM}}^T \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{3}{4}e^2 & -\frac{1}{2}\sqrt{\frac{3}{2}}e^2 \\ 0 & -\frac{1}{2}\sqrt{\frac{3}{2}}e^2 & 1+\frac{1}{2}e^2 \end{pmatrix} V_{\text{TBM}}.$$

- It's too large to be consistent with the experimental data if $e \sim \mathcal{O}(1)$. Therefore, we should take as $e \ll 1$. The neutrino mass eigenvalues are

$$m_1 = 0, \quad \frac{m_2}{m_3} \simeq \frac{3}{4}e^2 \equiv r.$$

- The additional 2-3 mixing angle is

$$\tan(2\phi) \simeq -\sqrt{\frac{3}{2}}e^2.$$

- The relevant mixing matrix elements:

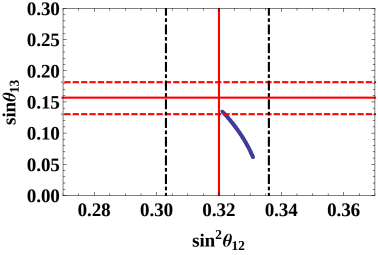
$$|U_{e2}| \simeq \frac{1}{\sqrt{3}}\sqrt{1-\frac{2}{3}r^2}, \quad |U_{e3}| \simeq \left|-\frac{\sqrt{2}}{3}r\right|, \quad |U_{\mu 3}| \simeq \left|-\frac{\sqrt{2}}{3}r + \frac{1}{\sqrt{2}}\sqrt{1-\frac{2}{3}r^2}\right|.$$

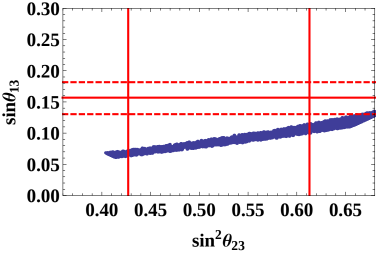
Numerical results

- (i) $b + c = 0$ and $f = 0$:

$$M_D = \begin{pmatrix} 0 & \frac{e}{2} \\ b & e \\ -b & 0 \end{pmatrix} v \to \begin{pmatrix} 0 & \frac{e}{2} \\ \frac{1}{\sqrt{2}} & e \\ -\frac{1}{\sqrt{2}} & 0 \end{pmatrix} v.$$

- "e" is fitted by neutrino mass ratio $\Delta m^2_{\mathrm{sol}}/\Delta m^2_{\mathrm{atm}}$.





- In this case, θ_{13} is marginal.

- In case (ii), the left-handed Majorana neutrino mass matrix:

$$M_\nu = \frac{v^2}{M_R} V_{\text{TBM}}^T \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{3}{4} f^2 & \frac{1}{2}\sqrt{\frac{3}{2}} f^2 \\ 0 & \frac{1}{2}\sqrt{\frac{3}{2}} f^2 & 1 + \frac{1}{2} f^2 \end{pmatrix} V_{\text{TBM}}.$$

- It's too large to be consistent with the experimental data if $f \sim \mathcal{O}(1)$, and thus we must take $f \ll 1$ as well as $e \ll 1$ in the previous case (i).
- Neutrino mass eigenvalues and additional mixing angle:

$$m_1 = 0, \quad \frac{m_2}{m_3} \simeq \frac{3}{4} f^2 \equiv r, \quad \tan(2\phi) \simeq \sqrt{\frac{3}{2}} f^2.$$

- The relevant mixing matrix elements:

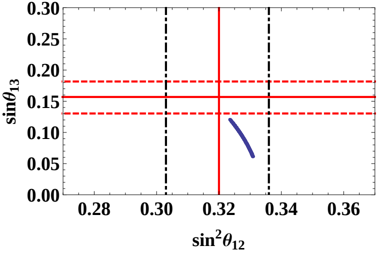
$$|U_{e2}| \simeq \frac{1}{\sqrt{3}} \sqrt{1 - \frac{2}{3} r^2}, \quad |U_{e3}| \simeq \frac{\sqrt{2}}{3} r, \quad |U_{\mu 3}| \simeq \left| \frac{\sqrt{2}}{3} r + \frac{1}{\sqrt{2}} \sqrt{1 - \frac{2}{3} r^2} \right| .$$

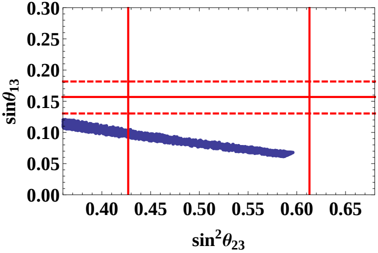
Numerical results

- (ii) $b + c = 0$ and $e = 0$:

$$M_D = \begin{pmatrix} 0 & \frac{f}{2} \\ b & 0 \\ -b & f \end{pmatrix} v \to \begin{pmatrix} 0 & \frac{f}{2} \\ \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & f \end{pmatrix} v.$$

- "f" is fitted by neutrino mass ratio $\Delta m^2_{\rm sol}/\Delta m^2_{\rm atm}$





- In this case, θ_{13} is unfavored.

- In case (iii), the left-handed Majorana neutrino mass matrix:

$$M_\nu = \frac{v2}{M_R} V_{\rm TBM}^T \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{3}{4}(f^2+1) & \frac{1}{2}\sqrt{\frac{3}{2}}(f^2-1) \\ 0 & \frac{1}{2}\sqrt{\frac{3}{2}}(f^2-1) & \frac{1}{2}(f^2+1) \end{pmatrix} V_{\rm TBM}.$$

- Since the neutrino mass eigenvalues can be obtained by

$$m_1 = 0, \qquad \frac{m_2}{m_3} = \frac{5+5f^2-\sqrt{25-46f^2+25f^4}}{5+5f^2+\sqrt{25-46f^2+25f^4}} \equiv r,$$

the parameter f is evaluated as

$$f^2 \simeq \frac{25}{24} r \quad \text{or} \quad \frac{24}{25r}.$$

- The additional mixing angle is too large.

$$\tan(2\phi) = \frac{2\sqrt{6}(1-f^2)}{1+f^2} \simeq -2\sqrt{6} \quad \text{or} \quad 2\sqrt{6}.$$

One zero texture in normal hierarchy

- The general texture:

$$M_D = \begin{pmatrix} \frac{b+c}{2} & \frac{e+f}{2} \\ b & e \\ c & f \end{pmatrix} v.$$

One zero texture in normal hierarchy

- The general texture:

$$M_D = \begin{pmatrix} \frac{b+c}{2} & \frac{e+f}{2} \\ b & e \\ c & f \end{pmatrix} v.$$

- (1) $b + c = 0$:

$$M_D = \begin{pmatrix} 0 & \frac{e+f}{2} \\ b & e \\ -b & f \end{pmatrix} v \to \begin{pmatrix} 0 & \frac{e+f}{2} \\ \frac{1}{\sqrt{2}} & e \\ -\frac{1}{\sqrt{2}} & f \end{pmatrix} v.$$

One zero texture in normal hierarchy

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- (2) $c = 0$:

$$M_D = \begin{pmatrix} \frac{b}{2} & \frac{e+f}{2} \\ b & e \\ 0 & f \end{pmatrix} v \to \begin{pmatrix} \frac{1}{2} & \frac{e+f}{2} \\ 1 & e \\ 0 & f \end{pmatrix} v.$$

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- The general texture:

$$M_D = \begin{pmatrix} \frac{b+c}{2} & \frac{e+f}{2} \\ b & e \\ c & f \end{pmatrix} v.$$

- (1) $b + c = 0$:

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- (2) $c = 0$:

$$M_D = \begin{pmatrix} \frac{b}{2} & \frac{e+f}{2} \\ b & e \\ 0 & f \end{pmatrix} v \to \begin{pmatrix} \frac{1}{2} & \frac{e+f}{2} \\ 1 & e \\ 0 & f \end{pmatrix} v.$$

- (3) $b = 0$:

$$M_D = \begin{pmatrix} \frac{c}{2} & \frac{e+f}{2} \\ 0 & e \\ c & f \end{pmatrix} v \to \begin{pmatrix} \frac{1}{2} & \frac{e+f}{2} \\ 0 & e \\ 1 & f \end{pmatrix} v.$$

One zero texture in normal hierarchy

- The general texture:

$$M_D = \begin{pmatrix} \frac{b+c}{2} & \frac{e+f}{2} \\ b & e \\ c & f \end{pmatrix} v.$$

- (1) $b + c = 0$:

$$M_D = \begin{pmatrix} 0 & \frac{e+f}{2} \\ b & e \\ -b & f \end{pmatrix} v \to \begin{pmatrix} 0 & \frac{e+f}{2} \\ \frac{1}{\sqrt{2}} & e \\ -\frac{1}{\sqrt{2}} & f \end{pmatrix} v.$$

- (2) $c = 0$:

$$M_D = \begin{pmatrix} \frac{b}{2} & \frac{e+f}{2} \\ b & e \\ 0 & f \end{pmatrix} v \to \begin{pmatrix} \frac{1}{2} & \frac{e+f}{2} \\ 1 & e \\ 0 & f \end{pmatrix} v.$$

- (3) $b = 0$:

$$M_D = \begin{pmatrix} \frac{c}{2} & \frac{e+f}{2} \\ 0 & e \\ c & f \end{pmatrix} v \to \begin{pmatrix} \frac{1}{2} & \frac{e+f}{2} \\ 0 & e \\ 1 & f \end{pmatrix} v.$$

- We focus on (1) texture.

- (1) $b + c = 0$: $M_R^{2\times 2} = M_R \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\quad M_D = \begin{pmatrix} 0 & \frac{e+f}{2} \\ \frac{1}{\sqrt{2}} & e \\ -\frac{1}{\sqrt{2}} & f \end{pmatrix} v.$

- The left-handed Majorana neutrino mass matrix:

$$M_\nu = M_D (M_R^{2\times 2})^{-1} M_D^T = \begin{pmatrix} \frac{1}{4}(e+f)^2 & \frac{1}{2}e(e+f) & \frac{1}{2}(e+f)f \\ \frac{1}{2}e(e+f) & \frac{1}{2}+e^2 & -\frac{1}{2}+ef \\ \frac{1}{2}(e+f)f & -\frac{1}{2}+ef & \frac{1}{2}+f^2 \end{pmatrix} \frac{v^2}{M_R}.$$

- Rotating tri-bimaximal mixing matrix V_{TBM}:

$$M_\nu = V_{\text{TBM}}^T \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{3}{4}(e+f)^2 & \frac{1}{2}\sqrt{\frac{3}{2}}(e-f)(e+f) \\ 0 & \frac{1}{2}\sqrt{\frac{3}{2}}(e-f)(e+f) & 1+\frac{1}{2}(e-f)^2 \end{pmatrix} \frac{v^2}{M_R} V_{\text{TBM}},$$

$$V_{\text{TBM}} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}.$$

- The neutrino mass eigenvalues: $\frac{m_2}{m_3} \simeq \frac{3}{4}(e+f)^2 \equiv r.$

- The lepton mixing:

$$U_{\rm PMNS} = V_{\rm TBM} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{pmatrix}, \quad \tan 2\phi \simeq \sqrt{\frac{3}{2}}(e-f)(e+f) \equiv \sqrt{6}\lambda.$$

- Reparametrization including phases:

$$e = \frac{2re^{2i\alpha} - 3\lambda e^{i\delta}}{2\sqrt{3re^{2i\alpha}}}, \quad f = \frac{2re^{2i\alpha} + 3\lambda e^{i\delta}}{2\sqrt{3re^{2i\alpha}}}.$$

- The relevant mixing matrix elements:

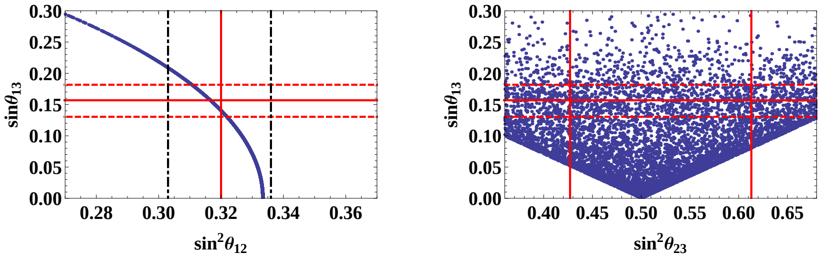
$$|U_{e2}| = \left|\frac{\cos\phi}{\sqrt{3}}\right| \simeq \sqrt{\frac{1}{3} - \frac{\lambda^2}{2}}, \quad |U_{e3}| = \left|\frac{\sin\phi}{\sqrt{3}}\right| \simeq \frac{\lambda}{\sqrt{2}},$$

$$|U_{\mu 3}| = \left|\frac{\cos\phi}{\sqrt{2}} + \frac{\sin\phi}{\sqrt{3}}\right| \simeq \left|\sqrt{\frac{1}{2} - \frac{3\lambda^2}{4}} + \frac{\lambda}{\sqrt{2}}\right|.$$

Numerical analysis

We take the parameters:

$$0.1\sqrt{\frac{\Delta m^2_{\rm sol}}{\Delta m^2_{\rm atm}}} < r < 2\sqrt{\frac{\Delta m^2_{\rm sol}}{\Delta m^2_{\rm atm}}}, \quad -0.4 < \lambda < 0.4, \quad -\frac{\pi}{2} < \alpha < \frac{\pi}{2}, \quad -\pi < \delta < \pi.$$



$$\Delta m^2_{\rm atm} = (2.55^{+0.19}_{-0.24}) \times 10^{-3}\ {\rm eV}^2, \quad \Delta m^2_{\rm sol} = (7.62^{+0.58}_{-0.50}) \times 10^{-5}\ {\rm eV}^2,$$

$$\sin^2\theta_{12} = 0.320 \pm 0.05, \quad \sin^2\theta_{23} = 0.427\ (0.613)\ 0.36\text{–}0.68, \quad \sin^2\theta_{13} = 0.0246^{+0.0084}_{-0.0076}\ .$$

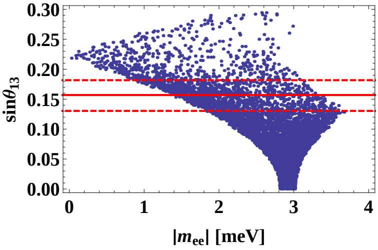
$\nu 0\beta\beta$ and CP violation

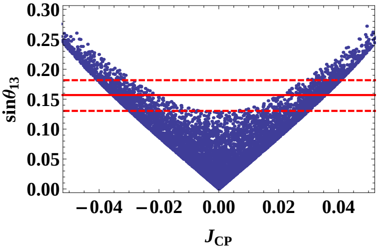
- $\nu 0\beta\beta$:

$$|m_{ee}| = \sum_i^3 \left| m_i U_{ei}^2 \right| .$$

- CP violation:

$$J_{CP} = \mathrm{Im}\left[U_{\mu 3} U_{\tau 3}^* U_{\mu 2}^* U_{\tau 2} \right] .$$





Inverted hierarchy

- The condition of 2-3 deviation:

$$a+b+c=0, \quad b=c, \quad \text{and} \quad 2d-e-f=0.$$

- The left-handed Majorana neutrino mass matrix:

$$M_\nu = V_{\text{TBM}}^T \begin{pmatrix} 6b^2 & 0 & 0 \\ 0 & \frac{3}{4}(e+f)^2 & \frac{1}{2}\sqrt{\frac{3}{2}}(e-f)(e+f) \\ 0 & \frac{1}{2}\sqrt{\frac{3}{2}}(e-f)(e+f) & \frac{1}{2}(e-f)^2 \end{pmatrix} \frac{v^2}{M_R} V_{\text{TBM}}.$$

- The right-handed Majorana and Dirac neutrino mass matrices:

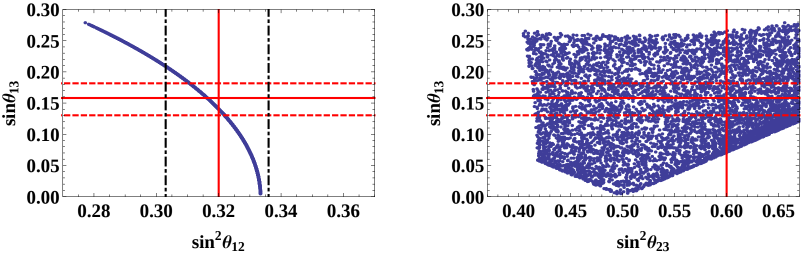
$$M_R^{2\times 2} = M_R \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad M_D = \begin{pmatrix} -2b & \frac{e+f}{2} \\ b & e \\ b & f \end{pmatrix} v \quad \to \quad \begin{pmatrix} -2 & \frac{e+f}{2} \\ 1 & e \\ 1 & f \end{pmatrix} v.$$

- Neutrino mass eigenvalues:

$$m_3 = 0, \quad \frac{m_2}{m_1} = \frac{1}{24}\left(5e^2 + 2ef + 5f^2\right) \equiv r'.$$

- The additional 2-3 mixing angle ϕ:

$$\tan 2\phi = \sqrt{\frac{2}{3}}\left(\frac{-e+f}{e+f}\right).$$



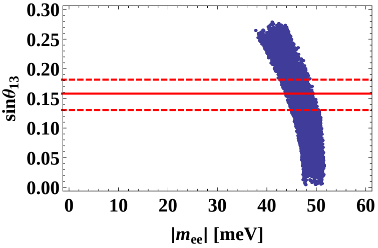
$\nu 0\beta\beta$ and CP violation

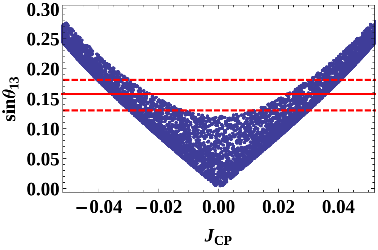
- $\nu 0\beta\beta$:

$$|m_{ee}| = \sum_i^3 |m_i U_{ei}^2| .$$

- CP violation:

$$J_{CP} = \text{Im}\left[U_{\mu 3} U_{\tau 3}^* U_{\mu 2}^* U_{\tau 2}\right] .$$





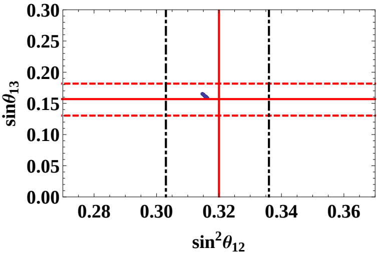
One zero texture with the Cabibbo angle

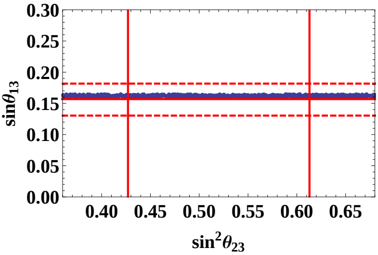
- The one zero texture (normal hierarchy) (1) $b + c = 0$:

$$M_R^{2\times 2} = M_R \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad M_D = \begin{pmatrix} 0 & \frac{e+f}{2} \\ \frac{1}{\sqrt{2}} & e \\ -\frac{1}{\sqrt{2}} & f \end{pmatrix} v,$$

$$e = \frac{2re^{2i\alpha} - 3\lambda e^{i\delta}}{2\sqrt{3re^{2i\alpha}}}, \quad f = \frac{2re^{2i\alpha} + 3\lambda e^{i\delta}}{2\sqrt{3re^{2i\alpha}}}.$$

- We take λ the Cabibbo angle ($\lambda = 0.225$).





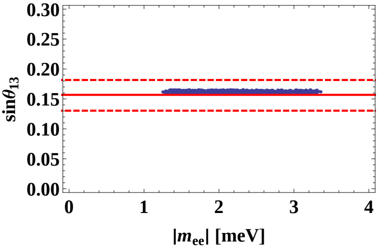
$\nu 0\beta\beta$ and CP violation

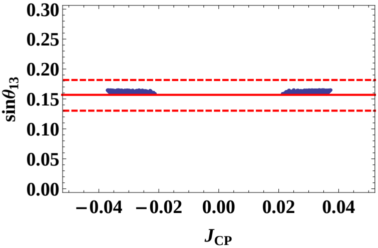
- $\nu 0\beta\beta$:

$$|m_{ee}| = \sum_i^3 |m_i U_{ei}^2| .$$

- CP violation:

$$J_{CP} = \mathrm{Im}\left[U_{\mu 3} U_{\tau 3}^* U_{\mu 2}^* U_{\tau 2}\right] .$$





The Cabibbo angle and neutrino mass ratio

- The one zero texture (normal hierarchy) (1) $b + c = 0$:

$$M_R^{2\times 2} = M_R \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad M_D = \begin{pmatrix} 0 & \frac{e+f}{2} \\ \frac{1}{\sqrt{2}} & e \\ -\frac{1}{\sqrt{2}} & f \end{pmatrix} v,$$

$$e = \frac{2re^{2i\alpha} - 3\lambda e^{i\delta}}{2\sqrt{3re^{2i\alpha}}}, \quad f = \frac{2re^{2i\alpha} + 3\lambda e^{i\delta}}{2\sqrt{3re^{2i\alpha}}}.$$

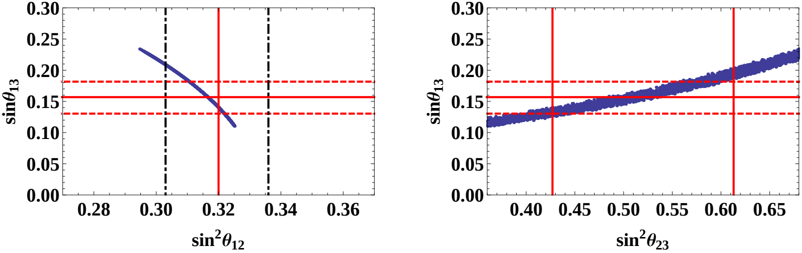
- $\lambda/\sqrt{2} = r$, $\delta = 2\alpha$:

$$e = \frac{(2 - 3\sqrt{2})\sqrt{r}e^{i\alpha}}{2\sqrt{3}}, \quad f = \frac{(2 + 3\sqrt{2})\sqrt{r}e^{i\alpha}}{2\sqrt{3}}.$$

Numerical result

- Neutrino Texture:

$$M_R^{2\times 2} = M_R \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad M_D = \begin{pmatrix} 0 & \frac{1}{\sqrt{3}}\sqrt{r}e^{i\alpha} \\ \frac{1}{\sqrt{2}} & \frac{2-3\sqrt{2}}{2\sqrt{3}}\sqrt{r}e^{i\alpha} \\ -\frac{1}{\sqrt{2}} & \frac{2+3\sqrt{2}}{2\sqrt{3}}\sqrt{r}e^{i\alpha} \end{pmatrix} v.$$



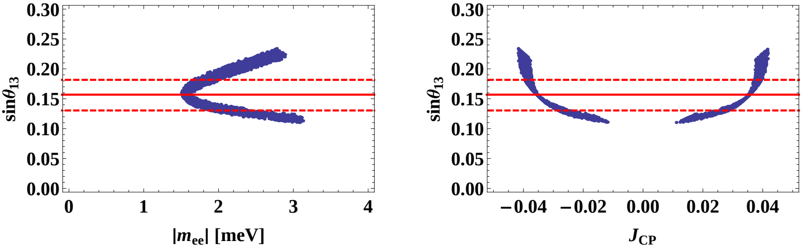
$\nu 0\beta\beta$ and CP violation

- $\nu 0\beta\beta$:

$$|m_{ee}| = \sum_i^3 \left| m_i U_{ei}^2 \right| .$$

- CP violation:

$$J_{CP} = \mathrm{Im}\left[U_{\mu 3} U_{\tau 3}^* U_{\mu 2}^* U_{\tau 2} \right] .$$



4. Summary

Conclusion

- The large θ_{13} is given impact for us.
- We propose minimal texture which makes the connection between masses and mixing angles.

$$r = \sqrt{\frac{\Delta m^2_{\text{sol}}}{\Delta m^2_{\text{atm}}}} = \frac{\lambda}{\sqrt{2}} = \sin\theta_{13}.$$

- These textures are motivated for model building.

We have presented the texture realized large θ_{13}!!

$$M_R^{2\times 2} = M_R \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad M_D = \begin{pmatrix} 0 & \frac{e+f}{2} \\ \frac{1}{\sqrt{2}} & e \\ -\frac{1}{\sqrt{2}} & f \end{pmatrix}.$$

Future works

- We want to build the concrete model not only lepton sector but also quark sector.
  (e.g. non-Abelian discrete symmetry A_4, S_4, $\Delta(54)$ ...)
- Please discuss with me the flavor physics.
  (e.g. lepton flavor, CP violation in B mesons ...)

Danke Schön!!