Patterns of Remnant Discrete Symmetries

Roland Schieren

in collaboration with

Rolf Kappl, Patrick Vaudrevange, Michael Ratz and Björn Petersen

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General considerations

- A general finite, abelian group is always of the form
  (fundamental theorem of finite abelian groups)

$$\mathbb{Z}_{d_1} \times \ldots \times \mathbb{Z}_{d_N}$$

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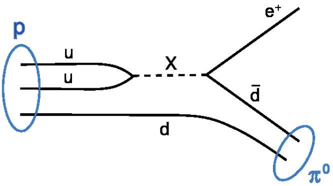
  - Example: matter parity to suppress proton decay ~~UDD~~



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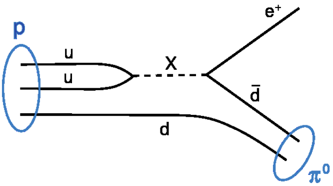

- abelian, discrete groups have only one-dimensional irreducible representations

  ↷ cannot explain mixing angles, etc.

Origin of abelian discrete symmetries

Arguments against imposing discrete symmetries:

- their breaking leads to domain walls — Zeldovich et.al (1974)

  $\curvearrowright$ solutions:
  - symmetry is not exact
  - break before inflation
  - embedd in gauge symmetry
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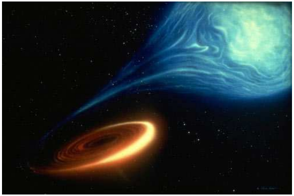
- quantum gravity is expected to violate global symmetries
  (black hole only carry gauged charges) — L. Krauss and F. Wilczek (1989)



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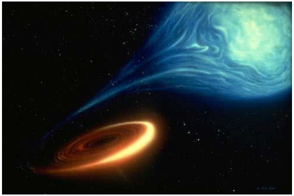

- in string theory all symmetries must be gauged — Polchinski

Review: $\mathrm{U}(1) \to \mathbb{Z}_N$

	ϕ	ψ
$\mathrm{U}(1)$	3	1

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gets a VEV (ϕ)

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Calculation

1. usual ansatz:

$$e^{\mathrm{i}\,3\,\alpha(x)}\phi = \phi$$

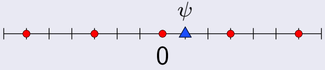
$$\Rightarrow \quad \alpha = 2\pi\frac{n}{3}$$

2. transformation of ψ:

$$\psi \to e^{2\pi\mathrm{i}\,\frac{n}{3}}\psi$$

Visualization

1. ϕ defines a lattice



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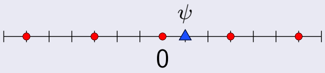

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Unbroken symmetry: $\mathbb{Z}_3$

The general case: Breaking $\mathrm{U}(1)^N$

- Notation: VEV fields $\phi^{(i)}$ with $\mathrm{U}(1)_j$ charge $q_j(\phi^{(i)})$ and other fields $\psi^{(k)}$

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- charges of the $\psi^{(i)}$: $\quad Q'_\psi = Q_\psi\,B$

A two-dimensional example

	$\mathrm{U}(1)_1$	$\mathrm{U}(1)_2$
$\phi^{(1)}$	8	-2
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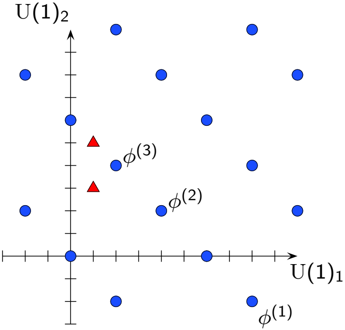

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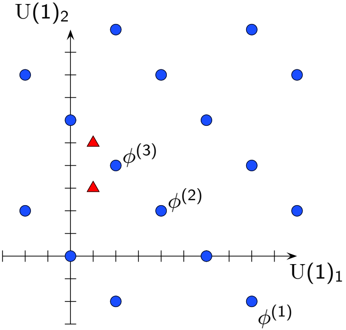

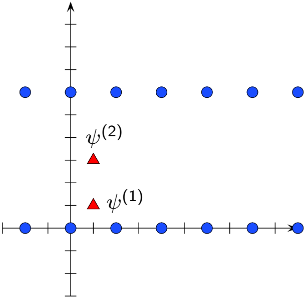

Redundancies

A trivial example

- $\mathbb{Z}_6$ and a field ψ having charge 4:

$$\psi \longrightarrow e^{2\pi i\, 4\, \frac{n}{6}}\psi = e^{2\pi i\, 2\, \frac{n}{3}}\psi$$

- One cannot perform all symmetry transformations. $n = 0, 1, 2$
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For a general group $\mathbb{Z}_{d_1} \times \ldots \times \mathbb{Z}_{d_N}$ there are two cases:

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Complication

for a general group there a very many equivalent charge assigments
(corresponds to the automorphisms of the group)
e.g. $\mathbb{Z}_2 \times \mathbb{Z}_4 \times \mathbb{Z}_4$ has 1536 equivalent charge assigments

Two Examples

Case 1

	$\mathbb{Z}_4$	$\mathbb{Z}_8$
$\psi^{(1)}$	2	4
$\psi^{(2)}$	3	3

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All three tables show physically equivalent symmetry groups

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Case 2

	$\mathbb{Z}_2$	$\mathbb{Z}_6$
$\psi^{(1)}$	1	1
$\psi^{(2)}$	1	3

	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_3$
$\psi^{(1)}$	1	1	2
$\psi^{(2)}$	1	1	0

	$\mathbb{Z}_6$
$\psi^{(1)}$	1
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Generalization

So far: $\mathrm{U}(1)^N \longrightarrow \mathbb{Z}_{d_1} \times \ldots \times \mathbb{Z}_{d_N}$

- Mixed cases: $\mathbb{Z}_d \times \mathrm{U}(1) \longrightarrow \mathbb{Z}_{d_1} \times \mathbb{Z}_{d_2}$
  Write as $\mathrm{U}(1) \times \mathrm{U}(1)$ and introduce dummy field ϕ with charge $(d, 0)$
- R-symmetry breaking: Introduce dummy field Ω which has the same charge as the superpotential

Discrete Anomalies

$\mathbb{Z}_\mathbf{N}$ anomalies

The setup

- gauge theory with simple gauge group G_{gauge}
- a $\mathbb{Z}_N$ symmetry
- fields $\psi^{(i)}$ in irreducible representation $\mathbf{r}^{(i)}$ of G_{gauge}; $\mathbb{Z}_N$ charge $q^{(i)}$

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Under a $\mathbb{Z}_N$ transformation

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T. Araki et. al. (2008)

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$n_{\text{gauge}} \in \mathbb{Z}$
Note: $n_{\text{gauge}} = 0$ for $G_{\text{gauge}} = \mathrm{U}(1)$

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Dynkin index

factor 2 comes from normalization $\ell(\mathbf{M}) = \frac{1}{2}$ for $\mathrm{SU}(M)$

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Anomaly condition: $\sum_i q^{(i)}\ell(\mathbf{r}^{(i)}) = 0 \mod \frac{N}{2}$

Consequences of discrete anomalies

instantons induce an effective term in the Lagrangian

$$\mathscr{L} \supset e^{-\frac{8\pi^2}{g^2}k} \prod_{i=1} \left(\psi^{(i)}\right)^{2\ell(\mathbf{r}^{(i)})}$$

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anomaly $\longrightarrow$ $\boxed{\psi^{(1)}\psi^{(2)}}$

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An $\mathrm{SO}(10)$ SUSY GUT example

The model

R. Mohapatra, M. Ratz (2007)

	ψ_m	H	H'	ψ_H	$\overline{\psi}_H$	A	S
SO(10)	**16**	**10**	**10**	**16**	$\overline{\mathbf{16}}$	**45**	**54**
$\mathbb{Z}_6$	1	4	2	4	2	0	0

- ψ_m, H and H' are SM matter and Higgses
- ψ_H, $\overline{\psi}_H$, A and S break $\mathrm{SO}(10) \to G_{\mathrm{SM}}$

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- ψ_H, $\overline{\psi}_H$, A and S break $\mathrm{SO}(10) \to G_{\mathrm{SM}}$
- non-anomalous $\mathbb{Z}_6$ suppresses proton decay
- There is an additional $\mathrm{U}(1)_X$

$$\mathrm{SO}(10) \to \mathrm{SU}(5) \times \mathrm{U}(1)_X$$
$$\mathbf{16} \to \mathbf{10}_{-1} + \overline{\mathbf{5}}_3 + \mathbf{1}_{-5}$$

- Singlet obtains a VEV

$\mathrm{U}(1)_X \times \mathbb{Z}_6$ broken by fields with charge $(\pm 5, \pm 2)$

The model II

$$\boxed{\mathrm{U}(1)_X \times \mathbb{Z}_6 \text{ broken by fields with charge } (\pm 5, \pm 2)}$$

- Naive expectation: $\mathbb{Z}_5 \times \mathbb{Z}_2$

The model II

$\mathrm{U}(1)_X \times \mathbb{Z}_6$ broken by fields with charge $(\pm 5, \pm 2)$

- Naive expectation: $\cancel{\mathbb{Z}_5 \times \mathbb{Z}_2}$

	Q	$\bar{U}$	$\bar{D}$	L	$\bar{E}$	H_U	H_D
$\mathbb{Z}_{30}$	23	23	1	1	23	14	6
$\mathbb{Z}_5 \times \mathbb{Z}_6$	(3,1)	(3,1)	(1,5)	(1,5)	(3,1)	(4,4)	(1,0)

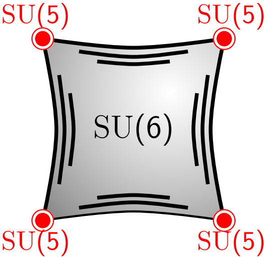
- This field content is anomalous!
- Either more light states are present or the $\mathbb{Z}_6$ is not exact.

A string theory example

The model

M. Blaszczyk, S. Groot Nibbelink, M. Ratz, F. Ruehle, M. Trapletti, P. Vaudrevange (2009)

- $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold compactification of the heterotic string
- massless spectrum: 3 × generations + vector-like
- (local) $\mathrm{SU}(5)$ GUT structure
- 4D gauge group



$$\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \left[\mathrm{SU}(3) \times \mathrm{SU}(2)^2 \times \mathrm{U}(1)^8\right]$$

- discrete R-symmetries $\mathbb{Z}_2^R \times \mathbb{Z}_2^R \times \mathbb{Z}_2^R$

The spectrum

massless spectrum:

#	representation	label	#	representation	label
3	$(\mathbf{3},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{\frac{1}{6}}$	q	3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{-\frac{2}{3}}$	$\overline{u}$
$3+1$	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{\frac{1}{3}}$	$\overline{d}$	1	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{-\frac{1}{3}}$	d
$3+1$	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{-\frac{1}{2}}$	ℓ	1	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{\frac{1}{2}}$	ℓ
3	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{1}$	$\overline{e}$	33	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{0}$	s
3	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{-\frac{1}{2}}$	h	3	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{\frac{1}{2}}$	$\overline{h}$
4	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{\frac{1}{3}}$	$\overline{\delta}$	4	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{-\frac{1}{3}}$	δ
5	$(\mathbf{1},\mathbf{1};\mathbf{3},\mathbf{1},\mathbf{1})_{0}$	x	5	$(\mathbf{1},\mathbf{1};\overline{\mathbf{3}},\mathbf{1},\mathbf{1})_{0}$	$\overline{x}$
6	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{2})_{0}$	y	6	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2},\mathbf{1})_{0}$	z

- some SM singlets get VEV
- many choices (string landscape)

Search for a good vacuum

We look for vacua with the following properties:

- all exotics decouple
- non-trivial Yukawa couplings
- matter parity

Our vacuum has the following properties:

- 21 fields $\widetilde{\phi}$ which get a VEV
- the $\widetilde{\phi}$ break $\mathrm{U}(1)^8 \times \mathbb{Z}_2^R \times \mathbb{Z}_2^R \times \mathbb{Z}_2^R \quad \rightarrow \quad \mathbb{Z}_2^{\mathcal{M}} \times \mathbb{Z}_4^R$

$\mathbb{Z}_2^{\mathcal{M}} \times \mathbb{Z}_4^R$ quantum numbers & implications

quarks and leptons						Higgs and exotics					
q_1	1	2	$\bar{u}_1$	1	2	$\bar{h}_1$	0	2	h_1	0	2
q_2	1	2	$\bar{u}_2$	1	2	$\bar{h}_2$	0	0	h_2	0	2
q_3	1	0	$\bar{u}_3$	1	0	$\bar{h}_3$	0	0	h_3	0	2
$\bar{d}_1$	1	2	ℓ_1	1	2	$\bar{\delta}_1$	0	2	δ_1	0	2
$\bar{d}_2$	1	2	ℓ_2	1	2	$\bar{\delta}_2$	0	0	δ_2	0	0
$\bar{d}_3$	1	0	ℓ_3	1	0	$\bar{\delta}_3$	0	2	δ_3	0	0
$\bar{d}_4$	1	0	ℓ_4	1	0	$\bar{\delta}_4$	0	2	δ_4	0	0
d_1	1	2	$\bar{\ell}_1$	1	2						
$\bar{e}_1$	1	2									
$\bar{e}_2$	1	2									
$\bar{e}_3$	1	0									

$\mathbb{Z}_2^{\mathcal{M}} \times \mathbb{Z}_4^{R}$ quantum numbers & implications

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q_1	1	2	$\bar{u}_1$	1	2	$\bar{h}_1$	0	2	h_1	0	2
q_2	1	2	$\bar{u}_2$	1	2	$\bar{h}_2$	0	0	h_2	0	2
q_3	1	0	$\bar{u}_3$	1	0	$\bar{h}_3$	0	0	h_3	0	2
$\bar{d}_1$	1	2	ℓ_1	1	2	$\bar{\delta}_1$	0	2	δ_1	0	2
$\bar{d}_2$	1	2	ℓ_2	1	2	$\bar{\delta}_2$	0	0	δ_2	0	0
$\bar{d}_3$	1	0	ℓ_3	1	0	$\bar{\delta}_3$	0	2	δ_3	0	0
$\bar{d}_4$	1	0	ℓ_4	1	0	$\bar{\delta}_4$	0	2	δ_4	0	0
d_1	1	2	$\bar{\ell}_1$	1	2						
$\bar{e}_1$	1	2									
$\bar{e}_2$	1	2									
$\bar{e}_3$	1	0									

One massless Higgs pair: $\mathcal{M}_h = \begin{pmatrix} 0 & 0 & 0 \\ * & * & * \\ * & * & * \end{pmatrix}$

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q_3	1	0	$\bar{u}_3$	1	0	$\bar{h}_3$	0	0	h_3	0	2
$\bar{d}_1$	1	2	ℓ_1	1	2	$\bar{\delta}_1$	0	2	δ_1	0	2
$\bar{d}_2$	1	2	ℓ_2	1	2	$\bar{\delta}_2$	0	0	δ_2	0	0
$\bar{d}_3$	1	0	ℓ_3	1	0	$\bar{\delta}_3$	0	2	δ_3	0	0
$\bar{d}_4$	1	0	ℓ_4	1	0	$\bar{\delta}_4$	0	2	δ_4	0	0
d_1	1	2	$\bar{\ell}_1$	1	2						
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$\mathrm{SU}(3)_C$ exotics δ decouple

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q_1	1	2	$\bar{u}_1$	1	2	$\bar{h}_1$	0	2	h_1	0	2
q_2	1	2	$\bar{u}_2$	1	2	$\bar{h}_2$	0	0	h_2	0	2
q_3	1	0	$\bar{u}_3$	1	0	$\bar{h}_3$	0	0	h_3	0	2
$\bar{d}_1$	1	2	ℓ_1	1	2	$\bar{\delta}_1$	0	2	δ_1	0	2
$\bar{d}_2$	1	2	ℓ_2	1	2	$\bar{\delta}_2$	0	0	δ_2	0	0
$\bar{d}_3$	1	0	ℓ_3	1	0	$\bar{\delta}_3$	0	2	δ_3	0	0
$\bar{d}_4$	1	0	ℓ_4	1	0	$\bar{\delta}_4$	0	2	δ_4	0	0
d_1	1	2	$\bar{\ell}_1$	1	2						
$\bar{e}_1$	1	2									
$\bar{e}_2$	1	2									
$\bar{e}_3$	1	0									

Yukawas have realistic structure $Y_u = \begin{pmatrix} * & * & 0 \\ * & * & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$\mathbb{Z}_2^{\mathcal{M}} \times \mathbb{Z}_4^R$ quantum numbers & implications

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q_1	1	2	$\bar{u}_1$	1	2	$\bar{h}_1$	0	2	h_1	0	2
q_2	1	2	$\bar{u}_2$	1	2	$\bar{h}_2$	0	0	h_2	0	2
q_3	1	0	$\bar{u}_3$	1	0	$\bar{h}_3$	0	0	h_3	0	2
$\bar{d}_1$	1	2	ℓ_1	1	2	$\bar{\delta}_1$	0	2	δ_1	0	2
$\bar{d}_2$	1	2	ℓ_2	1	2	$\bar{\delta}_2$	0	0	δ_2	0	0
$\bar{d}_3$	1	0	ℓ_3	1	0	$\bar{\delta}_3$	0	2	δ_3	0	0
$\bar{d}_4$	1	0	ℓ_4	1	0	$\bar{\delta}_4$	0	2	δ_4	0	0
d_1	1	2	$\bar{\ell}_1$	1	2						
$\bar{e}_1$	1	2									
$\bar{e}_2$	1	2									
$\bar{e}_3$	1	0									

SU(5) relations $Y_e = Y_d$: good for 3$^{\text{rd}}$ generation but bad for 1$^{\text{st}}$ & 2$^{\text{nd}}$

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$\bar{d}_4$	1	0	ℓ_4	1	0	$\bar{\delta}_4$	0	2	δ_4	0	0
d_1	1	2	$\bar{\ell}_1$	1	2						
$\bar{e}_1$	1	2									
$\bar{e}_2$	1	2									
$\bar{e}_3$	1	0									

$q_i \, q_j \, q_k \, \ell_m$

proton decay operators involve always 3rd generation field

$\curvearrowright$ proton stable at this level

$\mathbb{Z}_4^R$ anomaly

$\mathbb{Z}_4^R$ is anomalous (partly descends from the so-called 'anomalous $\mathrm{U}(1)$')

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Extra terms at the non-perturbative level:

- proton decay operators

$$[q\,q\,q\,\ell]_{\text{light generations}} \sim \widetilde{\phi}^{15}\,\mathrm{e}^{-a\,S}$$

- mixing between first two and third generations

$$(Y_u)_{13} \sim \widetilde{\phi}^{4}\,\mathrm{e}^{-a\,S}$$

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'Anomalous' $\mathbb{Z}_4^R$ explains suppressed μ term and relates the suppression of proton decay operators to mixing between first two and third generations
Many similar configurations. . .

Summary

We have shown

- how to obtain any discrete, abelian symmetry by spontaneous breaking $\mathrm{U}(1)^N$

  There are of course other ways, e.g. extra dimensions
- how to eleminate redundancies for discrete, abelian symmetries
- how discrete anomalies influence a model
- how all this can be applied in model building

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We have shown

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Thank You!