Nuclear Matrix Elements for $0\nu\beta\beta$ decay

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MPIK, Heidelberg, 13/12/2010

Outline

- Introduction
- Nuclear matrix elements for $\beta\beta$ decay
  Status of calculations
- Can one measure $M^{0\nu}$?
- Conclusions

Neutrinos

massive particles

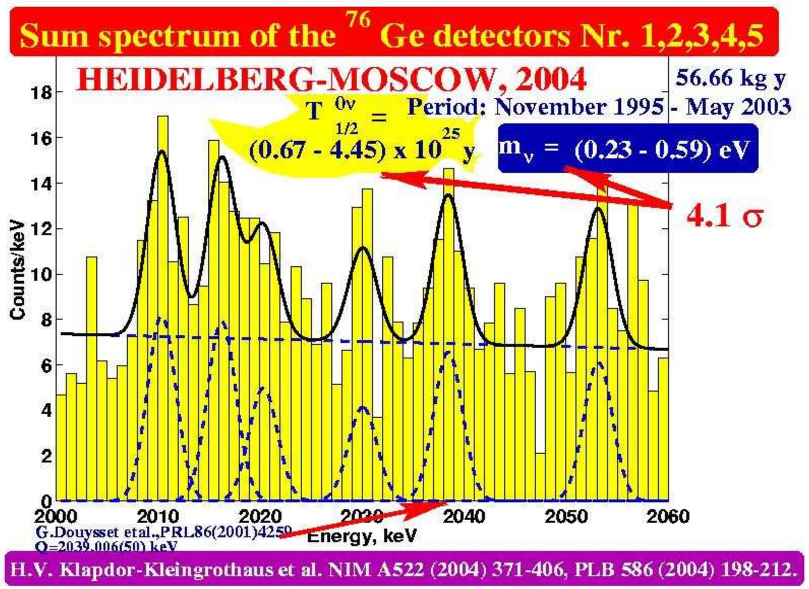
Dirac vs. Majorana

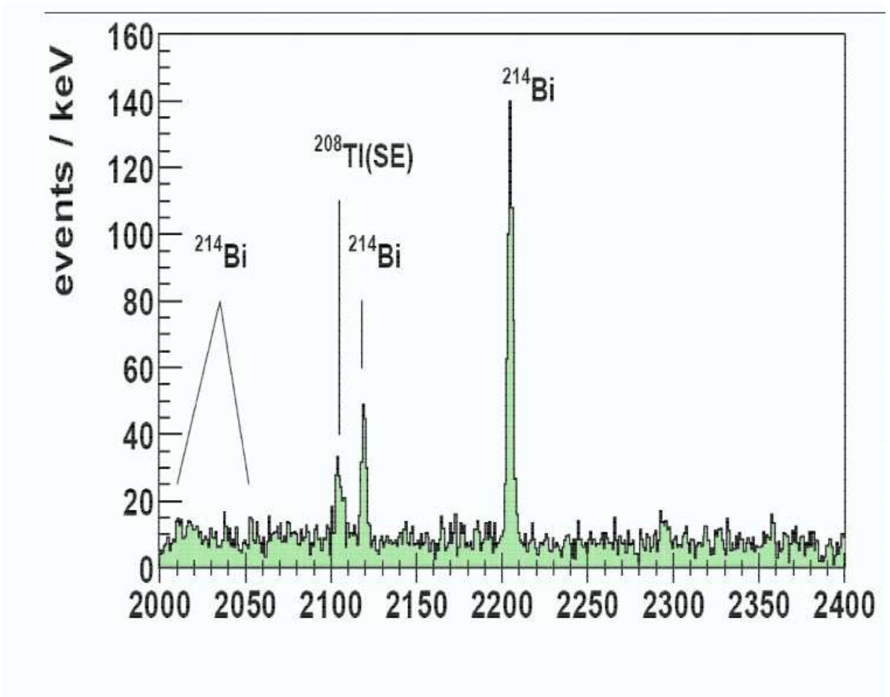
$\overline{\nu} \neq \nu$ $\overline{\nu} = \nu$

$$0\nu\beta\beta \Longleftrightarrow \overline{\nu} = \nu,\, m_\nu > 0$$

Heidelberg-Moscow Ge experiment, $T^{0\nu}_{1/2}$:

$2.23^{+0.44}_{-0.31} \cdot 10^{25}$ y $\quad (m_{\beta\beta} \approx 0.3 \div 0.5$ eV)
(Heidelberg, Klapdor et al. hep-ph/0512263)

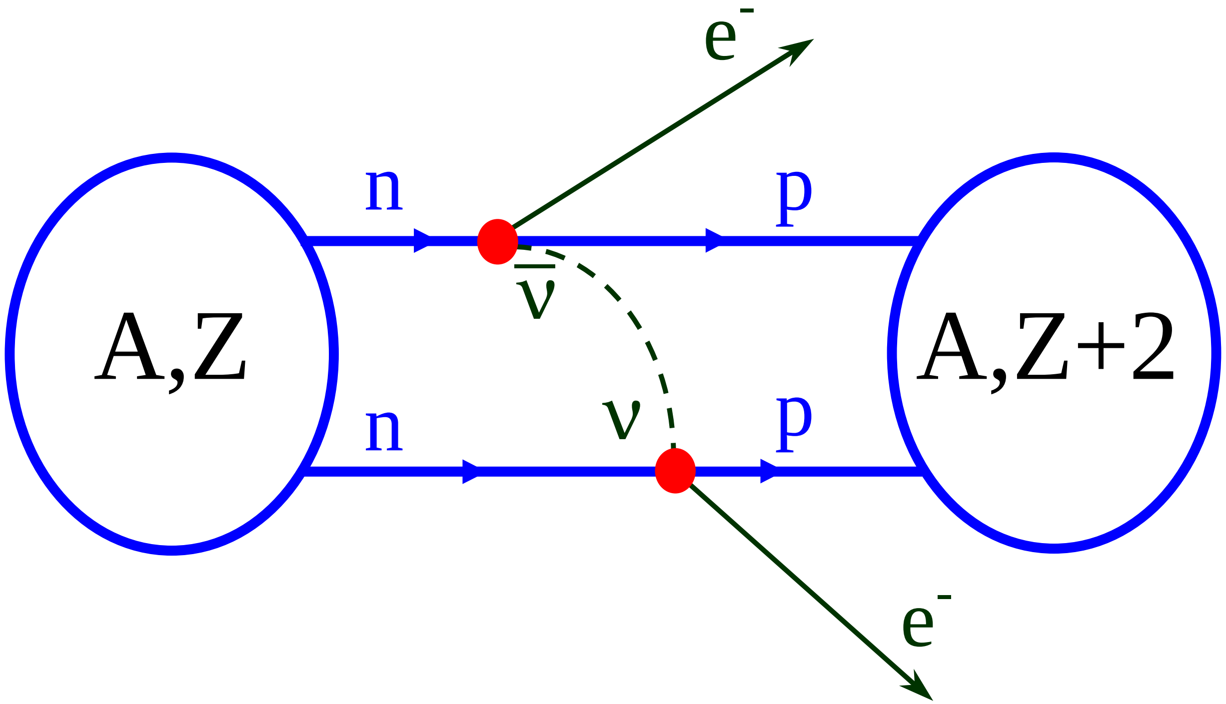




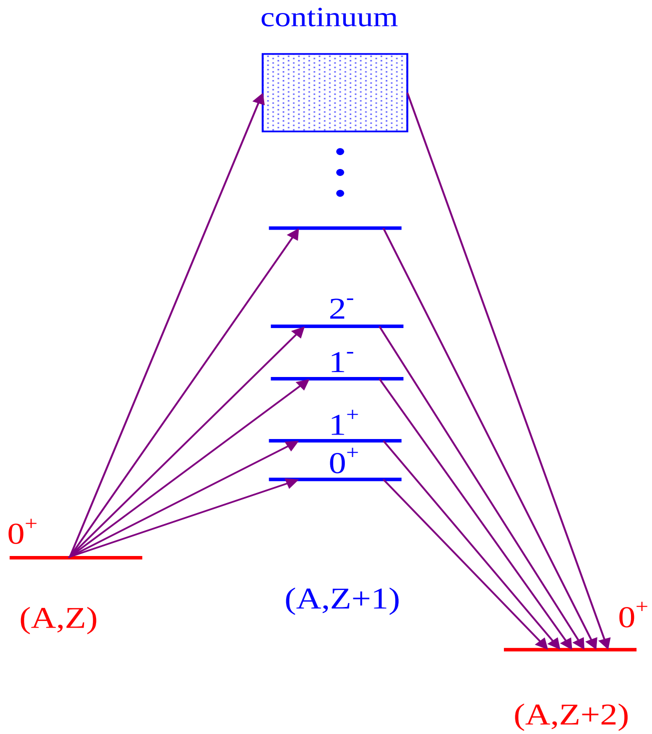
$\geq 1.6 \cdot 10^{25}$ y $\quad$ (90% C.L.)
(Moscow, Belyaev et al.)

$\Rightarrow$ GERDA I (next year)

Nuclear $0\nu\beta\beta$-decay ($\overline{\nu} = \nu$)



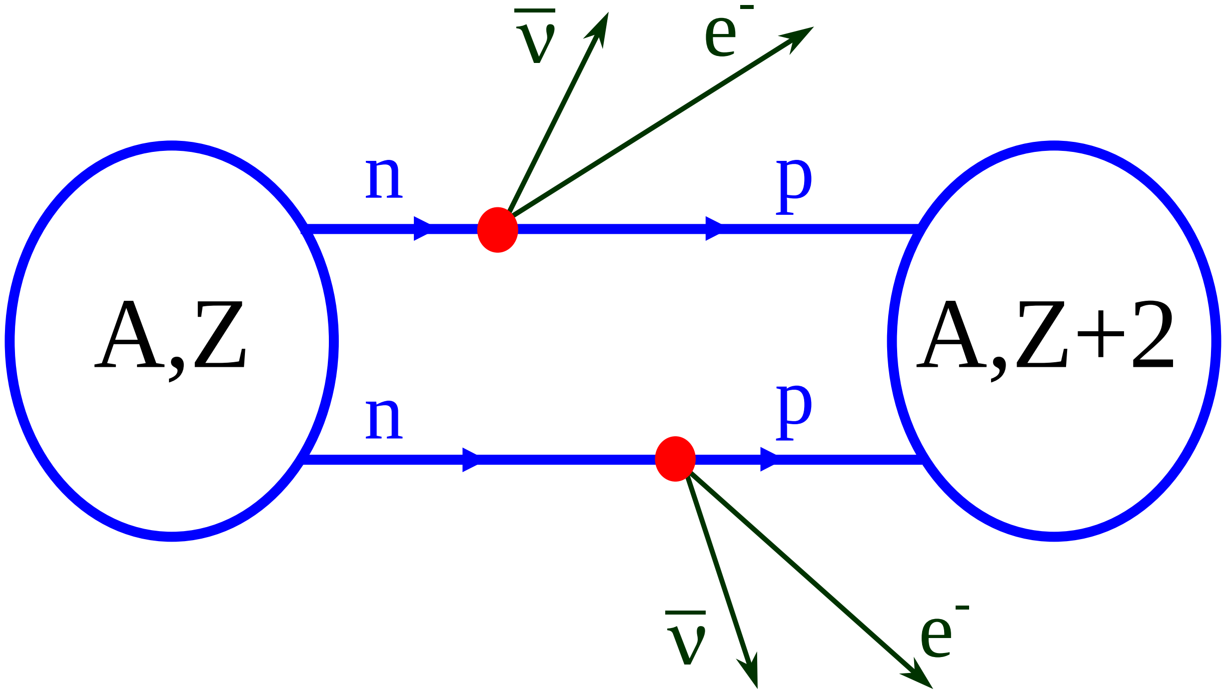
Light neutrino
exchange mechanism



virtual excitation
of states of all multipolarities
in (A,Z+1) nucleus

Nuclear $2\nu\beta\beta$-decay

second order weak process
within SM



measured $T_{1/2}^{2\nu}$ (compilation of A. Barabash, 2005)

Isotope	$T_{1/2}^{2\nu}$, in 10^{19} y
^{48}Ca	$4.2^{+2.1}_{-1.0}$
^{76}Ge	150 ± 10
^{82}Se	9.2 ± 0.7
^{96}Zr	2.0 ± 0.3
^{100}Mo	0.71 ± 0.04
^{116}Cd	3.0 ± 0.2
^{128}Te	$(2.5 \pm 0.3) \times 10^5$
^{130}Te	90 ± 10
^{136}Xe	> 81 (90% CL)
^{150}Nd	0.78 ± 0.07
^{238}U	200 ± 60

$2\nu\beta\beta$ $0\nu\beta\beta$

Inverse Half-Lives $[T_{1/2}(0^+ \to 0^+)]^{-1}$

$$G^{2\nu}(Q,Z)\,|M_{GT}^{2\nu}|^2$$

$$m_{\beta\beta}^2\, G^{0\nu}(Q,Z)\left|M_{GT}^{0\nu} - \frac{g_V^2}{g_A^2} M_F^{0\nu}\right|^2$$

Eff. neutrino mass $m_{\beta\beta} = \sum_j m_j U_{ej}^2$

U_{ej} — first raw of the neutrino mixing matrix

$2\nu\beta\beta$ $0\nu\beta\beta$

Nuclear Matrix Elements

$$M_{GT}^{2\nu} = \sum_s \frac{\langle 0_f || \hat{\beta}^- || s \rangle \langle s || \hat{\beta}^- || 0_i \rangle}{E_s - (M_i + M_f)/2}$$

$$\hat{\beta}^- = \sum_k \sigma_k \tau_k^-$$

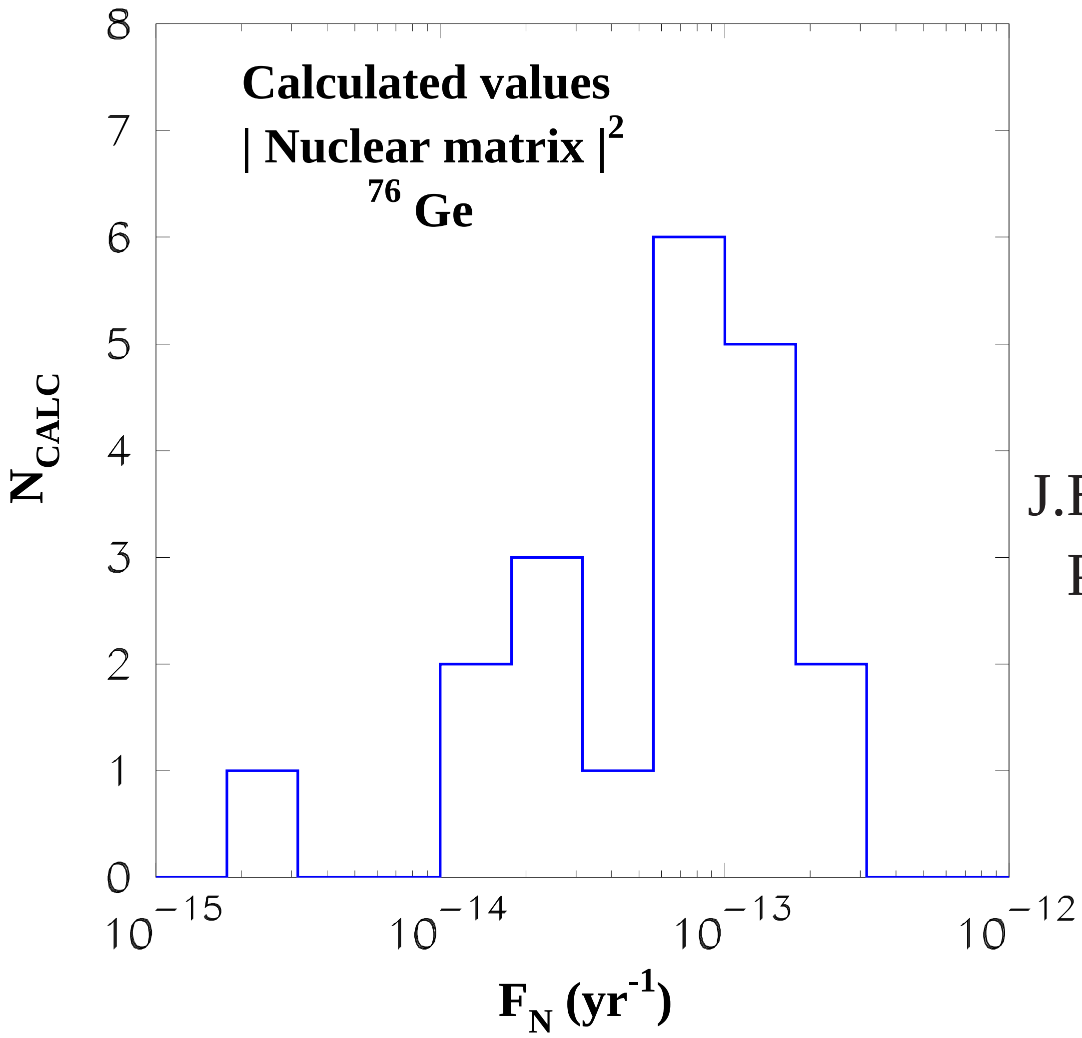
$$M_{GT}^{0\nu} = \langle 0_f | \sum_{ik} P_\nu(r_{ik}, \overline{\omega}) \tau_i^- \tau_k^- \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_k | 0_i \rangle$$

Neutrino potential : $P_\nu(r, \overline{\omega}) =$

$$\frac{2R}{\pi r} \int_0^\infty dq \frac{q \sin(qr)}{\omega(\omega + \overline{\omega})}$$

$$\approx \frac{R}{r} \phi(\overline{\omega} r)$$

QRPA vs. SM

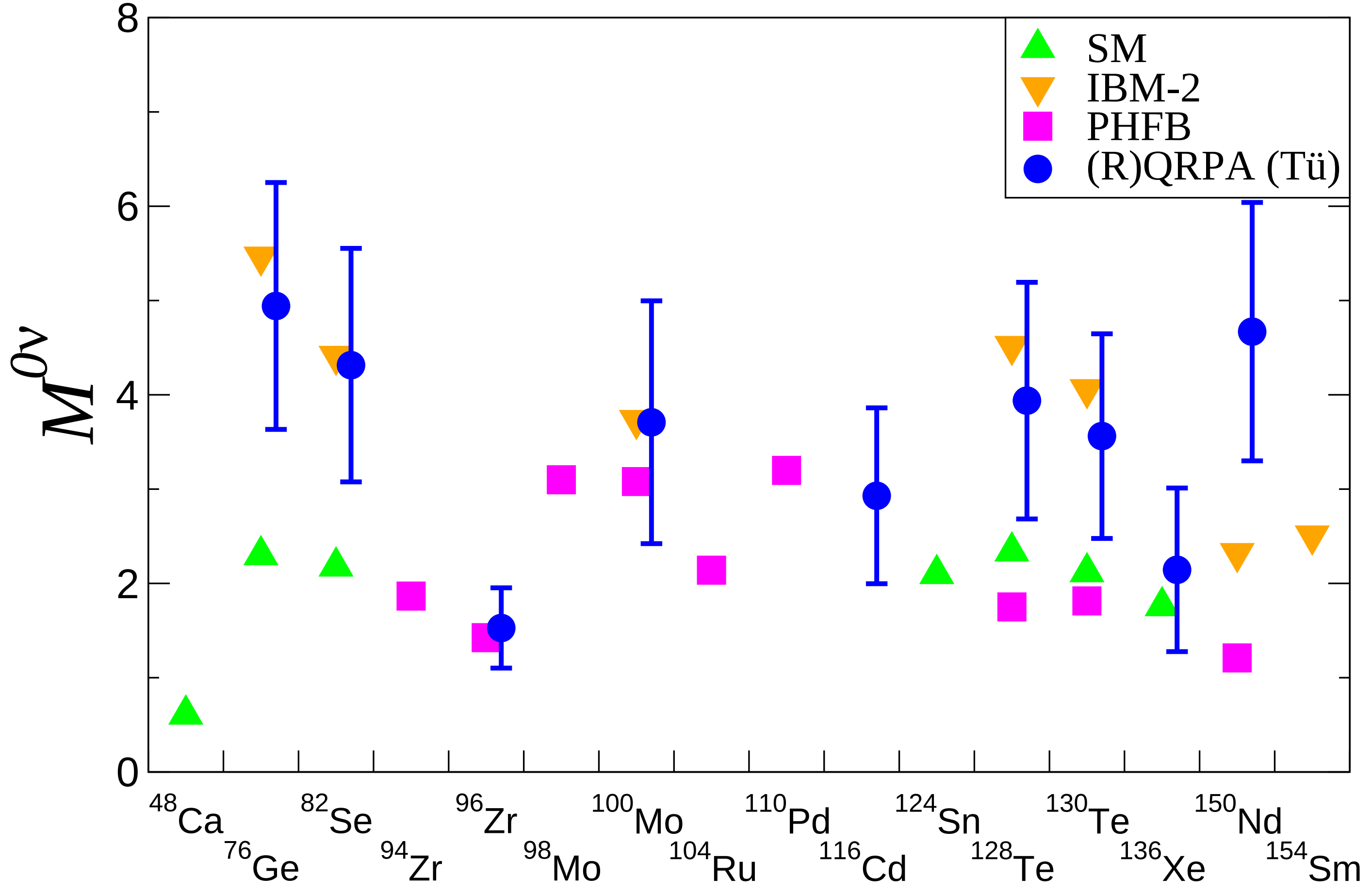


J.Bahcall, H.Murayama, C.Pena-Garay
PRD 70 (2004)

“In the foreseeable future, it does not seem possible to derive in a direct and controlled manner from QCD nuclear matrix elements for large *A*.
Thus, there is no way of quantifying with absolute confidence the range of uncertainties ..."

World status of $M^{0\nu}$, light neutrino mass mechanism

A. Escuderos, A. Faessler, V. R., F. Šimkovic, J.Phys.G37 (2010) arXiv:1001.3519 [nucl-th]



(R)QRPA (Tü) = F. Šimkovic, A. Faessler, V.R., P. Vogel and J. Engel, PRC **77** (2008)

SM = E. Caurier, J. Menendez, F. Nowacki, A. Poves, PRL **100** (2008)

IBM-2 = J. Barea and F. Iachello, PRC **79** (2009)

PHFB = K. Chaturvedi *et al.*, PRC **78** (2008)

QRPA vs. SM

Complete theory of nuclear structure does not exist!

Nuclear models to calculate $\beta\beta$-amplitudes

Mean field $\longrightarrow$ s.p. states $\longrightarrow$ configuration space $\longrightarrow$ diagonalization

	Tübingen; Jyväskylä	Strassbourg-Madrid
	QRPA	NSM
s.p. bases	$N\hbar\omega$	$0\hbar\omega$
configurations	limited	all

QRPA vs. SM

Shell Model

Model Space: ^{48}Ca — fp; ^{76}Ge, ^{82}Se — $p, f_{5/2}, g_{9/2}$

^{96}Zr, ^{100}Mo — s, d, g; ^{128,130}Te, ^{136}Xe — $s, d, g_{7/2}, h_{11/2}$

- A lot of GT strength is missing (Ikeda Sum Rule is violated up to 40%)

  several spin-orbit partners are missing even in $0\hbar\omega$ model space

- Many $0\nu\beta\beta$-transitions via negative parity intermediate states (dipole, spin-dipole etc.) are missing. They contribute a lot to $M^{0\nu}$ (shown by QRPA)

QRPA vs. SM

QRPA

- Works quite well when applied to description of collective states

- Fulfills exactly various model-independent sum rules

$M^{0\nu}$ and $M^{2\nu}$ are integral quantities (sums over all intermediate states)
challenge for experimental verification, but favors QRPA description

g_{pp}-problem
$2\nu\beta\beta$ is sensitive to degree of violation of the Wigner SU(4)
symmetry $\Rightarrow$ g_{pp}-sensitivity is unavoidable!

Systematic study of QRPA uncertainties

V.R., A. Faessler, F. Šimkovic, P. Vogel, PRC 68 (2003); NPA 766 (2006); NPA 793 (2007); PRC **77** (2008)

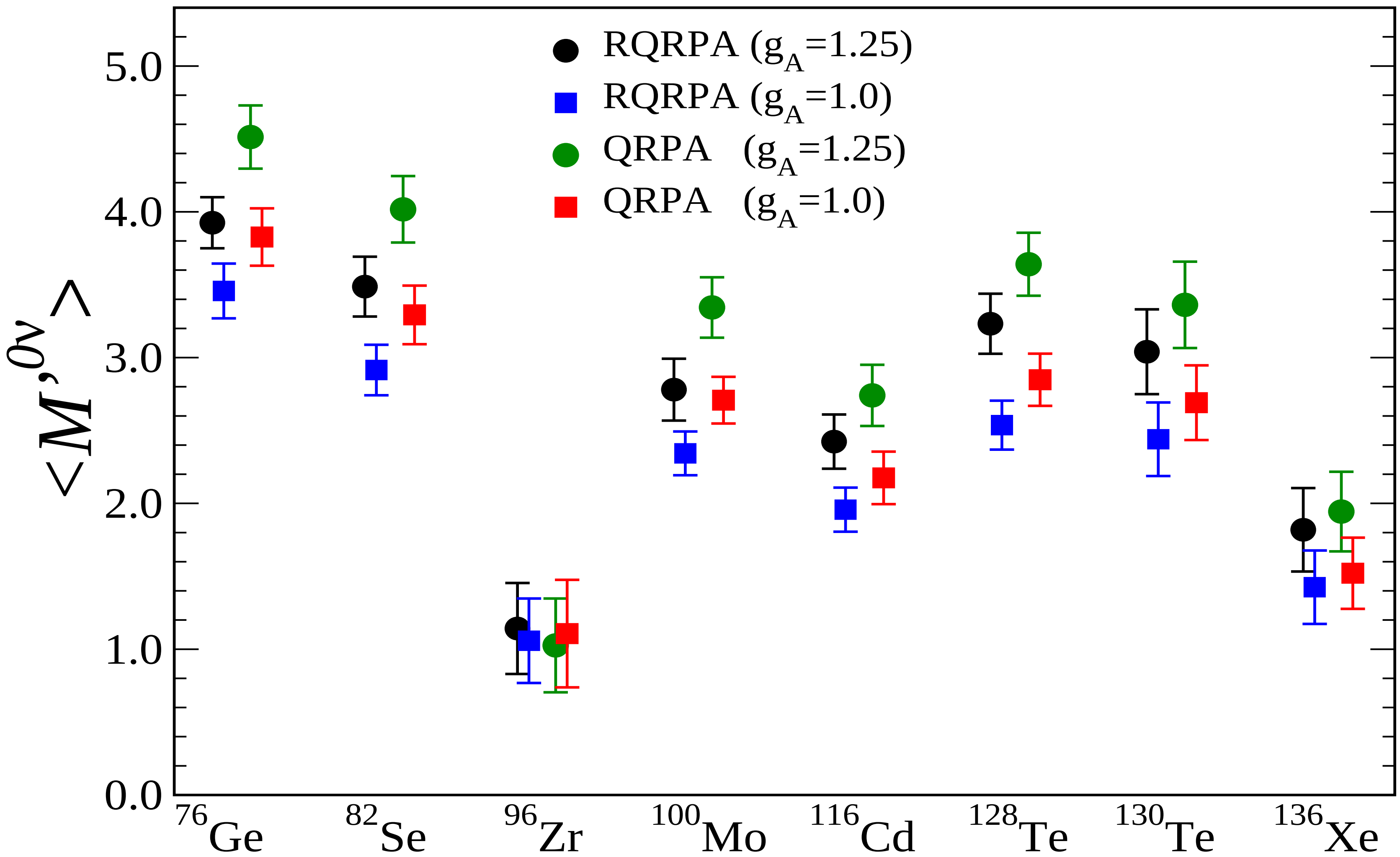
g_{pp} fitted to $2\nu\beta\beta$-decay half-life $\Rightarrow$ stable $M^{0\nu}$

Tested sensitivity of $M^{0\nu}$ to:

- size of the single-particle basis $(2, 3, 5\ \hbar\omega)$
- different realistic representations of the nucleon G-matrix (Bonn-CD, Argon, Nijmegen)
- quenching of the axial vector strength g_A

QRPA vs. SM

Light Majorana Neutrino Exchange Mechanism



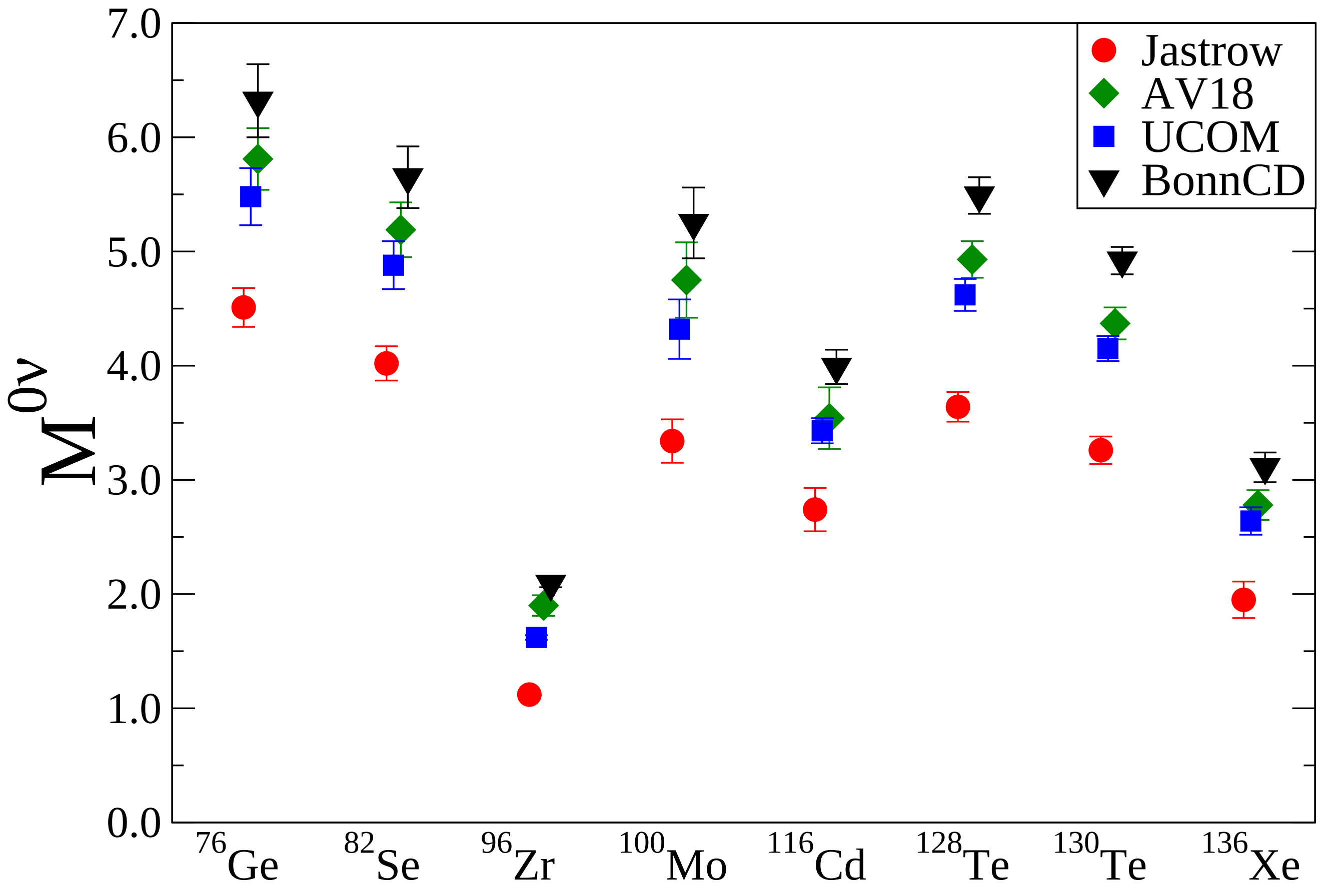
9 calculations for every point

Errors: 1σ "theory" + induced 1σ experiment

QRPA vs. SM

Effect of short range correlation

F. Šimkovic, A. Faessler, H. Muether, V.R., M. Stauf, PRC **79** (2009)



Partial contributions to $M^{0\nu}$

- $M^{0\nu} = \sum_{J^\pi} M^{0\nu}_{J^\pi}$ — particle-hole channel

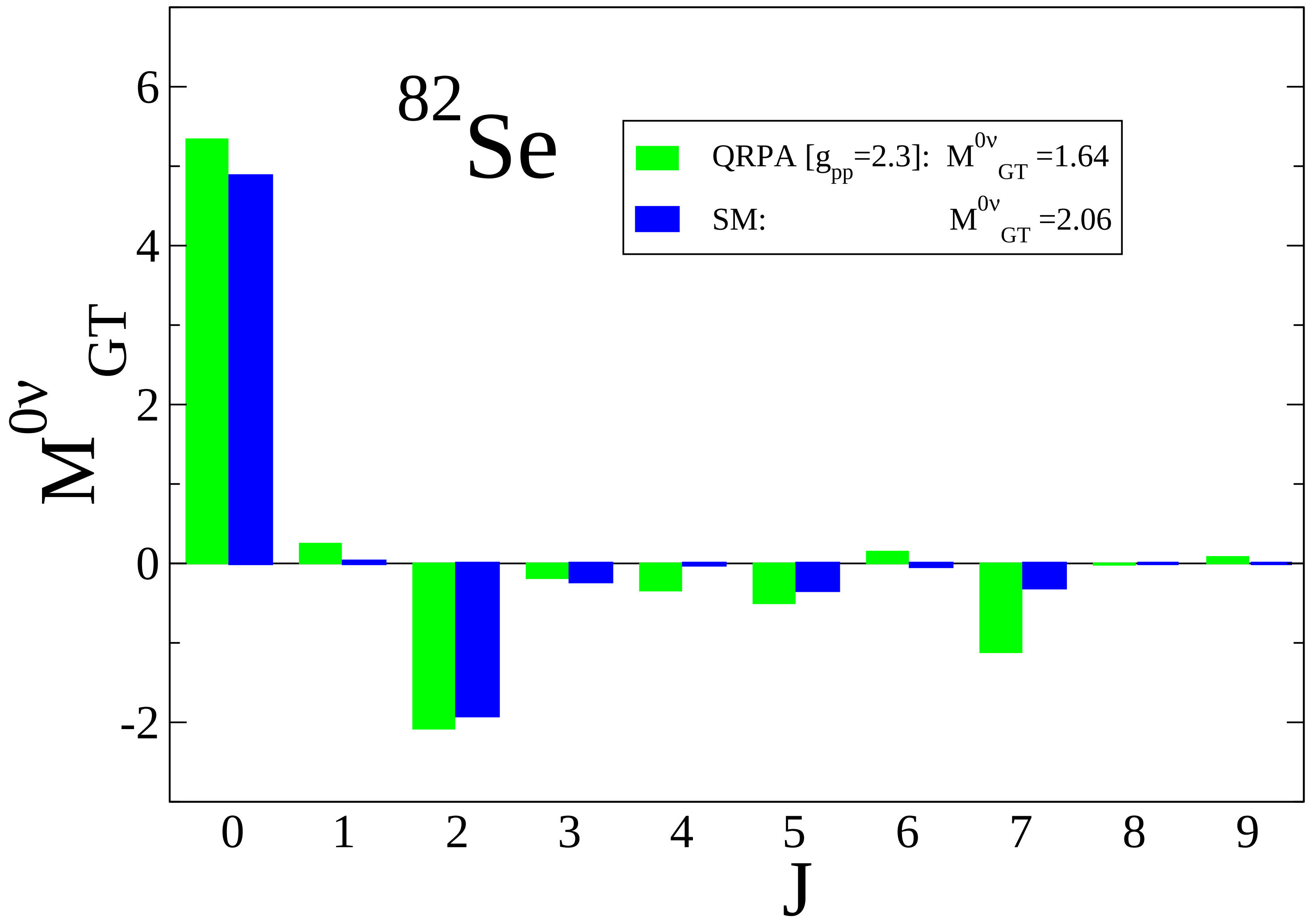
$$M^{0\nu}_{J^\pi} = \sum_{pnp'n'} a_{pnp'n'} \langle 0_f | \left[c^\dagger_p \tilde{c}_n \right]_J \left[c^\dagger_{p'} \tilde{c}_{n'} \right]_J | 0_i \rangle$$

- $M^{0\nu} = \sum_{\mathcal{J}^\pi} M^{0\nu}_{\mathcal{J}^\pi}$ —particle-particle channel

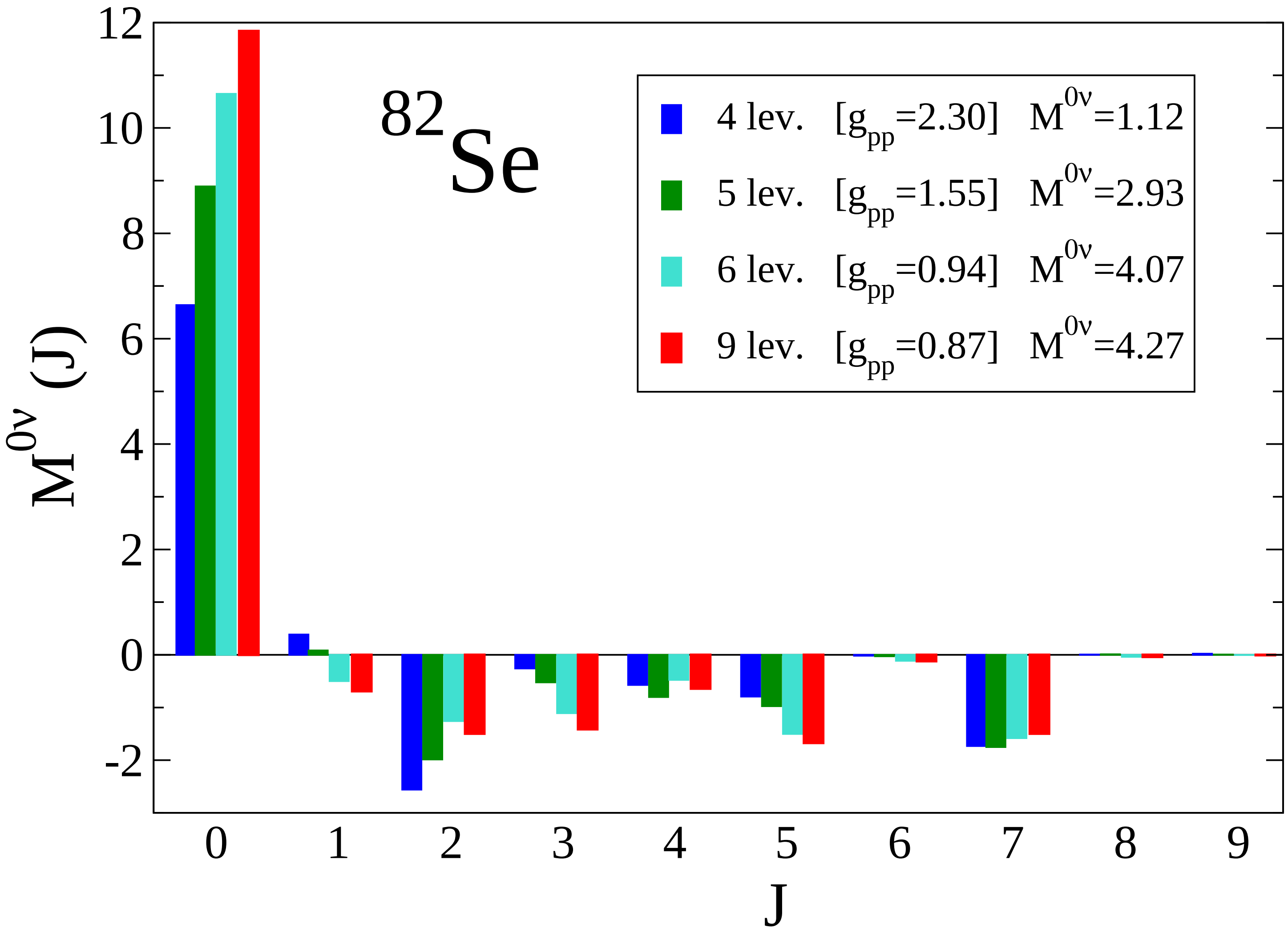
$$M^{0\nu}_{\mathcal{J}^\pi} = \sum_{pnp'n'} b_{pnp'n'} \langle 0_f | \left[c^\dagger_p c^\dagger_{p'} \right]_{\mathcal{J}} \left[c_n c_{n'} \right]_{\mathcal{J}} | 0_i \rangle$$

QRPA vs. SM

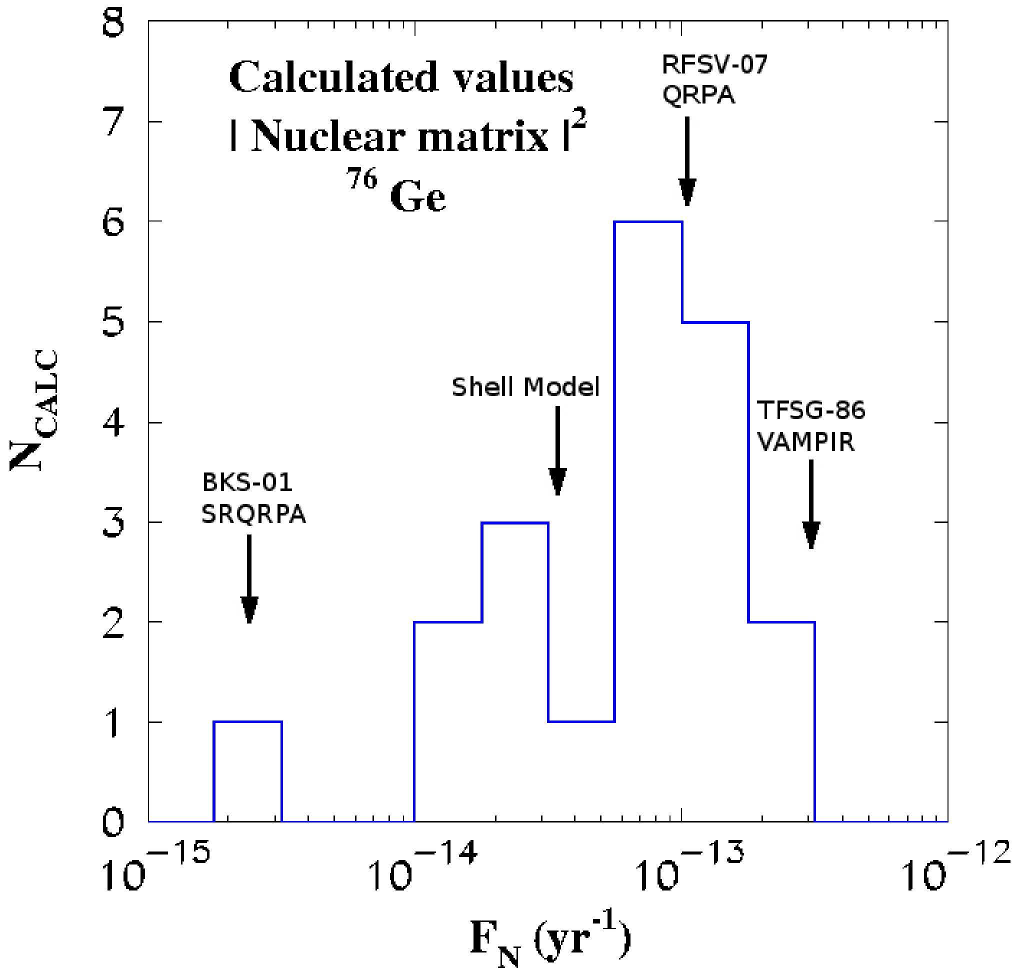
A. Escuderos, A. Faessler, V. R., F. Šimkovic, J.Phys.G37 (2010) arXiv:1001.3519 [nucl-th]



QRPA vs. SM


82Se
4 lev. [gpp=2.30] M0ν=1.12
5 lev. [gpp=1.55] M0ν=2.93
6 lev. [gpp=0.94] M0ν=4.07
9 lev. [gpp=0.87] M0ν=4.27
M0ν (J)
J
12
10
8
6
4
2
0
-2
0 1 2 3 4 5 6 7 8 9

QRPA vs. SM



BKS-01 = A. Bobyk, W. Kaminski, F. Simkovic, PRC**63** (2001) ⇐ $2\nu\beta\beta$ 20 times too slow

Shell Model = E. Caurier, G. Martinez-Pinedo, F. Nowacki, A. Poves A. Zuker, RMP**77** (2005)

RFSV-07 = V.R., A. Faessler, F. Simkovic, P. Vogel, NPA**793** (2007) (2003) ⇐ $2\nu\beta\beta$ fitted

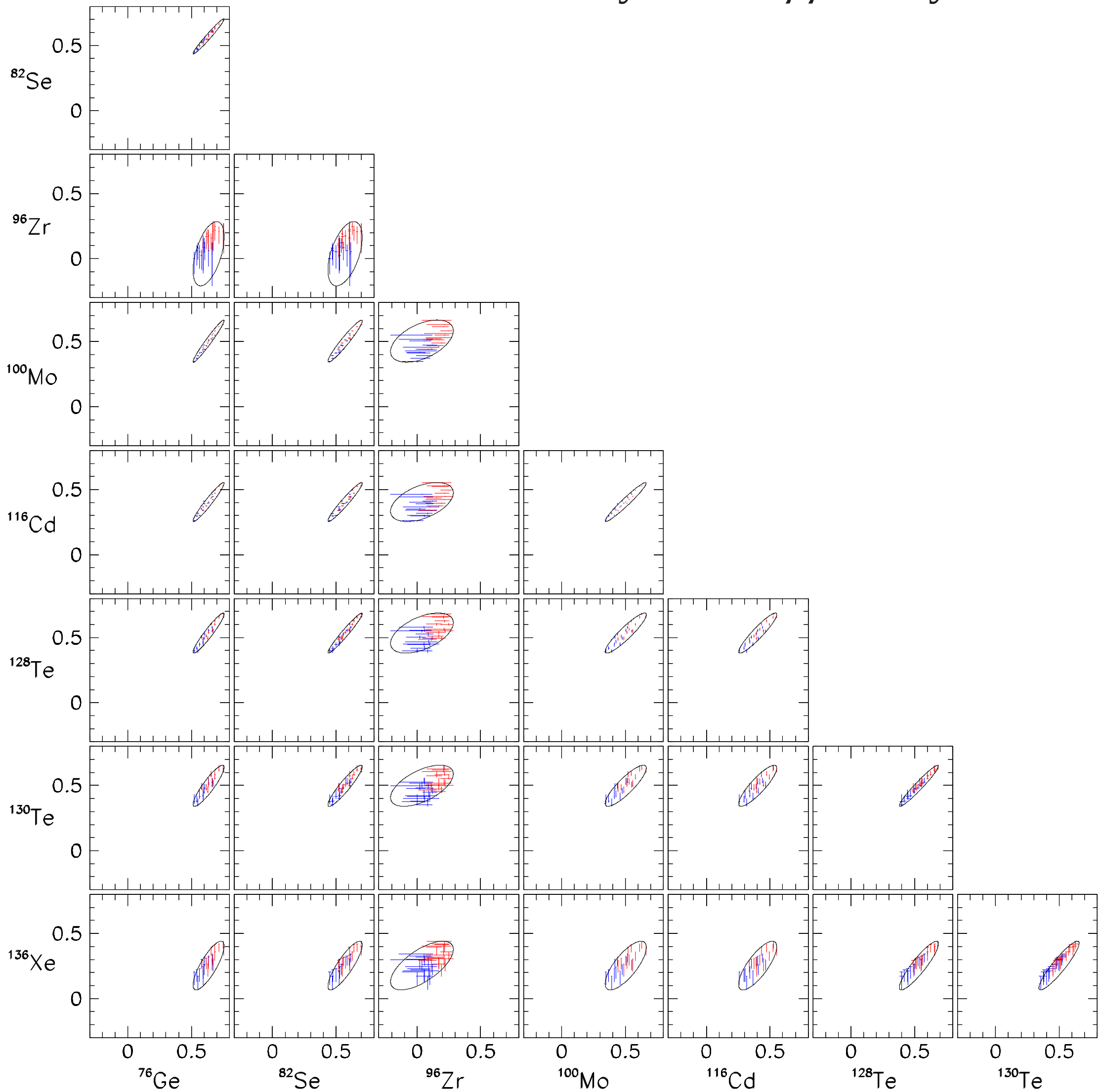
TFSG-86 = T. Tomoda, A. Faessler, K. W. Schmid, F. Grummer, NPA**452** (1986)

⇐ $2\nu\beta\beta$ 8 times too fast

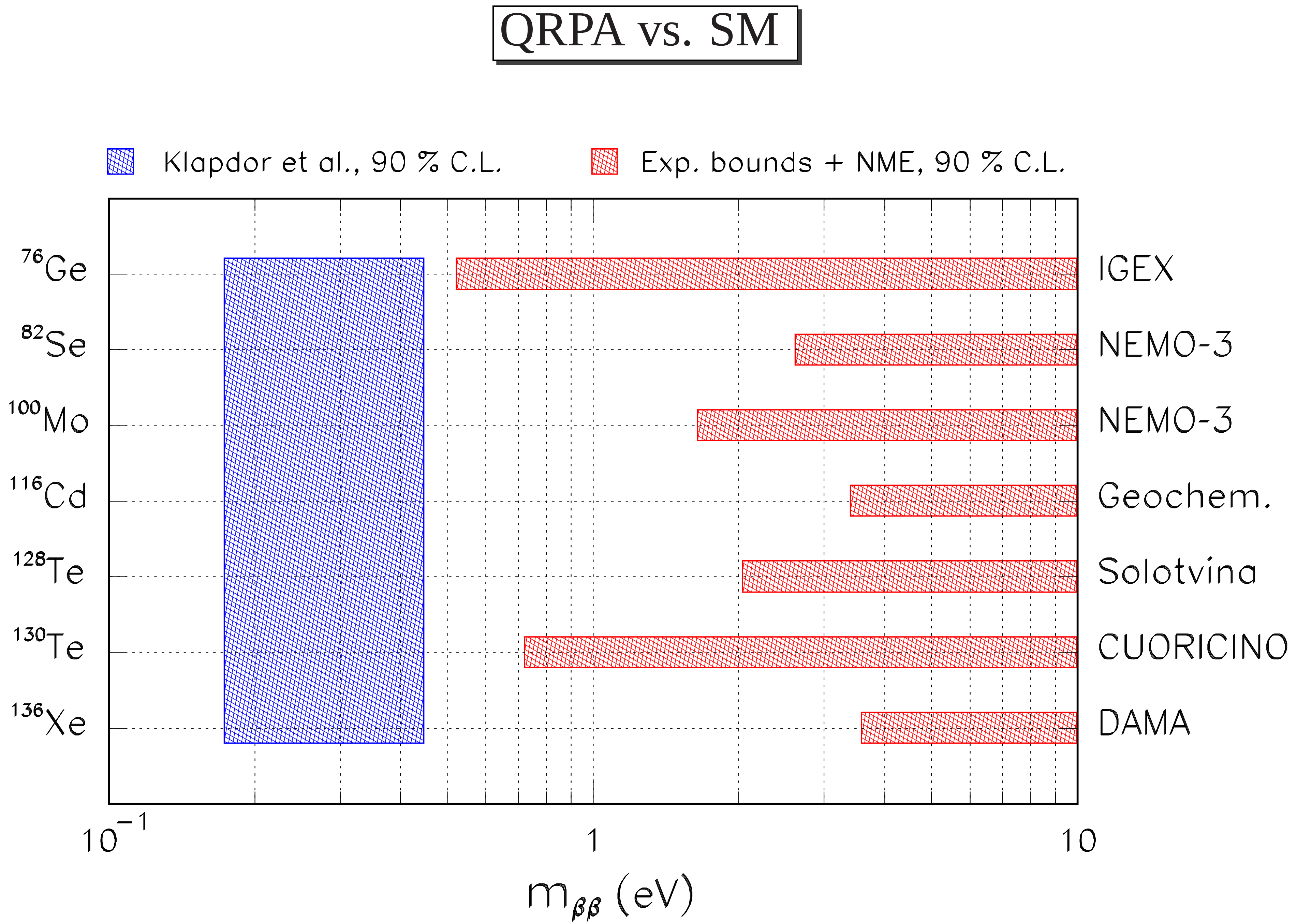
QRPA vs. SM

A. Faessler, G. L. Fogli, E. Lisi, V. R., A. M. Rotunno and F. Simkovic,

"QRPA uncertainties and their correlations in the analysis of $0\nu\beta\beta$ decay" arXiv:0810.5733 [hep-ph]



Scatter plot of QRPA $\eta = log M^{0\nu}$, with 1σ error ellipse

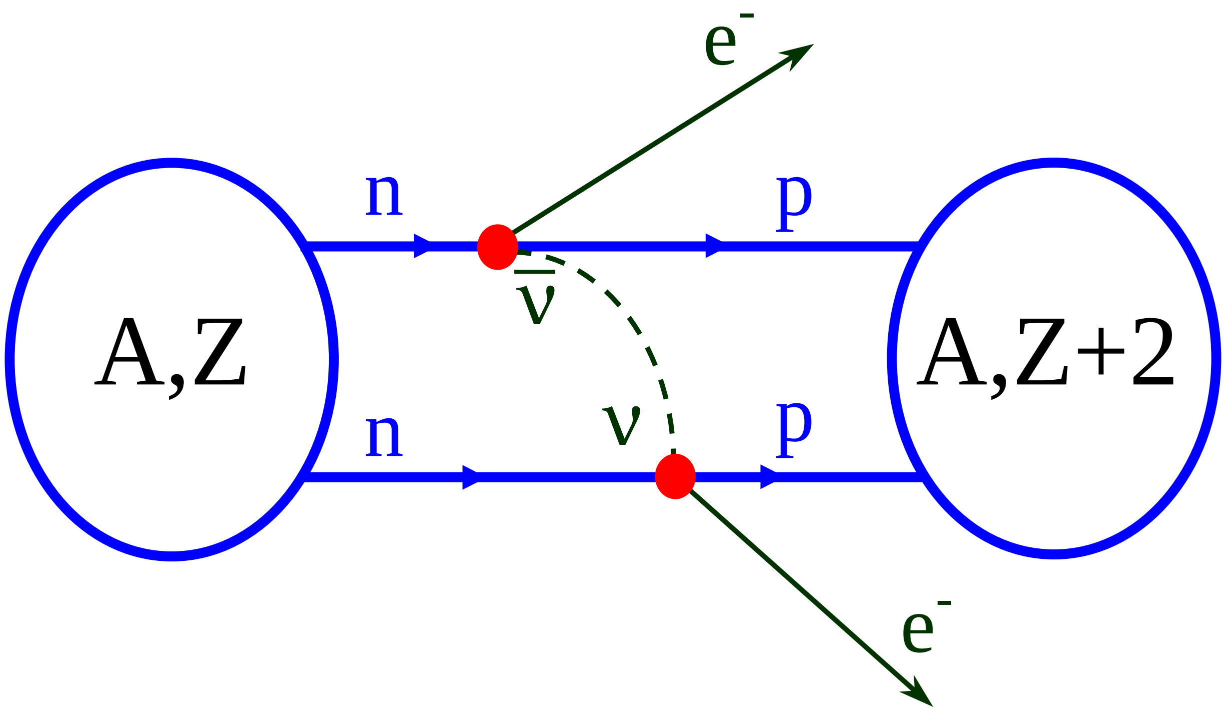

QRPA vs. SM
Klapdor et al., 90 % C.L.
Exp. bounds + NME, 90 % C.L.
^{76}Ge
^{82}Se
^{100}Mo
^{116}Cd
^{128}Te
^{130}Te
^{136}Xe
IGEX
NEMO-3
NEMO-3
Geochem.
Solotvina
CUORICINO
DAMA
10^{-1}
1
10
$m_{\beta\beta}$ (eV)

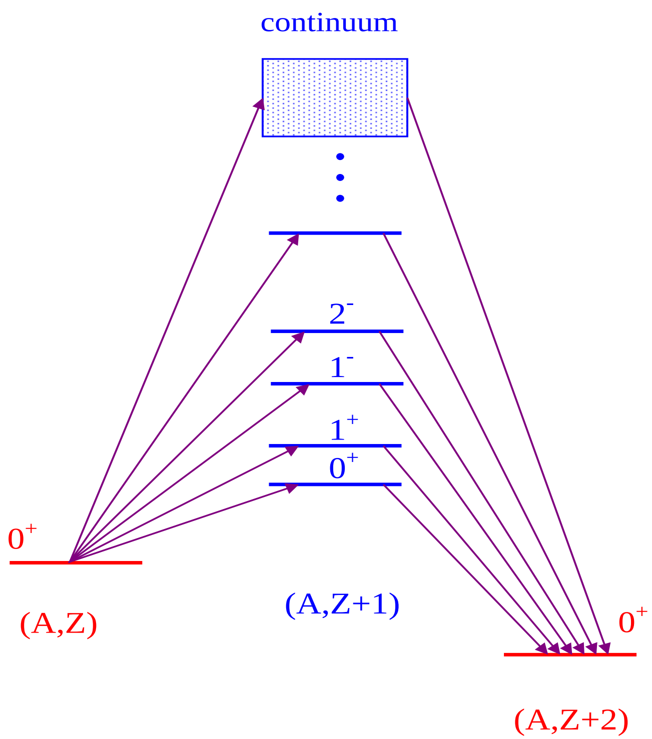
Can one measure nuclear matrix elements of neutrinoless double beta decay?

V.R., A. Faessler, PRC **80** , 041302(R) (2009) [arXiv:0906.1759 [nucl-th]]

Nuclear $0\nu\beta\beta$-decay ($\overline{\nu} = \nu$)

Light neutrino
exchange mechanism

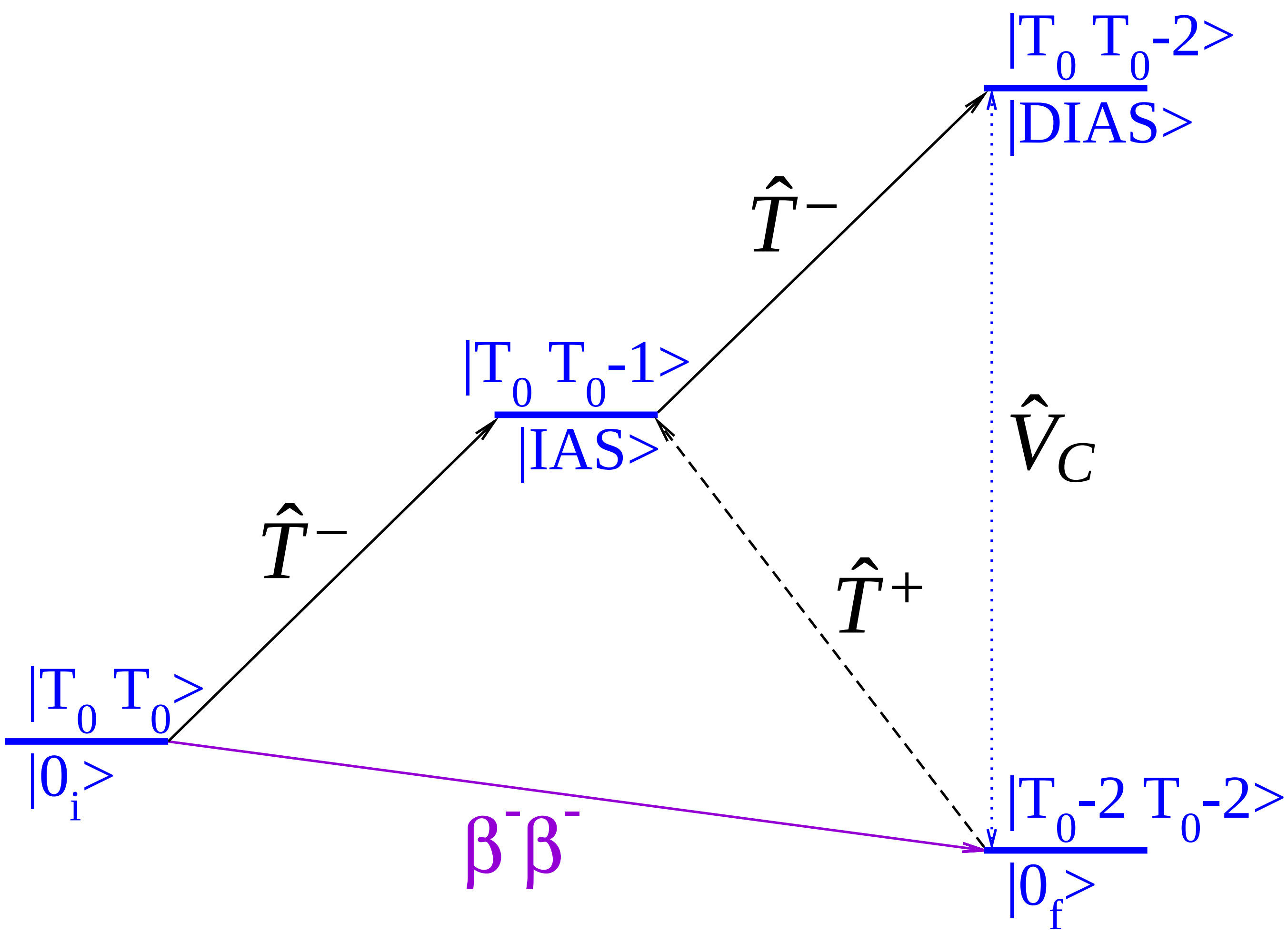




B(GT) of 1^+ — from charge-exchange reactions
(D. Frekers, H. Sakai, R. Zegers, et al.)

Measuring $M_F^{0\nu}$

Double Fermi transition ($J_s^\pi = 0^+$)



$M_F^{2\nu} = 0$ if isospin SU(2) symmetry is exact — Violated by Coulomb

Measuring $M_F^{0\nu}$

$$\hat{W}_F^{0\nu} = \sum_{ab} P_\nu(r_{ab})\tau_a^- \tau_b^- = \frac{1}{e^2}\left[\hat{T}^-, [\hat{T}^-, \hat{V}_C]\right]$$

Isospin lowering operator $\hat{T}^- = \sum_a \tau_a^-$; Coulomb interaction $\hat{V}_C = \frac{e^2}{8}\sum_{a\neq b}\frac{(1-\tau_a^{(3)})(1-\tau_b^{(3)})}{r_{ab}}$

Measuring $M_F^{0\nu}$

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Isospin lowering operator $\hat{T}^- = \sum_a \tau_a^-$; Coulomb interaction $\hat{V}_C = \frac{e^2}{8}\sum_{a\neq b}\frac{(1-\tau_a^{(3)})(1-\tau_b^{(3)})}{r_{ab}}$

$$\hat{V}_C = \hat{V}_C^{(0)} + \hat{V}_C^{(1)} + \hat{V}_C^{(2)}$$

$\hat{V}_C^{(0)} = \frac{e^2}{8}\sum_{a\neq b}\frac{1+\frac{\boldsymbol{\tau}_a\boldsymbol{\tau}_b}{3}}{r_{ab}}$ $\hat{V}_C^{(1)} = -\frac{e^2}{8}\sum_{a\neq b}\frac{\tau_a^{(3)}+\tau_b^{(3)}}{r_{ab}}$ $\hat{V}_C^{(2)} = \frac{e^2}{8}\sum_{ab}\frac{T_{ab}^{(2)}}{r_{ab}}$ $(T_{ab}^{(2)} \equiv \tau_a^{(3)}\tau_b^{(3)} - \frac{\boldsymbol{\tau}_a\boldsymbol{\tau}_b}{3})$

Only isotensor $\hat{V}_C^{(2)}$ contributes to $\left[\hat{T}^-, [\hat{T}^-, \hat{V}_C]\right]$

Measuring $M_F^{0\nu}$

$$\hat{H}_{tot} = \hat{T} + \hat{H}_{str} + \hat{V}_C$$

If $\hat{H}_{str}$ exactly isospin-symmetric: $\left[\hat{T}^-, \hat{H}_{str}\right] = 0$

$$\Downarrow$$

$$\hat{W}_F^{0\nu} = \frac{1}{e^2}\left[\hat{T}^-, [\hat{T}^-, \hat{H}_{tot}]\right]$$

Measuring $M_F^{0\nu}$

$$M_F^{0\nu} =$$

$$-\frac{2}{e^2}\sum_s \bar{\omega}_s \langle 0_f^+|\hat{T}^-|0_s^+\rangle\langle 0_s^+|\hat{T}^-|0_i^+\rangle$$

$$\bar{\omega}_s = E_s - (E_{0_i^+} + E_{0_f^+})/2$$

Measuring $M_F^{0\nu}$

$$M_F^{0\nu} \approx -\frac{2}{e^2}\,\overline{\omega}_{IAS}\langle 0_f^+|\hat{T}^-|IAS\rangle\langle IAS|\hat{T}^-|0_i^+\rangle$$

$$\approx \frac{1}{e^2}\langle 0_f^+|\hat{V}_C^{(2)}|DIAS\rangle\langle DIAS|\left(\hat{T}^-\right)^2|0_i^+\rangle$$

Measuring $M_F^{0\nu}$

Measure the $\Delta T = 2$ isospin-forbidden matrix element $\langle 0_f^+ | \hat{T}^- | IAS \rangle$

charge-exchange (n, p)-type reaction

Challenge: $\langle 0_f^+ | \hat{T}^- | IAS \rangle \sim 0.005$

$\langle IAS | \hat{T}^- | 0_i^+ \rangle \approx \sqrt{N - Z} \sim 5$

$$M_F^{0\nu}(QRPA)/M_F^{0\nu}(SM) \approx 3 \div 5$$

Measuring $M_F^{0\nu}$

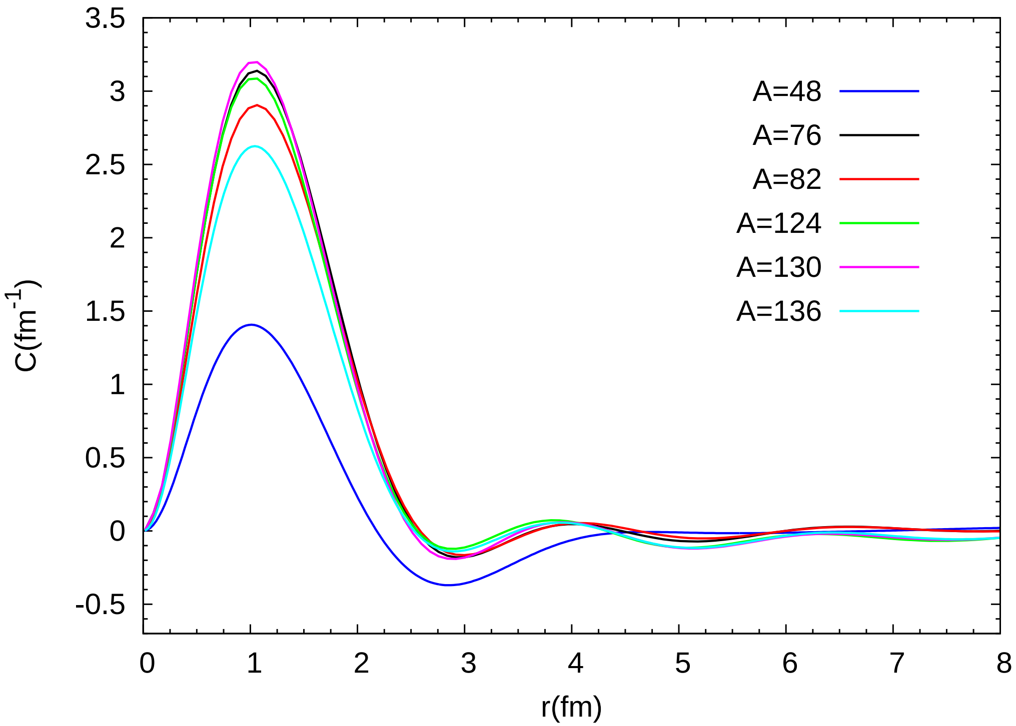
But $M_F^{0\nu}/M_{GT}^{0\nu} \approx 0.3$

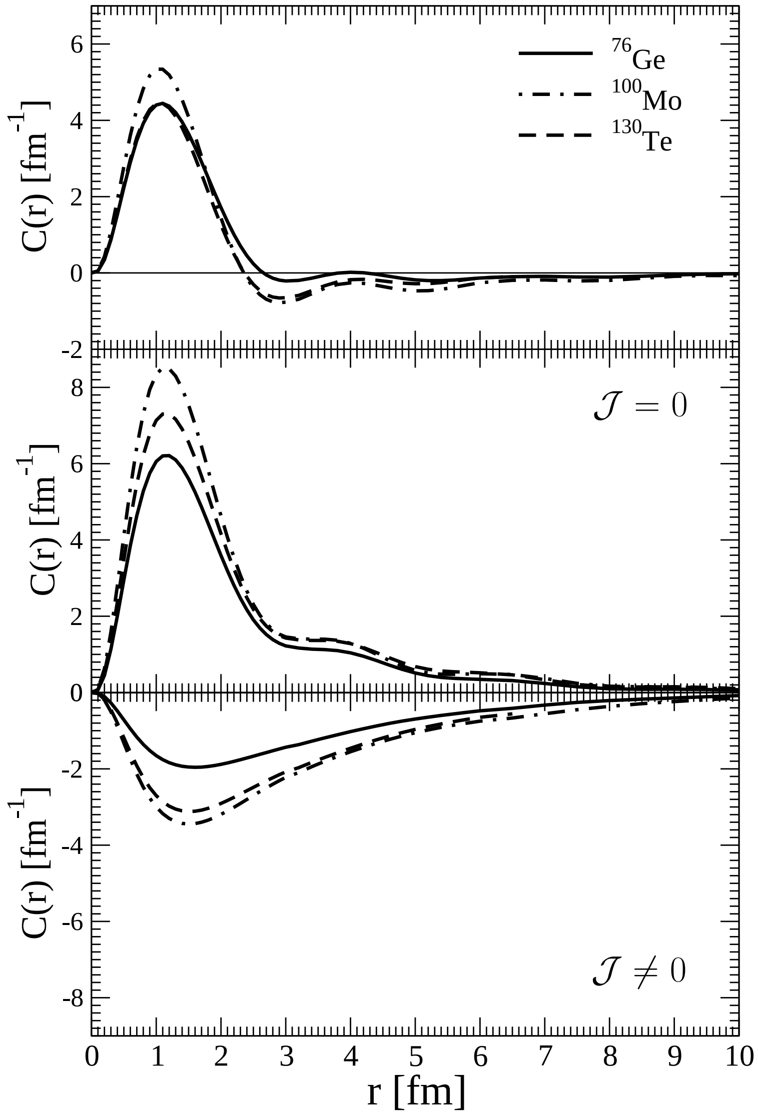
Ratio $M_F^{0\nu}/M_{GT}^{0\nu}$

may be more reliably calculable than $M_F^{0\nu}$ and $M_{GT}^{0\nu}$ separately

Measuring $M_F^{0\nu}$

$$\int_0^\infty C(r)dr = M^{0\nu}$$





Measuring $M_F^{0\nu}$

Only small r_{ab} ~1–2 fm determine $M^{0\nu}$

$\Rightarrow$ nucleon pairs in the relative s-wave contribute $\Rightarrow$ $T = 1$, $S = 0$ pairs

$$\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 |S = 0, T = 1\rangle = -3|S = 0, T = 1\rangle$$

$$\Downarrow$$

$$M_{GT}^{0\nu} = -3 M_F^{0\nu}$$

provided the neutrino potential is the same in both F and GT cases

High-order terms of nucleon weak current $\Rightarrow$ $M_{GT}^{0\nu}/M_F^{0\nu} \approx -2.5$

Basic requirements for a charge-exchange probe

Measure cross section ≡ Know $\langle IAS|\hat{T}^{+}|0^{+}_{f}\rangle$
???

Reaction analysis

Any hadronic probe adds isospin to nuclear system
(weak interaction probe would be ideal)

to probe small admixture of $|DIAS\rangle$ to $|0_f^+\rangle$
$\Rightarrow$ must be forbidden to connect in reaction
main components of $|IAS\rangle$ and $|0_f^+\rangle$ ($\Delta T = 2$)

Only $T = \frac{1}{2}$ probes ((n, p), $(t, {}^3\mathrm{He})$, ...)

Reaction analysis

$$\sigma_{np}(0_f^+ \rightarrow IAS) \propto \langle IAS | \hat{T}^+ | 0_f^+ \rangle$$

???

$$|0_i^+\rangle = |T_0\, T_0\rangle; \qquad |IAS\rangle = \frac{\hat{T}^-}{\sqrt{2T_0}}|0_i^+\rangle + \alpha\,|T_0-1\; T_0-1\rangle$$

$$|0_f^+\rangle = |T_0-2\; T_0-2\rangle + \beta\,|T_0-1\; T_0-2\rangle + \gamma\,\frac{(\hat{T}^-)^2}{\sqrt{4T_0(2T_0-1)}}|0_i^+\rangle$$

$$= |DIAS\rangle$$

Reaction analysis

^{82}Se

$\sigma_{np}(\gamma DIAS \rightarrow IAS)$ is 10 times large than other mechanisms

IAS of ^{48}Ca ($T = 4, T_z = 3$) in ^{48}Sc

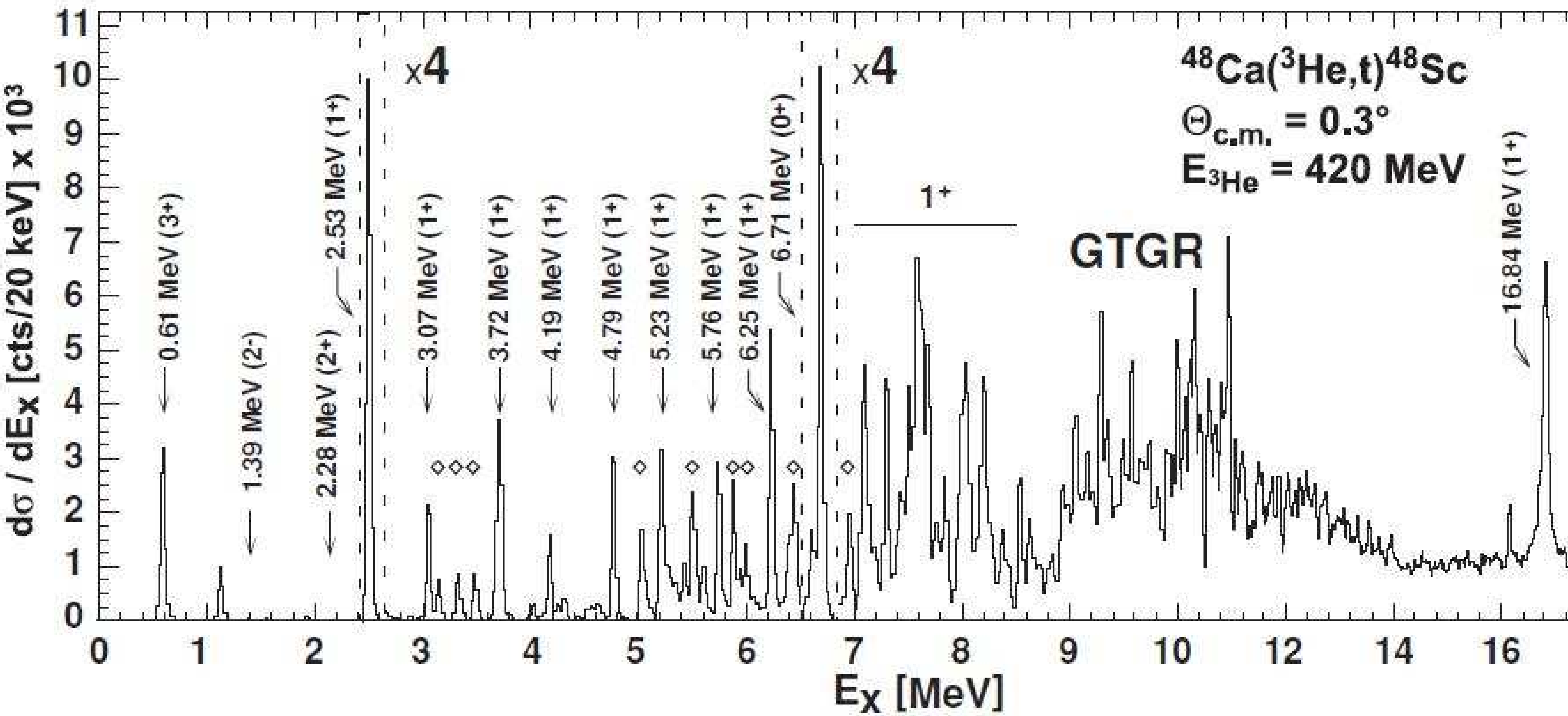
1. locates at E_x =6.678 MeV
2. 100% γ-decay to 1^+ state at E_x =2.517 MeV (E_γ=4.160 MeV)

Reaction analysis

E. W. Grewe, D. Frekers *et al.*, PRC **76**, 054307 (2007)

Resolution = 40 keV

IAS↓



Conclusions

- $0\nu\beta\beta$-decay is an *experimentum crucis* for revealing the Majorana nature of neutrinos and a feasible way to determine the absolute neutrino mass scale down to 10 meV's.

- Uncertainties in the QRPA calculations of $M^{0\nu}$ could be greatly reduced by using the experimental data on $2\nu\beta\beta$.

- The $M^{0\nu}$ of the SM are substantially smaller than the QRPA and other $M^{0\nu}$. Reason for such a deviation is under active study now (too small basis of the SM?).

Conclusions

- Reliable $M^{0\nu}$:
  Understanding essential nuclear physics of $0\nu\beta\beta$-decay
  Devising nuclear model capable of catching this physics
  Trying to separate out less model-dependent contributions to $M^{0\nu}$
  Comparison with relevant experimental data is indispensible

- Non-accelerator methods playing more and more important role in deciphering new physics.
  $M^{0\nu}$ — very probably not the last input needed from nuclear physics.

Supported by: DFG SFB TR27 "Neutrinos and beyond"