Models for neutrino mass

Stefano Morisi
AHEP Group, IFIC, CISIC, University of Valencia

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Outline

-lepton mixing

-tri-bimaximal ansatz (TBM)

-neutrino mass hierarchy: absolute scale & 0nubb

-Majorana mass and seesaw mechanisms

-neutrino mass matrix and TBM

-discrete groups

-a prototype A4 model

-A4 breaking & vacuum alignment problem

-an A4 model with accidental TBM

-conclusions

Lepton mixing

$$\mathfrak{L} = ig2^{-1/2}W^-_\mu \sum_{a=1}^{n} \overline{E}_{aL}\gamma_\mu \rho_{aL} + \text{H.c.}$$

$$= ig2^{-1/2}W^-_\mu \sum_{a,b,\alpha} \overline{e}_{bL}\gamma_\mu \Omega^*_{ab} U_{a\alpha}\nu_{\alpha L} + \text{H.c.}$$

$$\Rightarrow \quad K_{b\alpha} = \sum_{c=1}^{n} (\Omega^\dagger)_{bc} U_{c\alpha}$$

$$\begin{matrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{matrix} \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix} \times \text{diag}(e^{i\alpha_1/2},\ e^{i\alpha_2/2},\ 1)$$

Lepton mixing

parameter	Ref. [1] best fit$\pm 1\sigma$	Ref. [1] 3σ interval	Ref. [2] (MINOS updated) best fit$\pm 1\sigma$	Ref. [2] (MINOS updated) 3σ interval
Δm^2_{21} [10^{-5}eV^2]	$7.65^{+0.23}_{-0.20}$	7.05–8.34	$7.67^{+0.22}_{-0.21}$	7.07–8.34
Δm^2_{31} [10^{-3}eV^2]	$\pm 2.40^{+0.12}_{-0.11}$	$\pm$(2.07–2.75)	-2.39 ± 0.12 $+2.49 \pm 0.12$	$-$(2.02–2.79) $+$(2.13–2.88)
$\sin^2\theta_{12}$	$0.304^{+0.022}_{-0.016}$	0.25–0.37	$0.321^{+0.023}_{-0.022}$	0.26–0.40
$\sin^2\theta_{23}$	$0.50^{+0.07}_{-0.06}$	0.36–0.67	$0.47^{+0.07}_{-0.06}$	0.33–0.64
$\sin^2\theta_{13}$	$0.01^{+0.016}_{-0.011}$	≤ 0.056	0.003 ± 0.015	≤ 0.049

Schwetz, Tortola, Valle,NJP10 (08')

Gonzalez,Maltoni, PR460 (08')

Table 1: Global 3ν oscillation analysis (2008): best-fit values and allowed n_σ ranges, from Ref. [4].

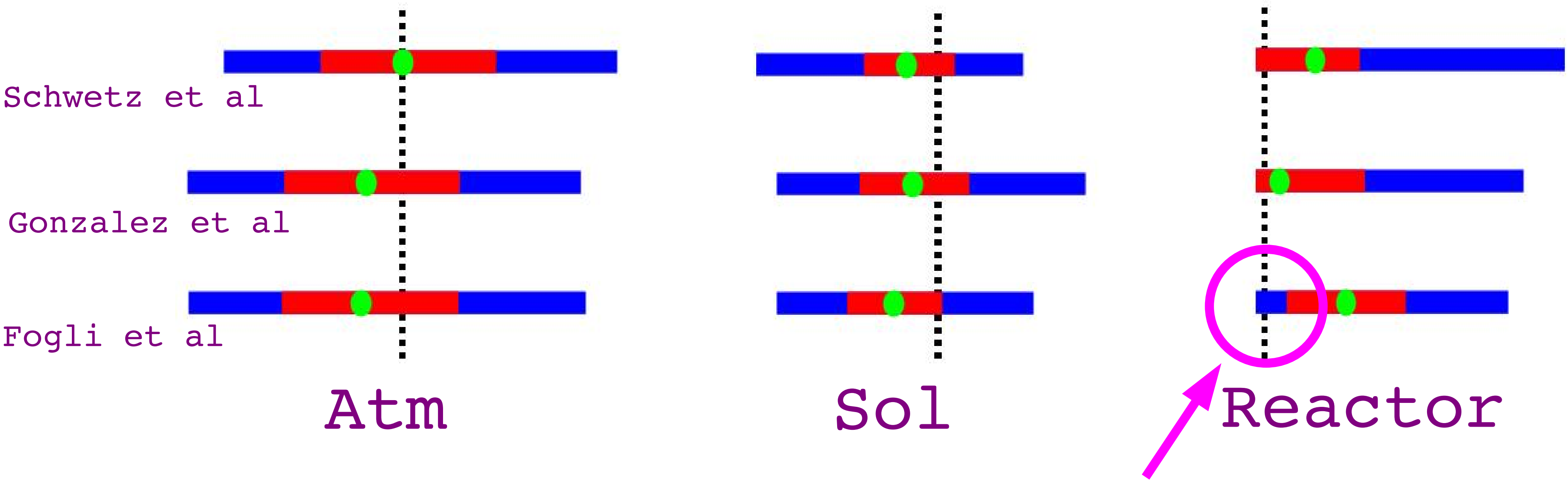
Parameter	$\delta m^2/10^{-5}$ eV^2	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	$\Delta m^2/10^{-3}$ eV^2
Best fit	7.67	0.312	0.016	0.466	2.39
1σ range	7.48 – 7.83	0.294 – 0.331	0.006 – 0.026	0.408 – 0.539	2.31 – 2.50
2σ range	7.31 – 8.01	0.278 – 0.352	< 0.036	0.366 – 0.602	2.19 – 2.66
3σ range	7.14 – 8.19	0.263 – 0.375	< 0.046	0.331 – 0.644	2.06 – 2.81

Fogli,Lisi,Marrone,Palazzo, PPNP57(06')

Tri-Bimaximal ansatz

Harrison, Perkins, Scott, 2002

$$\sin^2\theta_{23} = 0.5 \qquad \sin^2\theta_{12} = 1/3 \qquad \sin^2\theta_{13} = 0$$



with this exception

tri-bimaximal is in good agreement within one sigma

Tri-Bimaximal mixing

$$U_{\rm HPS} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

Bimaximal

$$\nu_3 = \frac{1}{\sqrt{2}}(-\nu_\mu + \nu_\tau)$$

trimaximal

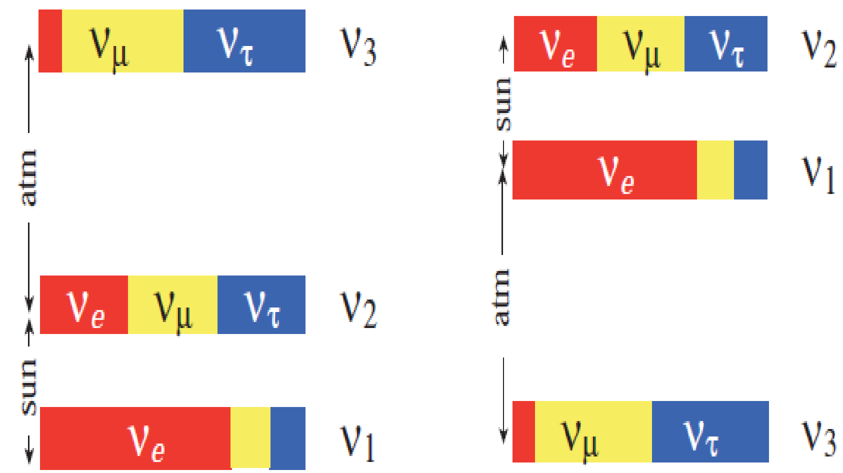
$$\nu_2 = \frac{1}{\sqrt{3}}(\nu_e + \nu_\mu + \nu_\tau)$$

other ansatz

Albright,Dueck,Rodejohann 1004.2798

- mu-tau
- trimaximal
- tetramaximal
- symmetric mixing
- bimaximal
- hexagon mixing
- golden
- quark-lepton complementarity

mass hierarchies



NH **IH**

parameter	best fit	2σ	3σ
$\Delta m^2_{21}\ [10^{-5}\mathrm{eV}^2]$	$7.59^{+0.23}_{-0.18}$	7.22–8.03	7.03–8.27
$\lvert\Delta m^2_{31}\rvert\ [10^{-3}\mathrm{eV}^2]$	$2.40^{+0.12}_{-0.11}$	2.18–2.64	2.07–2.75

absolute value

$$\lambda_C < \frac{m_2^\nu}{m_3^\nu} < 1 \qquad 0 < \frac{m_1^\nu}{m_2^\nu} < 1$$

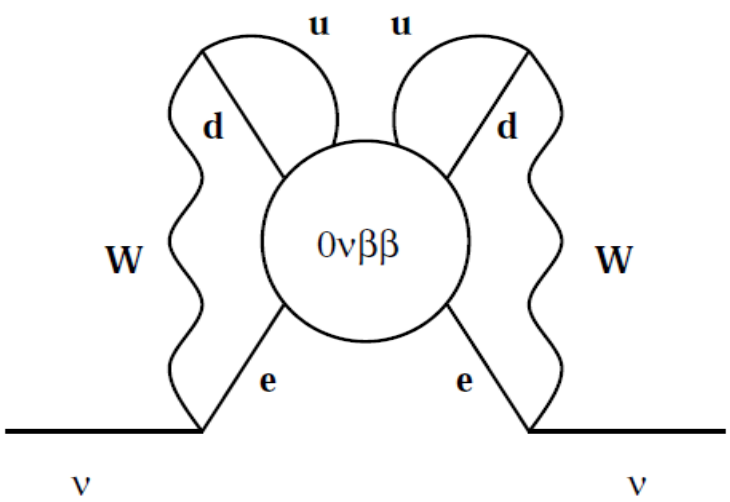
Absolute neutrino mass scale unknown

charged fermions

$$\frac{m_2^u}{m_3^u} \approx \lambda_C^4 \qquad \frac{m_1^u}{m_2^u} \approx \lambda_C^3$$

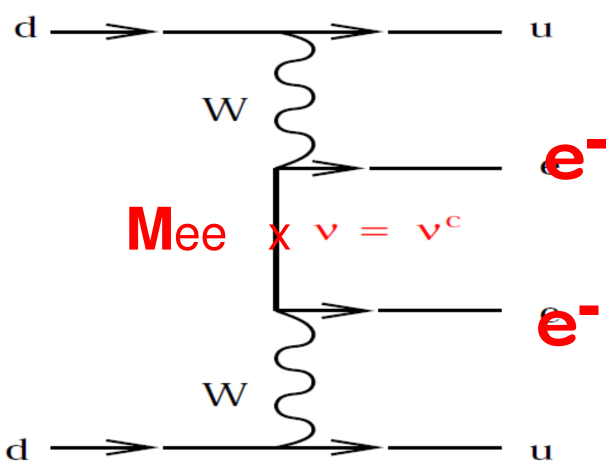
$$\frac{m_2^{d,l}}{m_3^{d,l}} \approx \lambda_C^2 \qquad \frac{m_1^{d,l}}{m_2^{d,l}} \approx \lambda_C^2$$

absolute neutrino mass scale



BLACK BOX THEOREM:
If neutrinoless double beta decay is observed, neutrino has Majorana mass

Schechter, Valle PRD25 (82')



$$M_{ee} = |m_1 U_{e1}^2 + m_2 U_{e2}^2 + m_3 U_{e3}^2|$$

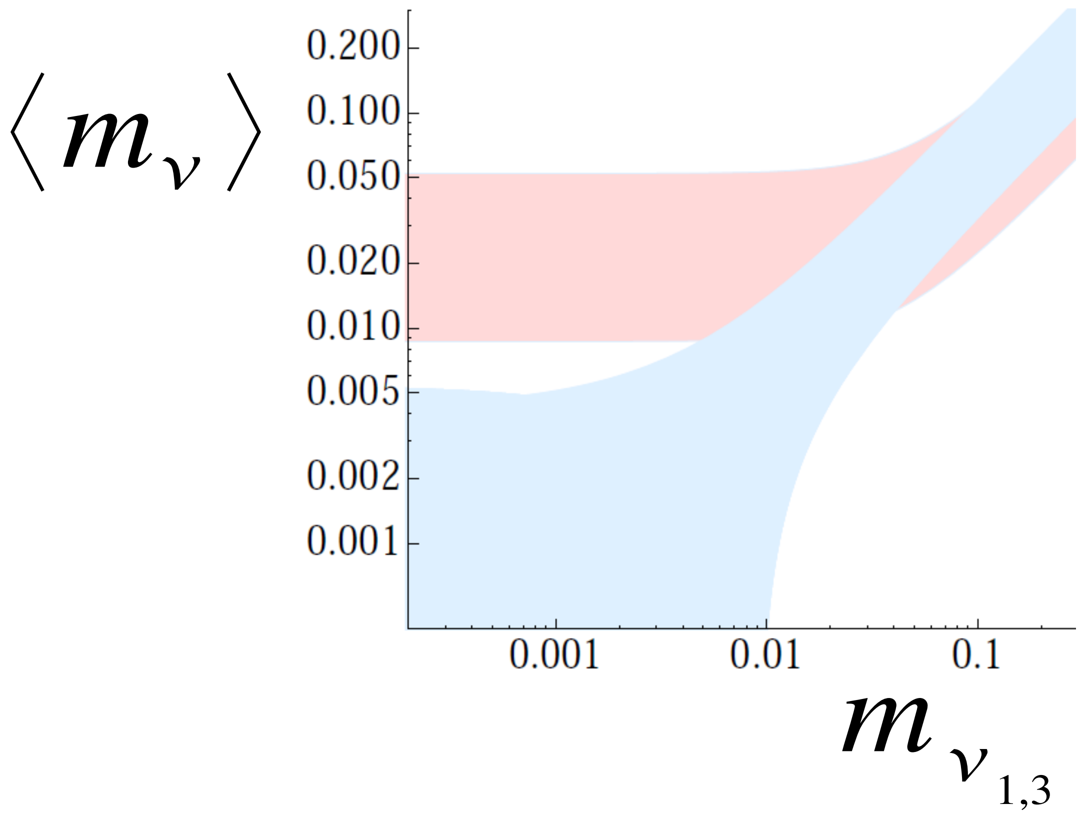
ΔL =2

0nubb

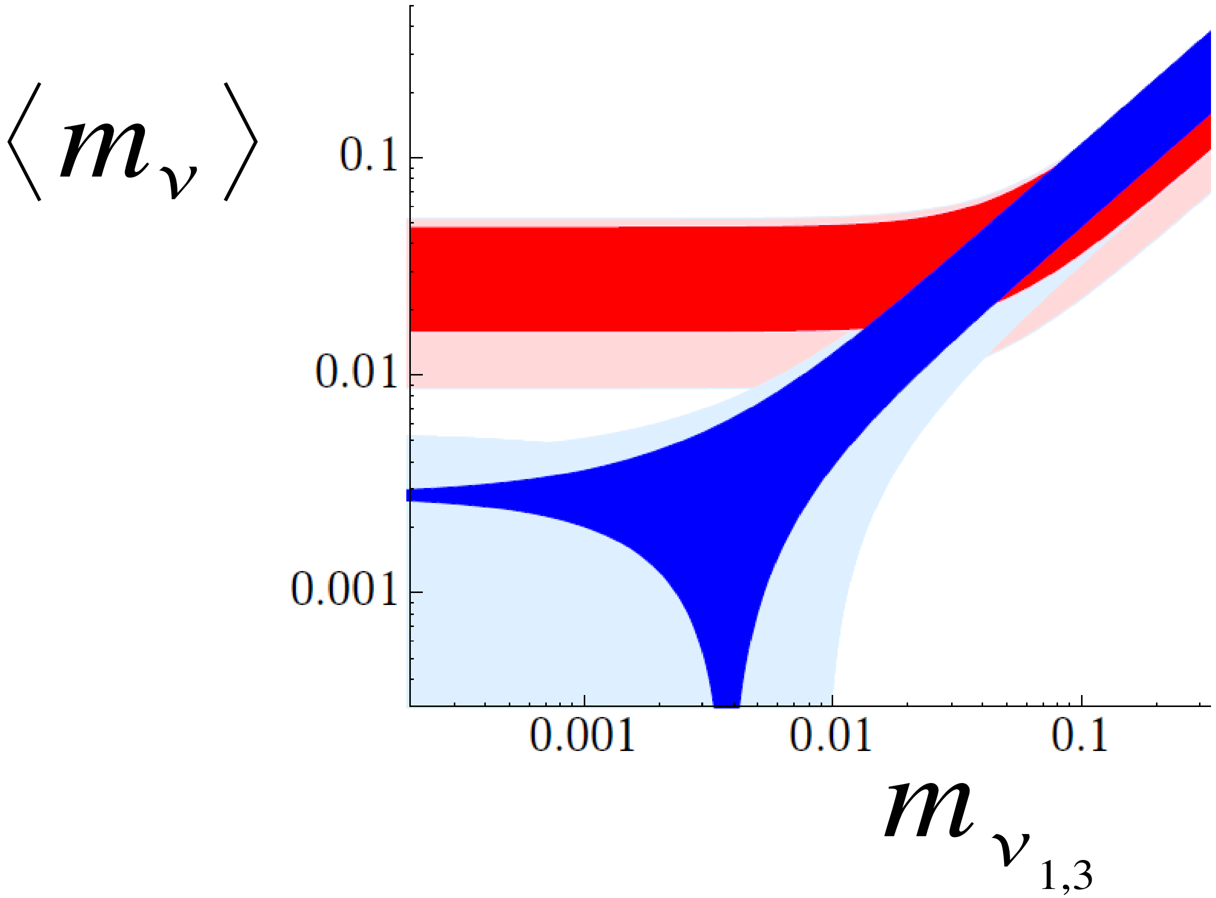
Hirsch 0609146
Petcov 0504166

$$\langle m_\nu \rangle = c_\odot^2 c_R^2 m_{\nu_1} + s_\odot^2 c_R^2 e^{i\alpha}\sqrt{m_{\nu_1}^2 + \Delta m_\odot^2} + s_R^2 e^{i\beta}\sqrt{m_{\nu_1}^2 + \Delta m_\odot^2 + \Delta m_{\rm Atm}^2},$$ normal hierarchy

$$\langle m_\nu \rangle = c_\odot^2 c_R^2 \sqrt{m_{\nu_3}^2 - \Delta m_\odot^2 + \Delta m_{\rm Atm}^2} + s_\odot^2 c_R^2 e^{i\alpha}\sqrt{m_{\nu_3}^2 + \Delta m_{\rm Atm}^2} + s_R^2 e^{i\beta} m_{\nu_3}$$ inverse hierarchy

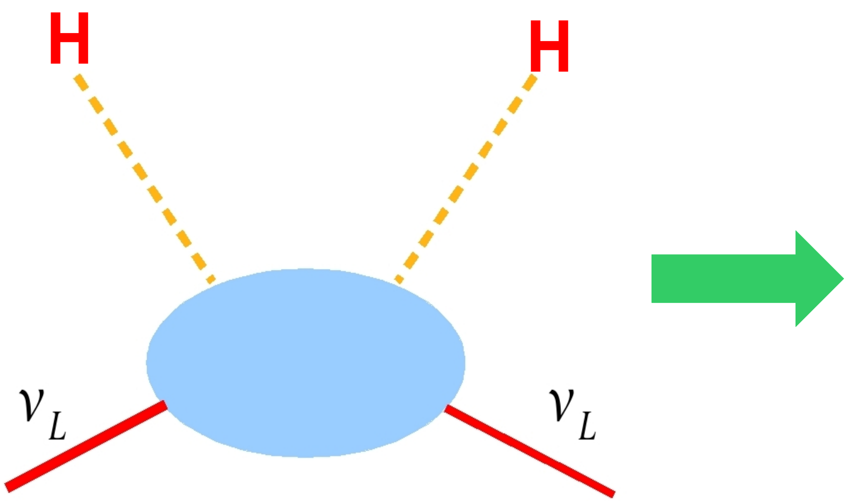


0nubb & TBM



Majorana mass

If lepton number is violated



$$LH\,LH\,/\,\mathcal{M}$$

Dim-5 operator, Weinberg (80)

There are many seesaw realizations:

type-I, II, III, linear, inverse seesaw,
R parity breaking, radiatively

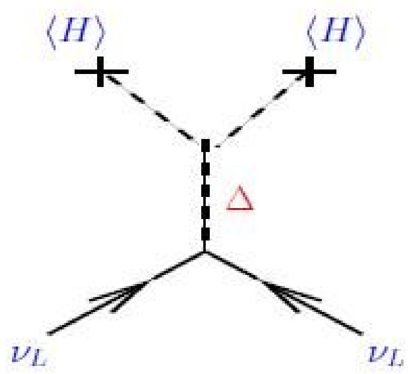
Seesaw type-I & II

In the basis $\nu_L, \ \nu_L^c$

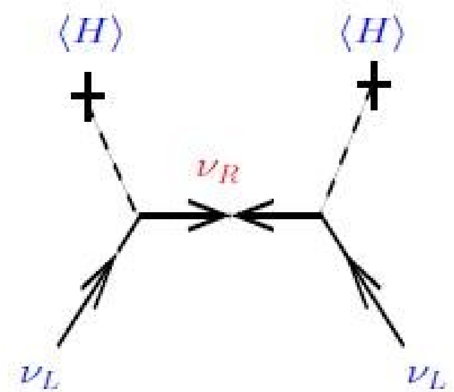
$$M_\nu = \begin{pmatrix} M_L & M_D \\ M_D^T & M_N \end{pmatrix}$$

Minkowski 77
Yanagita 79
Gell-Mann,Ramond,Slansky 79
Mohapatra,Senjanovic 80

$$m_\nu = -M_D M_N^{-1} M_D^T$$



Schechter,Valle 80,82
Cheng,Li 80
Mohapatra, Senjanovic 81



type-III

Similar to type-I:
right-handed neutrino replaced
with a SU(2)-triplet *Y=0*

$$\Sigma = \begin{pmatrix} \Sigma^0/\sqrt{2} & \Sigma^+ \\ \Sigma^- & -\Sigma^0/\sqrt{2} \end{pmatrix}$$

Foot et al 88
Ma, 98
Ma,Roy, 02
Bajc,Senjanovic, 07
Abada et al, 07

$$M_\nu = \begin{pmatrix} 0 & M_D \\ M_D^T & M_\Sigma \end{pmatrix} \Rightarrow m_\nu = -M_D^T M_\Sigma^{-1} M_D$$

Inverse seesaw

Mohapatra,Valle PRD34

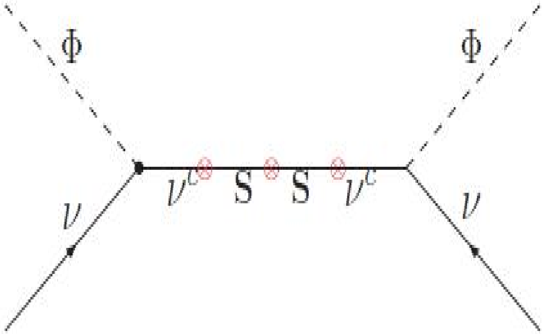
Extra singlets ***S***

$$\nu,\ \nu^c,\ S$$

$$M_\nu = \begin{pmatrix} 0 & M_D & 0 \\ M_D^T & 0 & M \\ 0 & M^T & \mu \end{pmatrix}$$

μ **Breaks lepton number**

M at EW or TeV scale
μ at KeV scale



$$m_\nu = M_D {M^T}^{-1} \mu M^{-1} M_D^T$$

$\mu \to 0$ **Lepton number is conserved** → Three massless neutrinos

Linear seesaw

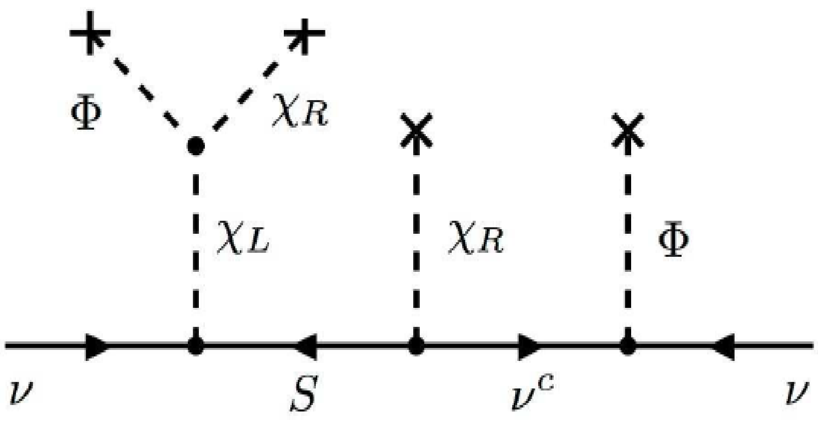
Malinsky, Romao, Valle, PRL95 (05)

$$SO(10) \xrightarrow{M_G} \mathbf{3}_c\mathbf{2}_L\mathbf{2}_R\mathbf{1}_{B-L} \xrightarrow{V_R} \mathbf{3}_c\mathbf{2}_L\mathbf{1}_{I_{3R}}\mathbf{1}_{B-L} \xrightarrow{v_R} \mathbf{3}_c\mathbf{2}_L\mathbf{1}_Y$$

Breaks B-L

$$M_\nu = \begin{pmatrix} 0 & Yv & Fv_L \\ Y^T v & 0 & \tilde{F} v_R \\ F^T v_L & \tilde{F}^T v_R & 0 \end{pmatrix}$$

$$v_L \sim \frac{v\, v_R}{M_G}$$



Linear in Y

$$M_\nu \simeq \frac{v^2}{M_G}\rho\left[Y(F\tilde{F}^{-1})^T + (F\tilde{F}^{-1})Y^T\right]$$

Neutrino mass is suppressed by M_G irrespectively how low is *B-L breaking scale*

Neutrino mass matrix and mu-tau

Grimus, Joshipura, Kaneko, Lavoura,Tanimoto JHEP0407

$$m = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix}$$

$$\mu \leftrightarrow \tau$$

$$O = \begin{pmatrix} -\cos\theta & \sin\theta & 0 \\ \frac{1}{\sqrt{2}}\sin\theta & \frac{1}{\sqrt{2}}\cos\theta & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}\sin\theta & \frac{1}{\sqrt{2}}\cos\theta & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\sin^2\theta_{13} = 0$$

$$\sin^2\theta_{12} = 0.5$$

Neutrino mass matrix and TBM

$$m = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix}$$

$$m_{\nu_{1,1}} + m_{\nu_{1,3}} = m_{\nu_{2,2}} + m_{\nu_{2,3}}$$

$$\sin^2 \theta_{23} = 1/3$$

The Flavor Problem:

-why fermion mass hierarchies?
-why flavor eigenstates are mixed?
-why quarks and leptons mixing are so different?

-Why tri-bimaximal lepton mixing?

$$SU_c(3) \times SU_L(2) \times U_Y(1) \times G_f$$

Gf = SO(3) or SU(3) ⇒ discrete subgroups

spontaneously

Adulpravitchai, Blum, Lindner JHEP 0909
Berger, Grossman JHEP 1002

Discrete groups

A group **G={A,B,C,...}** which consist of a finite number of elements **g** is a *finite group if*

- the set is close with respect to the composition law
- associative
- cancelling rule: A X = B X and Y A = Y B ⇒ A = B

To each finite group correspond a multiplication table

g=6	I	A	B	C	D	E
I	I	A	B	C	D	E
A	A	B	I	E	C	D
B	B	I	A	D	E	C
C	C	D	E	I	A	B
D	D	E	C	B	I	A
E	E	C	D	A	B	I

Not all the product are independent:

A C = E, C B = E, B B = A, C B = E, A E = D ➡ C A = D

It exist a set of **elements** and a set of **independent relations** associated to each multiplication table

Generators of the group

Set of elements

A, C

Set of relations

$$A^3 = C^2 = (AC)^2 = I$$

$$I, A, A^2, C, AC, CA$$

		I	A	B	C	D	E
$I =$	I	I	A	B	C	D	E
$A =$	A	A	B	I	E	C	D
$A^2 =$	B	B	I	A	D	E	C
$C =$	C	C	D	E	I	A	B
$CA =$	D	D	E	C	B	I	A
$AC =$	E	E	C	D	A	B	I

Classification of the group of order < 32

Frampton and Kephart, PRD64 (01)

order	groups
6	$S_3 \equiv D_3$
8	$D_4,\ Q = Q_4$
10	D_5
12	$D_6,\ Q_6,\ T \equiv A_4$
14	D_7
16	$D_8,\ Q_8,\ Z_2 \times D_4,\ Z_2 \times Q$
18	$D_9,\ Z_3 \times D_3$
20	$D_{10},\ Q_{10}$
22	D_{11}
24	$D_{12}, Q_{12}, Z_2 \times D_6, Z_2 \times Q_6, Z_2 \times T, Z_3 \times D_4, Z_3 \times Q, Z_4 \times D_3, S_4$
26	D_{13}
28	$D_{14},\ Q_{14}$
30	$D_{15},\ D_5 \times Z_3,\ D_3 \times Z_5$

For a review of discrete group see also
Ishimori et al 1003.3552

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18	$D_9,\ Z_3 \times D_3$
20	$D_{10},\ Q_{10}$
22	D_{11}
24	$D_{12}, Q_{12}, Z_2 \times D_6, Z_2 \times Q_6, Z_2 \times T, Z_3 \times D_4, Z_3 \times Q, Z_4 \times D_3, S_4$
26	D_{13}
28	$D_{14},\ Q_{14}$
30	$D_{15},\ D_5 \times Z_3,\ D_3 \times Z_5$

$$\langle A, B \mid A^2, B^3, (AB)^4 \rangle$$

S_n **Pemutation group of n objects of order n!**

Classification of the group of order < 32

Frampton and Kephart, PRD64 (01)

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6	$S_3 \equiv D_3$
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18	$D_9, Z_3 \times D_3$
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22	D_{11}
24	$D_{12}, Q_{12}, Z_2 \times D_6, Z_2 \times Q_6, Z_2 \times T, Z_3 \times D_4, Z_3 \times Q, Z_4 \times D_3, S_4$
26	D_{13}
28	D_{14}, Q_{14}
30	$D_{15}, D_5 \times Z_3, D_3 \times Z_5$

Hypercomplex number

$$a + bi + cj + dk$$

$$i^4 = 1, i^2 = j^2, ji = i^3 j$$

⇕

$$A^4 = I, A^2 = B^2, AB = B^3 A$$

Q_n **Quaternion series of order 2 n**

Classification of the group of order < 32

Frampton and Kephart, PRD64 (01)

order	groups
6	$S_3 \equiv D_3$
8	$D_4, Q = Q_4$
10	D_5
12	$D_6, Q_6, T \equiv A_4$
14	D_7
16	$D_8, Q_8, Z_2 \times D_4, Z_2 \times Q$
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24	$D_{12}, Q_{12}, Z_2 \times D_6, Z_2 \times Q_6, Z_2 \times T, Z_3 \times D_4, Z_3 \times Q, Z_4 \times D_3, S_4$
26	D_{13}
28	D_{14}, Q_{14}
30	$D_{15}, D_5 \times Z_3, D_3 \times Z_5$

$$\left\langle A, B \,\middle|\, A^n, B^2, (AB)^2 \right\rangle$$

D_n **Dihedral series of order 2 n**

Classification of the group of order < 32

Frampton and Kephart, PRD64 (01)

order	groups
6	$S_3 \equiv D_3$
8	$D_4,\ Q = Q_4$
10	D_5
12	$D_6,\ Q_6,\ T \equiv A_4$
14	D_7
16	$D_8,\ Q_8,\ Z_2 \times D_4,\ Z_2 \times Q$
18	$D_9,\ Z_3 \times D_3$
20	$D_{10},\ Q_{10}$
22	D_{11}
24	$D_{12}, Q_{12}, Z_2 \times D_6, Z_2 \times Q_6, Z_2 \times T, Z_3 \times D_4, Z_3 \times Q, Z_4 \times D_3, S_4$
26	D_{13}
28	$D_{14},\ Q_{14}$
30	$D_{15},\ D_5 \times Z_3,\ D_3 \times Z_5$

Smallest group with triplet irrep 3

$$\langle S, T \mid S^2, T^3, (ST)^3 \rangle$$

A_n **Alterning series of order n!/2**

A4 group

12 elem.

C1: I

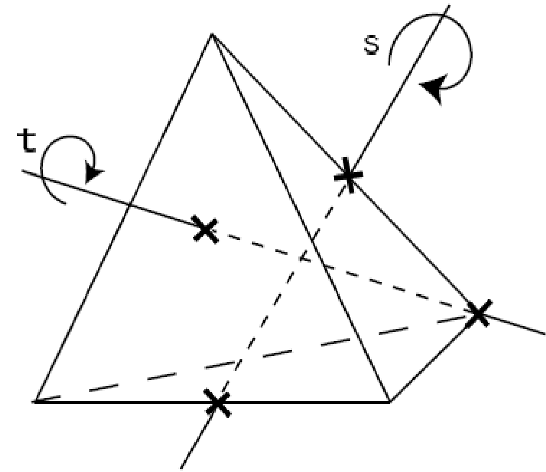
C2: **T**, ST, TS, STS

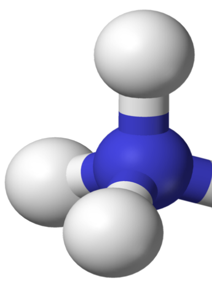
C3: TT, STT,TTS,TST

C4: **S**, TTST, TSTT

Isomorphic to group of thetraedron rotations

12 rotations





CH4

$$\langle S, T \mid S^2=I, T^3=I, (ST)^3=I \rangle$$

Z_2 ;C2 Z_3 ;C4

Subgroups of A4

A4 product rules

Class	χ^1	$\chi^{1'}$	$\chi^{1"}$	χ^3
C_1	1	1	1	3
C_2	1	ω	ω^2	0
C_3	1	ω^2	ω	0
C_4	1	1	1	-1

$$1' \times 1' = 1'', \; 1' \times 1'' = 1, \; 1'' \times 1'' = 1' \text{ etc.}$$

$$3 \times 3 = 1 + 1' + 1'' + 3 + 3$$

$\omega^3 = 1$

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \; T = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

S-diag basis

$$1 = a_1 b_1 + a_2 b_2 + a_3 b_3$$

$$1' = a_1 b_1 + \omega^2 a_2 b_2 + \omega a_3 b_3$$

$$1" = a_1 b_1 + \omega a_2 b_2 + \omega^2 a_3 b_3$$

$$3 \sim (a_2 b_3, a_3 b_1, a_1 b_2)$$

$$3 \sim (a_3 b_2, a_1 b_3, a_2 b_1)$$

A4 product rules

$$T' \;=\; VTV^\dagger = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix},$$

$$S' \;=\; VSV^\dagger = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

$$1 = a_1b_1 + a_2b_3 + a_3b_2$$

$$1' = a_3b_3 + a_1b_2 + a_2b_1$$

$$1'' = a_2b_2 + a_1b_3 + a_3b_3$$

$$3_{symm} \sim \frac{1}{3}(2a_1b_1 - a_2b_3 - a_3b_2, 2a_3b_3 - a_1b_2 - a_2b_1, 2a_2b_2 - a_1b_3 - a_3b_1)$$

$$3_{antisymm} \sim \frac{1}{2}(a_2b_3 - a_3b_2, a_1b_2 - a_2b_1, a_1b_3 - a_3b_1)$$

Models with A4 symmetry: matter assignments

Type	L_i	ℓ_i^c	ν_i^c	Δ
A1	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$	-	-
A2				$\underline{1}, \underline{1}', \underline{1}'', \underline{3}$
B1	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{3}$	-
B2				$\underline{1}, \underline{3}$
C1	$\underline{3}$	$\underline{3}$	-	-
C2				$\underline{1}$
C3				$\underline{1}, \underline{3}$
C4				$\underline{1}, \underline{1}', \underline{1}'', \underline{3}$
D1	$\underline{3}$	$\underline{3}$	$\underline{3}$	-
D2				$\underline{1}$
D3				$\underline{1}'$
D4				$\underline{1}', \underline{3}$
E	$\underline{3}$	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$	-
F	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{3}$	$\underline{3}$	$\underline{1}$ or $\underline{1}'$
G	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{1}, \underline{1}', \underline{1}''$	-
H	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	-	-
I	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	$\underline{1}, \underline{1}, \underline{1}$	-
J	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	$\underline{3}$	-

Barry, Rodejohann 1003.2385
(Morisi 0807.4013)

Models with A4 symmetry

Ma, Rajasekaran PRD64(01')
Babu, Ma, Valle PLB552(02')

	L	l_1^c	l_2^c	l_3^c	ν_1^c	H	ϕ	ϕ'
$SU(2)$	2	1	1	1	1	2	1	1
A_4	3	1	$1''$	$1'$	3	1	3	3

- Altarelli, Feruglio NPB720 (05')

$$\mathcal{L} = y_e(L\phi')l_1^c + y_\mu(L\phi')'l_2^c + y_\tau(L\phi')''l_3^c + y_D(L\nu^c)H + y_a(\nu^c\nu^c) + y_b(\nu^c\nu^c\phi)$$

$$\phi \Longleftrightarrow \phi'$$

We assume extra Abelian symmetries Z3, Z4,...
to forbid unwanted terms

Models with A4 symmetry

	L	l_1^c	l_2^c	l_3^c	ν_1^c	H	ϕ	ϕ'
$SU(2)$	2	1	1	1	1	2	1	1
A_4	3	1	$1''$	$1'$	3	1	3	3

$$\mathcal{L} = y_e(L\phi')l_1^c + y_\mu(L\phi')'l_2^c + y_\tau(L\phi')''l_3^c + y_D(L\nu^c)H + y_a(\nu^c\nu^c) + y_b(\nu^c\nu^c\phi)$$

$$(L\phi') = L_1\phi'_1 + L_2\phi'_3 + L_3\phi'_2$$

Models with A4 symmetry

	L	l_1^c	l_2^c	l_3^c	ν_1^c	H	ϕ	ϕ'
$SU(2)$	2	1	1	1	1	2	1	1
A_4	3	1	$1''$	$1'$	3	1	3	3

$$\mathcal{L} = y_e(L\phi')l_1^c + y_\mu(L\phi')'l_2^c + y_\tau(L\phi')''l_3^c + y_D(L\nu^c)H + y_a(\nu^c\nu^c) + y_b(\nu^c\nu^c\phi)$$

$$(L\phi') = L_1\phi_1' + L_2\phi_3' + L_3\phi_2'$$

$$\langle\phi'\rangle \sim (1,0,0)$$

Models with A4 symmetry

	L	l_1^c	l_2^c	l_3^c	ν_1^c	H	ϕ	ϕ'
$SU(2)$	2	1	1	1	1	2	1	1
A_4	3	1	$1''$	$1'$	3	1	3	3

$$\mathcal{L} = y_e(L\phi')l_1^c + y_\mu(L\phi')'l_2^c + y_\tau(L\phi')''l_3^c + y_D(L\nu^c)H + y_a(\nu^c\nu^c) + y_b(\nu^c\nu^c\phi)$$

$$(L\phi') = L_1\phi'_1 + L_2\phi'_3 + L_3\phi'_2$$

$$\langle\phi'\rangle \sim (1,0,0)$$

$$M_l = \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

Models with A4 symmetry

	L	l_1^c	l_2^c	l_3^c	ν_1^c	H	ϕ	ϕ'
$SU(2)$	2	1	1	1	1	2	1	1
A_4	3	1	$1''$	$1'$	3	1	3	3

$$\mathcal{L} = y_e(L\phi')l_1^c + y_\mu(L\phi')'l_2^c + y_\tau(L\phi')''l_3^c + y_D(L\nu^c)H + y_a(\nu^c\nu^c) + y_b(\nu^c\nu^c\phi)$$

$$(L\phi')' = L_3\phi_3' + L_1\phi_2' + L_2\phi_1'$$

$$\langle\phi'\rangle \sim (1,0,0)$$

$$M_l = \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

Models with A4 symmetry

	L	l_1^c	l_2^c	l_3^c	ν_1^c	H	ϕ	ϕ'
$SU(2)$	2	1	1	1	1	2	1	1
A_4	3	1	$1''$	$1'$	3	1	3	3

$$\mathcal{L} = y_e(L\phi')l_1^c + y_\mu(L\phi')'l_2^c + y_\tau(L\phi')''l_3^c + y_D(L\nu^c)H + y_a(\nu^c\nu^c) + y_b(\nu^c\nu^c\phi)$$

$$(L\phi')'' = L_2\phi_2' + L_1\phi_3' + L_3\phi_1'$$

$$\langle\phi'\rangle \sim (1,0,0)$$

$$M_l = \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

Models with A4 symmetry

	L	l_1^c	l_2^c	l_3^c	ν_1^c	H	ϕ	ϕ'
$SU(2)$	2	1	1	1	1	2	1	1
A_4	3	1	$1''$	$1'$	3	1	3	3

$$\mathcal{L} = y_e(L\phi')l_1^c + y_\mu(L\phi')'l_2^c + y_\tau(L\phi')''l_3^c + y_D(L\nu^c)H + y_a(\nu^c\nu^c) + y_b(\nu^c\nu^c\phi)$$

$$\psi_1\varphi_1 + \psi_2\varphi_3 + \psi_3\varphi_2 \sim 1$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Models with A4 symmetry

	L	l_1^c	l_2^c	l_3^c	ν_1^c	H	ϕ	ϕ'
$SU(2)$	2	1	1	1	1	2	1	1
A_4	3	1	$1''$	$1'$	3	1	3	3

$$\mathcal{L} = y_e (L\phi') l_1^c + y_\mu (L\phi')' l_2^c + y_\tau (L\phi')'' l_3^c + y_D (L\nu^c) H + y_a (\nu^c \nu^c) + y_b (\nu^c \nu^c \phi)$$

$$(\nu^c \nu^c)_3 = \begin{pmatrix} 2\nu_1^c \nu_1^c - \nu_2^c \nu_3^c - \nu_3^c \nu_2^c \\ 2\nu_3^c \nu_3^c - \nu_1^c \nu_2^c - \nu_2^c \nu_1^c \\ 2\nu_2^c \nu_2^c - \nu_1^c \nu_3^c - \nu_3^c \nu_1^c \end{pmatrix}$$

Models with A4 symmetry

	L	l_1^c	l_2^c	l_3^c	ν_1^c	H	ϕ	ϕ'
$SU(2)$	2	1	1	1	1	2	1	1
A_4	3	1	$1''$	$1'$	3	1	3	3

$$\mathcal{L} = y_e(L\phi')l_1^c + y_\mu(L\phi')'l_2^c + y_\tau(L\phi')''l_3^c + y_D(L\nu^c)H + y_a(\nu^c\nu^c) + y_b(\nu^c\nu^c\phi)$$

$$(\nu^c\nu^c)_3 = \begin{pmatrix} 2\nu_1^c\nu_1^c - \nu_2^c\nu_3^c - \nu_3^c\nu_2^c \\ 2\nu_3^c\nu_3^c - \nu_1^c\nu_2^c - \nu_2^c\nu_1^c \\ 2\nu_2^c\nu_2^c - \nu_1^c\nu_3^c - \nu_3^c\nu_1^c \end{pmatrix} \qquad \langle\phi\rangle \sim (1,1,1)$$

$$M_R = \qquad + b\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$$

Models with A4 symmetry

	L	l_1^c	l_2^c	l_3^c	ν_1^c	H	ϕ	ϕ'
$SU(2)$	2	1	1	1	1	2	1	1
A_4	3	1	$1''$	$1'$	3	1	3	3

$$\mathcal{L} = y_e(L\phi')l_1^c + y_\mu(L\phi')'l_2^c + y_\tau(L\phi')''l_3^c + y_D(L\nu^c)H + y_a(\nu^c\nu^c) + y_b(\nu^c\nu^c\phi)$$

$$(\nu^c\nu^c)_1 = (\nu_1^c\nu_1^c + \nu_2^c\nu_3^c + \nu_3^c\nu_2^c)$$

$$M_R = a\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + b\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$$

Models with A4 symmetry

$$M_R = a\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + b\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$$

$$m_D \frac{1}{M_R} m_D^T \sim \begin{pmatrix} -a^2 + 2ab + 3b^2 & b(-a+3b) & b(-a+3b) \\ b(-a+3b) & b(2a+3b) & -a^2 - ab + 3b^2 \\ b(-a+3b) & -a^2 - ab + 3b^2 & b(2a+3b) \end{pmatrix}$$

$$\mu \leftrightarrow \tau$$

$$m_{\nu_{1,1}} + m_{\nu_{1,3}} = m_{\nu_{2,2}} + m_{\nu_{2,3}}$$

→ **TBM**

Models with A4 symmetry

$$M_R = a\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + b\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$$

$$m_D \frac{1}{M_R} m_D^T \sim \begin{pmatrix} -a^2 + 2ab + 3b^2 & b(-a+3b) & b(-a+3b) \\ b(-a+3b) & b(2a+3b) & -a^2 - ab + 3b^2 \\ b(-a+3b) & -a^2 - ab + 3b^2 & b(2a+3b) \end{pmatrix}$$

Only three free parameters, predicts three angles and absolute neutrino mass scale

A4 breaking

$$T' = VTV^\dagger = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix},$$

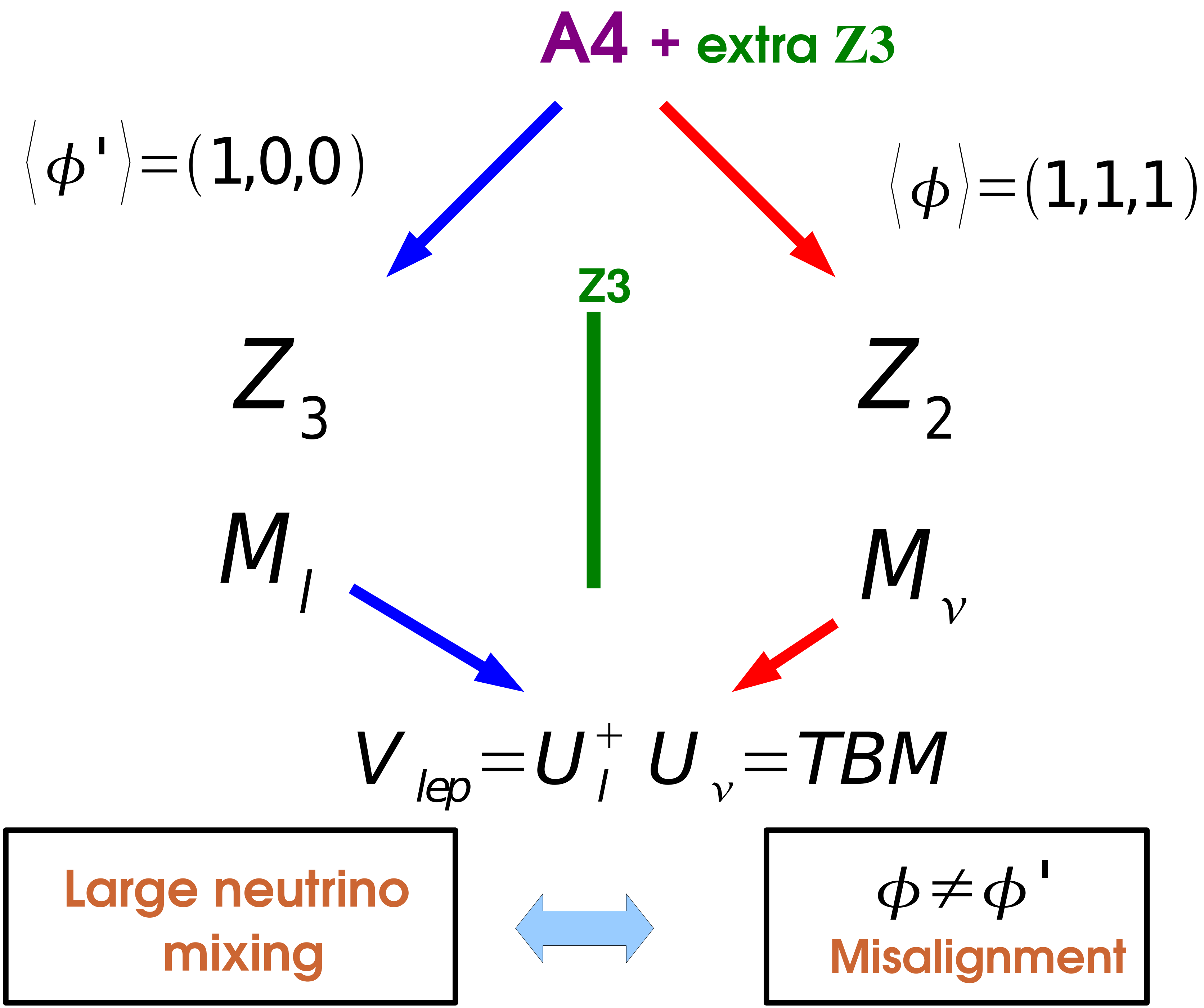
⇒ invariant $\langle \phi' \rangle \sim (1,0,0)$

$$S' = VSV^\dagger = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

⇒ $\langle \phi \rangle \sim (1,1,1)$

$T\,T\,T = I$ $\qquad A4 \xrightarrow{\phi'} Z3$

$S\,S = I$ $\qquad A4 \xrightarrow{\phi} Z2$



A4 + extra Z3
⟨φ'⟩=(1,0,0)
⟨φ⟩=(1,1,1)
Z3
Z_3
Z_2
M_l
M_ν
$V_{lep}=U_l^+ U_\nu = TBM$
Large neutrino mixing
$\phi \neq \phi'$
Misalignment

Vacuum alignment problem

Altarelli, Feruglio, NPB 720 05'

$$
\begin{aligned}
B_1 &= \varphi_1^2 + \varphi_2^2 + \varphi_3^2 \\
B_2 &= {\varphi'_1}^2 + {\varphi'_2}^2 + {\varphi'_3}^2 \\
T_1 &= \varphi_1\varphi_2\varphi_3 \\
T_2 &= \varphi_1\varphi'_2\varphi'_3 + \varphi_2\varphi'_3\varphi'_1 + \varphi_3\varphi'_1\varphi'_2 \\
Q_1 &= \varphi_1^2\varphi_2^2 + \varphi_2^2\varphi_3^2 + \varphi_3^2\varphi_1^2 \\
Q_2 &= |\varphi_1^2 + \omega^2\varphi_2^2 + \omega\varphi_3^2|^2 \\
Q_3 &= {\varphi'_1}^2{\varphi'_2}^2 + {\varphi'_2}^2{\varphi'_3}^2 + {\varphi'_3}^2{\varphi'_1}^2 \\
Q_4 &= |{\varphi'_1}^2 + \omega^2{\varphi'_2}^2 + \omega{\varphi'_3}^2|^2 \\
Q_5 &= \varphi_1\varphi_2\varphi'_1\varphi'_2 + \varphi_2\varphi_3\varphi'_2\varphi'_3 + \varphi_3\varphi_1\varphi'_3\varphi'_1 \\
Q_6 &= (\varphi_1^2 + \varphi_2^2 + \varphi_3^2)({\varphi'_1}^2 + {\varphi'_2}^2 + {\varphi'_3}^2) \\
Q_7 &= (\varphi_1^2 + \omega^2\varphi_2^2 + \omega\varphi_3^2)({\varphi'_1}^2 + \omega{\varphi'_2}^2 + \omega^2{\varphi'_3}^2)
\end{aligned}
$$

$$
\begin{aligned}
V &= \frac{M_1^2}{2}B_1^2 + \frac{M_2^2}{2}B_2^2 + \mu_1 T_1 + \mu_2 T_2 \\
&+ c_1 Q_1 + c_2 Q_2 + c_3 Q_3 + c_4 Q_4 \\
&+ c_5 Q_5 + c_6 Q_6 + (c_7 Q_7 + c.c) \quad ,
\end{aligned}
$$

Vacuum alignment problem

Altarelli, Feruglio, NPB 720 05'

$$\begin{aligned} \langle\varphi\rangle &= (v, v, v) \\ \langle\varphi'\rangle &= (v', 0, 0) \end{aligned}$$

$$\begin{aligned}
\frac{\partial V}{\partial \varphi_1} &= M_1^2 v + \mu_1 v^2 + 4c_1 v^3 + 2c_6 v v'^2 + 2(c_7 + \bar{c}_7) v v'^2 = 0 \\
\frac{\partial V}{\partial \varphi_2} &= M_1^2 v + \mu_1 v^2 + 4c_1 v^3 + 2c_6 v v'^2 + 2(\omega^2 c_7 + \omega \bar{c}_7) v v'^2 = 0 \\
\frac{\partial V}{\partial \varphi_3} &= M_1^2 v + \mu_1 v^2 + 4c_1 v^3 + 2c_6 v v'^2 + 2(\omega c_7 + \omega^2 \bar{c}_7) v v'^2 = 0 \\
\frac{\partial V}{\partial \varphi'_1} &= M_2^2 v' + 4c_4 v'^3 + 6c_6 v^2 v' = 0 \\
\frac{\partial V}{\partial \varphi'_2} &= \mu_2 v v' + c_5 v^2 v' = 0 \\
\frac{\partial V}{\partial \varphi'_3} &= \mu_2 v v' + c_5 v^2 v' = 0 \quad .
\end{aligned}$$

Incompatible unless c7=0 → v, v' = 0

Vacuum alignment problem

Altarelli, Feruglio, NPB 720 05'

$$\langle\varphi\rangle = (v, v, v)$$
$$\langle\varphi'\rangle = (v', 0, 0)$$

$$\frac{\partial V}{\partial \varphi_1} = M_1^2 v + \mu_1 v^2 + 4c_1 v^3 + 2c_6 v v'^2$$
$$\frac{\partial V}{\partial \varphi_2} = M_1^2 v + \mu_1 v^2 + 4c_1 v^3 + 2c_6 v v'^2$$
$$\frac{\partial V}{\partial \varphi_3} = M_1^2 v + \mu_1 v^2 + 4c_1 v^3 + 2c_6 v v'^2$$
$$\frac{\partial V}{\partial \varphi'_1} = M_2^2 v' + 4c_4 v'^3 + 6c_6 v^2 v' = 0$$
$$\frac{\partial V}{\partial \varphi'_2} = \mu_2 v v' + c_5 v^2 v' = 0$$
$$\frac{\partial V}{\partial \varphi'_3} = \mu_2 v v' + c_5 v^2 v' = 0 \quad .$$

c7=0

v, v' different from zero if c6 = 0, mu2 = 0, c5 = 0

Vacuum alignment problem

Altarelli, Feruglio, NPB 720 05'

$$\begin{aligned} V &= \frac{M_1^2}{2} B_1^2 + \frac{M_2^2}{2} B_2^2 + \mu_1 T_1 + \mu_2 T_2 \\ &+ c_1 Q_1 + c_2 Q_2 + c_3 Q_3 + c_4 Q_4 \\ &+ c_5 Q_5 + c_6 Q_6 + (c_7 Q_7 + c.c) \quad , \end{aligned}$$

c7 = 0, c6 = 0, mu2 = 0, c5 = 0

$$\begin{aligned} B_1 &= \varphi_1^2 + \varphi_2^2 + \varphi_3^2 \\ B_2 &= {\varphi'_1}^2 + {\varphi'_2}^2 + {\varphi'_3}^2 \\ T_1 &= \varphi_1 \varphi_2 \varphi_3 \\ T_2 &= \varphi_1 \varphi'_2 \varphi'_3 + \varphi_2 \varphi'_3 \varphi'_1 + \varphi_3 \varphi'_1 \varphi'_2 \\ Q_1 &= \varphi_1^2 \varphi_2^2 + \varphi_2^2 \varphi_3^2 + \varphi_3^2 \varphi_1^2 \\ Q_2 &= |\varphi_1^2 + \omega^2 \varphi_2^2 + \omega \varphi_3^2|^2 \\ Q_3 &= {\varphi'_1}^2 {\varphi'_2}^2 + {\varphi'_2}^2 {\varphi'_3}^2 + {\varphi'_3}^2 {\varphi'_1}^2 \\ Q_4 &= |{\varphi'_1}^2 + \omega^2 {\varphi'_2}^2 + \omega {\varphi'_3}^2|^2 \\ Q_5 &= \varphi_1 \varphi_2 \varphi'_1 \varphi'_2 + \varphi_2 \varphi_3 \varphi'_2 \varphi'_3 + \varphi_3 \varphi_1 \varphi'_3 \varphi'_1 \\ Q_6 &= (\varphi_1^2 + \varphi_2^2 + \varphi_3^2)({\varphi'_1}^2 + {\varphi'_2}^2 + {\varphi'_3}^2) \\ Q_7 &= (\varphi_1^2 + \omega^2 \varphi_2^2 + \omega \varphi_3^2)({\varphi'_1}^2 + \omega {\varphi'_2}^2 + \omega^2 {\varphi'_3}^2) \end{aligned}$$

we don't want terms mixing

$\varphi \quad \varphi'$

Abelian symmetries do not forbid such a terms

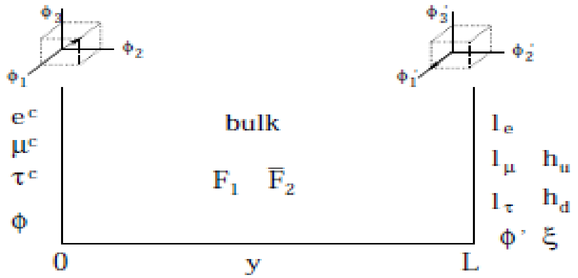
Vacuum alignment problem

- SUSY:
  the renormalizable superpotential only contains cubic terms like

Altarelli, Feruglio NPB741

$$\varphi\varphi\varphi'$$ not invariant under some abelian symm

- Extra Dimension



Altarelli, Feruglio NPB720

- Enlarge the flavor group

$A_4 \ltimes (Z_2)^3$ each component of the scalar triplet is glue to a corresponding right-handed field

$$(L_1\varphi_1 + L_2\varphi_3 + L_3\varphi_2) l_1^c$$

Grimus, Lavoura JHEP0904
Mohapatra, Nasri PLB639
Morisi PRD79

Vacuum alignment problem

- SUSY:
  the renormalizable superpotential only contains cubic terms like

  Altarelli, Feruglio NPB741

  $\varphi\varphi\varphi'$ not invariant under some abelian symm

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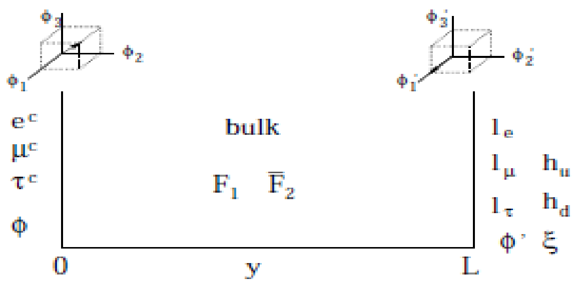

Altarelli, Feruglio NPB720

- Enlarge the flavor group

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$$(L_1\varphi_1 + L_2\varphi_3 + L_3\varphi_2)\, l_1^c$$

Grimus, Lavoura JHEP0904
Mohapatra, Nasri PLB639
Morisi PRD79

charged leptons diagonal
also with $\langle\varphi\rangle = (v, v, v)$

TBM as accidental mixing

Abbs, Smirnov 1004.0099

Morisi, Peinado PRD80 (09')

fields	L_i	l_i^c	H_i
$SU(2)_L$	2	1	2
A_4	3	3	3

- -no flavons
- -three Higgs doublets
- -no extra abelian symmetry
- -neutrino mass from dim 5 oper

Assuming the vevs complex, the minimization of the potential gives

$$\langle \phi_1 \rangle = v_1, \quad \langle \phi_2 \rangle = v e^{i\alpha/2}, \quad \langle \phi_3 \rangle = v e^{-i\alpha/2}$$

Lavoura,kuhbock EPJC55

$$\begin{aligned} L_{\text{Yukawa}} &= y_1 \left(\bar{L}_1 \phi_3 l_2^c + \bar{L}_2 \phi_1 l_3^c + \bar{L}_3 \phi_2 l_1^c \right) + \\ &+ y_2 \left(\bar{L}_1 \phi_2 l_3^c + \bar{L}_2 \phi_3 l_1^c + \bar{L}_3 \phi_1 l_2^c \right) \end{aligned}$$

$$M_l = \begin{pmatrix} 0 & a e^{i\alpha} & b e^{-i\alpha} \\ b e^{i\alpha} & 0 & a r \\ a e^{-i\alpha} & b r & 0 \end{pmatrix}$$

charged leptons

$$M_l M_l^T = \begin{pmatrix} a^2+b^2 & abr & abr \\ abr & b^2+a^2r^2 & ab \\ abr & ab & a^2+b^2r^2 \end{pmatrix}$$

$$r \approx \frac{m_\tau}{\sqrt{m_e m_\mu}}\sqrt{1-\frac{m_e^2 m_\mu^2}{m_\tau^4}},$$
$$a \approx \frac{m_\mu}{m_\tau}\sqrt{m_e m_\mu}\left[1+\frac{1}{2}\frac{m_\mu^2}{m_\tau^2}\right],$$
$$b \approx \sqrt{m_e m_\mu}\left[1-\frac{1}{2}\frac{m_\mu^2}{m_\tau^2}\right].$$

$$a < b \ll r$$

$$O_{l_{12}} \approx \frac{b}{a} r^{-1}, \quad O_{l_{13}} \approx \frac{a}{b} r^{-1}, \quad O_{l_{23}} \approx \frac{a}{b} r^{-2}$$

$$O_l = \begin{pmatrix} 0.997 & 0.069 & 2.44\times10^{-4} \\ -0.069 & 0.997 & 1.075\times10^{-6} \\ -2.439\times10^{-4} & -1.800\times10^{-5} & 0.999 \end{pmatrix}$$

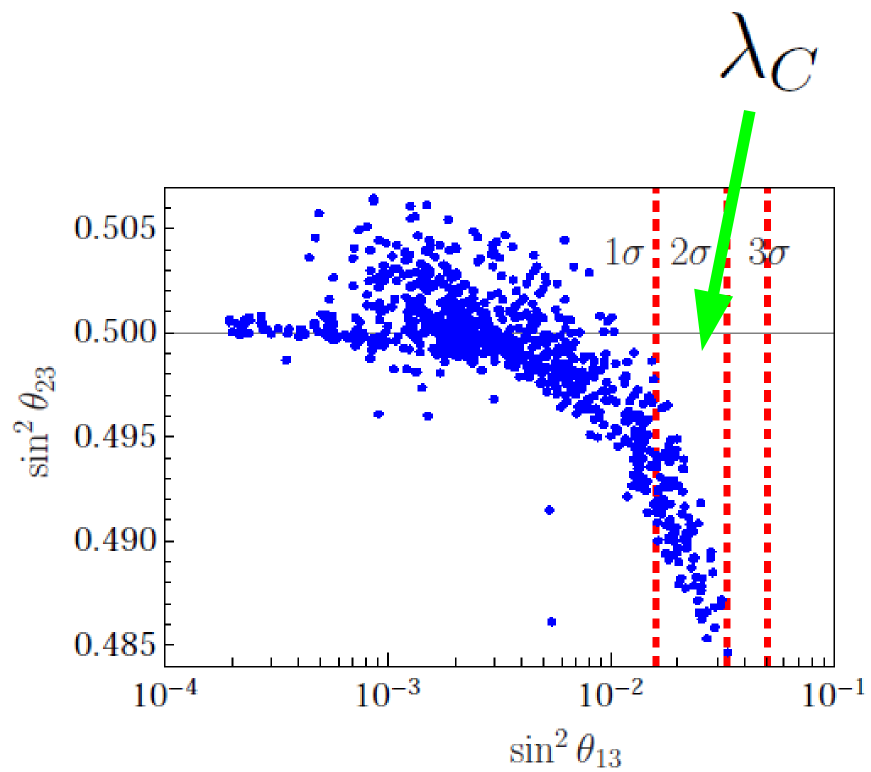
Neutrino mass matrix

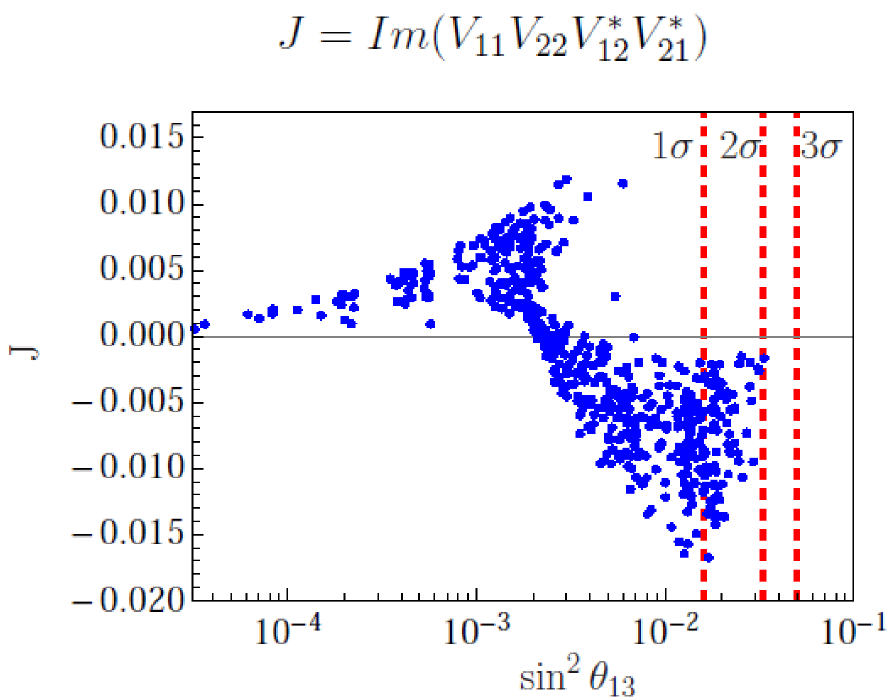
$$\begin{aligned}\mathcal{L}_{5d} &= \beta\,(LL)_3\,(HH)_3 + k\,(LL)_1\,(HH)_1 + \alpha'\,(LL)_{1'}\,(HH)_{1''} + \alpha''\,(LL)_{1''}\,(HH)_{1'} + \\ &+ \left[a\,(LH)_{3^a}\,(LH)_{3^a} + b\,(LH)_{3^a}\,(LH)_{3^b} + c\,(LH)_{3^b}\,(LH)_{3^a} + d\,(LH)_{3^b}\,(LH)_{3^b}\right] + \\ &+ l\,(LH)_1\,(LH)_1 + l'\left[(LH)_{1'}\,(LH)_{1''} + (LH)_{1''}\,(LH)_{1'}\right],\end{aligned}$$

$$M_\nu = \begin{pmatrix} xr^2 & \kappa r e^{-i\alpha} & \kappa r e^{i\alpha} \\ \kappa r e^{-i\alpha} & zr^2 & \kappa \\ \kappa r e^{i\alpha} & \kappa & yr^2 \end{pmatrix}$$

we have six free parameters, **r** is already fixed

Neutrino phenomenology



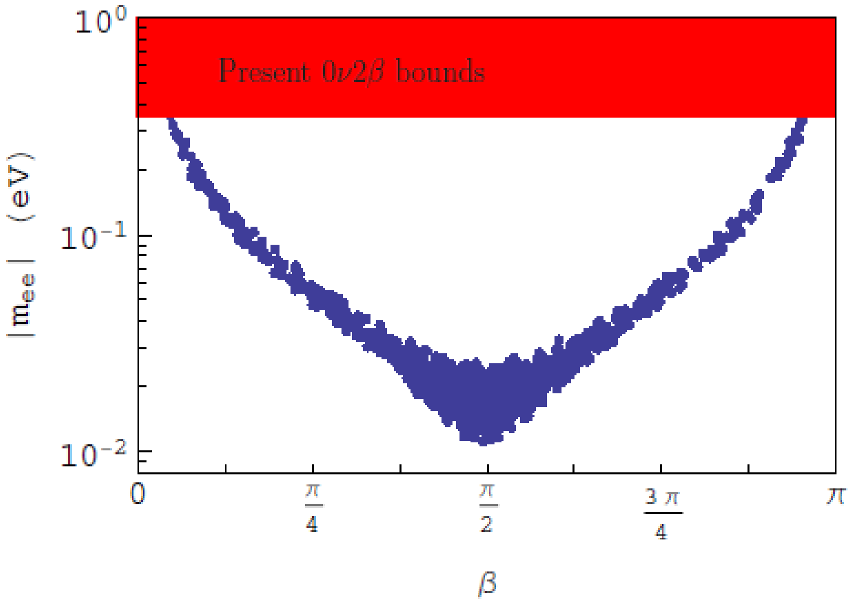


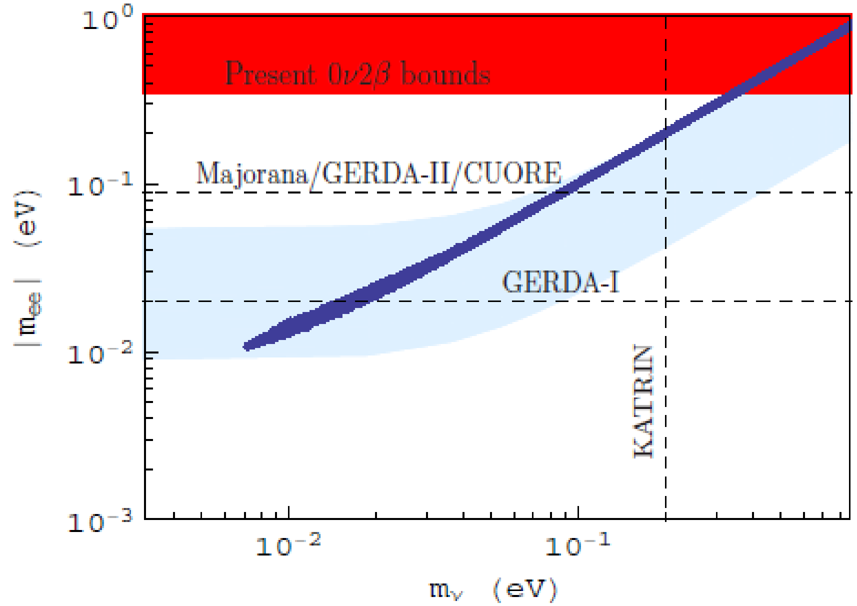
-in general all mixing angle receive corrections of the same order

-allowed departures of the solar angle from the best fit $\mathcal{O}(\lambda_C^2)$

-then TBM models give corrections for the reactor angle of the same order

Neutrino phenomenology





$$m_{\text{light}} > 0.008$$

conclusions

Which is the flavor symmetry?

If tri-bimaximal will be confirmed from future experiments

may be a non-Abelian discrete group of SU(2), SU(3),...

however...

conclusions

A4

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In general not possible to distinguish experimentally

conclusions

If tri-bimaximal will be ruled out

non-Abelian discrete groups are useful to have

small reactor and large atmospheric angles

in general models with discrete group have few

free parameters and are predictive