Supersymmetric Seesaws

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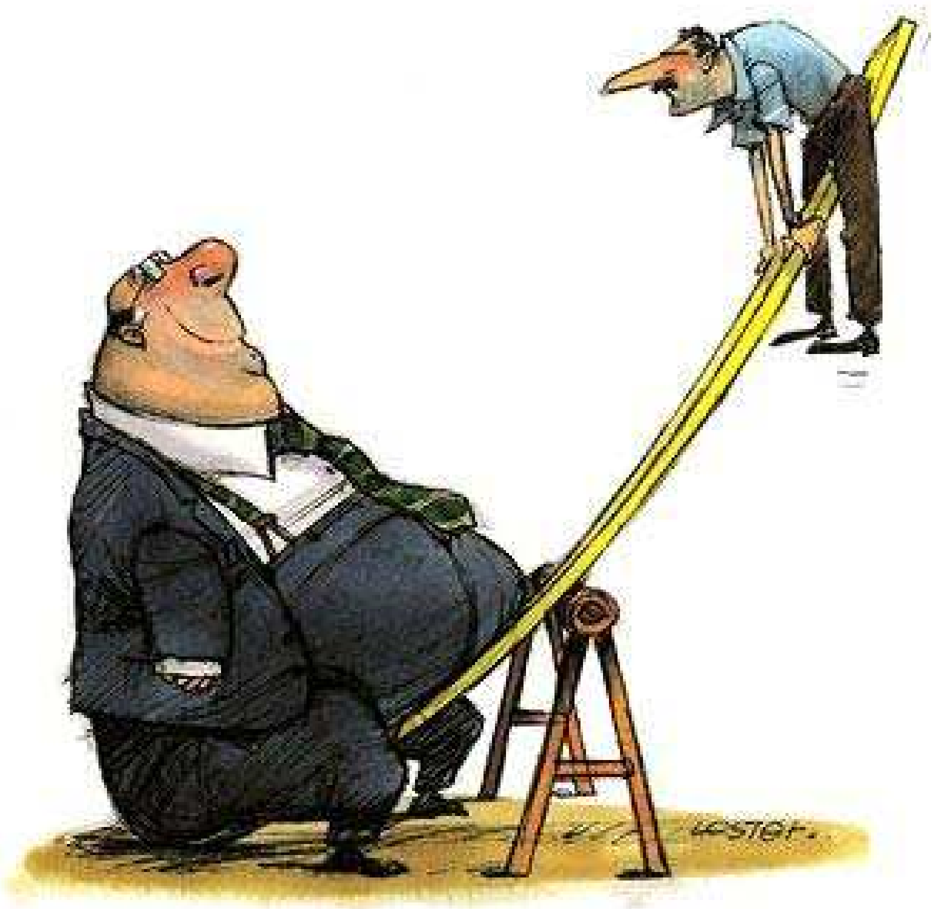
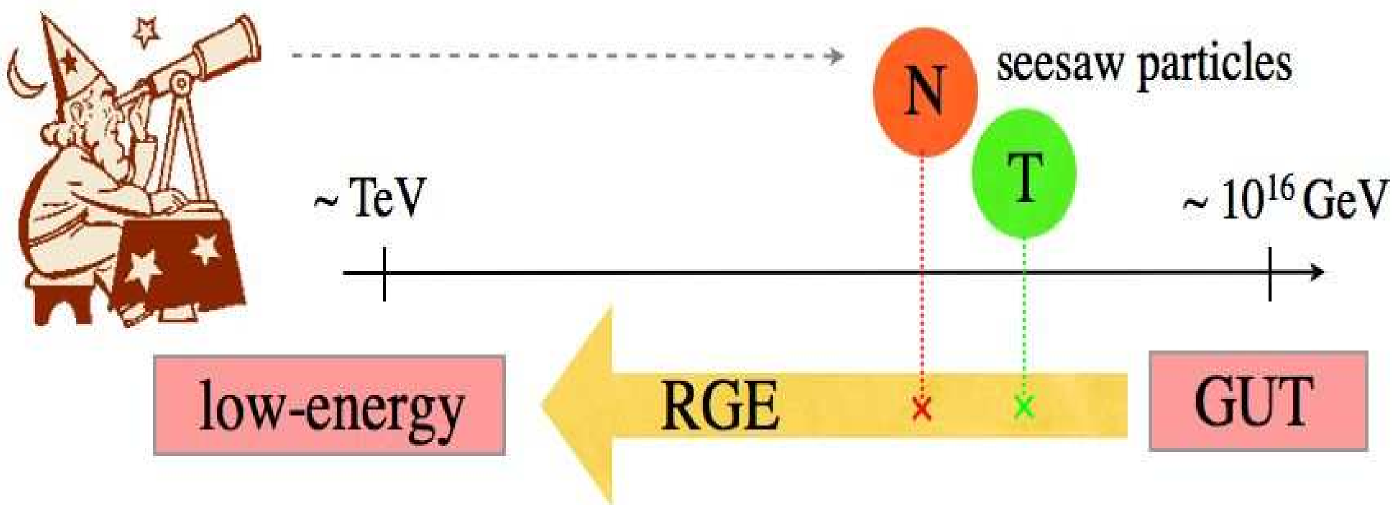
The seesaw mechanism



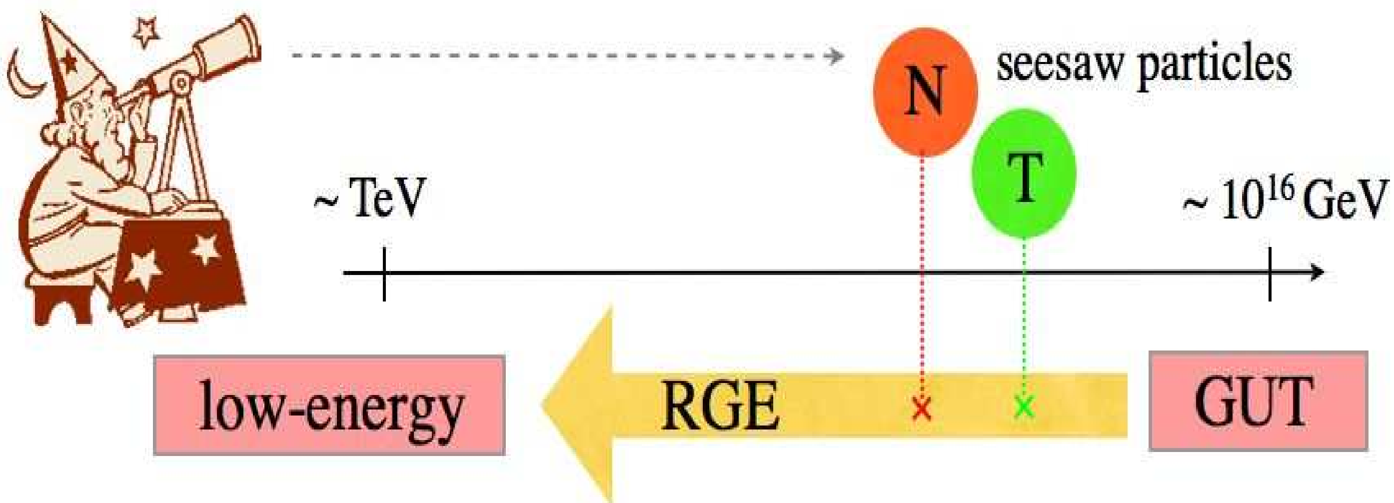
Supersymmetric seesaw



Supersymmetric seesaw

??? LHC = SUSY telescope ???

⇓



Outline

$\mathcal{I}$. Introduction

$\mathcal{II}$. Supersymmetric seesaws

$\mathcal{III}$. LFV in seesaw

$\mathcal{IV}$. SUSY spectra and the seesaw scale

$\mathcal{I}.$

Introduction

Neutrino oscillation data

Neutrino masses are non-zero:

Parameter	Best fit	3 σ c.l.
$\Delta m^2_\odot$ (10^{-5} eV^2)	$7.59^{+0.23}_{-0.18}$	7.03 - 8.27
$\Delta m^2_{\rm Atm}$ (10^{-3} eV^2)	$2.40^{+0.12}_{-0.11}$	2.07 - 2.75
$\sin^2\theta_\odot$	$0.318^{+0.019}_{-0.016}$	0.27 - 0.38
$\sin^2\theta_{\rm Atm}$	$0.50^{+0.07}_{-0.06}$	0.36 - 0.67
$\sin^2\theta_{13}$	$0.013^{+0.013}_{-0.009}$	$\leq$ 0.053

Data from updated global fit:
Schwetz, Tórtola & Valle, New J Phys 10:113011, 2008;
arXiv:0808.2016 (hep-ph) updated V3: 11 Feb 2010

Hint for no-zero θ_{13} at 1.5 σ? - Fogli et al., 2008

Neutrino angles

Very good first approximation to this data is the so-called tri-bimaximal mixing ansatz of Harrison, Perkins and Scott, PL**B530**:

$$\mathcal{U}_\nu^{\rm TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Corresponding to

$$\tan^2\theta_{\rm Atm} = 1 \quad , \quad \tan^2\theta_\odot = \frac{1}{2} \quad , \quad \sin^2\theta_{\rm R} = 0$$

Absolute mass scale

Tritium decay end point searches:

$$m_\nu^\beta = \sqrt{\sum_i |U_{ei}|^2 m_i^2} \leq 2.2 \text{ eV}$$

Double beta decay: Majorana neutrino!

$$m_\nu^{\beta\beta} = \sum_i U_{ei}^2 m_i \leq (0.5 - 1.0) \text{ eV}$$

Cosmology (CMB + LSS + $\cdots$):

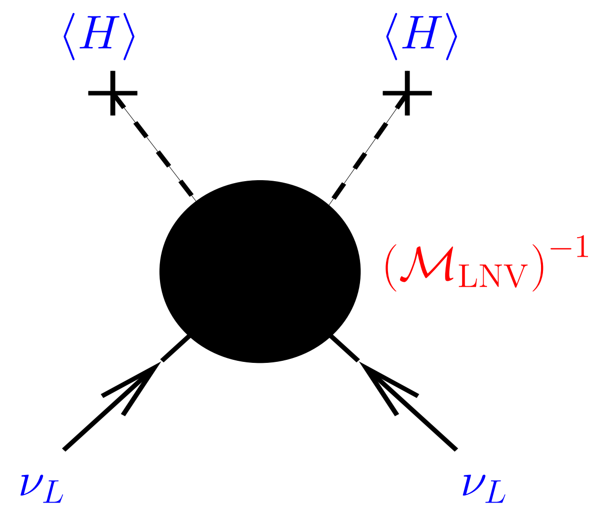
$$\sum_i m_{\nu_i} \leq (0.4 - 1.0) \text{ eV}$$

$\Rightarrow$ Recall for hierarchical neutrinos:

$$\sqrt{\Delta m^2_{\rm Atm}} \sim 50 \text{ meV} \quad \text{and} \quad \sqrt{\Delta m^2_\odot} \sim 9 \text{ meV}$$

Majorana $\mathcal{M}_\nu$

If Lepton Number is Violated:



Weinberg, 1979

$$m_\nu = \frac{1}{\mathcal{M}_{LNV}}(LH)(LH)$$

Many possible models:

(i) Seesaw mechanism: Type-I, Type-II, Type-III, Inverse seesaw, etc ...
(ii) Radiative models: Zee, Babu, LQs ...
(iii) SUSY neutrino masses: $R\!\!\!/_p$
(iv) $\cdots$

Majorana $\mathcal{M}_\nu$

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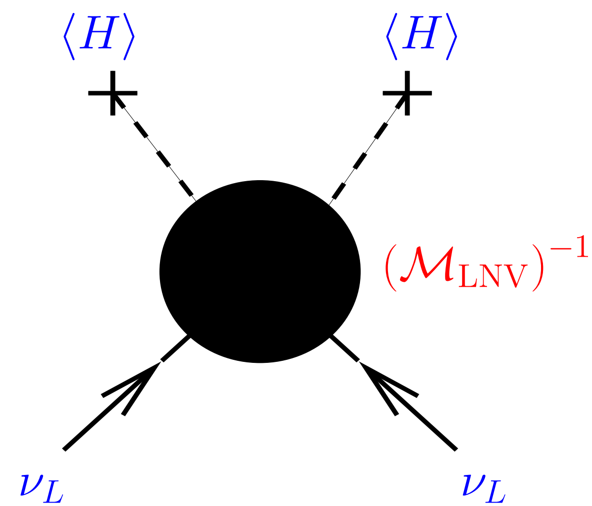

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Many possible models:

(i) Seesaw mechanism: Type-I, Type-II, Type-III, Inverse seesaw, etc ...
(ii) Radiative models: Zee, Babu, LQs ...
(iii) SUSY neutrino masses: $\not{R}_p$
(iv) $\cdots$

E. Ma, PRL81, 1998:

At tree level
only three realizations
of seesaw operator

'Classical' Seesaw

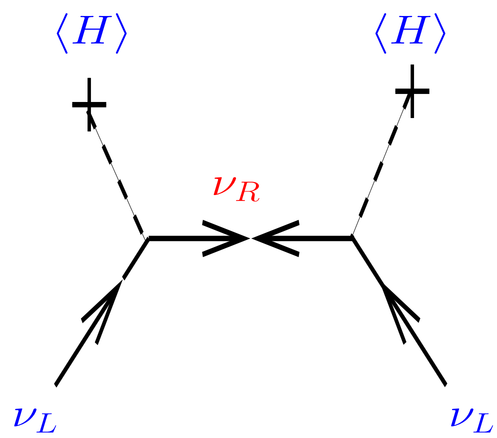
In the basis (ν_L, ν_R) write mass matrix:

Minkowski, 1977
Yanagida, 1979
Gell-Mann, Ramond & Slansky, 1979
Mohapatra & Senjanovic, 1980
... ...

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m_D \\ m_D & M_M \end{pmatrix} .$$

If $m_D \ll M_M$:

$$m_{1/2} \simeq (-\frac{m_D^2}{M_M}, M_M)$$



Santamaria, 1993

⇒ For 3 ν_R 21 parameters

⇒ At low energy 12 parameters measurable:

3 m_{l_i}, 3 m_{ν_i}, 3 angles & 3 phases

⇒ Predictive power: -9

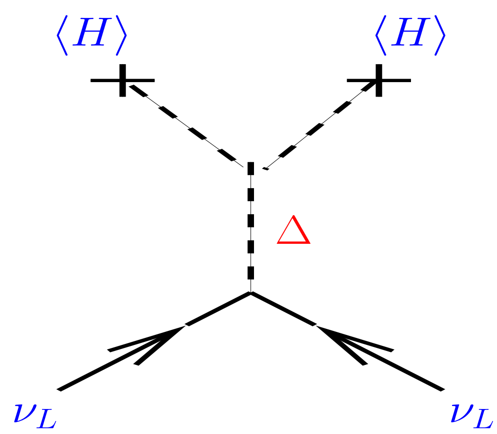
Seesaw: Type II

Schechter & Valle, 1980, 1982
Cheng & Li, 1980
Mohapatra, Senjanovic, 1981
...

$$\mathcal{M}_\nu = m_M$$

with:

$$m_M \simeq Y^\nu \langle \Delta_L^0 \rangle$$



Example:

$SU(5)$ with $\mathbf{15}$:

$$\langle \Delta_L^0 \rangle \sim \frac{\langle h^0 \rangle^2}{m_{\mathbf{15}}}$$

⇒ With 2 triplets (SUSY) 15 parameters

⇒ Predictive power (low energy): -3

Seesaw: Type-III

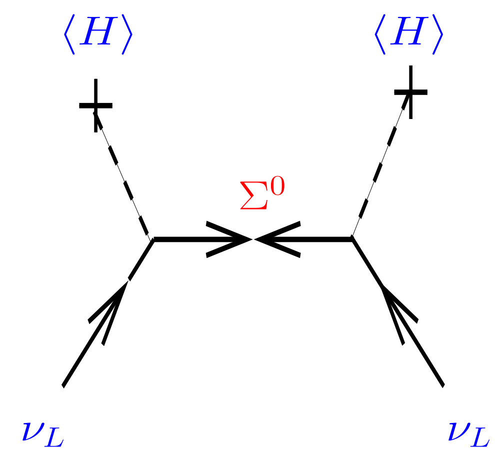
As in seesaw type-I. Replace ν_R by $\mathbf{\Sigma} = (\Sigma^+, \Sigma^0, \Sigma^-)$.
In the basis (ν_L, Σ^0) write mass matrix:

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m_D \\ m_D & M_\Sigma \end{pmatrix}.$$

R. Foot et al., 1988
E. Ma, 1998

If $m_D \ll M_\Sigma$:

$$m_{1/2} \simeq (-\frac{m_D^2}{M_\Sigma}, M_\Sigma)$$



$\Rightarrow$ For 3 Σ 21 parameters
$\Rightarrow$ Predictive power: -9

$\mathcal{II}.$

Supersymmetric seesaws

The Setup @ GUT scale

- Type-I

$$W_{\rm RHN} = \mathbf{Y}_N^{\rm I}\, N^c \cdot \bar{5} \cdot 5_H + \frac{1}{2}\, M_R\, N^c N^c$$

- Type-II

$$W_{\rm 15H} = \frac{1}{\sqrt{2}} \mathbf{Y}_N^{\rm II}\, \bar{5} \cdot 15 \cdot \bar{5} + \frac{1}{\sqrt{2}} \lambda_1 \bar{5}_H \cdot 15 \cdot \bar{5}_H + \frac{1}{\sqrt{2}} \lambda_2 5_H \cdot \overline{15} \cdot 5_H$$
$$+ \mathbf{Y}_5 10 \cdot \bar{5} \cdot \bar{5}_H + \mathbf{Y}_{10} 10 \cdot 10 \cdot 5_H + M_{15} 15 \cdot \overline{15} + M_5 \bar{5}_H \cdot 5_H$$

- Type-III

$$W_{\rm 24M} = \sqrt{2}\, \mathbf{Y_5} \bar{5} \cdot 10 \cdot \bar{5}_H - \frac{1}{4} \mathbf{Y_{10}} 10 \cdot 10 \cdot 5_H + Y_N^{III} 5_H \cdot 24_M \cdot \bar{5}$$
$$+ \frac{1}{2} 24_M M_{24} 24_M$$

The $SU(5)$-broken phase

Under $SU(3) \times SU_L(2) \times U(1)_Y$

- The $\bar{5}$ contains:

$$\bar{5} = (d^c, L)$$

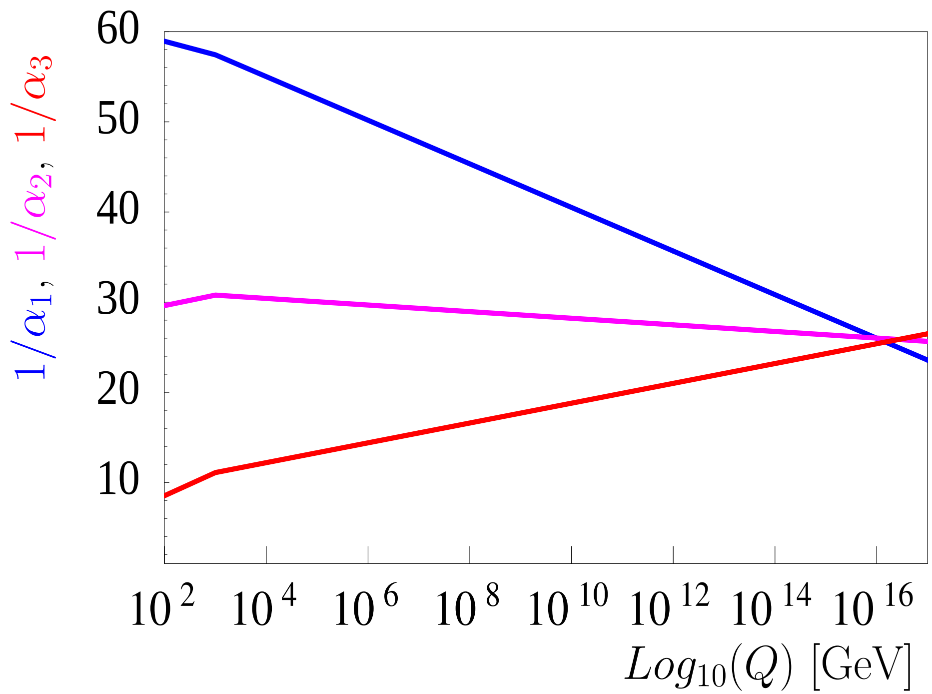
- The $\mathbf{15}$ decomposes as

$$\mathbf{15}_H = S(6,1,-\frac{2}{3}) + T(1,3,1) + Z(3,2,\frac{1}{6})$$

- The $\mathbf{24}$ decomposes as

$$\mathbf{24}_M = W_M(1,3,0) + B_M(1,1,0) + \overline{X}_M(3,2,-\frac{5}{6}) + X_M(\bar{3},2,\frac{5}{6}) + G_M(8,1,0)$$

Gauge coupling unification

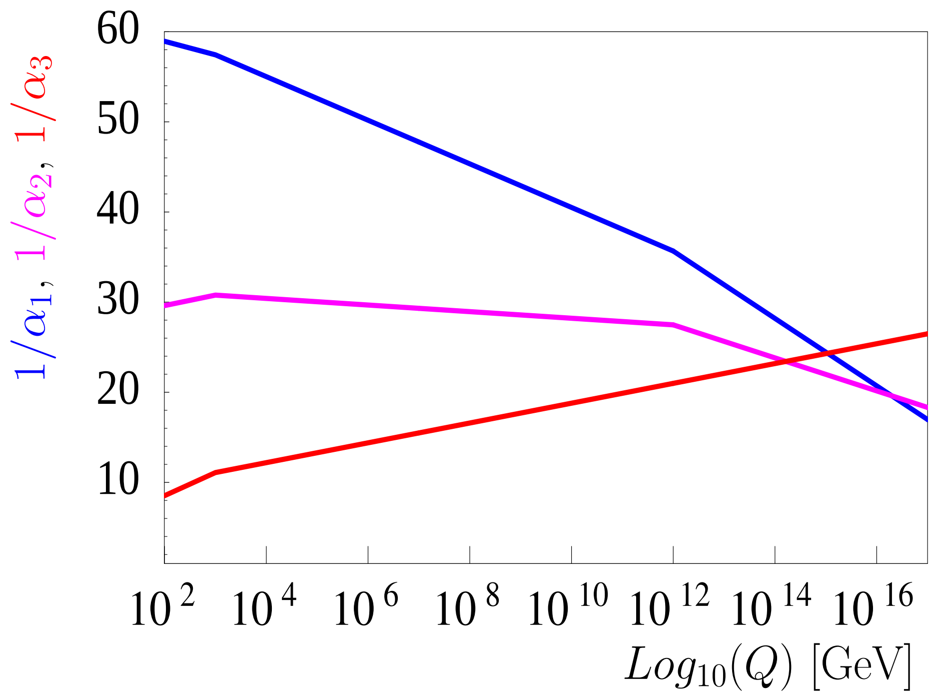


In the MSSM (nearly) perfect unification of gauge couplings

$\Rightarrow$ Evolution of inverse of gauge couplings $\alpha_i = \frac{g_i^2}{4\pi}$:

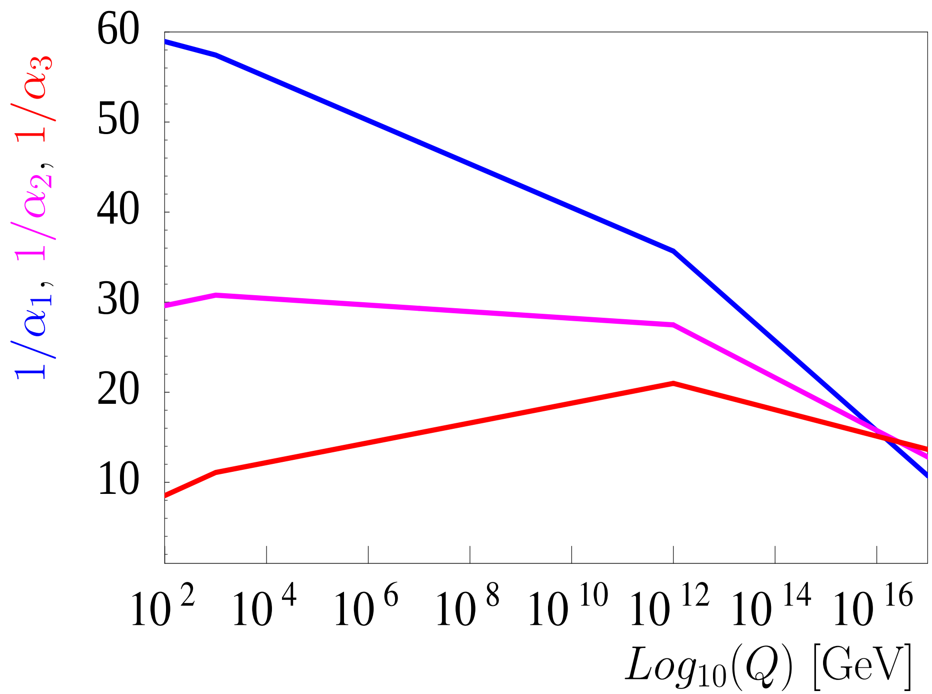
$\Rightarrow$ Note change in slope at $Q = 1TeV$

Gauge coupling unification



Adding a (pair of) triplet fields T and $\bar{T}$ at any $Q \lesssim (\text{few})\ 10^{15}$ GeV destroys unification of gauge couplings

Gauge coupling unification

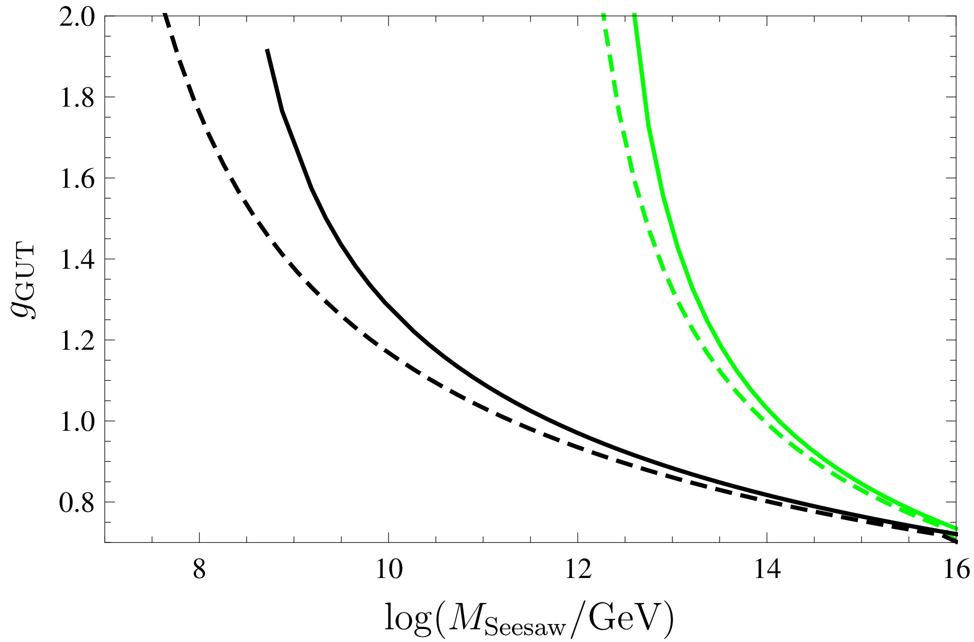


Adding complete $SU(5)$ multiplets changes $\alpha(M_{GUT})$, but maintains unification of gauge couplings

$\Rightarrow$ Note change in $\alpha(m_{GUT})$

$\Rightarrow$ Note change in slope at $Q = 10^{12}$ GeV

Landau poles



Seesaw type-II

Seesaw type-III

Perturbativity of $\alpha(M_{GUT})$ gives lower limit for $M_{\rm Seesaw}$

$\Rightarrow$ dashed lines: 1-loop; full lines: full 2-loop RGEs

mSugra

Boundary conditions: mSUGRA ("minimal Supergravity") :

$M_1 = M_2 = M_3 = M_{1/2}$,
$m^2_{H_u} = m^2_{H_d} = m^2_0$,
$M^2_{\tilde{Q}} = M^2_{\tilde{U}} = M^2_{\tilde{D}} = M^2_{\tilde{L}} = M^2_{\tilde{E}} = m^2_0 1_3$,
$A_d = A_0 Y_d$, $A_u = A_0 Y_u$, $A_e = A_0 Y_e$.

$\Rightarrow$ # of parameters: $4\frac{1}{2}$ (m_0, $M_{1/2}$, A_0, $\tan\beta$, $sgn(\mu)$)

$\Rightarrow$ Sometimes also called the CMSSM (C = constrained)

$\Rightarrow$ All low energy masses can then be calculated by RGE ("renormalization group equations")

$\Rightarrow$ No neutrino masses and no LFV

$\Rightarrow$ More complicated SUSY breaking schemes could be studied, however need: $\Lambda_{SUSY} > M_{\rm Seesaw}$

$V_{\rm soft}$ & *the seesaw-II scale*

RGEs allow to calculate low-scale SUSY masses. Example, only rough estimate:

$$
\begin{aligned}
m^2_{\tilde{L}} &\simeq m_0^2 + 0.5 M^2_{1/2} \\
m^2_{\tilde{E}} &\simeq m_0^2 + 0.15 M^2_{1/2} \\
M_1 &\simeq 0.45 M_{1/2}
\end{aligned}
$$

Form "invariants":

$$(m^2_{\tilde{L}} - m^2_{\tilde{E}})/M_1^2 \simeq 1.7$$

Buckley & Murayama, 2006

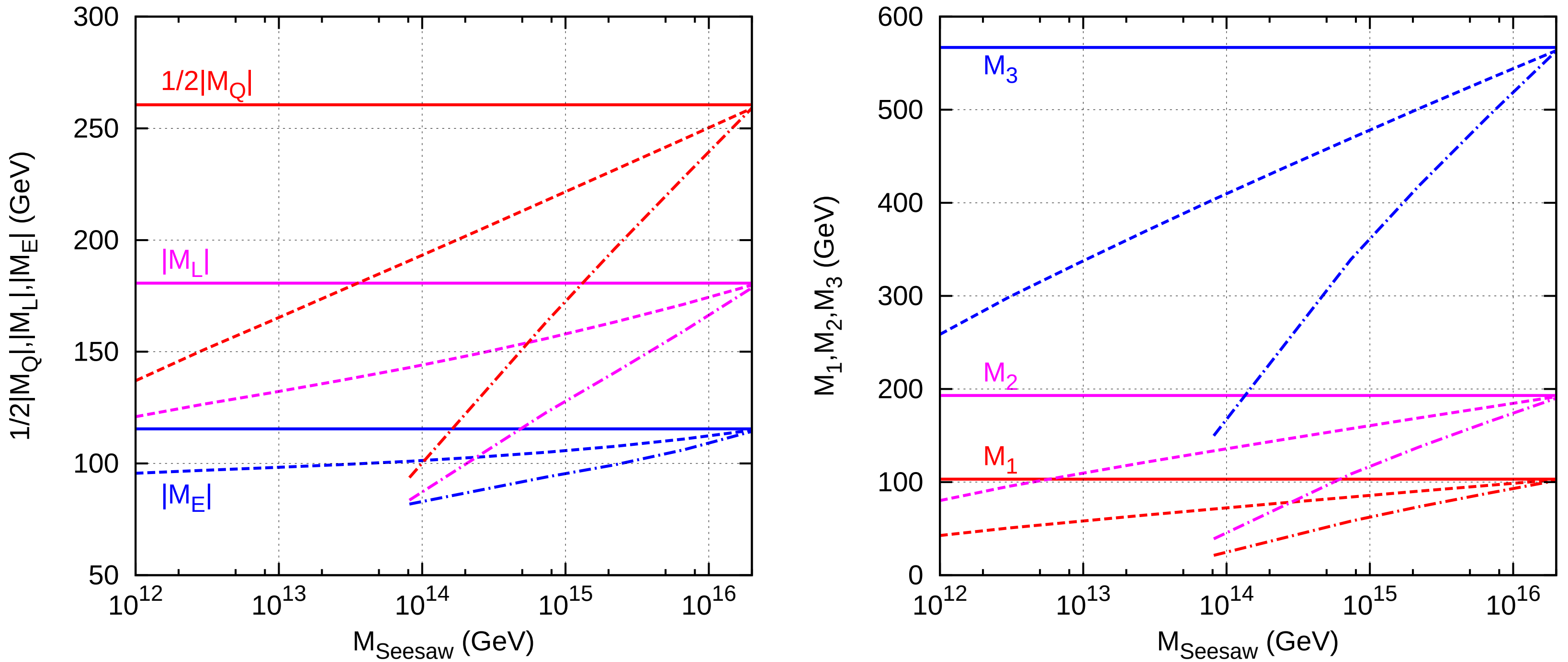
⇒ Different "invariants" can be defined

⇒ To first approximation no dependence on m_0 and $M_{1/2}$

⇒ Departure from mSugra expecation contains info on high energy physics!

Seesaw & running of $V_{\rm soft}$

Just one example (full => type-I; dotted => type-II; dash-dotted => type-III):

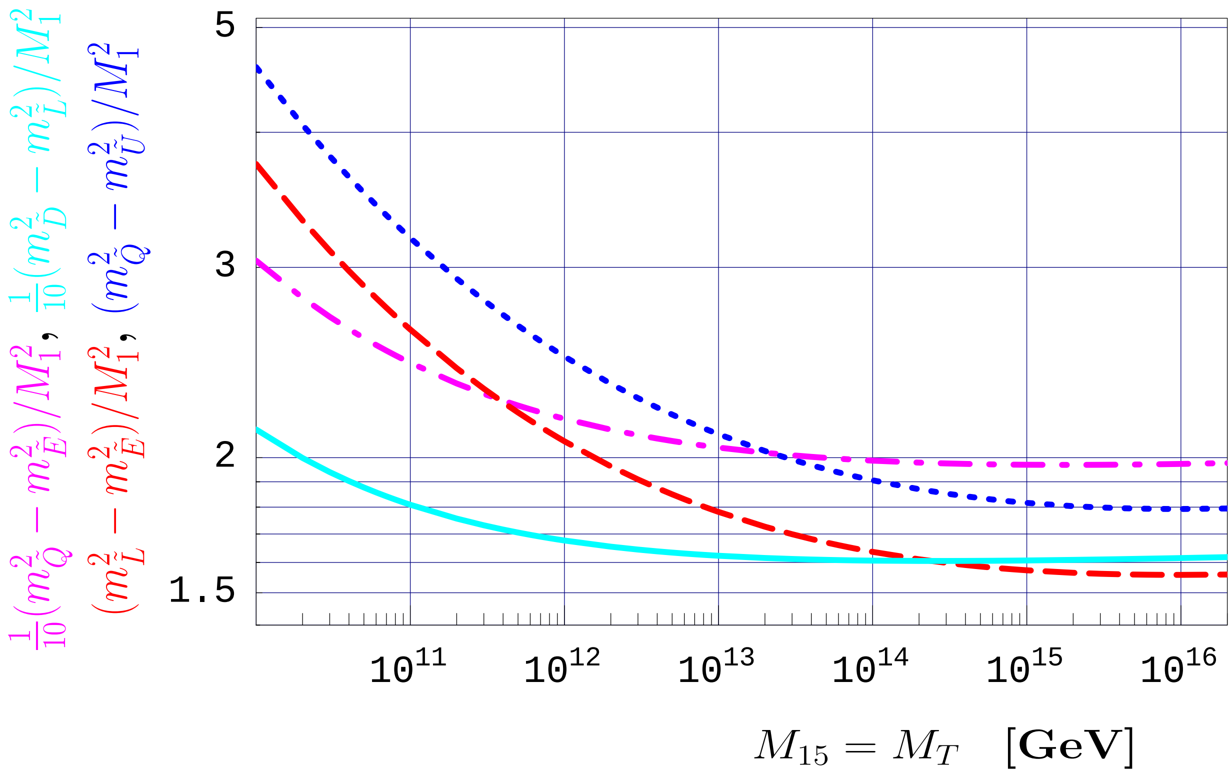


⇒ Gaugino mass parameters run faster then sfermion masses

⇒ type-III changes faster than type-II; type-I: no change

Soft masses and seesaw-II

Four examples of "invariants" as function of $M_{15} = M_T$:



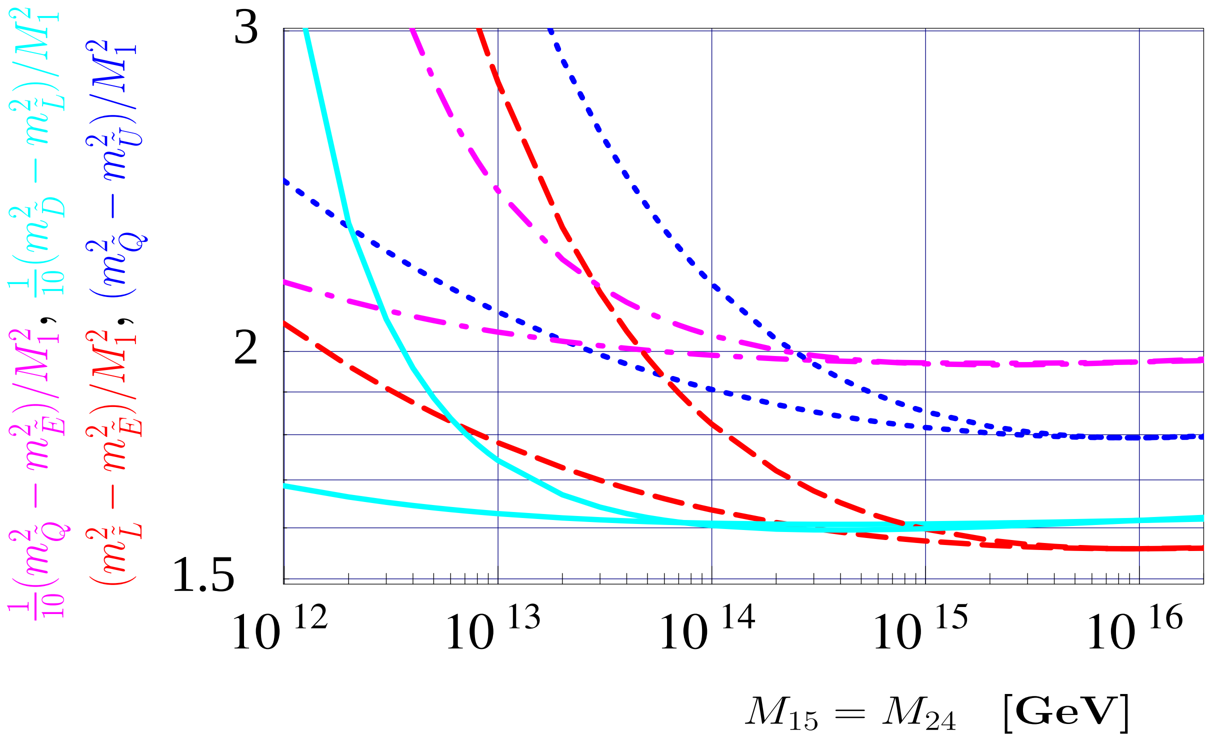
Analytic
1-loop
leading-log
approximation

⇒ Consistent departures from mSugra point to M_T

⇒ Dependence on M_T only $\log(M_T)$

Compare type-II and type-III

Four examples of "invariants" as function of $M_{15} = M_{24}$:

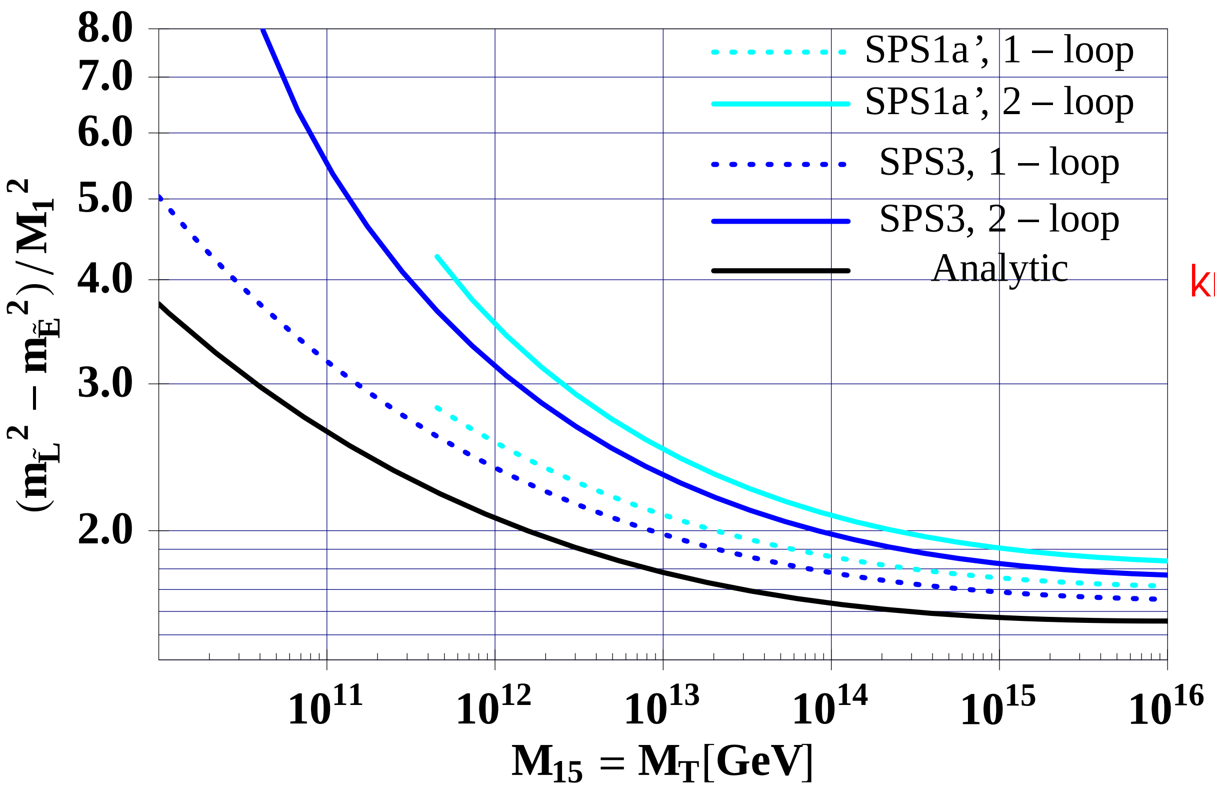


Analytic
1-loop
leading-log
approximation

⇒ Type-III seesaw "invariants" run faster

⇒ Dependence on $M_{\rm Seesaw}$ only $\log(M_{\rm Seesaw})$

Soft masses and seesaw-II



Quantitative analysis requires accurate knowledge of complete SUSY spectrum

Difficult at LHC!

$\Rightarrow$ "Invariant" $(m^2_{\tilde{L}} - m^2_{\tilde{E}})/M^2_1$, calculated with $Y_T \simeq 0$

$\Rightarrow$ "Analytic": Leading-log 1-loop

$\Rightarrow$ 1-loop and 2-loop: numerical calculation

$\mathcal{III}$.

LFV & SUSY Seesaw

Lepton flavour violation

Decay	Current	Future (?)
$\tau \to \mu\gamma$	$6.8 \cdot 10^{-8}$	$\sim 10^{-8}$
$\tau \to e\gamma$	$1.1 \cdot 10^{-7}$	$\sim 10^{-8}$
$\mu \to e\gamma$	$1.2 \cdot 10^{-11}$	$\sim 10^{-13}$
$\tau \to 3\mu$	$1.9 \cdot 10^{-7}$	$\sim 10^{-8}$
$\tau^- \to e^-\mu^+\mu^-$	$2 \cdot 10^{-7}$	$\sim 10^{-8}$
$\tau^- \to e^+\mu^-\mu^-$	$1.3 \cdot 10^{-7}$	$\sim 10^{-8}$
$\tau^- \to \mu^-e^+e^-$	$1.9 \cdot 10^{-7}$	$\sim 10^{-8}$
$\tau^- \to \mu^+e^-e^-$	$1.1 \cdot 10^{-7}$	$\sim 10^{-8}$
$\tau \to 3e$	$1.1 \cdot 10^{-7}$	$\sim 10^{-8}$
$\mu \to 3e$	$1 \cdot 10^{-12}$	$\sim 10^{-13}$

PDG 2006

In the SM:

all Br $\equiv$ 0!

In SM + Seesaw I:
unmeasurably small:

Br$(\mu \to e\gamma) \lesssim 10^{-40}$

mSugra and RGEs

Seesaw type-I: Borzumati & Masiero, 1986

$$(\Delta M^2_{\tilde{L}})_{ij} \sim -\frac{1}{8\pi^2} f(m_0, A_0, M_{1/2}, ...)(Y_\nu^\dagger L Y_\nu)_{ij}$$

Note: $L_i = \log[M_G/M_i]$.

$\Rightarrow$ 9 new independent parameters

Seesaw type-II:

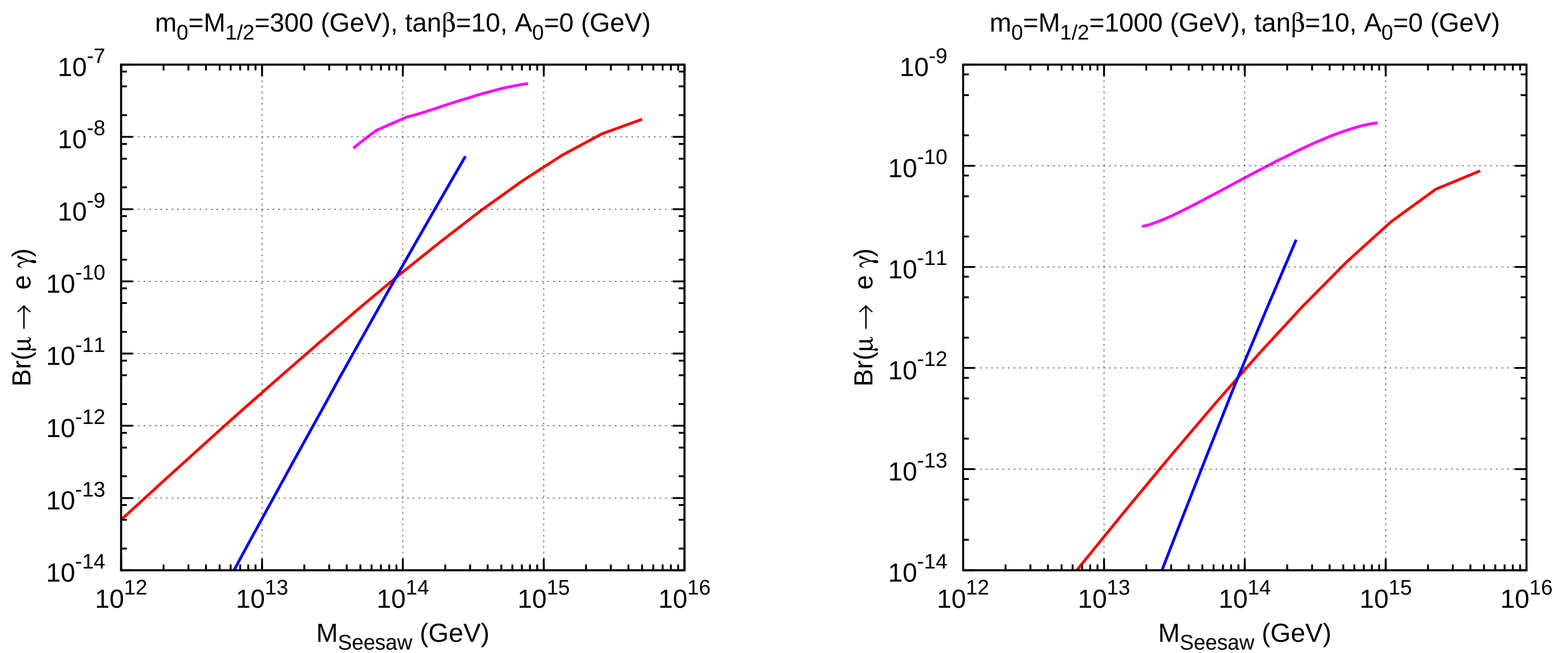
$$(\Delta M^2_{\tilde{L}})_{ij} \sim -\frac{1}{8\pi^2} g(m_0, A_0, M_{1/2}, ...)(Y_T^\dagger Y_T)_{ij} \log(M_G/M_T)$$

$\Rightarrow$ 9+12=21, but only 15 (14) parameters

$\Rightarrow$ Measuring all entries in $(\Delta M^2_{\tilde{L}})_{ij}$ "over-constrains" type-II seesaw ???

Note: type-III equation as type-I, but larger LFV ... see below

$\mu \to e\gamma$ in mSugra sessaw

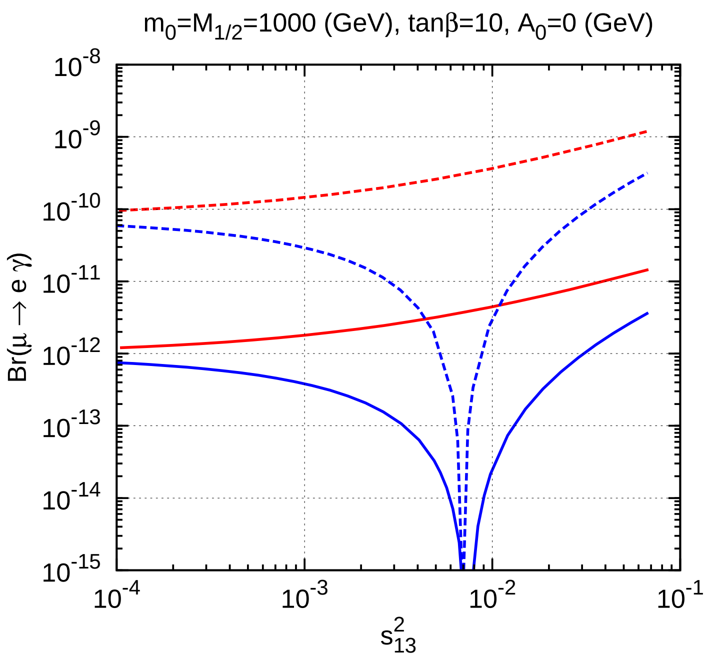


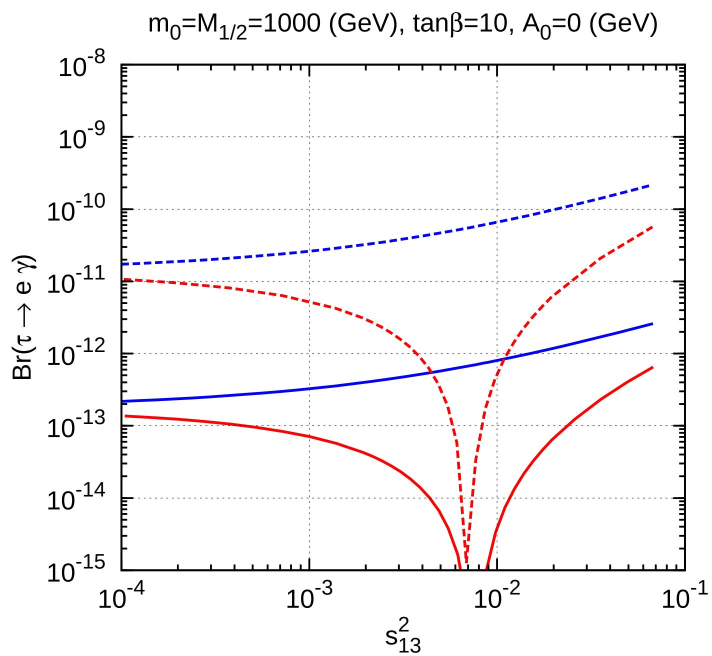
⇒ The three different seesaws are: type-III, type-II and type-I

⇒ General expectation: "Large" LFV for "large" $M_{\rm Seesaw}$

⇒ General expectation LFV in type-III $\gg$ type-I

Type-I and type-III only





Note: This example as function of s_{13}, but
cancellations independent of any low-energy neutrino physics
can be found in fine-tuned portions of parameter space

No prediction of LFV possible, but:
Observation of LFV $=>$ constraints on RH sector

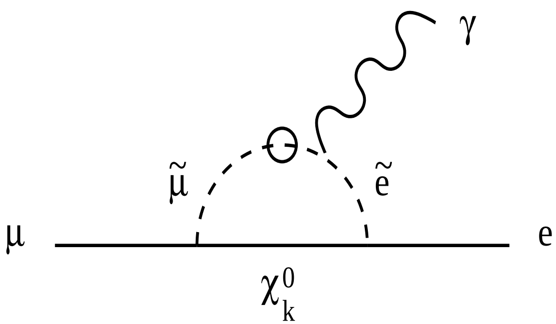
Collider versus low energy

Soft SUSY breaking:

Hinchliffe and Page, 2001
Porod and Majerotto, 2002
$\cdots$ many others $\cdots$

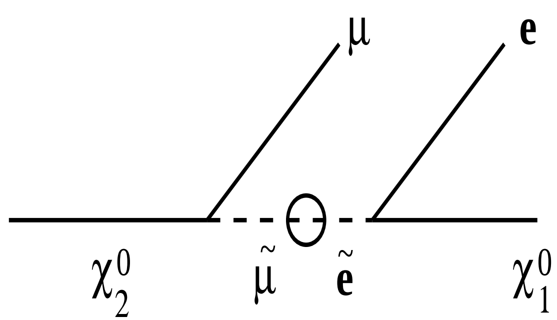
$$V = (m^2_{\tilde{L}})_{ij} \tilde{L}^*_i \tilde{L}_j + \cdots$$

Off-diagonal elements induce decays:



$\sim (m^2_{\tilde{L}})_{21}$

Slepton decays and $l_i \to l_j \gamma$ related:

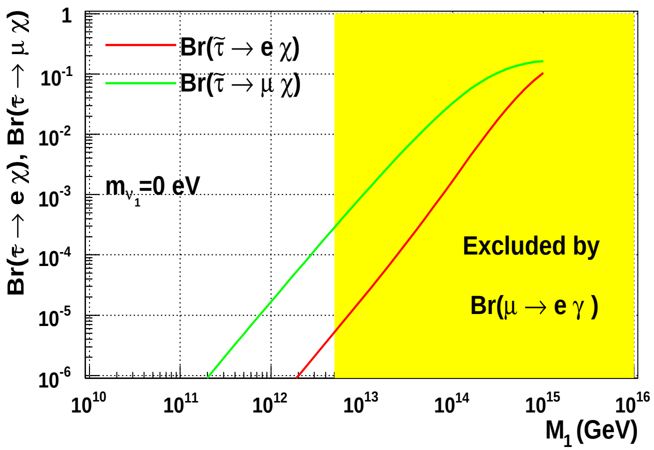


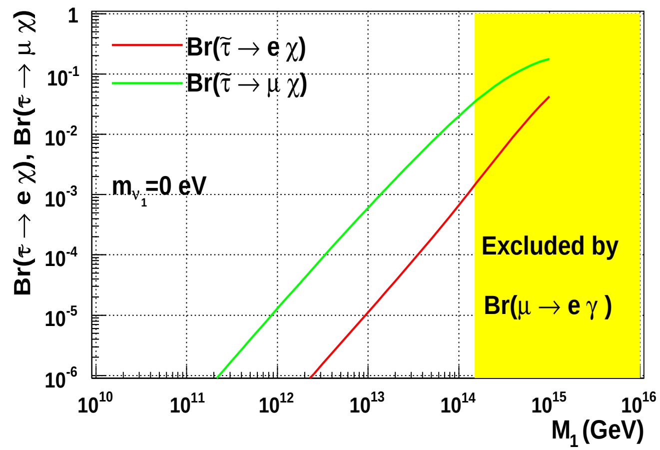
$\sim (m^2_{\tilde{L}})_{21}$

Numerical results: LHC

Seesaw-I:

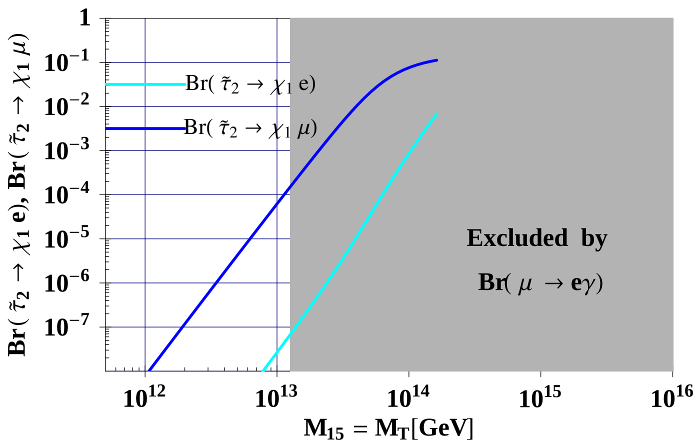
Left: SPS1a′ Right: SPS3

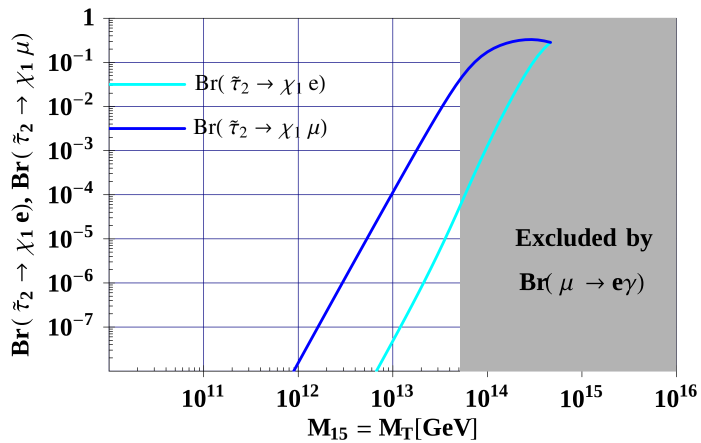




Seesaw-II:

Left: SPS1a′ Right: SPS3





$\Rightarrow$ LHC can see LFV (if SPS3-like ...)

(Small detour)

SUSY Left-right symmetric model

SUSY LR model

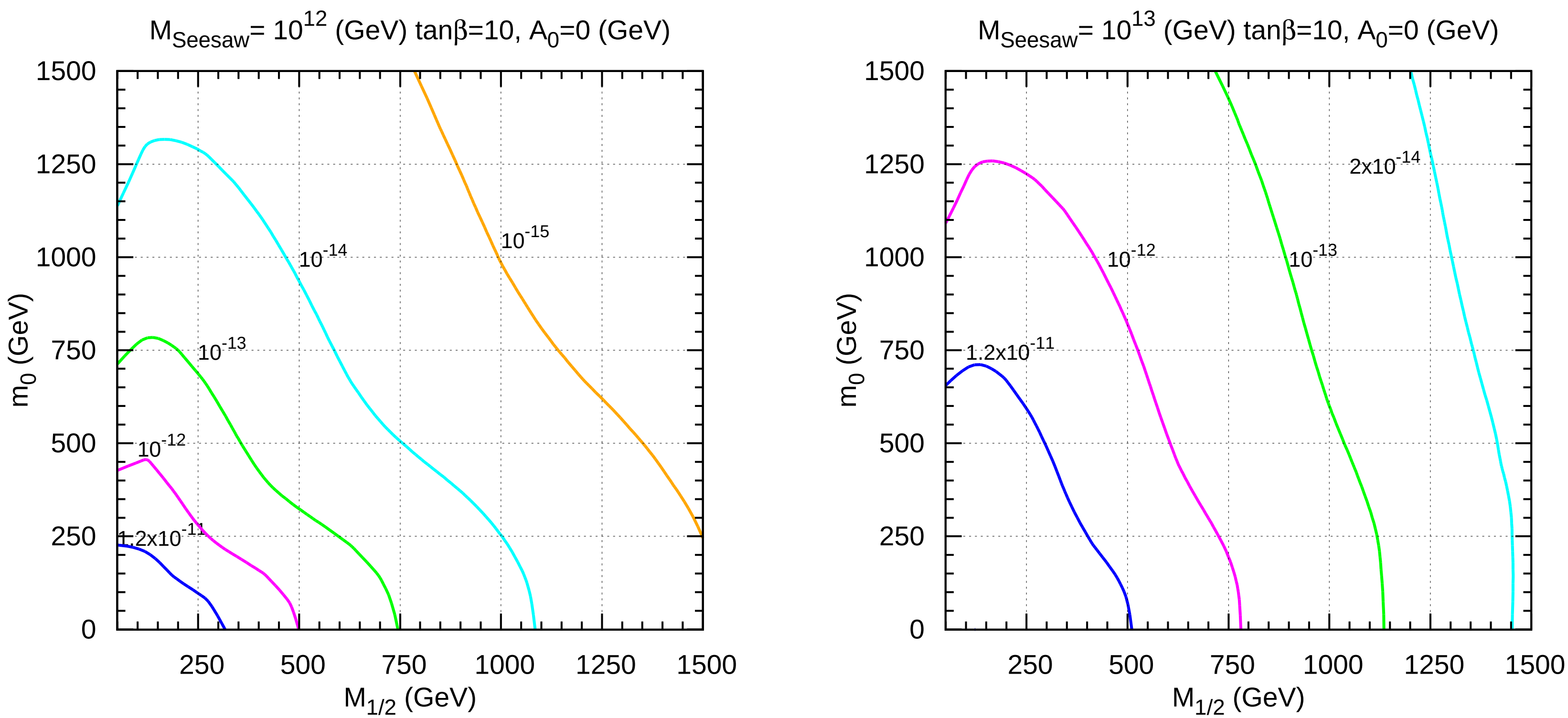
Consider gauge group:

$$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

Advantages:

- . Restoration of parity at high energy
- . Generates seesaw: N^c is part of theory
- . Provides (potentially) solution to CP problems
- . Can be embedded in SO(10)
- . R-parity conservation can be automatic

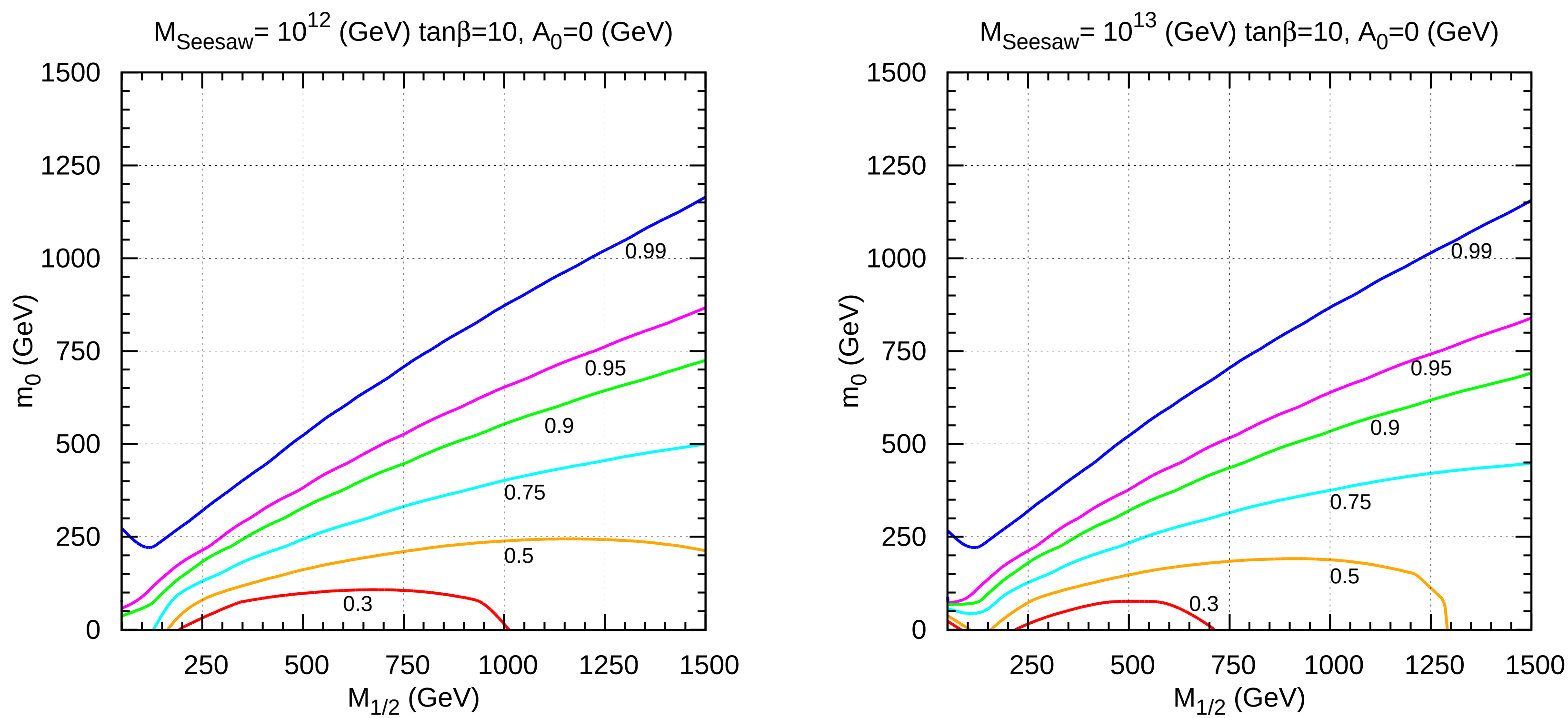
LFV in SUSY LR model



$\Rightarrow$ As in seesaw Br($\mu^+ \to e^+\gamma$) strong function of M_{Seesaw}

... but ...

LFV in SUSY LR model



Asymmetry:

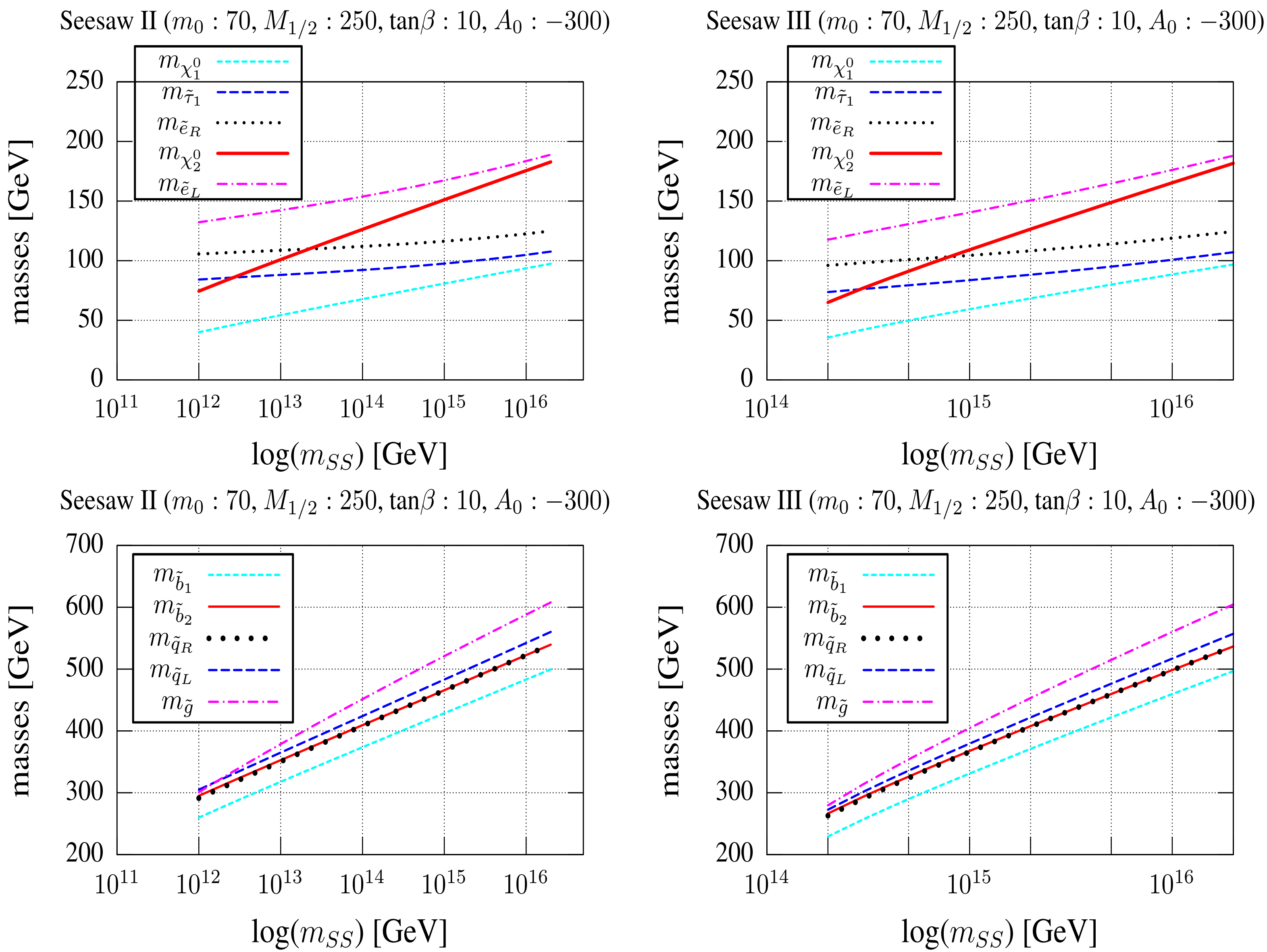
$$\mathcal{A}(\mu^+ \to e^+\gamma) = \frac{|A_L|^2 - |A_R|^2}{|A_L|^2 + |A_R|^2},$$

$\Rightarrow$ Note: In mSugra seesaw $\mathcal{A} = 1$ always

$\mathcal{IV}$.

SUSY spectra & seesaw scale

SUSY mass spectra



LHC observables - I

⇒ In R_P SUSY LHC does not measure SUSY masses directly

⇒ Consider, for example, decay chain:

$$\tilde{q}_L \to \chi_2^0 + q$$
$$\to \chi_2^0 \to l + \tilde{l} \to l^\pm + l^\mp + \chi_1^0$$

Bachacou, Hinchliffe & Paige, PRD62

⇒ Signal: Two opposite sign leptons + jets + missing energy

⇒ 5 independent kinematical variables:

$$(m_{ll})^{edge},\ (m_{lq})^{\text{edge}}_{\text{low}},\ (m_{lq})^{\text{edge}}_{\text{high}},$$
$$(m_{llq})_{\text{edge}} \text{ and } (m_{llq})_{\text{thresh}}$$

⇒ ... but only 4 unknown masses involved!

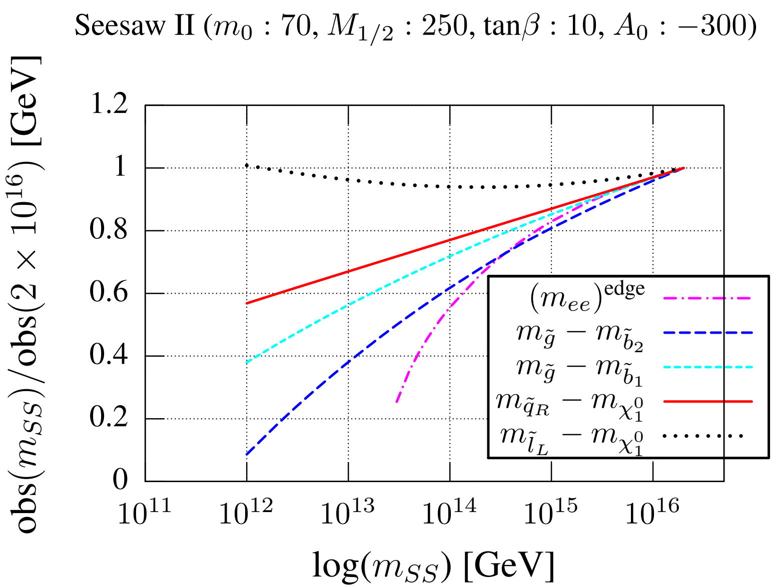
LHC observables - II

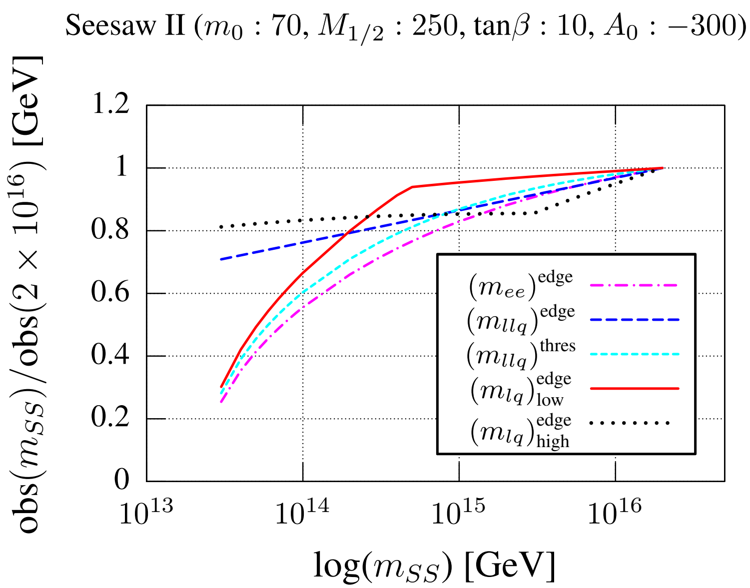
Variable	Value (GeV)	Error (GeV)
m_{ll}^{max}	77.07	0.08
m_{llq}^{max}	428.5	4.5
m_{lq}^{low}	300.3	3.1
m_{lq}^{high}	378.0	3.9
m_{llq}^{min}	201.9	2.6
m_{llb}^{min}	183.1	4.1
$m(\tilde{l}_L) - m(LSP)$	106.1	1.6
$m_{ll}^{max}(\chi_4^0)$	280.9	2.3
$m_{\tau\tau}^{max}$	80.6	5.1
$m(\tilde{g}) - 0.99 \times m(LSP)$	500.0	6.4
$m(\tilde{q}_R) - m(LSP)$	424.2	10.9
$m(\tilde{g}) - m(\tilde{b}_1)$	103.3	1.8
$m(\tilde{g}) - m(\tilde{b}_2)$	70.6	2.6

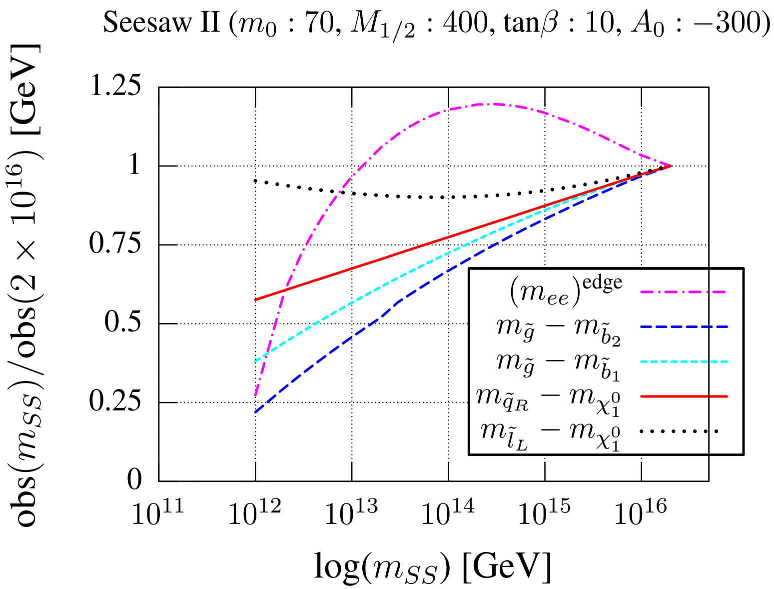
G. Weiglein et al.
Phys. Rep. 426

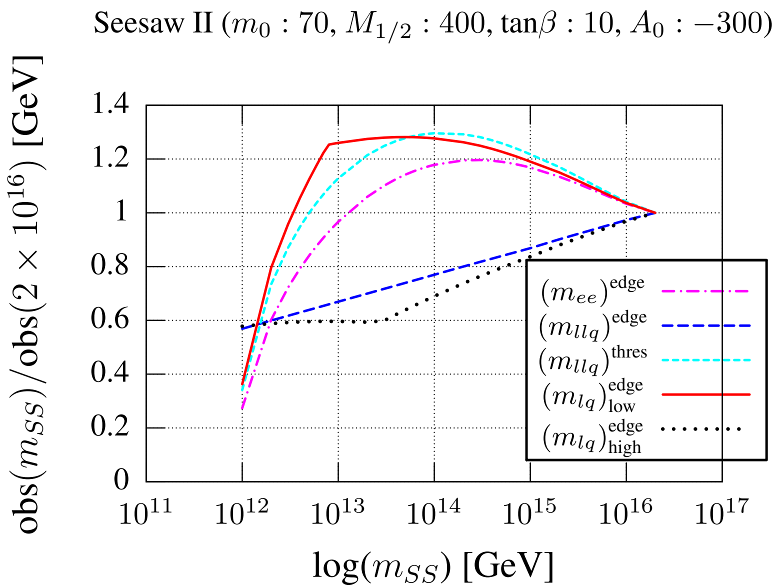
Values correspond to:
mSugra point SPS1a

LHC observables - III









LHC & ILC combined

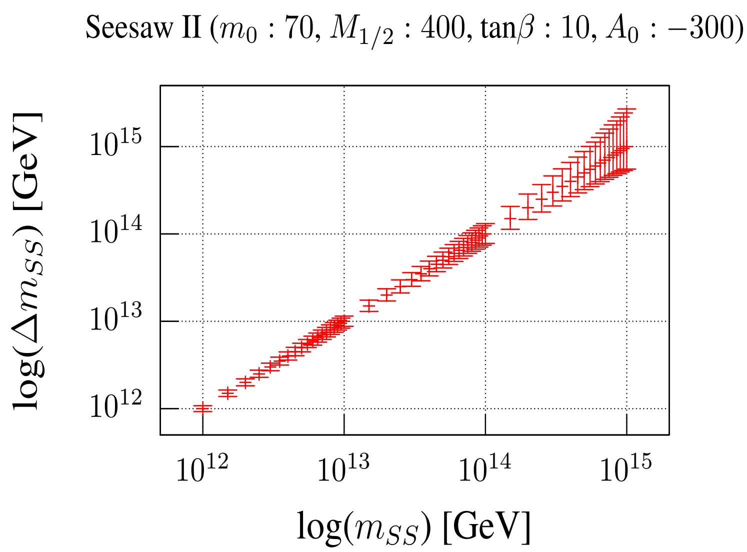
Particle	Mass	"LHC"	"ILC"	"LHC+ILC"
h^0	116.0	0.25	0.05	0.05
H^0	425.0		1.5	1.5
$\tilde{\chi}_1^0$	97.7	4.8	0.05	0.05
$\tilde{\chi}_2^0$	183.9	4.7	1.2	0.08
$\tilde{\chi}_4^0$	413.9	5.1	$3-5$	2.5
$\tilde{\chi}_1^\pm$	183.7		0.55	0.55
$\tilde{e}_R$	125.3	4.8	0.05	0.05
$\tilde{e}_L$	189.9	5.0	0.18	0.18
$\tilde{\tau}_1$	107.9	$5-8$	0.24	0.24
$\tilde{q}_R$	547.2	$7-12$	–	$5-11$
$\tilde{q}_L$	564.7	8.7	–	4.9
$\tilde{t}_1$	366.5		1.9	1.9
$\tilde{b}_1$	506.3	7.5	–	5.7
$\tilde{g}$	607.1	8.0	–	6.5

Aguilar-Saavedra, et al.
Eur. Phys. J. **C46**

Values correspond to:
mSugra point SPS1a′

Results ILC+LHC

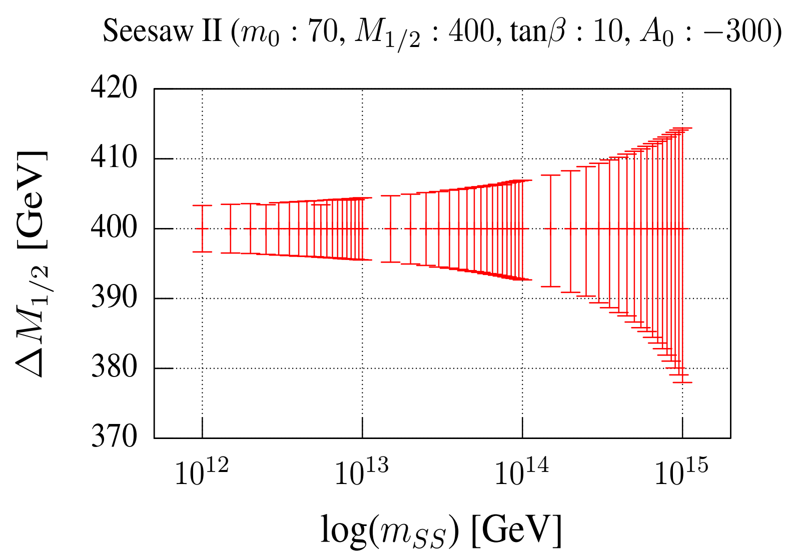
Seesaw type-II

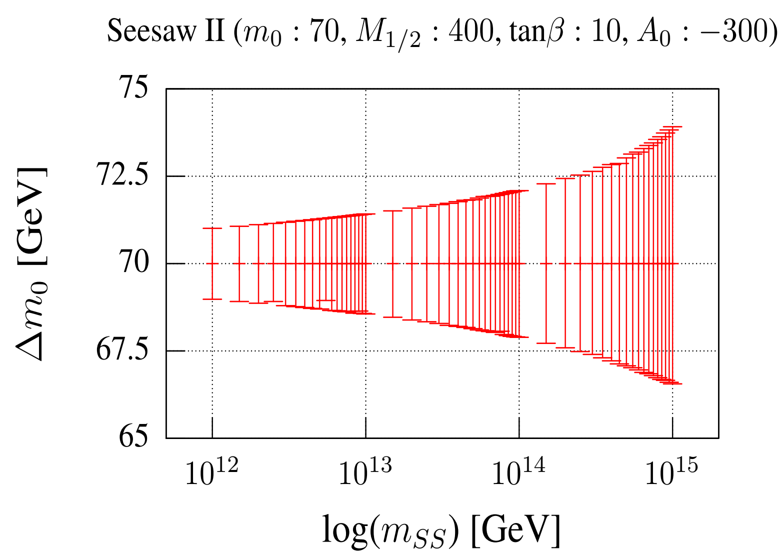


Note:

(i) $m_{SS} + \Delta(m_{SS}) = M_{GUT}$ at around $m_{SS} \sim 10^{15}$ GeV
1 σ c.l.

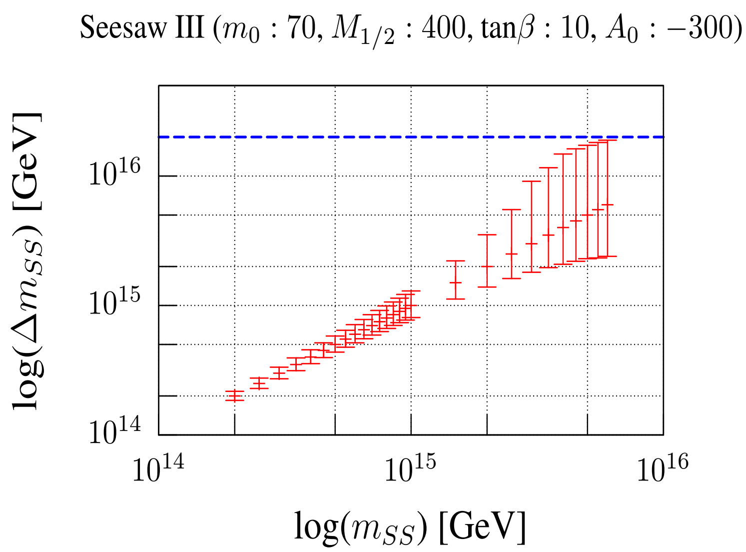
(ii) $\Delta(m_{SS})$ strongly dependent on m_{SS}





Results ILC+LHC

Seesaw type-III

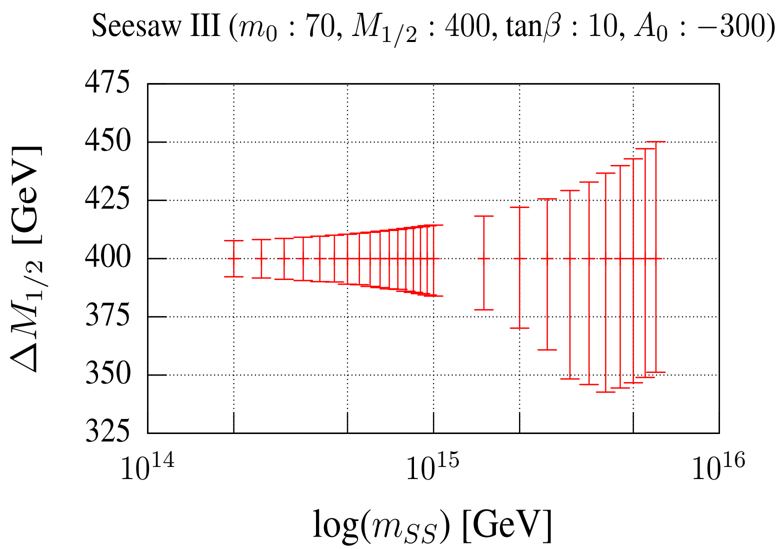


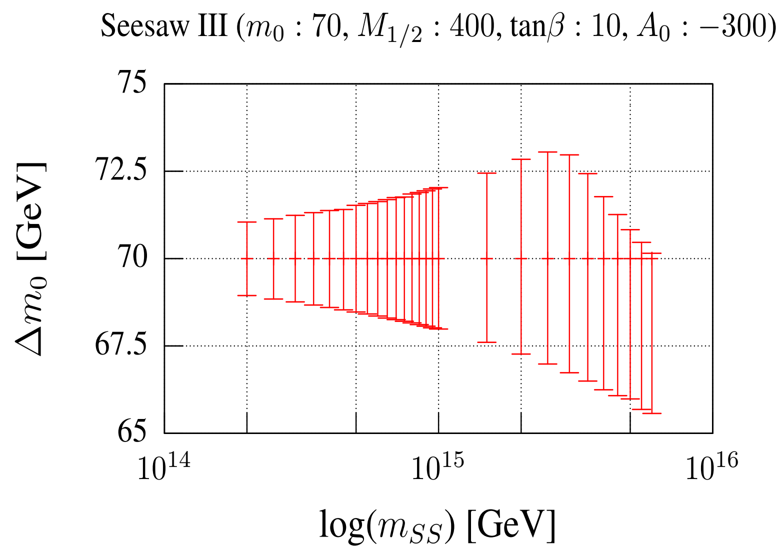
Note:

(i) change in scale: m_{SS}

(ii) $m_{SS} + \Delta(m_{SS}) = M_{GUT}$
at $m_{SS} \sim (6-7) \times 10^{15}$ GeV
1 σ c.l.

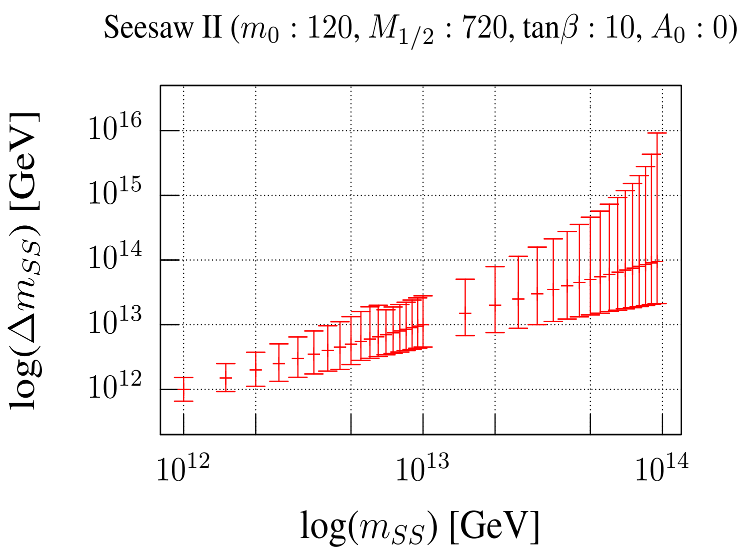
(iii) $\Delta(m_{SS})$ strongly
dependent on m_{SS}





Results LHC only

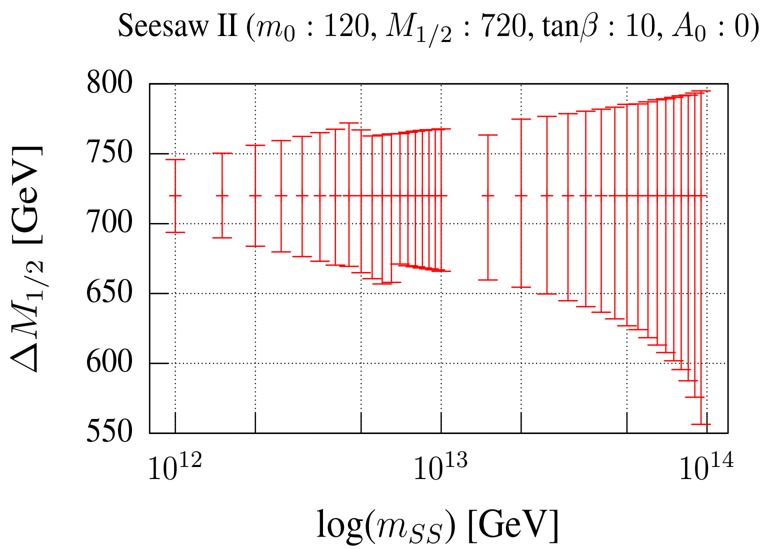
Seesaw type-II

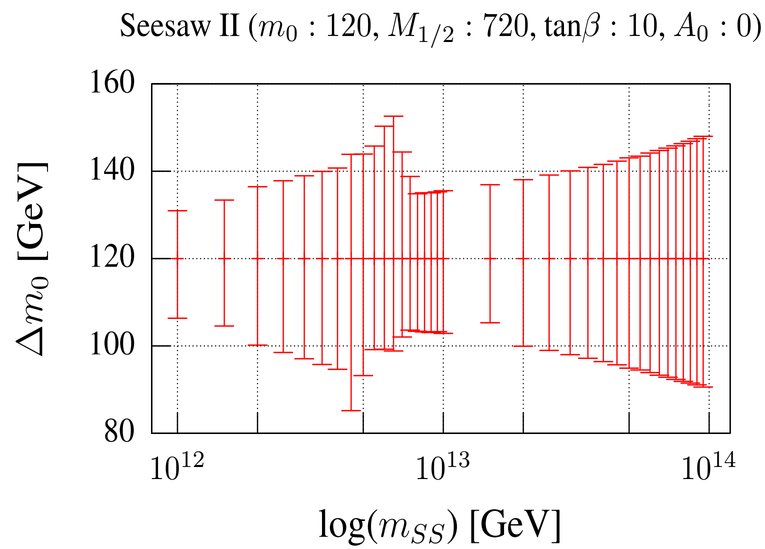


Note:

(i) $m_{SS} + \Delta(m_{SS}) = M_{GUT}$ at around $m_{SS} \sim 10^{14}$ GeV 1 σ c.l.

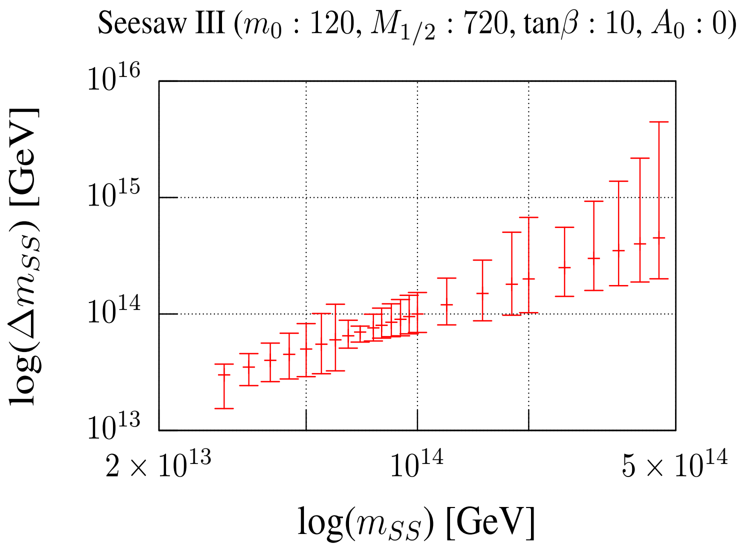
(ii) All error bars much larger than for LHC + ILC





Results LHC only

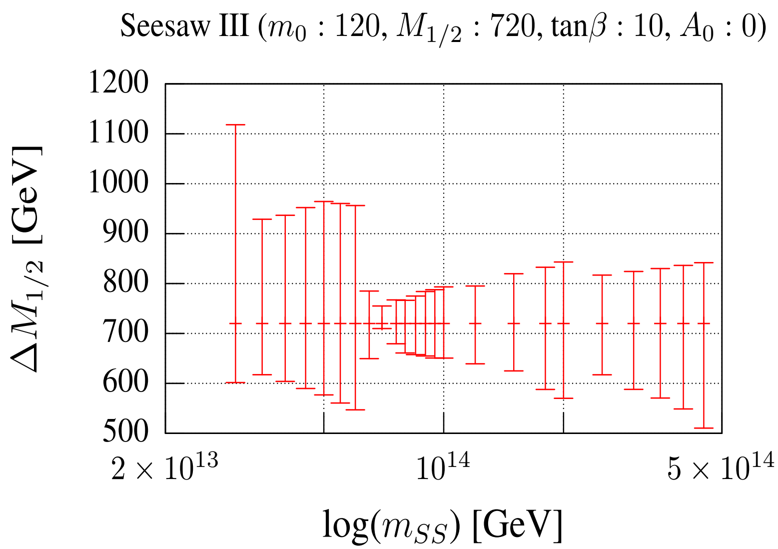
Seesaw type-III

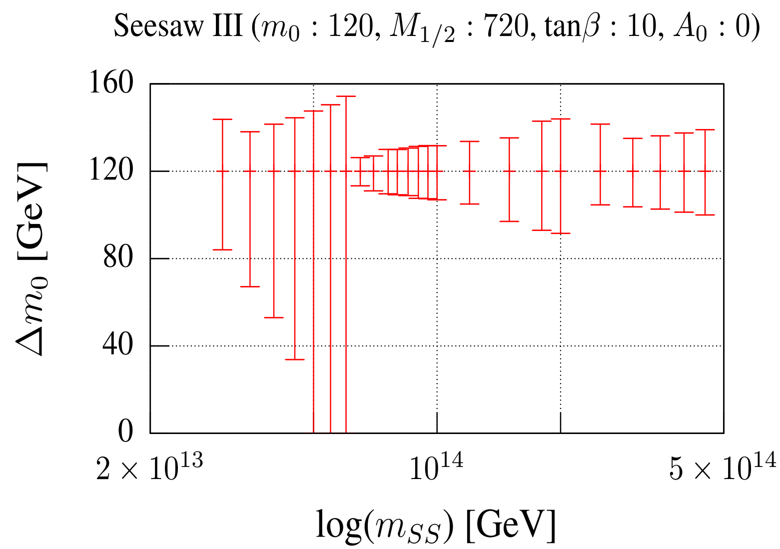


Note:

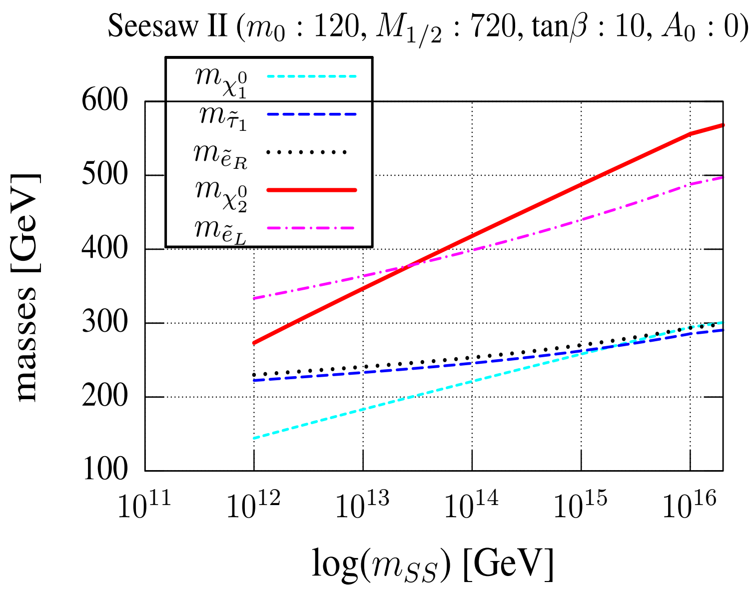
(i) $m_{SS} + \Delta(m_{SS}) = M_{GUT}$ at $m_{SS} \sim 5 \times 10^{14}$ GeV 1 σ c.l.

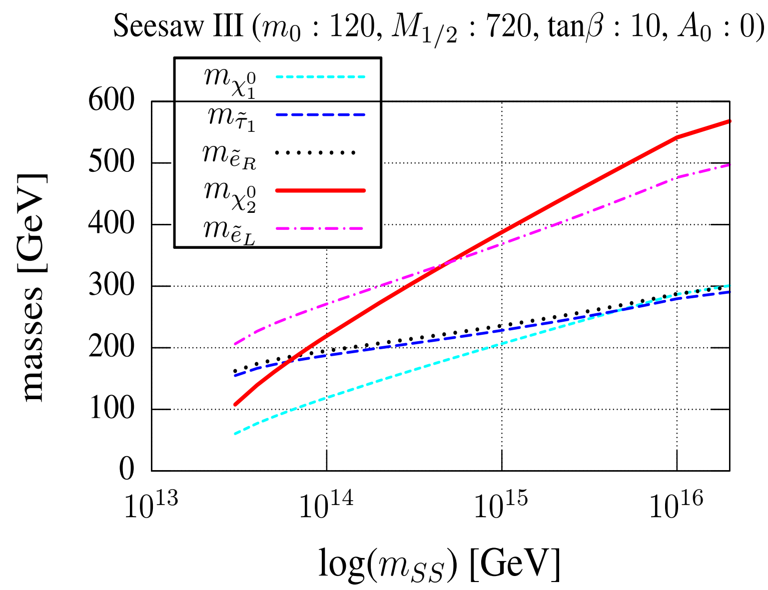
(ii) All error bars much larger than for LHC + ILC





Results LHC only

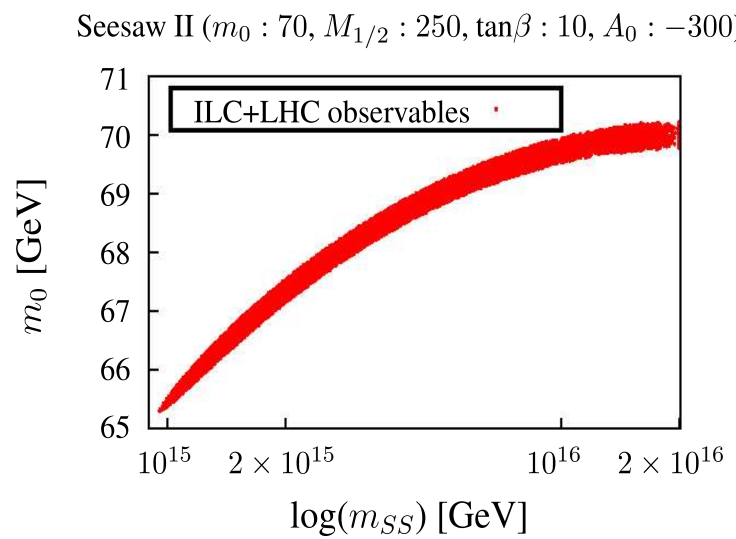


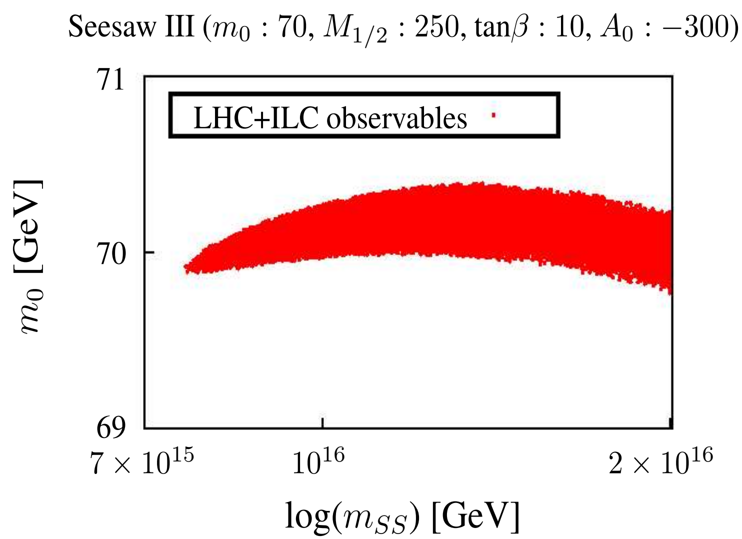


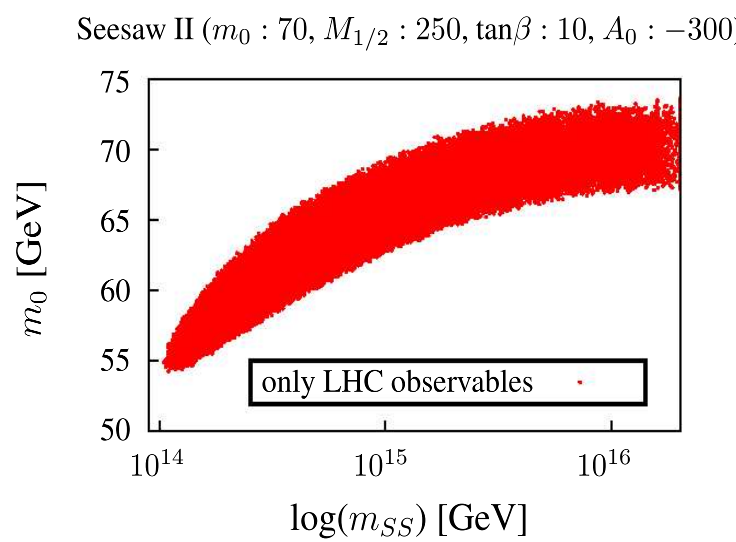
$\Rightarrow$ As function of m_{SS} mass ordering changed!

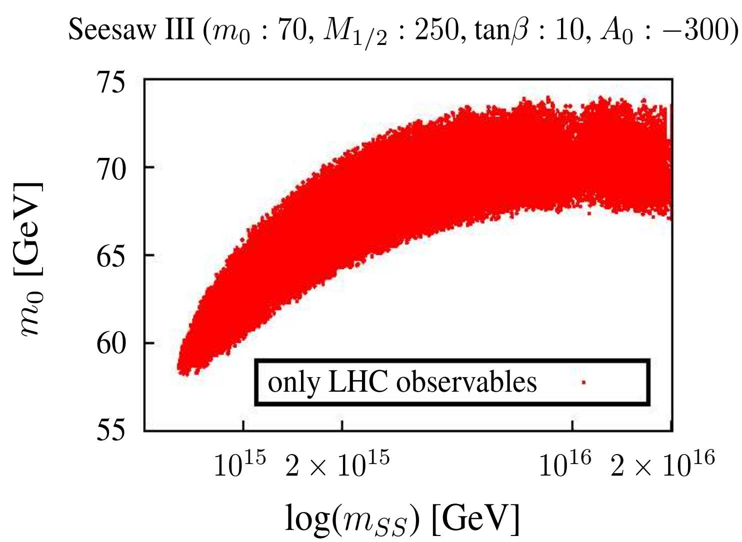
$\Rightarrow$ "Edge variables" crucial for LHC

Fitting mSugra with seesaw

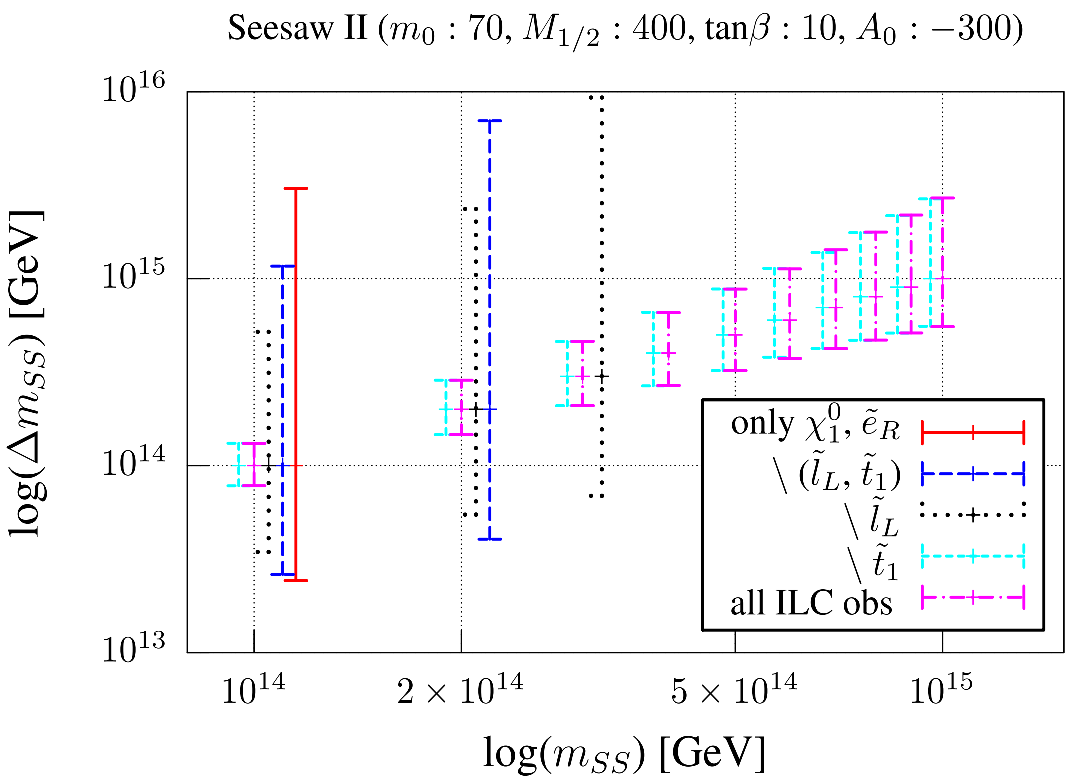








ILC errors & observables

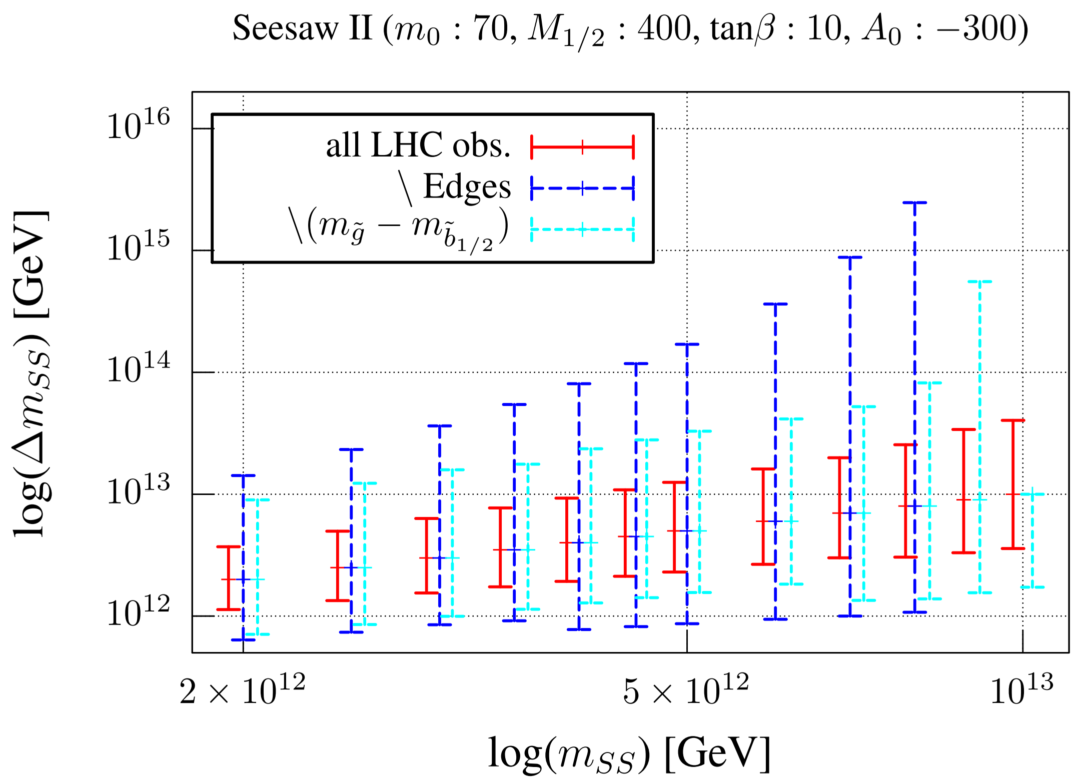


⇒ Switching off artificially different observables

⇒ left-slepton mass measurement most important

⇒ for $m_{SS} \lesssim 10^{14}$ measuring χ_1^0 + $\tilde{l}_R$ “sufficient”

LHC errors & observables



⇒ Switching off artificially different observables

⇒ "Edges" at LHC most important

⇒ Accurate $m_{\tilde{g}} - m_{\tilde{b}_1}$ brings significant improvement

Conclusions

Conclusions

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Backup slides

Analysis outline

Make forecast of expected sensitivity of mass measurements to M_{Seesaw} with:

$\Rightarrow$ For any set of mSugra + seesaw parameters calculate observables

$\Rightarrow$ Assume *relative errors* as in study points

$\Rightarrow$ Calculate χ^2 - distribution, varying 5 parameters

$\Rightarrow$ $\Delta\chi^2 \sim 5.9$ approximately expected 1 σ c.l. errors on parameters

$\Rightarrow$ type-II seesaw with one pair of $15, \overline{15}$

$\Rightarrow$ type-II seesaw 3 copies of 24_M

$\Rightarrow$ All calculations using 2-loop RGEs (SPheno)

$\Rightarrow$ Two "data sets": (i) LHC+ILC and (ii) LHC only