Phenomenology of the type-III seesaw model

Thomas Hambye
Université Libre de Bruxelles, Belgium

In collaboration with:

A. Abada, C. Biggio, F. Bonnet and B. Gavela: arXiv:0707.0633 (hep-ph), JHEP '07
arXiv:0803.0481 (hep-ph), PLB '08

R. Franceschini and A. Strumia, arXiv:0805.1613 (hep-ph), PLB '08

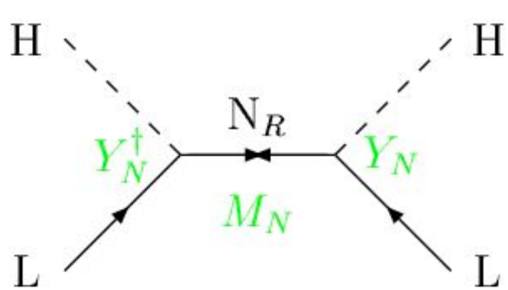
Y. Lin, A. Notari, M. Papucci, A. Strumia, arXiv: hep-ph/0312203, NPB '03

MPI-Heidelberg, 29/06/2009

The 3 basic seesaw models

i.e. tree level ways to generate the dim 5 operator

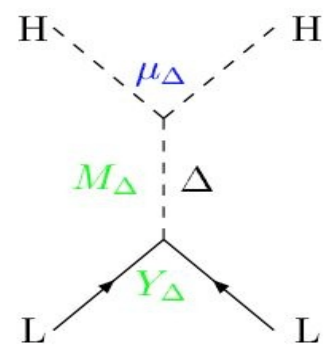
Right-handed singlet:
(type-I seesaw)



$$m_\nu = Y_N^T \frac{1}{M_N} Y_N v^2$$

Minkowski; Gellman, Ramon, Slansky; Yanagida;Glashow; Mohapatra, Senjanovic

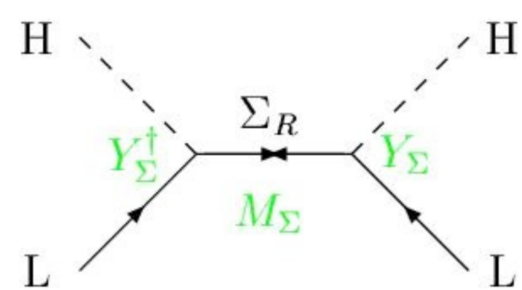
Scalar triplet:
(type-II seesaw)



$$m_\nu = Y_\Delta \frac{\mu_\Delta}{M_\Delta^2} v^2$$

Magg, Wetterich; Lazarides, Shafi; Mohapatra, Senjanovic; Schechter, Valle

Fermion triplet:
(type-III seesaw)



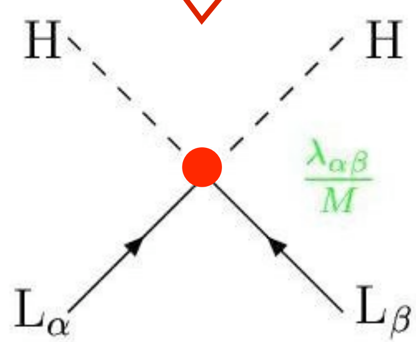
$$m_\nu = Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma v^2$$

Foot, Lew, He, Joshi; Ma; Ma, Roy;T.H., Lin, Notari, Papucci, Strumia; Bajc, Nemevsek, Senjanovic; Dorsner, Fileviez-Perez;....

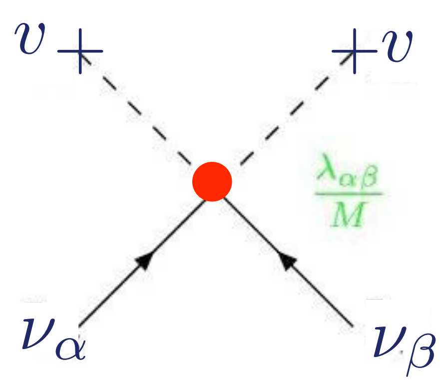
Dimension 5 operator

there is only one: $\mathcal{L} \ni \frac{\lambda}{M}(LLHH) = c_{d=5}(LLHH)$

$\Downarrow$



$\Downarrow$



$\Longrightarrow \quad m_\nu = \frac{\lambda}{M} v^2$

Majorana

$\Longrightarrow$ dim 5 operator $\Longleftrightarrow$ neutrino masses

Type-III seesaw Lagrangian

hyperchargeless triplet(s) of fermions: $\Sigma^+, \Sigma^0, \Sigma^-$

$$\Sigma = \begin{pmatrix} \Sigma^0/\sqrt{2} & \Sigma^+ \\ \Sigma^- & -\Sigma^0/\sqrt{2} \end{pmatrix}$$

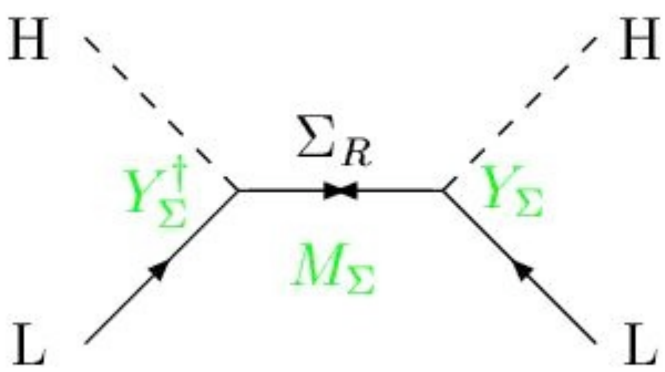
$$\mathcal{L} = \mathcal{L}_{SM} + Tr[\overline{\Sigma} i \not{D} \Sigma] - \frac{1}{2} Tr[\overline{\Sigma} M_\Sigma \Sigma^c + \overline{\Sigma^c} M_\Sigma^* \Sigma] - \tilde{\phi}^\dagger \overline{\Sigma} \sqrt{2} Y_\Sigma L + h.c.$$

Majorana mass term

kinetic term: interactions with W and Z bosons

Yukawa interactions

Neutrino masses



$$m_\nu = Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma v^2$$

(Σ^0 for neutrino masses is just like a right-handed neutrino)

Main differences between type-I and type-III models

triplets unlike N singlets:

- have gauge interactions:

$$\bar{\Sigma}^{-}\Sigma^{-}Z\,,\ \bar{\Sigma}^{+}\Sigma^{+}Z\,,\ \bar{\Sigma}^{0}\Sigma^{+}W^{-}\,,\ \bar{\Sigma}^{0}\Sigma^{-}W^{+}$$

$$(+h.c.)$$

- induce mixing of the charged leptons with new physics states: $\Sigma^{+c}-l^{-}$ mixing (much easier to see than mixing of neutral neutrino states)

$$\mathcal{L} \ni vY_{\Sigma}\Sigma^{+}l^{-}$$

production at colliders and rare decays

Testing the type-III seesaw models

- production at colliders

  at LHC for M_Σ up to $\sim 1.5\,\mathrm{TeV}$

- rare leptonic decays

  up to $M_\Sigma \sim 200\,\mathrm{TeV}$

Rare leptonic processes

→ to calculate them one first needs to diagonalize the mass matrix of charged and neutral leptons non-diagonal due to Yukawa interactions

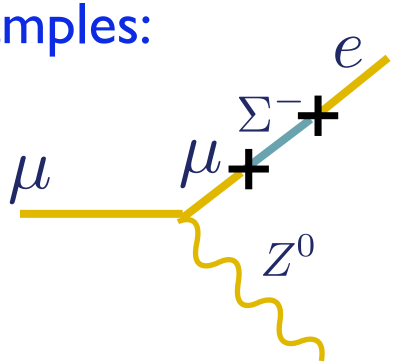
→ $\mathcal{L} \ni v Y_\Sigma \Sigma^+ l^- + v Y_\Sigma \Sigma^0 \nu + M_\Sigma \bar{\Sigma}\Sigma$

↑

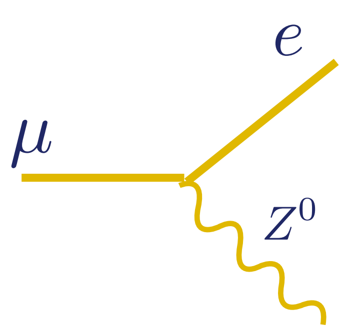
$\Sigma^0 - \nu$ and $\Sigma^{+c} - l^-$ mixings of $\mathcal{O}\left(\frac{Y_\Sigma v}{M_\Sigma}\right)$

$\Longrightarrow$ generates flavour violating vertices in the mass eigenstates basis:

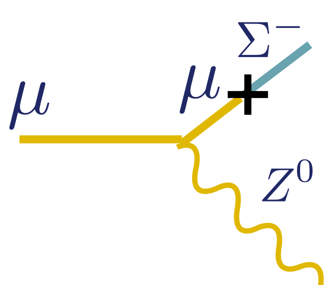
examples:



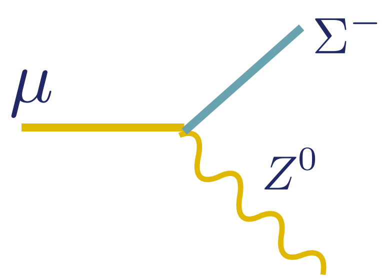
$\Longrightarrow$



$$\propto \frac{v^2}{2}\left(Y_\Sigma^\dagger \frac{1}{M_\Sigma^\dagger M_\Sigma} Y_\Sigma\right)_{e\mu}$$

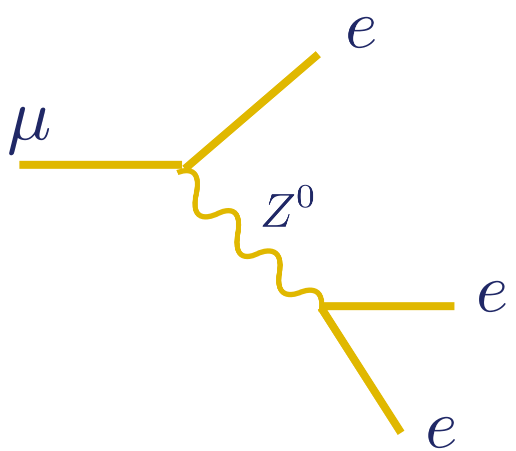


$\Longrightarrow$



$$\propto \frac{Y_\Sigma v}{M_\Sigma}$$

$\mu \to eee$



$$\Longrightarrow \quad Br(\mu \to eee) = \left(\frac{v^2}{2} Y_\Sigma^\dagger \frac{1}{M_\Sigma^\dagger M_\Sigma} Y_\Sigma\right)^2_{e\mu} \cdot (3\sin^4\theta_W - 2\sin^2\theta_W + 1/2)$$

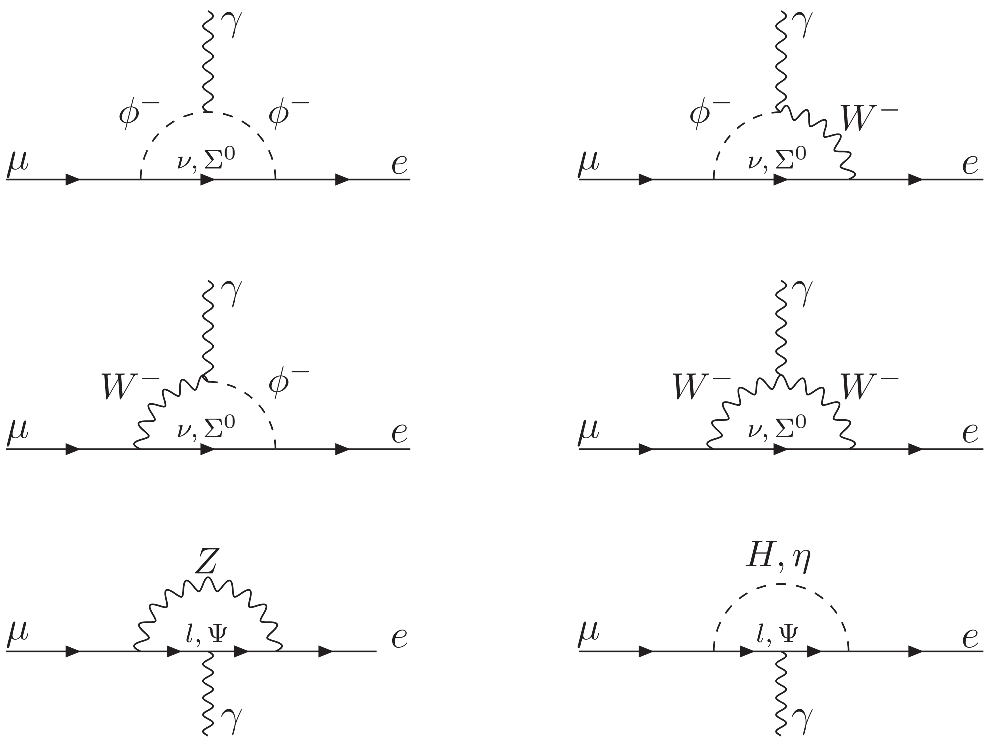
$$\Downarrow \quad \longleftarrow Br(\mu \to eee)_{exp} < 1.0 \cdot 10^{-12}$$

$$\left(\frac{v^2}{2} Y_\Sigma^\dagger \frac{1}{M_\Sigma^\dagger M_\Sigma} Y_\Sigma\right)_{e\mu} < 1.1 \cdot 10^{-6}$$

$\Longrightarrow$ $Y_\Sigma <\sim 10^{-2} \cdot (\frac{M_\Sigma}{1\,\mathrm{TeV}})$ (or if $Y_\Sigma \simeq 1 : M_\Sigma > 170\,\mathrm{GeV}$)

better than in the type-I model where this process can be induced only at one-loop level (because no charged lepton mixing)

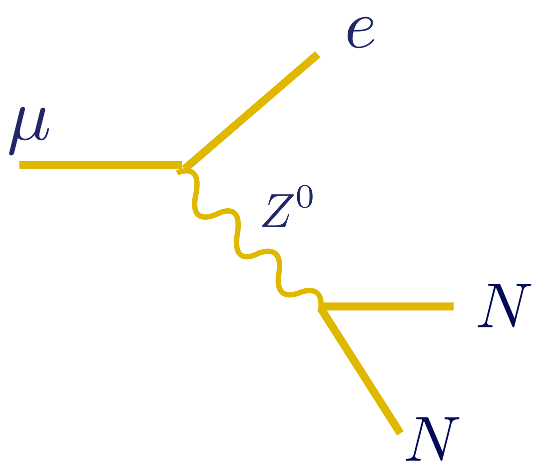
$\mu \to e\gamma$



$$\Rightarrow Br(\mu \to e\gamma) = \frac{3}{32}\frac{\alpha}{\pi}\left(\frac{13}{3} - 6.56\right)\left(\frac{v^2}{2} Y_\Sigma^\dagger \frac{1}{M_\Sigma^\dagger M_\Sigma} Y_\Sigma\right)^2_{e\mu}$$

$$\left(\frac{v^2}{2} Y_\Sigma^\dagger \frac{1}{M_\Sigma^\dagger M_\Sigma} Y_\Sigma\right)_{e\mu} < 1.1 \cdot 10^{-4}$$

$\mu \longrightarrow e$ conversion in atomic nuclei



$\Longrightarrow$ same vertex as for $\mu \longrightarrow eee$

$R_{\mu\to e}$ in ${}^{48}_{22}Ti$ gives: $\frac{v^2}{2}\left(Y_\Sigma^\dagger \frac{1}{M_\Sigma^\dagger M_\Sigma} Y_\Sigma\right)_{e\mu} < 1.7\cdot 10^{-7}$

Summary of constraints

Constraints on	Process	Bound
$\lvert\varepsilon_{e\mu}\rvert$	$\mu^- \to e^+e^-e^-$	$< 1.1 \cdot 10^{-6}$
$\lvert\varepsilon_{e\tau}\rvert$	$\tau^- \to e^+e^-e^-$	$< 1.2 \cdot 10^{-3}$
$\lvert\varepsilon_{\mu\tau}\rvert$	$\tau^- \to \mu^+\mu^-\mu^-$	$< 1.2 \cdot 10^{-3}$
$\lvert\varepsilon_{\tau e}\rvert$	$\tau^- \to \mu^+\mu^-e^-$	$< 1.6 \cdot 10^{-3}$
$\lvert\varepsilon_{\tau\mu}\rvert\lvert\varepsilon_{e\mu}\rvert$	$\tau^- \to e^+\mu^-\mu^-$	$< 3.1 \cdot 10^{-4}$
$\lvert\varepsilon_{\tau\mu}\rvert$	$\tau^- \to e^+e^-\mu^-$	$< 1.5 \cdot 10^{-3}$
$\lvert\varepsilon_{\tau e}\rvert\lvert\varepsilon_{\mu e}\rvert$	$\tau^- \to \mu^+e^-e^-$	$< 2.9 \cdot 10^{-4}$
$\lvert\varepsilon_{e\mu}\rvert$	$\mu \to e\gamma$	$< 1.1 \cdot 10^{-4}$
$\lvert\varepsilon_{\mu\tau}\rvert$	$\tau \to \mu\gamma$	$< 1.5 \cdot 10^{-2}$
$\lvert\varepsilon_{e\tau}\rvert$	$\tau \to e\gamma$	$< 2.4 \cdot 10^{-2}$
$\lvert\varepsilon_{e\mu}\rvert$	$R_{\mu\to e}$	$< 1.7 \cdot 10^{-7}$

$$\varepsilon_{\alpha\beta} = \left(\frac{v^2}{2} Y_\Sigma^\dagger \frac{1}{M_\Sigma^\dagger M_\Sigma} Y_\Sigma\right)_{\alpha\beta}$$

$\Longrightarrow$ combining all constraints:

$$\frac{v^2}{2}\, |Y_\Sigma^\dagger \frac{1}{M_\Sigma^\dagger} \frac{1}{M_\Sigma} Y_\Sigma|_{\alpha\beta} \lesssim \begin{pmatrix} 3 \cdot 10^{-3} & < 1.7 \cdot 10^{-7} & < 1.2 \cdot 10^{-3} \\ < 1.1 \cdot 10^{-6} & 4 \cdot 10^{-3} & < 1.2 \cdot 10^{-3} \\ < 1.2 \cdot 10^{-3} & < 1.2 \cdot 10^{-3} & 4 \cdot 10^{-3} \end{pmatrix}$$

Rare processes ratio predictions

$\mu \to eee$

$\mu \to e\gamma$ all depend on the same combination $\frac{v^2}{2}\left(Y_\Sigma^\dagger \frac{1}{M_\Sigma^\dagger M_\Sigma} Y_\Sigma\right)_{e\mu}$

$R_{\mu\to e}$

$\Longrightarrow$ predictions for the ratios

$$Br(\mu \to e\gamma) = 1.3 \cdot 10^{-3} \cdot Br(\mu \to eee) = 2.4 \cdot 10^{-1} \cdot R_{\mu\to e}$$

the observation of $\mu \to e\gamma$ in the near future would rule out the type-III model

similarly

$$\begin{aligned} Br(\tau \to \mu\gamma) &= 1.3 \cdot 10^{-3} \cdot Br(\tau \to \mu\mu\mu) = 2.1 \cdot 10^{-3} \cdot Br(\tau^- \to e^- e^+ \mu^-) \\ Br(\tau \to e\gamma) &= 1.3 \cdot 10^{-3} \cdot Br(\tau \to eee) = 2.1 \cdot 10^{-3} \cdot Br(\tau^- \to \mu^- \mu^+ e^-) \end{aligned}$$

Only one dimension 6 operator in type III seesaw

Model	Effective Lagrangian $\mathcal{L}_{eff} = c_i \mathcal{O}_i$		
	$c^{d=5}$	$c_i^{d=6}$	$\mathcal{O}_i^{d=6}$
Fermionic Singlet	$Y_N^T \frac{1}{M_N} Y_N$	$Y_N^\dagger \frac{1}{\vert M_N\vert^2} Y_N$	$\left(\overline{L}\widetilde{H}\right) i\partial\!\!\!/ \left(\widetilde{H}^\dagger L\right)$
Fermionic Triplet	$Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma$	$Y_\Sigma^\dagger \frac{1}{\vert M_\Sigma\vert^2} Y_\Sigma$	$\left(\overline{L}\vec{\tau}\widetilde{H}\right) iD\!\!\!\!/ \left(\widetilde{H}^\dagger \vec{\tau} L\right)$
Scalar Triplet	$4Y_\Delta \frac{\mu_\Delta}{\vert M_\Delta\vert^2}$	$Y_\Delta^\dagger \frac{1}{2\vert M_\Delta\vert^2} Y_\Delta$	$\left(\overline{\widetilde{L}}\vec{\tau}L\right)\left(\overline{L}\vec{\tau}\widetilde{L}\right)$
		$\frac{\vert\mu_\Delta\vert^2}{\vert M_\Delta\vert^4}$	$\left(H^\dagger\vec{\tau}\widetilde{H}\right)\left(\overleftarrow{D}_\mu \overrightarrow{D}^\mu\right)\left(\widetilde{H}^\dagger\vec{\tau}H\right)$
		$-2\left(\lambda_3+\lambda_5\right)\frac{\vert\mu_\Delta\vert^2}{\vert M_\Delta\vert^4}$	$\left(H^\dagger H\right)^3$

Do we expect the rare processes rates to be close to the experimental bounds??

e.g. no! ← would require low $M_\Sigma \sim \mathrm{TeV}$ and large $Y_\Sigma > 10^{-2}$

$$m_\nu = -\frac{v^2}{2} Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma \sim \mathrm{MeV}$$

but not at all excluded:

- rare processes: do not violate L: $\sim \frac{v^2}{2} Y_\Sigma^\dagger \frac{1}{M_\Sigma^\dagger M_\Sigma} Y_\Sigma$
- neutrino masses: do break L: $\sim -\frac{v^2}{2} Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma$

$\Rightarrow$ approximate L conservation framework:

- first assume low $M_\Sigma \sim TeV$ and large Yukawas with L conserved → large rare processes, $m_\nu = 0$
- then introduce a small perturbation breaking L → $m_\nu \neq 0$

Production at LHC

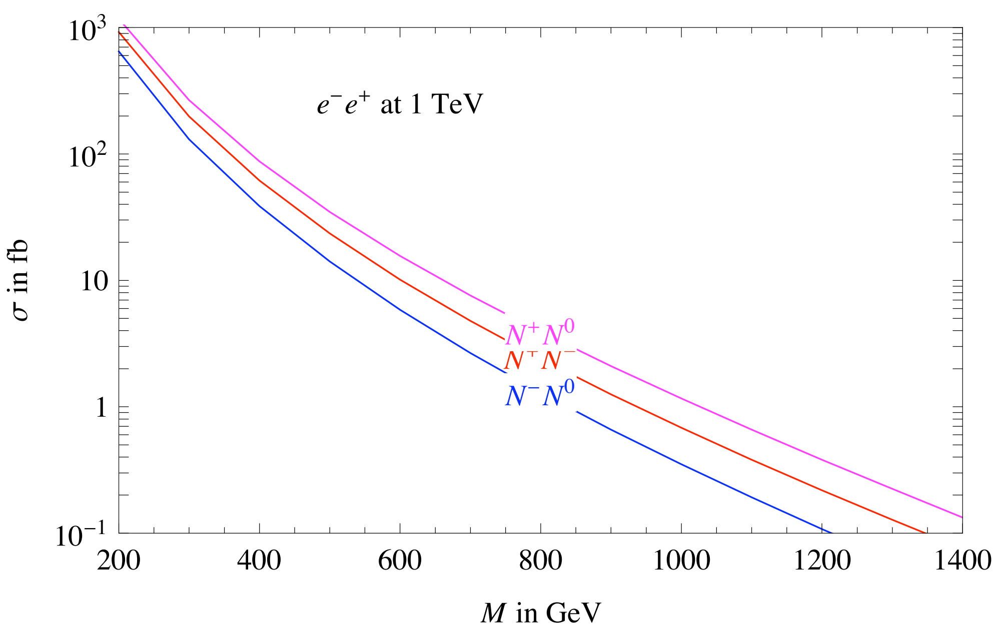
via gauge interactions: $\bar{\Sigma}^{-}\Sigma^{-}Z\,,\ \bar{\Sigma}^{+}\Sigma^{+}Z\,,\ \bar{\Sigma}^{0}\Sigma^{+}W^{-}\,,\ \bar{\Sigma}^{0}\Sigma^{-}W^{+}$

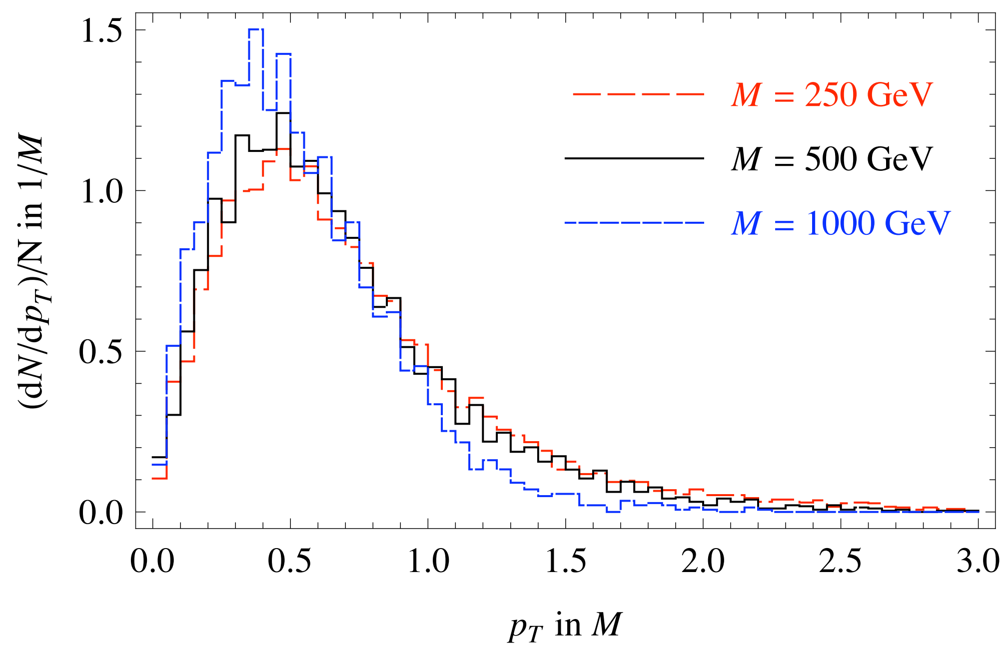
$(+h.c.)$

$$q\bar{q} \to Z \to \Sigma^{+}\Sigma^{-}$$
$$q\bar{q} \to W^{\pm} \to \Sigma^{\pm}\Sigma^{0}$$

Ma, Roy '02

Franceschini, TH, Strumia '08





$\Rightarrow$ for $M_\Sigma = 250\,\mathrm{GeV}$: 3000 triplet pairs for $\mathcal{L} = 3\,\mathrm{fb}^{-1}$

$M_\Sigma = 1\,\mathrm{TeV}$: 10 triplet pairs for $\mathcal{L} = 3\,\mathrm{fb}^{-1}$ $\Rightarrow$ up to $M_\Sigma \sim 1.5\,\mathrm{TeV}$

$\Rightarrow$ to determine M_Σ, establish it is a fermion produced via gauge interaction

Decays at LHC

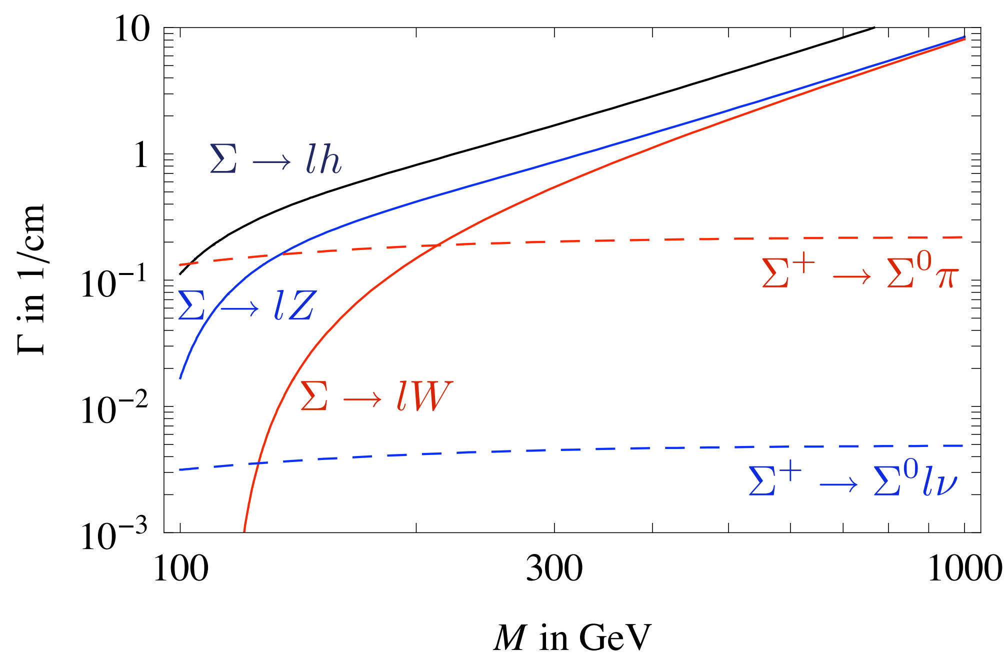
necessary to establish the Yukawa interactions and their flavour structure

Bajc, Nemevsek, Senjanovic '07

$$\left.\begin{array}{l}\Sigma^0 \to \nu h \\ \Sigma^0 \to Z\nu \\ \Sigma^0 \to W^- l^+\end{array}\right\} \propto Y_\Sigma^2$$

$$\left.\begin{array}{l}\Sigma^+ \to l^+ h \\ \Sigma^+ \to Z l^+ \\ \Sigma^+ \to W^+ \nu\end{array}\right\} \propto Y_\Sigma^2$$

Franceschini, TH, Strumia '08



+ decay to pion (allowed because $M_{\Sigma^+} - M_{\Sigma^0} = 166\,\mathrm{MeV} > m_{\pi^+}$):

$\Sigma^+ \to \Sigma^0 \pi^+ \propto$ gauge couplings only $\Rightarrow$ dominant for small Yukawa's in case all Σ^+ become Σ^0 (pion to soft to be observed)

L violation

last necessary ingredient to establish that the triplets have contributions to neutrino masses

requires to observe the decays of both triplets

$$pp \longrightarrow (\Sigma^+ \longrightarrow l_1^+ Z) + (\Sigma^0 \longrightarrow l_2^+ W^-)$$

Yukawa couplings + L-violation: neutrino masses

The signal with highest rate

$$pp \to (\Sigma^+ \to \bar{\nu} W^+) + (\Sigma^0 \to W^\pm l^\mp) \to 4\,\text{jets} + l^\mp + \text{missing energy}$$

100 events for $M_\Sigma = 250\,\text{GeV}$ and $\mathcal{L} = 3\,\text{fb}^{-1}$ $(\sigma \sim \text{fb})$

clean because background can be distinguished kinematically

$$(V = W, Z)$$

$$pp \to (V \to 2\,\text{jets}) + (V \to 2\,\text{jets}) + (W^- \to l^- \bar{\nu}) \qquad (\sigma \sim 17\,\text{fb})$$

$$pp \to 4\,\text{QCD jets} + (W^- \to l^- \bar{\nu}) \qquad (\sigma \sim 4500\,\text{fb})$$

$$pp \to (t \to bW^- \to b l^- \bar{\nu}) + (\bar{t} \to \bar{b} + 2\,\text{jets}) \qquad (\sigma \sim 160\,\text{pb})$$

$l^- \bar{\nu}$ pairs from W unlike signal $\Rightarrow$ require $m_T(l^- \bar{\nu}) > m_W$

$$pp \to 4\,\text{QCD jets} + (Z \to \nu \bar{\nu}) + (W^- \to l^- \bar{\nu}) \qquad (\sigma \sim 200\,\text{fb})$$

Lepton flavour violating signals

$$pp \to l_1 \bar{l}_2 Z W^+ \quad (Z \to \mu^+\mu^-, e^+e^-, 2\,\text{jets};\, W^+ \to l^+\bar{\nu}, 2\text{jets})$$

$$pp \to l_1 \bar{l}_2 Z Z \quad (Z \to \mu^+\mu^-, e^+e^-, 2\,\text{jets})$$

background under control $\longleftarrow$ see Franceschini, TH, Strumia '08

Lepton number violating signals

$$pp \rightarrow (\Sigma^+ \rightarrow l_1^+ Z) + (\Sigma^0 \rightarrow l_2^+ W^-)$$
$$pp \rightarrow (\Sigma^- \rightarrow l_1^- Z) + (\Sigma^0 \rightarrow l_2^- W^+)$$

background from

$$pp \rightarrow VV + (W^+ \rightarrow l_1^+ \nu) + (W^+ \rightarrow l_2^+ \nu) \qquad (\sigma \simeq 1\,\mathrm{fb})$$
$$pp \rightarrow 4\,\mathrm{QCD\ jets} + (W^+ \rightarrow l_1^+ \nu) + (W^+ \rightarrow l_2^+ \nu) \qquad (\sigma \simeq 20\,\mathrm{fb})$$

So far: M_Σ, Y_Σ flavour structure, L violation

but still not the absolute scale of Yukawa's $\longleftarrow$ absolute scale of m_ν

$\uparrow$

displaced vertices

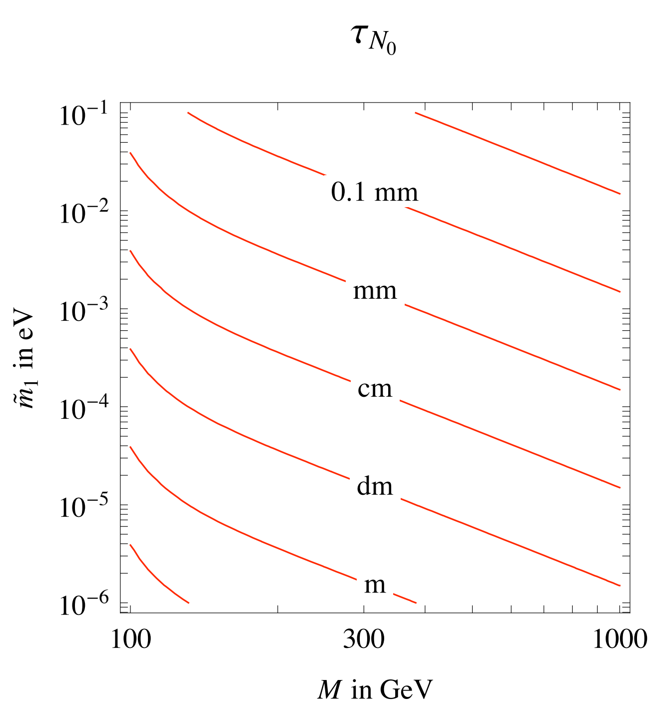
Displaced vertices

possible in type-III model unlike in most TeV scale new physics models

$M_\Sigma \sim 1\,\mathrm{TeV},\ m_\nu \sim 10^{-1}\,\mathrm{eV}$ requires $Y_\Sigma > \sim 10^{-6}$

2 body decays e.g. far too fast at this scale

slow decay: $\tau_\Sigma = \dfrac{8\pi}{|Y_{\Sigma i}|^2 M_\Sigma} = 0.3\,\mathrm{mm} \cdot \dfrac{10^{-12}}{|Y_{\Sigma i}|^2} \cdot \dfrac{100\,\mathrm{GeV}}{M_\Sigma}$



$$\tilde{m} = \frac{|Y_{\Sigma i}|^2\, v^2}{M_\Sigma}$$

$$\tau_\Sigma = \frac{8\pi v^2}{\tilde{m} M_\Sigma^2} = 0.3\,\mathrm{mm} \cdot \frac{\sqrt{\delta m^2_{atm}}}{\tilde{m}} \cdot \left(\frac{100\,\mathrm{GeV}}{M_\Sigma}\right)^2$$

LHC detectors: better than $0.1\,\mathrm{mm}$ in the transverse direction

Determining the neutrino hierarchy from measuring displaced vertices

- In full generality: $\tilde{m} > m_{\nu_1}$

for example for $M_\Sigma = 100\,\mathrm{GeV}$ if the averaged displaced vertex is measured larger than $0.3\,\mathrm{mm}$ it means $(\Delta m^2_{atm})^{1/2} > \tilde{m} > m_{\nu_1}$ $\Longrightarrow$ hierarchical neutrino spectrum

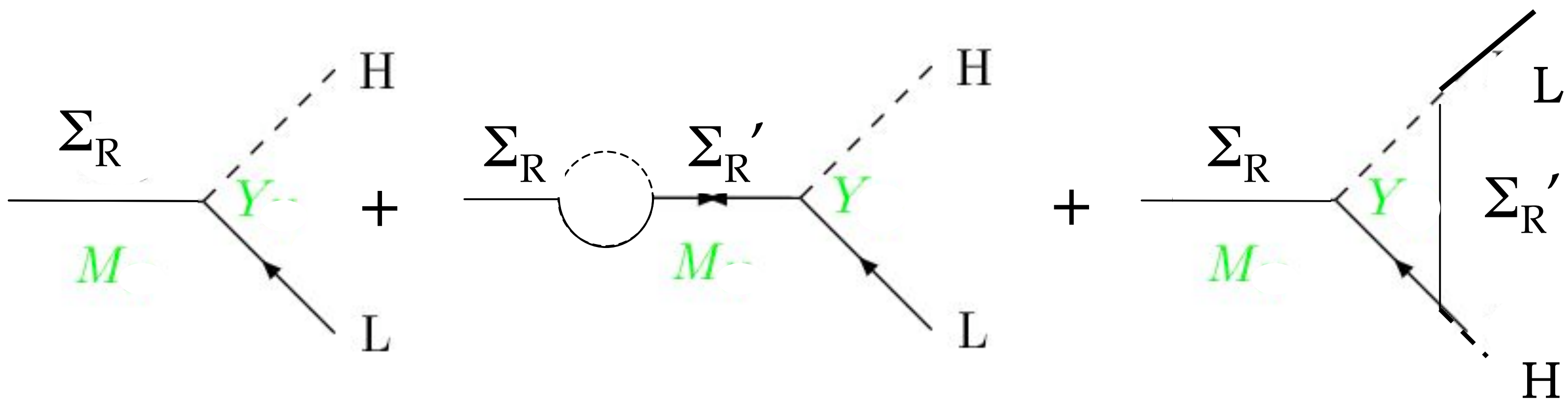
$$\tau_\Sigma = \frac{8\pi v^2}{\tilde{m} M_\Sigma^2} = 0.3\,\mathrm{mm} \cdot \frac{\sqrt{\delta m^2_{atm}}}{\tilde{m}} \cdot \left(\frac{100\,\mathrm{GeV}}{M_\Sigma}\right)^2$$

- Similarly if one observes several triplets one can distinguish a normal and inverted hierarchies $\longleftarrow$ $\sum_i \tilde{m}_i > \sum_i |m_{\nu_i}|$

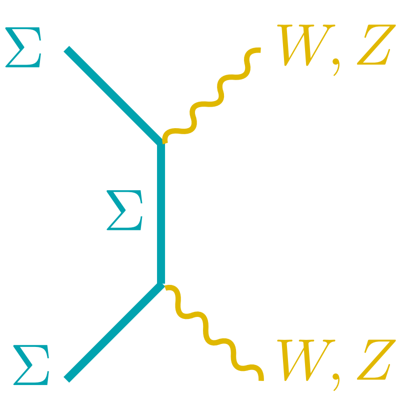
Leptogenesis in the type-III seesaw model

TH, Lin, Notari, Papucci, Strumia '03

from triplets decays to leptons and Higgses



⇒ one important difference with type-I seesaw leptogenesis:
gauge interactions:



→ put the triplets into closer thermal equilibrium

suppress the L asymmetry produced

Lower bounds on the triplet mass for leptogenesis

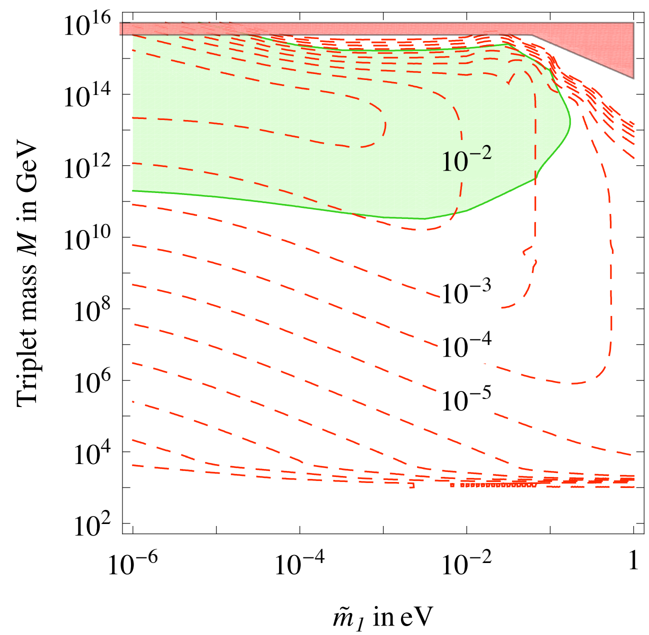
- Hierarchical triplet mass spectrum:

$$M_\Sigma > 1.5 \cdot 10^{10}\,\mathrm{GeV}$$

- Quasi-degenerate triplet mass spectrum

$$M_\Sigma > 1.6\,\mathrm{TeV}$$

TH, Lin, Notari, Papucci, Strumia '03
Strumia '08



Summary

- Type-III seesaw model: perfectly viable origin of neutrino masses

  e.g. expected at a very scale, but nothing forbids it around the TeV scale

- Rich phenomenology: - rare lepton processes $\mu \to eee,\ \mu \to e\gamma,$
  $\mu \to e\,\text{conversion}, \ldots$

  $\Downarrow$

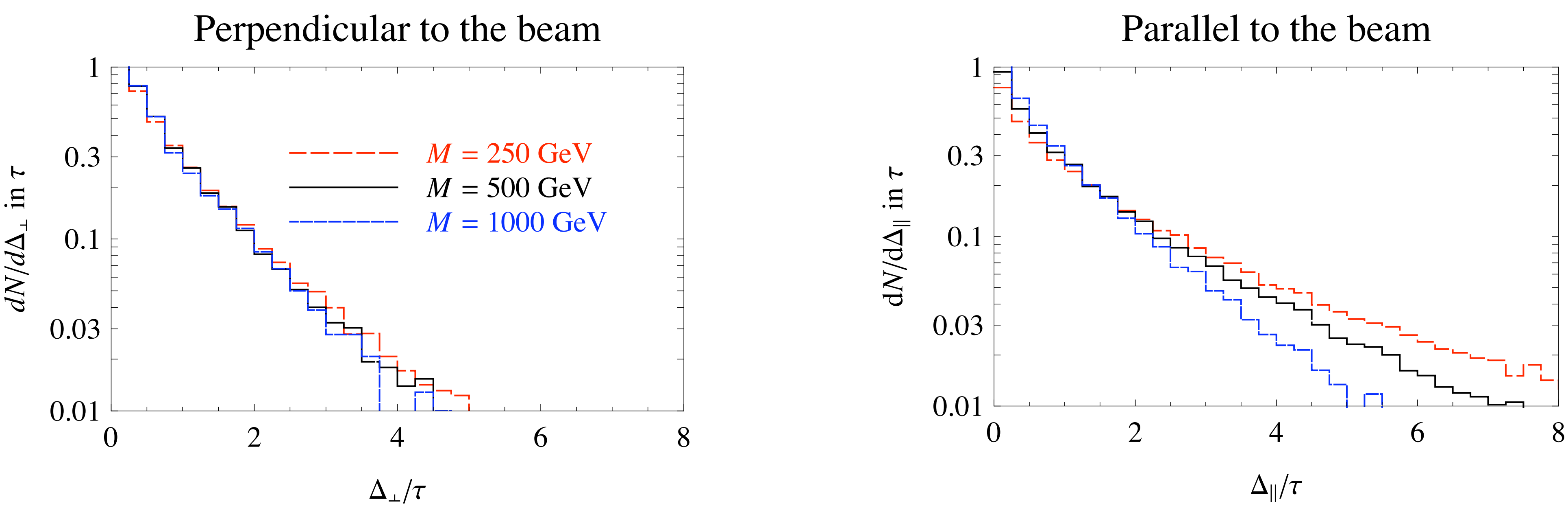
  definite predictions for the ratios

  (large if approximate L conservation framework)

  - fully testable at LHC up to $M_\Sigma \sim 1.5\,\text{TeV}$

- Allow successful leptogenesis for $M_\Sigma > 1.5 \cdot 10^{10}\,\text{GeV}$ (hierarchical)

  $M_\Sigma > 1.6\,\text{TeV}$ (quasi-degenerate)



Distribution of the displacement of the secondary vertex from the interaction point.

Do we expect the rare processes rates to be close to the experimental bounds??

e.g. no! ← would require low $M_\Sigma \sim \mathrm{TeV}$ and large $Y_\Sigma > 10^{-2}$

$$m_\nu = -\frac{v^2}{2} Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma \sim \mathrm{MeV}$$

but not at all excluded:

- rare processes: do not violate L: $\sim \frac{v^2}{2} Y_\Sigma^\dagger \frac{1}{M_\Sigma^\dagger M_\Sigma} Y_\Sigma$
- neutrino masses: do break L: $\sim -\frac{v^2}{2} Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma$

$\Rightarrow$ approximate L conservation framework:

- first assume low $M_\Sigma \sim TeV$ and large Yukawas with L conserved → large rare processes, $m_\nu = 0$
- then introduce a small perturbation breaking L → $m_\nu \neq 0$

Dim 5 and dim 6 operator summary

Model	Effective Lagrangian $\mathcal{L}_{eff} = c_i \mathcal{O}_i$		
	$c^{d=5}$	$c_i^{d=6}$	$\mathcal{O}_i^{d=6}$
Fermionic Singlet	$Y_N^T \frac{1}{M_N} Y_N$	$Y_N^\dagger \frac{1}{\lvert M_N\rvert^2} Y_N$	$\left(\overline{L}\widetilde{H}\right) i\not\partial \left(\widetilde{H}^\dagger L\right)$
Fermionic Triplet	$Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma$	$Y_\Sigma^\dagger \frac{1}{\lvert M_\Sigma\rvert^2} Y_\Sigma$	$\left(\overline{L}\vec{\tau}\widetilde{H}\right) i\not{D} \left(\widetilde{H}^\dagger \vec{\tau} L\right)$
Scalar Triplet	$4 Y_\Delta \frac{\mu_\Delta}{\lvert M_\Delta\rvert^2}$	$Y_\Delta^\dagger \frac{1}{2\lvert M_\Delta\rvert^2} Y_\Delta$	$\left(\overline{\widetilde{L}}\vec{\tau}L\right)\left(\overline{L}\vec{\tau}\widetilde{L}\right)$
		$\frac{\lvert\mu_\Delta\rvert^2}{\lvert M_\Delta\rvert^4}$	$\left(H^\dagger\vec{\tau}\widetilde{H}\right)\left(\overleftarrow{D}_\mu\overrightarrow{D}^\mu\right)\left(\widetilde{H}^\dagger\vec{\tau}H\right)$
		$-2\left(\lambda_3+\lambda_5\right)\frac{\lvert\mu_\Delta\rvert^2}{\lvert M_\Delta\rvert^4}$	$\left(H^\dagger H\right)^3$

What about the size of dim 6 effects??

expected e.g. very suppressed:

$$\left.\begin{aligned} c_{d=6} &\sim Y_N^\dagger \frac{1}{M_N^2} Y_N \\ c_{d=5} &\sim Y_N^T \frac{1}{M_N} Y_N \end{aligned}\right\} \Rightarrow \quad c_{d=6} \sim \frac{c_{d=5}}{M_N} \sim \underbrace{\frac{m_\nu}{M_N}}\frac{1}{v^2}$$

$$\sim 10^{-13} \quad (M_N \sim \mathrm{TeV})$$

$$\downarrow$$

$$Y_N \sim 10^{-6}$$

but not necessarily so:

- $c_{d=5}$ and $c_{d=6}$: not same Yukawa combination
- $c_{d=5}$ breaks lepton number but $c_{d=6}$ do not!

$\Rightarrow$ there is no symmetry reasons why if $c_{d=5}$ is suppressed $c_{d=6}$ should also be

“Direct Lepton number Violation,,

assume a L conserving setup with not too large $M_{N,\Delta,\Sigma}$ and large Yukawas

$M_{N,\Delta,\Sigma} \sim 100\,\mathrm{GeV} - 100\,\mathrm{TeV}$

$$\Downarrow$$

$$c_{d=6} \sim \frac{Y^2}{M^2} \longleftarrow \text{large}$$

$$c_{d=5} = m_\nu = 0 \longleftarrow \text{no L violation}$$

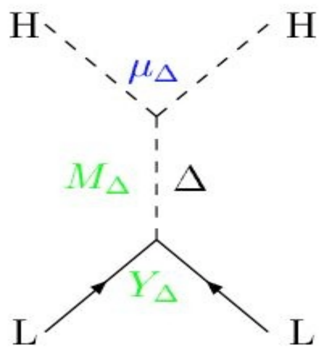
assume L is broken by a small perturbation μ

$$\Downarrow$$

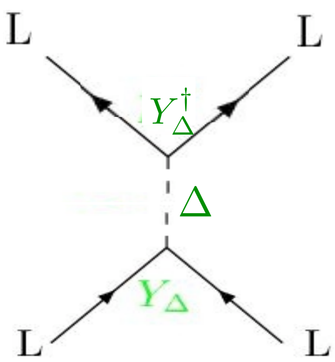
$$m_\nu = f(Y)\,\frac{\mu}{M^2}\,v^2 \longleftarrow$$

neutrino masses directly proport. to a small source of L violation μ rather than inversely proport. to a large mass M

Direct Lepton number Violation in type-II model



$$\Longrightarrow \quad c_{d=5} = \frac{Y_\Delta \mu_\Delta}{M_\Delta^2} \quad \leftarrow \left(= \frac{m_\nu}{v^2} \right)$$



$$\Longrightarrow \quad c_{d=6} = \frac{Y_\Delta^\dagger Y_\Delta}{M_\Delta^2}$$

if Y_Δ large, μ_Δ small $\Longrightarrow$ small enough neutrino masses with large dim 6 effects

if $\mu_\Delta = 0$ no L violation

DLV in type-I (and type-III) model

example with one light neutrino and 2 N:

$$
\begin{array}{c|ccc}
 & \nu_L & N_1 & N_2 \\
\nu_L & & & \\
N_1 & & & \\
N_2 & & &
\end{array}
\quad
\begin{array}{c}
 \\ \nu_L \\ N_1 \\ N_2
\end{array}
\begin{pmatrix}
0 & Y_N \frac{v}{\sqrt{2}} & 0 \\
Y_N \frac{v}{\sqrt{2}} & 0 & M_N \\
0 & M_N & 0
\end{pmatrix}
$$

"inverse seesaw" as in
Gonzalez-Garcia, Valle '89
Kersten, Smirnov '07
Abada, Biggio, Bonnet, Gavela, T.H. '07

if Y_N is large, M_N not too high:

$c_{d=6}$ large with $c_{d=5} = 0$ (L is conserved)

$L(\nu) = 1,\ L(N_1) = -1,\ L(N_2) = 1$

Phenomenology of dim 6 operators

long list of effects depending on the seesaw model:

-rare lepton decays: $\mu \to e\gamma,\ \tau \to e\gamma, \tau \to \mu\gamma, \mu \to eee, \tau \to 3\,l$

-universality tests: $W \to l\bar{\nu},\ \pi \to l\bar{\nu},\ \tau \to l\nu\bar{\nu}, \ldots$

-Z and W decays: $Z \to l\bar{l},\ W \to l\nu$

-Z invisible width: $Z \to \nu\bar{\nu}$

- ρ parameter

- W mass

- ....

Bounds on Yukawa couplings from dim 6 operator induced processes: type-II model

Process	Constraint on	Bound $\left(\times \left(\frac{M_\Delta}{1\,\mathrm{TeV}}\right)^2\right)$
M_W	$\lvert Y_{\Delta\mu e}\rvert^2$	$< 7.3 \times 10^{-2}$
$\mu^- \to e^+e^-e^-$	$\lvert Y_{\Delta\mu e}\rvert\lvert Y_{\Delta ee}\rvert$	$< 1.2 \times 10^{-5}$
$\tau^- \to e^+e^-e^-$	$\lvert Y_{\Delta\tau e}\rvert\lvert Y_{\Delta ee}\rvert$	$< 1.3 \times 10^{-2}$
$\tau^- \to \mu^+\mu^-\mu^-$	$\lvert Y_{\Delta\tau\mu}\rvert\lvert Y_{\Delta\mu\mu}\rvert$	$< 1.2 \times 10^{-2}$
$\tau^- \to \mu^+e^-e^-$	$\lvert Y_{\Delta\tau\mu}\rvert\lvert Y_{\Delta ee}\rvert$	$< 9.3 \times 10^{-3}$
$\tau^- \to e^+\mu^-\mu^-$	$\lvert Y_{\Delta\tau e}\rvert\lvert Y_{\Delta\mu\mu}\rvert$	$< 1.0 \times 10^{-2}$
$\tau^- \to \mu^+\mu^-e^-$	$\lvert Y_{\Delta\tau\mu}\rvert\lvert Y_{\Delta\mu e}\rvert$	$< 1.8 \times 10^{-2}$
$\tau^- \to e^+e^-\mu^-$	$\lvert Y_{\Delta\tau e}\rvert\lvert Y_{\Delta\mu e}\rvert$	$< 1.7 \times 10^{-2}$
$\mu \to e\gamma$	$\lvert\Sigma_{l=e,\mu,\tau} Y^\dagger_{\Delta l\mu} Y_{\Delta el}\rvert$	$< 4.7 \times 10^{-3}$
$\tau \to e\gamma$	$\lvert\Sigma_{l=e,\mu,\tau} Y^\dagger_{\Delta l\tau} Y_{\Delta el}\rvert$	< 1.05
$\tau \to \mu\gamma$	$\lvert\Sigma_{l=e,\mu,\tau} Y^\dagger_{\Delta l\tau} Y_{\Delta \mu l}\rvert$	$< 8.4 \times 10^{-1}$

Combined bounds		
Process	Yukawa	Bound $\left(\times \left(\frac{M_\Delta}{1\,\mathrm{TeV}}\right)^4\right)$
$\mu \to e\gamma$	$\lvert Y^\dagger_{\Delta\mu\mu} Y_{\Delta\mu e} + Y^\dagger_{\Delta\tau\mu} Y_{\Delta\tau e}\rvert$	$< 4.7 \times 10^{-3}$
$\tau \to e\gamma$	$\lvert Y^\dagger_{\Delta\tau\tau} Y_{\Delta\tau e}\rvert$	< 1.05
$\tau \to \mu\gamma$	$\lvert Y^\dagger_{\Delta\tau\tau} Y_{\Delta\tau\mu}\rvert$	$< 8.4 \times 10^{-1}$

Abada, Biggio, Bonnet, Gavela, T.H. '07

Partly from: Barger et al '82; Pal '83; Bernabeu et al '84, '86; Bilenky, Petcov'87; Gunion et al '89, '06; Swartz '89; Mohapatra '92

Bounds on Yukawa couplings from dim 6 operator induced processes: type-I model

Antusch, Biggio, Fernandez-Martinez, Lopez-Pavon, Gavela '06

$$\frac{v^2}{2}|c^{d=6}|_{\alpha\beta} = \frac{v^2}{2}|Y_N^\dagger \frac{1}{|M_N|^2} Y_N|_{\alpha\beta} \lesssim \begin{pmatrix} 10^{-2} & 7.2\cdot 10^{-5} & 1.6\cdot 10^{-2} \\ 7.2\cdot 10^{-5} & 10^{-2} & 1.1\cdot 10^{-2} \\ 1.6\cdot 10^{-2} & 1.1\cdot 10^{-2} & 10^{-2} \end{pmatrix}$$

Abada, Biggio, Bonnet, Gavela, T.H. '07

effects come mostly from the mixings between the ν and N which induce modifications of W couplings to leptons and Z couplings to ν (through non-unitarity effects)

-rare lepton decays: $\mu \to e\gamma,\ \tau \to e\gamma, \tau \to \mu\gamma, \mu \to eee, \tau \to 3\,l$

-universality tests: $W \to l\bar{\nu}$

-Z and W decays: $Z \to l\bar{l},\ W \to l\nu$

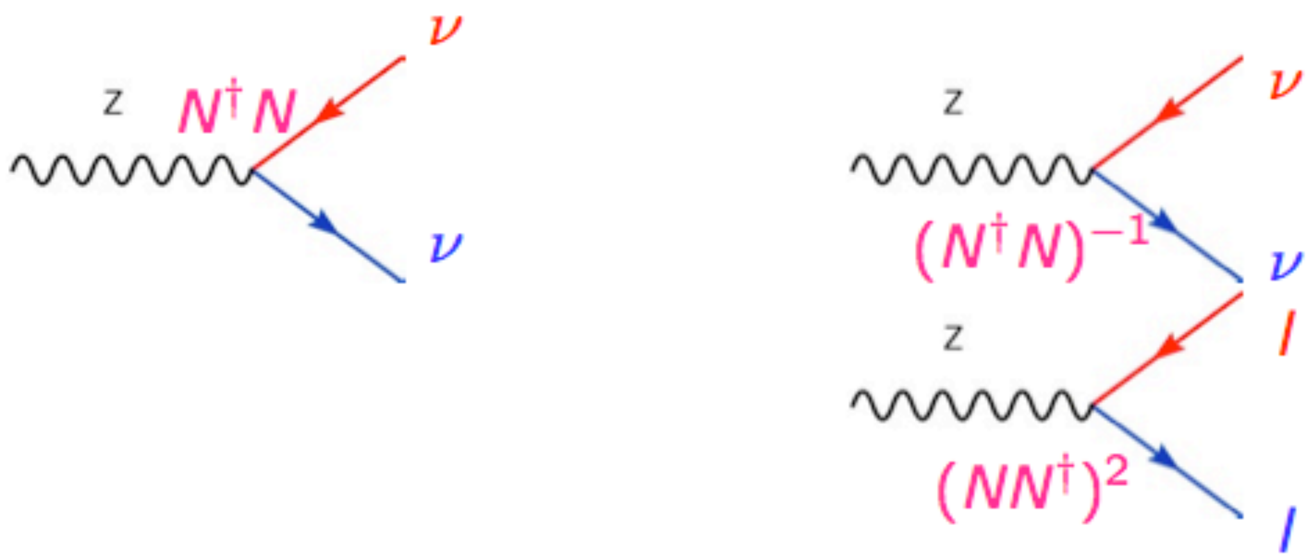
-Z invisible width: $Z \to \nu\bar{\nu}$

3 nus +3 N DFV case

$$\begin{array}{c} (\nu_e, \nu_\mu, \nu_\tau, N_1, N_2, N_3) \\ \begin{pmatrix} 0 & 0 & 0 & c & 0 & 0 \\ 0 & 0 & 0 & d & 0 & 0 \\ 0 & 0 & 0 & e & 0 & 0 \\ c & d & e & 0 & 0 & a \\ 0 & 0 & 0 & 0 & b & 0 \\ 0 & 0 & 0 & a & 0 & 0 \end{pmatrix} \end{array}$$

Singlet and triplet Seesaws differ in the the pattern of the Z couplings

Singlet	Triplet
$J_\mu^{-\,CC} \equiv \overline{l_L}\,\gamma_\mu\, N\,\nu$	$J_\mu^{-\,CC} \equiv \overline{l_L}\,\gamma_\mu\, N\,\nu$
$J_\mu^{NC} \equiv \frac{1}{2}\overline{\nu}\,\gamma_\mu (N^\dagger\, N)\,\nu$	$J_\mu^{Z}(\text{neutrinos}) \equiv \frac{1}{2}\overline{\nu}\,\gamma_\mu (N^\dagger\, N)^{-1}\,\nu$
	$J_\mu^{3}(\text{leptons}) \equiv \frac{1}{2}\overline{l}\,\gamma_\mu (N N^\dagger)^2\, l$



l->l' gamma versus l->3l' ratios are predicted to fixed values in the type-III seesaw model

$$
\begin{aligned}
Br(\mu \to e\gamma) &= 1.3 \cdot 10^{-3} \cdot Br(\mu \to eee)\,, \\
Br(\tau \to \mu\gamma) &= 1.3 \cdot 10^{-3} \cdot Br(\tau \to \mu\mu\mu) = 2.1 \cdot 10^{-3} \cdot Br(\tau^- \to e^- e^+ \mu^-) \\
Br(\tau \to e\gamma) &= 1.3 \cdot 10^{-3} \cdot Br(\tau \to eee) = 2.1 \cdot 10^{-3} \cdot Br(\tau^- \to \mu^- \mu^+ e^-)
\end{aligned}
$$

l->l' gamma:

$$
\begin{aligned}
|\epsilon_{e\mu}| &= \frac{v^2}{2} |Y_\Sigma^\dagger \frac{1}{M_\Sigma^\dagger} \frac{1}{M_\Sigma} Y_\Sigma|_{\mu e} \lesssim 1.1 \cdot 10^{-4} \\
|\epsilon_{\mu\tau}| &= \frac{v^2}{2} |Y_\Sigma^\dagger \frac{1}{M_\Sigma^\dagger} \frac{1}{M_\Sigma} Y_\Sigma|_{\tau\mu} \lesssim 1.5 \cdot 10^{-2} \\
|\epsilon_{e\tau}| &= \frac{v^2}{2} |Y_\Sigma^\dagger \frac{1}{M_\Sigma^\dagger} \frac{1}{M_\Sigma} Y_\Sigma|_{\tau e} \lesssim 2.4 \cdot 10^{-2}
\end{aligned}
$$

l->3l':

Constraints on	Process	Bound
$\lvert(NN^\dagger)_{e\mu}\rvert$	$\mu^- \to e^+ e^- e^-$	$< 1.1 \cdot 10^{-6}$
$\lvert(NN^\dagger)_{e\tau}\rvert$	$\tau^- \to e^+ e^- e^-$	$< 1.2 \cdot 10^{-3}$
$\lvert(NN^\dagger)_{\mu\tau}\rvert$	$\tau^- \to \mu^+ \mu^- \mu^-$	$< 1.2 \cdot 10^{-3}$
$\lvert(NN^\dagger)_{\tau e}\rvert$	$\tau^- \to \mu^+ \mu^- e^-$	$< 1.6 \cdot 10^{-3}$
$\lvert(NN^\dagger)_{\tau\mu}\rvert\lvert(NN^\dagger)_{e\mu}\rvert$	$\tau^- \to e^+ \mu^- \mu^-$	$< 3.1 \cdot 10^{-4}$
$\lvert(NN^\dagger)_{\tau\mu}\rvert$	$\tau^- \to e^+ e^- \mu^-$	$< 1.5 \cdot 10^{-3}$
$\lvert(NN^\dagger)_{\tau e}\rvert\lvert(NN^\dagger)_{\mu e}\rvert$	$\tau^- \to \mu^+ e^- e^-$	$< 2.9 \cdot 10^{-4}$
$\lvert(NN^\dagger)_{e\mu}\rvert$	$\mu \to e\gamma$	$2.8 \cdot 10^{-5}$
$\lvert(NN^\dagger)_{\mu\tau}\rvert$	$\tau \to \mu\gamma$	$5.2 \cdot 10^{-3}$
$\lvert(NN^\dagger)_{e\tau}\rvert$	$\tau \to e\gamma$	$6.6 \cdot 10^{-3}$