Vortices, Superfluid turbulence
& Nonthermal Fixed Points in Bose Gases

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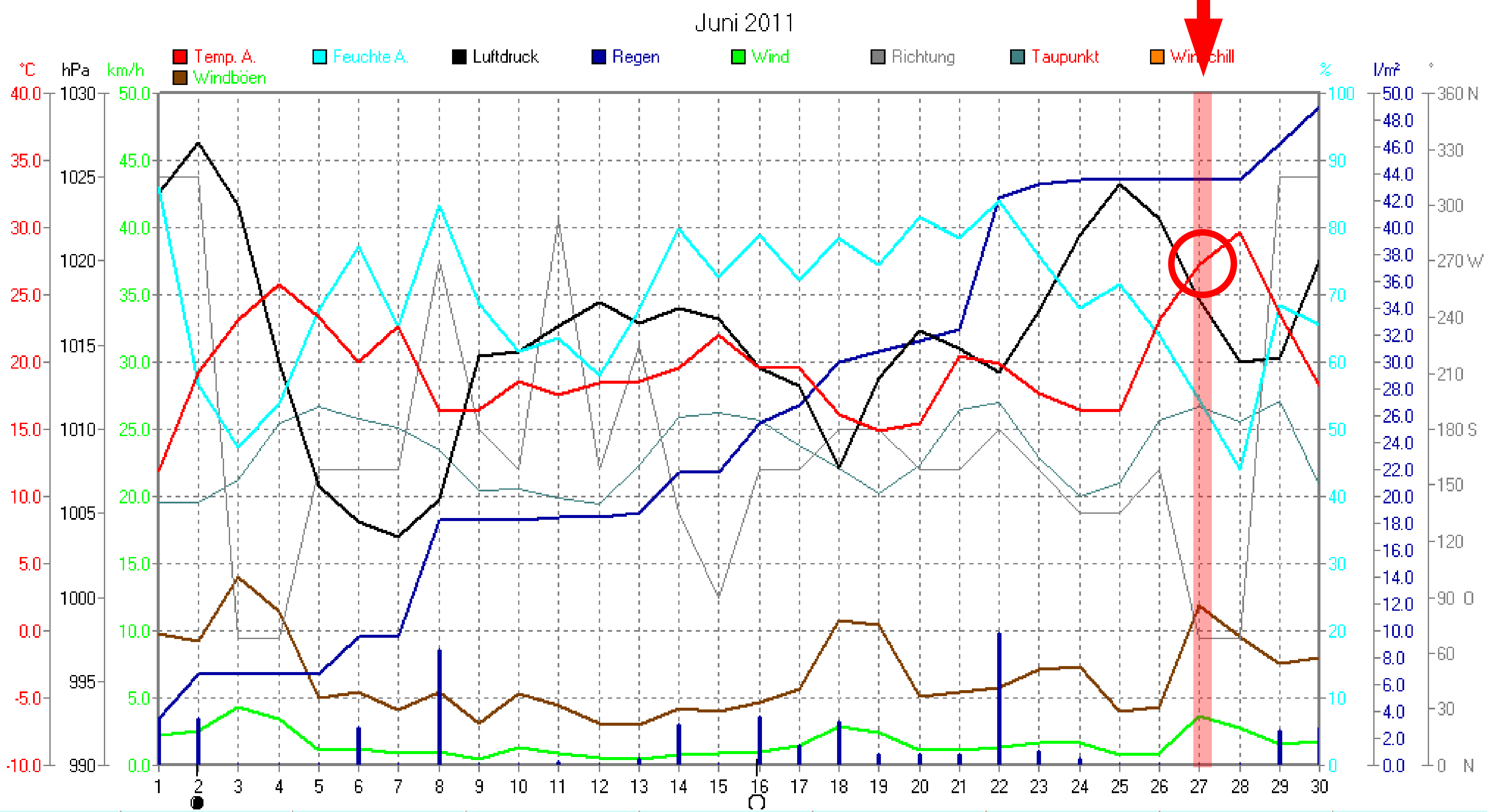
Center for Quantum Dynamics

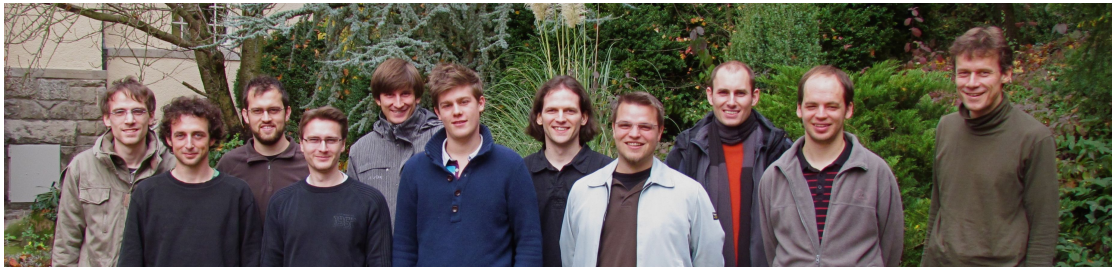
EMMI

HD Weather on 27/06/2011



Sensor	Temp. A.	°C	Feuchte A.	%	Luftdruck	hPa	Wind	km/h	Richtung		Windböen	km/h	Regen	l/m²	PMV+2:18
MinWert	19.06. 03:45	10.8	28.06. 17:45	29	07.06. 23:50	999.9	30.06. 05:10	0.0	01.06. 20:25	N	30.06. 05:35	0.0	Regentage: 18		WC 18.2 °C
MaxWert	28.06. 16:55	35.7	22.06. 08:15	96	02.06. 10:30	1028.5	06.06. 12:2S-SO	12.0	30.06. 12:55	N	19.06. 13:05 W	32.2	22.06. 05:55	9.8	TP 10.7 °C
Durchschnitt		19.72		69		1015.5	199,0 km	1.6		S-SO	199,0 km	6.6	Gesamt:	49.0	

Thanks & credits to...



...my work group in Heidelberg:

Sebastian Bock
Sebastian Erne
Martin Gärttner
Roman Hennig
Markus Karl
Steven Mathey
Boris Nowak
Nikolai Philipp
Maximilian Schmidt
Jan Schole
Dénes Sexty
Martin Trappe
Jan Zill

€€€...

Alexander von Humboldt Stiftung / Foundation

Heisenberg-Programm
DFG Deutsche Forschungsgemeinschaft

RUPRECHT-KARLS-UNIVERSITÄT HEIDELBERG

LGFG BaWue

DAAD Deutscher Akademischer Austausch Dienst
German Academic Exchange Service

Center for Quantum Dynamics

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HELMHOLTZ GEMEINSCHAFT

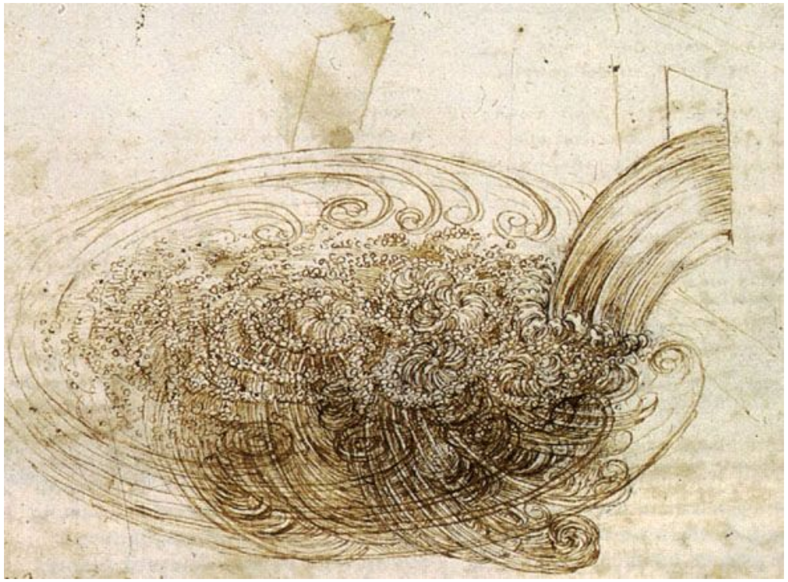
...my former students:

Cédric Bodet (→ NEC), Alexander Branschädel (→ KIT Karlsruhe), Stefan Keßler (→ U Erlangen), Matthias Kronenwett (→ R. Berger), **Christian Scheppach** (→ Cambridge, UK), Philipp Struck (→ Konstanz), Kristan Temme (→ Vienna)

Equilibration

Transient, metastable state
e.g. Turbulence
Non-thermal fixed point

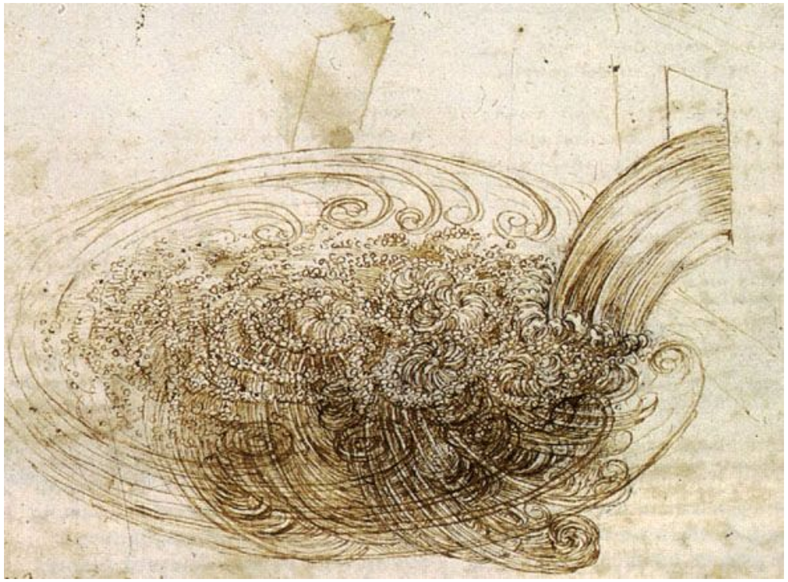
Classical Turbulence

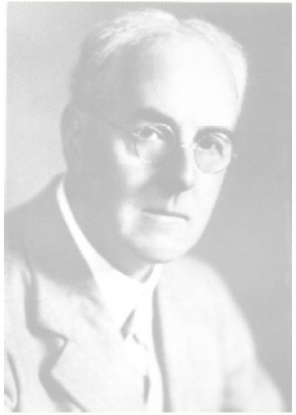


Kinetic energy cascade

large scales (source)
$\rightarrow$ small scales (sink)

Classical Turbulence





Lewis F. Richardson
(1881-1953)

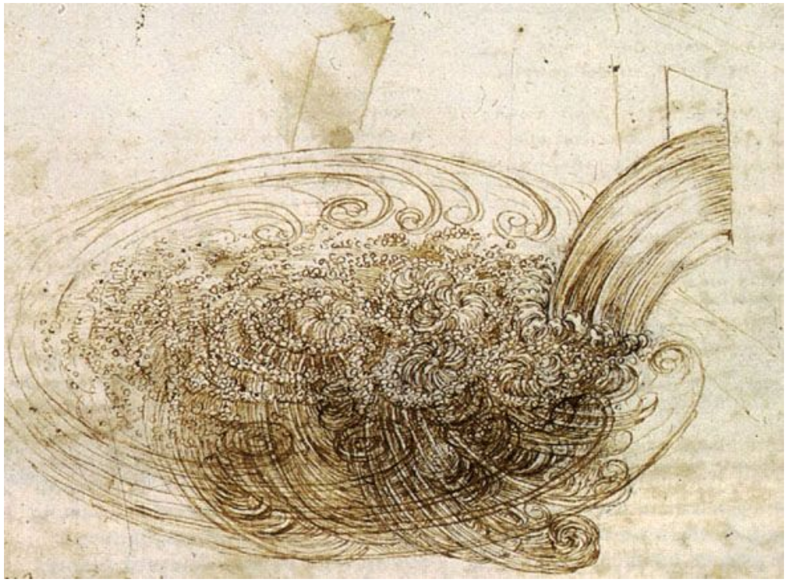
Kinetic energy cascade

large scales (source)
$\rightarrow$ small scales (sink)

"Big whirls have little whirls that feed on their velocity, and little whirls have lesser whirls and so on to viscosity."

(Richardson, 1920)

Classical Turbulence



Andrey N. Kolmogorov
(1903-1987)

Kinetic energy cascade

large scales (source)
$\rightarrow$ small scales (sink)

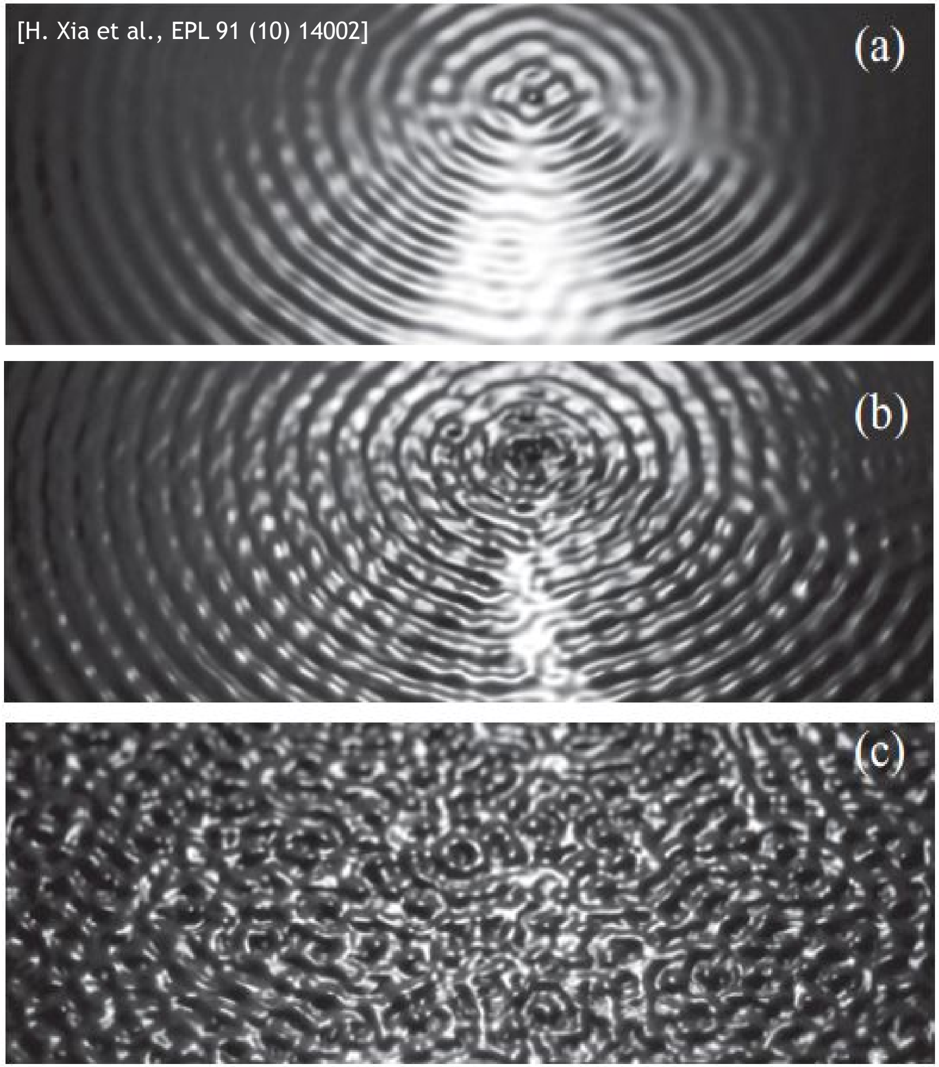
*"Big whirls have little whirls that feed on their velocity,
and little whirls have lesser whirls and so on to viscosity."*

(Richardson, 1920)

Kolmogorov (1941) $E(k) \sim k^{-5/3}$ (dynamical critical phenomenon)

Wave turbulence

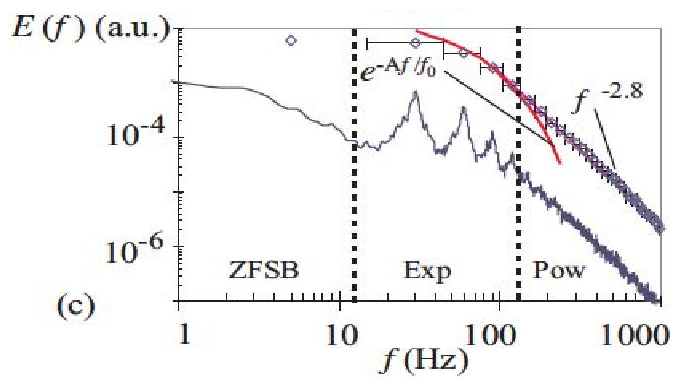
Wave Turbulence – e.g. on water



Theory prediction:

$$E_\omega \sim \omega^{-17/6}.$$

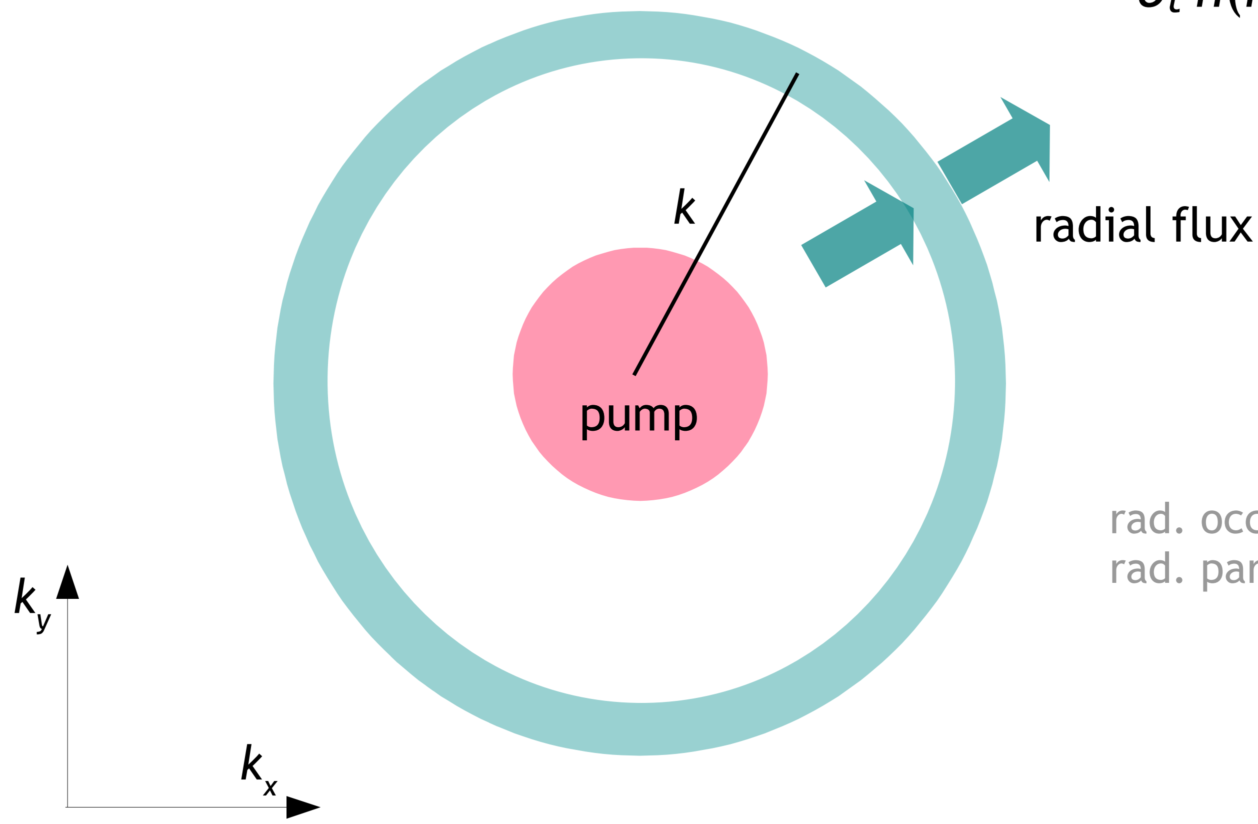
[Zakharov & Filonenko (67)]



Transport in momentum space

Imagine you had a balance equation for the radial flux

$$\partial_t\, n(k) = -\,\partial_k\, Q(k)$$



rad. occupation no. n
rad. particle flux Q

Transport in momentum space

Transport equation (Quantum Boltzmann eq.):

$$\partial_t n(k) = -\partial_k Q(k) \sim k^{d-1} J(k)$$

$$= k^{d-1}\, d\Omega_k \int \mathrm{d}^d p\, \mathrm{d}^d q\, \mathrm{d}^d r\, \underbrace{|T_{\mathbf{kpqr}}|^2}_{\textit{coupling}} \underbrace{\delta(\mathbf{k}+\mathbf{p}-\mathbf{q}-\mathbf{r})}_{\textit{mom. conservation}} \underbrace{\delta(\omega_{\mathbf{k}}+\omega_{\mathbf{p}}-\omega_{\mathbf{q}}-\omega_{\mathbf{r}})}_{\textit{energy conservation}}$$

$$\times \Big[\underbrace{(n_{\mathbf{k}}+1)(n_{\mathbf{p}}+1)n_{\mathbf{q}}n_{\mathbf{r}}}_{\textit{in-scattering rate}} - \underbrace{n_{\mathbf{k}}n_{\mathbf{p}}(n_{\mathbf{q}}+1)(n_{\mathbf{r}}+1)}_{\textit{out-scattering rate}}\Big]$$

dilute Bose gas: $T_{\mathbf{kpqr}} \equiv g = 4\pi a_0/m = \textit{const}.$

Transport in momentum space

Radial transport equation (Quantum Boltzmann):

$$\partial_t n(k) = -\partial_k Q(k) \sim k^{d-1} J(k)$$

$$= k^{d-1} \mathrm{d}\Omega_k \int \mathrm{d}^d p \, \mathrm{d}^d q \, \mathrm{d}^d r \, |T_{\mathbf{kpqr}}|^2 \delta(\mathbf{k}+\mathbf{p}-\mathbf{q}-\mathbf{r}) \, \delta(\omega_{\mathbf{k}}+\omega_{\mathbf{p}}-\omega_{\mathbf{q}}-\omega_{\mathbf{r}})$$

coupling *mom. conservation* *energy conservation*

$$\times \left[(n_{\mathbf{k}}+1)(n_{\mathbf{p}}+1)n_{\mathbf{q}}n_{\mathbf{r}} - n_{\mathbf{k}}n_{\mathbf{p}}(n_{\mathbf{q}}+1)(n_{\mathbf{r}}+1)\right]$$

in-scattering rate *out-scattering rate*

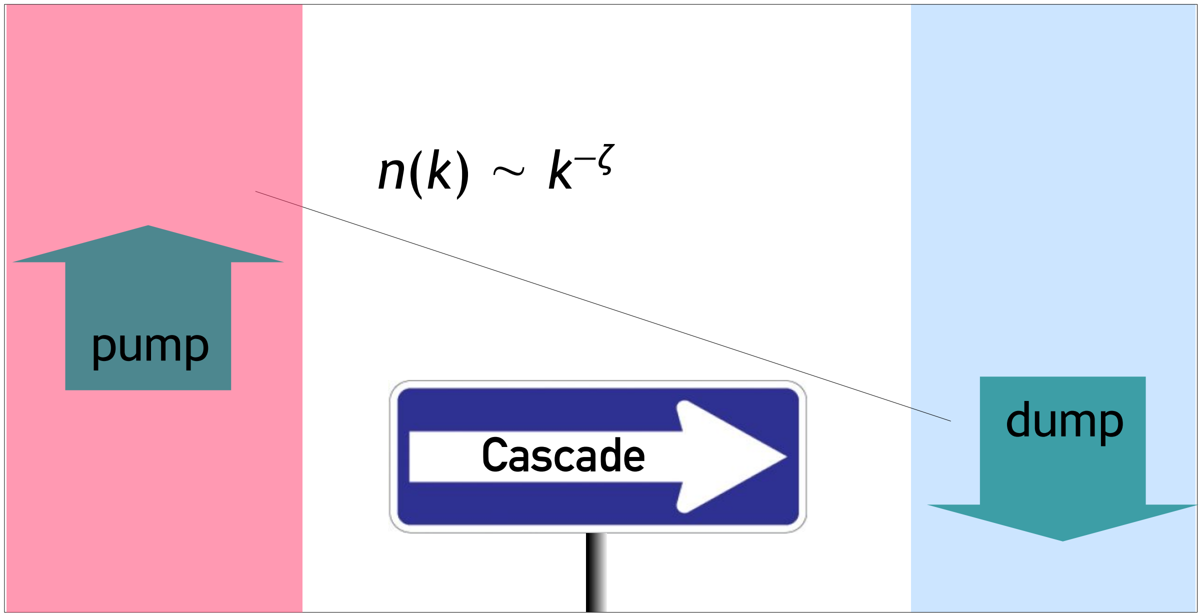
Stationary distribution $n(k,t) \equiv n(k)$ if $Q(k) \equiv Q$

This requires a particular scaling of $n(k) \sim k^{-\zeta}$

Wave turbulence

Stationary scaling $n(k)$ within inertial region:

log $n(k)$



log k

Wave turbulence
in an ultracold Bose gas

Dilute ultracold Bose Gas

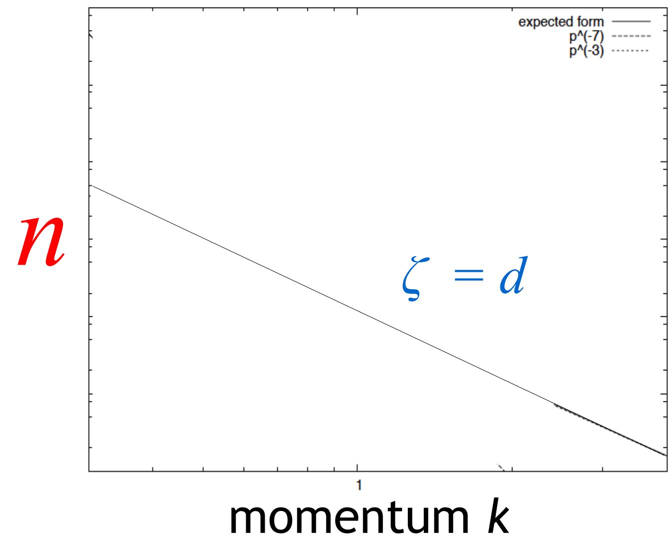
Gross-Pitaevskii Equation: ($g = 4\pi a_0/m$)

$$i\frac{\partial\Psi(\boldsymbol{\rho},t)}{\partial t} = \left(-\frac{\nabla^2}{2} + g|\Psi(\boldsymbol{\rho},t)|^2\right)\Psi(\boldsymbol{\rho},t)$$

Momentum spectrum:

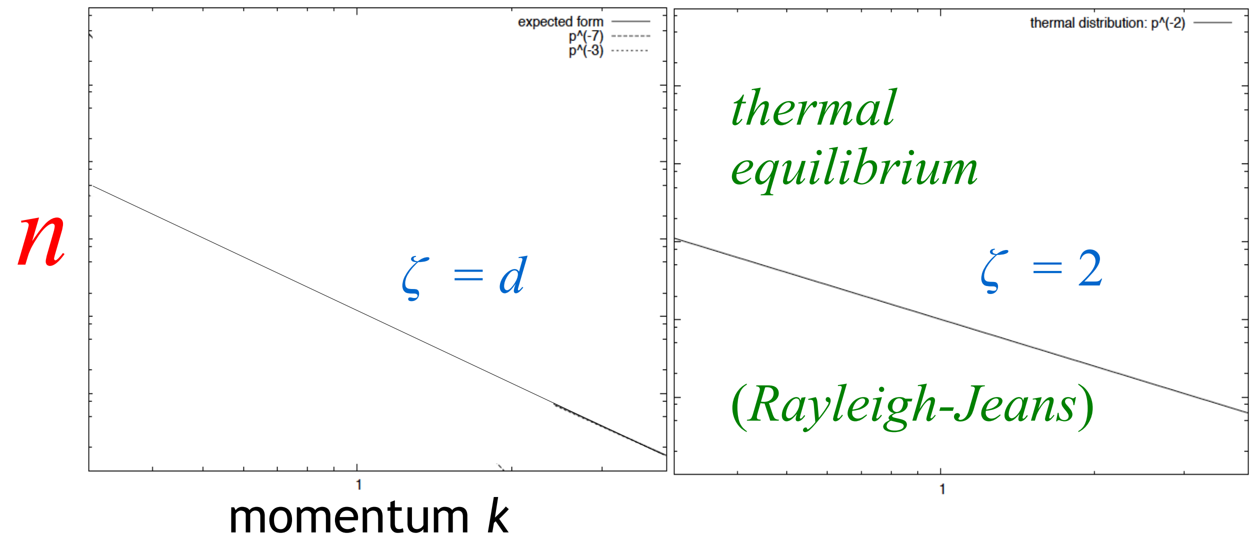
$$n(\mathbf{k}) = \langle \Psi^*(\mathbf{k})\Psi(\mathbf{k})\rangle$$

Bose gas in d spatial dimensions $n \sim k^{-\zeta}$



J. Berges, A. Rothkopf, J. Schmidt, PRL **101** (08) 041603
C. Scheppach, J. Berges, TG PRA **81** (10) 033611

Bose gas in d spatial dimensions $n \sim k^{-\zeta}$





J. Berges, A. Rothkopf, J. Schmidt, PRL **101** (08) 041603
C. Scheppach, J. Berges, TG PRA **81** (10) 033611

Transport in momentum space

Quantum Boltzmann breaks down for large n, once $|T_{kpqr}|n_k \gg O(1)$

$$\partial_t n(k) = -\partial_k Q(k) \sim k^{d-1} J(k)$$

$$= k^{d-1}\, d\Omega_k \int d^dp\, d^dq\, d^dr\, |T_{\mathbf{kpqr}}|^2 \delta(\mathbf{k}+\mathbf{p}-\mathbf{q}-\mathbf{r})\, \delta(\omega_{\mathbf{k}}+\omega_{\mathbf{p}}-\omega_{\mathbf{q}}-\omega_{\mathbf{r}})$$

coupling *mom. conservation* *energy conservation*

$$\times \left[(n_{\mathbf{k}}+1)(n_{\mathbf{p}}+1)n_{\mathbf{q}}n_{\mathbf{r}} - n_{\mathbf{k}}n_{\mathbf{p}}(n_{\mathbf{q}}+1)(n_{\mathbf{r}}+1)\right]$$

in-scattering rate *out-scattering rate*

here: $T_{\mathbf{kpqr}} \equiv g = const.$

Cured by
effective many-body T-Matrix:

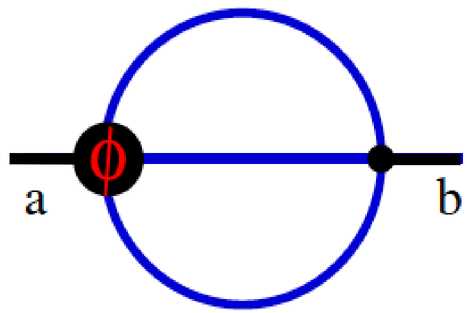
$$|T|^2 = g^2 \rightarrow |T_k^{MB}|^2 \sim \frac{g^2}{1+(g k n_k)^2}$$

Dyn. QFT: Resummed Vertex

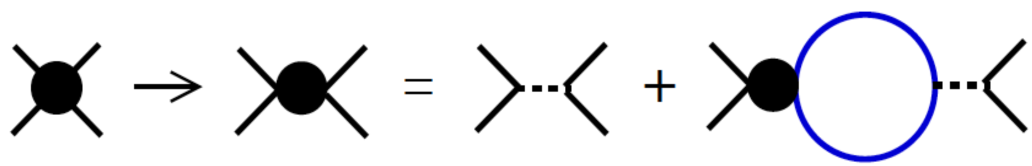
$p = (p_0, \mathbf{p})$:

$$J(p) := \Sigma^{\rho}_{ab}(p) F_{ba}(p) - \Sigma^{F}_{ab}(p)\rho_{ba}(p) \stackrel{!}{=} 0$$

$\Sigma_{ab}(x,y) =$



Vertex bubble resummation:
(e.g. 2PI to NLO in $1/N$)

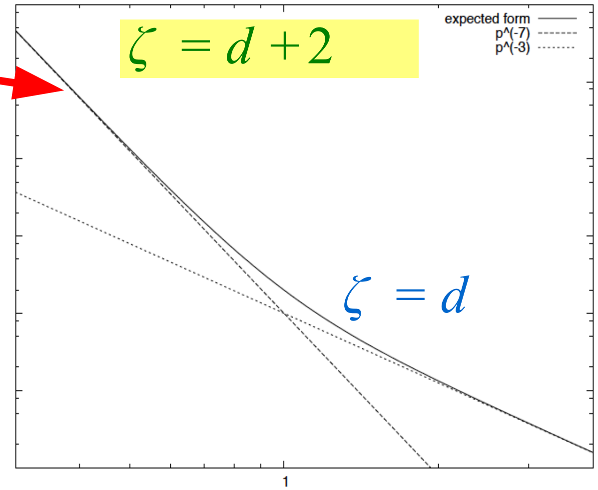


[*Dynamics:* J. Berges, (02); G. Aarts et al., (02);
Nonthermal fixed points: J. Berges, A. Rothkopf, J. Schmidt, PRL (08)]

Bose gas in d spatial dimensions $n \sim k^{-\zeta}$

New exponent
beyond
Quantum Boltzmann!

n



momentum k

J. Berges, A. Rothkopf, J. Schmidt, PRL **101** (08) 041603; J. Berges, G. Hoffmeister, NPB 813, 383 (2009)
C. Scheppach, J. Berges, TG PRA **81** (10) 033611

Vortices in a superfluid ultracold Bose gas

Superfluid hydro of Bose-condensed Gas

The Gross-Pitaevskii Equation, ($g = 4\pi a_0/m$)

$$i\frac{\partial\Psi(\boldsymbol{\rho},t)}{\partial t} = \left(-\frac{\nabla^2}{2} + g|\Psi(\boldsymbol{\rho},t)|^2\right)\Psi(\boldsymbol{\rho},t)$$

using defs.

$$\Psi(\boldsymbol{\rho},t) = \sqrt{n(\boldsymbol{\rho},t)}\exp[i\varphi(\boldsymbol{\rho},t)]$$

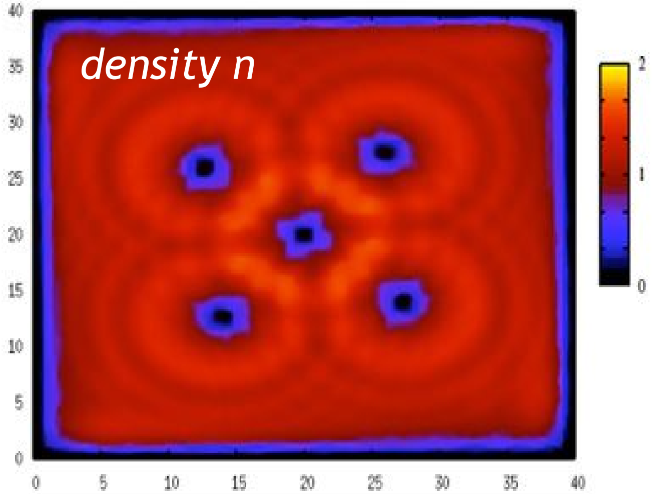
$$Q = gn \qquad \mathbf{u}(\boldsymbol{\rho},t) = \nabla\varphi(\boldsymbol{\rho},t)$$

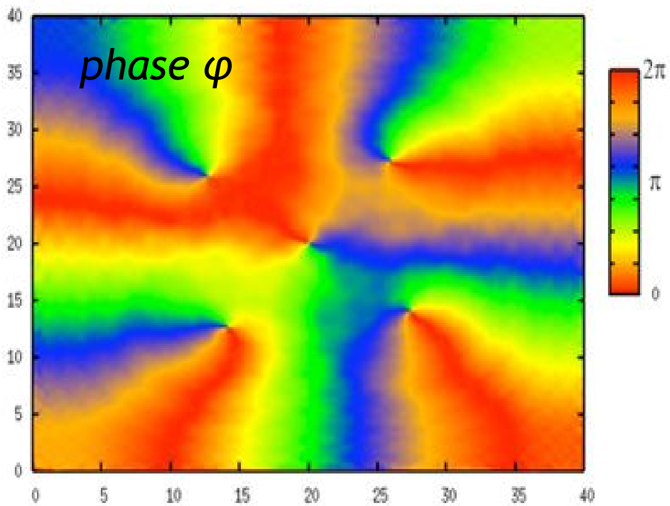
can be written as

$$\frac{\partial}{\partial t}n + \nabla\cdot(n\mathbf{u}) = 0 \qquad \frac{\partial}{\partial t}\mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla Q$$

Euler equation

Quantum Vortices





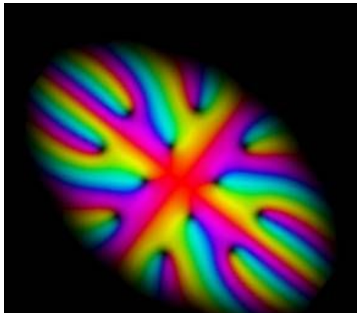
[W. P. Reinhardt, Seattle]

$$\Psi(\boldsymbol{\rho},t)=\sqrt{n(\boldsymbol{\rho},t)}\exp[i\varphi(\boldsymbol{\rho},t)]$$

complex field

$$\mathbf{u}(\boldsymbol{\rho},t)=\nabla\varphi(\boldsymbol{\rho},t)$$

velocity



[Wolfgang Ketterle, MIT]

Vortices in a Na condensate

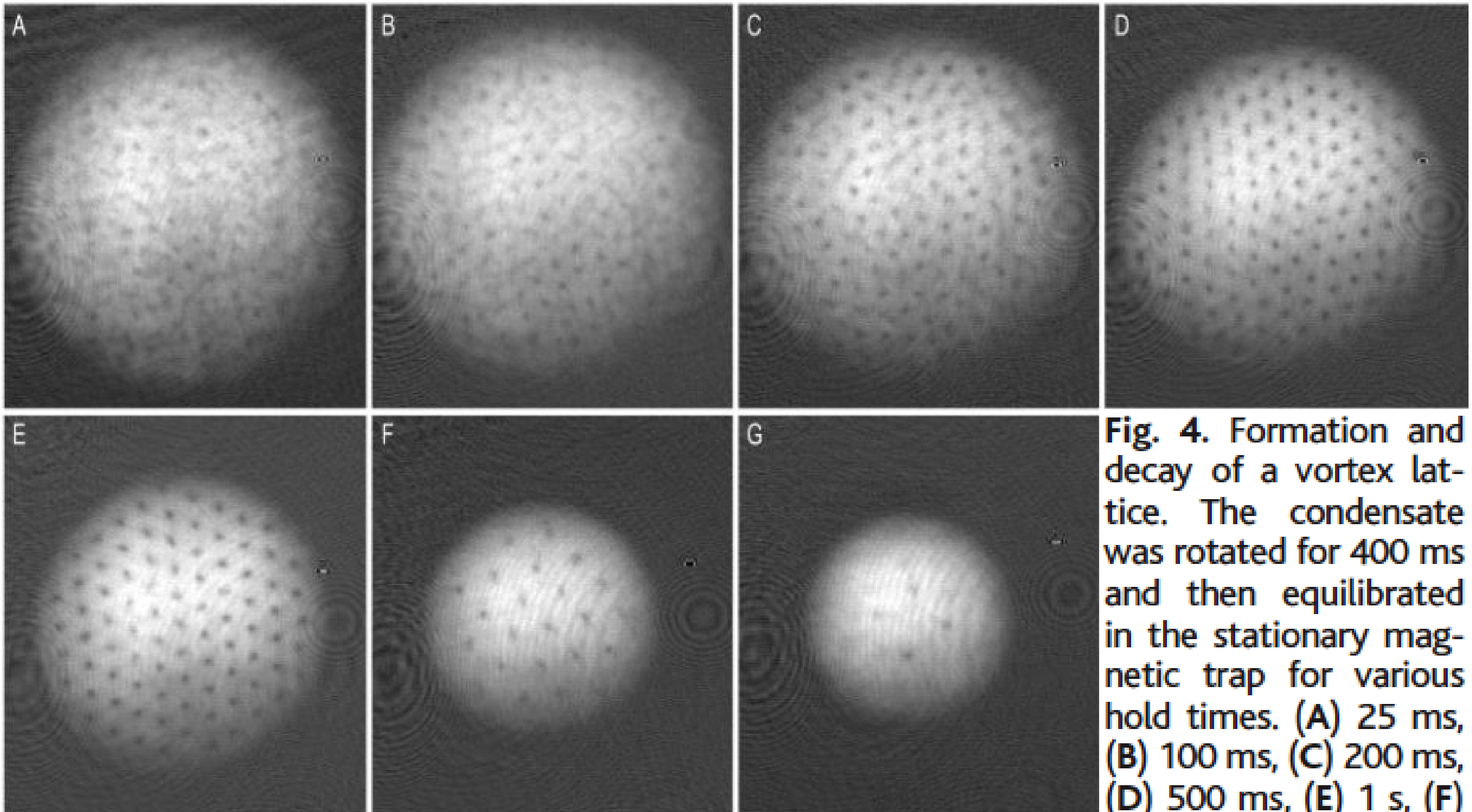


Fig. 4. Formation and decay of a vortex lattice. The condensate was rotated for 400 ms and then equilibrated in the stationary magnetic trap for various hold times. (A) 25 ms, (B) 100 ms, (C) 200 ms, (D) 500 ms, (E) 1 s, (F) 5 s, (G) 10 s,

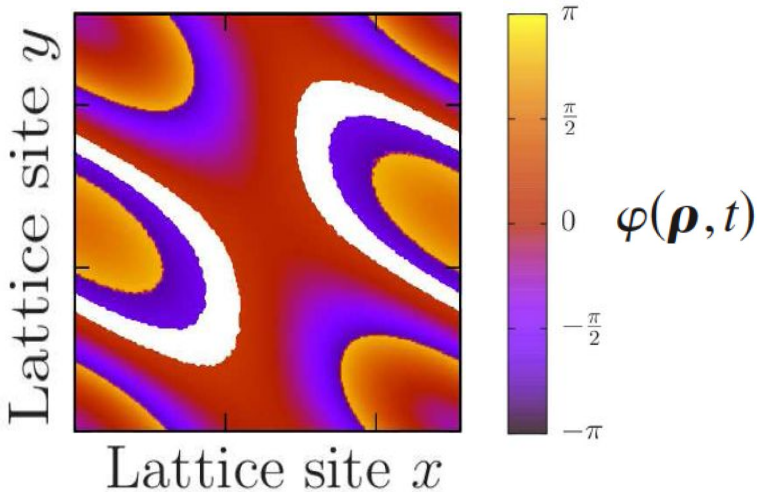
J. R. Abo-Shaeer, C. Raman, J. M. Vogels, W. Ketterle
20 APRIL 2001 VOL 292 SCIENCE

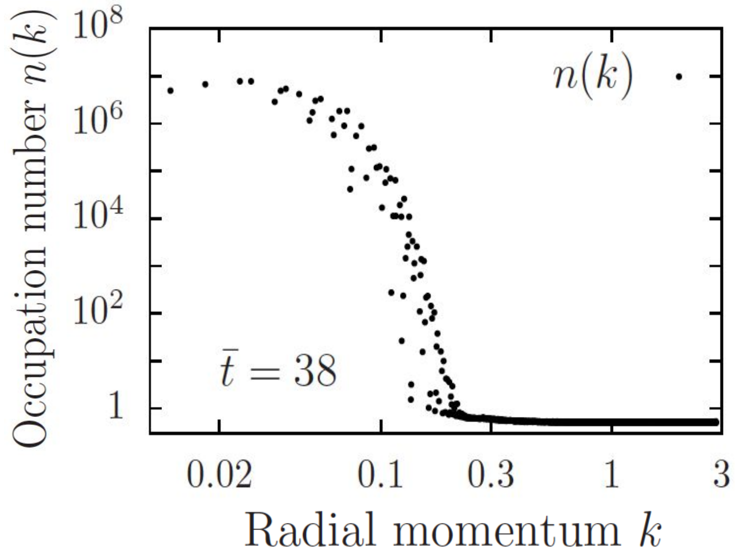
Gross-Pitaevskii Simulations for an ultracold Bose gas

Movie 1: Phase evolution & Spectrum

$$\Psi(\boldsymbol{\rho},t) = \sqrt{n(\boldsymbol{\rho},t)}\exp[i\varphi(\boldsymbol{\rho},t)]$$

$$n(k) = \langle \Psi^*(\mathbf{k})\Psi(\mathbf{k})\rangle\big|_{\text{angle average}}$$





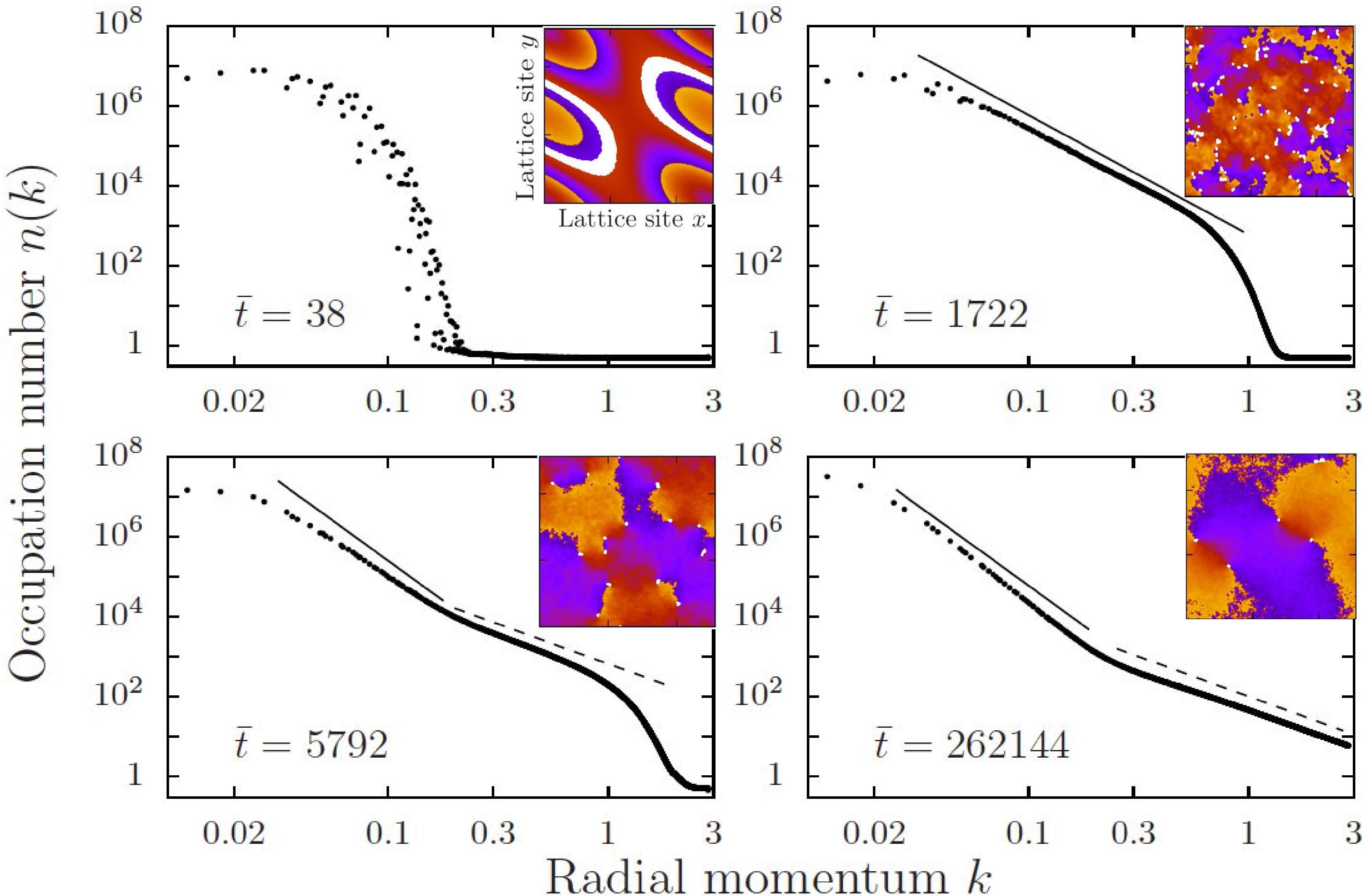
Movie by Jan Schole

Movie 2: Vortex “gas” & Spectrum

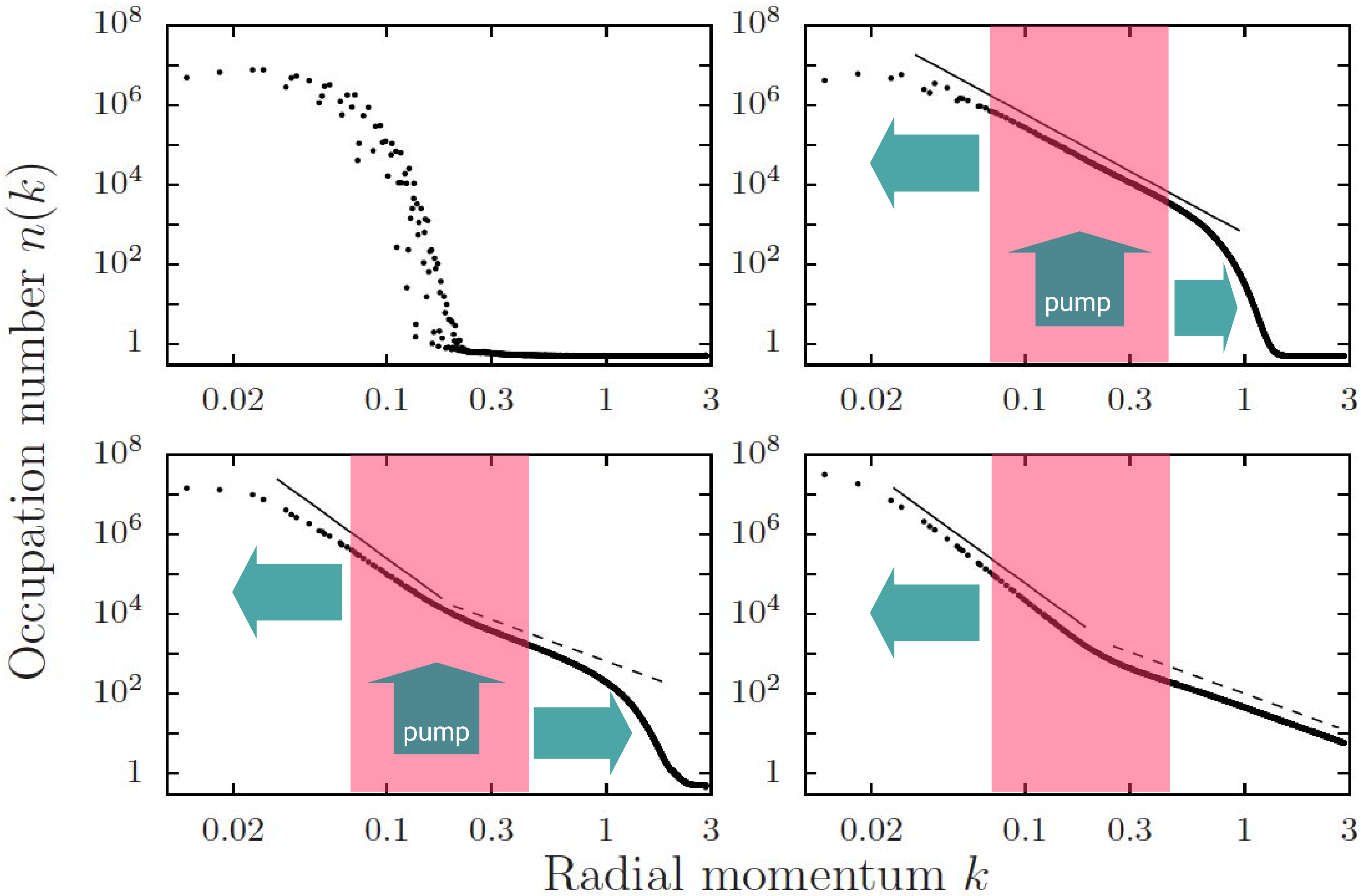
$$n(k) = \langle \Psi^*(\mathbf{k})\Psi(\mathbf{k})\rangle\big|_{\text{angle average}}$$

B. Nowak, D. Sexty, TG, PRB **84**: 020506(R), 2011

Spectrum in 2+1 D

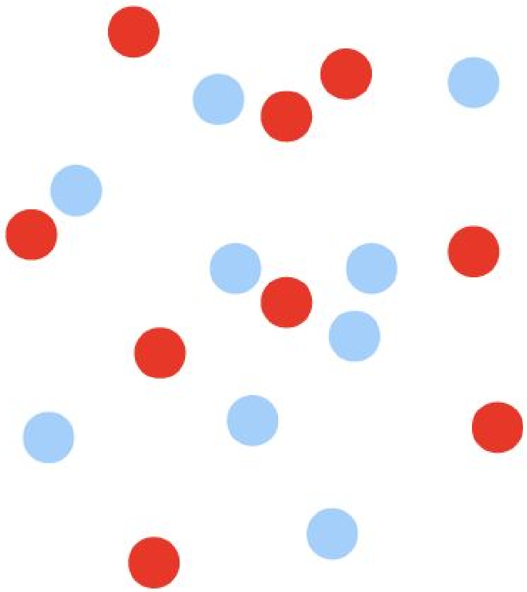


Cascades in 2+1 D

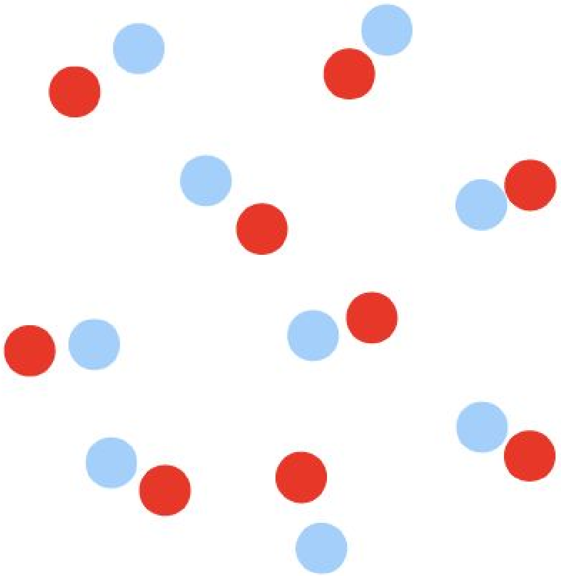


Interpretation: Random vortex distributions

Point vortex model



$$n_k \sim k^{-4}$$

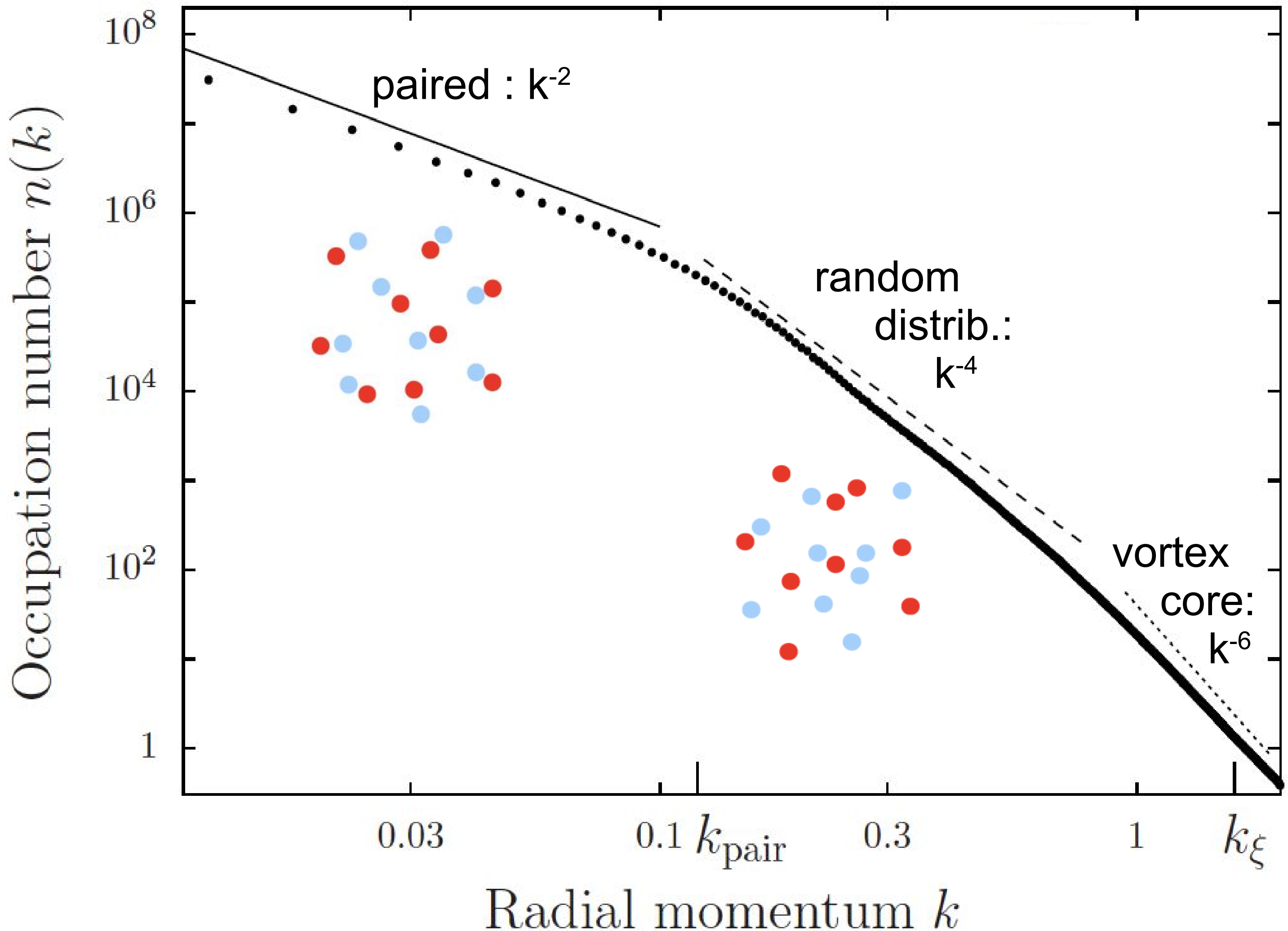


$$n_k \sim k^{-2}, \;\; k < k_{\mathrm{pair}}$$

$$n_k \sim k^{-4}, \;\; k > k_{\mathrm{pair}}$$

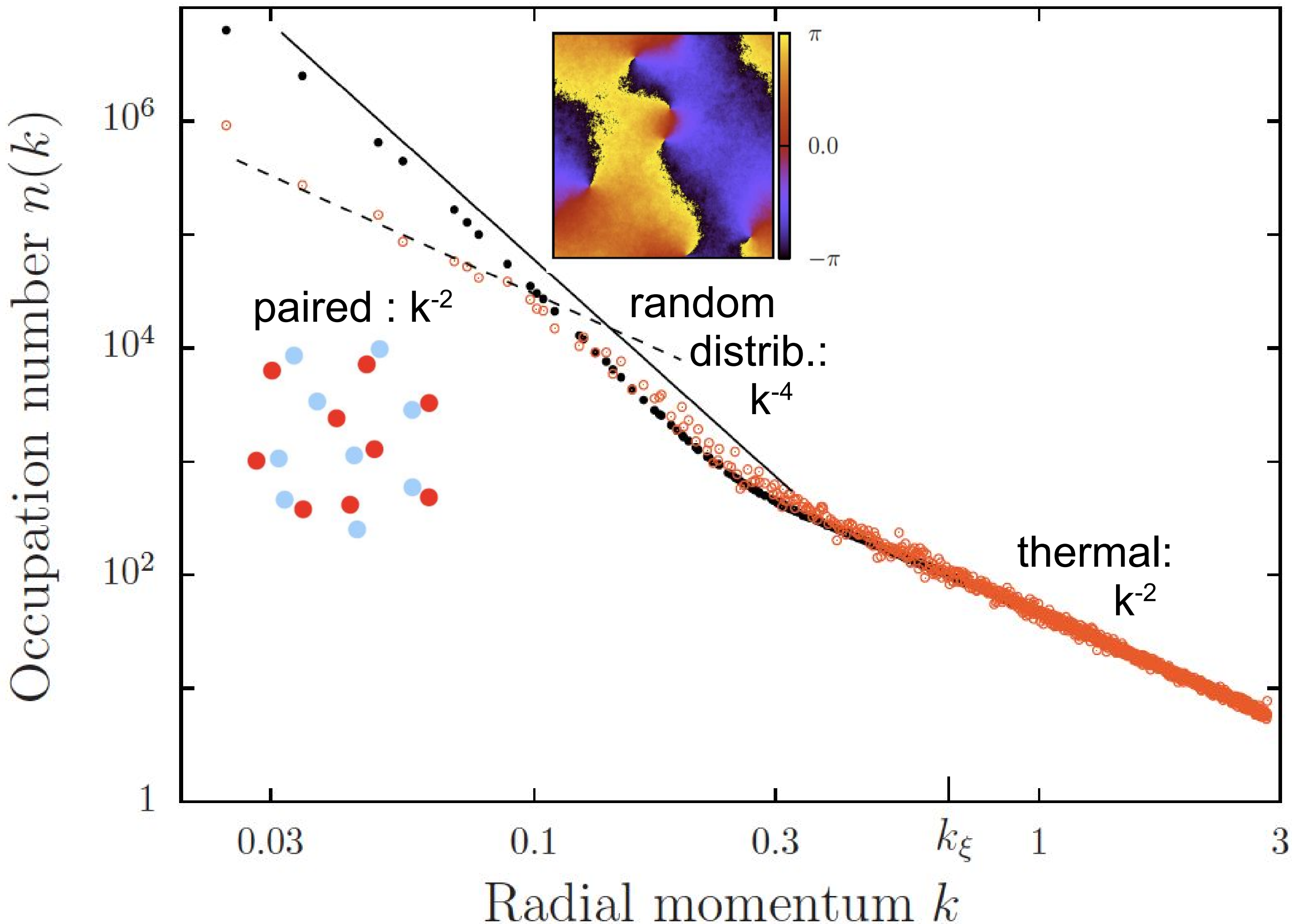
B. Nowak, J. Schole, D. Sexty, TG, arXiv:1111.61XX [cond-mat.quant-gas]

Point vortex model in 2+1 D



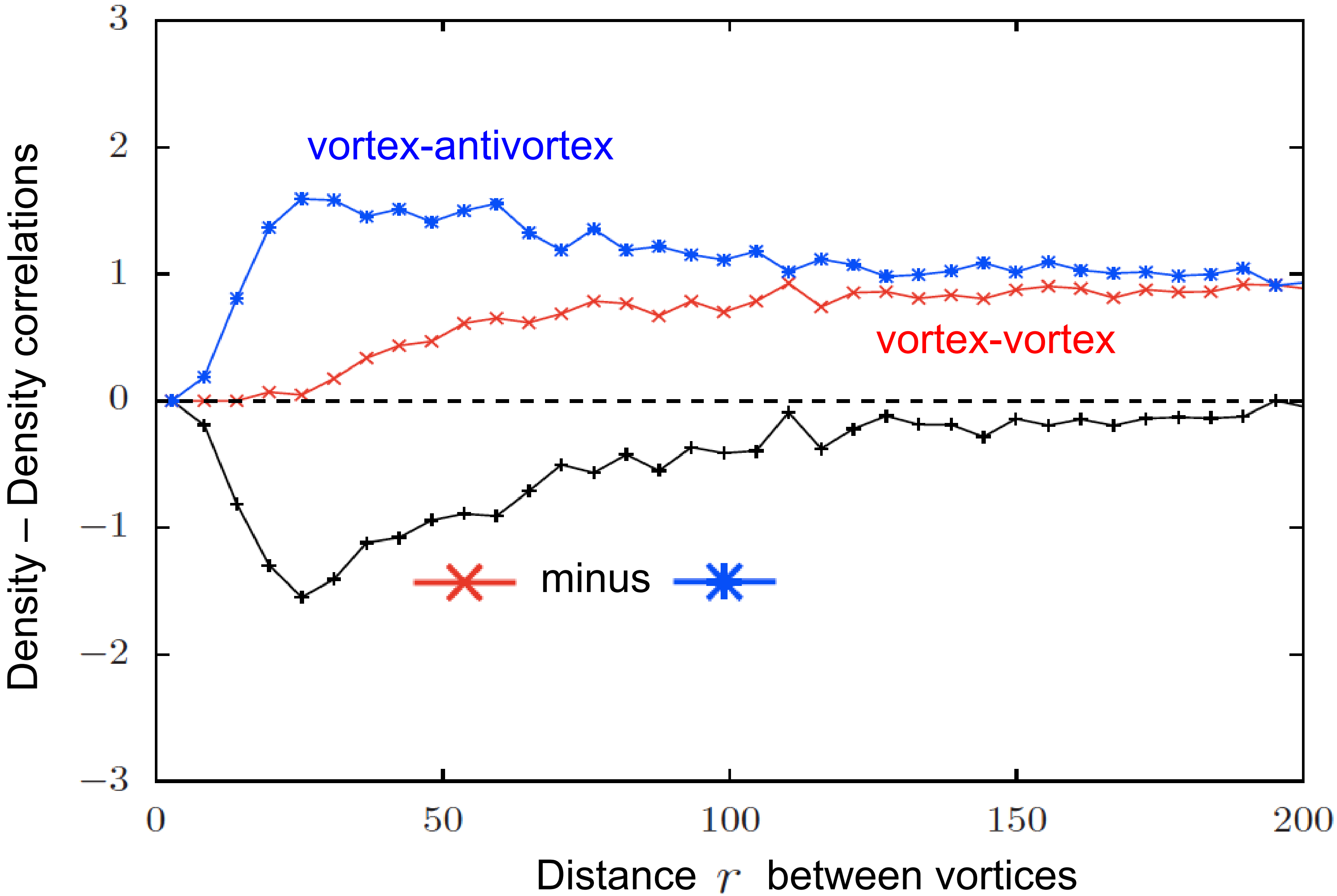
B. Nowak, J. Schole, D. Sexty, TG, arXiv:1111.61XX [cond-mat.quant-gas]

Simulations in 2+1 D



B. Nowak, J. Schole, D. Sexty, TG, arXiv:1111.61XX [cond-mat.quant-gas]

Vortex position correlations



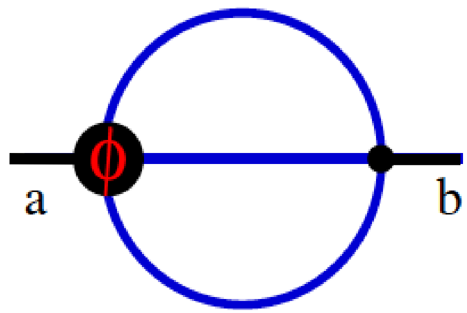
B. Nowak, J. Schole, D. Sexty, TG, arXiv:1111.61XX [cond-mat.quant-gas]

Dyn. QFT: Resummed Vertex

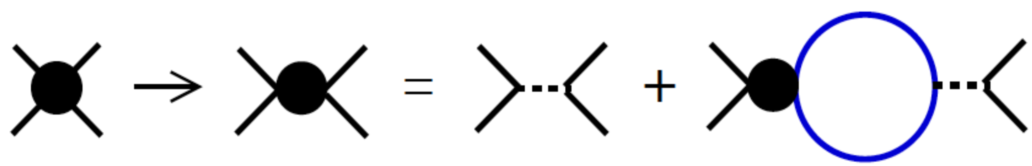
$p = (p_0, \mathbf{p})$:

$$J(p) := \Sigma^{\rho}_{ab}(p)F_{ba}(p) - \Sigma^{F}_{ab}(p)\rho_{ba}(p) \stackrel{!}{=} 0$$

$\Sigma_{ab}(x,y)$ =



Vertex bubble resummation:
(e.g. 2PI to NLO in $1/N$)



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Dyn. QFT: Resummed Vertex

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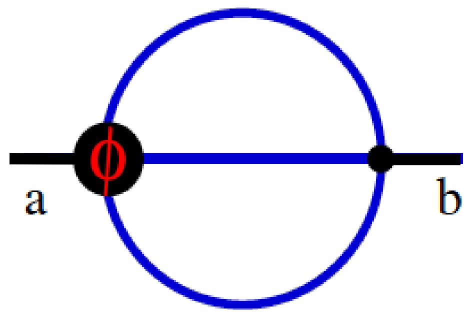
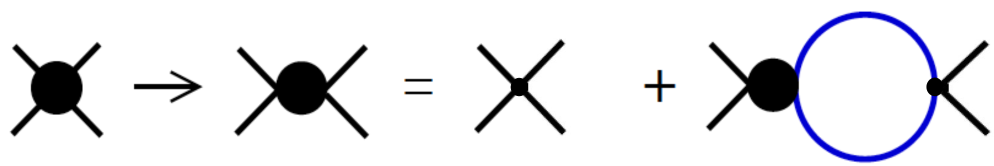
$\Sigma_{ab}(x,y) =$



Vertex bubble resummation:

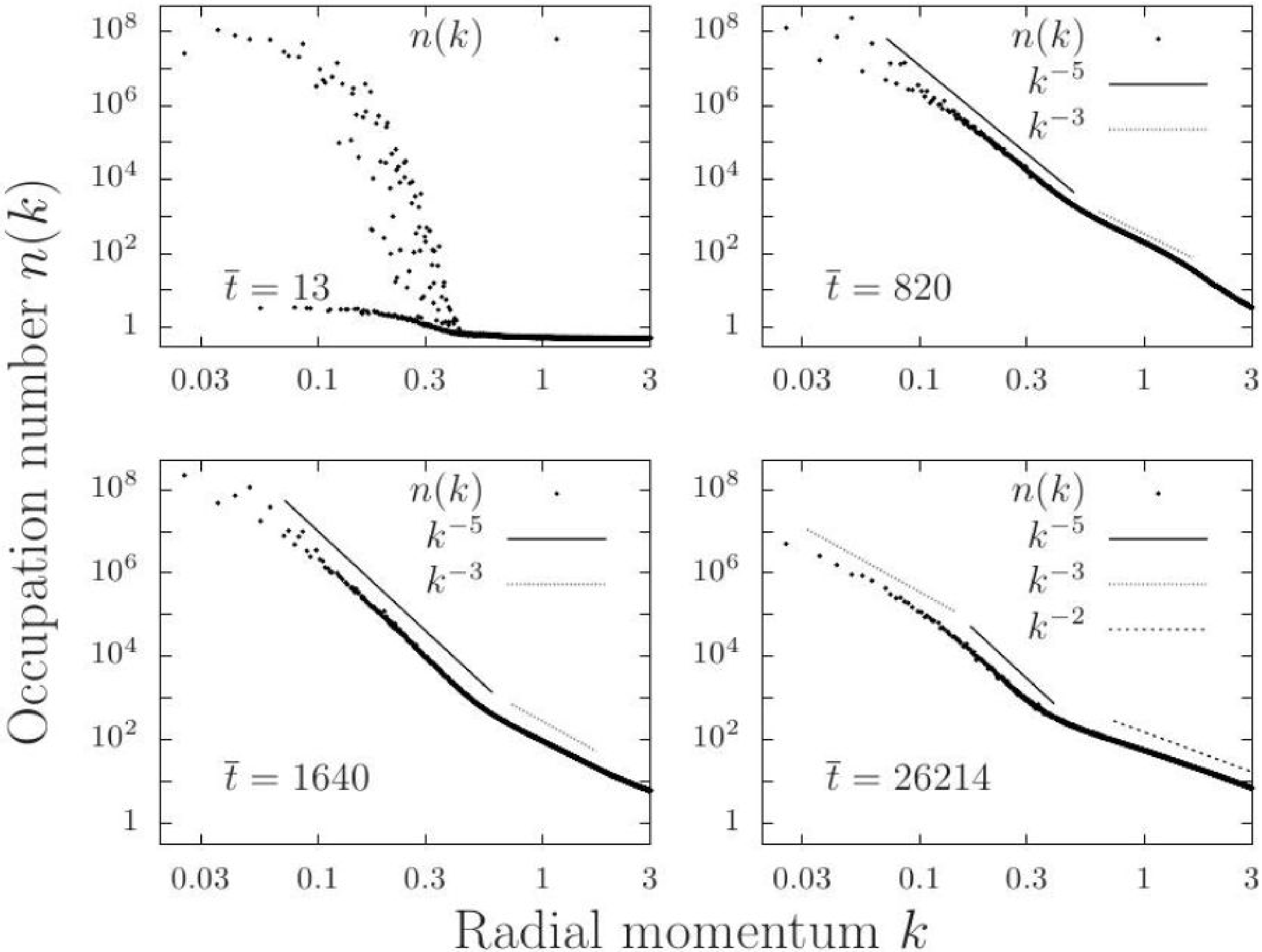


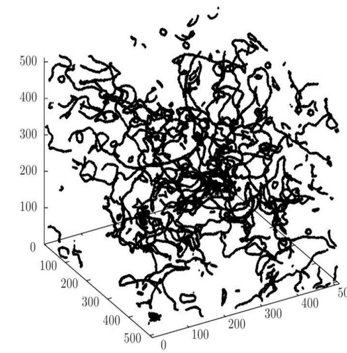
Movie 3: Vortex Lines in 3+1 D

$$n(k) = \langle \Psi^*(\mathbf{k})\Psi(\mathbf{k})\rangle\big|_{\text{angle average}}$$

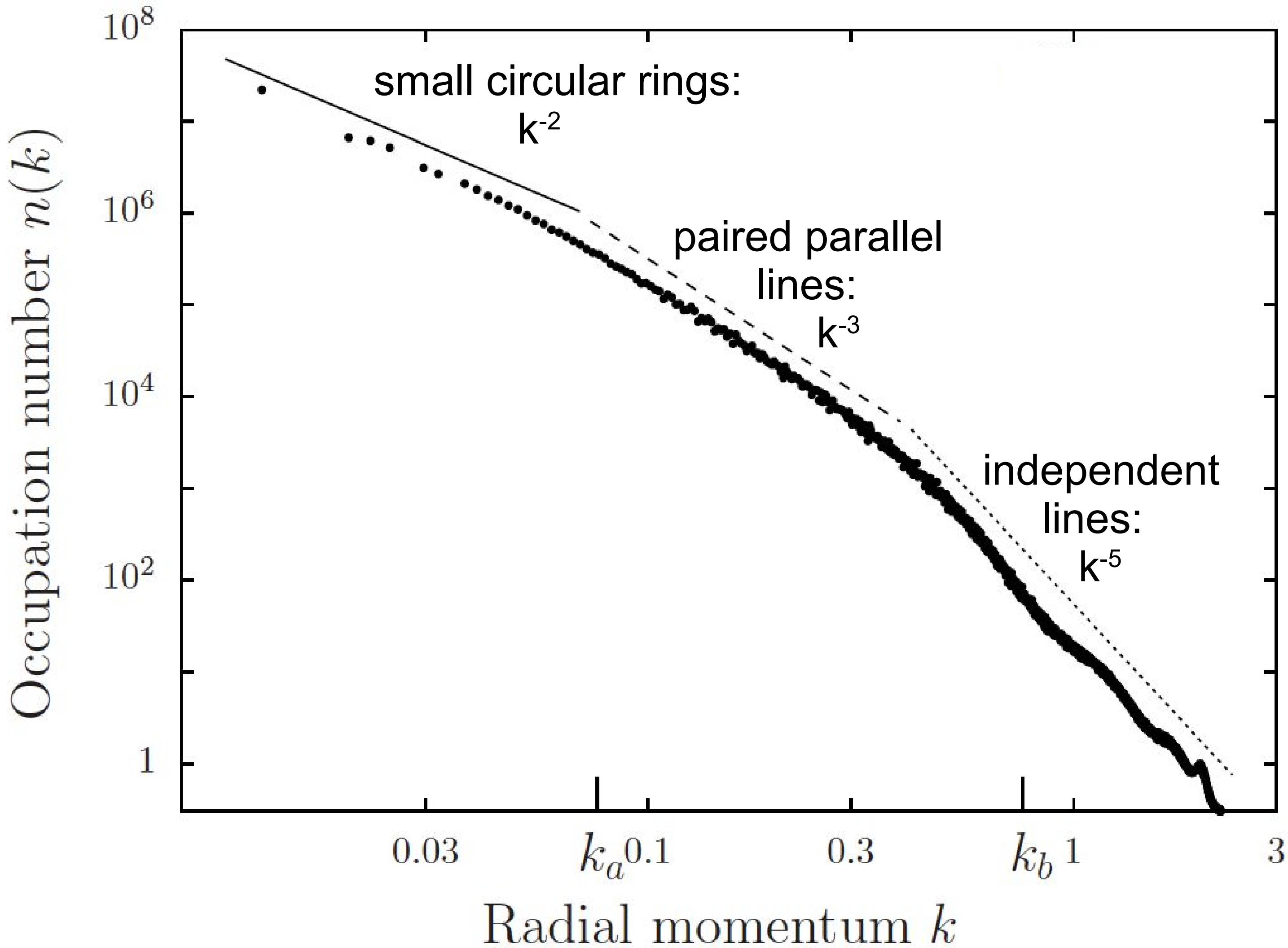
B. Nowak, D. Sexty, TG, PRB **84**: 020506(R), 2011

3+1 D simulations

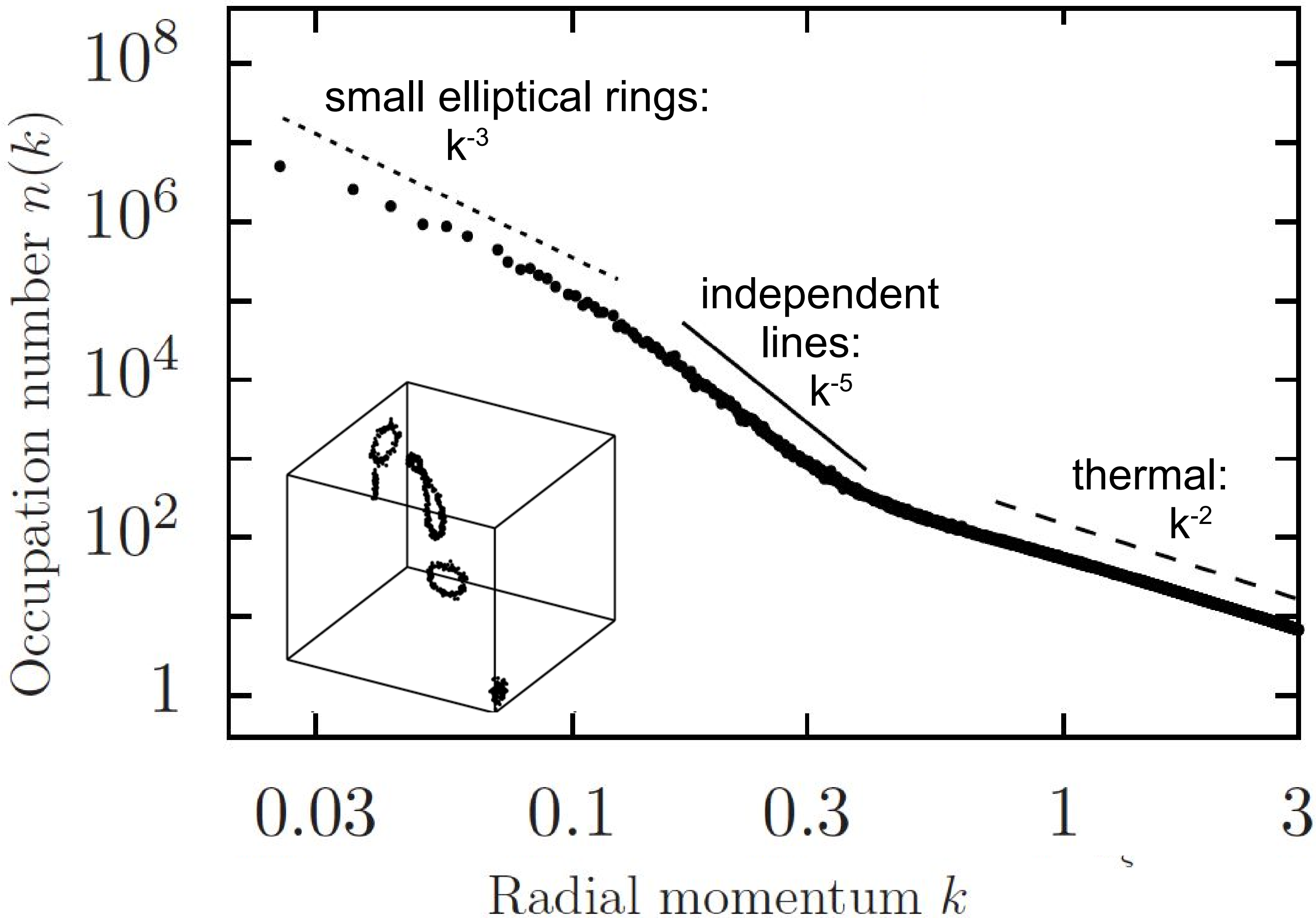




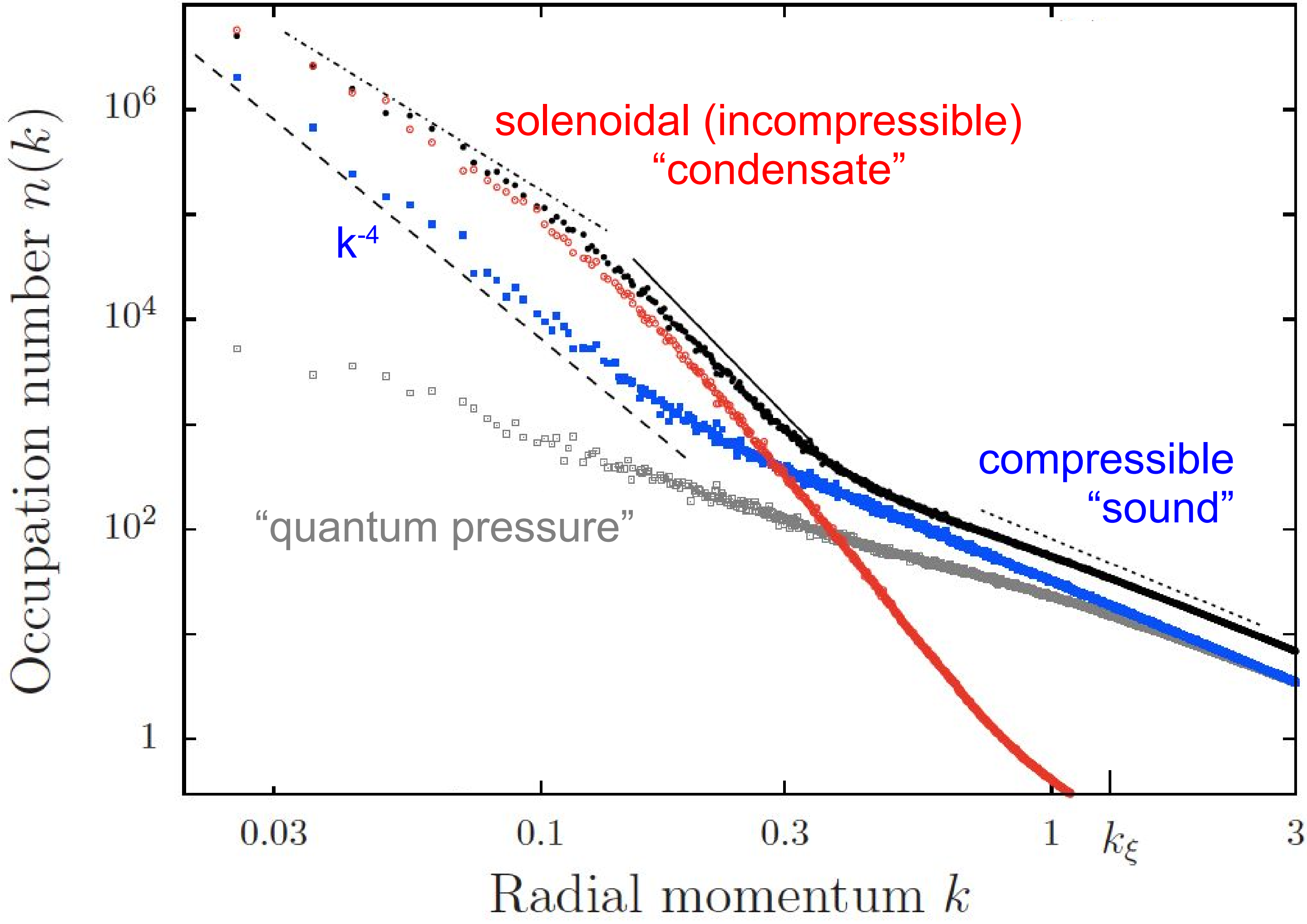
Line vortex model in 3+1 D



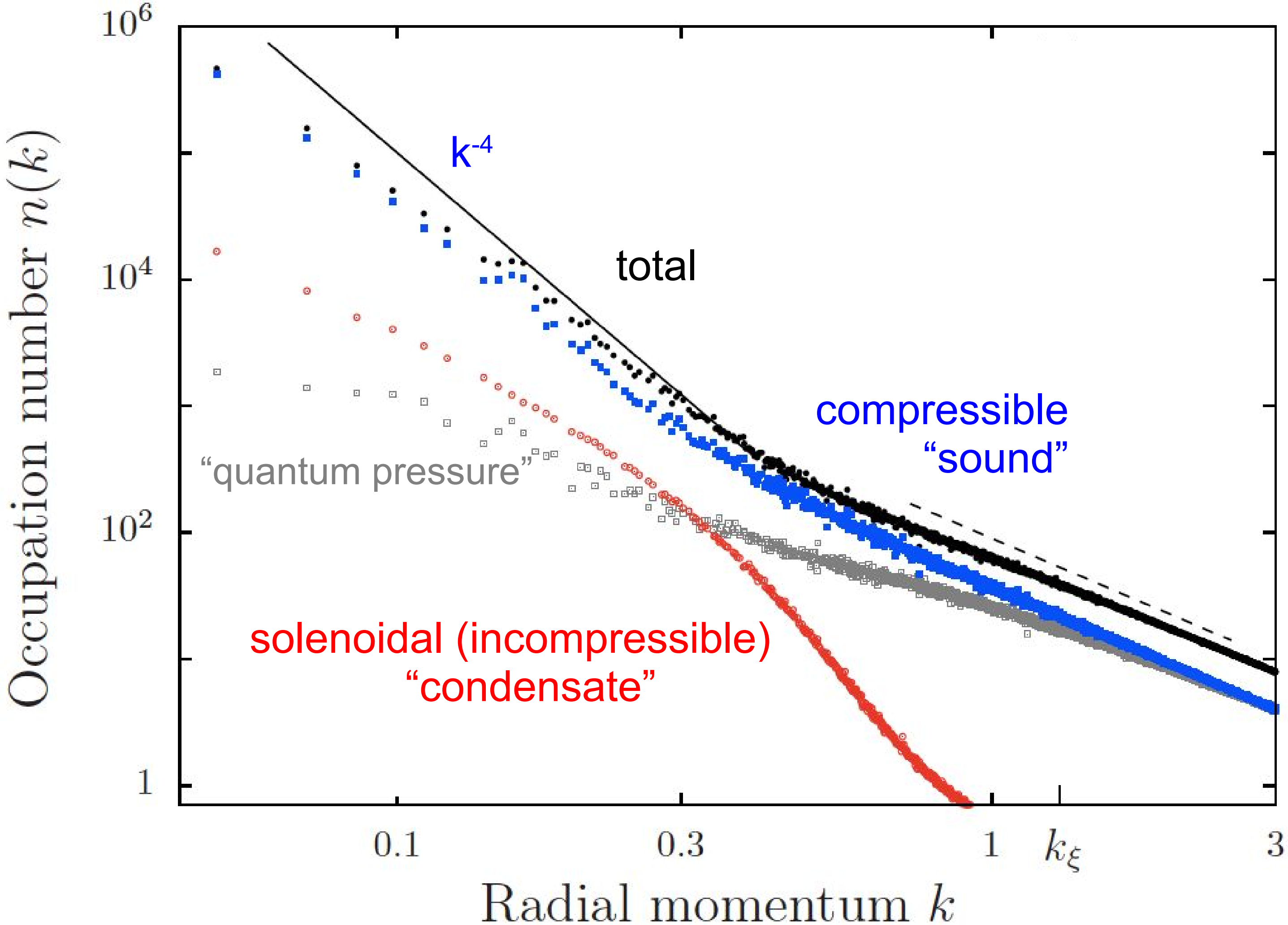
Simulations in 3+1 D



Decomposition of flow

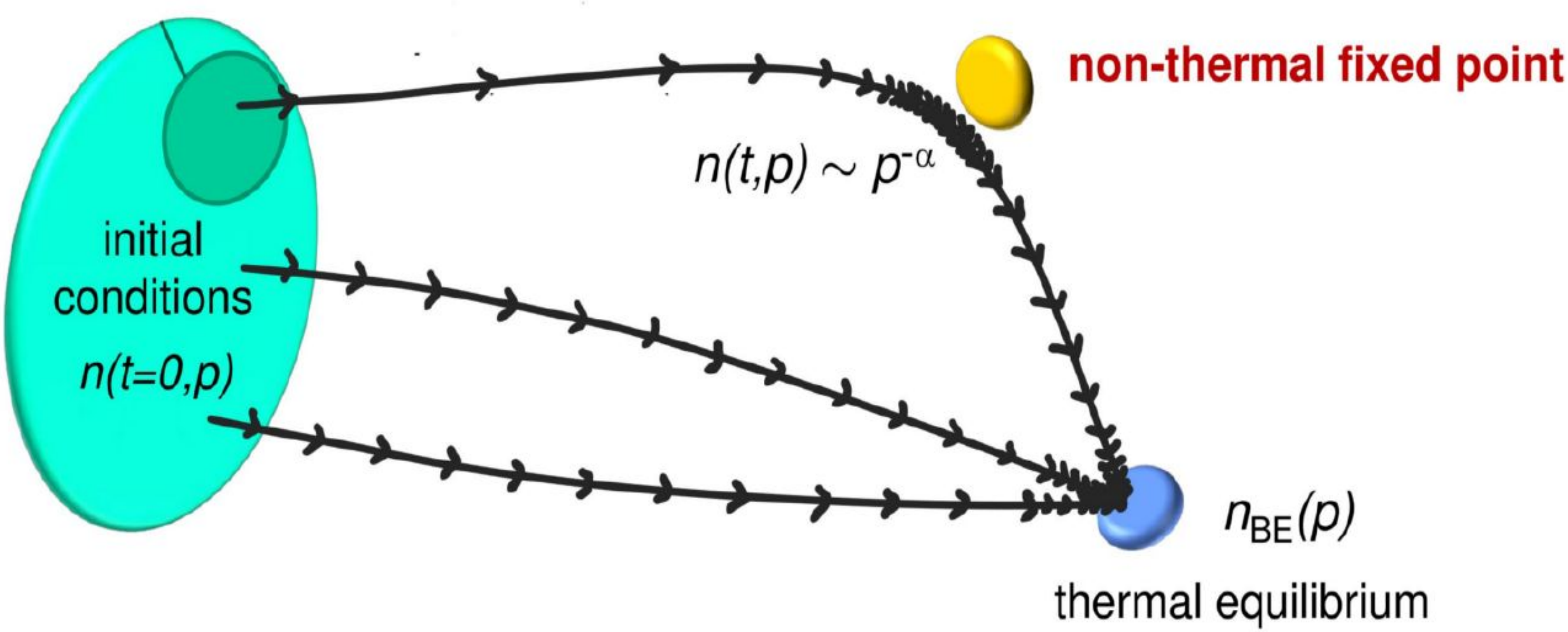


Acoustic turbulence



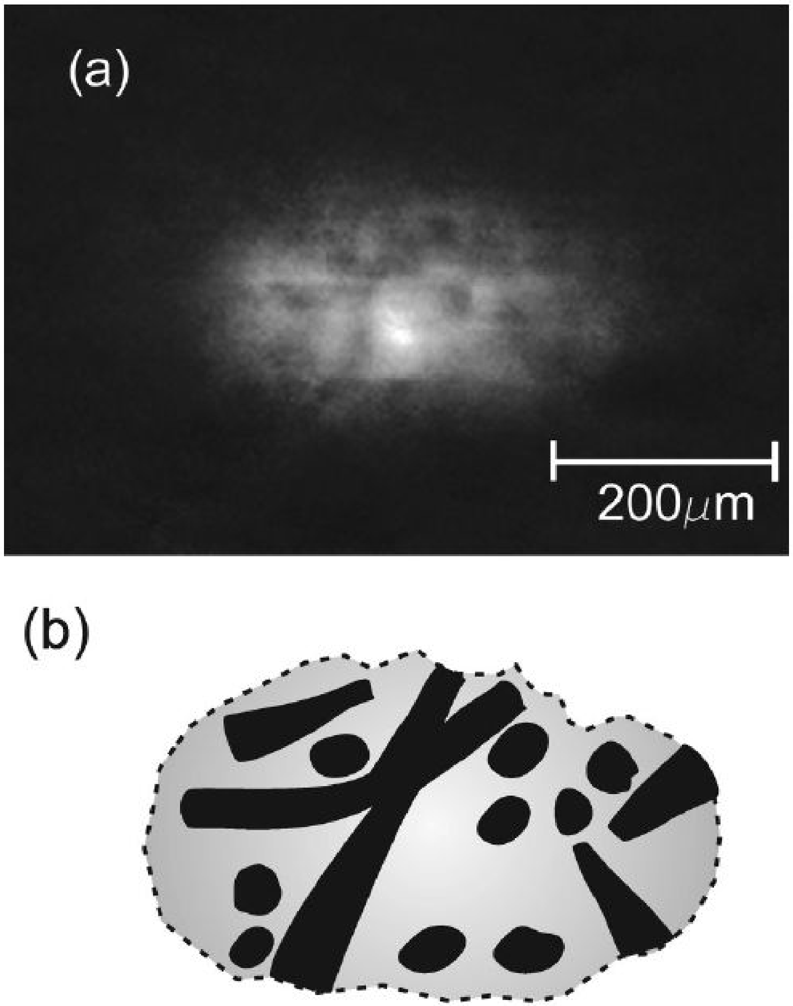
B. Nowak, J. Schole, D. Sexty, TG, arXiv:1111.61XX [cond-mat.quant-gas]

Non-thermal fixed point

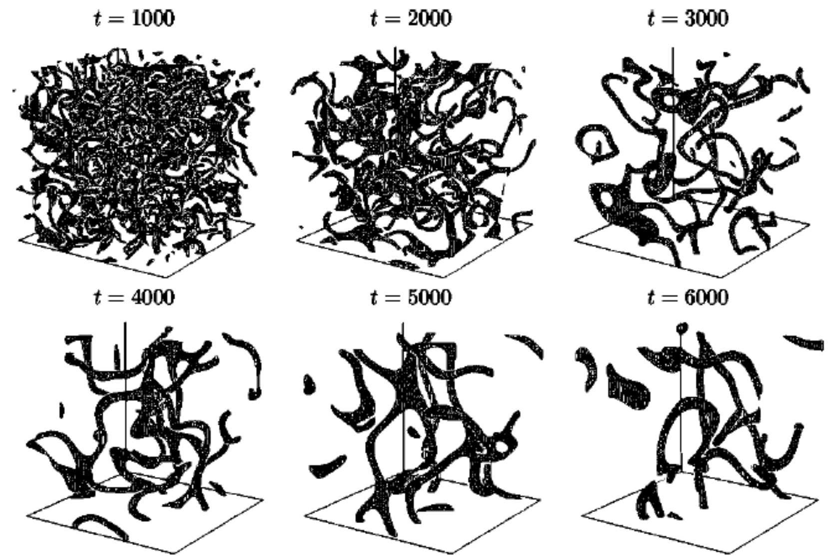


[Fig. courtesy: J. Berges '08]

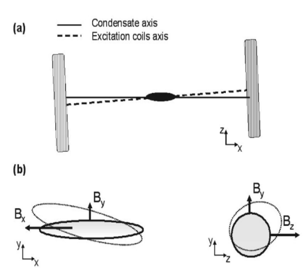
Vortex tangles in Bose Einstein Condensates



[E.A.L. Henn et al. PRL **103** (09)]



[N. Berloff & B. Svistunov, PRA (02)]



Relativistic scalar field

Strong Turbulence

Simulations of the non-linear Klein-Gordon equation, O(2) symmetry

$$(\partial_t^2 - \partial_x^2)\varphi(x,t) + \lambda\varphi^3(x,t) = 0$$

Initial condition: Highly occupied zero mode, Unoccupied modes with k>0

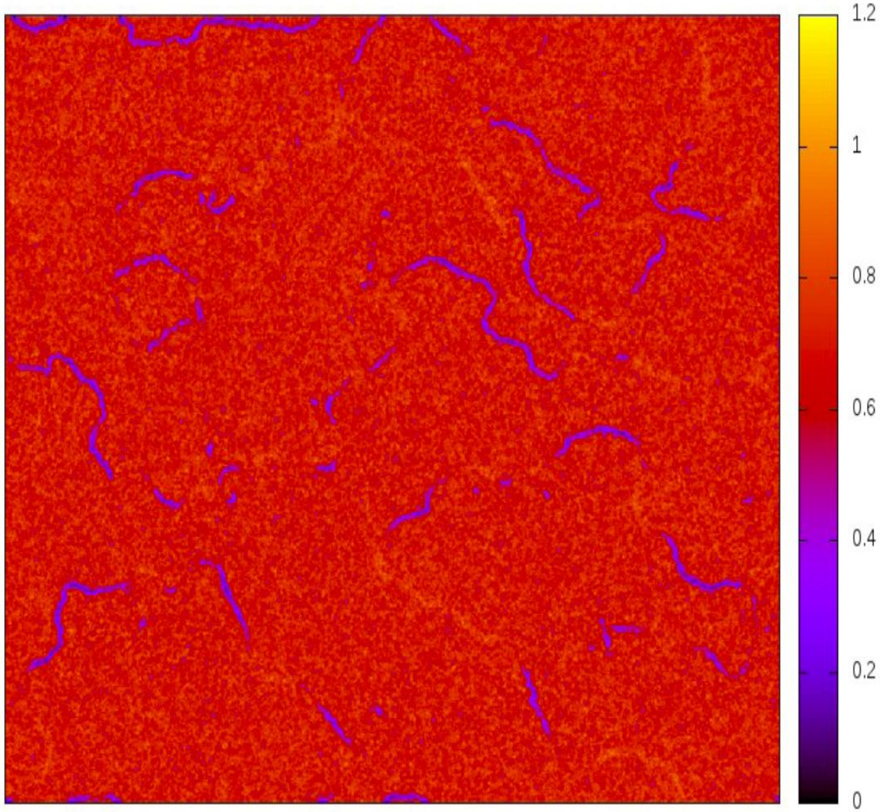
(video)

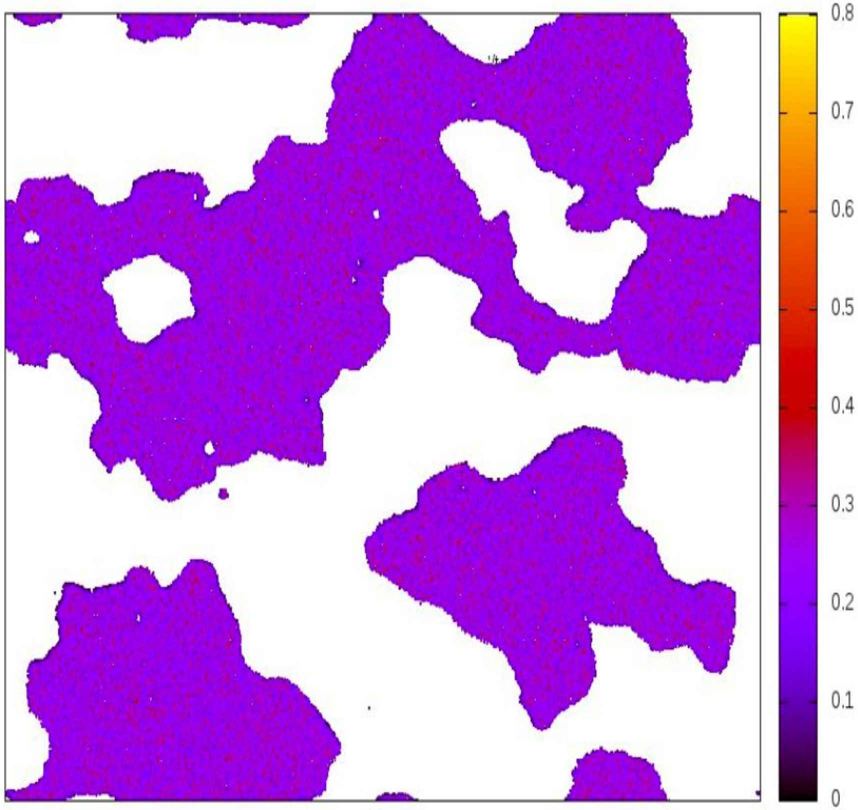
See also: http://www.thphys.uni-heidelberg.de/~sexty/videos

TG, B. Nowak, D. Sexty, arXiv:1108.0541 [hep-ph]

Strong Turbulence = Charge Separation

Modulus of complex field $|\varphi|$ vs. mean charge distribution



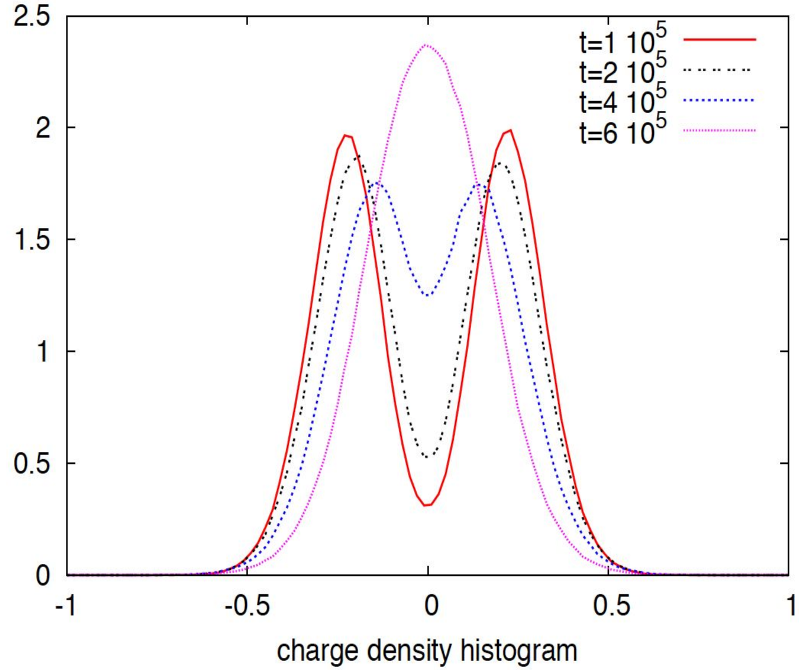


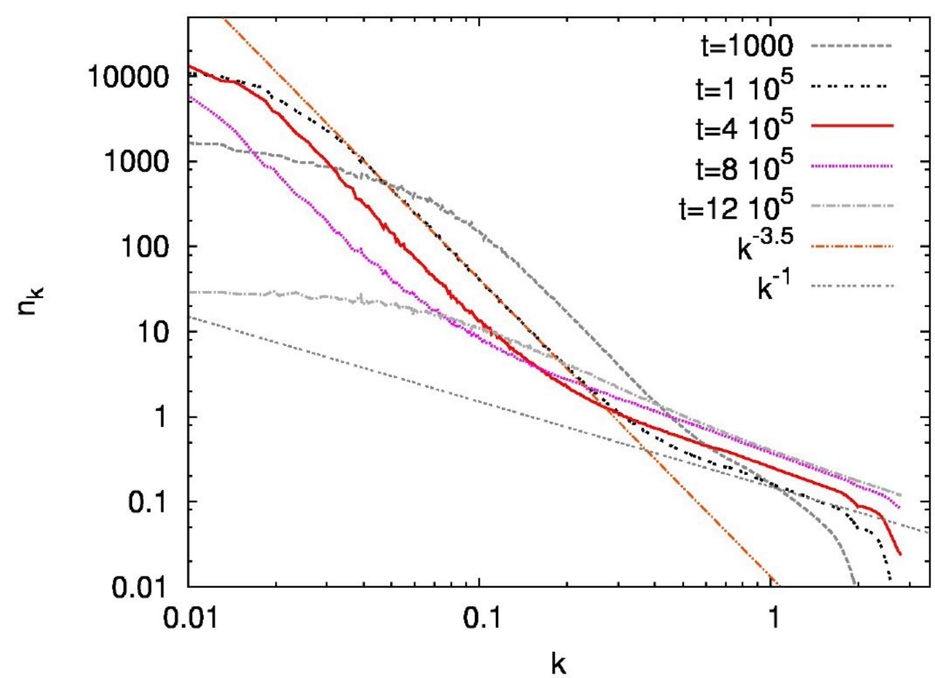
TG, B. Nowak, D. Sexty, arXiv:1108.0541 [hep-ph]
cf. also Tkachev, Kofman, Starobinsky, Linde (1998)

Strong Turbulence = Charge Separation

Charge density distribution vs. power spectrum

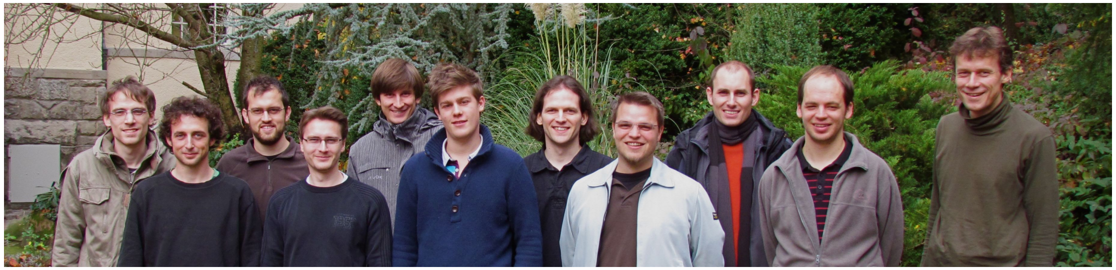
(d = 2, N = 2)





TG, B. Nowak, D. Sexty, arXiv:1108.0541 [hep-ph]

Thanks & credits to...



...my work group in Heidelberg:

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€€€...

Heisenberg-Programm
DFG Deutsche Forschungsgemeinschaft
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LGFG BaWue
DAAD Deutscher Akademischer Austausch Dienst German Academic Exchange Service
Center for Quantum Dynamics
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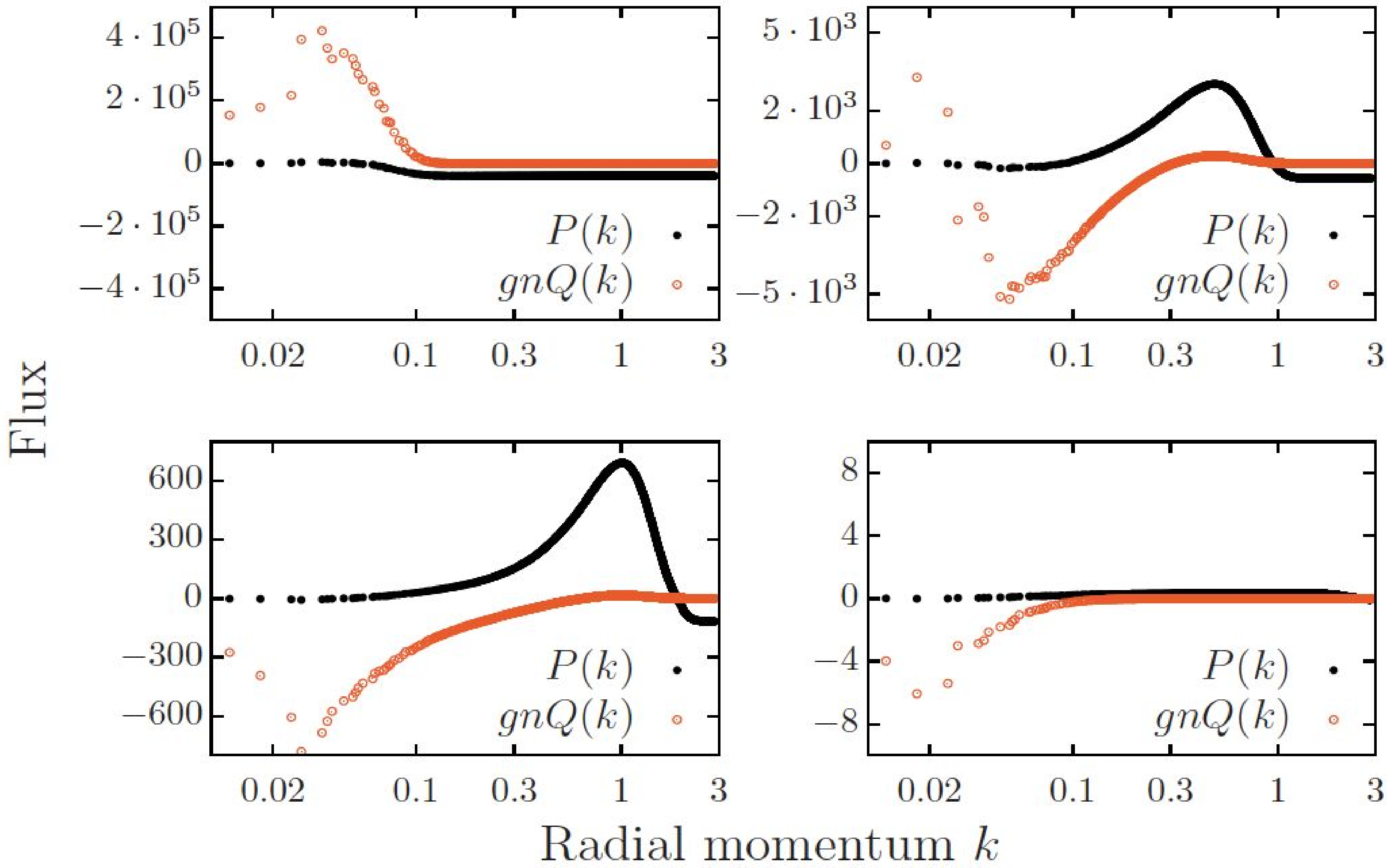
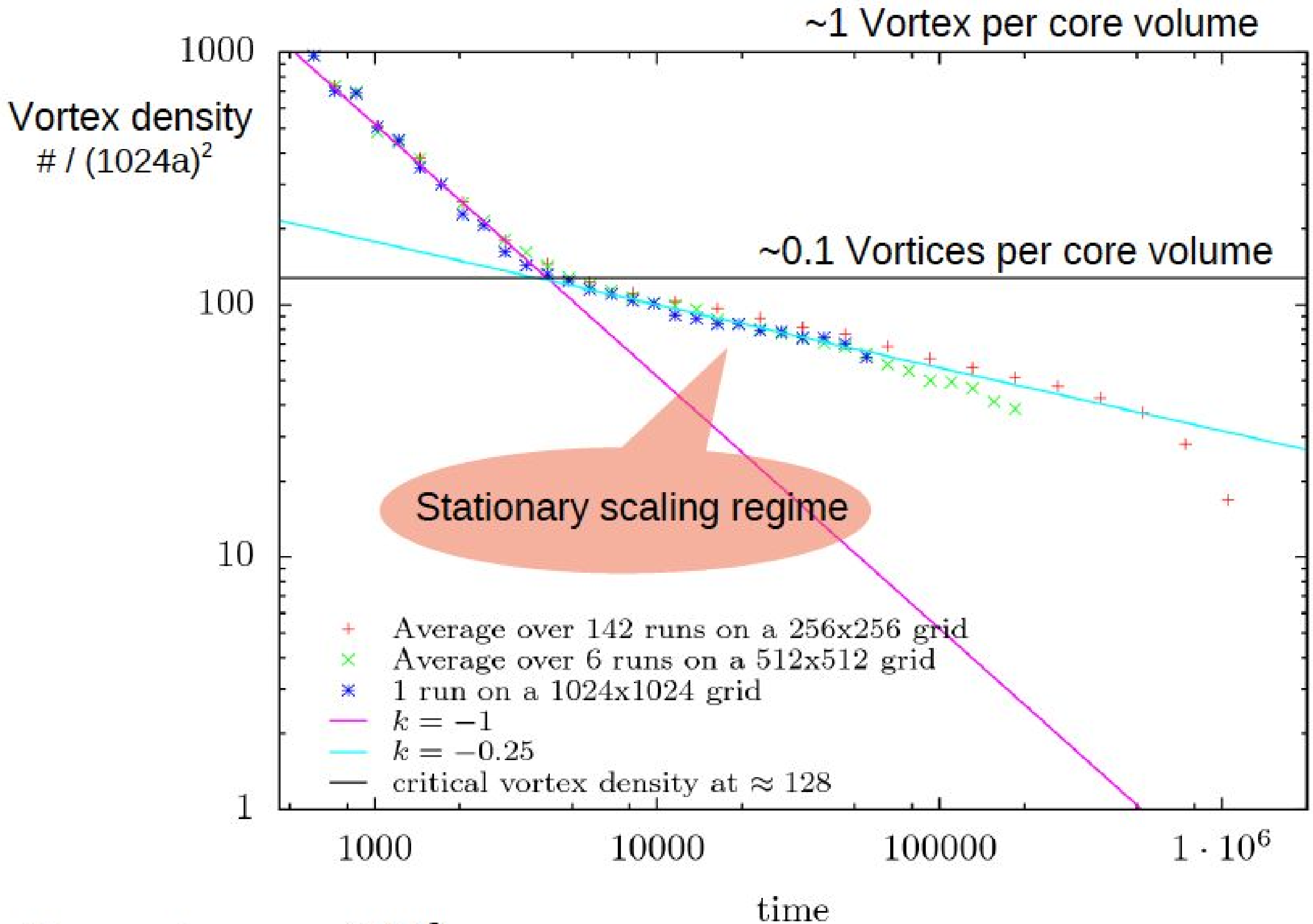
Supplementary slides

Cascades in 2+1 D: Fluxes



Time evolution of vortex density



Core volume ~ $\pi(3\xi)^2$

J. Schole, B. Nowak, D. Sexty, TG (unpublished)

Decomposition of Energy

$$E_{tot} = \int \left(\frac{1}{2} |\nabla \sqrt{n} e^{-i\varphi}|^2 + \frac{1}{2} g n^2 \right) d\boldsymbol{\rho}$$

$$= E_{kin} + E_q + E_{int}$$

$$\mathbf{u}(\boldsymbol{\rho}, t) = \nabla \varphi(\boldsymbol{\rho}, t)$$

$$\nabla \times (\sqrt{n}\mathbf{u})^c = 0$$

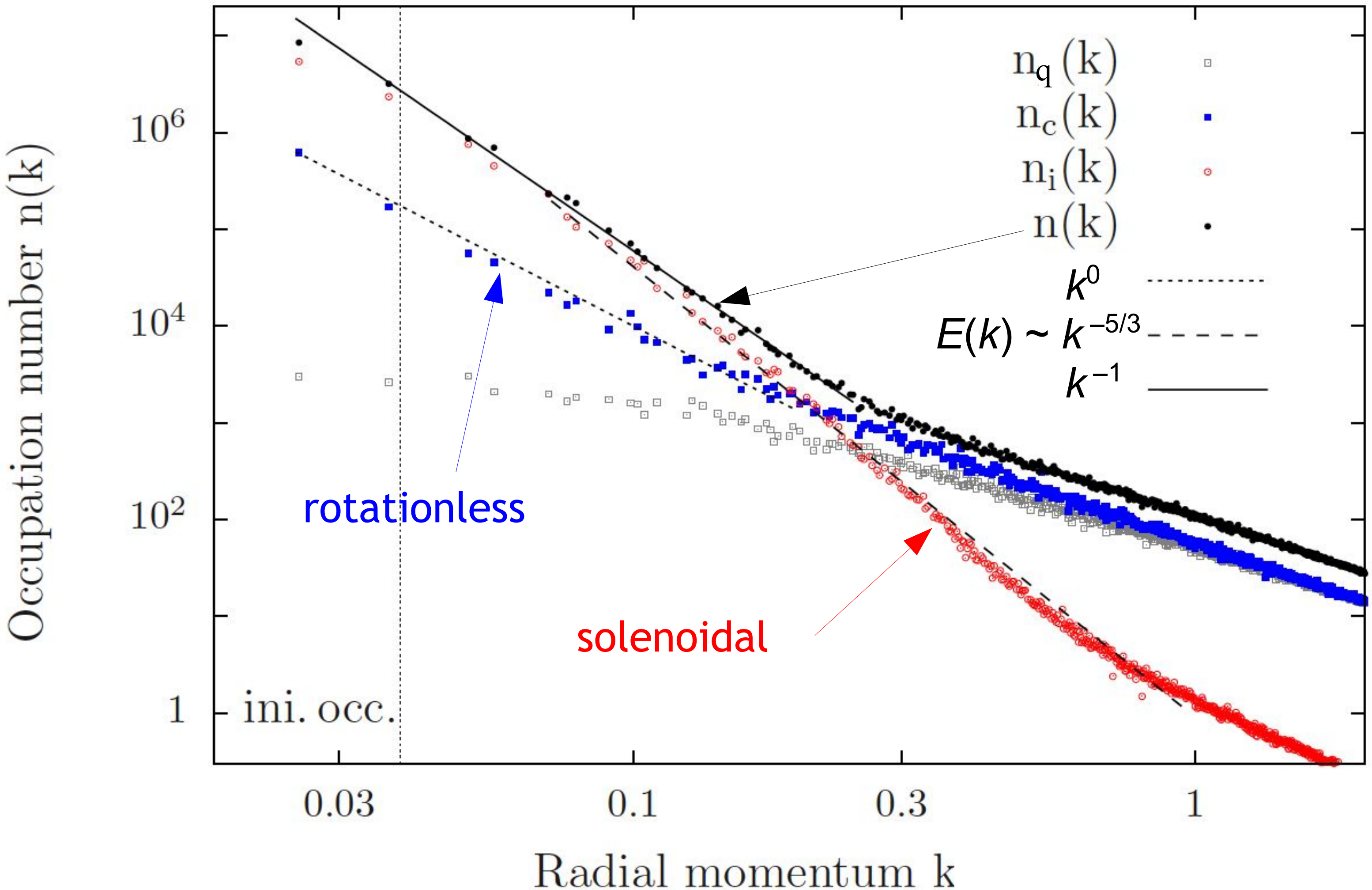
$$E_{kin} = \frac{1}{2} \int |\sqrt{n}\mathbf{u}|^2 d\boldsymbol{\rho} \quad = E^i_{kin} + E^c_{kin}$$

$$\nabla \cdot (\sqrt{n}\mathbf{u})^i = 0$$

$$E_q = \frac{1}{2} \int (\nabla \sqrt{n})^2 d\boldsymbol{\rho}$$

Simulations in 2+1 D

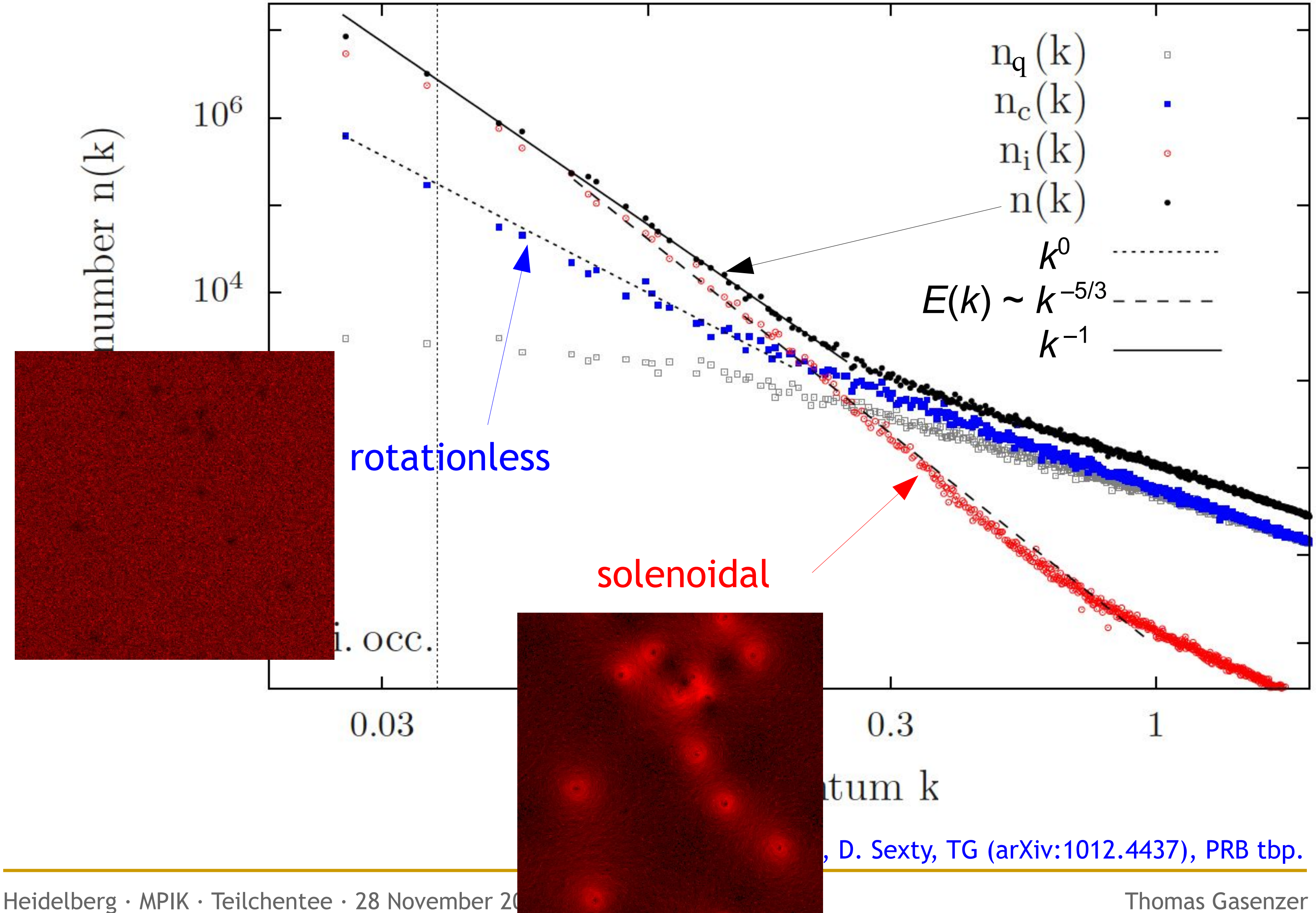
$$E(k) = \omega(k)k^{d-1}n(k)$$



B. Nowak, D. Sexty, TG (arXiv:1012.4437), PRB tbp.

Simulations in 2+1 D

$$E(\mathrm{k}) = \omega(\mathrm{k})\mathrm{k}^{d-1}\mathrm{n}(\mathrm{k})$$



, D. Sexty, TG (arXiv:1012.4437), PRB tbp.

Lewis Fry Richardson, FRS (1881-1953)

Big whirls have little whirls that feed on their velocity,
and little whirls have lesser whirls and so on to viscosity.

(L.F. Richardson, *The supply of energy from and to Atmospheric Eddies*, 1920)

Great fleas have little fleas upon their backs to bite 'em,
And little fleas have lesser fleas, and so ad infinitum.
And the great fleas themselves, in turn, have greater fleas to go on;
While these again have greater still, and greater still, and so on.

(Augustus de Morgan, *A Budget of Paradoxes, 1872, p. 370*)

So, naturalists observe, a flea
Has smaller fleas that on him prey;
And these have smaller still to bite 'em;
And so proceed ad infinitum.

(Jonathan Swift: *Poetry, a Rhapsody*, 1733)