Renormalization techniques for Kadanoff-Baym equations

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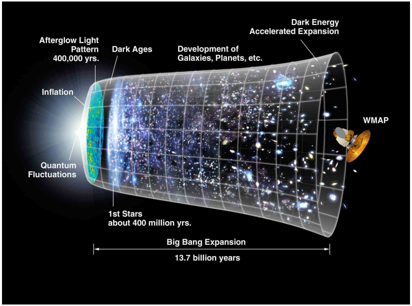
Outline

Renormalization techniques for Kadanoff-Baym equations

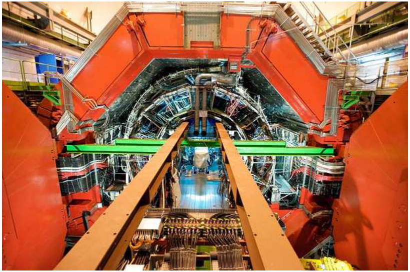
- Nonequilibrium dynamics at high energy
- The classical approach: Boltzmann
- Nonequilibrium quantum field theory: Kadanoff-Baym
- Renormalization techniques: Non-Gaussian initial states

Nonequilibrium dynamics at high energy



Heavy Ion Collisions

- LHC: ALICE
- RHIC

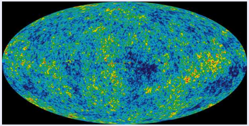


Early universe

- Reheating after Inflation
- Baryogenesis
- ...

Nonequilibrium dynamics at high energy

Baryon asymmetry



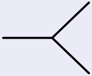
$$4.7 \cdot 10^{-10} < \eta < 6.5 \cdot 10^{-10} \text{ (95\% CL)}$$

Baryogenesis: three Sakharov conditions

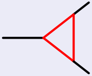
- baryon number violation
- CP violation
- deviation from thermal equilibrium

Example: decay of heavy right-handed neutrino ν_R

$$\mathcal{M}_{\nu_R \to \nu_L H} =$$



$$+$$



$$+ \dots$$

CP violation in decay described by loop process

Nonequilibrium dynamics at high energy

Nonequilibrium processes at high energy

Name:	Baryogenesis	Reheating	Heavy Ion Collisions
Process:	Generation of baryon asymmetry	Decay of oscillating scalar field	Formation of QGP
Description:	Boltzmann Equations	Mean-field approximation	Hydrodynamic Equations
Assumption:	On-shell	Collisionless	LTE

Quantum nonequilibrium effects ?

The classical approach: Boltzmann equations

Boltzmann equation for 2-to-2 scattering processes

$$
\begin{aligned}
k^\mu \cdot \partial_{x^\mu} f(t, \mathbf{x}, \mathbf{k}) \; = \\
& \int \frac{d^3p}{(2\pi)^3 2E_p} \int \frac{d^3q}{(2\pi)^3 2E_q} \int \frac{d^3r}{(2\pi)^3 2E_r} \left|\mathcal{M}_{2\to 2}\right|^2 \\
& \times \delta\left(\mathbf{k} + \mathbf{p} - \mathbf{q} - \mathbf{r}\right) \delta\left(E_k + E_q - E_p - E_r\right) \\
& \times \Big((1 + \eta f_k)(1 + \eta f_p) f_q f_r - f_k f_p (1 + \eta f_q)(1 + \eta f_r) \Big)
\end{aligned}
$$

$f(t, \mathbf{x}, \mathbf{k}) =$ one-particle distribution function

$$
\eta = \begin{cases} +1 & \text{for bosons} \\ -1 & \text{for fermions} \end{cases}
$$

Quantum effects in Boltzmann equations ?

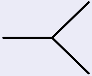
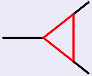
Pauli blocking and Bose enhancement $[N^{-1/3} \sim \lambda_{deBroglie}]$

Nordheim-Uehling-Uhlenbeck corrections (1933)

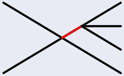
$$\Big(f_q f_r - f_k f_p\Big) \quad \rightarrow \quad \Big((1+\eta f_k)(1+\eta f_p) f_q f_r - f_k f_p (1+\eta f_q)(1+\eta f_r)\Big)$$

Off-shell effects $[\lambda_{mfp} \sim \lambda_{deBroglie}]$

Loop corrections [e.g. CP violation in leptogenesis]

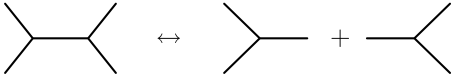
$\mathcal{M}_{\nu_R \to \nu_L H} =$  $+$  $+ \dots$

New processes [e.g. chemical equilibration in real scalar Φ^4 theory]

$\mathcal{M}_{2\to 4} =$ 

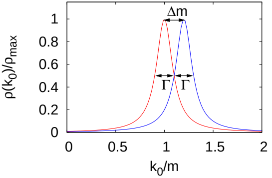
Limitations of Quantum Boltzmann equations

- Double Counting Problem



Example: Leptogenesis [RIS-subtraction] *Pilaftsis,...*

- Spectral function $\neq$ quasi-particles



Example: Mixing fields in Resonant Leptogenesis *Pilaftsis, Underwood*

- Memory & Correlation effects, Higher Gradients
- No controlled expansion

Limitations of Quantum Boltzmann equations

Can one describe nonequilibrium within QFT ?

Nonequilibrium Quantum Field Theory

Statistical ensemble described by density matrix ρ

$$\rho_{non-equilibrium}(t_{init} = 0) \quad \rightarrow \quad \rho_{equilibrium}(t \rightarrow \infty)$$

The *two-time* approach: Kadanoff-Baym Equations

- Time-evolution of one- and two point function

$$\begin{aligned} \phi(x) &\equiv \langle \Phi(x) \rangle \\ G(x,y) &\equiv \langle (\Phi(x) - \phi(x))(\Phi(y) - \phi(y)) \rangle \end{aligned}$$

- Non-secular
- Controlled Approximation
- Describes thermalization
- Off-shell and Memory effects
- Applicable far from equilibrium

Equations of motion from 2PI effective action

1PI: $$\frac{\delta\Gamma_{1PI}[\phi]}{\delta\phi} = 0$$

2PI: $$\frac{\delta\Gamma_{2PI}[\phi, G]}{\delta\phi} = 0, \quad \frac{\delta\Gamma_{2PI}[\phi, G]}{\delta G} = 0$$

Self-consistent Schwinger-Dyson equation

Cornwall, Jackiw, Tomboulis (1974)

$$\frac{\delta\Gamma_{2PI}}{\delta G} = 0 \quad \Leftrightarrow \quad G^{-1} = G_0^{-1} - \Pi[G]$$

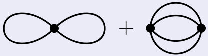
$$\begin{aligned} \Pi[G] &= \delta\Gamma_2/\delta G \\ \Gamma_2[G] &= \text{sum of 2PI Feynman diagrams w/o legs} \end{aligned}$$

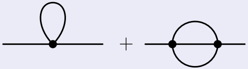
Nonequilibrium Quantum Field Theory

Controlled approximation...

... by truncation of $\Gamma_2[\phi, G]$

Example: Three-loop truncation in $\lambda\Phi^4$-theory (for $\phi = 0$)

$\Gamma_2[G] =$ 

$\Pi[G] =$ 

Nonequilibrium Quantum Field Theory

Kadanoff-Baym equations for $G_{F/\rho}(x,y) = \langle [\Phi(x), \Phi(y)]_\pm \rangle$

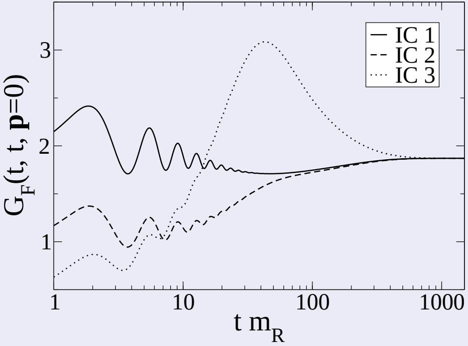
$$\left(\Box_x + m^2 + \frac{\lambda}{2} G(x,x) \right) G_F(x,y) = \int_0^{y^0} d^4z \, \Pi_F(x,z) G_\rho(z,y) - \int_0^{x^0} d^4z \, \Pi_\rho(x,z) G_F(z,y)$$

$$\left(\Box_x + m^2 + \frac{\lambda}{2} G(x,x) \right) G_\rho(x,y) = \int_{x_0}^{y^0} d^4z \, \Pi_\rho(x,z) G_\rho(z,y)$$

- Memory integrals
- Spectral function includes off-shell effects

Example: Thermalization in Φ^4-theory

Lindner, Müller Phys.Rev. (2006)



Nonequilibrium Physics

- Quantum thermalization
  Berges, Cox (2001); Berges, Borsanyi, Serreau (2003); Juchem, Cassing, Greiner (2004); Arrizabalaga, Smit, Tranberg (2005); Lindner, Müller (2006); Anisimov, Buchmüller, Drewes, Mendizabal (2008); Gasenzer, Pawlowski (2008)
- Prethermalization
  Berges, Borsanyi, Wetterich (2004)
- Parametric Resonance
  Berges, Serreau (2003)
- Nonequilibrium Instabilities
  Aarts, Tranberg (2007); Berges, Rothkopf, Schmidt (2008); Berges, Pruschke, Rothkopf (2009)
- Curved Spacetime
  Tranberg (2008); Hohenegger, Kartavtsev, Lindner (2008)

Renormalization

- 2PI Thermal field theory: established

  Hees, Knoll (2001, 2002); Blaizot, Iancu, Reinosa (2003); Berges, Borsanyi, Reinosa, Serreau (2004, 2005); Reinosa, Serreau (2006, 2007, 2009)

- 2PI Nonequilibrium field theory

  Borsanyi, Reinosa (2008); MG (2008); MG, Müller (2009)

Renormalization techniques for Kadanoff-Baym equations

Why renormalization?

- Numerical solutions use approximate renormalization
- Substantial cutoff-dependence
- Renormalization required for *quantitative* comparison with Boltzmann equations

Problem

- Standard Kadanoff-Baym equations: Gaussian initial states
- Incompatible with renormalization

State of the Art: Gaussian initial state
e.g. Berges, Cox (2001)

All n-point correlation functions vanish at $t = t_{init}$ for $n \geq 3$

$$\alpha_n(x_1, \ldots, x_n) = 0 \quad \text{for } n \geq 3$$

Physical initial state

n-point correlation functions asymptotically agree with vacuum correlations at short distances [for $n \leq 4$]

$$\alpha_n(x_1, \ldots, x_n) = \alpha_n^{th}(x_1, \ldots, x_n) + \Delta\alpha_n(x_1, \ldots, x_n)$$

Why does the Gaussian initial state lead to singularities ?

$$E_{total} = E_{kin}(t) + E_{corr}(t)$$

$$\begin{aligned}
E_{kin}(t) &= \frac{1}{2}\int\frac{d^3k}{(2\pi)^3}\Big[\partial_{x^0}\partial_{y^0} + \delta Z\partial_{x^0}\partial_{y^0} + \mathbf{k}^2 + \delta Z\mathbf{k}^2 + m_R^2 + \delta m^2 \\
&\quad + \frac{\lambda_R + \delta\lambda}{4}\int\frac{d^3q}{(2\pi)^3}G_F(t,t,\mathbf{q})\Big] G_F(x^0,y^0,\mathbf{k})|_{x^0=y^0=t} + \delta\Lambda \\
E_{corr}(t) &= -\frac{\lambda_R^2}{6}\int_0^t dz^0 \int\frac{d^3k}{(2\pi)^3}\int\frac{d^3p}{(2\pi)^3}\int\frac{d^3q}{(2\pi)^3}\, G_F(t,z^0,-\mathbf{k})G_\rho(t,z^0,\mathbf{k}-\mathbf{p}-\mathbf{q}) \\
&\quad \times\Big(G_F(t,z^0,\mathbf{p})G_F(t,z^0,\mathbf{q}) - \frac{1}{4}G_\rho(t,z^0,\mathbf{p})G_\rho(t,z^0,\mathbf{q})\Big)
\end{aligned}$$

- $E_{corr}(t)$ contains divergences
- These have to be cancelled by $\delta\lambda, \delta Z, \delta m^2, \delta\Lambda$ from $E_{kin}(t)$
- However: $E_{corr}(t)|_{t=0} = 0$ for Gaussian initial state
- $\Rightarrow$ unbalanced divergence at $t = 0$

Renormalization techniques *MG, Müller (2009)*

- Kadanoff-Baym equations for Non-Gaussian IC
- Vacuum/thermal initial correlations for $\lambda\Phi^4$

$$\alpha_n^{th}(x_1, \ldots, x_n)$$

  within nonequilibrium 2PI framework
- Kadanoff-Baym equations for correlated vacuum/thermal IC
- Leading Non-Gaussian correction

Why study the equilibrium limit of Kadanoff-Baym equations ?

- renormalization

$$\alpha_n(x_1,\ldots,x_n) = \alpha_n^{th}(x_1,\ldots,x_n) + \Delta\alpha_n(x_1,\ldots,x_n)$$

- controlled nonequilibrium dynamics

$$\Delta\alpha_n(x_1,\ldots,x_n) \to 0$$

- important for validation
- thermal equilibrium should be accessible as a special case within nonequilibrium QFT

Challenge: 2PI is non-perturbative

$\alpha_n^{th}(x_1,\ldots,x_n)$ have to match 2PI truncation underlying Kadanoff-Baym equations

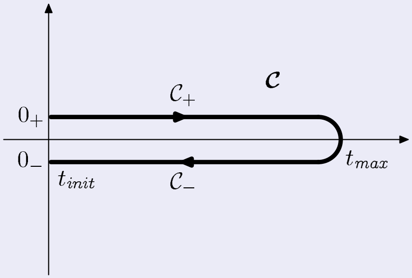
Initial state: density matrix $\rho = \sum_i p_i |i\rangle\langle i|$

Schwinger, Keldysh

Generating functional:

$$e^{iW[J]} \;=\; \int \mathcal{D}\varphi \, \langle \varphi_+ | \rho | \varphi_- \rangle \exp\left(i \int_{\mathcal{C}} d^4x \, \mathcal{L}(x) + i \int_{\mathcal{C}} d^4x \, J(x)\varphi(x) \right)$$

where $\Phi(0, \mathbf{x})|\varphi_\pm\rangle = \varphi(0_\pm, \mathbf{x})|\varphi_\pm\rangle$



Kadanoff-Baym equations for Non-Gaussian initial states

State of the Art: Gaussian initial state

e.g. Berges, Cox (2001)

$$\langle \varphi_+ |\rho| \varphi_- \rangle = \exp\left(i\alpha_0 + i\int_{\mathcal{C}} d^4x\, \alpha_1(x)\varphi(x) + \frac{i}{2}\int_{\mathcal{C}} d^4x \int_{\mathcal{C}} d^4y\; \varphi(x)\alpha_2(x,y)\varphi(y) \right)$$

...depends only on $\varphi(0_\pm, \mathbf{x}) \Rightarrow$

$$\begin{aligned} \alpha_1(x) &= \alpha_1^+(\mathbf{x})\delta(x^0 - 0_+) + \alpha_1^-(\mathbf{x})\delta(x^0 - 0_-) \\ \alpha_2(x,y) &= \sum_{\epsilon_i \in \pm} \alpha_1^{\epsilon_1\epsilon_2}(\mathbf{x},\mathbf{y})\delta(x^0 - 0_{\epsilon_1})\delta(x^0 - 0_{\epsilon_2}) \end{aligned}$$

Kadanoff-Baym equations for Non-Gaussian initial states

Non-Gaussian initial density matrix

Calzetta, Hu (1988)

$$\langle \varphi_+ | \rho | \varphi_- \rangle = \exp\left(iF[\varphi]\right)$$

$$\begin{aligned} F[\varphi] &= \sum_{n=0}^{\infty} \int_{\mathcal{C}} d^4x_1 \ldots d^4x_n \, \alpha_n(x_1, \ldots, x_n)\varphi(x_1)\cdots\varphi(x_n) \\ &= F_{Gauss}[\varphi] + F_{Non-Gauss}[\varphi] \end{aligned}$$

$$\alpha_n(x_1, \ldots, x_n) = \sum_{\epsilon_i \in \pm} \alpha_n^{\epsilon_1, \ldots, \epsilon_n}(\mathbf{x_1}, \ldots, \mathbf{x_n})\delta(x_1^0 - 0_{\epsilon_1})\cdots\delta(x_n^0 - 0_{\epsilon_n})$$

Kadanoff-Baym equations for Non-Gaussian initial states

Generating functional for Non-Gaussian initial states:

$$e^{iW[J,K]} = \int \mathcal{D}\varphi \, \langle \varphi_+ | \rho | \varphi_- \rangle \exp\left(iS[\varphi] + iJ\varphi + \frac{i}{2}\varphi K \varphi \right)$$

$$= \int \mathcal{D}\varphi \, \exp\left(i(S[\varphi] + F_{Non-Gauss}[\varphi] + \alpha_0) + i(J + \alpha_1)\varphi + \frac{i}{2}\varphi(K + \alpha_2)\varphi \right)$$

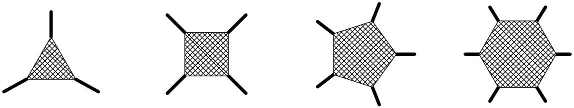
2PI effective action contains vertices derived from

$$\tilde{S}[\varphi] \equiv S[\varphi] + F_{Non-Gauss}[\varphi]$$

Local standard vertices: $\delta^n S[\varphi]/\delta\varphi^n (n > 2)$

$-i\lambda\phi(x) =$  $\quad -i\lambda =$ 

Effective non-local vertices: $\delta^n F_{Non-Gauss}[\varphi]/\delta\varphi^n = \alpha_n(x_1, \ldots, x_n)$



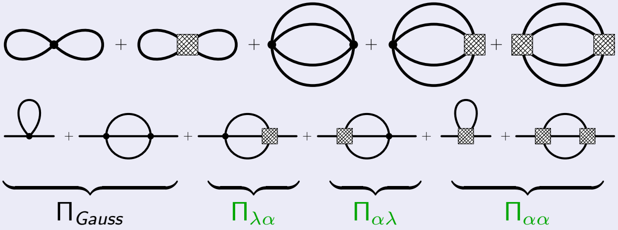
. . . encode the Non-Gaussian initial correlations

Kadanoff-Baym equations for Non-Gaussian initial states

Example: Initial 4-point correlation, 2PI three-loop truncation

$\Gamma_2[G, \alpha_4] =$

$\Pi[G, \alpha_4] =$



Initial-time Surface contribution

For the example:

$$\Pi_{\lambda\alpha}(x, z) = -\frac{\lambda}{6} \int_{\mathcal{C}} d^4 y_{123} G(x, y_1) G(x, y_2) G(x, y_3) \alpha_4(y_1, y_2, y_3, z)$$

In general:

$$\Pi_{\lambda\alpha}(x, z) = \Pi^{+}_{\lambda\alpha}(x, \mathbf{z}) \delta(z^0 - 0_+) + \Pi^{-}_{\lambda\alpha}(x, \mathbf{z}) \delta(z^0 - 0_-)$$

Kadanoff-Baym equations for Non-Gaussian initial states

$$\left(\Box_x + m^2 + \frac{\lambda}{2}G(x,x)\right) G_F(x,y) = \int_0^{y^0} d^4z\, \Pi_F(x,z)G_\rho(z,y)$$

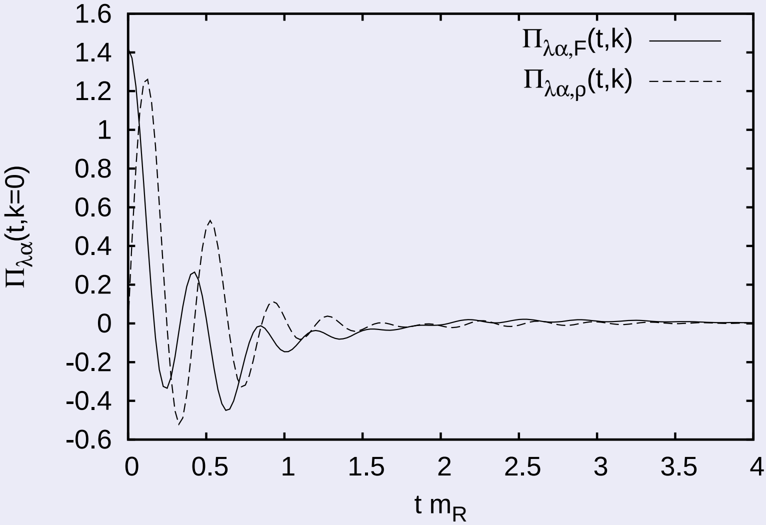
$$- \int_0^{x^0} d^4z\, \Pi_\rho(x,z)G_F(z,y) - \int_{\mathcal{C}} d^4z\, \Pi_{\lambda\alpha}(x,z)G(z,y)$$

$$\left(\Box_x + m^2 + \frac{\lambda}{2}G(x,x)\right) G_\rho(x,y) = \int_{x_0}^{y^0} d^4z\, \Pi_\rho(x,z)G_\rho(z,y)$$

For example:

$$\Pi_{\lambda\alpha}(x,z) = \int_{\mathcal{C}} d^4y_{123}\, G(x,y_1)G(x,y_2)G(x,y_3)\alpha_4(y_1,y_2,y_3,z)$$

Example: Initial 4-point correlation, 2PI three-loop truncation



Thermal Initial Correlations

Thermal density matrix of the interacting theory

$$\rho_{th} = \frac{1}{Z} e^{-\beta H}, \qquad H = H_0 + H_{int}$$

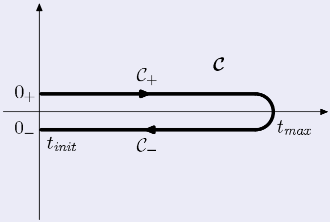
$\Rightarrow$ Compute the corresponding initial correlations

$$\langle \varphi_+ | \rho_{th} | \varphi_- \rangle = \exp\left(i \sum_{n=0}^{\infty} \int d^4x_{12\ldots n} \, \alpha_n^{th}(x_1, \ldots, x_n) \varphi(x_1)\varphi(x_2) \cdots \varphi(x_n) \right)$$

- Can be done order-by-order in H_{int}
- Problem: Need approximations compatible with 2PI formalism
- Solution: Match 2PI on closed real-time path with 2PI thermal (imaginary-time) field theory *MG, Müller (2009)*

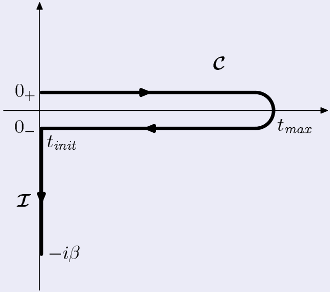
Thermal Initial Correlations

Closed time path $\mathcal{C}$ with α_n



$$\langle \varphi_+ | \rho_{th} | \varphi_- \rangle = \exp\left(i \sum_{n=0}^{\infty} \alpha^{th}_{12\cdots n} \varphi_1 \varphi_2 \cdots \varphi_n \right)$$

Thermal time path $\mathcal{C} + \mathcal{I}$



$$\langle \varphi_+ | \rho_{th} | \varphi_- \rangle = \int_{\varphi(0,\mathbf{x})=\varphi_-(\mathbf{x})}^{\varphi(-i\beta,\mathbf{x})=\varphi_+(\mathbf{x})} \mathcal{D}\varphi \ \exp\left(i \int_{\mathcal{I}} d^4x \, \mathcal{L}(x) \right)$$

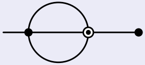
Thermal Initial Correlations

Thermal time path $\mathcal{C}+\mathcal{I}$

Self-consistent Schwinger-Dyson equation

$$G_{th}^{-1}(x,y) = G_{0,th}^{-1}(x,y) - \Pi_{th}(x,y) \qquad \Leftrightarrow$$

$$(\Box_x + m^2)G_{th}(x,y) = -i\delta_{\mathcal{C}+\mathcal{I}}(x-y) - i \underbrace{\int_{\mathcal{C}+\mathcal{I}} d^4z \Pi_{th}(x,z) G_{th}(z,y)}$$



Closed time path $\mathcal{C}$ with initial correlations α

Kadanoff-Baym equation for a Non-Gaussian initial state

$$\begin{aligned}(\Box_x + m^2)G(x,y) &= -i\delta_{\mathcal{C}}(x-y) \\ &\quad - i \int_{\mathcal{C}} d^4z \left[\Pi_{Gauss}(x,z) + \Pi_{non-Gauss}(x,z)\right] G(z,y)\end{aligned}$$

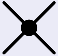
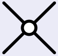
Thermal Initial Correlations

Thermal time path $\mathcal{C}+\mathcal{I}$

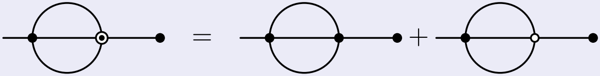
The lines represent the *complete* propagator

$G_{th}^{\mathcal{C}\mathcal{C}}(x,y) =$  $G_{th}^{\mathcal{C}\mathcal{I}}(x,y) =$ 

$G_{th}^{\mathcal{I}\mathcal{I}}(x,y) =$  $G_{th}^{\mathcal{I}\mathcal{C}}(x,y) =$ 

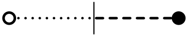
$-i\lambda\int_{\mathcal{C}} d^4x =$  $-i\lambda\int_{\mathcal{I}} d^4x =$  $-i\lambda\int_{\mathcal{C}+\mathcal{I}} d^4x =$ 

Example: 2PI three-loop truncation



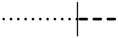
Thermal Initial Correlations: Perturbation Theory

$$G_{0,th}^{\mathcal{IC}}(-i\tau, y^0, \mathbf{k}) = \int_{\mathcal{C}} dt\, \Delta_0(-i\tau, t, \mathbf{k})\, G_{0,th}^{\mathcal{CC}}(t, y^0, \mathbf{k})$$

 = 

Free 'connection'

$$\Delta_0(-i\tau, z^0, \mathbf{k}) = \Delta_0^+(-i\tau, \mathbf{k})\, \delta_{\mathcal{C}}(z^0 - 0_+) + \Delta_0^-(-i\tau, \mathbf{k})\, \delta_{\mathcal{C}}(z^0 - 0_-)$$

= 

where

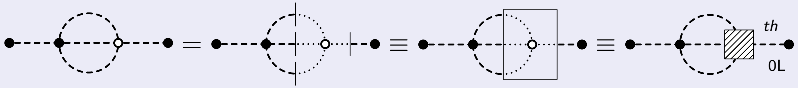
$$\Delta_0^+(-i\tau, \mathbf{k}) = \frac{\sinh(\omega_{\mathbf{k}}\tau)}{\sinh(\omega_{\mathbf{k}}\beta)} = \frac{G_{0,th}^{\mathcal{II}}(-i\tau, 0, \mathbf{k})}{2G_{0,th}(0,0,\mathbf{k})} + \partial_\tau G_{0,th}^{\mathcal{II}}(-i\tau, 0, \mathbf{k})$$

$$\Delta_0^-(-i\tau, \mathbf{k}) = \frac{\sinh(\omega_{\mathbf{k}}(\beta-\tau))}{\sinh(\omega_{\mathbf{k}}\beta)} = \frac{G_{0,th}^{\mathcal{II}}(-i\tau, 0, \mathbf{k})}{2G_{0,th}(0,0,\mathbf{k})} - \partial_\tau G_{0,th}^{\mathcal{II}}(-i\tau, 0, \mathbf{k})$$

Thermal Initial Correlations: Perturbation Theory

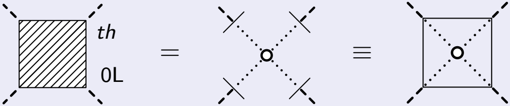
Example

MG, Müller (2009)



Perturbative initial 4-point correlation

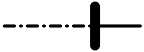
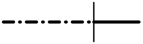
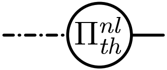
$$\alpha^{th}_{4,0L}(z_1, z_2, z_3, z_4) \quad = \quad -i\lambda \int_{\mathcal{I}} d^4 v \; \Delta_0(v, z_1)\Delta_0(v, z_2)\Delta_0(v, z_3)\Delta_0(v, z_4)$$



$$G_{th}^{\mathcal{I}\mathcal{C}}(-i\tau, y^0, \mathbf{k}) = \int_{\mathcal{C}} dt\, \tilde{\Delta}(-i\tau, t, \mathbf{k}) G_{th}^{\mathcal{C}\mathcal{C}}(t, y^0, \mathbf{k})$$

 = 

Complete ‘connection’:

$\tilde{\Delta}(-i\tau, z^0, \mathbf{k}) =$  =  + 

$\Delta(-i\tau, z^0, \mathbf{k}) =$ 

$$= \Delta^+(-i\tau, \mathbf{k})\, \delta_{\mathcal{C}}(z^0 - 0_+) + \Delta^-(-i\tau, \mathbf{k})\, \delta_{\mathcal{C}}(z^0 - 0_-)$$

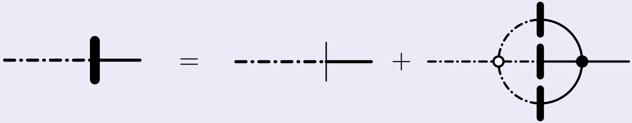
where

$$\Delta^+(-i\tau, \mathbf{k}) = \frac{G_{th}^{\mathcal{I}\mathcal{I}}(-i\tau, 0, \mathbf{k})}{2G_{th}(0, 0, \mathbf{k})} + \partial_\tau G_{th}^{\mathcal{I}\mathcal{I}}(-i\tau, 0, \mathbf{k})$$

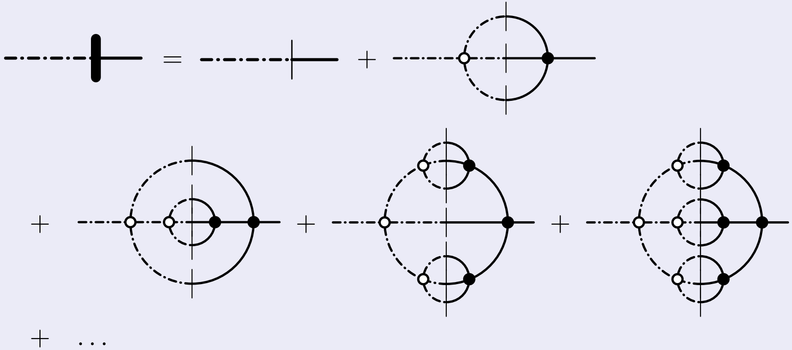
$$\Delta^-(-i\tau, \mathbf{k}) = \frac{G_{th}^{\mathcal{I}\mathcal{I}}(-i\tau, 0, \mathbf{k})}{2G_{th}(0, 0, \mathbf{k})} - \partial_\tau G_{th}^{\mathcal{I}\mathcal{I}}(-i\tau, 0, \mathbf{k})$$

Example: 2PI three-loop truncation

MG, Müller (2009)



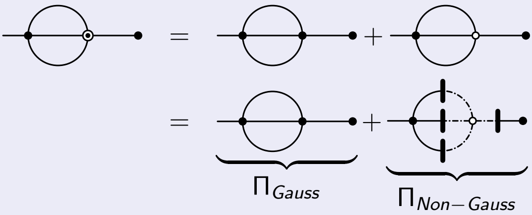
Iterative Solution:



Thermal Initial Correlations: 2PI

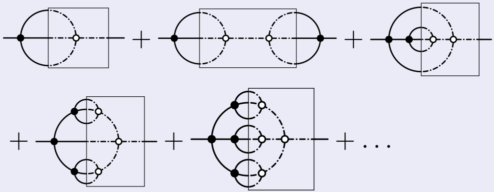
Example: 2PI three-loop truncation

MG, Müller (2009)



Non-Gaussian self-energy contains $\alpha_n^{th}(x_1, \ldots, x_n)$ for all $n \geq 4$

$\Pi_{non-Gauss}(x, z) \quad =$

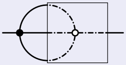


Kadanoff-Baym equations with thermal initial correlations contain

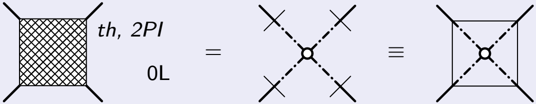
$$\alpha_n^{th}(x_1, \ldots, x_n) \qquad \text{for all } n \geq 4$$

The leading non-Gaussian correction is the 4-point correlation

$$\Pi^{leading}_{non-Gauss}(x, z) \quad =$$



$$\alpha^{th,2PI}_{4,0L}(z_1, z_2, z_3, z_4) \quad = \quad -i\lambda \int_{\mathcal{I}} d^4v \; \Delta(v, z_1)\Delta(v, z_2)\Delta(v, z_3)\Delta(v, z_4)$$

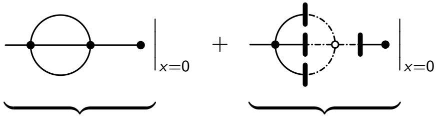


Thermal Initial Correlations: 2PI

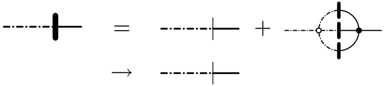
MG, Müller (2009)

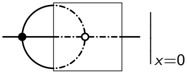
Correlation energy at initial time is non-zero !

$$E^{eq}_{corr}(t=0) = \frac{i}{4}\int_{\mathcal{C}} d^4z\,[\Pi_{Gauss}(x,z)+\Pi_{non-Gauss}(x,z)]\,G(z,x)\Big|_{x=0}$$

$=$ 

$$\int_0^{x^0} dz^0 \;\to\; 0$$



$=$ 

$$= \frac{\lambda}{4!}\int_{\mathcal{C}} d^4x_{1234}\; G_{th}(x,x_1)G_{th}(x,x_2)G_{th}(x,x_3)\, i\alpha_4^{th}(x_1,x_2,x_3,x_4)\,G_{th}(x_4,x)\Big|_{x=0}$$

$$= -\frac{\lambda^2}{4!}\int_{kpq} G_{th}(p)G_{th}(q)G_{th}(k-p-q)G_{th}(-k) \;=\; E^{eq}_{4-p.\,corr}(t=0)$$

...and given by the thermal 4-point correlation

Leading Non-Gaussian correction

Gaussian IC

$$\begin{aligned} G(x,y)|_{x^0,y^0=0} &= G_{th}(x,y)|_{x^0,y^0=0} \\ \alpha_4(x_1,\ldots,x_4) &= 0 \\ \alpha_n(x_1,\ldots,x_n) &= 0 \quad \text{for } n>4 \end{aligned}$$

Non-Gaussian IC with α_4^{th}

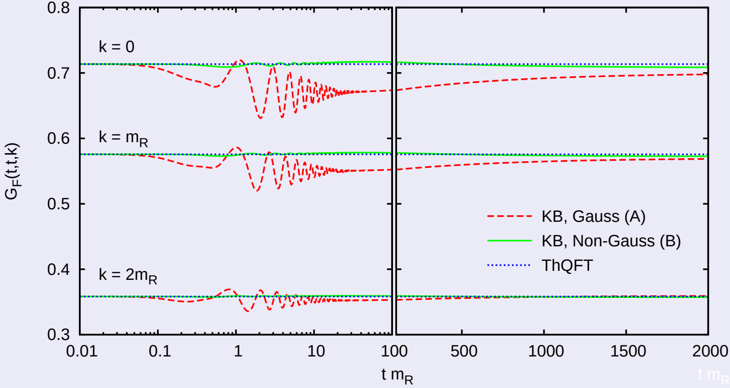
$$\begin{aligned} G(x,y)|_{x^0,y^0=0} &= G_{th}(x,y)|_{x^0,y^0=0} \\ \alpha_4(x_1,\ldots,x_4) &= \alpha_4^{th}(x_1,\ldots,x_4) \\ \alpha_n(x_1,\ldots,x_n) &= 0 \quad \text{for } n>4 \end{aligned}$$

- Truncate thermal initial correlations
- $\Rightarrow$ *nonequilibrium* initial states
- The upper states are ‘as thermal as possible’
- Expectation: Non-Gaussian state more close to equilibrium

Leading Non-Gaussian correction

Minimal offset from thermal equilibrium

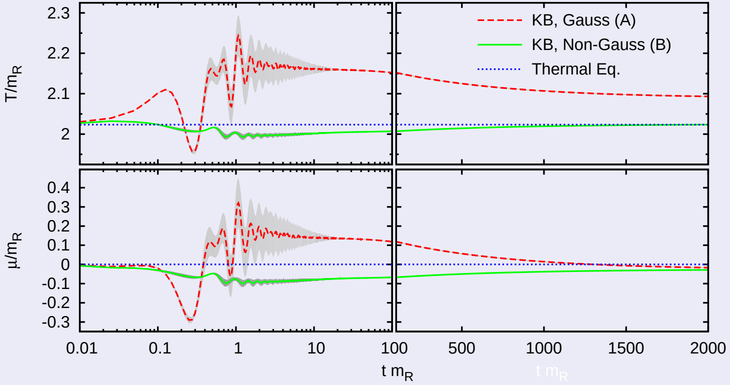
MG, Müller (2009)



Leading Non-Gaussian correction

Offset between initial and final temperature

MG, Müller (2009)



build-up kinetic - chemical equilibration

Leading Non-Gaussian correction

Gaussian IC

$$\begin{aligned} E_{total} &= E^{eq}_{kin}(T_{init}) \\ &= E^{eq}_{kin}(T_{final}) + E^{eq}_{corr}(T_{final}) \end{aligned}$$

$E^{eq}_{corr} \neq 0 \Rightarrow$

$$T_{init} \neq T_{final}$$

Non-Gaussian IC with α_4^{th}

$$\begin{aligned} E_{total} &= E^{eq}_{kin}(T_{init}) + E^{eq}_{4-p.\,corr}(T_{init}) \\ &= E^{eq}_{kin}(T_{final}) + E^{eq}_{corr}(T_{final}) \end{aligned}$$

$E^{eq}_{4-p.\,corr} = E^{eq}_{corr} \Rightarrow$

$$T_{init} = T_{final}$$

Leading Non-Gaussian correction

Gaussian IC

$$E_{total} = E^{eq}_{kin}(T_{init})$$
$$= E^{eq}_{kin}(T_{final}) + E^{eq}_{corr}(T_{final})$$

$E^{eq}_{corr} \sim \Lambda^4 + T^2\Lambda^2 + \dots \Rightarrow$

$$T_{init} \neq T_{final}$$

Cutoff-dependence

$$|T_{init} - T_{final}| \sim \Lambda^2$$

Non-Gaussian IC with α_4^{th}

$$E_{total} = E^{eq}_{kin}(T_{init}) + E^{eq}_{4-p.\,corr}(T_{init})$$
$$= E^{eq}_{kin}(T_{final}) + E^{eq}_{corr}(T_{final})$$

$E^{eq}_{4-p.\,corr} = E^{eq}_{corr} \Rightarrow$

$$T_{init} = T_{final}$$

No Cutoff-dependence

$$E_{total} = E^{eq}(T_{init}) = E^{eq}(T_{final}) = \text{finite}$$

Conclusion

Quantum nonequilibrium dynamics

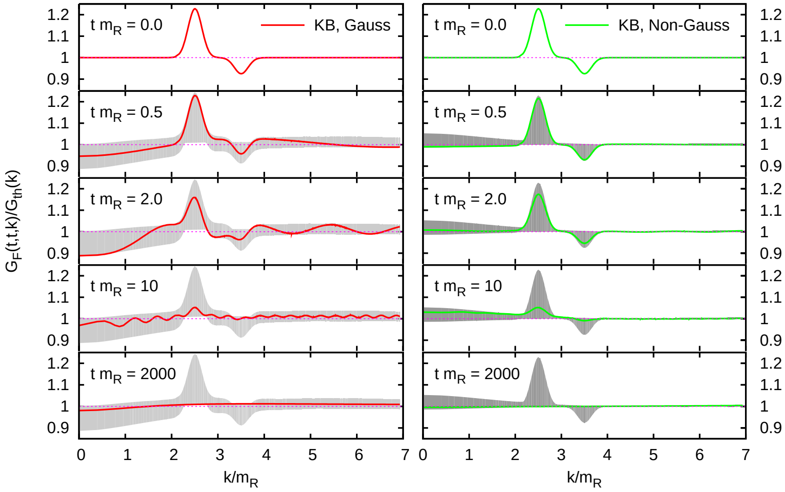
- Kadanoff-Baym equations
- Incorporate off-shell and memory effects
- Challenge: find renormalizable initial state

Renormalization techniques

MG, Müller arXiv:0904.3600

- Kadanoff-Baym equations for Non-Gaussian initial states
- Thermal/Vacuum initial correlations
- Leading Non-Gaussian correction removes singularity

Outlook: Renormalized Nonequilibrium Dynamics



Renormalization techniques for Kadanoff-Baym equations

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für Kernphysik
Heidelberg

MG, Markus Michael Müller, arXiv:0904.3600