Phenomenology of Higgs bosons at one loop in the triplet model

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M.A., S. Kanemura, M. Kikuchi, K. Yagyu, PLB714 (2012) 279
M.A., S. Kanemura, M. Kikuchi, K. Yagyu, To appear in PRD (arXiv:1204.1951 [hep-ph])

I. *Introduction*

The SM-like Higgs boson was discovered at the LHC with a mass of around 126 GeV.

The SM Higgs sector is very simple, but ...

Extended Higgs sector

SM Higgs boson (iso-doublet) + iso-singlets / iso-doublets / higher isospin multiplet ?

Additional role to the Higgs sector :

Beyond the SM : neutrino masses, dark matter, baryon asymmetry, ...

In constructing the extended Higgs sector, the following two requirements from the experimental data should be taken into account.

ρ is very close to unity
FCNC is suppressed

$\rho_{\rm exp} = 1.0008^{+0.0017}_{-0.0007}$

ρ parameter **(** ***at the tree level*****):**

$$\rho_{tree} \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = \frac{\sum_k [4T_k(T_k+1) - Y_k^2] v_k^2 c_k}{\sum_k 2Y_k^2 v_k^2}$$

$c_k = 1$ (1/2) for a complex (real) representation

I. *Introduction*

- **The custodial symmetry ensures ρ=1 at the tree level.**

$G = SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$ **global SU(2) symmetry**

Extended Higgs sector

- **SM (one-Higgs doublet model)**
- **one-Higgs doublet + singlets**
- **multi Higgs doublet**

$\Rightarrow \rho^{tree} = 1$

My talk

- **one-Higgs doublet + Y=0 SU(2)$_L$ triplet**
- **one-Higgs doublet + Y=2 SU(2)$_L$ triplet**

$\Rightarrow \rho^{tree} \neq 1$

small VEV
$v_\xi \lesssim 12$ GeV
$v_\Delta \lesssim 8$ GeV

- **Georgi-Machacek model** *H.Georgi and M.Machacek NPB262 (1985)*

one Higgs doublet(Φ)+ Y=2 Higgs triplet (Δ) + Y=0 Higgs triplet (ξ)

$\Rightarrow \rho^{tree} = 1$

Impose the custodial symmetry in the Higgs potential

The Higgs sector of the GM model can be described by the form of SU(2)$_L$×SU(2)$_R$ multiplets;

$$\Phi = \begin{pmatrix} \phi^{0*} & \phi^+ \\ \phi^- & \phi^0 \end{pmatrix}, \quad \chi = \begin{pmatrix} \Delta^{0*} & \xi^+ & \Delta^{++} \\ \Delta^- & \xi^0 & \Delta^+ \\ \Delta^{--} & \xi^- & \Delta^0 \end{pmatrix} \longrightarrow \; <\Phi>= \begin{pmatrix} v_\phi/\sqrt{2} & 0 \\ 0 & v_\phi/\sqrt{2} \end{pmatrix}, \quad <\chi>= \begin{pmatrix} v_\chi & 0 & 0 \\ 0 & v_\chi & 0 \\ 0 & 0 & v_\chi \end{pmatrix}$$

$$\rho^{tree} = \frac{2v_\Delta^2 + 4v_\xi^2 + v_\phi^2}{4v_\Delta^2 + v_\phi^2} \qquad v_\xi = v_\chi, \quad v_\Delta = \sqrt{2} v_\chi \quad \Rightarrow \rho^{tree} = 1$$

v_χ can be taken of order 100 GeV.

I. *Introduction*

Higgs triplet model (HTM)

SM with Y=2 Higgs triplet field (Δ)

$$\Delta = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^0 & -\frac{\Delta^+}{\sqrt{2}} \end{pmatrix}$$

Important predictions

★ **The tree-level ρ parameter deviates from unity.**

$$\rho_{tree} = \frac{1+\frac{2v_\Delta^2}{v_\phi^2}}{1+\frac{4v_\Delta^2}{v_\phi^2}} \simeq 1 - \frac{2v_\delta^2}{v_\phi^2}$$

$$\rho_{\rm exp} \simeq 1.0008 \quad \rightarrow \quad v_\Delta \lesssim 8\text{GeV}$$

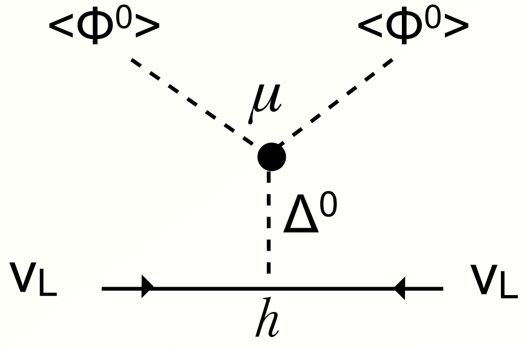
★ **Extra Higgs bosons**

Doubly-charged $H^{\pm\pm}$, Singly-charged $H^{\pm}$,
CP-odd neutral A, CP-even neutral H .

★ **Neutrino masses via Type II seesaw mechanism**

$$\mathcal{L}_Y = h_{ij}\overline{L_L^{ic}} i\tau_2 \Delta L_L^j + \text{h.c.}$$

$$M_\nu = \sqrt{2} h v_\Delta$$



I. *Introduction*

H⁺⁺ ⟷ **Neutrino**

e.g.)

- **$H^{++} \to l^+ l^+$**

LHC $\quad pp \to H^{++}H^{--} \to \ell_i^+ \ell_j^+ \ell_k^- \ell_l^-$

$\quad pp \to H^{++}H^{-} \to \ell_i^+ \ell_j^+ \ell_k^- \nu$

ILC $\quad e^+e^- \to H^{++}H^{--} \to \ell_i^+ \ell_j^+ \ell_k^- \ell_l^-$

$\quad e^-e^- \to H^{--} \to \ell_i^- \ell_j^-$

- **LFV**

  τ→lll, μ→eee at the tree level

- **ν0ββ**

- **inverse ν0ββ** $\quad e^-e^- \to H^{--} \to W^-W^-$

M.muhlleitner, M.Spira, PRD68 (2003)
A.G.Akeroyd, M.A., PRD72 (2005)
T. Han et al, PRD76 (2007)
M.Kadastik, M.Raidal, L.Rebane, PRD77(2008)
J. Garayoa, T. Schwetz, JHEP 0803 (2008)
A.G.Akeroyd, M.A., H. Sugiyama, PRD77 (2008)
M. Kadastik, M. Raidal, L.Rebane, PRD77 (2008)
P. Fileviez Perez et al, PRD78 (2008)
F. del Aguila, J.A.Aguilar-Saavedra, NPB813 (2009)
A.G. Akeroyd, C.W. Chiang, PRD80 (2009)
W. Rodejohann, H. Zhang, PRD83 (2011)

E.J.Chun,K.Y.Lee,C.S.Park,PLB566 (2003)
M.Kakizaki,Y.Ogura,F.Shima,PLB566 (2003)
A.G.Akeroyd, M.A., H.Sugiyama, PRD79 (2009)

S.T. Petcov, H. Sugiyama, Y. Takanishi, PRD80 (2009)
W. Rodejohann, PRD81 (2010)
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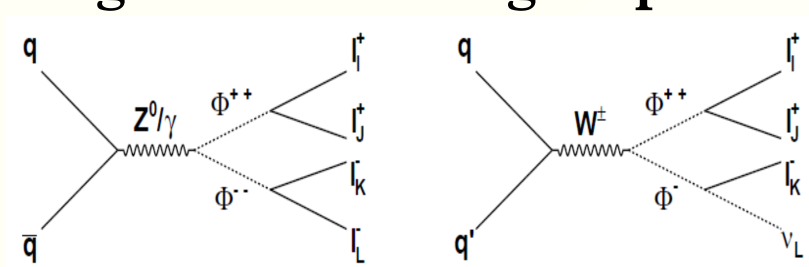
Limit on the mass of H++ @ LHC

Assuming 100% same-sign leptonic decay of the H++



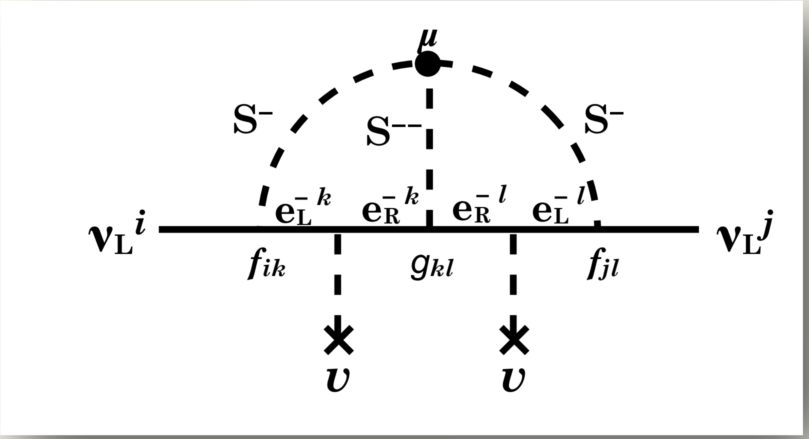
$$m_{H^{++}} \gtrsim 400 \text{ GeV}$$

I. *Introduction*

Models with doubly-charged scalars and neutrino masses

Isospin singlet fields with Y=4 : S++

Zee-Babu model *Zee, NPB264(1986)* *Babu, PLB203 (1988)*



SM + *singlet scalars* (S^-, S^{--})

#L=2

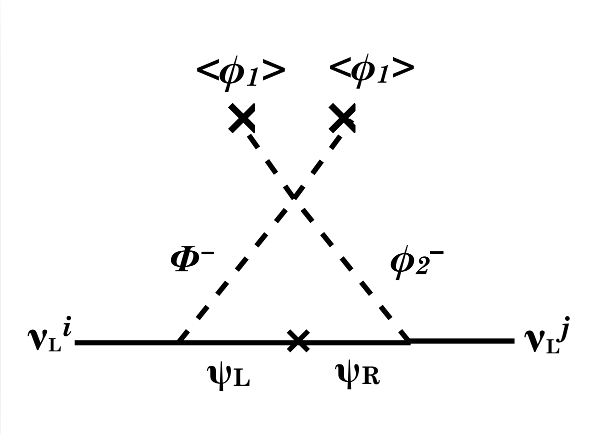
Isospin doublet fields with Y=3 : $\Phi_{Y=3}$

$$\Phi_{Y=3} = \begin{pmatrix} \Phi^{++} \\ \Phi^{+} \end{pmatrix}$$

~~$\overline{L_L^c} \cdot \Phi_{Y=3} \ell_R + h.c.$~~ $\Phi_{Y=3}$ ✗→ LL, VV

$\Phi_{Y=3}$ cannot decay into the SM particles.

	L	exact Z_2
ϕ_1	0	+
ϕ_2	0	−
$\Phi_{Y=3}$	-2	−
ψ^α	1	−



2HDM + $\Phi_{Y=3}$ *M.A., S.Kanemura, K.Yagyu, PLB702 (2011)*

$\Phi^{++} \to H^+W^+$

ψ: singlet Dirac fermion with Y=−1

I. *Introduction*

Evidence of the HTM

- **Relationship among the triplet-like Higgs masses**

$$m^2_{H++} - m^2_{H+} \simeq m^2_{H+} - m^2_{A/H}$$

- **Indirect signatures**

Deviations from the SM in the Higgs couplings (*hγγ, hZZ, hWW, hhh, hff*)

h→γγ: The current experimental value of the signal strength for the diphoton mode is 1.5-2 at the LHC

Accuracy of the measured deviations in the Higgs couplings

LHC-14TeV Lum= 300 fb^{-1} *hWW, hZZ, hγγ* → 10%, *htt, hbb*→ 20%, *hττ* → 10%

ILC-1TeV

Lum=500 fb-1 hWW, hZZ → less than 1%, *hγγ* → 5%, hbb, hττ → 2-3%, htt → 5-10%

M.E.Peskin, arXiv:1207.2516 [hep-ph]

Lum=2 ab-1 hhh is expected to be measured with about 20%.

K.Fujii, talk at the LCWS2012

Renormalization of the HTM

- **Radiative correction to the mass relationship.**
- **The deviations from the SM in *h→γγ* decay rate, *hZZ, hWW and hhh* couplings.**

Contents

II. *Higgs Triplet Model*

Relevant terms in the Lagrangian

$$\mathcal{L}_{\rm HTM} = \mathcal{L}_{\rm kin} + \mathcal{L}_Y - V(\Phi,\Delta)$$

The isospin doublet field Φ with Y=1 and the triplet field Δ with Y=2.

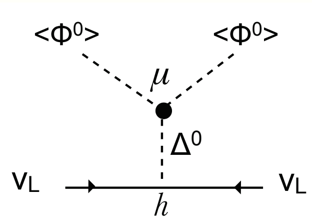
$$\Phi = \left[\begin{array}{c} \phi^+ \\ \frac{1}{\sqrt{2}}(\phi + v_\phi + i\chi) \end{array}\right], \quad \Delta = \left[\begin{array}{cc} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^0 & -\frac{\Delta^+}{\sqrt{2}} \end{array}\right] \text{ with } \Delta^0 = \frac{1}{\sqrt{2}}(\delta + v_\Delta + i\eta)$$

$$v^2 \equiv v_\phi^2 + 2v_\Delta^2 \simeq (246\ \text{GeV})^2 \qquad m_W^2 = \frac{g^2}{4}(v_\phi^2 + 2v_\Delta^2), \quad m_Z^2 = \frac{g^2}{4\cos^2\theta_W}(v_\phi^2 + 4v_\Delta^2)$$

$$V = m^2\Phi^\dagger\Phi + M^2\text{Tr}(\Delta^\dagger\Delta) + \left[\mu\Phi^T i\tau_2\Delta^\dagger\Phi + \text{h.c.}\right]$$
$$+ \lambda_1(\Phi^\dagger\Phi)^2 + \lambda_2\left[\text{Tr}(\Delta^\dagger\Delta)\right]^2 + \lambda_3\text{Tr}(\Delta^\dagger\Delta)^2 + \lambda_4(\Phi^\dagger\Phi)\text{Tr}(\Delta^\dagger\Delta) + \lambda_5\Phi^\dagger\Delta\Delta^\dagger\Phi$$

Neutrino masses:

$$(M_\nu)_{ij} = \sqrt{2}h_{ij}v_\Delta = h_{ij}\frac{\mu v_\phi^2}{M_\Delta^2} \qquad M_\Delta^2 \equiv \frac{v_\phi^2\mu}{\sqrt{2}v_\Delta}$$



The potential respects additional global symmetry.

- **μ term is absent**

→ **The potential respects the global U(1) symmetry which conserves the lepton number.**

Two couplings (λ_4, λ_5) determine the 4 masses.

$m_{H^{++}},\ m_{H^+},\ m_A,\ m_H$ **2 masses are independent.**

II. *Higgs Triplet Model*

The mass matrices for the scalar bosons can be diagonalized by rotating the scalar fields as following.

Doubly-charged: *Singly-charged:*

$$H^{\pm\pm} = \Delta^{\pm\pm}, \qquad \begin{pmatrix} \phi^{\pm} \\ \Delta^{\pm} \end{pmatrix} = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} G^{\pm} \\ H^{\pm} \end{pmatrix},$$

CP-odd: *CP-even:*

$$\begin{pmatrix} \chi \\ \eta \end{pmatrix} = \begin{pmatrix} \cos\beta' & -\sin\beta' \\ \sin\beta' & \cos\beta' \end{pmatrix} \begin{pmatrix} G^{0} \\ A \end{pmatrix}, \qquad \begin{pmatrix} \phi \\ \delta \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} h \\ H \end{pmatrix}$$

Mixing angles:

$$\tan\beta = \frac{\sqrt{2}v_\Delta}{v_\phi}, \quad \tan\beta' = \frac{2v_\Delta}{v_\phi}, \quad \tan 2\alpha = \frac{v_\Delta}{v_\phi}\frac{2v_\phi^2(\lambda_4+\lambda_5) - 4M_\Delta^2}{2v_\phi^2\lambda_1 - M_\Delta^2 - 2v_\Delta^2(\lambda_2+\lambda_3)}.$$

- ***β*** **and** ***β'*** **are different.**
- $v_\Delta^2/v_\phi^2 \lesssim 0.001$ $\rightarrow$ ***β, β'*** **and** ***α*** **are near zero.**

Physical states:

Triplet-like Higgs bosons: $H^{\pm\pm}$, $H^{\pm}$, A, H

SM-like Higgs boson: h

II. *Higgs Triplet Model*

Mass formulae:

$$
\begin{aligned}
m^2_{H^{++}} &= M^2_\Delta - v^2_\Delta \lambda_3 - \frac{\lambda_5}{2} v^2_\phi && \simeq M^2_\Delta - \frac{\lambda_5}{2} v^2_\phi \\
m^2_{H^{+}} &= \left(M^2_\Delta - \frac{\lambda_5}{4} v^2_\phi\right)\left(1 + \frac{2v^2_\Delta}{v^2_\phi}\right) && \simeq M^2_\Delta - \frac{\lambda_5}{4} v^2_\phi \\
m^2_A &= M^2_\Delta \left(1 + \frac{4v^2_\Delta}{v^2_\phi}\right) && \simeq M^2_\Delta \\
m^2_H &= \mathcal{M}^2_{11} \sin^2\alpha + \mathcal{M}^2_{22} \cos^2\alpha + \mathcal{M}^2_{12} \sin 2\alpha && \simeq M^2_\Delta \\
m^2_h &= \mathcal{M}^2_{11} \cos^2\alpha + \mathcal{M}^2_{22} \sin^2\alpha - \mathcal{M}^2_{12} \sin 2\alpha && \simeq 2\lambda_1 v^2_\phi
\end{aligned}
$$

$$M^2_\Delta \equiv \frac{v^2_\phi \mu}{\sqrt{2} v_\Delta}$$

$$
\begin{aligned}
\mathcal{M}^2_{11} &= 2v^2_\phi \lambda_1, \\
\mathcal{M}^2_{22} &= M^2_\Delta + 2v^2_\Delta(\lambda_2 + \lambda_3), \\
\mathcal{M}^2_{12} &= -\frac{2v_\Delta}{v_\phi} M^2_\Delta + v_\phi v_\Delta (\lambda_4 + \lambda_5).
\end{aligned}
$$

Neglecting the term with v^2_Δ / v^2_ϕ

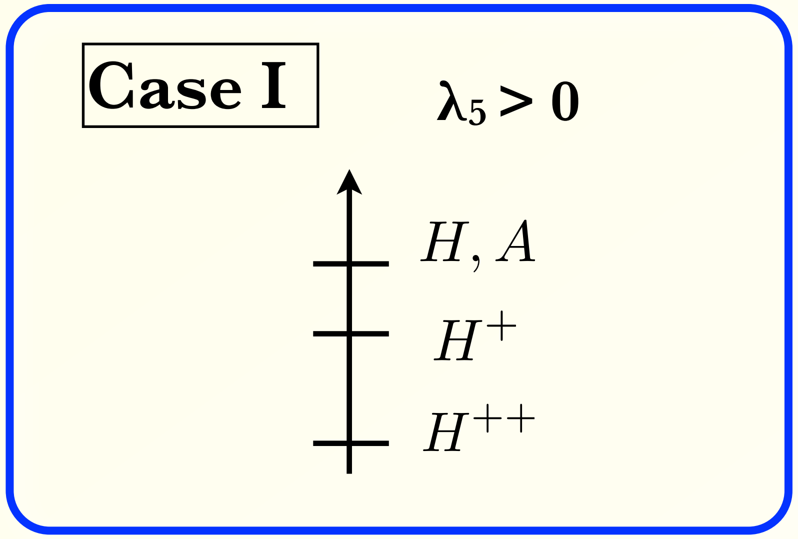
Relationships among the masses of the triplet-like Higgs bosons:

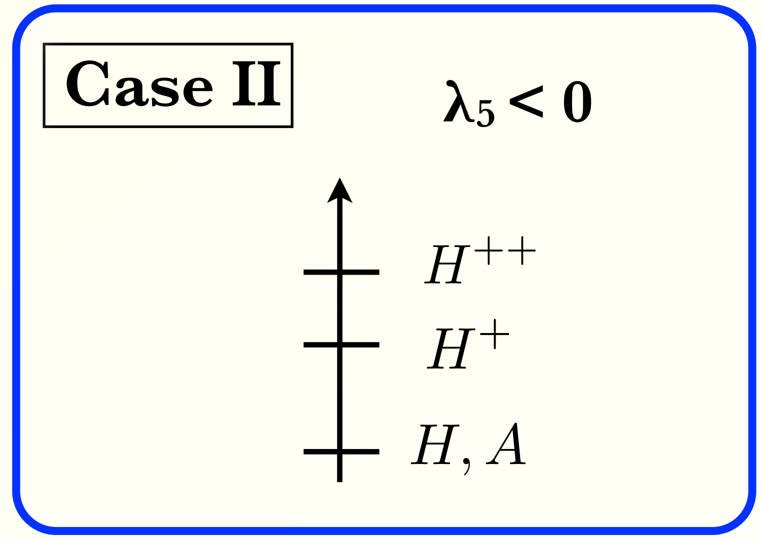
$$m^2_{H^{++}} - m^2_{H^{+}} = m^2_{H^{+}} - m^2_A \left(= -\frac{\lambda_5}{4} v^2_\phi\right),$$

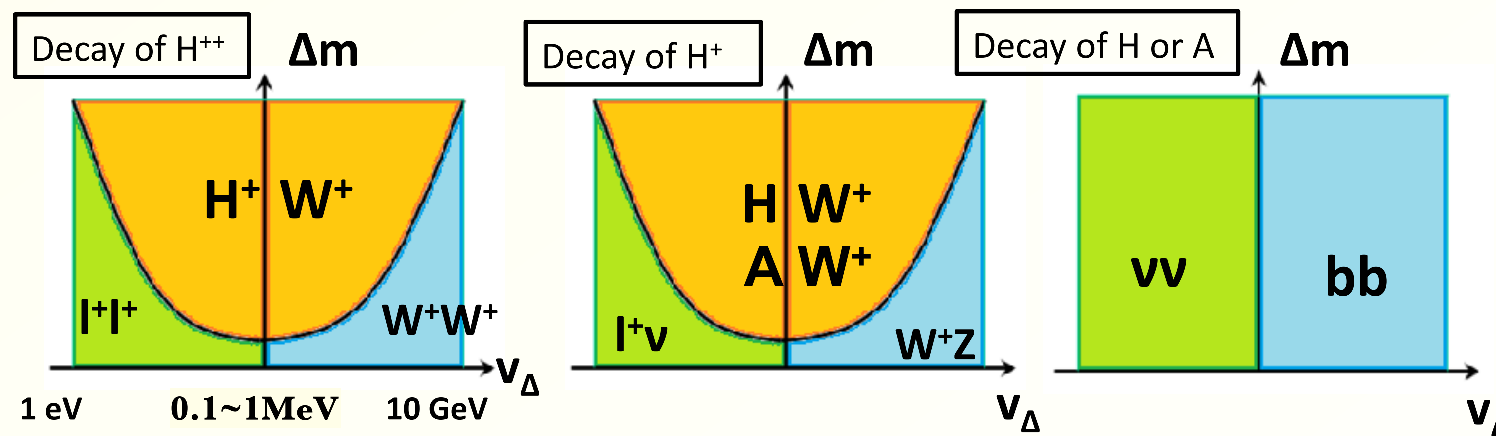
$$m^2_H = m^2_A \left(= M^2_\Delta\right).$$

In the limit of vΔ/vϕ→0, the mass parameters of the triplet-like Higgs bosons are determined by two parameters. This can be regarded as the consequence of the global U(1) symmetry in the Higgs potential.

II. *Higgs Triplet Model*







e.g.) $H(A) \to H^+W^{-(*)} \to H^{++}W^{-(*)}W^{-(*)}$

$H^+ \to H^{++}W^{-(*)}$

$H^{++} \to \ell^+\ell^+ \text{or} W^+W^+$

$H^{++} \to H^+W^{+(*)} \to H(A)W^{+(*)}W^{+(*)}$

$H^+ \to H(A)W^{-(*)}$

$H(A) \to \nu\nu \text{ or } bb$

A.G.Akeroyd, H.Sugiyama, PRD84(2011)

Cascade decays of the triplet-like scalar bosons become important.

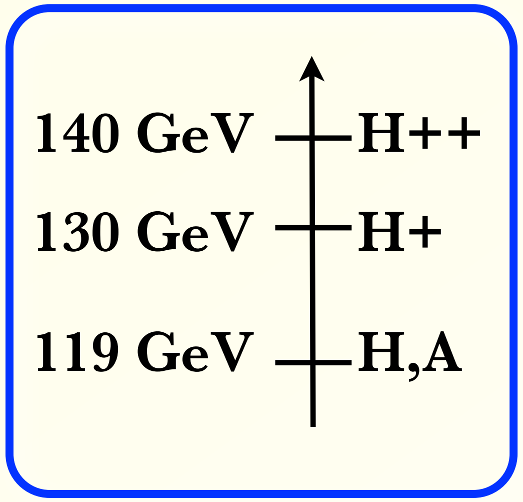
II. *Higgs Triplet Model*

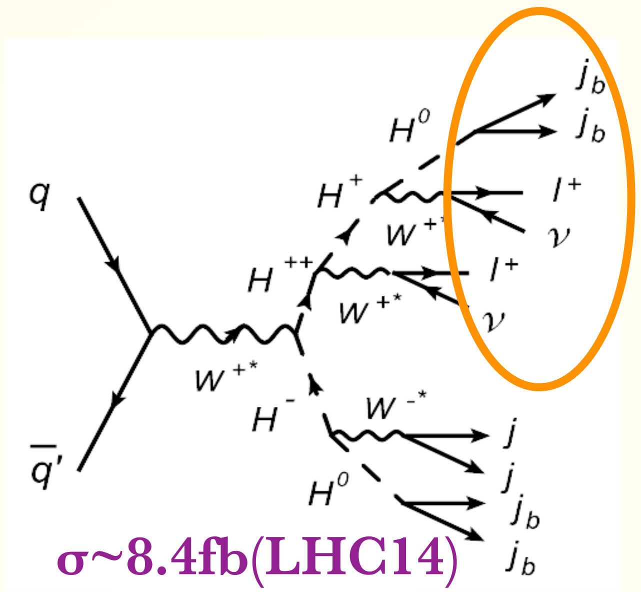
Mass reconstruction at LHC

M.A., S.Kanemura, K.Yagyu, PRD85(2012)

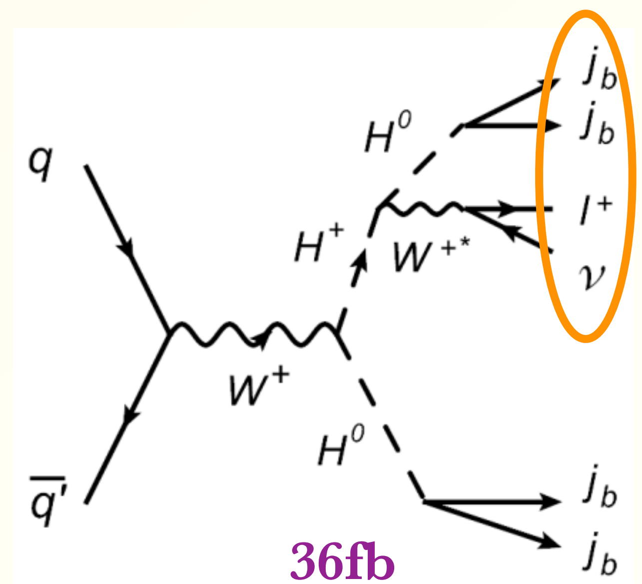
Case II

e.g.) $H^{++} \to H^{+}W^{+(*)} \to HW^{+(*)}W^{+(*)} \to b\bar{b}\ell^{+}\nu\ell^{+}\nu$

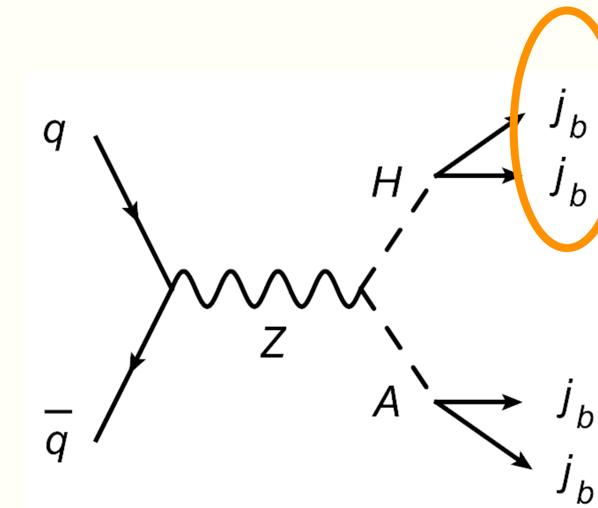




σ~8.4fb(LHC14)



36fb

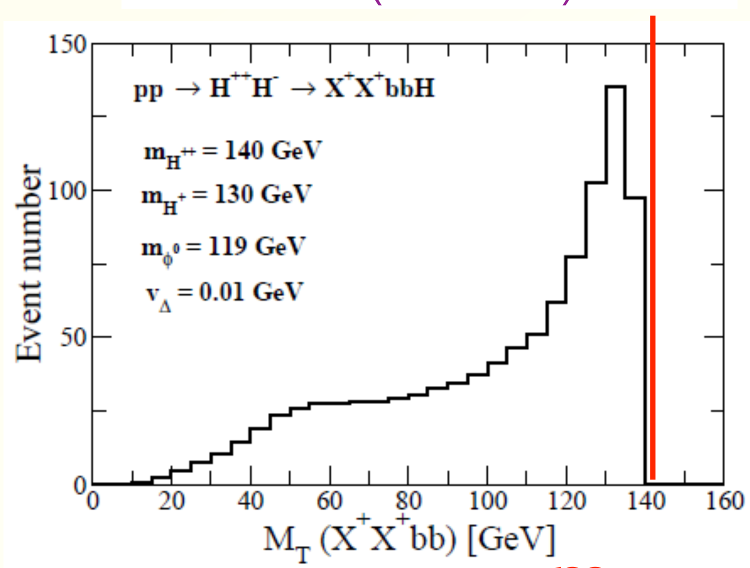


270fb

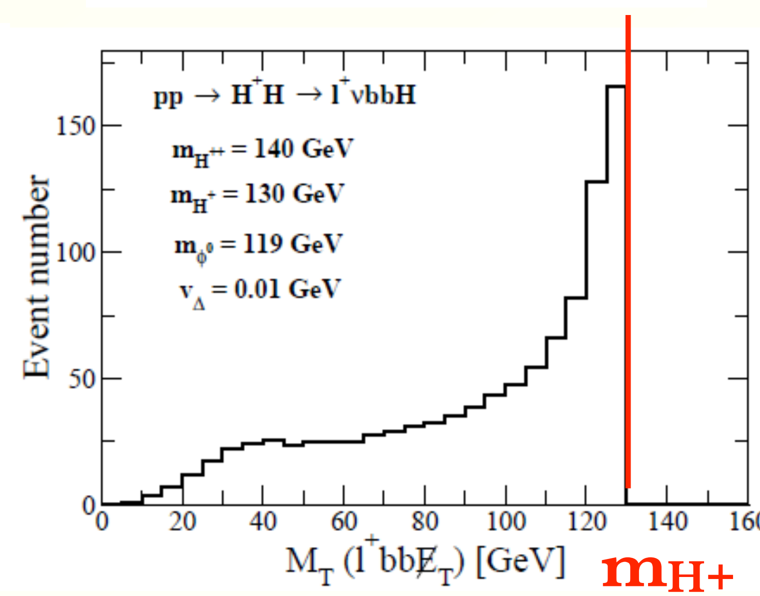
mh=114 GeV

v_Δ=10 MeV

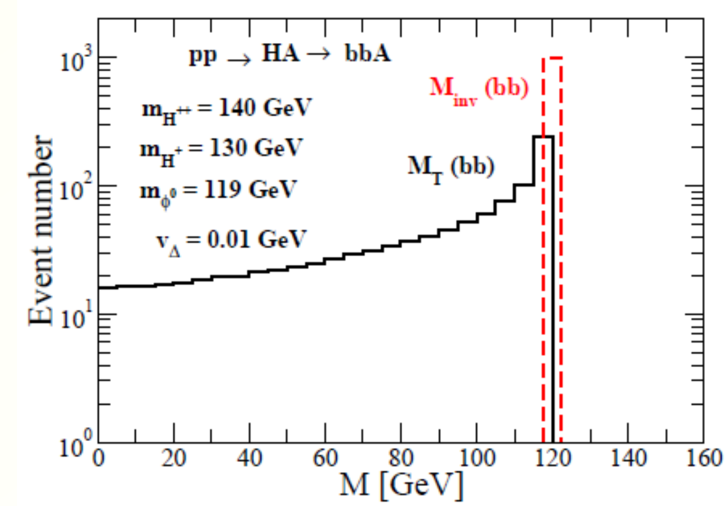
The total event number is assumed to be 1000.



m_{H++}



m_{H+}



m_H, m_A

Masses are determined by the transverse mass distributions.

$$M_T^2 = (\not{E}_T + p_T)^2 \simeq 2|\not{E}_T||p_T|(1-\cos\varphi)$$

II. *Theoretical bounds*

Vacuum stability bound

Arhrib et al., PRD84 (2011)

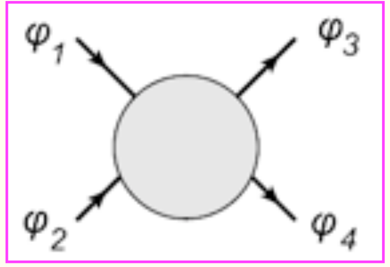
$V^{(4)}>0$

The Higgs potential is bounded from below in any direction of the large scalar fields region.

$$\lambda_1 > 0, \quad \lambda_2 + \mathrm{MIN}\left[\lambda_3,\ \frac{1}{2}\lambda_3\right] > 0, \quad \lambda_4 + \mathrm{MIN}[0,\ \lambda_5] + 2\mathrm{MIN}[\sqrt{\lambda_1(\lambda_2+\lambda_3)}, \sqrt{\lambda_1(\lambda_2+\lambda_3/2)}] > 0$$

Perturbative unitarity bound

Lee, Quigg, Thacker PRD16(1977)



The matrix of the S-wave amplitude for the elastic scatterings of two scalar boson states are satisfied $|\langle\varphi_3\varphi_4|a^0|\varphi_1\varphi_2\rangle| < 1$ or $1/2$.

φ_i ; the NG bosons and the physical Higgs bosons

- **There are 35 possible scattering processes in the HTM.** (**15 neutral, 10 singly-charged, 7 doubly-charged, 2 triply-charged, 1 quadruply-charged**)
- **12 eigenvalues can be regarded as independent eigenvalues.**

MA, Kanemura, PRD77(2008)
Arhrib et al., PRD84 (2011)

$$y_1 = 2\lambda_1, \quad y_2 = 2(\lambda_2+\lambda_3), \quad y_3 = 2\lambda_2,$$

$$y_4^{\pm} = \lambda_1 + \lambda_2 + 2\lambda_3 \pm \sqrt{\lambda_1^2 - 2\lambda_1(\lambda_2+2\lambda_3) + \lambda_2^2 + 4\lambda_2\lambda_3 + 4\lambda_3^2 + \lambda_5^2},$$

$$y_5^{\pm} = 3\lambda_1 + 4\lambda_2 + 3\lambda_3 \pm \sqrt{9\lambda_1^2 - 6\lambda_1(4\lambda_2+3\lambda_3) + 16\lambda_2^2 + 24\lambda_2\lambda_3 + 9\lambda_3^2 + 6\lambda_4^2 + 2\lambda_5^2},$$

$$y_6 = \lambda_4, \quad y_7 = \lambda_4 + \lambda_5, \quad y_8 = \frac{1}{2}(2\lambda_4 + 3\lambda_5), \quad y_9 = \frac{1}{2}(2\lambda_4 - \lambda_5), \quad y_{10} = 2\lambda_2 - \lambda_3$$

$$|y_i| < \zeta, \qquad \zeta = 16\pi \text{ or } 8\pi$$

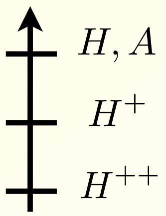
II. *Theoretical bounds*

vacuum stability and unitarity

$$V = m^2\Phi^\dagger\Phi + M^2\mathrm{Tr}(\Delta^\dagger\Delta) + \left[\mu\Phi^T i\tau_2\Delta^\dagger\Phi + \mathrm{h.c.}\right]$$
$$+\lambda_1(\Phi^\dagger\Phi)^2 + \lambda_2\left[\mathrm{Tr}(\Delta^\dagger\Delta)\right]^2 + \lambda_3\mathrm{Tr}(\Delta^\dagger\Delta)^2 + \lambda_4(\Phi^\dagger\Phi)\mathrm{Tr}(\Delta^\dagger\Delta) + \lambda_5\Phi^\dagger\Delta\Delta^\dagger\Phi$$

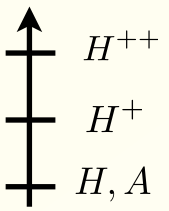
$\lambda_1 = m_h^2/(2v^2) \approx 0.13$

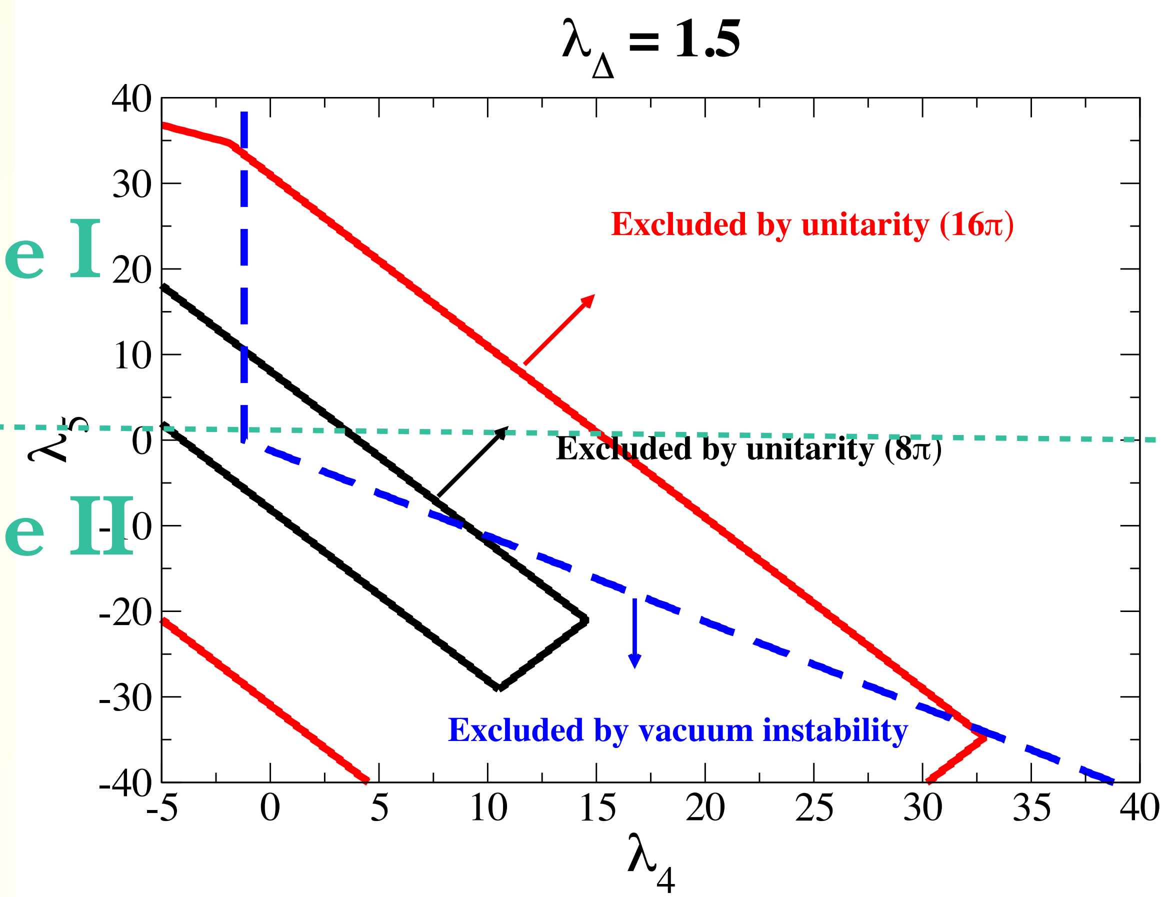
$\lambda_\Delta \equiv \lambda_2 = \lambda_3$

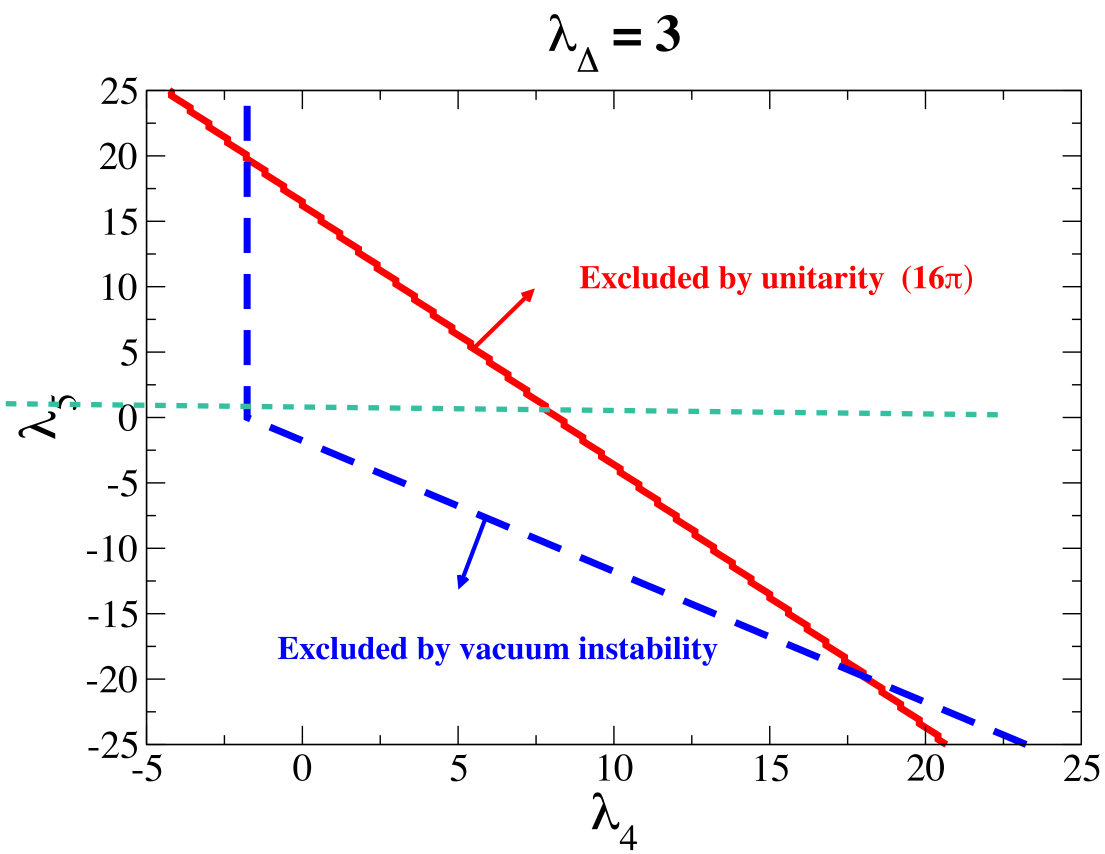


Case I
$\lambda_5 > 0$

Case II
$\lambda_5 < 0$







$\lambda_{4\,\mathrm{min}} \sim -1.3 \rightarrow \lambda_5 =$ 0-33 ($\zeta=16\pi$), 0-10 ($\zeta=8\pi$)

$\lambda_{4\,\mathrm{min}} \sim -1.7 \rightarrow \lambda_5 =$0-20 ($\zeta=16\pi$)

- $\lambda_4 < 0 \rightarrow$ negative values for λ_5 are strongly constrained.

$\rightarrow$ case II is disfavored.

III. Renormalization of the HTM

- *Renormalization of the EW sector*

- *Renormalization of parameters in the Higgs potential*

III. *Renormalization of the EW sector*

The renormalization prescription in models with $\rho_{\rm tree}\neq 1$ is different from that in models with $\rho_{\rm tree}=1$.

- **$\rho_{\rm tree}=1$ model: SM, 2HDM**

  3 input parameters

  $G_F,\ m_Z,\ \alpha_{em},$

  $$m_W^2 s_W^2 = \frac{\pi\alpha_{em}}{\sqrt{2}G_F},\quad s_W^2 = 1-\frac{m_W^2}{m_Z^2}$$

- **$\rho_{\rm tree}\neq 1$ model: HTM**

  4 input parameters are necessary to describe the electroweak parameters, because $m_W/m_Z=c_W$ does not hold.

  Blank, Hollik, NPB514 (1998)
  Chankowski, Pokorski, Wagner (2007)
  Chen, Dawson, Jackson (2008)
  Kanemura, Yagyu, PRD85 (2012)

Renormalized W boson mass

$$m_W^2 = \frac{\pi\alpha_{\rm em}}{\sqrt{2}G_F s_W^2}\frac{1}{1-\Delta r}$$

$\rho_{\rm tree}=1$ model

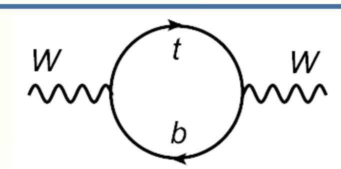
$$\Delta r = \Delta\alpha_{\rm em} - \frac{c_W^2}{s_W^2}\Delta\rho + \Delta r_{\rm rem}$$

The quadratic mass dependence appears in $\Delta\rho$.

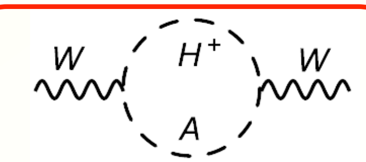
Violation of the custodial symmetry

2HDM

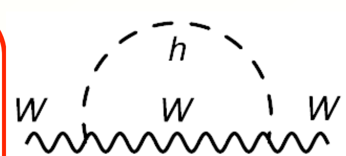
$$\Delta\rho \simeq \frac{1}{16\pi^2}\left[\frac{(m_t-m_b)^2}{m_W^2} + \frac{(m_{H^+}-m_A)^2}{m_W^2} - \ln\frac{m_h^2}{m_W^2}\right]$$



Custodial sym. breaking in the top-bottom sector



Custodial sym. breaking in the scalar sector



III. *Renormalization of the EW sector*

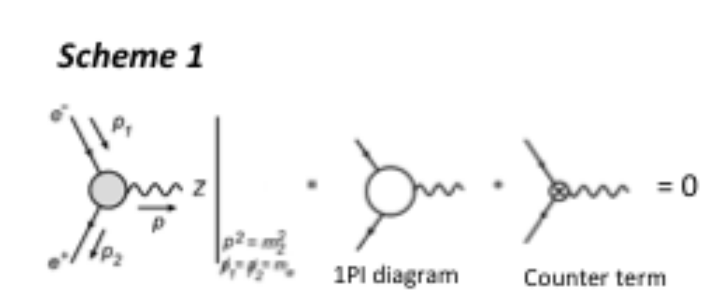
$\rho_{tree} \neq 1$ model

Scheme I G_F, m_Z, α_{em}, $\hat{s}_W^2$

$\hat{s}_W^2$ is defined as the ratio of the coefficients of the vector part and the axial vector part in the $\bar{e}eZ$ vertex.

Additional condition:

The mixing angle is not changed from the tree level prediction.



$$\Delta r^{\text{Scheme I}} = \Delta\alpha + \Delta r^{\text{Scheme I}}_{\text{rem}}$$

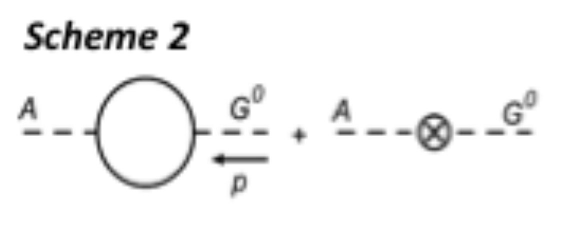
No $\Delta\rho$ → The renormalization of $\hat{s}_W^2$ absorbs the violation of the custodial symmetry.

Scheme II G_F, m_Z, α_{em}, β'

β' is obtained by the condition of the Higgs potential.

Additional condition:

No mixing between A and G^0.



$|_{p^2=0, m_A} = 0$

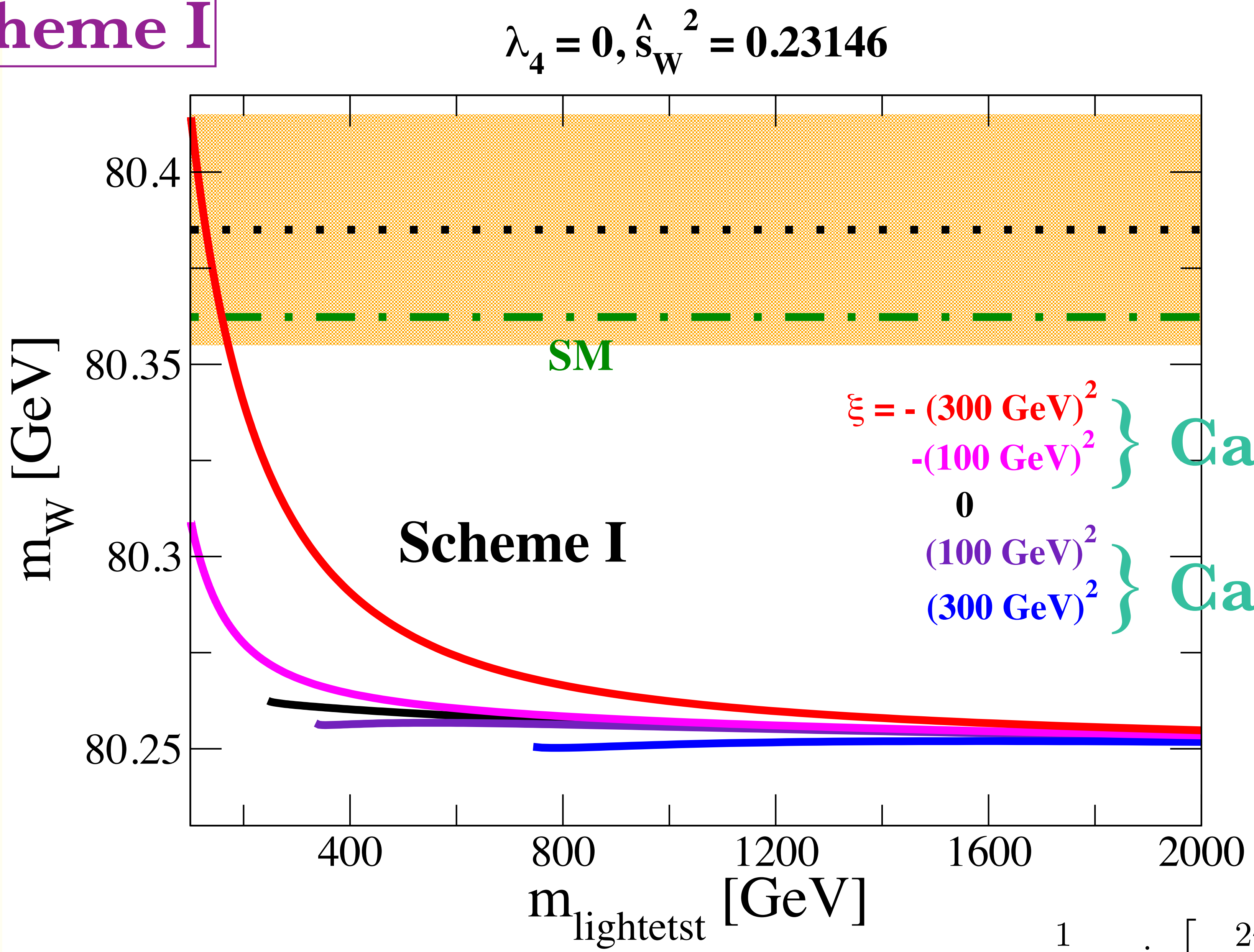
$$\Delta r^{\text{Scheme II}} = \Delta\alpha - \frac{\bar{c}_W^2}{\bar{s}_W^2}\Delta\rho + \Delta r^{\text{Scheme II}}_{\text{rem}}$$

The quadratic mass dependences due to the custodial symmetry breaking appear through $\Delta\rho$.

The mixing angle is not the independent parameter, but it is determined by $\bar{c}_W^2 = \dfrac{2m_W^2}{m_Z^2(1+c_{\beta'}^2)}$.
The C_W^2 in the $\rho_{tree}=1$ model can be reproduced by taking $\beta' \to 0$.

III. *Renormalization of the EW sector*

Scheme I



Case I

Case II

Input parameters

$m_{lightest}$, ξ, λ_4, s_W

$$\xi \equiv m_{H^{++}}^2 - m_{H^+}^2 = m_{H^+}^2 - m_A^2$$

$$\alpha = \frac{1}{2}\arcsin\left[\frac{2v_\Delta v}{m_h^2 - m_A^2}\left(\lambda_4 + \frac{2m_A^2 - 4m_{H^+}^2}{v^2}\right) + \mathcal{O}(v_\Delta^2/v^2)\right]$$

- m_W is asymptotically close to those in the case of ξ=0 in the large $m_{lightest}$ limit. However, the value is not consistent with the SM prediction.

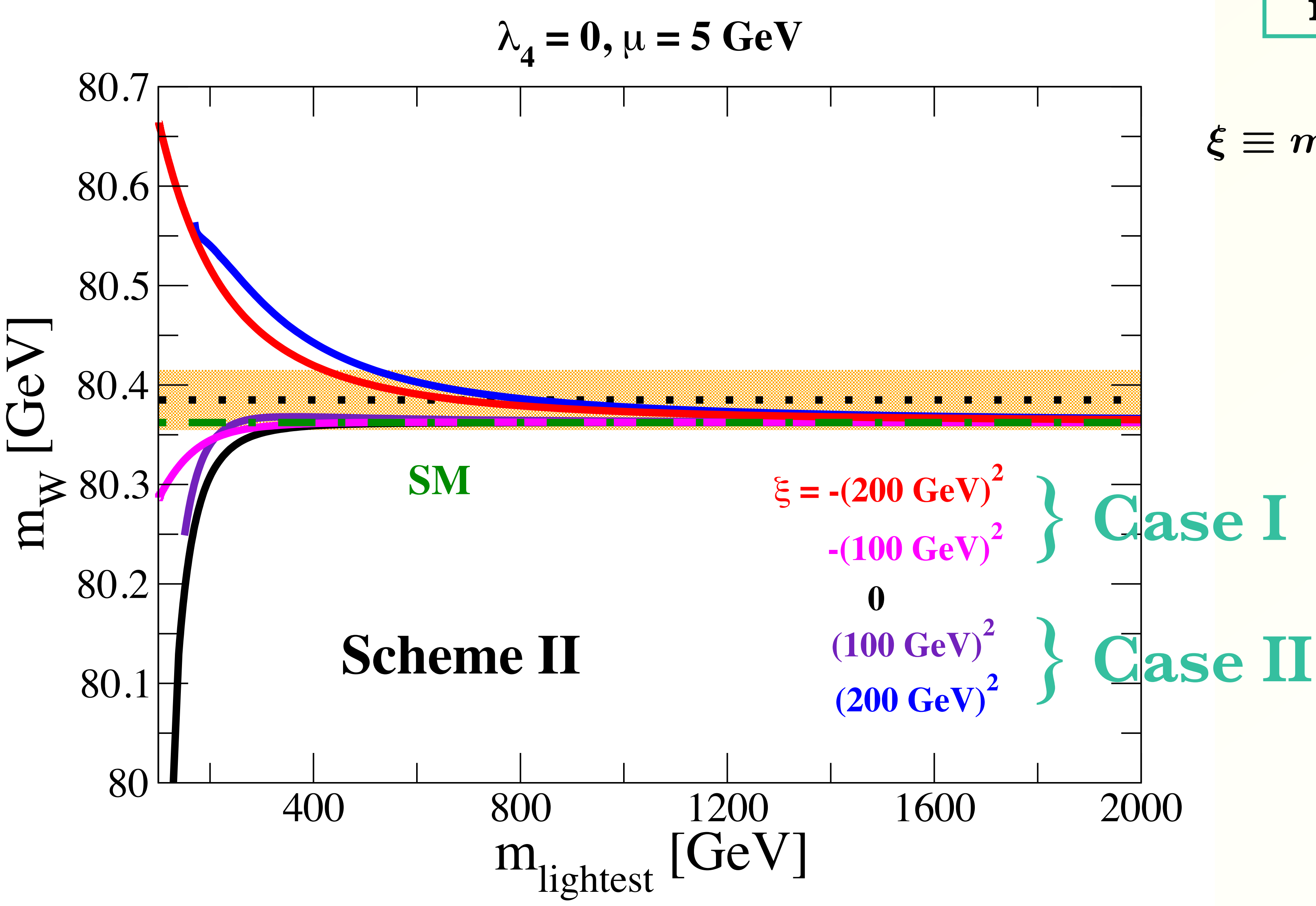
III. *Renormalization of the EW sector*

Scheme II

Input parameters

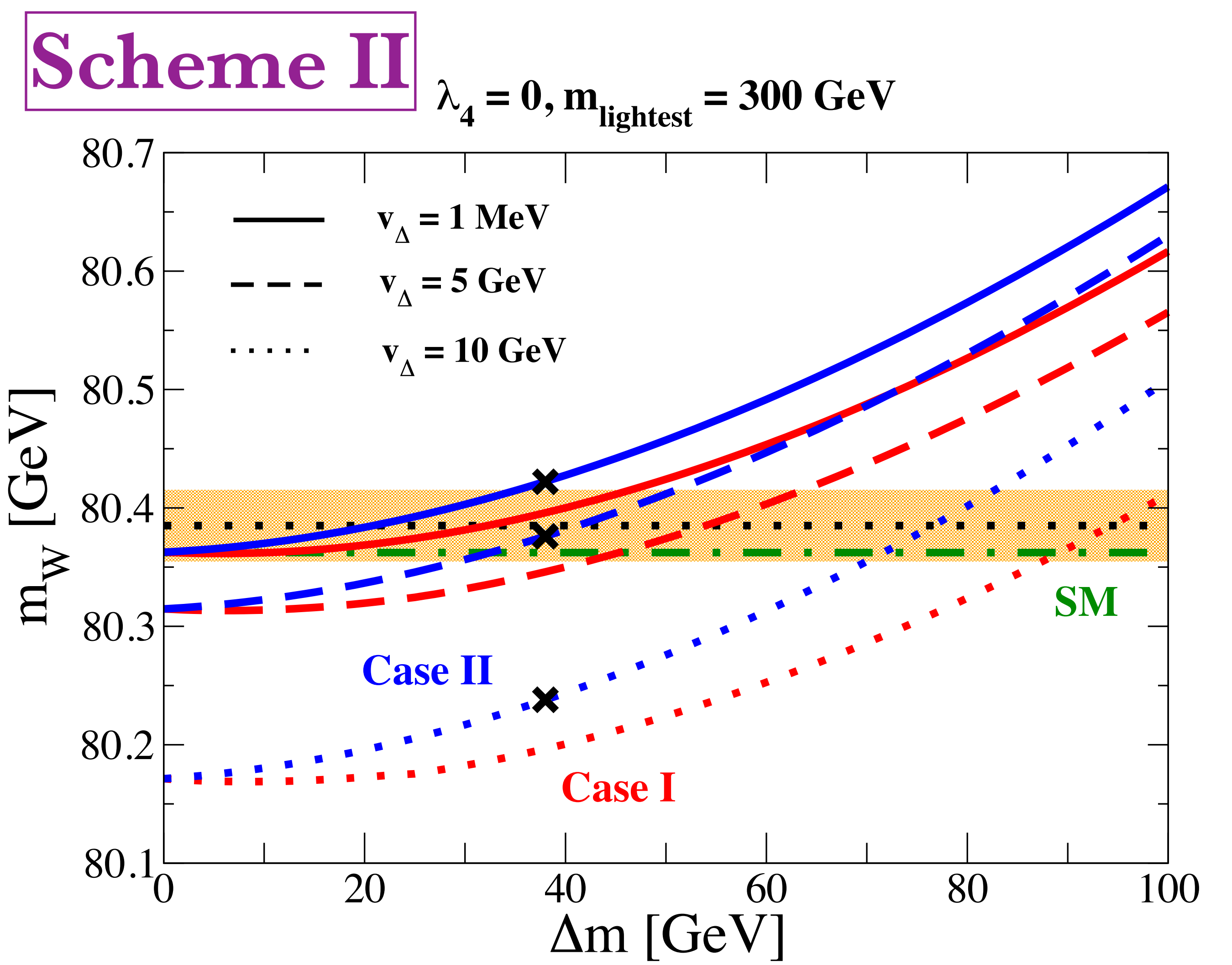
$m_{lightest}$, ξ, λ_4, μ

$$\xi \equiv m^2_{H++} - m^2_{H+} = m^2_{H+} - m^2_A$$



- m_W is asymptotically close to those in the case of ξ=0 in the large $m_{lightest}$ limit. The value is consistent with the SM prediction.

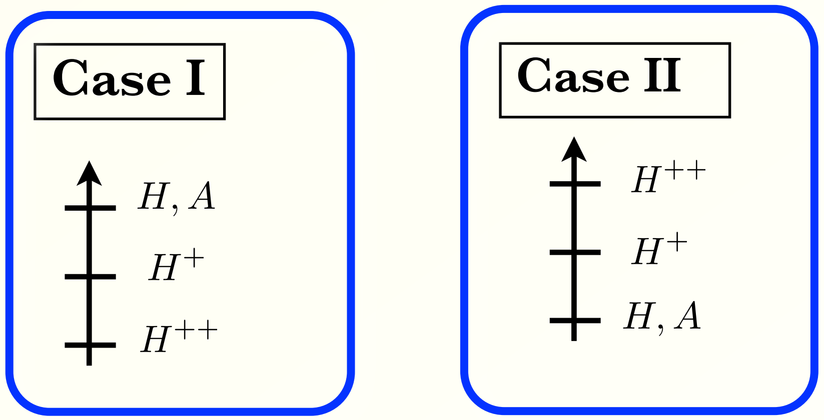
III. *Renormalization of the EW sector*



Input parameters

$m_{lightest}, \Delta m, \lambda_4, v_\Delta$

$$\Delta m \equiv m_{H^+} - m_{\text{lightest}}$$



· The cross marked points show the upper limit of Δm from the theoretical bounds.

Case II : $\Delta m \lesssim 40$ GeV for $\lambda_4=0$

· The dependence of λ4 on mW is quite small. But theoretical constraints depends on λ4.

The Δm is constrained by the constrains by the mW and theoretical bounds.

Case I : $\Delta m \lesssim 50$ GeV, 40-60 GeV, 85-100 GeV

Case II : $\Delta m \lesssim 30$ GeV, 30-50 GeV, 70-85 GeV

1MeV - 1GeV 5GeV 10GeV vΔ

III. *Renormalization of V_{Higgs}*

Parameters in the Higgs potential

$$v,\ \alpha,\ \beta,\ \beta',\ m_h^2,\ m_H^2,\ m_A^2,\ m_{H^+}^2,\ m_{H^{++}}^2.$$

The shift of the parameters :

Reno. of EW parameters → δv

Tadpoles $T_\Phi \to 0 + \delta T_\Phi,\ T_\Delta \to 0 + \delta T_\Delta,$ Vanishing 1-point function

→ $\delta T_\varphi,\ \delta T_\Delta$

VEV, mixing angles $v \to v + \delta v,\ \alpha \to \alpha + \delta\alpha,\ \beta \to \beta + \delta\beta,\ \beta' \to \beta' + \delta\beta'$

Masses $m_\varphi^2 \to m_\varphi^2 + \delta m_\varphi^2, \qquad \varphi = h, H, A, H^+ \text{ and } H^{++}$

The wave functions and the mixing parameters

$$H^{\pm\pm} \to \left(1 + \frac{1}{2}\delta Z_{H^{++}}\right) H^{\pm\pm},$$

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} \to \begin{pmatrix} 1 + \frac{1}{2}\delta Z_{G^+} & \delta\beta + \delta C_{GH} \\ -\delta\beta + \delta C_{GH} & 1 + \frac{1}{2}\delta Z_{H^+} \end{pmatrix} \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix},$$

$$\begin{pmatrix} G^0 \\ A \end{pmatrix} \to \begin{pmatrix} 1 + \frac{1}{2}\delta Z_{G^0} & \delta\beta' + \delta C_{GA} \\ -\delta\beta' + \delta C_{GA} & 1 + \frac{1}{2}\delta Z_A \end{pmatrix} \begin{pmatrix} G^0 \\ A \end{pmatrix},$$

$$\begin{pmatrix} h \\ H \end{pmatrix} \to \begin{pmatrix} 1 + \frac{1}{2}\delta Z_h & \delta\alpha + \delta C_{hH} \\ -\delta\alpha + \delta C_{hH} & 1 + \frac{1}{2}\delta Z_H \end{pmatrix} \begin{pmatrix} h \\ H \end{pmatrix}.$$

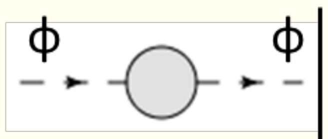
III. *Renormalization of V_{Higgs}*

The counter terms of the H++ mass & its wave function renormalization factor: $\delta m^2_{H++},\ \delta Z_{H++}$

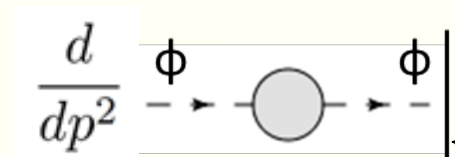
Renormalized H++ two point function at the one-loop level :

$$\hat{\Pi}_{H++H--}[p^2] = (p^2 - m^2_{H++})\delta Z_{H++} - \delta m^2_{H++} + \frac{\sqrt{2}}{s_\beta}\frac{\delta T_\Delta}{v} + \Pi^{1PI}_{H++H--}(p^2)$$

On-shell condition



$\Big|_{p^2=m_{H++}^2} = 0$



$\Big|_{p^2=m_{H++}^2} = 0$

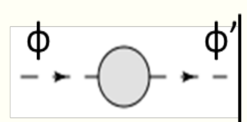
$$\hat{\Pi}_{H++H--}[m^2_{H++}] = 0, \quad \hat{\Pi}'_{H++H--}[m^2_{H++}] = 0$$

$$\delta m^2_{H++} = \frac{\sqrt{2}\delta T_\Delta}{v s_\beta} + \Pi^{1PI}_{H++H--}(m^2_{H++}), \quad \delta Z_{H++} = -\Pi^{1PI\prime}_{H++H--}(m^2_{H++})$$

The counter-terms related to the CP-odd scalar states $\delta m^2_A, \delta Z_{G^0}, \delta Z_A, \delta C_{GA}, \delta\beta'$

On-shell condition

No-mixing condition



$\Big|_{p^2=0,\ m_A^2} = 0$

$$\hat{\Pi}_{AA}[m^2_A] = 0, \quad \hat{\Pi}'_{AA}[m^2_A] = 0, \quad \hat{\Pi}'_{GG}[0] = 0,$$
$$\hat{\Pi}_{AG}[0] = 0, \quad \hat{\Pi}_{AG}[m^2_A] = 0.$$

The five counter-terms are obtained.

All counter-terms are determined by the renormalization conditions.
One-loop calculations for the other observables are now predictable.

IV. *Results*

- **Mass relationship**

We study the radiative correction to $R \equiv \dfrac{m^2_{H_{\pm\pm}} - m^2_{H_\pm}}{m^2_{H_\pm} - m^2_A}$

- **Indirect way**

We discuss the SM-like Higgs boson couplings, *hγγ, hZZ, hWW* and *hhh* at the one-loop level in the favored parameter regions by the unitarity bound, the vacuum stability bound and by the measured W boson mass discussed in previous sections.

IV. *Radiative correction to R*

$$R = \frac{m^2_{H\pm\pm} - m^2_{H\pm}}{m^2_{H\pm} - m^2_A} = 1 - 4\left(1 - \frac{\lambda_3}{\lambda_5}\right)\frac{v^2_\Delta}{v^2} + \mathcal{O}\left(\frac{v^4_\Delta}{v^4}\right)$$

One-loop quantity of R:

$$R^{\text{loop}} = R_{tree} + \Delta R - 4\left(1 - \frac{\lambda_3}{\lambda_5}\right)\frac{v^2_\Delta}{v^2} + \mathcal{O}\left(\frac{v^4_\Delta}{v^4}\right)$$

***R_{tree}* :**

$$(R)_{tree} = \frac{m^2_{H\pm\pm} - m^2_{H\pm}}{m^2_{H\pm} - (m^2_A)_{tree}} = 1 \quad \rightarrow \quad (m^2_A)_{tree} = 2m^2_{H\pm} - m^2_{H\pm\pm}$$

ΔR* : *One-loop correction to R in the limit of $(v_\Delta/v)^2 \to 0$.

In the limit $v\Delta/v \to 0$, m^2_A is not an independent parameter.

$$\Delta R = \frac{m^2_{H\pm\pm} - m^2_{H\pm}}{m^2_{H\pm} - (m^2_A)_{pole}} - 1$$

Predicted pole mass for A :

$$(m^2_A)_{\text{pole}} = (m^2_A)_{\text{tree}} - \frac{\delta T_\Delta}{v_\Delta} + \delta m^2_A - \Pi^{\text{1PI}}_{AA}[(m^2_A)_{\text{tree}}]$$

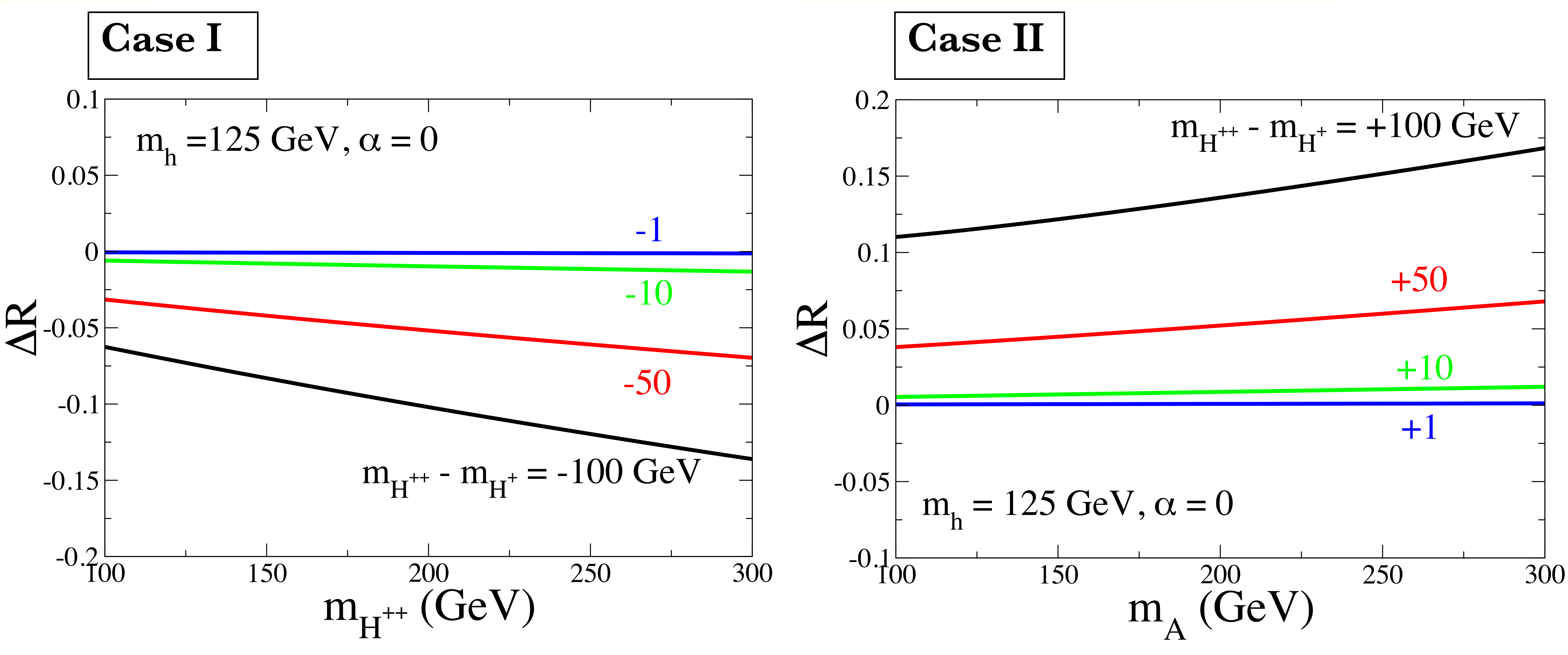
in the limit of $v_\Delta/v \to 0$.

$$\simeq (m^2_A)_{\text{tree}} + \Pi^{\text{1PI}}_{AA}[(m^2_A)_{\text{tree}}] + 2\Pi^{\text{1PI}}_{H^+H^-}[m^2_{H^+}] - \Pi^{\text{1PI}}_{H^{++}H^{--}}[m^2_{H^{++}}]$$

$$\Delta R = \frac{\Pi^{\text{1PI}}_{H^{++}H^{--}}[m^2_{H^{++}}] - 2\Pi^{\text{1PI}}_{H^+H^-}[m^2_{H^+}] + \Pi^{\text{1PI}}_{AA}[(m^2_A)_{\text{tree}}]}{m^2_{H^{++}} - m^2_{H^+}}$$

This is given by three input parameters, $m^2_{H^{++}},\ m^2_{H^+},\ m^2_h$.

IV. *Radiative correction to R*



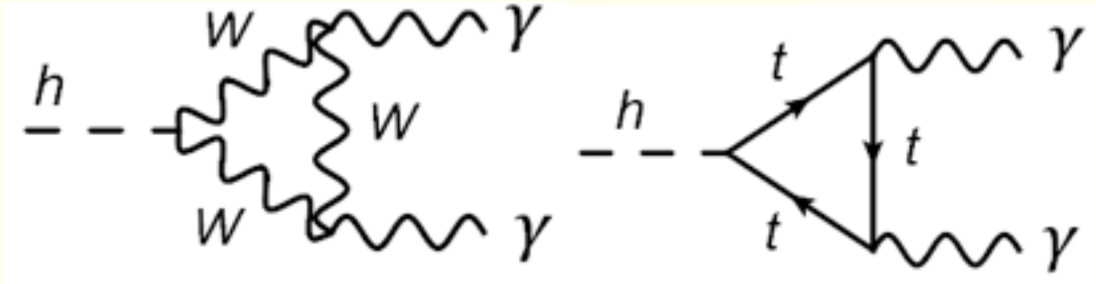
The contribution of ΔR to R is sizable, especially when the mass difference between the triplet fields is large.

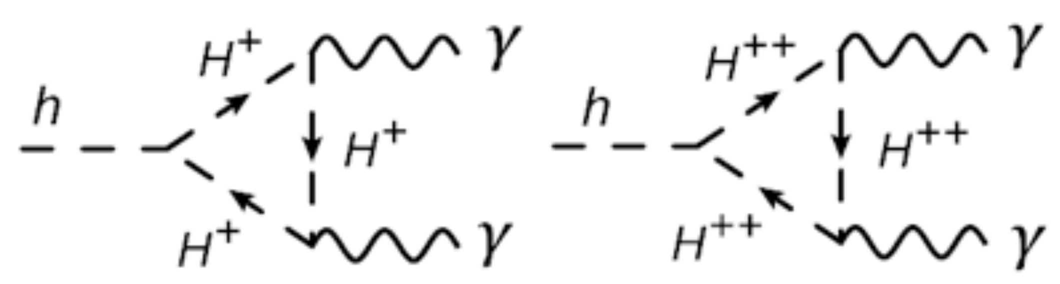
ΔR : $\gtrsim$10% for $|m_{H^{++}} - m_{H^{+}}| \sim$ 100 GeV

IV. $h \to \gamma\gamma$

$h \to \gamma\gamma$ in HTM

A.Arhrib, et al. JHEP04(2012)
S.Kanemura, K.Yagyu, PRD85(2012)
A.Akeroyd, S.Moretti, PRD86(2012)





SM contribution

Triplet-like Higgs loop contribution

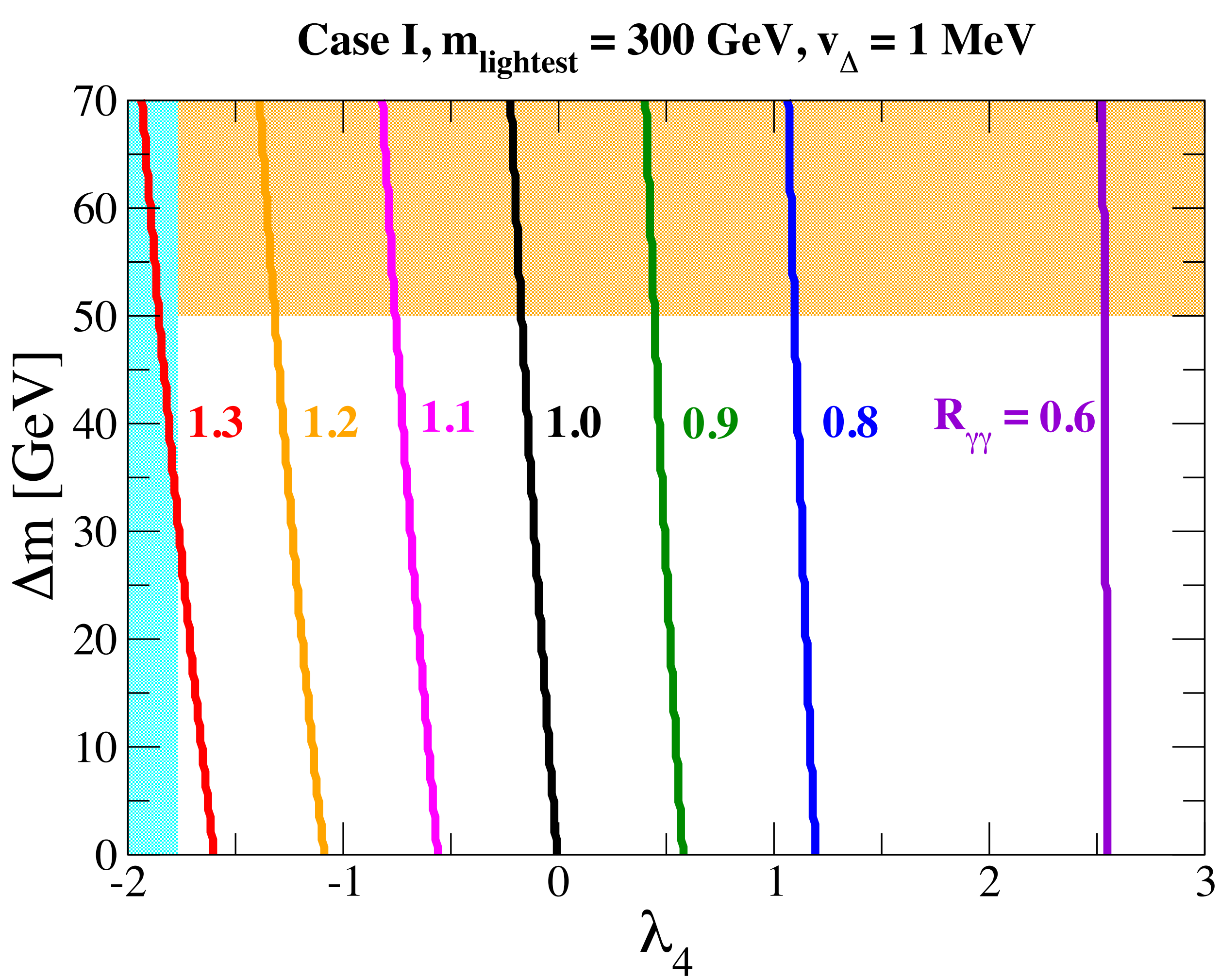
$\lambda_{hH+H-} \sim -v(2\lambda_4+\lambda_5)/2$ $\lambda_{hH++H--} \sim -v\lambda_4$

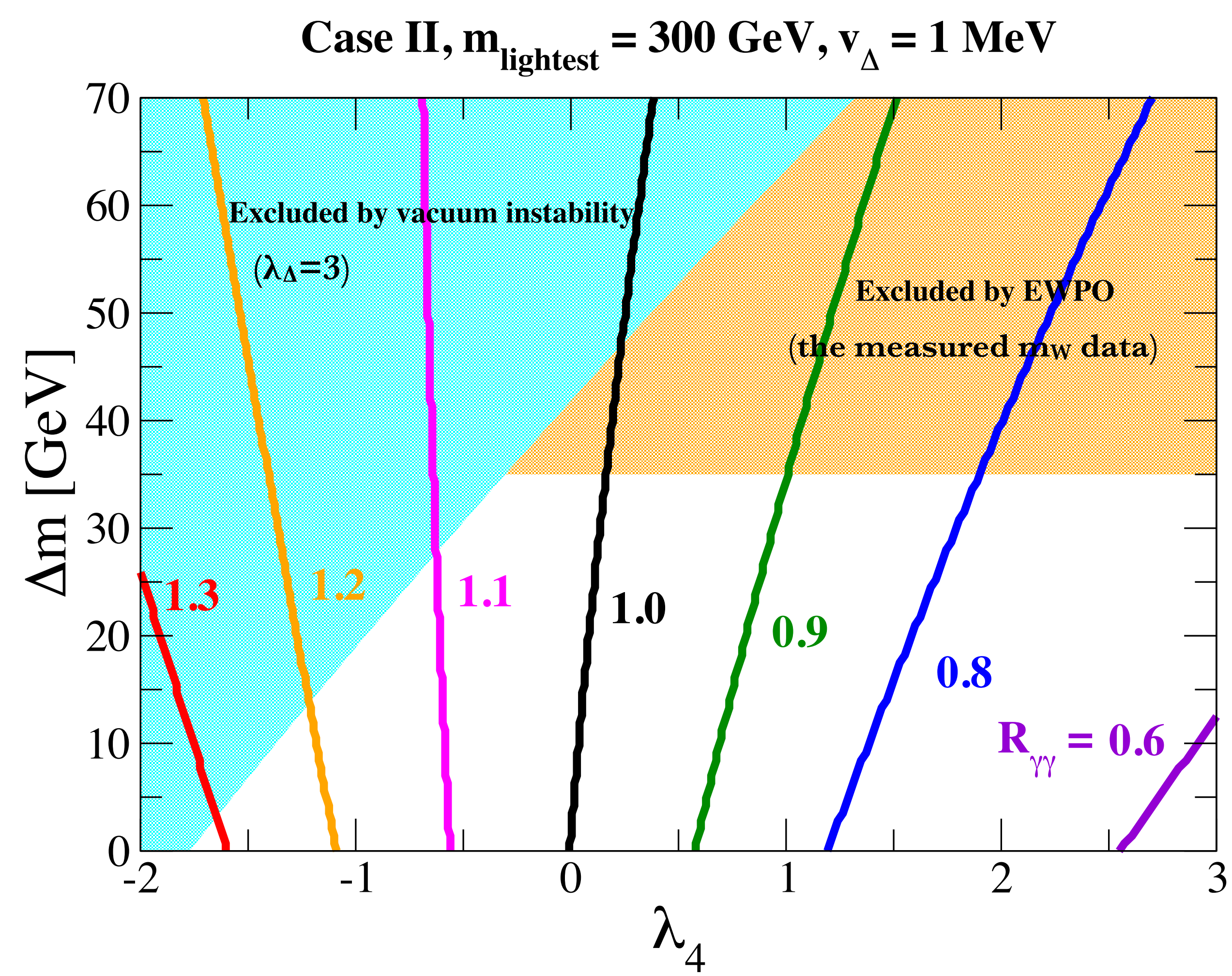
- **When the sign of the coupling $\lambda_{H++H--h}$ is positive (negative), then the H++ loop contribution has the same (opposite) sign of the W loop contribution.**
  $\lambda 4 < 0 \to$ The decay rate is enhanced.

- **$h \to \gamma\gamma$ is not sensitive to the magnitude of $\lambda 5$. So, the mass difference among the triplet-like Higgs boson (Case I or Case II) is not so important as long as we keep a fixed value of mH++.**

IV. $h \to \gamma\gamma$

$$\boxed{R_{\gamma\gamma} \equiv \frac{\sigma(gg\to h)_{\rm HTM}\times {\rm BR}(h\to\gamma\gamma)_{\rm HTM}}{\sigma(gg\to h)_{\rm SM}\times {\rm BR}(h\to\gamma\gamma)_{\rm SM}}} = \frac{c_\alpha^2 \times {\rm BR}(h\to\gamma\gamma)_{\rm HTM}}{s_\beta^2 \times {\rm BR}(h\to\gamma\gamma)_{\rm SM}}$$

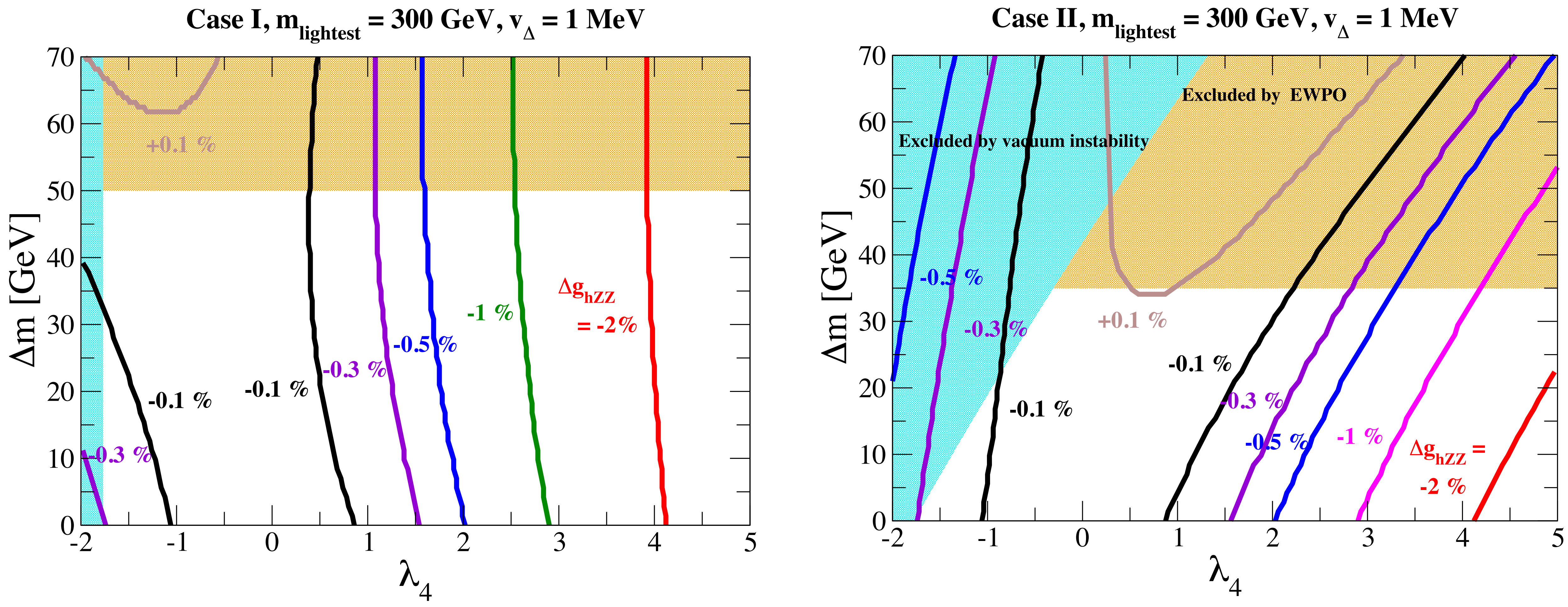




- **$R_{\gamma\gamma}$ can be greater than 1 for negative value of λ_4.**
- **$R_{\gamma\gamma}$ ~1.3 when λ_4~−1.7 , $R_{\gamma\gamma}$ ~0.6 when λ_4~3 in both Case I and Case II.**

IV. *hZZ*

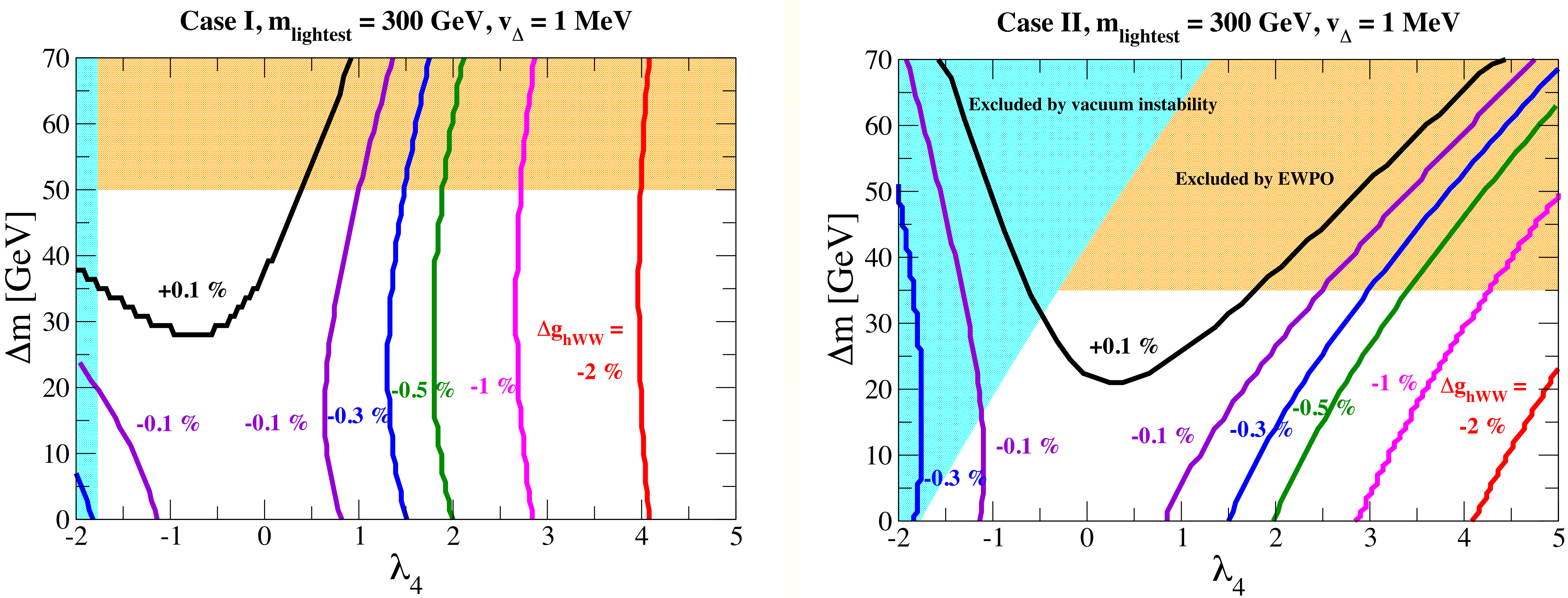
$$\Delta g_{hZZ} \equiv \frac{\mathrm{Re}M_1^{hZZ} - \mathrm{Re}M_1^{hZZ}(\mathrm{SM})}{\mathrm{Re}M_1^{hZZ}(\mathrm{SM})}$$



- For the smaller Δm, the magnitude of the negative correction is larger for positive larger value of λ_4.
- Δm ≳ 30 GeV, $\Delta g_{hZZ} \gtrsim 0$ appears.
- Δg_{hZZ} is predicted to be at most a few %.

IV. *hWW*

$$\Delta g_{hWW} \equiv \frac{\mathrm{Re}M_1^{hWW} - \mathrm{Re}M_1^{hWW}(\mathrm{SM})}{\mathrm{Re}M_1^{hWW}(\mathrm{SM})}$$



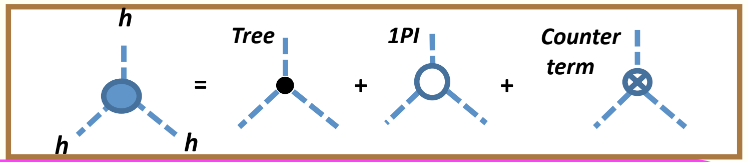
- **The behavior of Δg_{hWW} is similar to that of Δg_{hZZ}.
  But the correction can be positive for smaller values of Δm.**

IV. *hhh*

Finally, we show the numerical results for the deviation of the Higgs trilinear coupling from the SM prediction.

The renormalized *hhh* coupling Γ_{hhh}: $\frac{\partial^2 V_{\rm eff}}{\partial \varphi^3}|_{\varphi=v} = \frac{1}{3!}\Gamma_{hhh}$

$$\Gamma_{hhh}(p_1^2, p_2^2, q^2) = \Gamma^{\rm tree}_{hhh} + \delta\Gamma_{hhh} + \Gamma^{\rm 1PI}_{hhh}(p_1^2, p_2^2, q^2) \qquad q = p_1 + p_2$$



$v_\Delta/v \to 0$

tree : $\Gamma^{\rm tree}_{hhh} \to \frac{-3m_h^2}{v},$

Counter-term : $\delta\Gamma_{hhh} \to -\frac{3\delta m_h^2}{v} - \frac{9}{2}\frac{m_h^2}{v}\delta Z_h + \frac{3m_h^2}{v^2}\delta v.$

They are reduced to the same expressions in the SM.

1PI : **The *t* loop, the gauge boson loop** **→ the same as the SM.**

The triplet-like Higgs boson loop **→They can be remained even in this limit.**

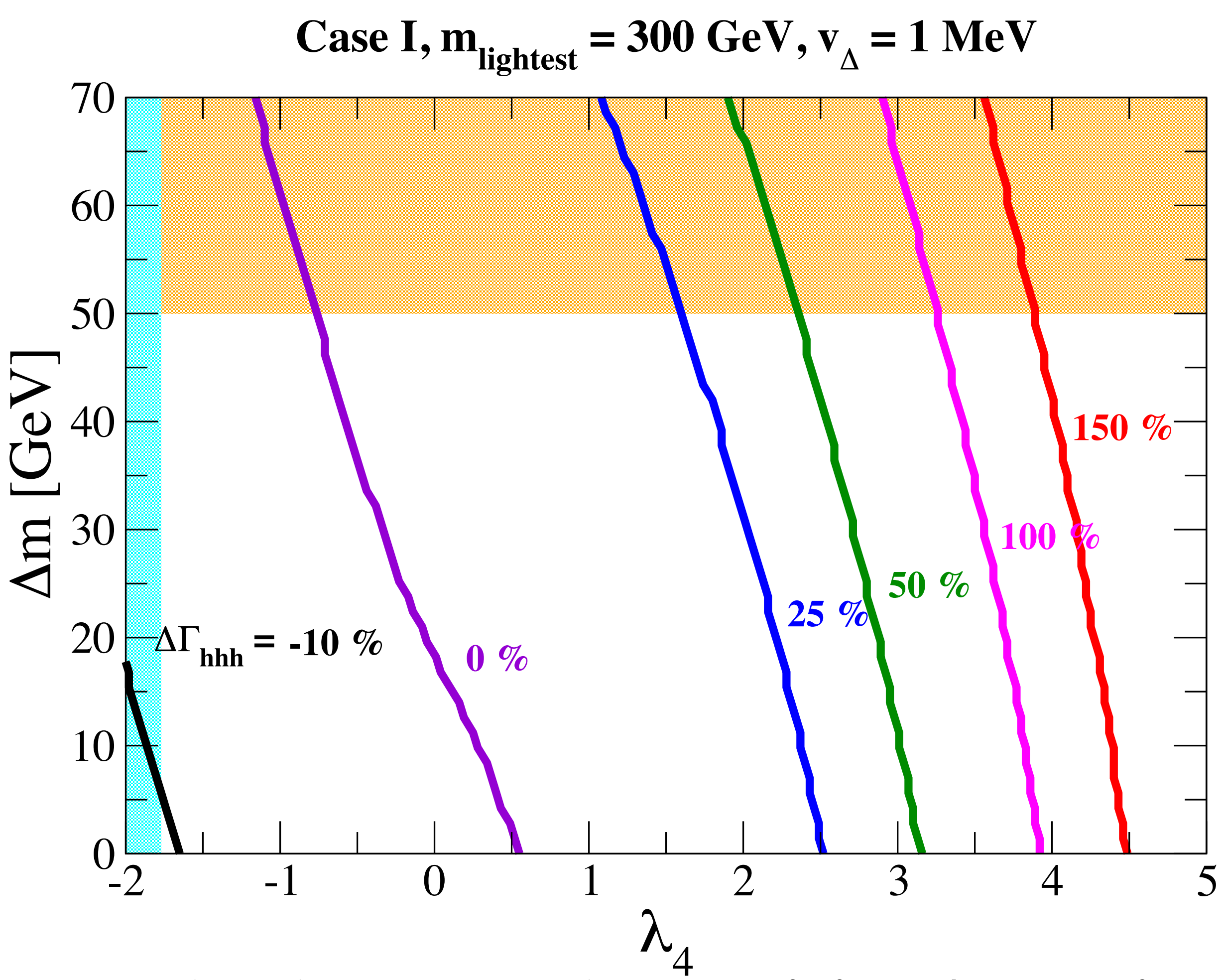
$$\Gamma_{hhh} \simeq -\frac{3m_h^2}{v}\left[1 - \frac{v}{48\pi^2 m_h^2}\left(\frac{\lambda^3_{H^{++}H^{--}h}}{m^2_{H^{++}}} + \frac{\lambda^3_{H^+H^-h}}{m^2_{H^+}} + \frac{4\lambda^3_{AAh}}{m_A^2} + \frac{4\lambda^3_{HHh}}{m_H^2}\right) + \cdots\right]$$

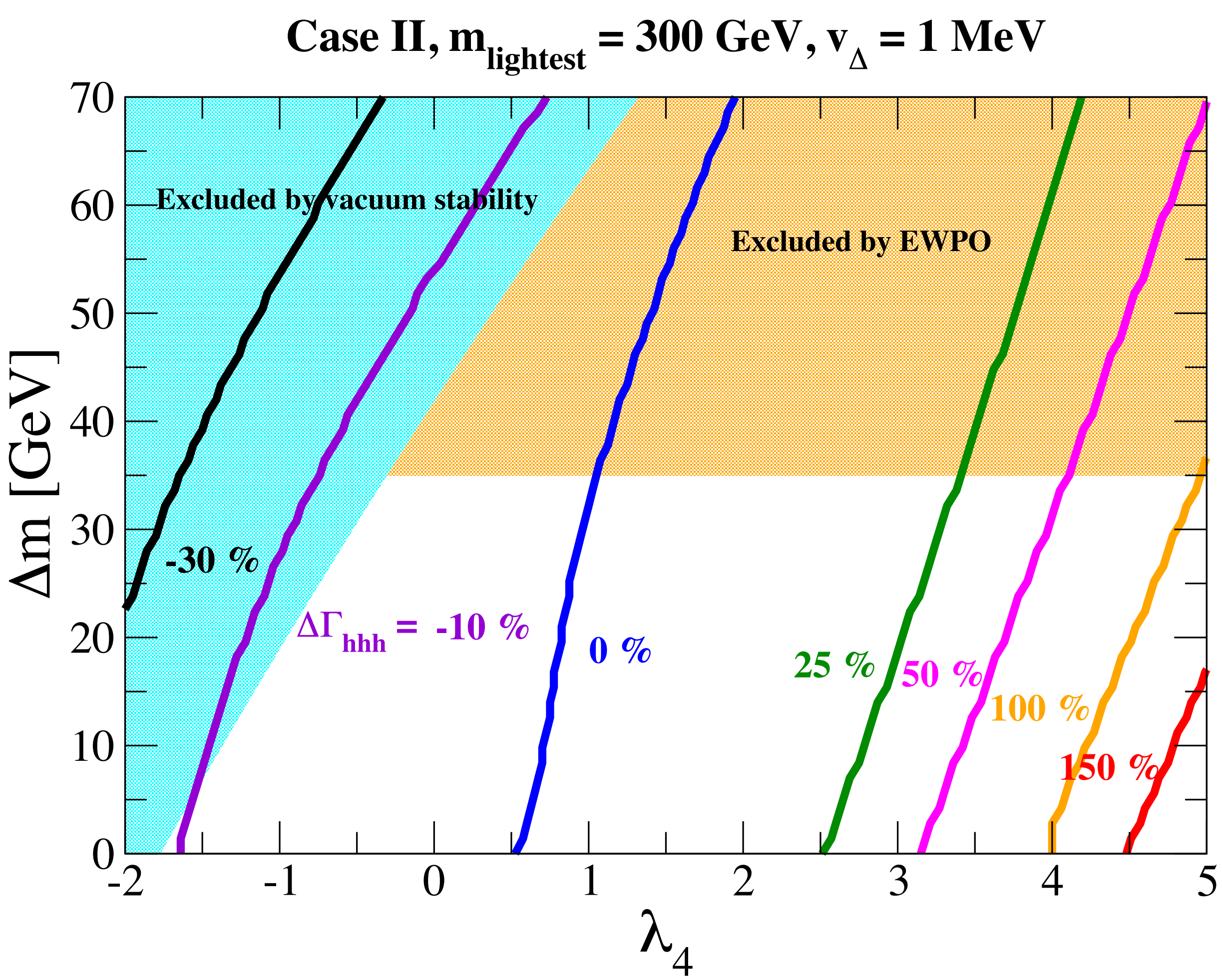
$$\simeq -\frac{3m_h^2}{v}\left\{1 + \frac{v^4}{48\pi^2 m_h^2}\left[\frac{\lambda_4^3}{m^2_{H^{++}}} + \frac{(\lambda_4 + \frac{\lambda_5}{2})^3}{m^2_{H^+}} + \frac{(\lambda_4+\lambda_5)^3}{2m_A^2} + \frac{(\lambda_4+\lambda_5)^3}{2m_H^2}\right] + \cdots\right\}$$

The triplet-like Higgs boson loop contribution gives a positive (negative) correction compared to the SM prediction when $\lambda 4 > 0$ ($\lambda 4 < 0$) and $\lambda 5 \simeq 0$.

IV. *hhh*

$$\Delta\Gamma_{hhh} \equiv \frac{\text{Re}\Gamma_{hhh} - \text{Re}\Gamma^{\text{SM}}_{hhh}}{\text{Re}\Gamma^{\text{SM}}_{hhh}}$$





- **In both cases, the positive (negative) values of $\Delta\Gamma_{hhh}$ are predicted in the case with a positive (negative) λ_4.**
- **The deviation from the SM prediction can be significant.**

Strong correlation in $R_{\gamma\gamma}$ and $\Delta\Gamma_{hhh}$ can be found.

$R_{\gamma\gamma} > 1$ ($R_{\gamma\gamma} < 1$), $\Delta\Gamma_{hhh}$ takes negative (positive) value.

V. Summary

We discussed the one-loop renormalization in the HTM.

- Characteristic mass relation

$$R \equiv \frac{m^2_{H^{\pm\pm}} - m^2_{H^\pm}}{m^2_{H^\pm} - m^2_A}$$

→ The ratio R can be modified around 10%.

- *$h \to \gamma\gamma$*
- ***Renormalized SM-like Higgs couplings hZZ, hWW and hhh***

Magnitudes of the deviations in these quantities from the SM predictions have been evaluated in the parameter regions where the unitarity and vacuum stability bounds are satisfied and the predicted W boson mass is consistent with the data.

Strong correlations among deviations in the Higgs boson couplings.

$h\gamma\gamma$	*hZZ, hWW*	*hhh*
$R_{\gamma\gamma}$	Δg_{hVV}	$\Delta\Gamma_{hhh}$
~ 1.3	~ −0.1%	~ −2%
~ 0.6	~ −10%	~ +150%

The HTM may be tested by measuring these couplings accurately at the future collider experiments, even when additional particles are not directly discovered.