Sterile neutrinos and oscillation coherence
for neutrinos produced in decays

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When are ν oscillations observable?

Observability conditions for ν oscillations:

- Coherence of ν production and detection: the produced and detected ν's are flavor eigenstates = coherent superpositions of different mass eigenstates
- Coherence of ν propagation: The produced neutrino state does not (irreversibly) lose coherence due to the wave packet separation in the course of propagation

Both conditions put upper limits on neutrino mass squared differences Δm^2 :

$$(1) \quad \Delta E_{jk} \sim \frac{\Delta m_{jk}^2}{2E} \ll \sigma_E; \qquad\qquad (2) \quad \frac{\Delta m_{jk}^2}{2E^2} L \ll \sigma_x \simeq v_g/\sigma_E$$

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Sterile neutrinos: hints from SBL accelerator experiments (LSND, MiniBooNE), reactor neutrino anomaly, keV sterile neutrinos, pulsar kicks, leptogenesis via ν oscillations, SN r-process nucleosynthesis, unconventional contributions to $2\beta 0\nu$ decay ...

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Production/detection coherence has to be re-checked – important implications for some neutrino experiments!

When are neutrino oscillations observable?

Keyword: *Coherence*

Neutrino flavour eigenstates ν_e, ν_μ and ν_τ are coherent superpositions of mass eigenstates ν_1, ν_2 and ν_3 $\Rightarrow$ oscillations are only observable if

- neutrino production and detection are coherent
- coherence is not (irreversibly) lost during neutrino propagation.

Possible decoherence at production (detection): If by accurate E and p measurements one can tell (through $E = \sqrt{p^2 + m^2}$) which mass eigenstate is emitted, the coherence is lost and oscillations disappear!

Full analogy with electron interference in double slit experiments: if one can establish which slit the detected electron has passed through, the interference fringes are washed out.

When are neutrino oscillations observable?

Another source of decoherence: wave packet separation due to the difference of group velocities Δv of different mass eigenstates.

If coherence is lost: Flavour transition can still occur, but in a non-oscillatory way. E.g. for $\pi \to \mu\nu_i$ decay with a subsequent detection of ν_i with the emission of e:

$$P \;\propto\; \sum_i P_{\rm prod}(\mu\,\nu_i) P_{\rm det}(e\,\nu_i) \;\propto\; \sum_i |U_{\mu i}|^2 |U_{ei}|^2$$

– the same result as for averaged oscillations.

The same is true for survival probabilities. In 2-flavour case:

$$P_{\mu\mu} = \cos^4\theta + \sin^4\theta + 2\sin^2\theta\,\cos^2\theta\,\cos(\Delta\phi)\,, \qquad \Delta\phi \equiv \frac{\Delta m^2}{2p}L$$

In the case of decoherence:

$$P_{\mu\mu} = \cos^4\theta + \sin^4\theta = P_{\mu\mu}^{av}$$

A manifestation of neutrino coherence

Even non-observation of neutrino oscillations at distances $L \ll l_{\rm osc}$ is a consequence of and an evidence for coherence of neutrino emission and detection! Two-flavour example (e.g. for ν_e emission and detection):

$$A_{\rm prod/det}(\nu_1) \sim U_{e1} = \cos\theta\,, \qquad A_{\rm prod/det}(\nu_2) \sim U_{e2} = \sin\theta \qquad \Rightarrow$$

$$A(\nu_e \to \nu_e) = \sum_{i=1,2} A_{\rm prod}(\nu_i) A_{\rm det}(\nu_i) = \cos^2\theta + e^{-i\Delta\phi}\sin^2\theta$$

Phase difference $\Delta\phi$ vanishes at short L $\Rightarrow$

$$P(\nu_e \to \nu_e) = (\cos^2\theta + \sin^2\theta)^2 = 1$$

If ν_1 and ν_2 were emitted and absorbed incoherently) $\Rightarrow$ one would have to sum probabilities rather than amplitudes:

$$P(\nu_e \to \nu_e) \sim \sum_{i=1,2} |A_{\rm prod}(\nu_i) A_{\rm det}(\nu_i)|^2 \sim \cos^4\theta + \sin^4\theta \; < \; 1$$

$\nu_{\rm ster}$ (de)coherence: decays of free π's and μ's

$\pi \to \mu\nu$ decay

For free pions: in the rest frame $\sigma_E \simeq \Gamma_0 \simeq 2.5 \times 10^{-8}$ eV.
Neutrino energy: $E_0 \simeq 30$ MeV. For a sterile neutrino with $\Delta m^2 \sim 2$ eV^2

$$\frac{\Delta m^2}{2E_0} \simeq 3.3 \times 10^{-8}\ \text{eV}; \qquad \text{compare with} \qquad \Gamma_0 \simeq 2.5 \times 10^{-8}\ \text{eV}$$

$\Rightarrow$ the coherence condition $\Delta m^2/2E \ll \sigma_E$ violated!

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$\mu \to e\nu_\mu\nu_e$ decay:

$\Gamma_0 \simeq 3 \times 10^{-10}$ eV. Neutrino energy: $E_0 \sim 40 - 50$ MeV. For $\Delta m^2 \sim 2$ eV^2

$$\frac{\Delta m^2}{2E_0} \simeq 2 \times 10^{-8}\ \mathrm{eV}^2; \qquad \text{compare with} \qquad \Gamma_0 \simeq 3 \times 10^{-10}\ \mathrm{eV}$$

$\Rightarrow$ expected violation of the the coherence condition is even much stronger!
Can occur even for smaller values of Δm^2.

What if prod./det. coherence is violated?

Should LSND & MiniBooNE results be reconsidered?
What about other experiments?

If production/detection coherence is strongly violated, the osc. probabilities $P_{\alpha\beta}$ take their averaged values (even for $L = 0$ – "zero distance effect").

$\Rightarrow$ No L/E dependence; 2-detector setups are useless!
No energy spectrum distortion!

$$\Downarrow$$

Careful examination within the QM wave packet formalism is necessary

◊ Consistent approaches:

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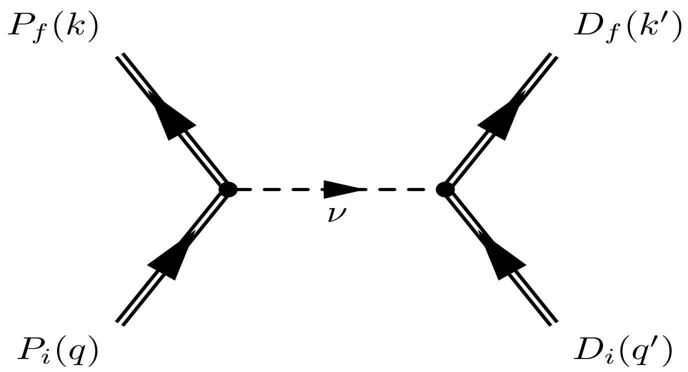
- QM wave packet approach – neutrinos described by wave packets rather than by plane waves
- QFT approach: neutrino production and detection explicitly taken into account. Neutrinos are intermediate particles described by propagators



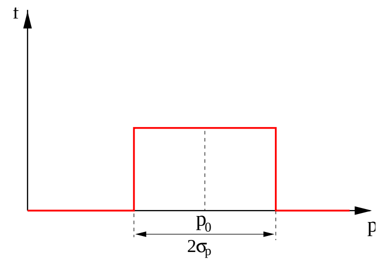
QM wave packet formalism

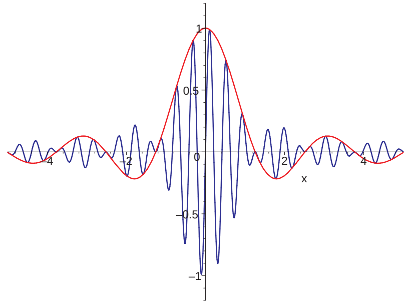
Propagating particles are described by wave packets. For a free particle:

$$\Psi(\vec{x},\, t) \;=\; \int \frac{d^3p}{(2\pi)^3}\, f(\vec{p},\vec{p}_0)\, e^{i\vec{p}\vec{x}-iE(p)t}$$

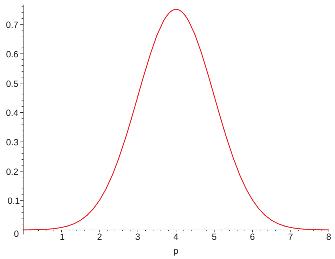
$f(\vec{p},\vec{p}_0)$ – amplitide of the momentum distribution function (= momentum space w. function of the particle); p_0 = peak momentum. Some examples:

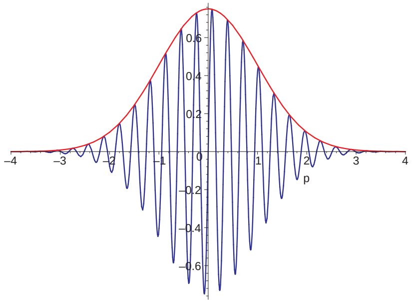
Rectangular mom. space w. packet





Gaussian mom. space w. packet:





$$\sigma_x \sigma_p \;=\; 1/2$$

Wave packets – contd.

Expand $E(p) = \sqrt{p^2 + m^2}$ near $p = p_0$:

$$E(p) = E(p_0) + v_g(\vec{p}_0)(\vec{p} - \vec{p}_0) + \dots, \qquad \vec{v}_g = \frac{\partial E(p)}{\partial \vec{p}} = \frac{\vec{p}}{E}$$

(higher order terms discarded $\Leftrightarrow$ w. packet spreading neglected)

$$\boxed{\Psi(\vec{x}, t) \simeq e^{i\vec{p}_0\vec{x} - iE(p_0)t} g(\vec{x} - \vec{v}_g t)}$$

"Shape factor" (envelope of the w. packet):

$$g(\vec{x} - \vec{v}_g t) = \int \frac{d^3 p_1}{(2\pi)^3} f(\vec{p}_1 + \vec{p}_0, \vec{p}_0)\, e^{i\vec{p}_1(\vec{x} - \vec{v}_g t)}$$

Peak of the wave packet: $\vec{x} - \vec{v}_g t = 0$.

Propagating wave packets

$$|\Psi(\vec{x},\, t)| \;=\; |\Psi(\vec{x}-\vec{v}_g t)|$$

$\Rightarrow$ propagation with velocity $\vec{v}_g$ with no change of shape

Example: Gaussian wave packets

Momentum-space distribution:

$$f(\vec{p},\vec{p}_0) \;=\; \frac{1}{(2\pi\sigma_p^2)^{3/4}}\exp\left\{-\frac{(\vec{p}-\vec{p}_0)^2}{4\sigma_p^2}\right\}$$

Momentum dispersion: $\langle\vec{p}^{\,2}\rangle - \langle\vec{p}\,\rangle^2 = \sigma_p^2$.

Coordinate-space wave packet:

$$\Psi(\vec{x},\, t) \;=\; e^{i\vec{p}_0\vec{x}-iE(p_0)t}\,\frac{1}{(2\pi\sigma_x^2)^{3/4}}\exp\left\{-\frac{(\vec{x}-\vec{v}_g t)^2}{4\sigma_x^2}\right\}, \qquad \sigma_x^2 = 1/(4\sigma_p^2)$$

$$\langle\vec{x}\,\rangle = \vec{v}_g t\,; \qquad \langle\vec{x}^{\,2}\rangle - \langle\vec{x}\,\rangle^2 = \sigma_x^2\,.$$

W. packets of ν's produced in π decays

Consider w. packets of neutrinos produced in decays of free pions confined to a decay tunnel of length l_p. Need the pion and muon w. functions!

Pions: produced by $pN \to \pi X$ processes; $\sigma_{x\pi} \sim \sigma_{xp} \lesssim 10^{-4}$ cm.
Muons: not detected ($\Leftrightarrow$ completely delocalized, $\sigma_{x\mu} \to \infty$).

$\sigma_{x\pi}$ completely negligible compared to all distances of interest ($l_{\rm osc},\ L,\ l_p$) $\Rightarrow$ can be set $\to 0$. The coordinate-state w. functions of the pion and muon:

$$\psi_\pi(x,t) = C_\pi\, e^{iQx - iE_\pi(Q)t - \Gamma t/2}\, \delta(x - v_\pi t)\, \mathrm{box}(x; l_p, 0)\,,$$

$$\psi_\mu(x,t) = C_\mu\, e^{iKx - iE_\mu(K)t}\,,$$

$$E_\pi(Q) = (Q^2 + m_\pi^2)^{1/2}, \quad E_\mu(K) = (K^2 + m_\mu^2)^{1/2}$$

$$\mathrm{box}(x; l_p, 0) = \begin{cases} 1\,, & l_p \ge x \ge 0\,, \\ 0\,, & \text{otherwise} \end{cases}$$

Pions assumed produced at $t = 0,\ x = 0$.

Neutrino wave packet – contd.

The amplitude of $\pi \to \mu\nu_\mu$ decay with production of muon with momentum K and mass-eigenstate neutrino ν_j with momentum p:

$$f_j^S(p) = M_P \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dx \; e^{iE_j(p)t - ipx} \, \psi_\mu(x,t)^* \psi_\pi(x,t) \,.$$

Here: $E_j(p) = (p^2 + m_j^2)^{1/2}$, M_P – mom.-space pion decay amplitude. For fixed K $\Rightarrow$ mom. distribution amplitude (mom.-space w. function) of ν_j.

$$\boxed{f_j^S(p) = C_j \, \frac{1 - e^{i[E_j(p) - E_P - v_\pi(p-P) + i\Gamma/2]\, l_p/v_\pi}}{E_j(p) - E_P - v_\pi(p-P) + i\Gamma/2}\,.}$$

$$P \equiv Q - K\,, \quad E_P \equiv E_\pi(Q) - E_\mu(K)\,,$$

Coordinate-space w. function of ν_j:

$$\psi_j^S(x,t) = \int \frac{dp}{2\pi} \, f_j^S(p) e^{-iE_j(p)t + ipx}$$

Coordinate-space ν wave function

The result:

$$\boxed{\psi_j^S(x,t) = Ce^{-iE_j(P_j)t+iP_jx}\left\{e^{-\frac{\Gamma}{2(v_j-v_\pi)}(v_jt-x)}\left[\theta(v_jt-x)-\theta(v_jt-x-\frac{v_j-v_\pi}{v_\pi}l_p)\right]\right\}}$$

$$P_j \equiv P + \frac{E_P - E_j(P)}{v_j - v_\pi}\,, \qquad \text{where} \qquad v_j \equiv \frac{\partial E_j(p)}{\partial p}\Big|_{p=P} = \frac{P}{E_j(P)}\,,$$

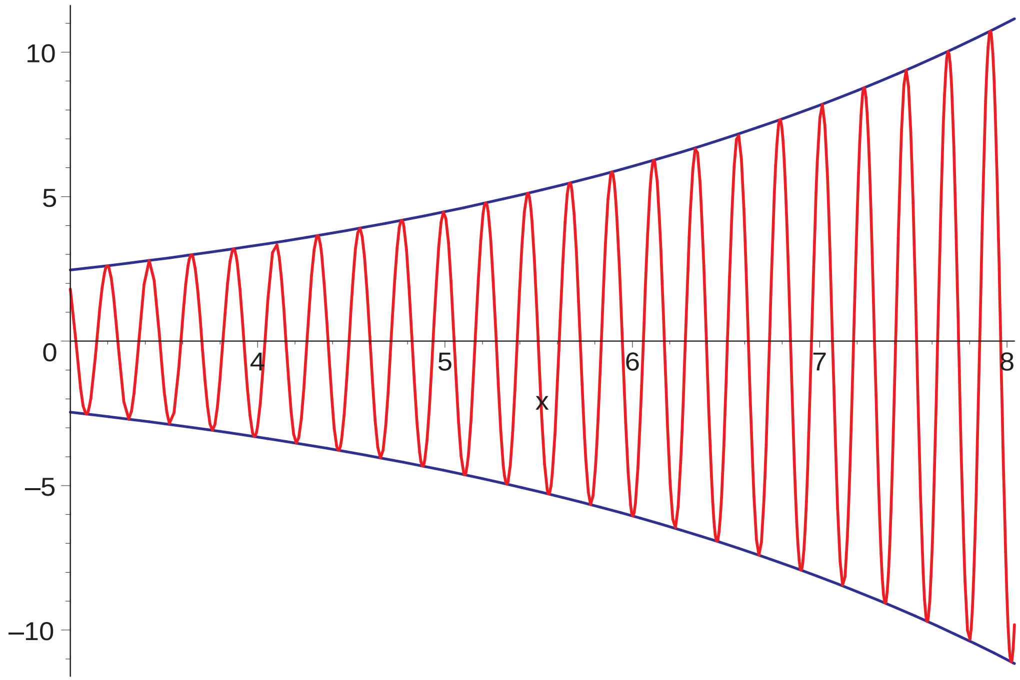
$$E_j(P_j) \simeq E_j(P) + v_j(P_j - P) = E_j(P) + v_j\frac{E_P - E_j(P)}{v_j - v_\pi}\,.$$

Neutrino wave packet:

- Has sharp edges
- Arrives at point x at $t_1 = x/v_j$; leaves it at $t_2 = x/v_j + (1/v_\pi - 1/v_j)l_p$
- Reaches its maximum at the front edge and exponentially decays towards the rear edge

$\Rightarrow$ An asymmetric wave packet!

Neutrino WF



Trumpet-like wave packet

Neutrino WF – contd.

The expectation value $\bar{x}$ of the neutrino coordinate in the state described by the neutrino wave packet:

$$\bar{x} = \frac{\int dx\, |\psi_j^S(x,t)|^2 x}{\int dx\, |\psi_j^S(x,t)|^2} = v_j t - \frac{v_j - v_\pi}{\Gamma}\left[1 - \frac{\Gamma l_p}{v_\pi}\,\frac{e^{-\Gamma l_p/v_\pi}}{1 - e^{-\Gamma l_p/v_\pi}}\right].$$

The width of the neutrino wave packet is given by the coordinate dispersion:

$$\sigma_{xj}^2 = \left(\overline{x^2} - \bar{x}^2\right) = \frac{\int dx\, |\psi_j^S(x,t)|^2 (x - \bar{x})^2}{\int dx\, |\psi_j^S(x,t)|^2}$$

$$= \left(\frac{v_j - v_\pi}{\Gamma}\right)^2 \left[1 - \left(\frac{\Gamma l_p}{v_\pi}\right)^2 \frac{e^{-\Gamma l_p/v_\pi}}{(1 - e^{-\Gamma l_p/v_\pi})^2}\right].$$

Neutrino WF – contd.

Limiting cases:

In the limit $\Gamma l_p/v_\pi \gg 1$ (l_p large compared to the pion decay length $l_{\rm decay} = v_\pi/\Gamma$ $\Rightarrow$ decay of unconfined free pions):

$$\bar{x} \approx v_j t - \frac{v_j - v_\pi}{\Gamma}\,, \qquad \sigma_{xj} \approx \frac{v_j - v_\pi}{\Gamma}$$

In the opposite limit, $\Gamma l_p/v_\pi \ll 1$:

$$\bar{x} \approx v_j t - \frac{v_j - v_\pi}{2v_\pi} l_p\,, \qquad \sigma_{xj} \approx \frac{1}{2\sqrt{3}}\frac{v_j - v_\pi}{v_\pi} l_p\,.$$

(In this limit only a small fraction of pions decays before being absorbed by the wall at the end of the decay tunnel).

Calculating the oscillation probabilities

The transition amplitude:

$$\mathcal{A}_{\alpha\beta}(L,t) = \sum_j U_{\alpha j}^* U_{\beta j} \mathcal{A}_j(L,t)$$

Contribution of ν_j:

$$\mathcal{A}_j(L,t) \equiv \int dx\, \psi_j^{D*}(x)\, \psi_j^S(x,t)\,.$$

The detected state $\nu_j^D(x)$: a wave packet centered on the point $x = L$.
Assume the detection process is well localized: $\psi_J^D(x,t) = \delta(x-L)$ $\Rightarrow$

$$\mathcal{A}_j(L,t) = \psi_j^S(L,t)\,.$$

The oscillation probability:

$$P_{\alpha\beta}(L) = \sum_{j,k} U_{\alpha j}^* U_{\beta j} U_{\alpha k} U_{\beta k}^*\, I_{jk}(L)\,,$$

Here:

$$I_{jk}(L) \equiv \int_{-\infty}^{\infty} dt\, \mathcal{A}_j(L,t)\mathcal{A}_k^*(L,t) = \int_{-\infty}^{\infty} dt\, \psi_j^S(L,t)\psi_k^{S*}(L,t)\,.$$

$$\Downarrow$$

$$\boxed{I_{jk}(L) = \frac{1}{(1-e^{-\Gamma l_p/v_\pi})}\cdot\frac{i\Gamma}{v_\pi\frac{\Delta m_{jk}^2}{2P}+i\Gamma}\left[e^{-i\frac{\Delta m_{jk}^2}{2P}L} - e^{-\Gamma l_p/v_\pi}e^{-i\frac{\Delta m_{jk}^2}{2P}(L-l_p)}\right]}$$

The absolute normalization fixed by imposing the unitarity constraint $\sum_\beta P_{\alpha\beta}(L) = 1 \quad \Rightarrow \quad I_{jj}(L) = 1$.

Consider SBL experiments in the 3+1 scheme (only $\Delta m_{41}^2 \equiv \Delta m^2$ can be considered to be nonzero) $\quad\Rightarrow\quad$ an effective two-flavour approximation

Survival probabilities $P_{\alpha\alpha}$: $\quad s = |U_{\alpha 4}|, \quad c = (1-|U_{\alpha 4}|^2)^{1/2}$.

Transition probability $P_{\alpha\beta}$: $\quad \sin^2 2\theta = 4|U_{\alpha 4}|^2|U_{\beta 4}|^2$.

ν_μ survival probability

$\diamondsuit$ $$P_{\mu\mu} = c^4 + s^4 + \frac{2c^2s^2}{\xi^2+1}\frac{1}{(1-e^{-\Gamma l_p/v_\pi})}\Big[\cos\phi + \xi\sin\phi$$

$$-e^{-\Gamma l_p/v_\pi}\left[\cos(\phi-\phi_p) + \xi\sin(\phi-\phi_p)\right]\Big]$$

Here:

$$\phi \equiv \frac{\Delta m^2}{2P}L\,, \qquad \phi_p \equiv \frac{\Delta m^2}{2P}l_p\,, \qquad \xi \equiv v_\pi\frac{\Delta m^2}{2P\Gamma}\,,$$

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N.B.: for non-zero ξ and $\Gamma l_p/v_\pi$ the probability $P_{\mu\mu} \neq 1$ even for $L=0$
– “zero-distance” effect, a consequence of production coherence violation.

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– "zero-distance" effect, a consequence of production coherence violation.

The parameter ξ: essentially the ratio of $\Delta m^2/2P$ and Γ – what we expected to be the (de)coherence parameter. For $\xi \gg 1$ the oscillating term strongly suppressed due to production decoherence (unless $\Gamma l_p/v_p \ll 1$).

A relation between ξ, ϕ_p and $\Gamma l_p/v_\pi$:

$$\xi\cdot\frac{\Gamma l_p}{v_\pi}=\phi_p\,.$$

In the limit $\Gamma l_p/v_p\ll 1$:

$$P_{\mu\mu}=c^4+s^4+\frac{2s^2c^2}{\phi_p}\big[\sin\phi-\sin(\phi-\phi_p)\big]\,.$$

The decoherence parameter is ϕ_p. For $\phi_p\gg 1$: $P_{\mu\mu}\simeq\bar{P}_{\mu\mu}=c^4+s^4$.
For $\phi_p\ll 1$:

$$P_{\mu\mu}=P_{\mu\mu}^{\rm stand}=c^4+s^4+2s^2c^2\cos\phi\,.$$

This result does not depend on whether ξ is small or large!
Two distinct regimes:

- $\Gamma l_p/v_\pi\ll 1$ (pion decay length large compared to l_p, decoherence parameter: ϕ_p
- $\Gamma l_p/v_\pi\gg 1$ (pion decay length small compared to l_p, decoherence parameter: ξ.

How can this be understood?

The prod. coherence condition (ensures that different neutrino mass eigenstates forming a flavor neutrino state are emitted coherently):

$$\Delta E \ll \sigma_E\,,$$

For a decay of a free particle in a box (e.g. decay tunnel):

$$\sigma_E \sim \max\{\Gamma, v_\pi/l_p\} \cdot \frac{v_g}{v_g - v_\pi}\,.$$

σ_E also determines the spatial width of the neutrino wave packet: $\sigma_x \simeq v_g/\sigma_E$

$\Rightarrow$ for $\Gamma l_p/v_\pi \gg 1$ the width of the wave packet $\sigma_x \sim (v_g - v_\pi)/\Gamma$;

for $\Gamma l_p/v_\pi \ll 1$ $\sigma_x \sim [(v_g - v_\pi)/v_\pi] l_p$.

$$\Delta E \equiv |E_j(P_j) - E_k(P_k)| \simeq \frac{\Delta m^2}{2P}\frac{v_\pi v_g}{(v_g - v_\pi)} \qquad \Rightarrow$$

The decoherence parameters

Two limiting regimes:

- $\Gamma l_p / v_\pi \ll 1$ (pion decay length $l_{\rm decay} = v_\pi/\Gamma \gg l_p$)

$$\frac{\Delta E}{\sigma_E} \simeq \frac{\Delta m^2}{2P} l_p = \phi_p \,.$$

- $\Gamma l_p / v_\pi \gg 1$ (pion decay length $l_{\rm decay} = v_\pi/\Gamma \ll l_p$)

$$\frac{\Delta E}{\sigma_E} \simeq \frac{\Delta m^2}{2P} \cdot \frac{v_\pi}{\Gamma} = \xi \,.$$

When expressed through $l_{\rm osc} = 4\pi E/\Delta m^2$:

$$\phi_p = 2\pi \frac{l_p}{l_{\rm osc}}\,, \qquad\qquad \xi = 2\pi \frac{l_{\rm decay}}{l_{\rm osc}}\,.$$

The meaning of the production coherence condition:

The osc. phase acquired over the neutrino production region must be small $\Leftrightarrow$ the production region must be small compared to $l_{\rm osc}/2\pi$.

The decoherence parameters – contd.

For $\Gamma l_p/v_\pi < 1$ ($l_p < l_{\rm decay}$) $\quad\Rightarrow\quad l_{\rm prod} \sim l_p$

For $\Gamma l_p/v_\pi > 1$ ($l_p > l_{\rm decay}$) $\quad\Rightarrow\quad l_{\rm prod} \sim l_{\rm decay}\;(< l_p)$.

The condition $l_p \ll l_{\rm osc}/2\pi$ in any case guarantees that the production coherence condition is satisfied.

What was wrong with the argument that the prod. coherence always requires $\xi \ll 1$? $\xi = v_P(\Delta m^2/2P\Gamma)$ becomes > 1 for small enough Γ (long-lived parent particle). But then $l_{\rm dec}$ may exceed l_p, and prod. coherence will be governed by ϕ_p rather than by ξ.

$$\xi > 1 \quad\Rightarrow\quad \frac{\Gamma}{v_\pi} < \frac{\Delta m^2}{2P}\,; \qquad\qquad l_p < l_{\rm dec} \quad\Rightarrow\quad \frac{\Gamma}{v_\pi} < l_p^{-1}\,.$$

The second cond. follows from the first if $\Delta m^2/2P < l_p^{-1}$, i.e. $\phi_p < 1$.
It is only in the case $\phi_p > 1$ ($\gg 1$) that the production coherence may be governed by ξ.

A completely different approach

Assume each neutrino production event is completely coherent $\Rightarrow$ neutrino flavor transitions are described by the standard oscillation formula. E.g. for $P_{\mu\mu}$:

$$P_{\mu\mu}(E,L) = P_{\mu\mu}^{\rm stand}(L,E) = c^4 + s^4 + 2s^2c^2\cos\phi\,.$$

Sum the effects of pion decays at different points along the decays tunnel at the probabilities level:

$$F_\mu(E,L) \;=\; F_\pi(E,0)\Gamma\int_0^{l_p} e^{-\frac{\Gamma x}{v_\pi}}\,P_{\mu\mu}^{\rm stand}(E,L-x)dx$$

The effective oscillation probability:

$$P_{\mu\mu}^{\rm eff}(L,E) \;\equiv\; F_\mu(L,E)/F_\mu^0(L,E)$$

(Hernandez & Smirnov, 2011; also performed a simplified calculation at the amplitude level)

Effective probability

The result:

$\diamondsuit$ $$P_{\mu\mu}^{\text{eff}} = c^4 + s^4 + \frac{2c^2 s^2}{\xi^2+1}\,\frac{1}{(1-e^{-\Gamma l_p/v_\pi})}\Big[\cos\phi + \xi\sin\phi$$
$$-e^{-\Gamma l_p/v_\pi}\left[\cos(\phi-\phi_p) + \xi\sin(\phi-\phi_p)\right]\Big]$$

Exactly the same expression as that obtained by summation (integration) at the amplitude level!

Effective probability

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Exactly the same expression as that obtained by summation (integration) at the amplitude level!

$$\Downarrow$$

The simple integration of the oscillation probability along the decay tunnel (source) – in general with the decay exponential taken into account – automatically takes care of the production coherence condition!

Prod. coherence $\Leftrightarrow$ localization of the ν production process.

Why does this happen?

It is well known that decoherence gives the same result as averaging; but why the exact probabilities (with arbitrary degree of coherence) exactly coincide in the two cases?

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Although the position of the pion decay (and neutrino production) point is not exactly known in the WP approach (uncertainty of order l_p for $\Gamma l_p/v_\pi \ll 1$ and $l_{\rm decay} = v_\pi/\Gamma$ for $\Gamma l_p/v_\pi \gg 1$), the *spatial size* of the production region is very small – given by the size of the smallest WP of the particles participating in neutrino production, in our case of the pion. Since we consider pions to be point-like, there is no interference between the amplitudes of neutrino production in different (even closely located) points.

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- $\Rightarrow$ The results of summation at the amplitude level and at the probabilities coincide.

For non-zero $\sigma_{x\pi}$ one can expect some differences between the results of the two approaches, but for $\sigma_{x\pi} \ll \min\{l_p,\, l_{\rm dec}\}$ they should be small.

By integrating first over time t (in the expression for $I_{jk}(L)$) and momenta (in Fourier transformations $f^S_{j,k}(p) \to \psi^S_{j,k}(x,t)$):

$$I_{jk}(L) = \frac{|M_P|^2}{v_g} e^{-i\frac{\Delta m^2_{jk}}{2P}L} \int dx_1 dt_1 \int dx_2\, dt_2\; g_\pi(x_1,t_1)\, g^*_\pi(x_2,t_2) e^{i\frac{\Delta m^2_{jk}}{2P}x_2 - \frac{\Gamma}{2}(t_1+t_2)}$$
$$\times e^{i[E_j(P)-E_P](t_1-t_2)}\, \delta[(x_1-x_2)-v_g(t_1-t_2)].$$

For pointlike pions: $g_\pi(x_1,t_1) \propto \delta(x_1 - v_\pi t_1)$, $g_\pi(x_2,t_2) \propto \delta(x_2 - v_\pi t_2) \quad \Rightarrow$

$$\delta[(x_1-x_2)-v_g(t_1-t_2)] = \delta[(v_g - v_\pi)(t_1-t_2)].$$

$\Rightarrow \quad t_1 = t_2 \quad \Rightarrow \quad x_1 = x_2$, i.e.

$$g_\pi(x_1,t_1)\, g^*_\pi(x_2,t_12) \quad \to \quad |g_\pi(x_1,t_1)|^2$$

No interf. terms in the expression for $|\mathcal{A}|^2$, summation at the probabilities level.

Finite-width pion WP

Two models of finite-size pion WP, Gaussian and box-type. For $\Gamma l_p/v_\pi \gg 1$:

$$\diamondsuit \quad P_{\mu\mu}^{\rm eff} = c^4 + s^4 + \frac{2c^2 s^2}{\xi^2+1}\left[(\cos\phi + \xi\sin\phi) - A_\pi \xi(\xi\cos\phi - \sin\phi)\right]$$

The parameter A_π:

$$A_{\pi{\rm box}} = \frac{v_g}{v_\pi}\frac{\Gamma}{v_g - v_\pi}\sigma_{x\pi}\,, \qquad A_{\pi{\rm Gauss}} = \frac{2}{\sqrt{2\pi}}\frac{v_g}{v_\pi}\frac{\Gamma}{v_g - v_\pi}\sigma_{x\pi}\,.$$

i.e. $A_\pi \sim (v_g/v_\pi)\sigma_{x\pi}/\sigma_{x\nu}$. The correction is of order

$$A_\pi \xi \sim \left[\frac{\Delta m^2}{2P}\sigma_{x\pi}\right]\cdot\frac{v_g}{v_g - v_\pi} = 2\pi\frac{\sigma_{x\pi}}{l_{\rm osc}}\cdot\frac{v_g}{v_g - v_\pi}$$

– small since $\sigma_{x\pi} \lll l_{\rm osc}$ (unless $v_\pi \simeq v_g$ to a very high accuracy).

An interesting point: summation at the probabilities level for finite-thickness $(= d)$ proton target and point-like neutrino production gives similar expression, but with $A_\pi \xi = (\Delta m^2/2P)d$ (no factor $[v_g/(v_g - v_\pi)]$).

Production coherence for some experiments

Unless otherwise specified, $\Delta m^2 = 2$ eV^2. For β-beams $E_0 = 2$ MeV, $\tau_0 = 1$s, $\gamma = 100$.

Experiment	$\langle E_\nu \rangle$(MeV)	L(m)	$l_p(m)$	$l_{\rm dec}$(m)	$l_{\rm osc}$(m)	ϕ	$\Gamma l_p / v_P$	ϕ_p	ξ
LSND	$\sim$40	30	0	0	50	3.8	-	0	0
KARMEN	$\sim$40	17.7	0	0	50	2.24	-	0	0
MiniBooNE	$\sim$800	541	50	89	992	3.43	0.56	0.32	0.56
NOMAD	$2.7 \cdot 10^4$	770	290	3009	33480	0.145	0.1	0.054	0.56
(20 eV^2)					3348	1.45	0.1	0.54	5.64
CCFR(10^2 eV^2)	$5 \cdot 10^4$	891	352	5570	1240	4.51	0.06	1.78	28.2
CDHS	3000	130	52	334	3720	0.22	0.155	0.088	0.56
(20 eV^2)					372	2.2	0.155	0.878	5.64
K2K	1500	300	200	167	1861	1.01	1.2	0.68	0.56
T2K	600	280	96	66.4	744	2.36	1.45	0.81	0.56
Minos	3300	1040	675	368	4092	1.6	1.84	1.04	0.56
NOνA	2000	1040	675	223	2480	2.64	3.03	1.71	0.56
β-beams	400	$1.3 \cdot 10^5$	2500	$3 \cdot 10^{10}$	496	1647	$8.3 \cdot 10^{-8}$	31.7	$3.8 \cdot 10^8$

Noticeable effects for MiniBooNE, NOMAD (20 eV^2), CCFR (100 eV^2), CDHS (20 eV^2), K2K, T2K, MINOS, NOνA, very large effects for β-beams

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The same applies to neutrino detection!

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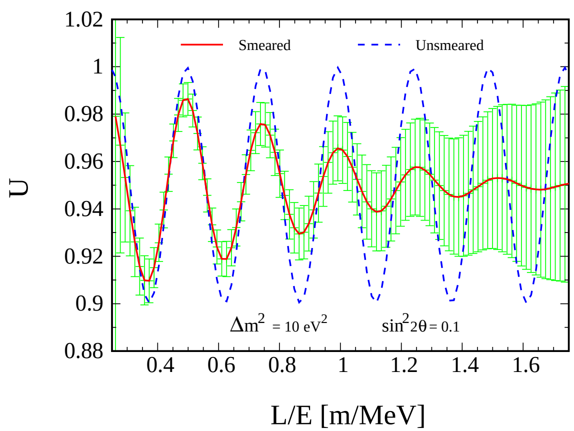
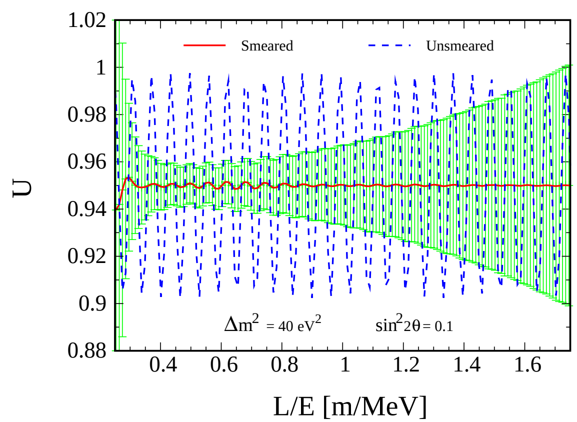
The analysis also applies to neutrinos produced in any other decay process (K-decay, μ-decay, β-decay, ...), provided that particles accompanying ν production are undetected. If they are detected, production coherence depends on the degree of their localization (coherence typically improved).

Examples of prod. coherence violation

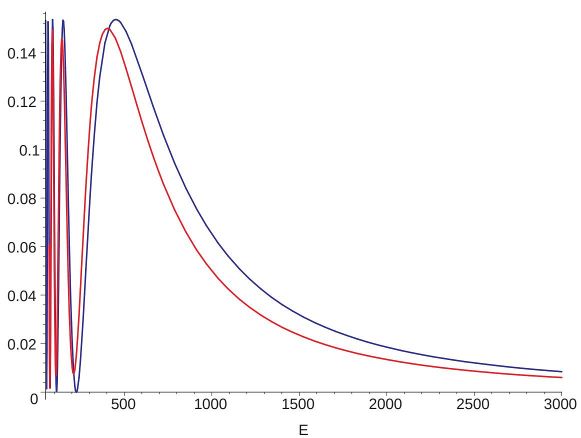
$\nu_e \to \nu_s$ oscillations in β-beam expts. (Agarwalla, Huber & Link, arXiv:0907.3145).

Ratio of oscillated and unoscillated fluxes ($\gamma = 30,\ l_p = 10$m, $L = 50$ m):

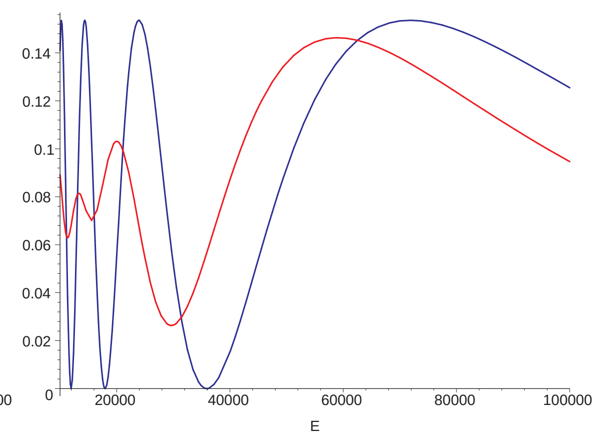




T2K



CCFR



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- For sterile neutrinos with $\Delta m^2 \gtrsim 1$ eV^2 production/detection coherence is no longer automatic and has to be carefully examined.

- QM wave packet formalism provides a consistent framework for that and allows to obtain the result through the summation of amplitudes corresponding to decays of parent particles in different points at the amplitude level.

- The production coherence depends on the oscillation phase acquired over the neutrino production region and is satisfied when this phase is small. The parameters governing the coherence are ϕ_p for $l_{\rm dec} > l_p$ and ξ for $l_{\rm dec} < l_p$.

Summary – contd.

- The obtained probability coincides exactly with the one obtained by incoherent summation of oscill. probabilities over the neutrino production points (with the proper decay exponential included). This coincidence is a consequence of vanishing spatial size of the wave packets of parent particles σ_{xP}. Approximately holds even if $\sigma_{xP} \neq 0$ provided that it is small compared to l_p and $l_{\rm osc}$.

Summary – contd.

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- When averaging over the neutrino source and neutrino detector is properly done, possible production/detection decoherence effects are automatically taken into account. No need for more sophisticated methods!

Backup slides

$$B_j \equiv \frac{E_P - E_j(P)}{v_j - v_\pi}\,.$$

Note a useful relation

$$E_j(P_j) - E_k(P_k) = v_\pi (B_j - B_k)\,.$$

From the definition of B_j:

$$B_j - B_k \simeq \frac{E_k(P) - E_j(P)}{v_g - v_\pi} + \left[E_P - \frac{E_j(P) + E_k(P)}{2}\right] \frac{v_k - v_j}{(v_g - v_\pi)^2}\,,$$

where v_g is the average group velocity of the neutrino mass eigenstates ν_j and ν_k, and terms $\sim (\Delta m_{jk}^2)^2$ have been discarded. The second term here has the smallness $\sim \Gamma/P$ compared to the first term and so can be safely neglected.

Lorentz invariance of oscillation probability

1. “Paradox” of neutrino w. packet length

For neutrino production in decays of unstable particles at rest (e.g. $\pi \to \mu\nu_\mu$):

$$\sigma_E \simeq \tau^{-1} = \Gamma_\pi \,, \qquad\qquad \sigma_x \simeq \frac{v_g}{\sigma_E} \simeq \frac{v_g}{\Gamma_\pi} \;(= v_g \tau)$$

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For decay in flight: $\Gamma'_\pi = (m_\pi/E_\pi)\Gamma_\pi$. One might expect

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On the other hand, if the decaying pion is boosted in the direction of the neutrino momentum, the neutrino w. packet should be Lorentz-contracted !

<u>The solution:</u> pion decay takes finite time. During the decay time the pion moves over distance $l = u\tau'$ ("chases" the neutrino if $u > 0$).

$$\sigma'_x \simeq v'_g/\Gamma' - l = v'_g\tau' - u\tau' = (v'_g - u)\gamma_u\tau = \frac{v_g\tau}{\gamma_u(1+v_g u)}\,,$$

[the relativ. law of addition of velocities: $v'_g = (v_g + u)/(1 + v_g u)$].

Lorentz invariance issues – contd.

That is

$$\sigma'_x = \frac{\sigma_x}{\gamma_u(1+v_g u)}$$

For relativistic neutrinos $v_g \approx v'_g \approx 1 \quad \Rightarrow$

$$\sigma'_x = \sigma_x \sqrt{\frac{1-u}{1+u}}$$

$\Rightarrow$ when the pion is boosted in the direction of neutrino emission $(u > 0)$ the neutrino wave packet gets contracted; when it is boosted in the opposite direction $(u < 0)$ – the wave packet gets dilated.

Coherence production conditions

Coherence production conditions:

$$|\Delta E| \ll \sigma_E\,, \qquad\qquad |\Delta p| \ll \sigma_p\,.$$

On the other hand:

$$\Delta E \simeq v_g \Delta p + \frac{\Delta m^2}{2E}.$$

Constraint $|\Delta E| \ll \sigma_E \quad \Rightarrow$

$$\left|\frac{v_g \Delta p}{\sigma_E} + \frac{\Delta m^2}{2E\sigma_E}\right| \ll 1. \qquad\qquad (*)$$

(a) The two terms in ΔE do not approximately cancel each other. $\Rightarrow$ $v_g|\Delta p| \ll \sigma_E \le \sigma_p$, i.e. for relativistic neutrinos $|\Delta p| \ll \sigma_p$ follows from $|\Delta E| \ll \sigma_E$.

(b1) There is a strong cancellation, but both terms on the l.h.s. of (*) are small – see case (a).

(b2) Strong cancellation, but both terms on the l.h.s. of (*) are $\gtrsim 1$: momentum condition is independent. But: the only known case – Mössbauer neutrinos.

When are neutrino oscillations observable?

How are the oscillations destroyed? Suppose by measuring momenta and energies of particles at neutrino production (or detection) we can determine its energy E and momentum p with uncertainties σ_E and σ_p. From $E_i^2 = p_i^2 + m_i^2$:

$$\sigma_{m^2} = \left[(2E\sigma_E)^2 + (2p\sigma_p)^2\right]^{1/2}$$

When are neutrino oscillations observable?

If $\sigma_{m^2} < \Delta m^2 = |m_i^2 - m_k^2|$ – one can tell which mass eigenstate is emitted.

$\sigma_{m^2} < \Delta m^2$ implies $2p\sigma_p < \Delta m^2$, or $\sigma_p < \Delta m^2/2p \simeq l_{\rm osc}^{-1}$.

But: To measure p with the accuracy σ_p one needs to measure the momenta of particles at production with (at least) the same accuracy $\Rightarrow$ uncertainty of their coordinates (and the coordinate of ν production point) will be

$$\sigma_{\rm x,\,prod} \gtrsim \sigma_p^{-1} > l_{\rm osc}$$

$\Rightarrow$ Oscillations washed out. Similarly for neutrino detection.

Natural necessary condition for coherence (observability of oscillations):

$$L_{\rm source} \ll l_{\rm osc}\,, \qquad L_{\rm det} \ll l_{\rm osc}$$

No averaging of oscillations in the source and detector

Satisfied with very large margins in most cases of practical interest

Wave packet separation

Wave packets representing different mass eigenstate components have different group velocities v_{gi} $\Rightarrow$ after time $t_{\rm coh}$ (coherence time) they separate $\Rightarrow$ Neutrinos stop oscillating! (Only averaged effect observable).

Coherence time and length:

$$\Delta v \cdot t_{\rm coh} \simeq \sigma_x\,; \qquad\qquad l_{\rm coh} \simeq v t_{\rm coh}$$

$$\Delta v = \frac{p_i}{E_i} - \frac{p_k}{E_k} \simeq \frac{\Delta m^2}{2E^2}$$

$$\boxed{l_{\rm coh} \simeq \frac{v}{\Delta v}\sigma_x = \frac{2E^2}{\Delta m^2}\,v\sigma_x}$$

The standard formula for $P_{\rm osc}$ is obtained when the decoherence effects are negligible.

Oscillations and QM uncertainty relations

Neutrino oscillations – a QM interference phenomenon, owe their existence to QM uncertainty relations

Neutrino energy and momentum are characterized by uncertainties σ_E and σ_p related to the spatial localization and time scale of the production and detection processes. These uncertainties

- allow the emitted/absorbed neutrino state to be a coherent superposition of different mass eigenstates
- determine the size of the neutrino wave packets $\Rightarrow$ govern decoherence due to wave packet separation

σ_E – the effective energy uncertainty, dominated by the smaller one between the energy uncertainties at production and detection. Similarly for σ_p.

The paradox of σ_E and σ_p

QM uncertainty relations: σ_p is related to the spatial localization of the production (detection) process, while σ_E to its time scale $\Rightarrow$ independent quantities.

On the other hand: Neutrinos propagating macroscopic distances are on the mass shell. For on-shell mass eigenstates $E^2 = p^2 + m_i^2$ means

$$E\sigma_E = p\sigma_p$$

How can this be understood?

The solution: At production, neutrinos are *not* on the mass shell. They go on shell only after they propagate $x \sim$ (a few)$\times$ De Broglie wavelengths. After that their energy and momentum get related by $E^2 = p^2 + m_i^2 \quad \Rightarrow$ the larger uncertainty shrinks towards the smaller one to satisfy $E\sigma_E = p\sigma_p$.

On-shell relation between E and p allows to determine the less certain of the two through the more certain one, reducing the error of the former.

What determines the length of ν w. packets?

The length of ν w. packets: $\sigma_x \sim 1/\sigma_p$. For propagating on-shell neutrinos:

$$\sigma_p \simeq \min\{\sigma_p^{\rm prod},\,(E/p)\sigma_E^{\rm prod}\} = \min\{\sigma_p^{\rm prod},\,(1/v_g)\sigma_E^{\rm prod}\}$$

Which uncertainty is smaller at production, $\sigma_p^{\rm prod}$ or $\sigma_E^{\rm prod}$?

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- If $T_S > \tau$ (quasi-free parent particle) $\Rightarrow$ $\sigma_E \simeq \tau^{-1} = \Gamma$.

  $\sigma_p \simeq [(p/E)\tau]^{-1} \simeq [(p/E)\sigma_E]^{-1}$, i.e. $\sigma_E \simeq (p/E)\sigma_p < \sigma_p$.

The length of ν w. packets – contd.

In both cases $\boxed{\sigma_E^{\rm prod} < \sigma_p^{\rm prod}}$ $\Leftarrow$ also when $\nu's$ are produced in collisions.

$$\Longrightarrow \qquad \sigma_{p\ \rm eff} \simeq \frac{\sigma_E}{v_g}\,, \qquad\qquad \boxed{\sigma_x \simeq \frac{v_g}{\sigma_E}}$$

In the stationary limit $(\sigma_E \to 0)$ one has $\sigma_{p\ \rm eff} \to 0$ even though σ_p is finite! Therefore $\sigma_x \to \infty$ and so the coherence length $l_{\rm coh} \to \infty$

– a well known result.