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Strong-field QED in intense laser fields

Antonino Di Piazza

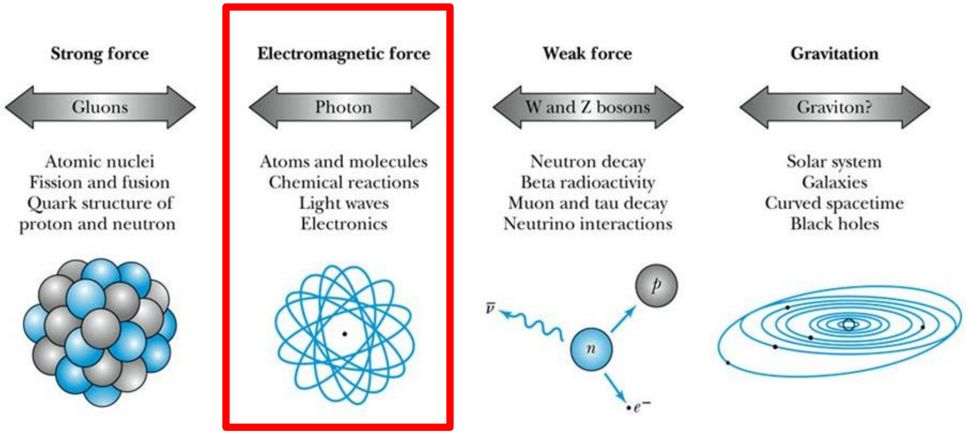
Particle and Astroparticle Theory Seminar
Max Planck Institute for Nuclear Physics, Heidelberg
July 10th 2017

Outline

- Introduction to classical electrodynamics (CED) and quantum electrodynamics (QED)
- Radiation reaction in CED and QED
- Strong-field QED in an intense laser field
- Nonlinear laser-neutrino interaction
- Conclusions

Electromagnetic interaction

- The electromagnetic interaction is one of the four fundamental interactions in Nature
- Limiting to the lightest charged particles (electrons and positrons), the Lagrangian of the theory depends on two parameters:



  - Electron mass m=9.1×10^{-28} g
  - Electron charge e, with $|e|$=4.8×10^{-10} statcoulomb
- The typical scales of classical electrodynamics (CED) are determined by combining m and e with another fundamental constant:
  - Speed of light c=3.0×10^{10} cm/s
- In quantum electrodynamics (QED) the dynamics is richer:
  - Reduced Planck constant $\hbar$ =1.1×10^{-27} erg s

Typical scales of CED and QED

	CED	QED
Energy	Electron's rest energy: $\varepsilon_0 = mc^2 = 0.5$ MeV	
Momentum	$p_0 = \varepsilon_0/c = 0.5$ MeV$/c$	
Length	Classical electron's radius: $r_0 = e^2/mc^2 = 2.8\times10^{-13}$ cm (from the Thomson cross section)	Compton wavelength: $\lambda_C = \hbar/p_0 = 3.9\times10^{-11}$ cm (from Heisenberg uncertainty principle)
Time	$\tau_0 = r_0/c = 1.0\times10^{-23}$ s	$\tau_C = \lambda_C/c = 1.3\times10^{-21}$ s

$r_0 = \alpha\lambda_C$, where $\alpha = e^2/\hbar c \approx 1/137$ is the fine-structure constant

Field scales of QED (critical or Schwinger field)

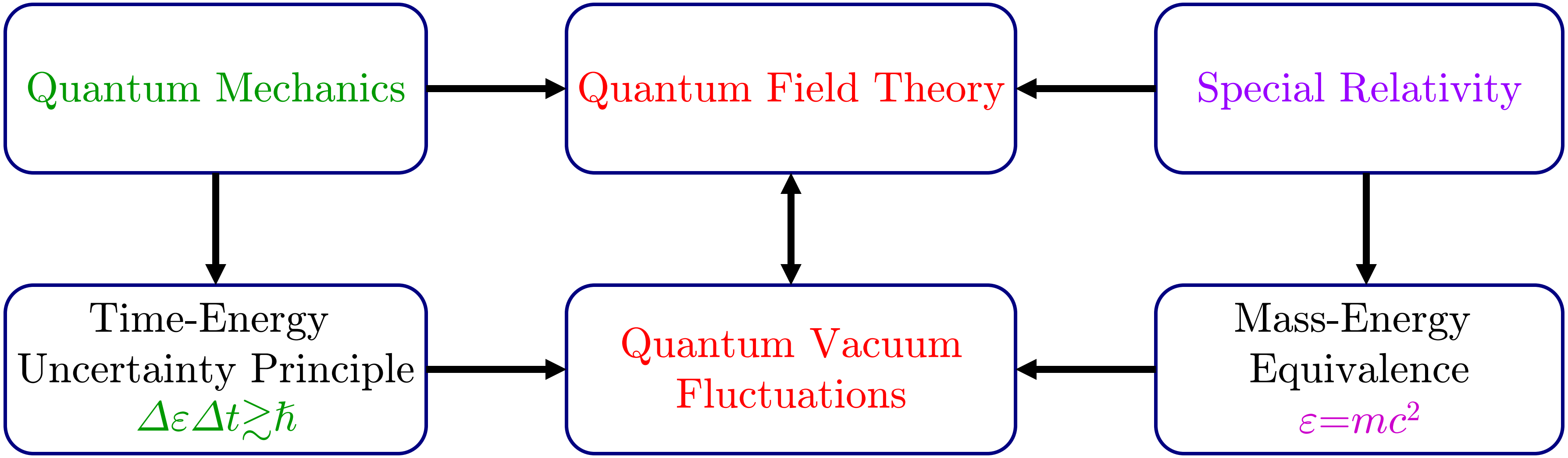
$$E_{cr} = \frac{m^2c^3}{\hbar|e|} = 1.3\times10^{16}\ \mathrm{V/cm}$$

$$B_{cr} = \frac{m^2c^3}{\hbar|e|} = 4.4\times10^{13}\ \mathrm{G}$$

$$\Rightarrow\quad I_{cr} = \frac{cE_{cr}^2}{4\pi} = 4.6\times10^{29}\ \mathrm{W/cm^2}$$

QED critical fields and vacuum physics

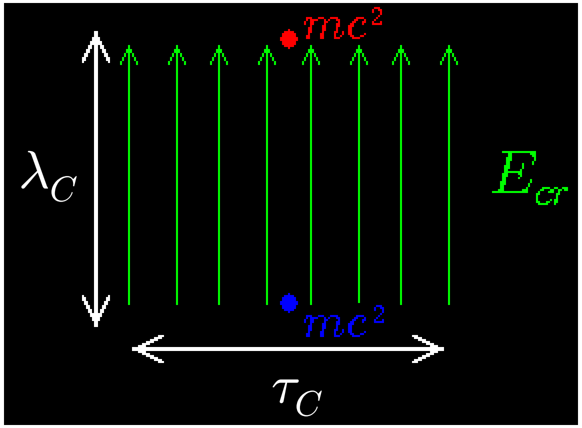
- The vacuum state is defined as the lowest-energy state where no particles are present and all fields vanish
- In quantum field theory no real particles are present in the vacuum and the average fields vanish but
  - "Fluctuations" of particles-antiparticles are present (variance of the fields)
  - They "live" for a very short time and cover a very short distance (for electrons and positrons $\tau_C = \hbar/mc^2 \sim 10^{-21}$ s and $\lambda_C = \hbar/mc \sim 10^{-11}$ cm, respectively)



- Physical meaning of the critical fields:

$$|e|E_{cr} \times \frac{\hbar}{mc} = mc^2 \quad \Rightarrow \quad |e|\mathcal{E}_0 \times r_0 = mc^2$$

$$\frac{|e|\hbar}{mc} \times B_{cr} = mc^2$$



- In the presence of background electromagnetic fields of the order of the critical ones a new regime of QED, the strong-field QED regime, opens:

  1. where the properties of the vacuum are substantially altered by the fields
  2. where a tight interplay unavoidably exists between collective (plasma-like) and quantum effects
  3. which is inaccessible to conventional accelerators because it requires coherent fields

- One can analogously introduce a critical electric field $\mathcal{E}_0$ (a critical magnetic field $\mathcal{B}_0$) of CED, which is $1/\alpha \approx 137$ times larger than E_{cr} (B_{cr})

- QED in vacuum is considered to be the most successful physical theory in terms of agreement with experiments
- Experiments on QED in the presence of intense background electromagnetic fields
  - ✓ aim at testing the theory in a sector complementary to the conventional high-energy/short-distance sector explored by means of accelerator facilities (interaction among many particles at the same time)
  - ✓ are not comparably numerous and accurate as those in vacuum
- The reasons are:
  1. on the experimental side, the critical electromagnetic field of QED is very "large"
  2. on the theoretical side, exact analytical calculations are feasible only for a few background electromagnetic fields: constant and uniform electric/magnetic field, Coulomb field, plane-wave field

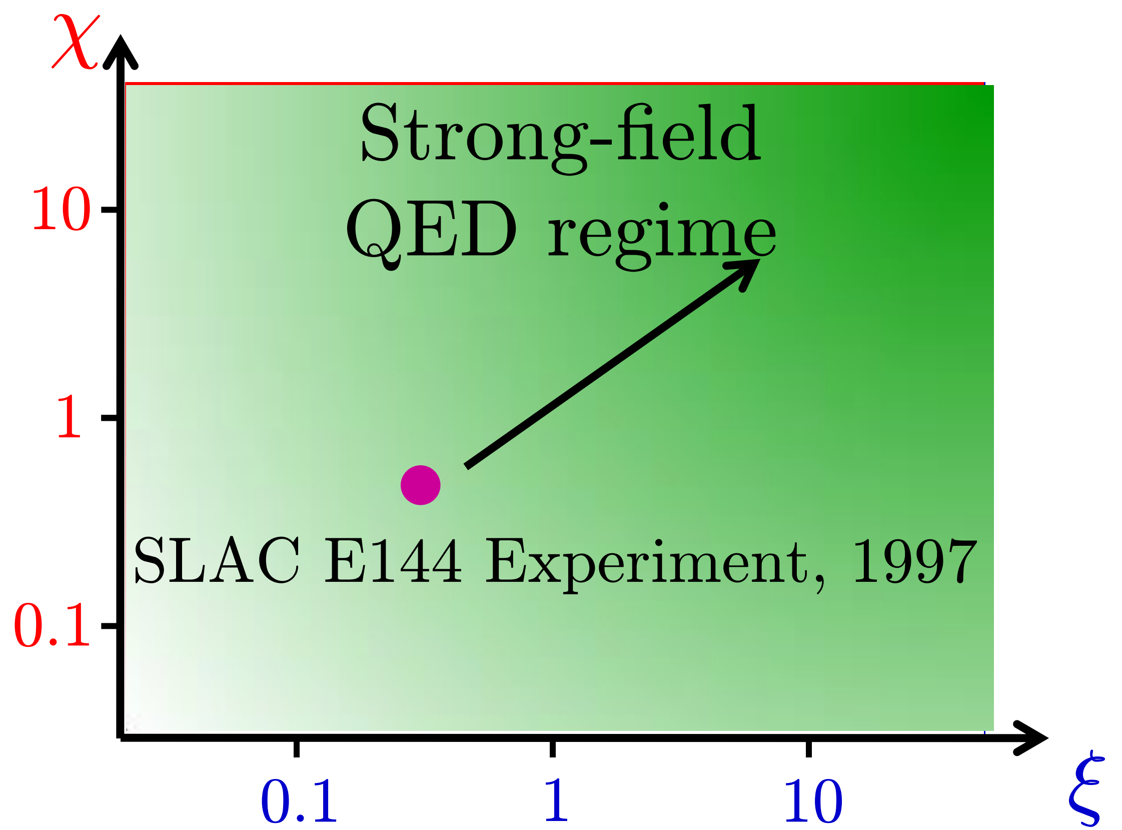
Strong-field QED in a strong laser field

- An electron with energy ε collides head-on with a plane wave with amplitude E_L and angular frequency ω_L (wavelength λ_L)

- The physical observables depend on the two Lorentz- and gauge-invariant parameters (Ritus 1985):

$$\xi = \frac{1}{2\pi}\frac{|e|E_L\lambda_L}{mc^2} = \frac{|e|E_L\lambda_C}{\hbar\omega_L}$$

$$\chi = \left.\frac{E_L}{E_{cr}}\right|_{\text{rest frame}} \approx \frac{2\varepsilon}{mc^2}\frac{E_L}{E_{cr}}$$



Optical laser technology

Optical laser technology ($\hbar\omega_L$=1 eV, λ_L=1 μm)	Energy (J)	Pulse duration (fs)	Spot radius (μm)	Intensity (W/cm^2)
State-of-art (Yanovsky et al., Opt. Express 2008)	10	30	1	2×10^{22}
Soon (APOLLON, Vulcan, ELI Beamlines etc...)	10÷100	10÷100	1	$10^{22}\div10^{23}$
Near future (ELI 4$^{\text{th}}$ pillar, XCELS)	10^4	10	1	$10^{25}\div10^{26}$

Electron accelerator technology

Electron accelerator technology	Energy (GeV)	Beam duration (fs)	Spot radius (μm)	Number of electrons
Conventional accelerators (PDG)	10÷50	$10^3\div10^4$	10÷100	$10^{10}\div10^{11}$
Laser-plasma accelerators (Leemans et al., Phys. Rev. Lett. 2014)	4.2	40	50	8×10^8

$$\xi = 7.5\frac{\sqrt{I_L[10^{20}\ \mathrm{W/cm^2}]}}{\hbar\omega_L[\mathrm{eV}]}$$

$$\chi = 5.9\times10^{-2}\,\varepsilon[\mathrm{GeV}]\sqrt{I_L[10^{20}\ \mathrm{W/cm^2}]}$$

Present technology allows in principle the experimental investigation of strong-field QED

Radiation reaction in CED

- What is the equation of motion of an electron in an external, given electromagnetic field $F^{\mu\nu}(x)$?

Units with $\hbar = c = 1$

- The Lorentz equation

$$m\frac{du^\mu}{ds} = eF^{\mu\nu}u_\nu$$

does not take into account that while being accelerated the electron generates an electromagnetic radiation field and it loses energy and momentum

- One has to solve self consistently the coupled Lorentz and Maxwell equations (Barut 1980)

$$m_0\frac{du^\mu}{ds} = eF_T^{\mu\nu}u_\nu$$

$$\partial_\mu F_T^{\mu\nu} = 4\pi e\int ds\delta(x - x(s))u^\nu$$

Lorenz gauge →

$$m_0\frac{du^\mu}{ds} = e(\partial^\mu A_T^\nu - \partial^\nu A_T^\mu)u_\nu$$

$$\Box A_T^\nu = 4\pi e\int ds\delta(x - x(s))u^\nu$$

where now m_0 is the electron's bare mass and $F_{T,\mu\nu} = \partial_\mu A_{T,\nu} - \partial_\nu A_{T,\mu}$ is the total electromagnetic field (external field plus the one generated by the electron)

- One first solves the inhomogeneous wave equation exactly with the Green's-function method

$$\Box A_T^\nu = 4\pi e \int ds \delta(x - x(s)) u^\nu = 4\pi j^\nu(x) \longrightarrow A_T^\nu(x) = A^\nu(x) + \int d^4x' \mathcal{D}_R(x - x') j^\nu(x')$$

and then re-substitute the solution into the Lorentz equation:

$$(m_0 + \delta m)\frac{du^\mu}{ds} = eF^{\mu\nu}u_\nu + \frac{2}{3}e^2\left(\frac{d^2u^\mu}{ds^2} + \frac{du^\nu}{ds}\frac{du_\nu}{ds}u^\mu\right)$$

where δm is a quantity which diverges for a pointlike charge

- After "classical mass renormalization" one obtains the Lorentz-Abraham-Dirac (LAD) equation

$$m\frac{du^\mu}{ds} = eF^{\mu\nu}u_\nu + \frac{2}{3}e^2\left(\frac{d^2u^\mu}{ds^2} + \frac{du^\nu}{ds}\frac{du_\nu}{ds}u^\mu\right)$$

- The LAD equation is plagued by serious inconsistencies: runaway solutions. Consider its three-dimensional non-relativistic limit

$$m\frac{d\boldsymbol{v}}{dt} = e(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) + \frac{2}{3}e^2\frac{d^2\boldsymbol{v}}{dt^2}$$

In the free case $\boldsymbol{E}=\boldsymbol{B}=\boldsymbol{0}$, it admits the solution $\boldsymbol{a}(t)=\boldsymbol{a}_0e^{t/\tau_0}$, where $\tau_0=(2/3)e^2/m\sim10^{-23}$ s.

- Avoiding the runaways: integro-differential LAD equation (Rohrlich 1961)

$$m\frac{du^\mu}{ds} = \frac{e^{s/\tau_0}}{\tau_0}\int_s^\infty ds' e^{-s'/\tau_0}\left(eF^{\mu\nu}u_\nu + \frac{2}{3}e^2\frac{du^\nu}{ds'}\frac{du_\nu}{ds'}u^\mu\right)$$

- Problem: preacceleration at time scales of the order of τ_0
- If $\max|F^{\mu\nu}(x)|_{\text{inst. rest-frame}} \ll \mathcal{F}_0^{\mu\nu}=(\mathcal{E}_0,\mathcal{B}_0)$, one can replace the four-acceleration du^μ/ds in the radiation-reaction force in the LAD equation

$$m\frac{du^\mu}{ds} = eF^{\mu\nu}u_\nu + \boxed{\frac{2}{3}e^2\left(\frac{d^2u^\mu}{ds^2} + \frac{du^\nu}{ds}\frac{du_\nu}{ds}u^\mu\right)}$$

with the zero-order four-acceleration $eF^{\mu\nu}u_\nu/m$ (Landau and Lifshitz 1947)
- Since $(\mathcal{E}_0,\mathcal{B}_0)=(E_{cr},B_{cr})/\alpha\approx 137(E_{cr},B_{cr})$, the above condition is always fulfilled by definition in the realm of CED
- The resulting equation

$$m\frac{du^\mu}{ds} = eF^{\mu\nu}u_\nu+\frac{2}{3}e^2\left[\frac{e}{m}(\partial_\alpha F^{\mu\nu})u^\alpha u_\nu-\frac{e^2}{m^2}F^{\mu\nu}F_{\alpha\nu}u^\alpha+\frac{e^2}{m^2}(F^{\alpha\nu}u_\nu)(F_{\alpha\lambda}u^\lambda)u^\mu\right]$$

is known as Landau-Lifshitz (LL) equation and there are proposals to test it experimentally

Radiation reaction in QED

- In CED solving the LAD (or the LL) equation is equivalent to solve self-consistently Maxwell and Lorentz equations

$$m_0\frac{du^\mu}{ds} = eF_T^{\mu\nu}u_\nu$$
$$\partial_\mu F_T^{\mu\nu} = e\int ds\delta(x-x(s))u^\nu$$
$$\longleftrightarrow \quad m\frac{du^\mu}{ds} = eF^{\mu\nu}u_\nu + \frac{2}{3}e^2\left(\frac{d^2u^\mu}{ds^2} + \frac{du^\nu}{ds}\frac{du_\nu}{ds}u^\mu\right)$$

- This corresponds in QED to determine the evolution of a single-electron state in background field+radiation field generated by the electron

$|t=-\infty\rangle = |e^-\rangle \longrightarrow$ Complete evolution operator (S-matrix) $\longrightarrow |t=+\infty\rangle$

- In strong-field QED including "radiation reaction" amounts in accounting for all possible quantum processes arising with an electron in the initial state (multiparticle effects)
- At $\chi \sim 0.1$ (moderately quantum regime) the multiple incoherent emission gives the main contribution and pair production is still negligible (Di Piazza et al., PRL 2010)

QED in a strong background field

Lagrangian density of QED in the presence of a background field $A_{B,\mu}(x)$ (Furry 1951)

$$\mathscr{L}_{QED} = \mathscr{L}_e + \mathscr{L}_\gamma + \mathscr{L}_{\text{int}}$$

$$\mathscr{L}_e = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi \qquad \mathscr{L}_e = \bar{\psi}[\gamma^\mu(i\partial_\mu - eA_{B,\mu}) - m]\psi$$

$$\mathscr{L}_\gamma = -\frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu} \quad \longrightarrow \quad \mathscr{L}_\gamma = -\frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu}$$

$$\mathscr{L}_{\text{int}} = -e\bar{\psi}\gamma^\mu\psi(A_\mu + A_{B,\mu}) \qquad \mathscr{L}_{\text{int}} = -e\bar{\psi}\gamma^\mu\psi A_\mu$$

- Only the interaction between the spinor and the radiation field is treated perturbatively

1. Solve the Dirac equation $[\gamma^\mu(i\partial_\mu - eA_{B,\mu}) - m]\psi=0$
   - find the "dressed" one-particle in- and out-electron states and the "dressed" electron propagator
2. Write the Feynman diagrams of the process at hand
3. Calculate the amplitude and then the cross sections (or the rates) using "dressed" states and propagators

QED in a strong laser field

- The laser field is approximated by a plane-wave field: $A_{B,\mu}(x) = A_{L,\mu}(\phi)$, $\phi = (k_L x) = \omega_L t - \boldsymbol{k}_L \cdot \boldsymbol{r}$ and $\omega_L = |\boldsymbol{k}_L|$

- One-particle states: Volkov states (Volkov 1936)

$$\psi_{p,\sigma} = \left[1 + \frac{e}{2(k_L p)} \hat{k}_L \hat{A}_L\right] \frac{u_{p,\sigma}}{\sqrt{2p_0}} \exp\left\{ \underbrace{-i(px) - i\int_{-\infty}^{\phi} d\phi' \left[\frac{e(pA_L)}{(k_L p)} - \frac{e^2 A_L^2}{2(k_L p)}\right]}_{i(\text{Classical action})} \right\}$$

Spin term

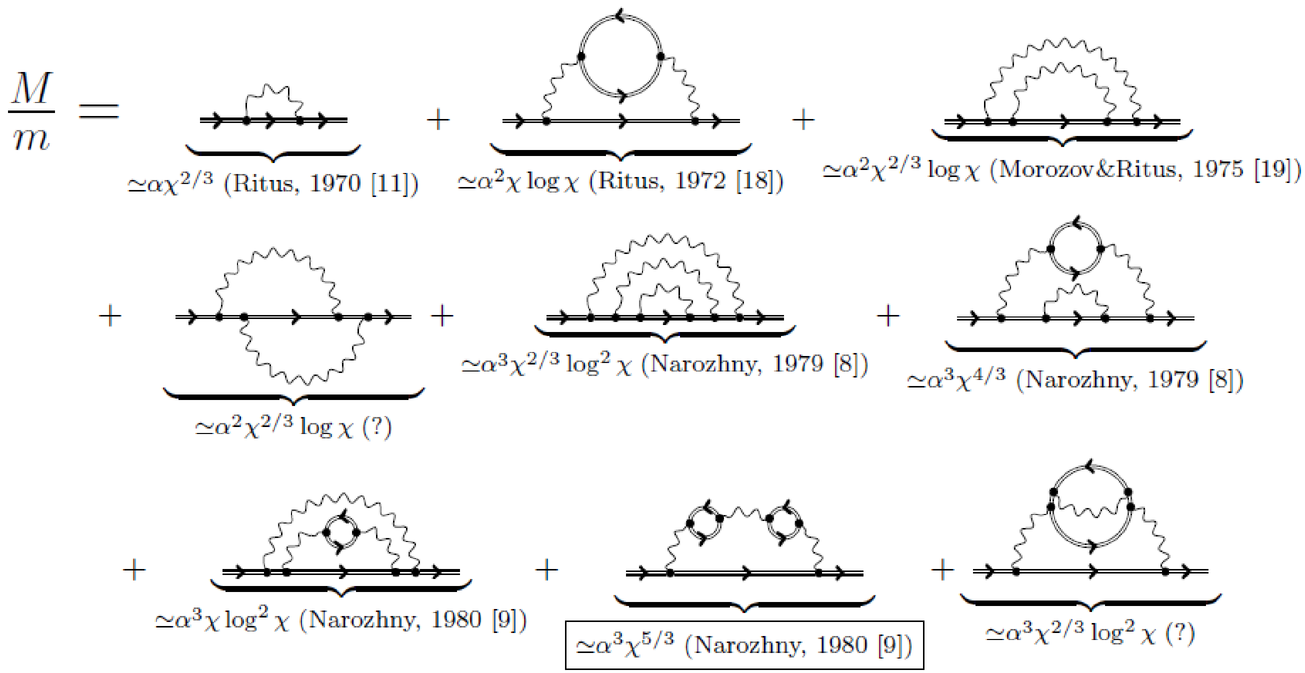
Electron momentum and spin at $t \to -\infty$

Free constant bi-spinor

- Technical notes:
  - Furry approach necessary for $\xi \gtrsim 1$ (Ritus 1985)
  - Volkov states are quasiclassical
  - The average spin four-vector fulfills the classical Bargmann-Michel-Telegdi (BMT) equation
  - In- and out-states are equivalent (differ only by a constant phase) as the vacuum in a plane wave is stable

Radiative corrections in strong-field QED

- A very interesting feature of strong-field QED is the scaling of radiative corrections at $\chi \gg 1$ (Fedotov 2016)



- In QED in vacuum the applicability of perturbation is ensured to be valid for any foreseeable energies $\varepsilon \gg m$ such that $(\alpha/3\pi)\log(\varepsilon^2/m^2) \ll 1$
- At $\chi \gg 1$ the effective coupling of strong-field QED seems to be $\alpha\chi^{2/3}$, and the theory becomes strongly coupled like QCD

Nonlinear neutrino-laser interaction

- If neutrinos have (different) masses they can decay electromagnetically in vacuum (Pal and Wolfenstein 1982)
- The decay rate in the Dirac case is given by

$$\Gamma_{21} = \frac{\alpha G_F^2}{128\pi^4}\left(\frac{m_2^2 - m_1^2}{m_2}\right)^3 (m_1^2 + m_2^2) \times \left|\sum_a U_{1a}U_{2a}^* F(r_a)\right|^2$$

Notation: G_F=1.2×10^{-5} GeV^{-2}=Fermi constant, m_1, m_2=neutrinos' masses, U_{ab}=unitary mixing matrix, $F(r)=-3/2+(3/4)r+\ldots$, $r_a=(m_{la}/m_W)^2$, with m_{la}(m_W=80.4 GeV) being the mass of the corresponding lepton (W-boson)

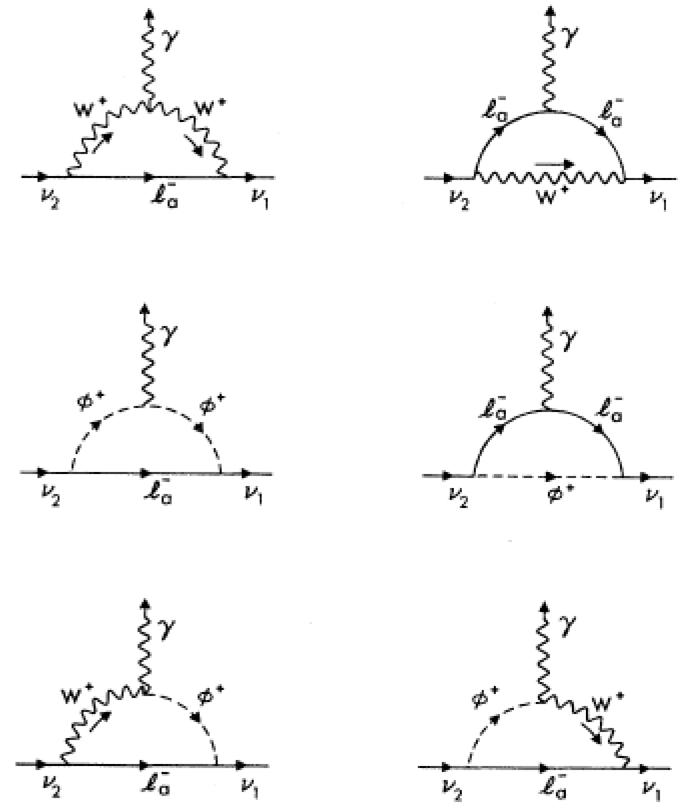


FIG. 1. Diagrams in the 't Hooft–Feynman gauge contributing to the process $\nu_2 \to \nu_1 + \gamma$ for Dirac neutrinos ν_2 and ν_1.

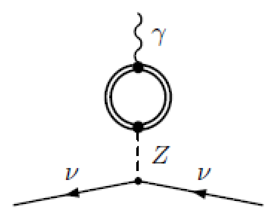
- Due to the unitarity of the mixing matrix, the decay is (further) suppressed: Glashow-Iliopoulos-Maiani (GIM) suppression

- In the presence of a background electromagnetic field each lepton family would contribute differently to the neutrino decay rate (Gvodzev et al. 1996)

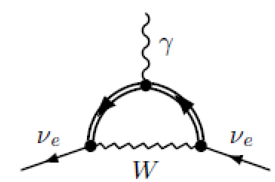
$$\frac{\Gamma_B}{\Gamma_0} \sim \chi_\nu^2 \left(\frac{m_W}{m_\tau}\right)^4$$

where $\chi_\nu = (E_\nu/m_\nu)(B/B_{cr})$, with $B_{cr} = m^2/|e| = 4.4\times10^{13}$ G being the (electron) critical field, and $m_\tau = 1.8$ GeV

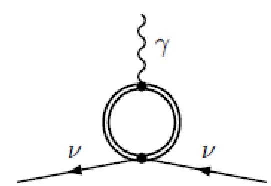
- We investigated the nonlinear interaction between a massless neutrino and a strong laser fields (Meuren et al. 2015)
- Due to the large masses of the gauge bosons, the leading contribution is given by three diagrams (the double lines indicated Volkov propagators)
- The contribution from the W-boson exchange has to be taken into account only for an electron neutrino



(a) Z boson exchange.



(b) W boson exchange.



(c) Local limit (Fermi description).

- Despite initial claims (Becker et al. 1981), laser-field effects on weak processes are expected to be tiny (Akhmedov 1983, 2010)

- In the Standard Model the lepton-gauge-boson interaction Lagrangians are

$$\mathcal{L}_{\rm L}^{\rm Z} = -\frac{g}{2\cos\theta_W} Z_\mu J_{\rm Z}^\mu, \qquad J_{\rm Z}^\mu = \bar{\psi}_f \left[g_v^{(f)} \gamma^\mu + g_a^{(f)} \gamma^\mu \gamma^5 \right] \psi_f$$

where θ_W is the Weinberg angle and $g_{v/a}^{(f)}$ are numerical coefficients depending on the lepton family, and

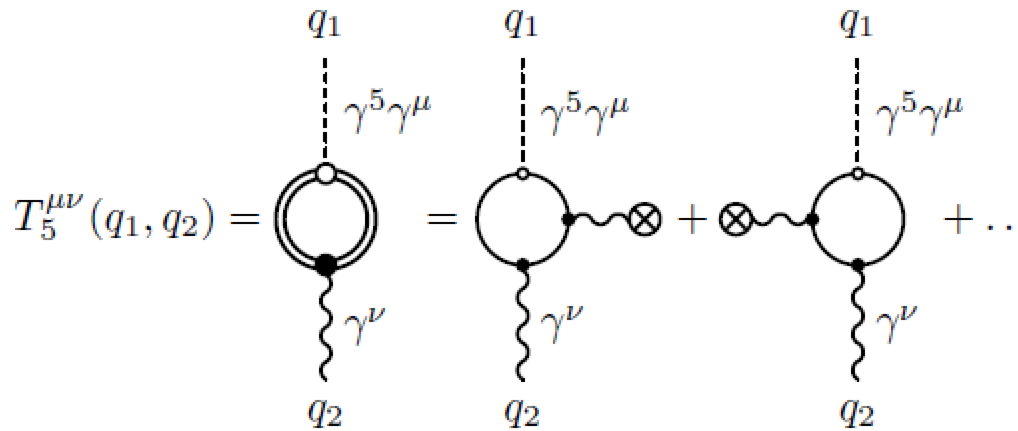
$$\mathcal{L}_e^{\rm W} = -\frac{g}{2\sqrt{2}} [W_\mu^+ J_{{\rm W},e}^\mu + W_\mu^- (J_{{\rm W},e}^\mu)^\dagger], \qquad J_{{\rm W},e}^\mu = \bar{\psi}_{\nu_e} \gamma^\mu (1 + \gamma^5) \psi_e.$$

- It is clear that in order to describe the interaction neutrino we need to compute the axial-vector-vector current coupling pseudo-tensor

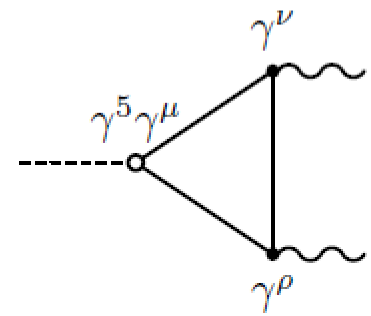
$$T_5^{\mu\nu}(q_1, q_2) = \int \frac{d^4p\, d^4p'}{(2\pi)^8} \,{\rm tr}\, \Gamma^\mu(p', q_1, p) \gamma^5 \frac{\not{p} + m}{p^2 - m^2 + i0} \Gamma^\nu(p, -q_2, p') \frac{\not{p}' + m}{p'^2 - m^2 + i0}$$

- The main differences with the usual polarization tensor are: 1) no regularization is required; 2) only diagrams with a total odd number of external photons contribute; 3) the appearance of the so-called Adler-Bell-Jackiw (ABJ) anomaly

- In the present calculation we have shown that an anomalous term also appears and that it is only due to the diagrams



- These diagrams correspond to the well-know triangle diagram, which gives rise to the anomaly in vacuum



- The anomalous contribution is given by

$$\mathfrak{T}_5^\nu(q_1, q_2) = -i\pi e^2 \delta^{(-,\perp)}(q_1 - q_2) 4e \int_{-\infty}^{+\infty} dx^- \, e^{i(q_2^+ - q_1^+)x^-} \, q_\mu F^{*\mu\nu}(kx)$$

- It is essentially important that the anomalous contribution does not depend on the electron mass as only in this way the sum of the anomalous contributions of all leptons adds to zero preserving the renormalizability of the Standard Model

Conclusions

- Present and next-generation lasers offer a unique possibility of accessing new extreme regimes of interaction of light with matter, where the effective strength of the electromagnetic fields becomes close to the critical fields of QED
- In these regimes, laser-matter interaction can be exploited as a new tool, alternative to conventional accelerators, for investigating fundamental physics
- In particular, the electron dynamics when radiation-reaction and quantum effects are not negligible can be investigated for the first time
- Theoretical perspectives on processes involving the nonlinear interaction of neutrinos with intense electromagnetic fields