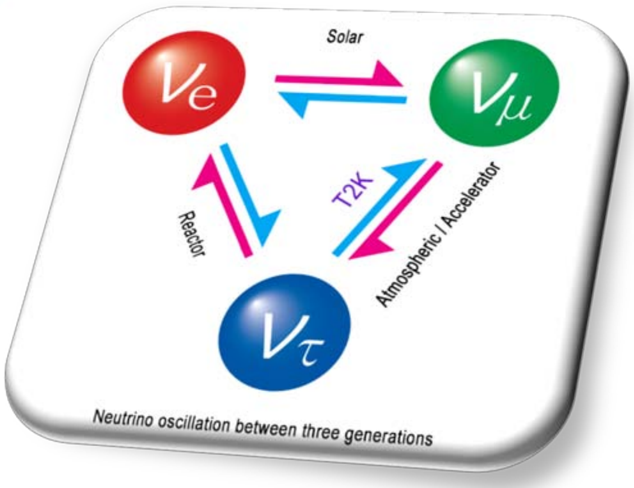

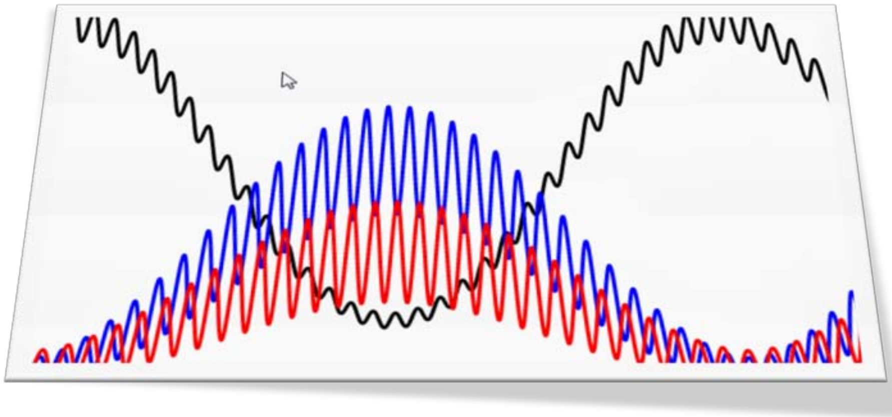

A Journey in Neutrino Oscillation

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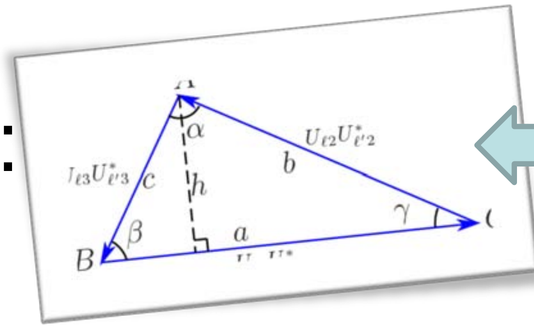
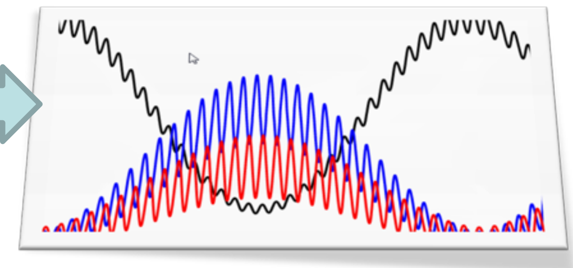
Based on 1311.4496(PRD), 1407.3736(JCAP), 1502.02503(JHEP), 1510.00666

Leaving cake: Next Friday

Outline

- Vacuum Osc:





- Vacuum Osc: Triangle games in IceCube
- Matter effect in T2K:
  - Why are you using Freund's formula?
- Matter effect in IceCube+Sterile:
  - Whoops! matter effect disappears….

How does 2-nu system oscillate?

$$P_{\nu_\ell \to \nu_{\ell'}} = A \sin^2 \frac{\Delta m^2 L}{4E}; \;\; P_{\nu_\ell \to \nu_\ell} = 1 - A \sin^2 \frac{\Delta m^2 L}{4E}$$

How does 3-nu system oscillate?

How does 2-nu system oscillate?

$$P_{\nu_\ell \to \nu_{\ell'}} = A \sin^2 \frac{\Delta m^2 L}{4E}; \;\; P_{\nu_\ell \to \nu_\ell} = 1 - A \sin^2 \frac{\Delta m^2 L}{4E}$$

How does 3-nu system oscillate?

$$\begin{aligned} P_{\nu_\ell \to \nu_{\ell'}} \quad = \quad & |U_{\ell 1} U_{\ell' 1}|^2 + |U_{\ell 2} U_{\ell' 2}|^2 + |U_{\ell 3} U_{\ell' 3}|^2 \\ & + 2|U_{\ell 2} U_{\ell' 2} U_{\ell 1} U_{\ell' 1}| \cos(\frac{\Delta m^2_{21} L}{2E} - \phi_{\ell' \ell;21}) \\ & + 2|U_{\ell 3} U_{\ell' 3} U_{\ell 2} U_{\ell' 2}| \cos(\frac{\Delta m^2_{32} L}{2E} - \phi_{\ell' \ell;32}) \\ & + 2|U_{\ell 3} U_{\ell' 3} U_{\ell 1} U_{\ell' 1}| \cos(\frac{\Delta m^2_{21} L}{2E} - \phi_{\ell' \ell;31}) \end{aligned}$$

PDG
p237

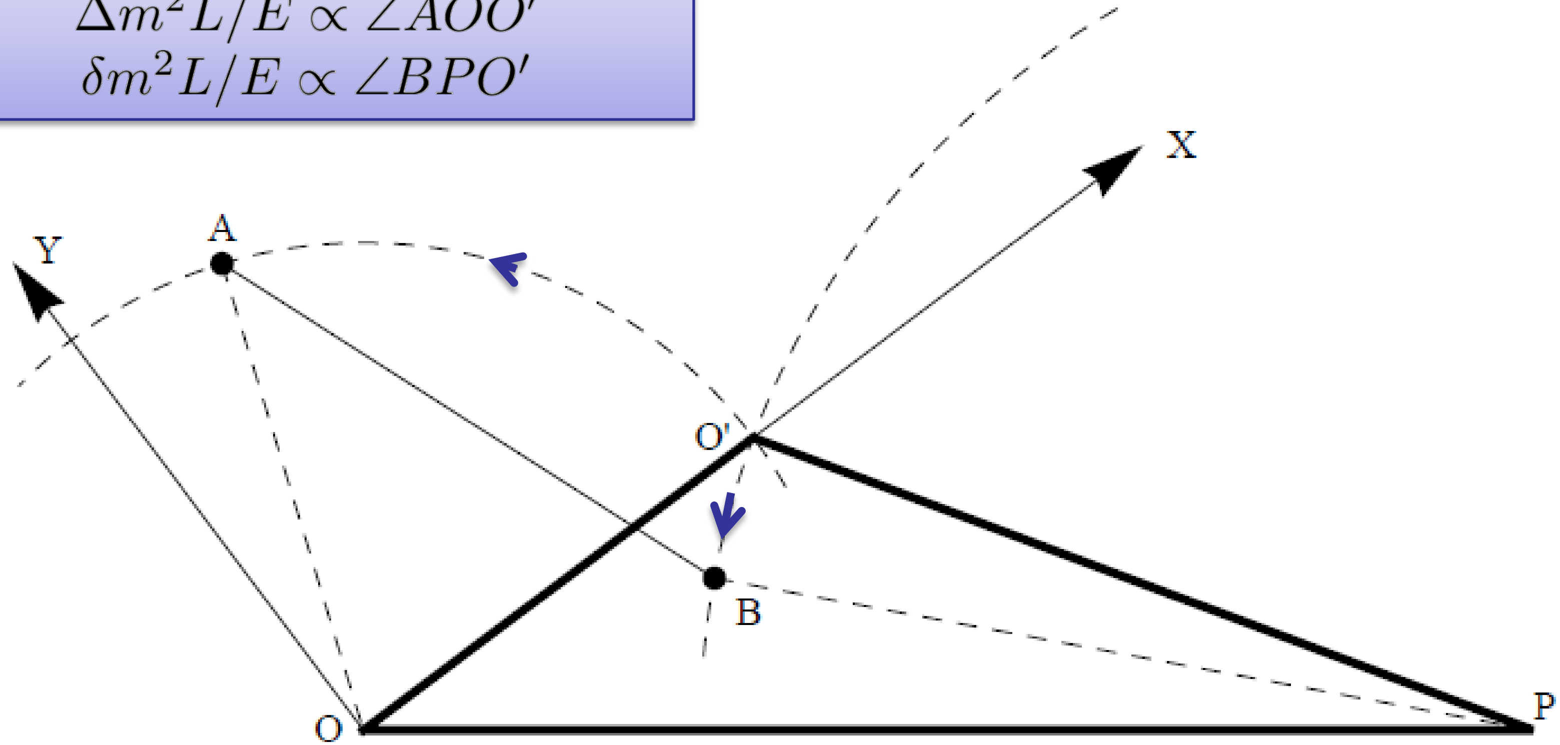
$$\phi_{\ell' \ell; jk} = \arg(U_{\ell' j} U^*_{\ell j} U_{\ell k} U^*_{\ell' k})$$

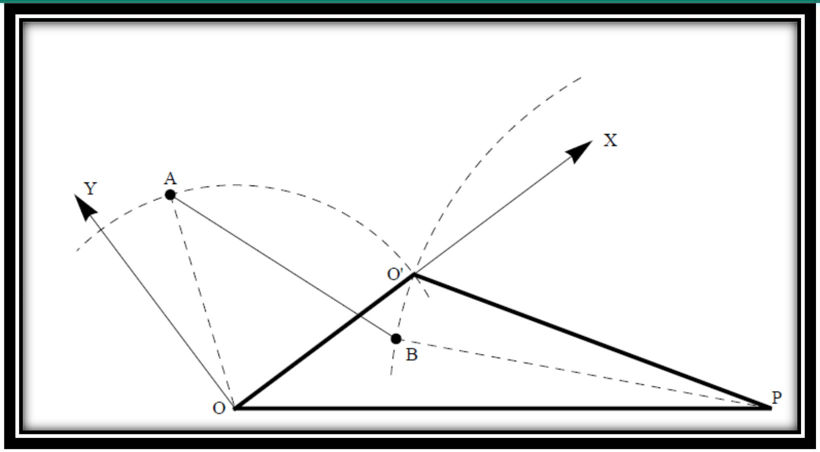
How does 3-nu system oscillate?

Probability = $|AB|^2$

$\Delta m^2 L/E \propto \angle AOO'$

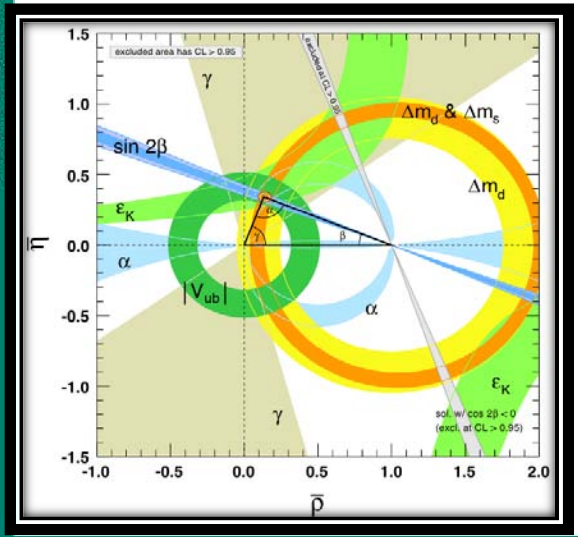
$\delta m^2 L/E \propto \angle BPO'$





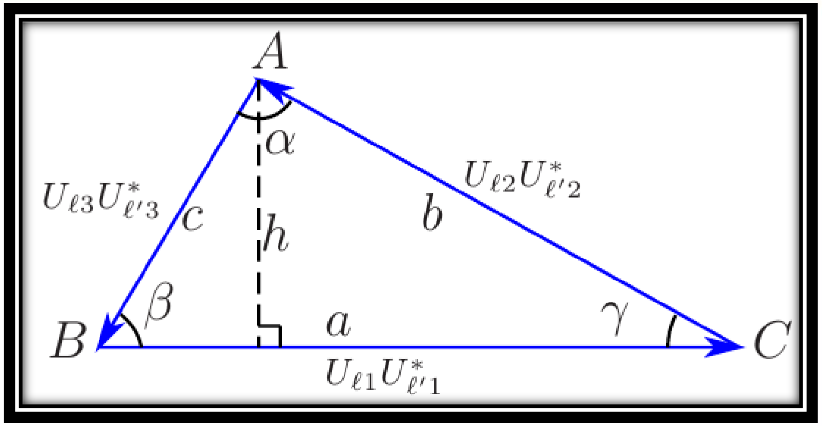
What is that triangle?

Answer: Unitarity Triangle (UT)



Why?

$$U_{\ell 1}U^*_{\ell' 1} + U_{\ell 2}U^*_{\ell' 2} + U_{\ell 3}U^*_{\ell' 3} = 0$$



Let's plot a UT

Hmm…seems amplitudes = side lengths

But…
How to about this?

$$\begin{aligned} P_{\nu_\ell \to \nu_{\ell'}} &= |U_{\ell 1}U_{\ell' 1}|^2 + |U_{\ell 2}U_{\ell' 2}|^2 + |U_{\ell 3}U_{\ell' 3}|^2 \\ &+2|U_{\ell 2}U_{\ell' 2}U_{\ell 1}U_{\ell' 1}| \cos\left(\frac{\Delta m^2_{21} L}{2E} - \phi_{\ell'\ell;21}\right) \\ &+2|U_{\ell 3}U_{\ell' 3}U_{\ell 2}U_{\ell' 2}| \cos\left(\frac{\Delta m^2_{32} L}{2E} - \phi_{\ell'\ell;32}\right) \\ &+2|U_{\ell 3}U_{\ell' 3}U_{\ell 1}U_{\ell' 1}| \cos\left(\frac{\Delta m^2_{21} L}{2E} - \phi_{\ell'\ell;31}\right) \end{aligned}$$

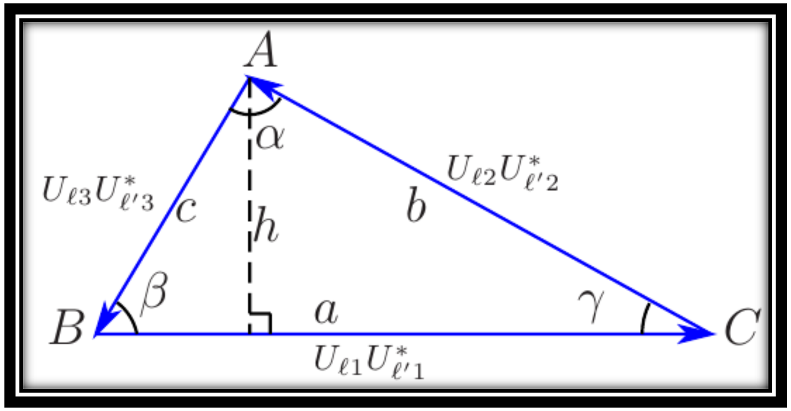
$$\boxed{\phi_{\ell'\ell;jk} = \arg(U_{\ell' j}U^*_{\ell j}U_{\ell k}U^*_{\ell' k})}$$

$$\phi_{\ell'\ell;jk} = \arg(U_{\ell' j} U^*_{\ell j} U_{\ell k} U^*_{\ell' k})$$

How to about this?

Ans: They are angles of the UT.

3 ϕ's=$\angle ABC, \angle BCA, \angle CAB$



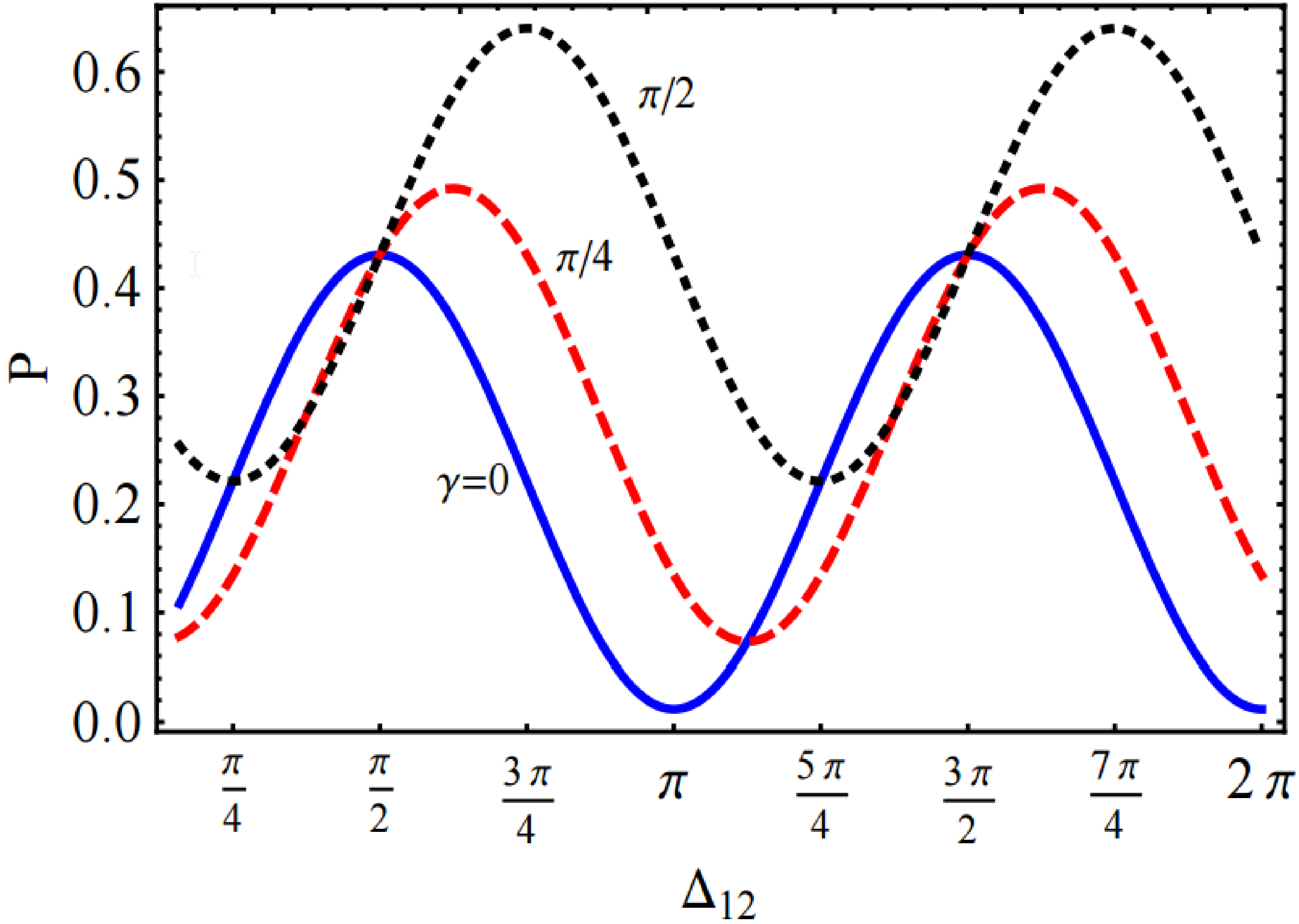
Physical meaning

Angles of UT = phase-shifts of Osc

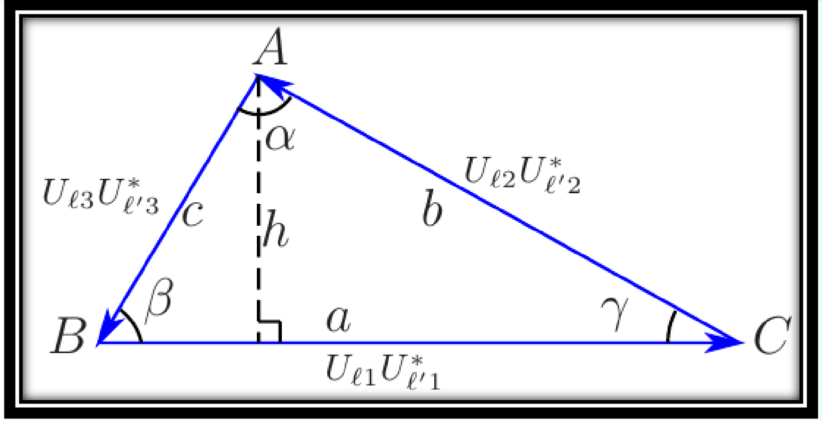
Side lengths of UT = amplitudes of Osc

$\Delta m^2 L/(2E)$ = phases of Osc

Change one angle of the UT, we can see the phase-shift



Now the complicated formula becomes much simpler…

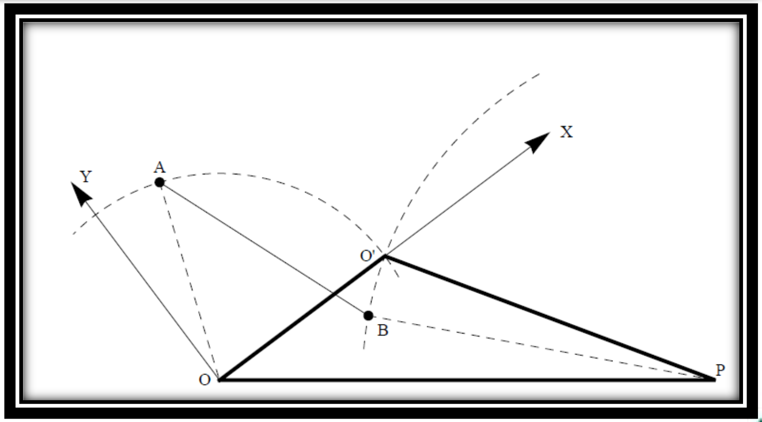


$$P_{\ell\to\ell'} = a^2 + b^2 + c^2 - 2ab\cos(2\Delta_{12} \pm \gamma)$$
$$-2bc\cos(2\Delta_{23} \pm \alpha) - 2ca\cos(2\Delta_{31} \pm \beta).$$

Or

$$P_{\ell\to\ell'} = 4ab\sin(\Delta_{12} \pm \gamma)\sin\Delta_{12}$$
$$+4bc\sin(\Delta_{23} \pm \alpha)\sin\Delta_{23}$$
$$+4ac\sin(\Delta_{31} \pm \beta)\sin\Delta_{31}\,.$$

I love pictures more than eqs



Depends on $\triangle$ rather than U

$$P_{\ell\to\ell'} = 4ab\sin(\Delta_{12}\pm\gamma)\sin\Delta_{12} + 4bc\sin(\Delta_{23}\pm\alpha)\sin\Delta_{23} + 4ac\sin(\Delta_{31}\pm\beta)\sin\Delta_{31}\,.$$

Some implications

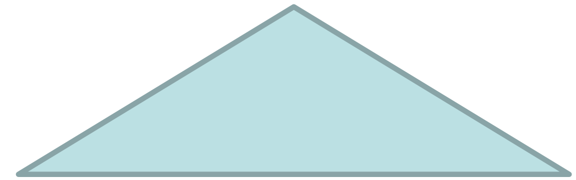
(1). Directly measure UT from Nu Osc

(2). 4 parameters $\rightarrow$ 3

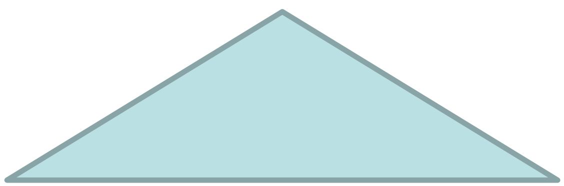
(3). CP violation, if phase-shift can be observed.

Game of Triangles

$$U = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$



But…



$$U = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

?

Can any triangles be a UT?

….Sometimes impossible.
An obvious example: what if one side larger than one?

UT can NOT be arbitrarily large !

Can be arbitrarily small ?

What is the condition for a triangle to be a UT ?

Answer: a triangle can be a UT
if and only if …

$$a, b, c \leqslant \frac{1}{2} \qquad a + b + c \leqslant \boxed{1}$$

Why?
Because
$P \leq 1$

Everyone knows:

$$a + b \geqslant c\,, \quad a + c \geqslant b\,, \quad b + c \geqslant a$$

Detailed proof in arXiv:1407.3736

A triangle can be a UT

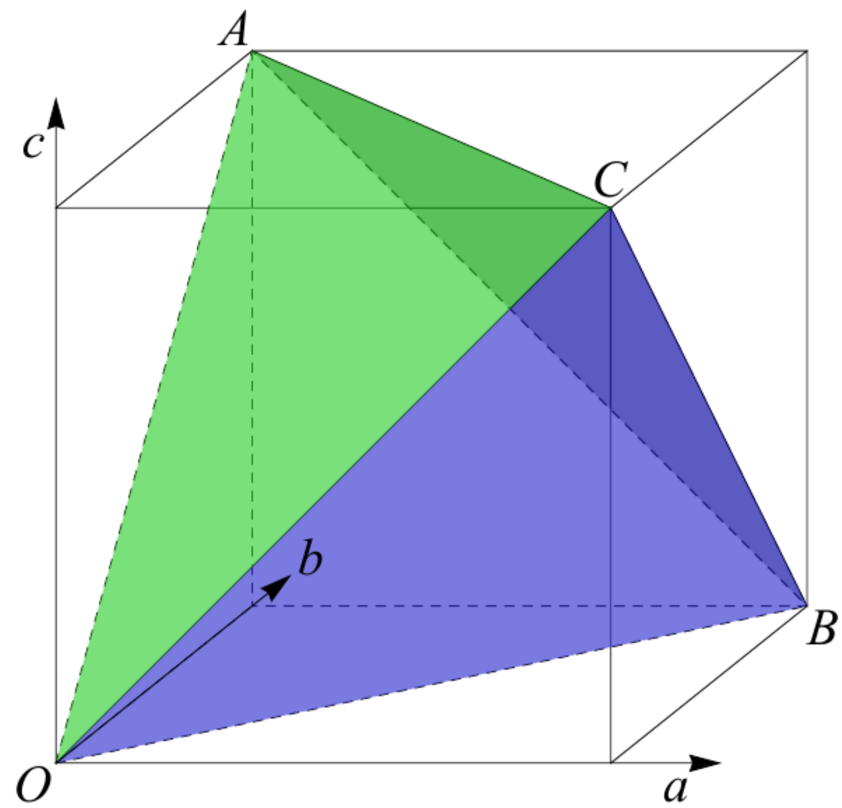
if and only if …

$$a, b, c \leqslant \frac{1}{2}$$

$$a + b + c \leqslant 1$$

$$\begin{aligned} a + b &\geq c \\ b + c &\geq a \\ c + a &\geq b \end{aligned}$$

$\Longleftrightarrow$



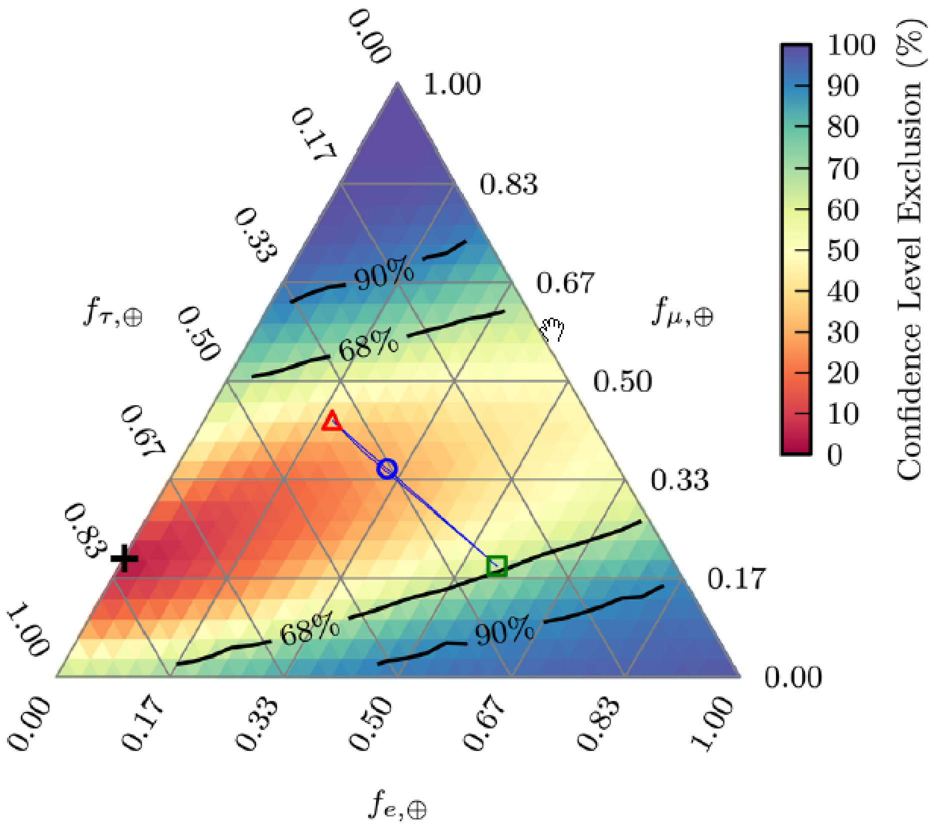
Again, I love pictures more than eqs

Werner: …applied to IceCube?

XJ: ….what is IceCube ?...

….

XJ: Hmm… seems IceCube is very hot. Let me point out an interesting region in this triangle.



1407.3736, with Werner and Hong-Jian

Flavor ratio of Astrophysical neutrinos

Very high energy 10TeV-PeV(IceCube). Maybe new physics.
Very far sources. $L\Delta m^2_{jk}/(4E)$ >>1

$$P_{\ell\to\ell'} = \sum_j |U_{\ell j} U_{\ell' j}|^2 \quad \boxed{= a^2 + b^2 + c^2}$$

$$\boxed{\begin{aligned} X &= a^2_{\mu\tau} + b^2_{\mu\tau} + c^2_{\mu\tau}\,, \\ Y &= a^2_{\tau e} + b^2_{\tau e} + c^2_{\tau e}\,, \\ Z &= a^2_{e\mu} + b^2_{e\mu} + c^2_{e\mu}\,. \end{aligned}}$$

Initial ratio:

$(\Phi_{e0}, \Phi_{\mu0}, \Phi_{\tau0})$

$$\mathbb{P} = \begin{pmatrix} 1-Y-Z & Z & Y \\ Z & 1-X-Z & X \\ Y & X & 1-X-Y \end{pmatrix}$$

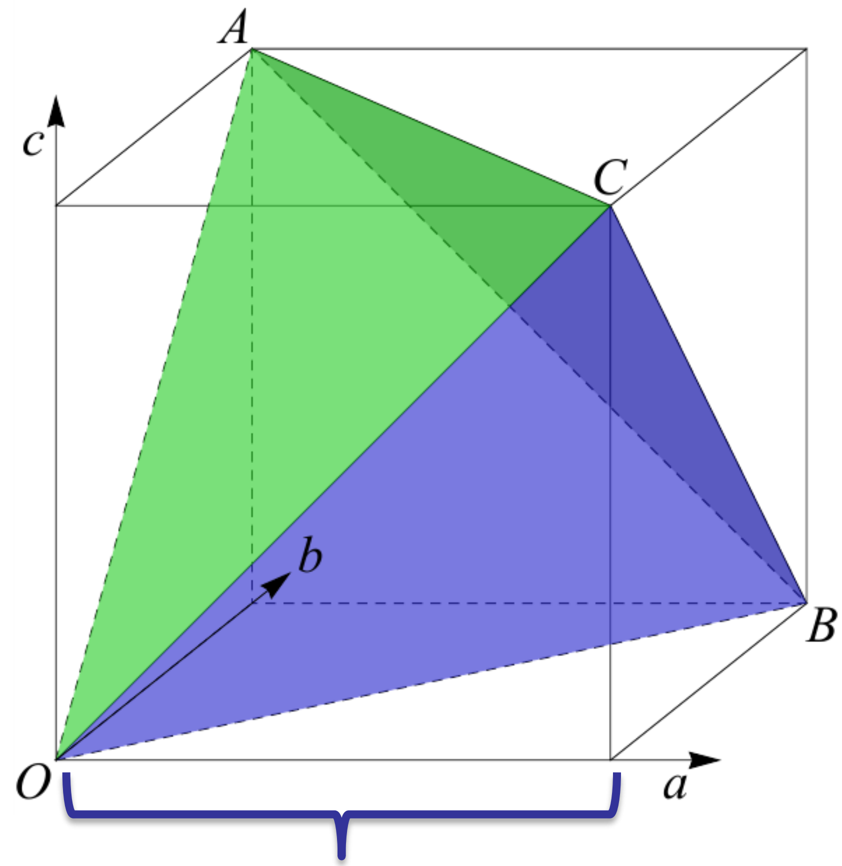
Final ratio:

$(\Phi_e, \Phi_\mu, \Phi_\tau)$

$$\boxed{(\Phi_e, \Phi_\mu, \Phi_\tau)^T \propto \mathbb{P}\,(\Phi_{e0}, \Phi_{\mu0}, \Phi_{\tau0})^T}$$

$$\mathbb{P} = \begin{pmatrix} 1-Y-Z & Z & Y \\ Z & 1-X-Z & X \\ Y & X & 1-X-Y \end{pmatrix}$$

$$\begin{aligned} X &= a_{\mu\tau}^2 + b_{\mu\tau}^2 + c_{\mu\tau}^2 \,, \\ Y &= a_{\tau e}^2 + b_{\tau e}^2 + c_{\tau e}^2 \,, \\ Z &= a_{e\mu}^2 + b_{e\mu}^2 + c_{e\mu}^2 \,. \end{aligned}$$



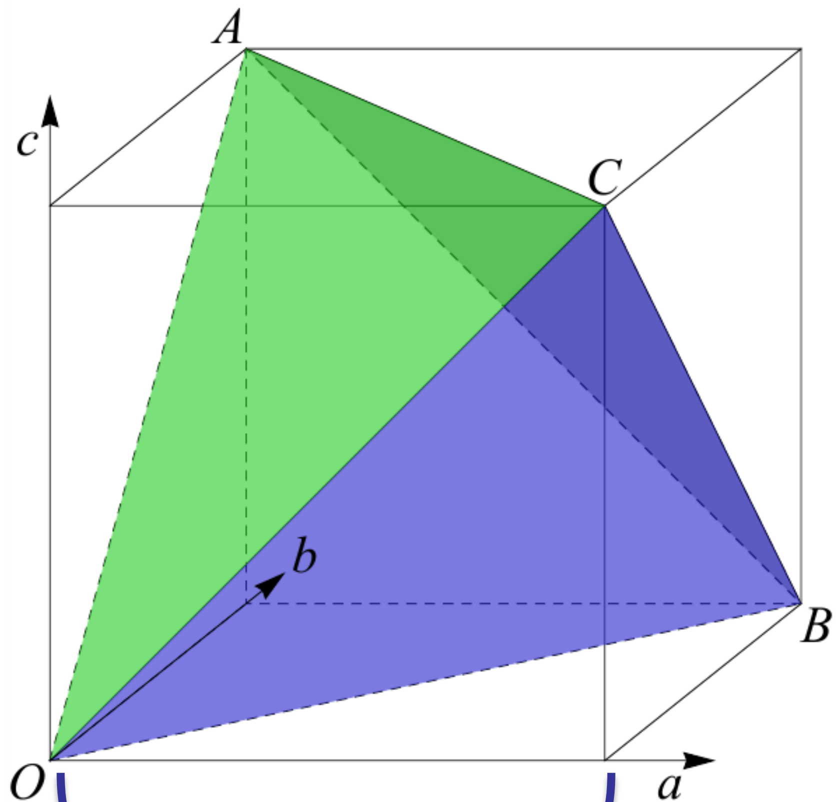
Side length=1/2

A good exercise: assume (a, b, c) is in the tetrahedron, then

$$a^2 + b^2 + c^2 \leq ?$$

$$\mathbb{P} = \begin{pmatrix} 1-Y-Z & Z & Y \\ Z & 1-X-Z & X \\ Y & X & 1-X-Y \end{pmatrix}$$

$$X = a_{\mu\tau}^2 + b_{\mu\tau}^2 + c_{\mu\tau}^2 \,,$$
$$Y = a_{\tau e}^2 + b_{\tau e}^2 + c_{\tau e}^2 \,,$$
$$Z = a_{e\mu}^2 + b_{e\mu}^2 + c_{e\mu}^2 \,.$$



Side length=1/2

$$\boxed{a^2 + b^2 + c^2 \le \left(\tfrac{1}{2}\right)^2 + \left(\tfrac{1}{2}\right)^2 + 0 = \tfrac{1}{2}}$$

Physical meaning:
The transition probability of astrophysical neutrinos $\le$ 50%.

Assume Pion Sources…

Neutrinos are produced by pions

$$\pi \rightarrow \mu + \nu_\mu \rightarrow e + \nu_e + 2\nu_\mu$$

Ignore diff between particles and anti-particles

Initial ratio $(\Phi_{e0} : \Phi_{\mu 0} : \Phi_{\tau 0})$ = $(1 : 2 : 0)$

Final ratio $(\Phi_e, \Phi_\mu, \Phi_\tau)^T \propto \mathbb{P}\,(\Phi_{e0}, \Phi_{\mu 0}, \Phi_{\tau 0})^T$

$$T = \frac{\Phi_\mu}{\Phi_{\text{tot}}}, \qquad S = \frac{\Phi_e}{\Phi_{\text{tot}}}$$

So we have $T = \frac{1}{3}(2 - 2X - Z), \quad S = \frac{1}{3}(1 - Y + Z).$

Question: $? \leq S \leq ?$

Note:

$X, Y, Z \leq 1/2$

Assume Pion Sources…

$$T = \frac{\Phi_\mu}{\Phi_{\rm tot}}\,, \quad S = \frac{\Phi_e}{\Phi_{\rm tot}}$$

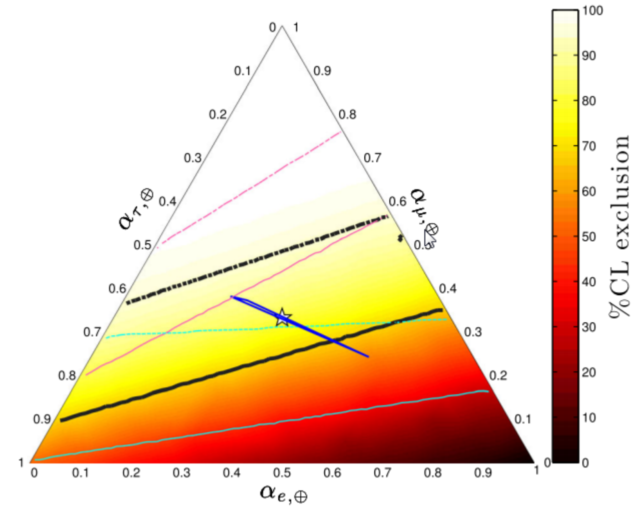
$$T \;=\; \tfrac{1}{3}(2-2X-Z)\,, \quad S \;=\; \tfrac{1}{3}(1-Y+Z)\,.$$

$$X, Y, Z \leq 1/2$$

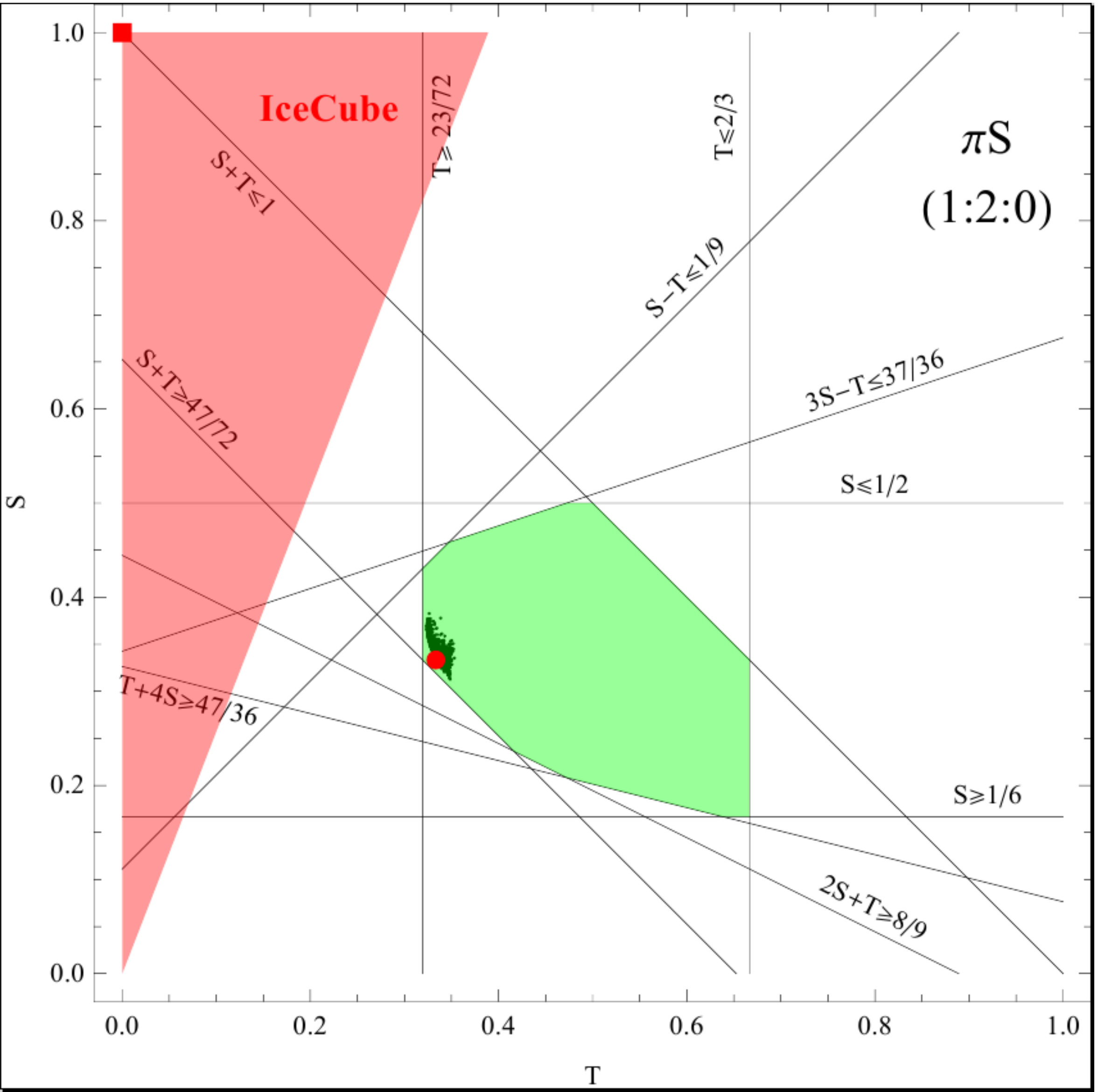
$$\boxed{\frac{1}{6} \leq S \leq \frac{1}{2}}$$

$$T = \frac{\Phi_\mu}{\Phi_{\text{tot}}}, \quad S = \frac{\Phi_e}{\Phi_{\text{tot}}}$$

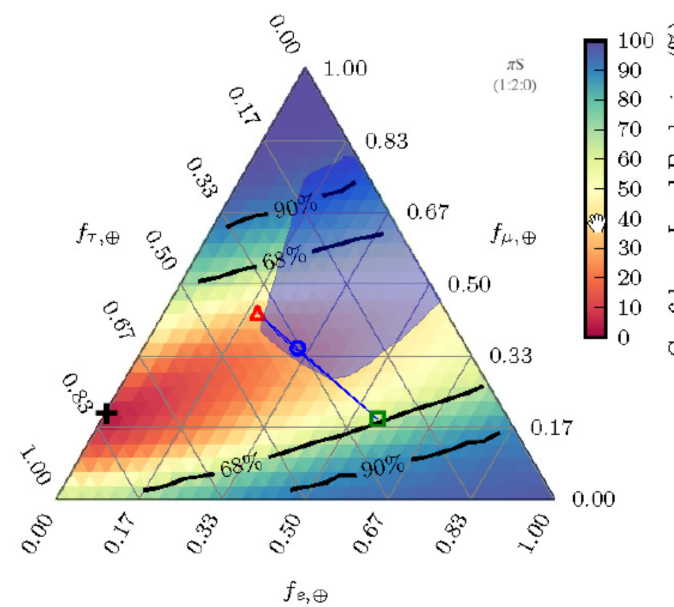
O. Mena, et al, PRL. (2014)



From 1407.3736



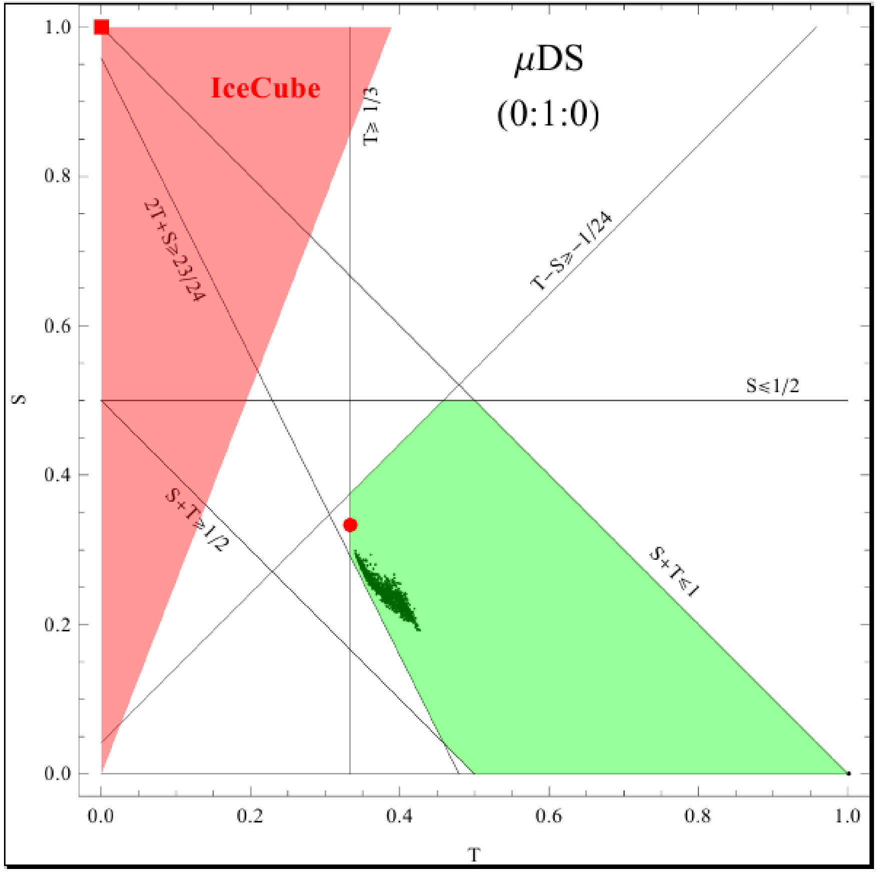
IC collaboration, PRL. (2015)

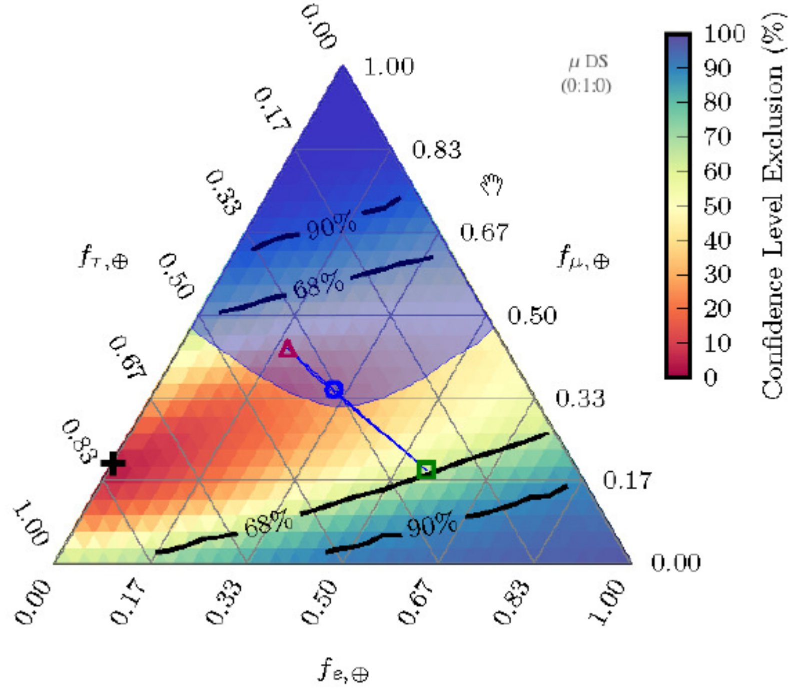


Ternary plot by Min-Shan Zheng

Other possible sources:

Muon Damped Sources (muDS)

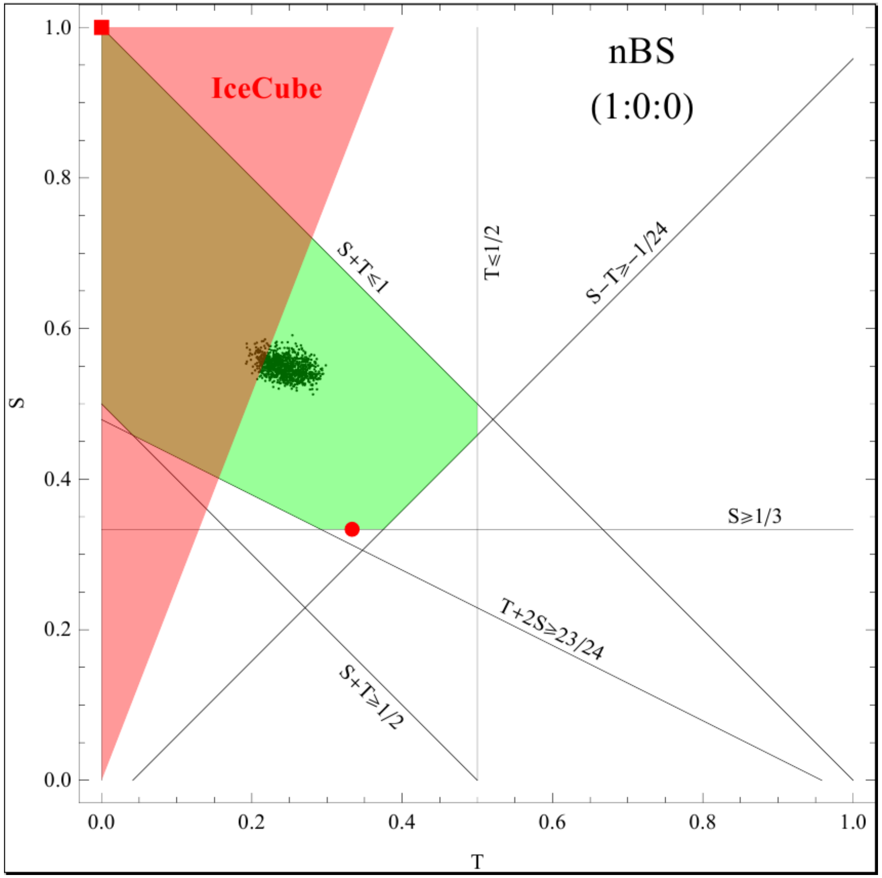


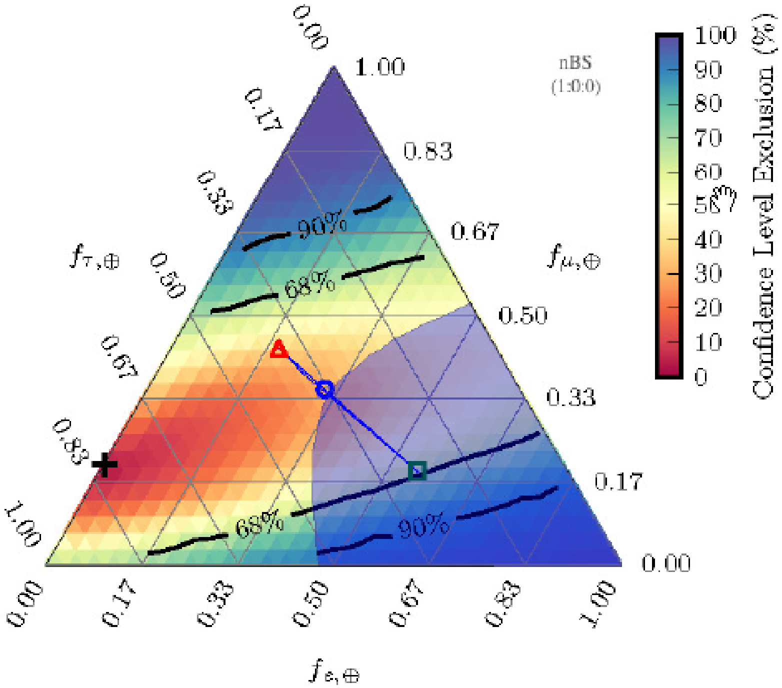


Ternary plot by Min-Shan Zheng

Other possible sources:

Neutron Beam Sources (nBS)

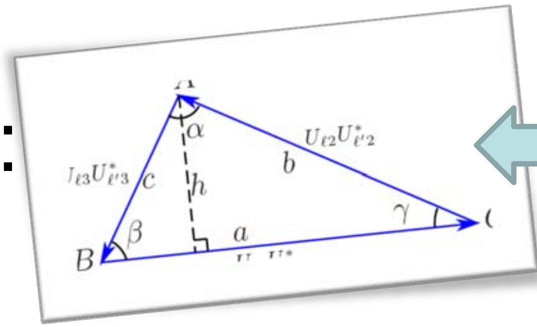


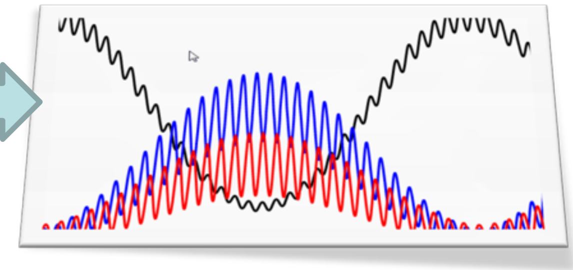


Ternary plot by Min-Shan Zheng

Outline

- Vacuum Osc:





- Vacuum Osc: Triangle games in IceCube
- Matter effect in T2K:
  - Why are you using Freund's formula?
- Matter effect in IceCube Sterile:
  - Whoops! matter effect disappears….

About T2K experiment

$$\nu_\mu(\overline{\nu}_\mu) \xrightarrow{295\text{km}} \nu_e(\overline{\nu}_e)$$

Energy: 0.1-1.2 GeV, peaked at 0.6 GeV

It can't be treated as:

2-nu Osc ✗

Sometimes
Contribution $\propto$ solar mixing >50%
(shown later)

Vacuum Osc ✗

Low energy, close to solar resonance, solar mixing is affected drastically by matter eff

Freund's formula

The electron neutrino appearance probability also includes subleading terms which depend on δ_{CP} and terms that describe matter interactions $\boxed{[31]}$:

$$
\begin{aligned}
P_{\nu_\mu \to \nu_e} = & \frac{1}{(A-1)^2} \sin^2 2\theta_{13} \sin^2 \theta_{23} \sin^2[(A-1)\Delta] \\
& -(+) \frac{\alpha}{A(1-A)} \cos\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \\
& \times \sin\delta_{CP} \sin\Delta \sin A\Delta \sin[(1-A)\Delta] + \frac{\alpha}{A(1-A)} \\
& \times \cos\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \\
& \times \cos\delta_{CP} \cos\Delta \sin A\Delta \sin[(1-A)\Delta] \\
& + \frac{\alpha^2}{A^2} \cos^2\theta_{23} \sin^2 2\theta_{12} \sin^2 A\Delta. \qquad (2)
\end{aligned}
$$

Here $\alpha = \frac{\Delta m^2_{21}}{\Delta m^2_{32}} \ll 1$, $\Delta = \frac{\Delta m^2_{32} L}{4E_\nu}$ and $A = 2\sqrt{2} G_F N_e \frac{E_\nu}{\Delta m^2_{32}}$, where N_e is the electron density of Earth's crust. In the

T2K PRD 2013

This formula is so accurate that T2K doesn't need to use any packages to compute P

But…

Max-Planck-Institut für Kernphysik

M.Freund PRD 2001

VII. CONCLUSIONS

The purpose of this work was to find approximate analytic expressions for the neutrino mixing parameters and oscillation probabilities in the presence of matter. It was stated that being interested in approximate solutions it is difficult to describe both the solar and the atmospheric resonance at the same time. Therefore, this work is restricted to energies above the solar resonance according to

$$|\hat{A}| \gtrsim |\alpha| \;\Rightarrow\; E_\nu \gtrsim 0.45\ \text{GeV}\left(\frac{\Delta m_{21}^2}{10^{-4}\ \text{eV}^2}\right)\left(\frac{2.8\ \text{g/cm}^3}{\rho}\right). \tag{45}$$

For this regime, the complete parameter mapping [Eqs. (27)] was given as series expansion in the small mass hierarchy parameter $\alpha = \Delta m_{21}^2/\Delta m_{31}^2$. It was shown, that the change of the CP phase δ in matter is triple suppressed by the mass hierarchy, the mixing angle θ_{13} and by θ_{23} being close to maximal. Furthermore, it was shown that in order $\Delta m_{21}^2/\Delta m_{31}^2$, the relevant contribution to the parameter map-

terms [Eqs. (38)] contributing to $P(\nu_e \to \nu_\mu)$:

$$P_0 = \sin^2\theta_{23}\frac{\sin^2 2\theta_{13}}{(\hat{A}-1)^2}\sin^2[(\hat{A}-1)\hat{\Delta}],$$

$$P_{\sin\delta} = \alpha\frac{\sin\delta\cos\theta_{13}\sin 2\theta_{12}\sin 2\theta_{13}\sin 2\theta_{23}}{\hat{A}(1-\hat{A})} \times \sin(\hat{\Delta})\sin(\hat{A}\hat{\Delta})\sin[(1-\hat{A})\hat{\Delta}],$$

$$P_{\cos\delta} = \alpha\frac{\cos\delta\cos\theta_{13}\sin 2\theta_{12}\sin 2\theta_{13}\sin 2\theta_{23}}{\hat{A}(1-\hat{A})} \times \cos(\hat{\Delta})\sin(\hat{A}\hat{\Delta})\sin[(1-\hat{A})\hat{\Delta}],$$

$$P_3 = \alpha^2\frac{\cos^2\theta_{23}\sin^2 2\theta_{12}}{\hat{A}^2}\sin^2(\hat{A}\hat{\Delta}),$$

and $\cot\delta = J_{CP}^{-1}\mathrm{Re}(U_{\mu 3}U^*_{e3}U_{e2}U^*_{\mu 2})$, $J_{CP} = \mathrm{Im}(U_{\mu 3}U^*_{e3}U_{e2}U^*_{\mu 2})$.
The analytic expression for $P_m^{3\nu\ man}(\nu_\mu \to \nu_e)$ given above is valid for [86] neutrino path lengths in the mantle ($L \leq 10660$ km) satisfying $L \lesssim 10560$ km $E[\mathrm{GeV}]\,(7.6\times 10^{-5}\ \mathrm{eV}^2/\Delta m^2_{21})$, and energies $E \gtrsim 0.34$ GeV$(\Delta m^2_{21}/7.6\times 10^{-5}\ \mathrm{eV}^2)\,(1.4\ \mathrm{cm}^{-3} N_A/N_e^{man})$. The expression for the $\bar\nu_\mu \to \bar\nu_e$ oscillation probability can be obtained

What does PDG say?

contribute and the CP violation effects due to the Dirac phase in the neutrino mixing matrix are taken into account, has the following form in the constant density approximation and keeping terms up to second order in the two small parameters $|\alpha| \equiv |\Delta m^2_{21}|/|\Delta m^2_{31}| \ll 1$ and $\sin^2\theta_{13} \ll 1$ [86]:

$$P_m^{3\nu\ man}(\nu_\mu \to \nu_e) \cong P_0 + P_{\sin\delta} + P_{\cos\delta} + P_3\,. \qquad (14.45)$$

Here

$$P_0 = \sin^2\theta_{23}\,\frac{\sin^2 2\theta_{13}}{(A-1)^2}\,\sin^2[(A-1)\Delta]$$

$$P_3 = \alpha^2\cos^2\theta_{23}\,\frac{\sin^2 2\theta_{12}}{A^2}\,\sin^2(A\Delta)\,, \qquad (14.46)$$

$$P_{\sin\delta} = -\,\alpha\,\frac{8\,J_{CP}}{A(1-A)}\,(\sin\Delta)\,(\sin A\Delta)\,(\sin[(1-A)\Delta])\,, \qquad (14.47)$$

$$P_{\cos\delta} = \alpha\,\frac{8\,J_{CP}\cot\delta}{A(1-A)}\,(\cos\Delta)\,(\sin A\Delta)\,(\sin[(1-A)\Delta])\,, \qquad (14.48)$$

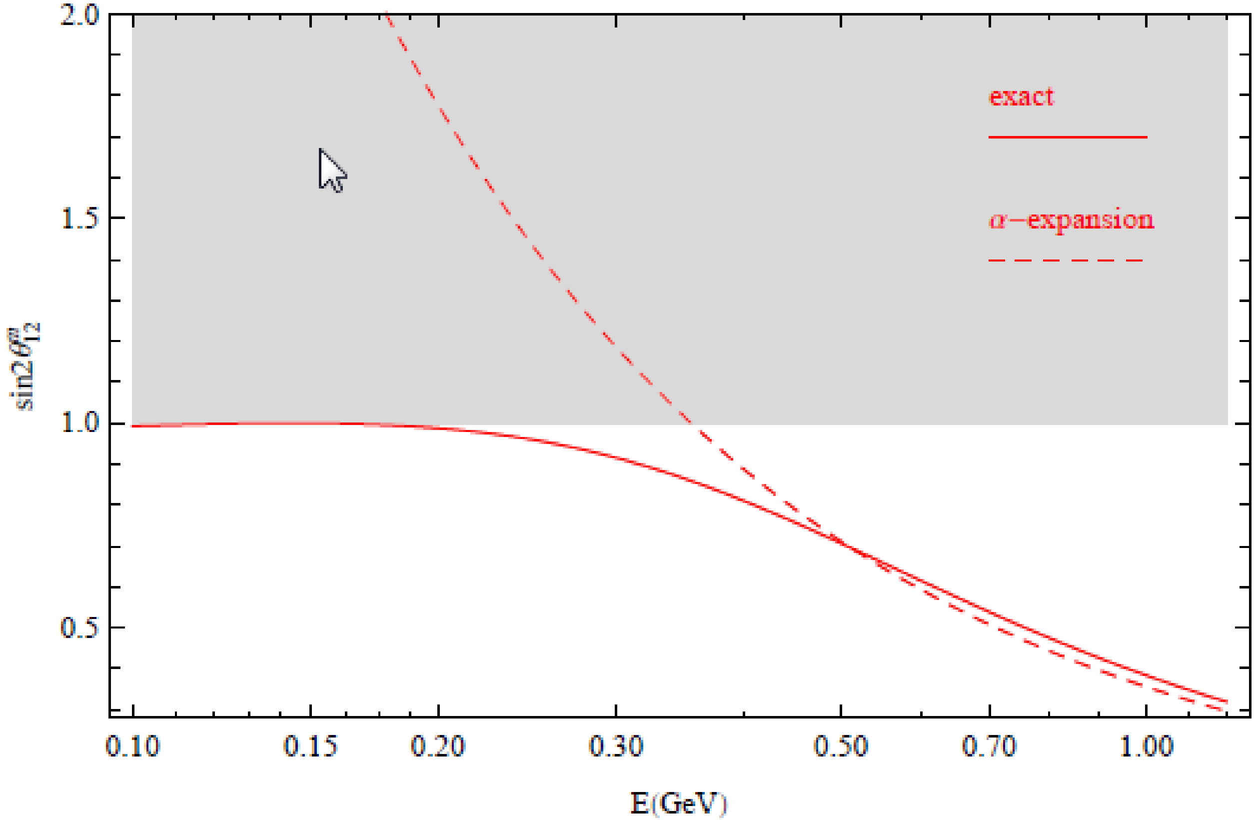
where

$$\alpha = \frac{\Delta m^2_{21}}{\Delta m^2_{31}}\,,\quad \Delta = \frac{\Delta m^2_{31}\,L}{4E}\,,\quad A = \sqrt{2}G_F N_e^{man}\,\frac{2E}{\Delta m^2_{31}}\,, \qquad (14.49)$$

and $\cot\delta = J_{CP}^{-1}\mathrm{Re}(U_{\mu 3}U^*_{e3}U_{e2}U^*_{\mu 2})$, $J_{CP} = \mathrm{Im}(U_{\mu 3}U^*_{e3}U_{e2}U^*_{\mu 2})$.
The analytic expression for $P_m^{3\nu\ man}(\nu_\mu \to \nu_e)$ given above is valid for [86] neutrino path lengths in the mantle ($L \leq 10660$ km) satisfying $L \lesssim 10560$ km $E[\mathrm{GeV}]\,(7.6\times 10^{-5}\ \mathrm{eV}^2/\Delta m^2_{21})$, and energies $E \gtrsim 0.34$ GeV$(\Delta m^2_{21}/7.6\times 10^{-5}\ \mathrm{eV}^2)\,(1.4\ \mathrm{cm}^{-3} N_A/N_e^{man})$. The expression for the $\bar\nu_\mu \to \bar\nu_e$ oscillation probability can be obtained

PDG 2014, p242

What's wrong below 0.34 GeV?

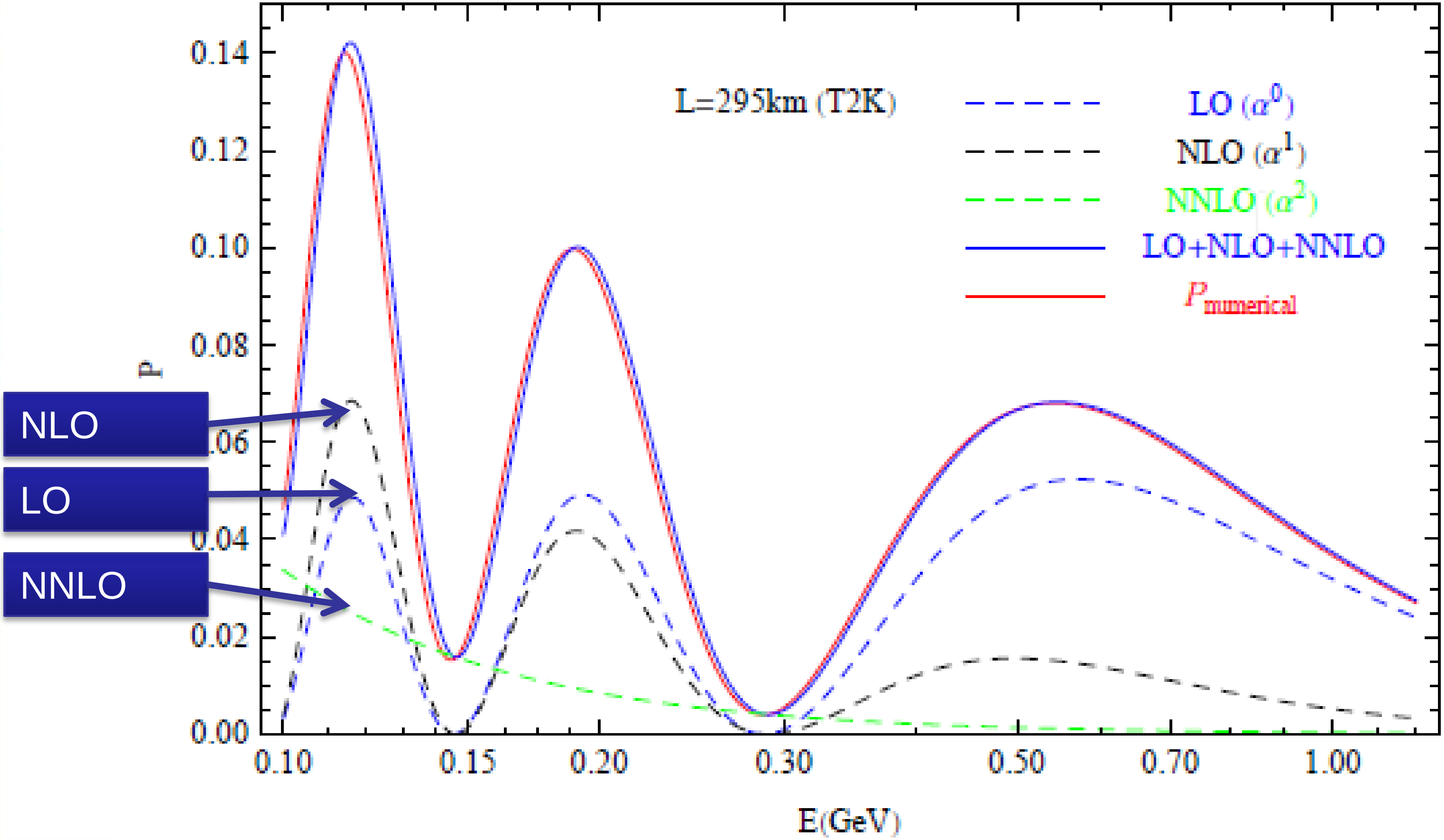


Which curve should we use ?

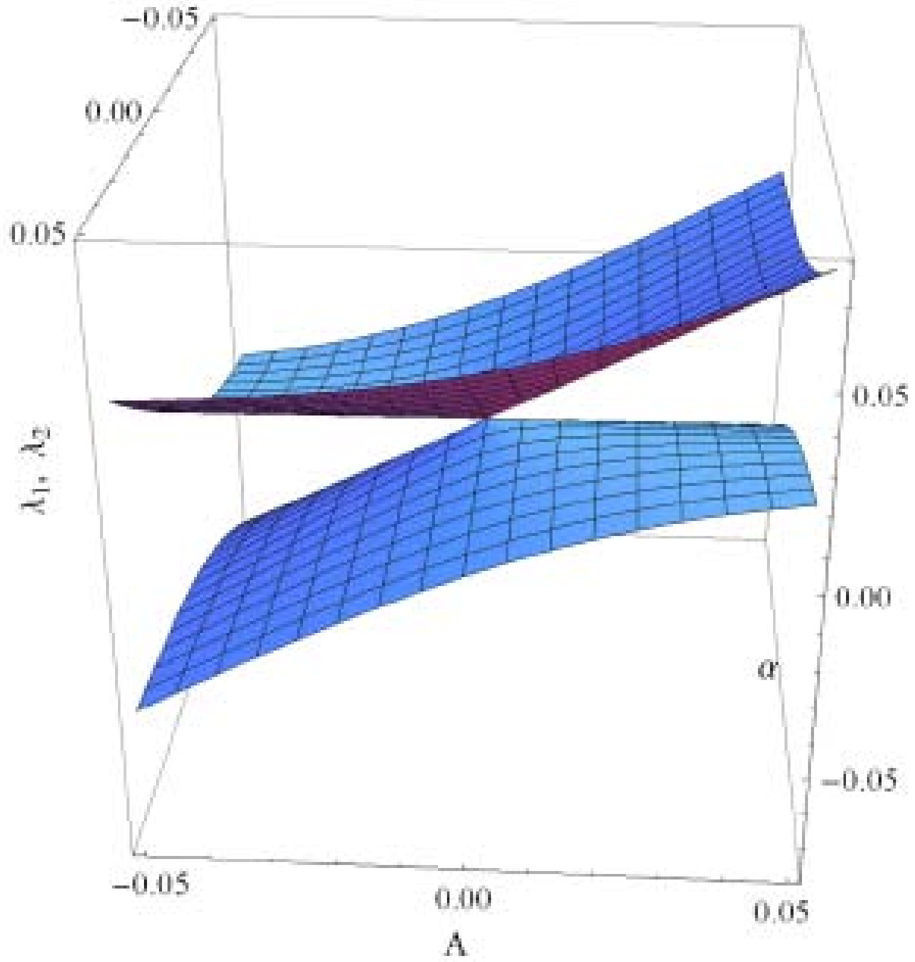
What if we take min(1, sin)?

To get an accurate result, we'd better use the wrong curve

How about NNNLO? LO<NLO, so is NNLO<NNNLO possible?



Eigenvalues as functions of A, α



Expansion far away from the singularity …only small curvature…. OK

How about near the singularity ?

MAX-PLANCK-INSTITUT FÜR KERNPHYSIK

Problem solved in 1502.02503(JHEP)

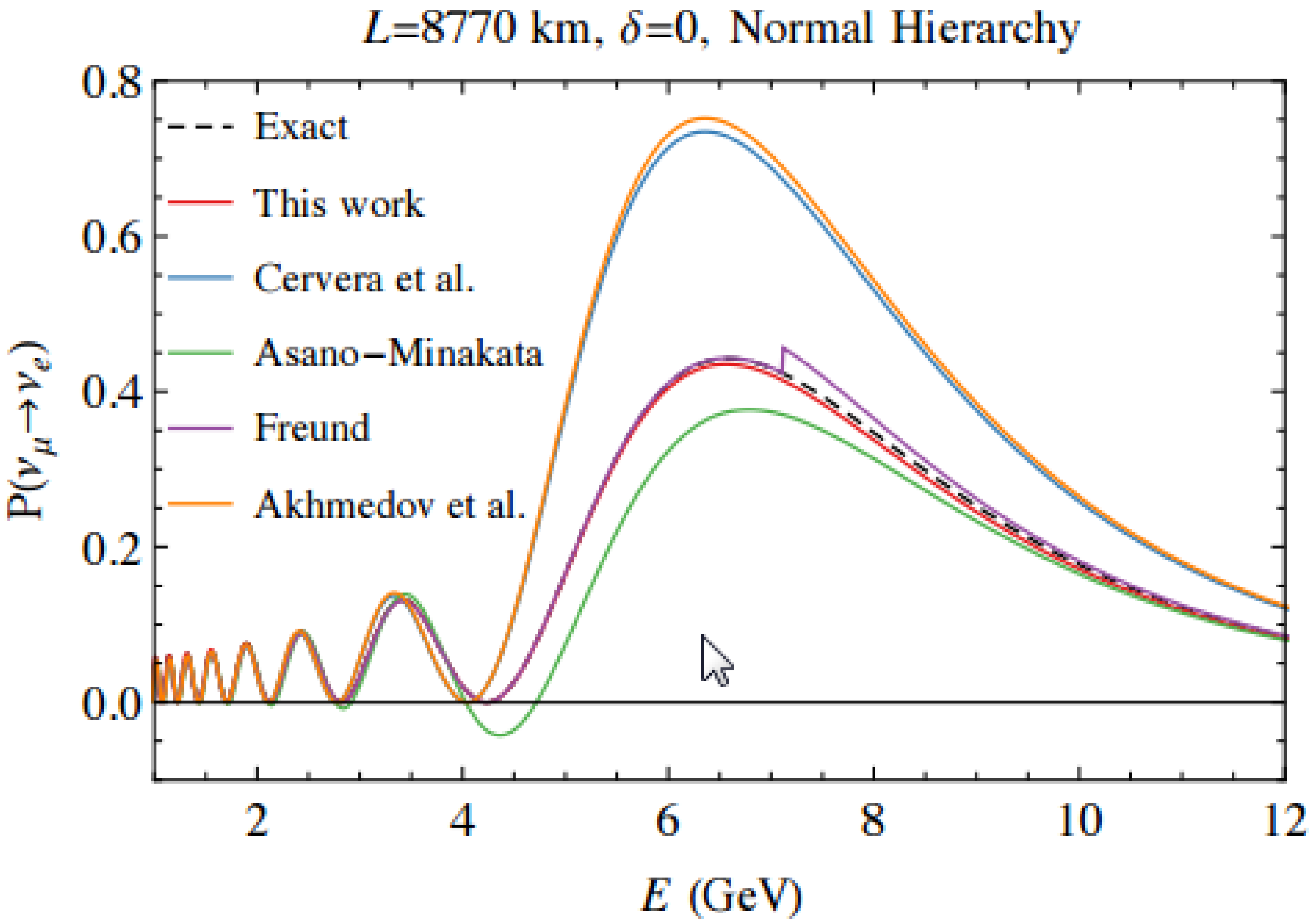
- It can be proven that the singularity originates from branch cut singularity.
- P , depends on λ_1, λ_2, but

$$\boxed{P(\lambda_1, \lambda_2) = \text{analytic function of } (\lambda_1 + \lambda_2),\ (\lambda_1 - \lambda_2)^2}$$

Both are free from branch cut singularity.
Exchanging two eigenvalues doesn't matter.

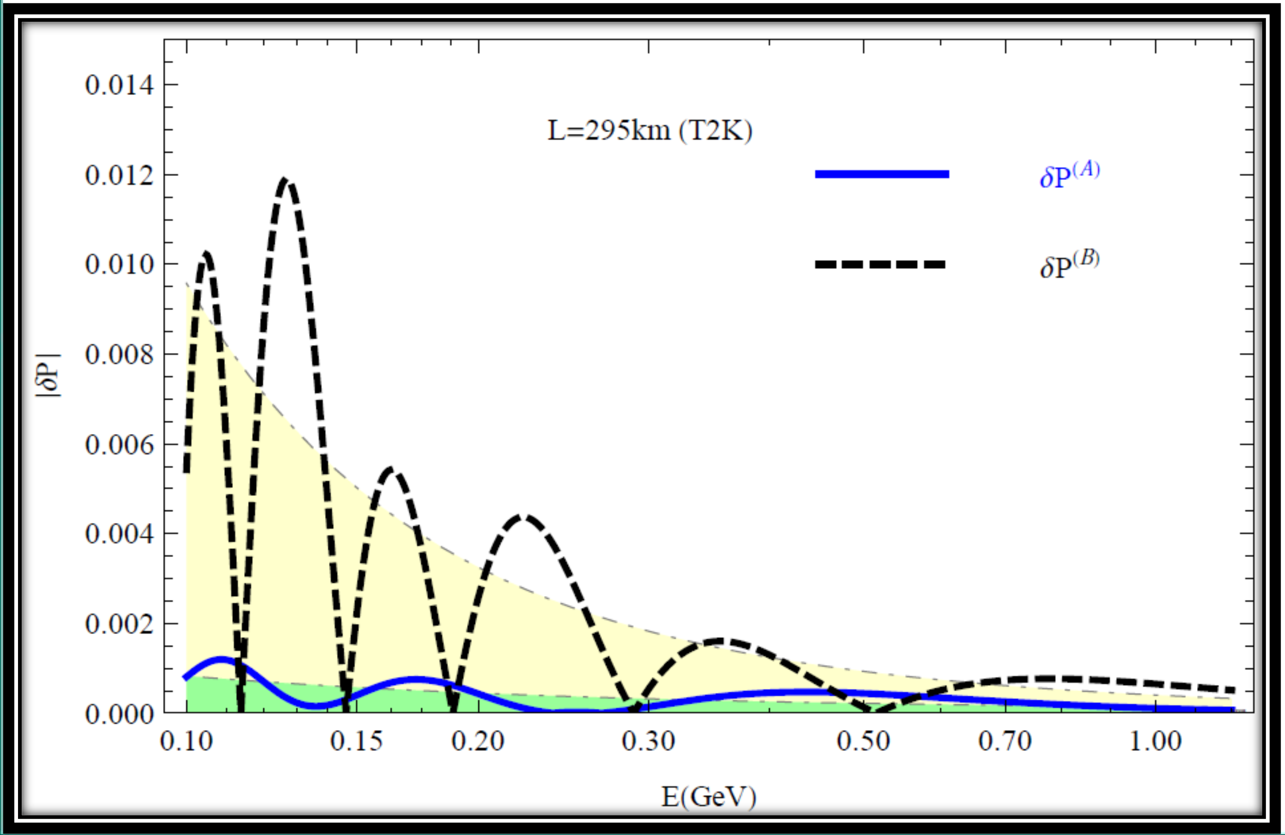
A mistake in 1302.6773

Agarwalla, Kao and Takeuchi, JHEP



A better formula

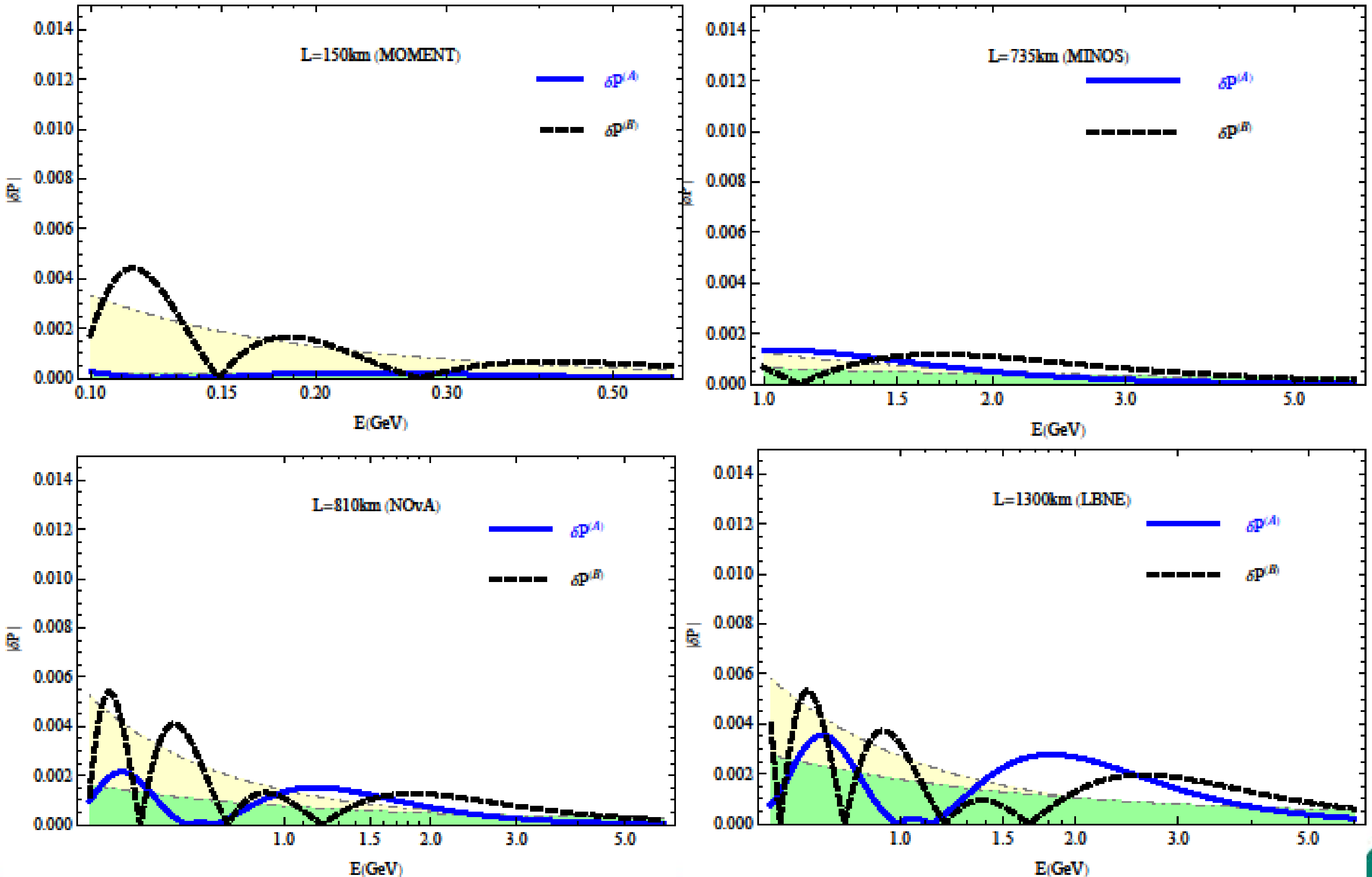
$$P^{(A)} = |p\frac{e^{-2i\Delta} - e^{-2iA\Delta}}{1-A} - 2iq\alpha e^{-i(A+\alpha)\Delta}\frac{\sin(\bar{A}\Delta)}{\bar{A}}|^2$$



1. More accurate

2. Higher speed (in χ^2-fit)

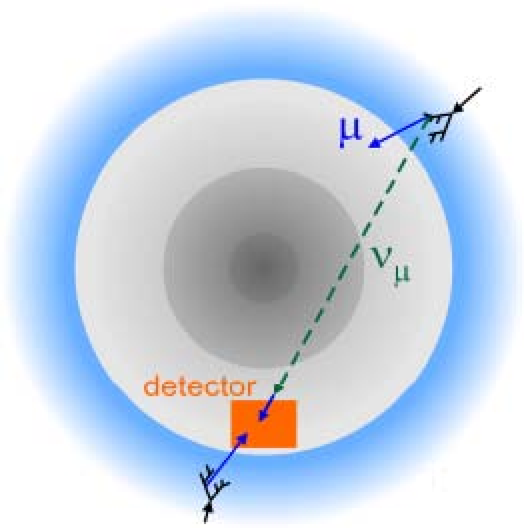
other exp?



L=150km (MOMENT)
L=735km (MINOS)
L=810km (NOvA)
L=1300km (LBNE)
$\delta P^{(A)}$
$\delta P^{(B)}$
$|\delta P|$
E(GeV)

Matter effect in IceCube-Sterile

Atmospheric neutrino oscillation for IC



The energy is so high that…

Nu osc only at $\Delta m^2 > 0.1\mathrm{eV}^2$

$$\boxed{i\frac{d}{dL}|\nu(L)\rangle = H|\nu(L)\rangle}$$

Constant density solution

$$\boxed{|\nu(L)\rangle = e^{-iHL}|\nu(0)\rangle}$$

$$H = \frac{1}{2E}U\begin{pmatrix} m_1^2 & & & \\ & m_2^2 & & \\ & & m_3^2 & \\ & & & m_4^2 \end{pmatrix}U^\dagger + \sqrt{2}G_F N_e \begin{pmatrix} 1 & & & \\ & 0 & & \\ & & 0 & \\ & & & \kappa \end{pmatrix}$$

S-matrix

$$\boxed{S \equiv e^{-iHL}}$$

Approximation

$U_{s4} = 0$ and $\Delta m^2_{21} = \Delta m^2_{31} = 0$

$$\boxed{S_{\mu\mu} = 1 - \left(1 - e^{-it}\right)\mu_4^2 + \frac{e^{-iAt} - (A-1)^2 - Ae^{-it}(2 + it - A(1+it))}{(A-1)^2}\mu_4^2 e_4^2 + \mathcal{O}(e_4^4)}$$

What a lengthy formula !

~O(1)

Thanks to Mathematica.
None of us did the calculation.

A is matter eff parameter.
What if $A \to 1$?

Matter effect suppressed by $e_4 \equiv U_{e4}$

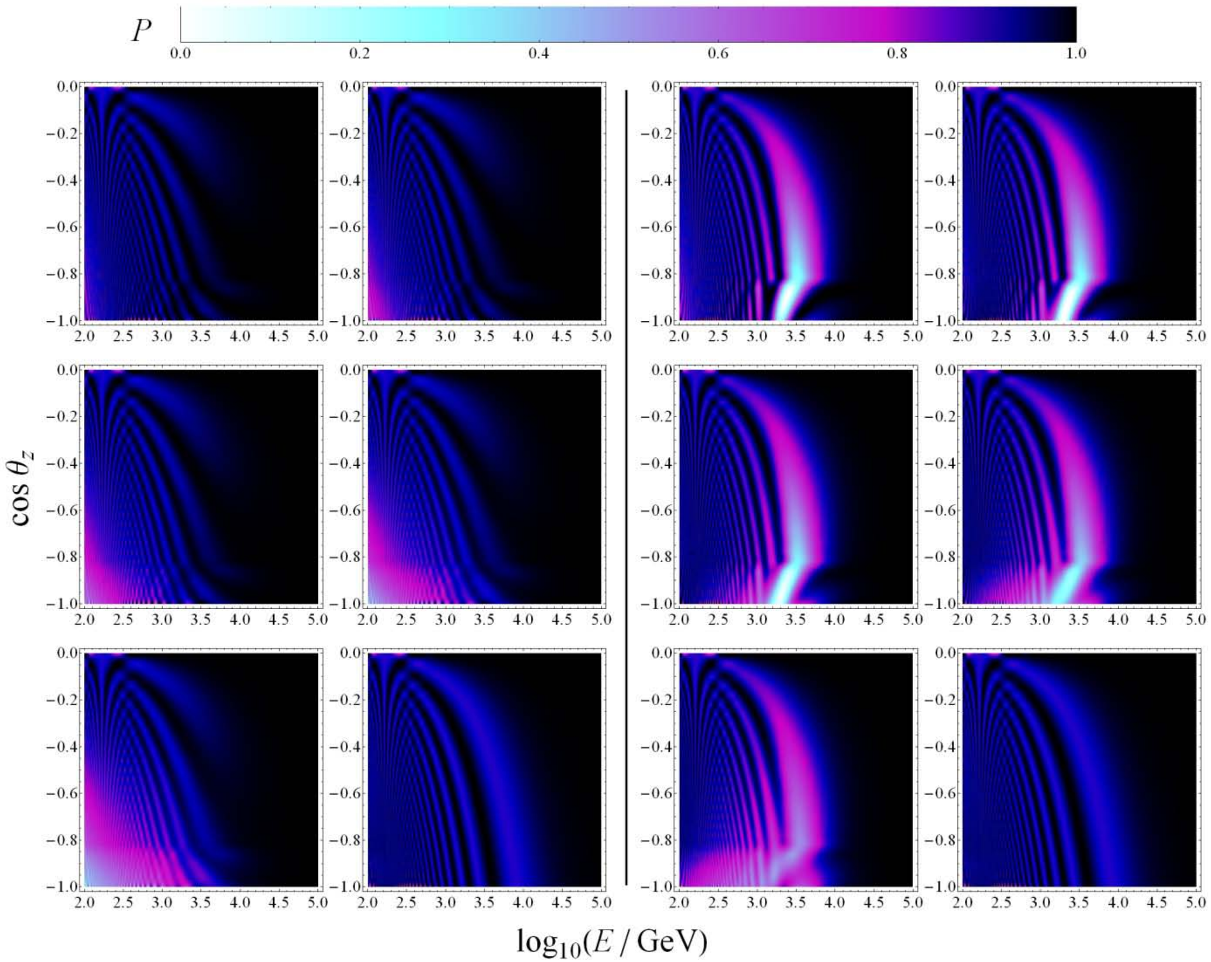
Thanks to my singularity work….

Matter effect decoupling

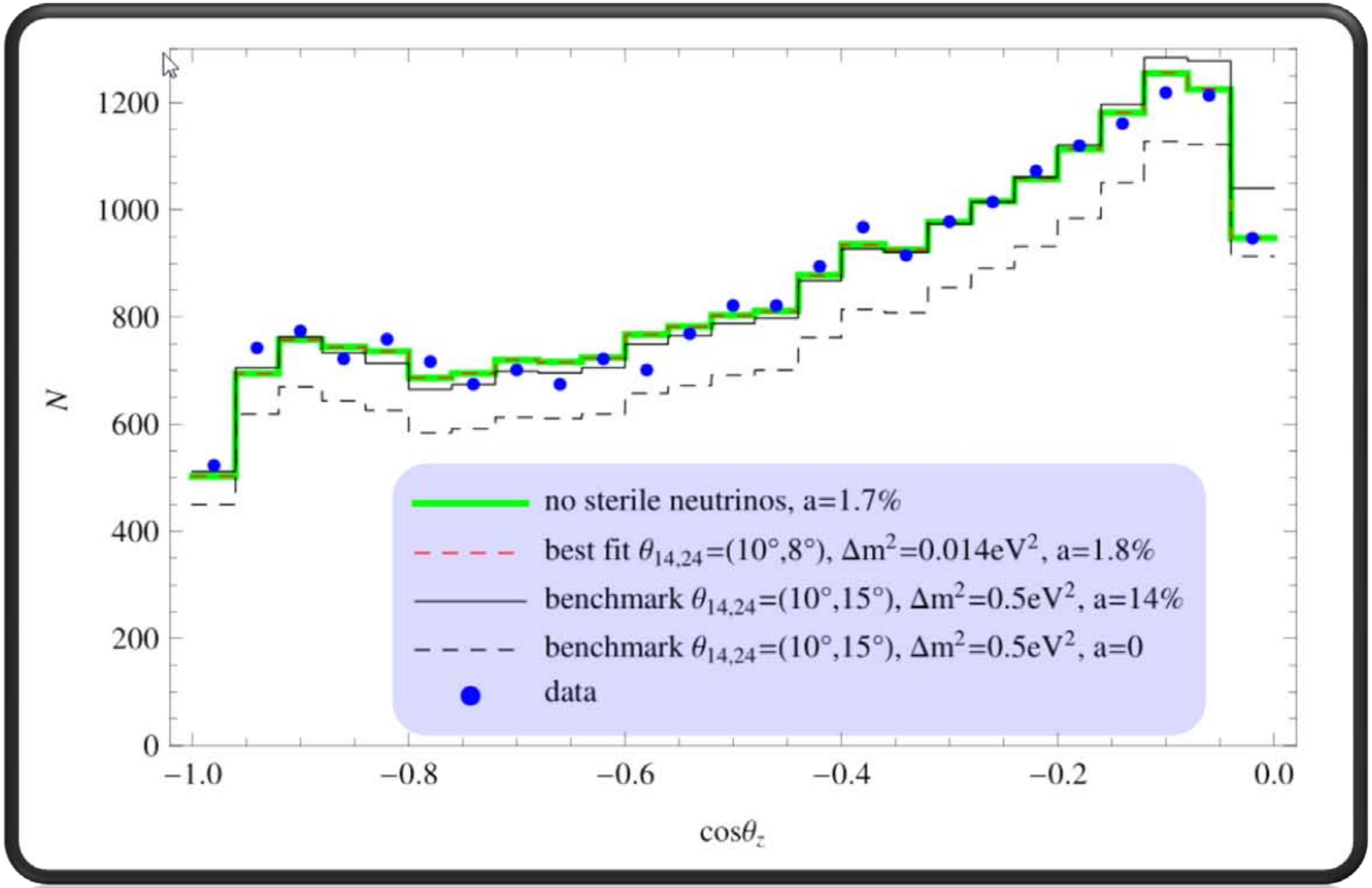
Nu

Anti-nu

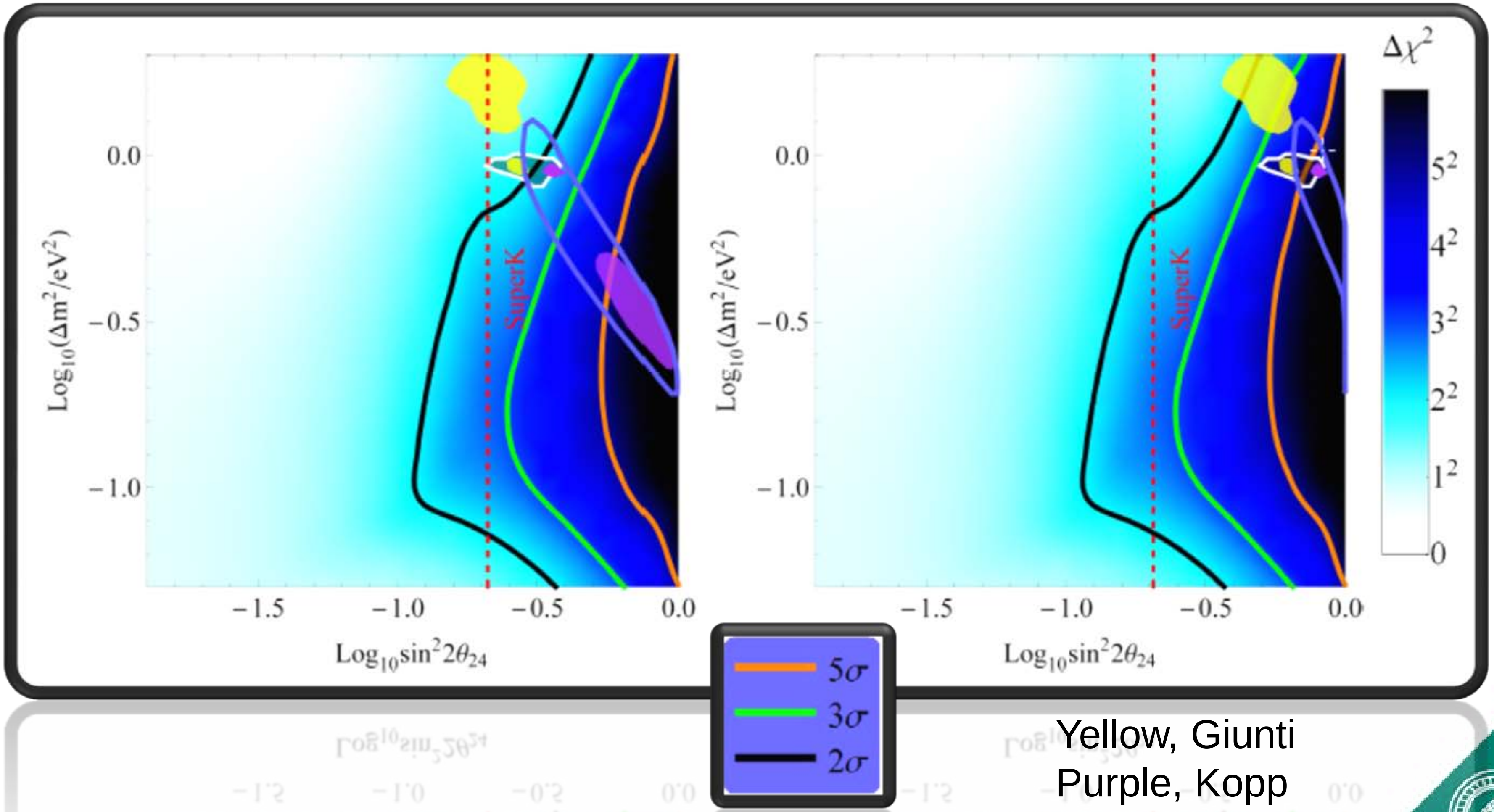
θ_{34}=
0°, 10°
20°, 30°
45°, 90°



Rule out/Find Sterile? -- Neither



Our result



Yellow, Giunti
Purple, Kopp
White, Conrad

Summary

I wish they would be easy to remember.

- α, β, γ = phase-shift
- $a+b+c \le 1,\ a, b, c \le 1/2$
- $P_{\alpha\to\alpha} \ge 1/3,\ P_{\alpha\to\beta} \le 1/2$ for astro nu
  - Some part in the flavor ratio of IC impossible
- Sometimes, Earth = vacuum, for TeV sterile OSC.
  - matter eff will not enhance the sensitivity of IC unless large 3-4 mixing angle is excluded.