

Quantum Tunneling in the Early Universe: Inflation and the Hubble Tension

Martin W. Winkler

in collaboration with K. Freese

based on Phys. Rev. D103 (2021) and arXiv:2102.13655



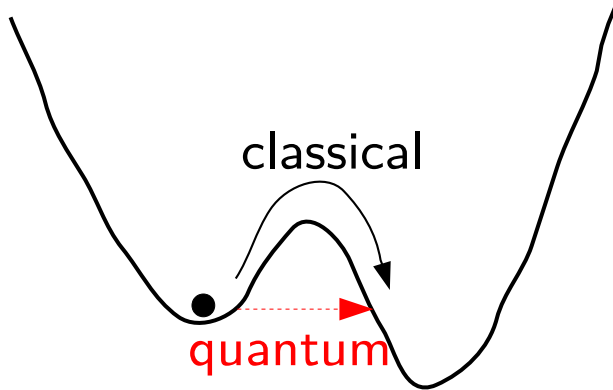
*MPIK Heidelberg
June 28, 2021*



- Quantum Tunneling
- Old and New Inflation
- Chain Inflation
- Hubble Tension
- Chain Early Dark Energy

Quantum Tunneling

Voloshin, Kobzarev, Okun 1974, Coleman 1977

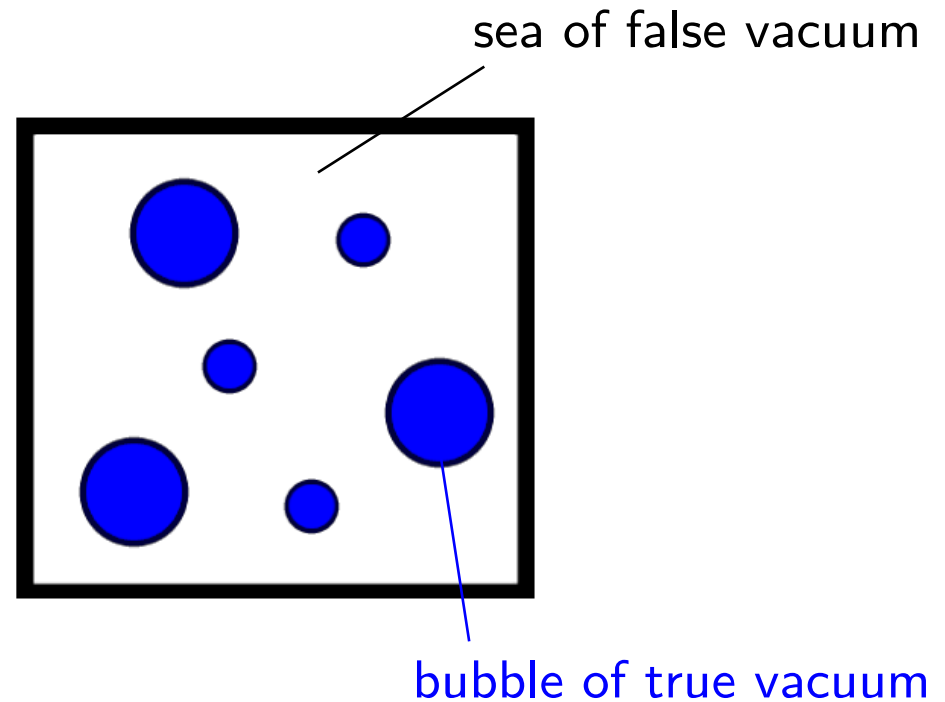


probability of vacuum decay per
volume and time: $\Gamma = Ae^{-S/\hbar}$
“tunneling rate”

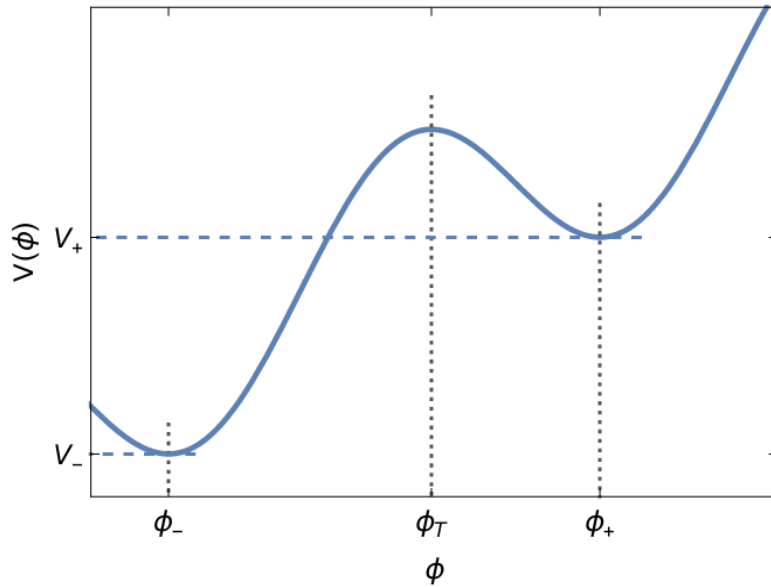
S : action of the bounce
(path of least resistance)

$$\frac{d^2\phi}{d\rho^2} + \frac{3}{\rho} \frac{d\phi}{d\rho} = V'(\phi)$$

$$S \propto \int_0^\infty d\rho \rho^3 \left[\frac{1}{2} \left(\frac{d\phi}{d\rho} \right)^2 + V \right]$$



Bounce Action



example potential:

$$V = \mu^3 \phi + \Lambda^4 \cos \frac{\phi}{f}$$

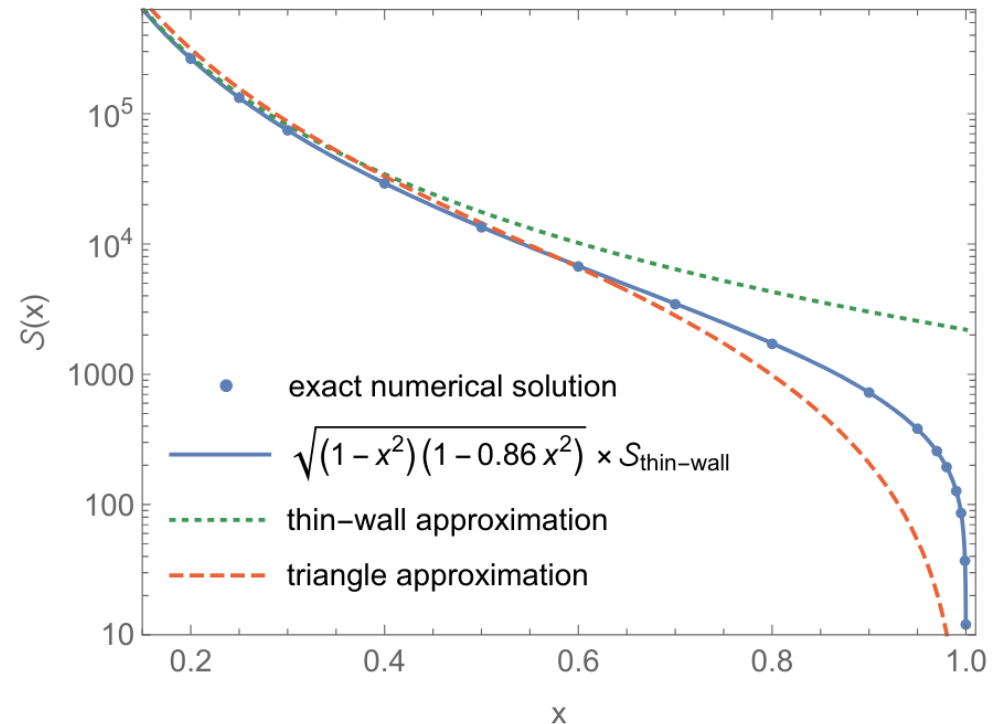
we define: $x = \frac{f\mu^3}{\Lambda^4}$

thin-wall approximation:

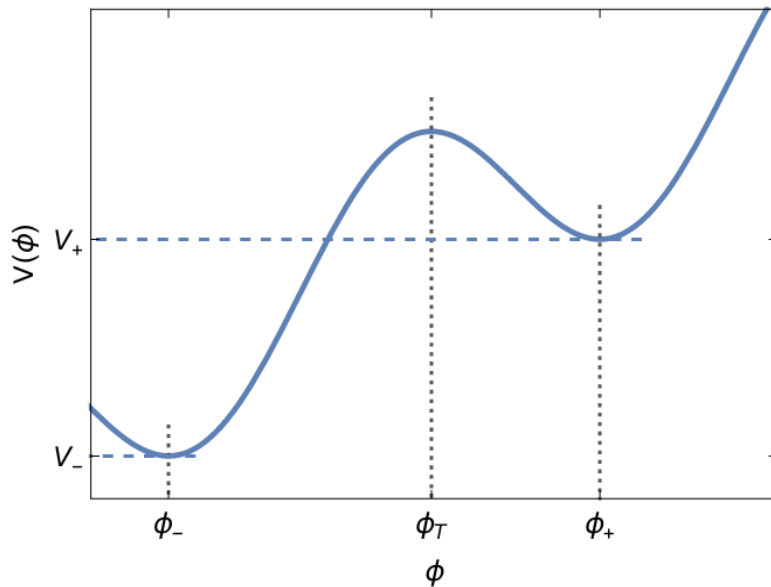
$$S_{\text{thin-wall}} = \frac{4}{\pi} \frac{f^4}{\Lambda^4} \left(\frac{12}{x} \right)^3$$

improved approximation:

$$S_{\text{improved}} = S_{\text{thin-wall}} \times \sqrt{(1-x^2)(1-0.86x^2)}$$



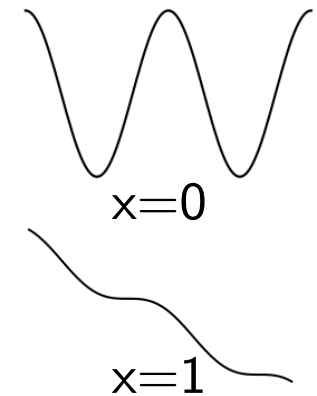
Bounce Action



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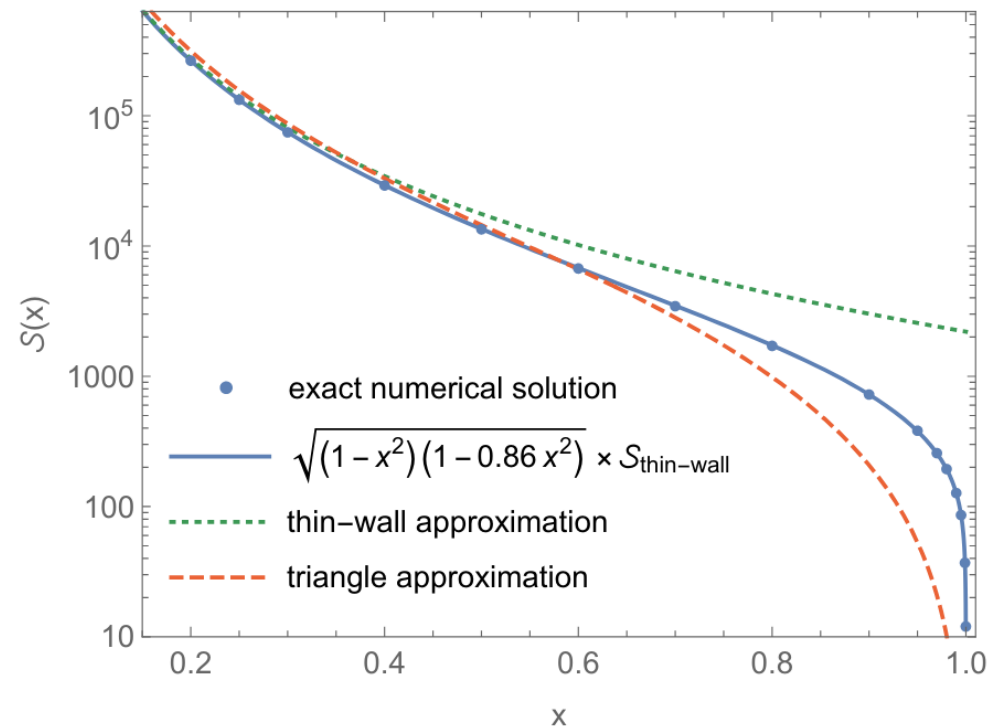


thin-wall approximation:

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improved approximation:

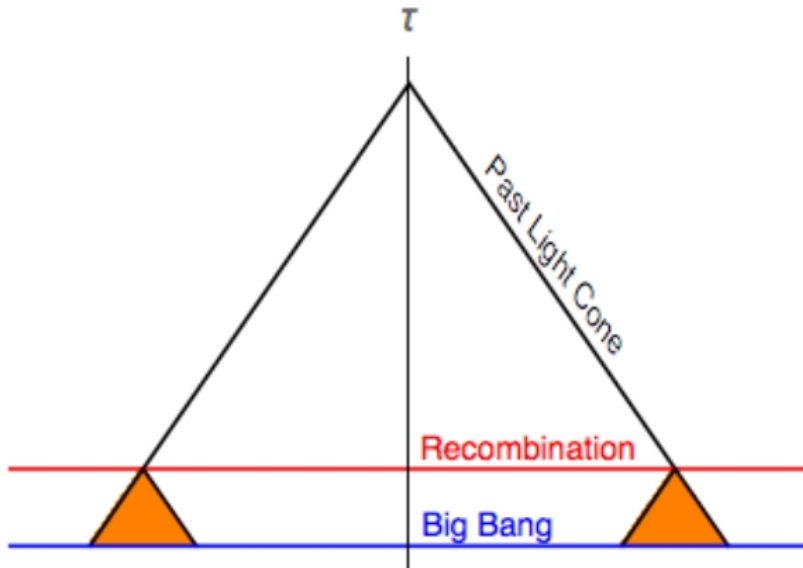
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Inflation

- rapid expansion of space

Guth 1981

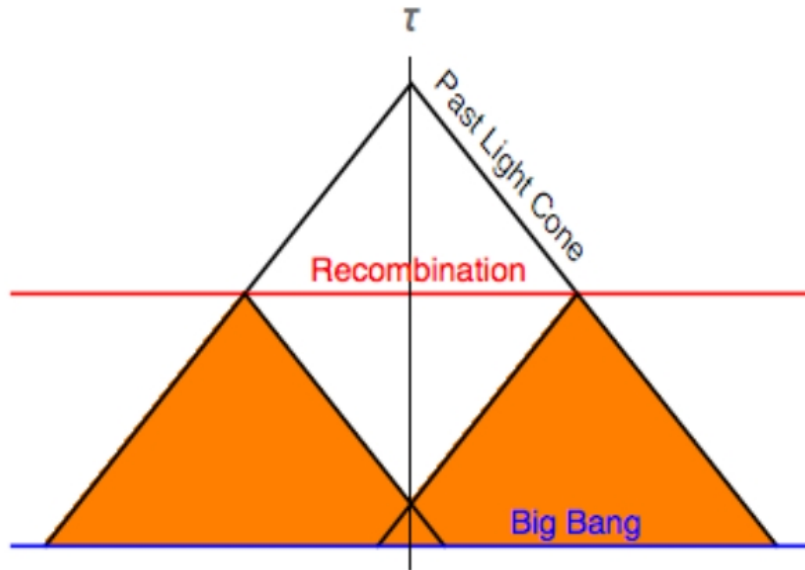


- solves horizon problem
- solves flatness problem
- dilutes dangerous relics

Inflation

- rapid expansion of space

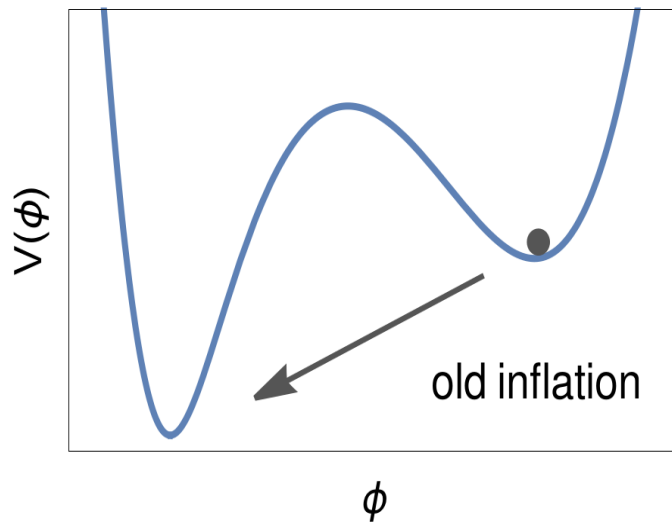
Guth 1981



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Old Inflation

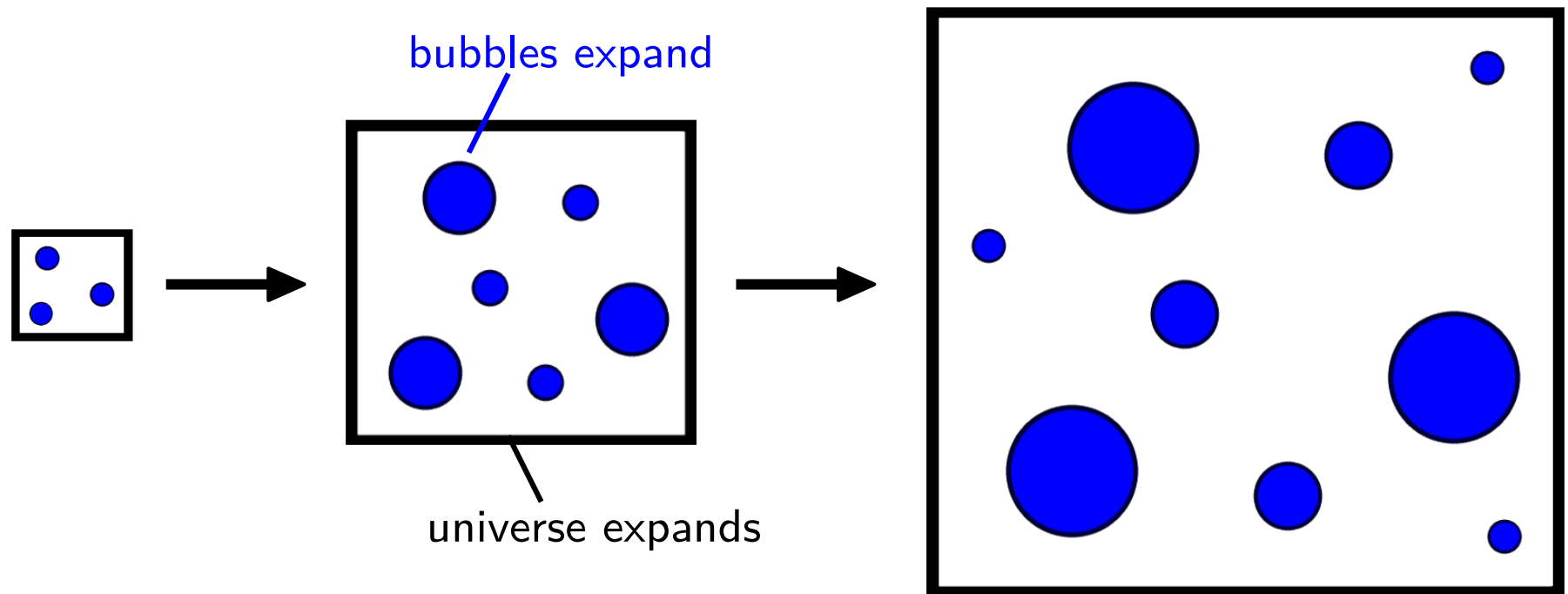
- inflation driven by potential energy of the inflaton field
- inflation models



Guth 1981

The Empty Universe Problem

- bubble formation rate = tunneling rate must be low enough to get 60 e-folds of inflation



bubbles don't percolate, no reheating, no particles

- volume of a single bubble created at time t_0

$$V_b \simeq \frac{4\pi}{9} e^{H(t-t_0)} \quad (\text{for } t \gg t_0)$$

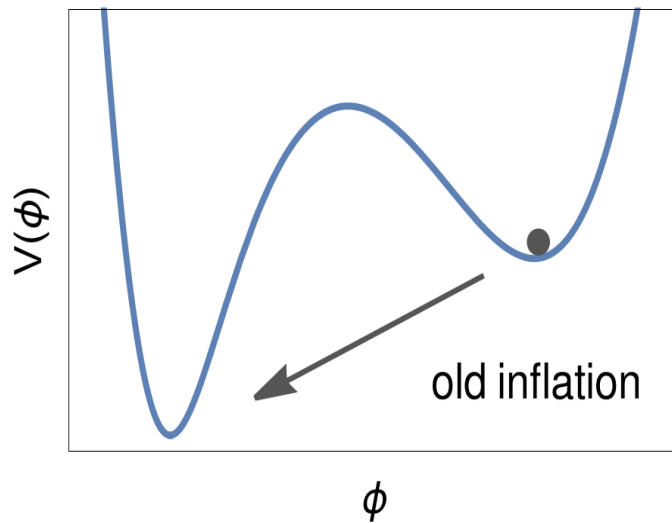
- volume of all bubbles vs. volume of the universe

$$\frac{\sum V_b}{V_{\text{tot}}} = \frac{4\pi}{9} \frac{\Gamma}{H^4}$$

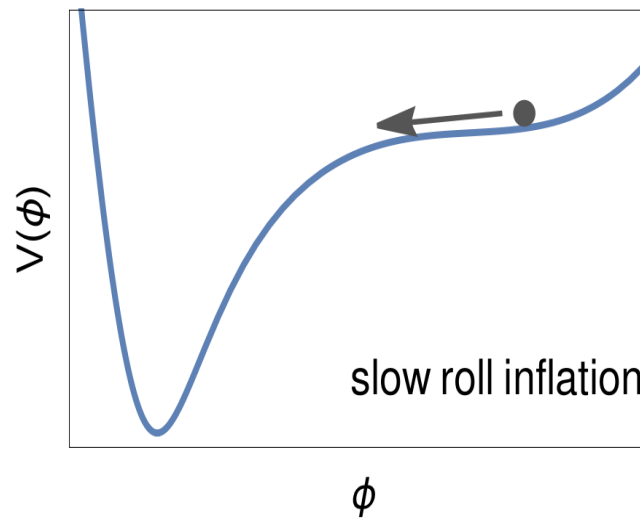
- bubble percolation hence requires $\Gamma/H^4 \gtrsim 1$
- inflation can only last for ~ 1 e-fold, **but need 60 e-folds**

Slow Roll Inflation

- inflation driven by potential energy of the inflaton field
- inflation models



Guth 1981

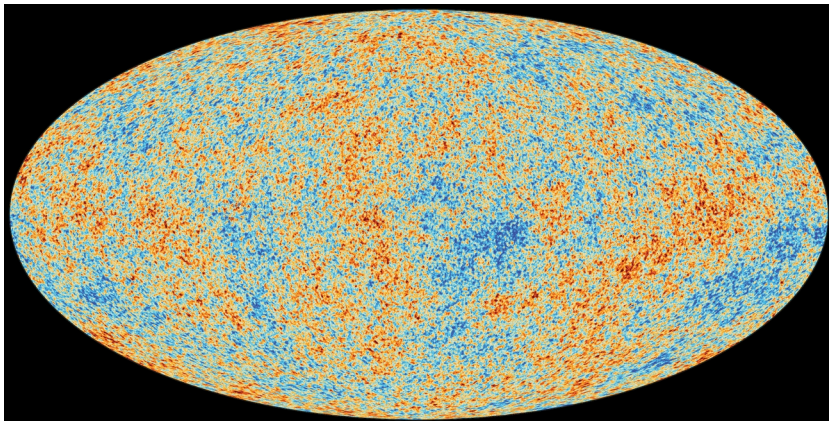
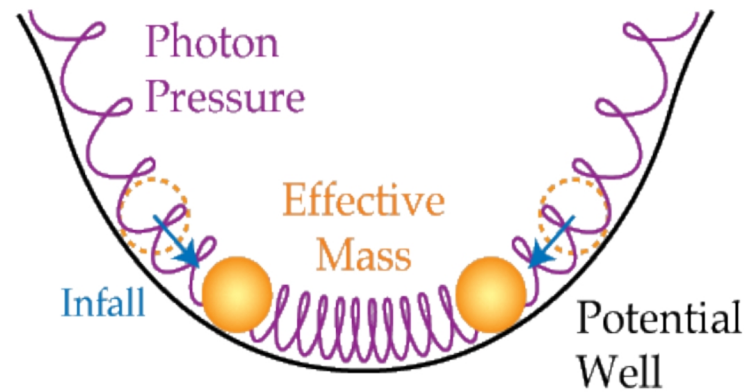


Linde 1982

Albrecht, Steinhardt 1982

Slow Roll Inflation

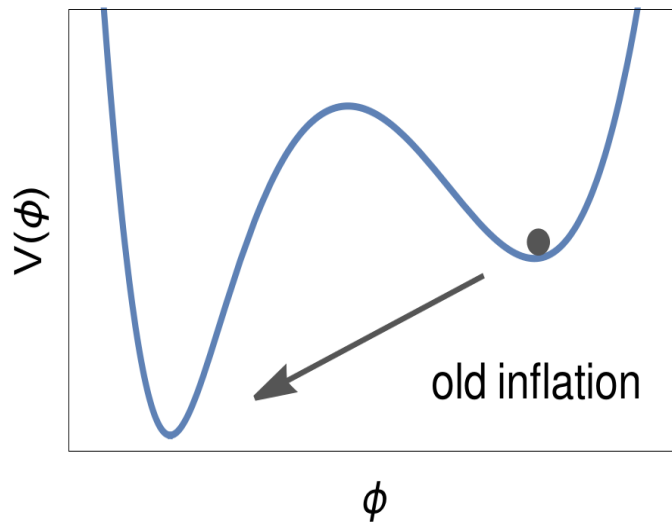
- quantum fluctuations of the inflaton are stretched, reheating converts them into density fluctuations



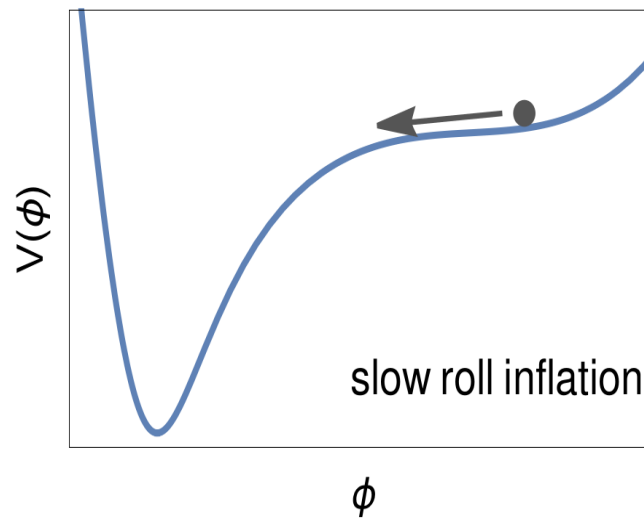
- create potential wells for baryon photon plasma
- acoustic oscillations due to radiation pressure
- CMB provides snapshot at last scattering

Chain Inflation

- inflation driven by potential energy of the inflaton field
- inflation models

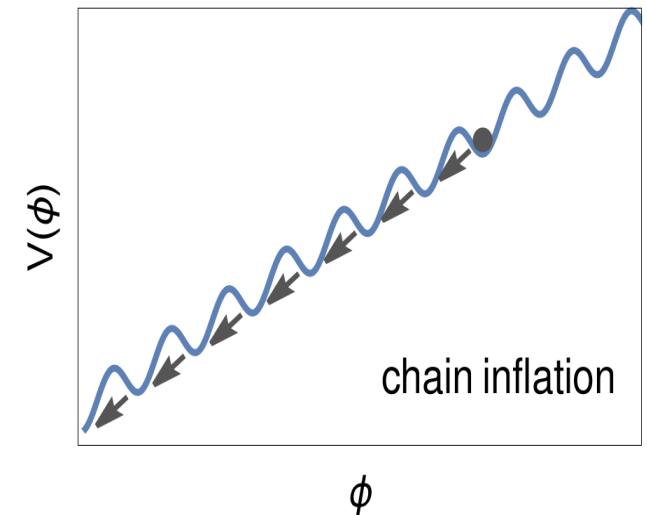


Guth 1981



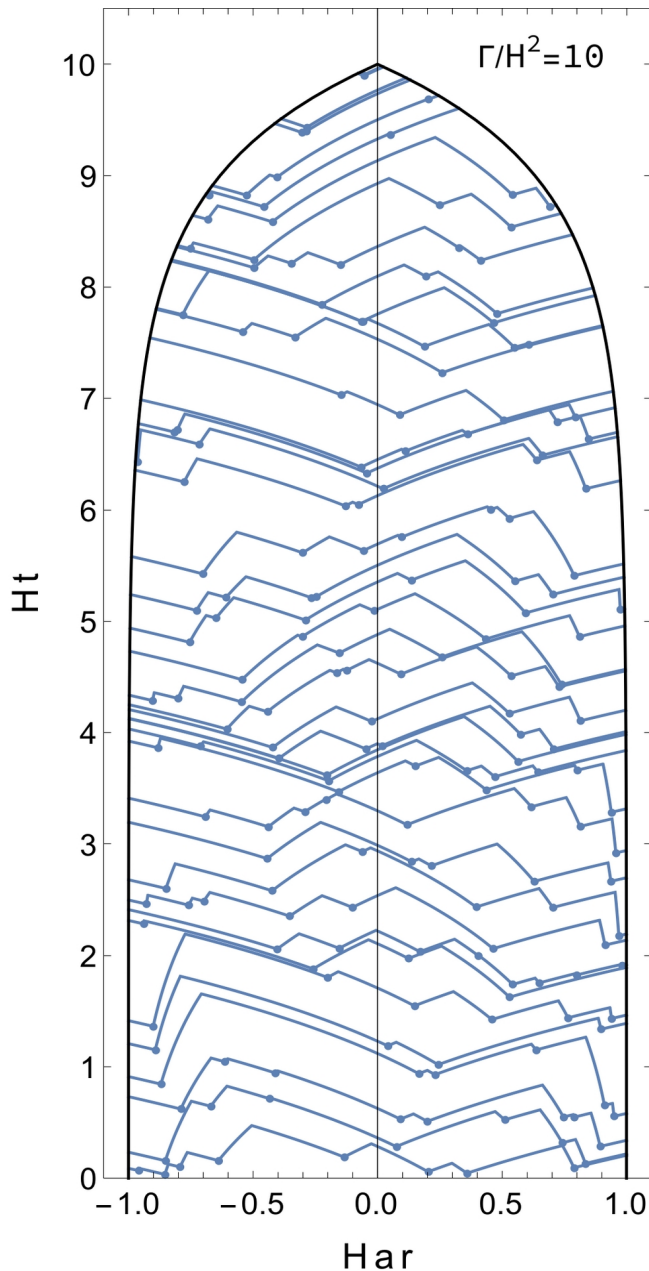
Linde 1982

Albrecht, Steinhardt 1982



Freese, Spolyar 2005

Chain Inflation



- inflaton tunnels along a series of false vacua of ever lower energy
- large Γ , bubbles are formed close to each other and percolate quickly
- bubble collisions create radiation which is quickly redshifted away
- what about the CMB?

CMB Anisotropies in Chain Inflation

origin of fluctuations:

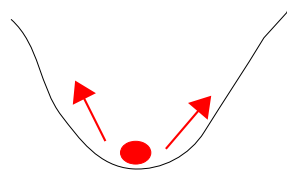
- quantum fluctuations of the inflaton

CMB Anisotropies in Chain Inflation

origin of fluctuations:

- ~~quantum fluctuations of the inflaton~~

suppressed by
inflaton mass

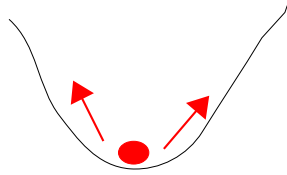


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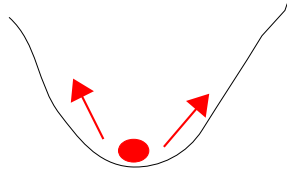
- bubble wall collisions

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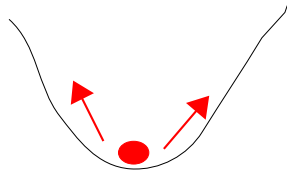
- (bubble wall collisions)
maybe, but complicated

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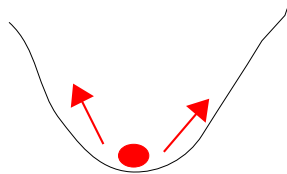
- probabilistic nature of tunneling

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- (bubble wall collisions)
maybe, but complicated

- probabilistic nature of tunneling

scalar power spectrum:

$$\Delta_{\mathcal{R}}^2 = \frac{H^2}{4\pi^2 c_s \epsilon} (-kc_s \eta)^{-2\epsilon}$$

$$\Delta_{\mathcal{R}}^2 = (0.04 \pm 0.02) \left(\frac{\Gamma}{H^4} \right)^{-0.42 \pm 0.03}$$

$$\Delta_{\mathcal{R}}^2 = \frac{H^2}{8\pi^2 \epsilon}$$

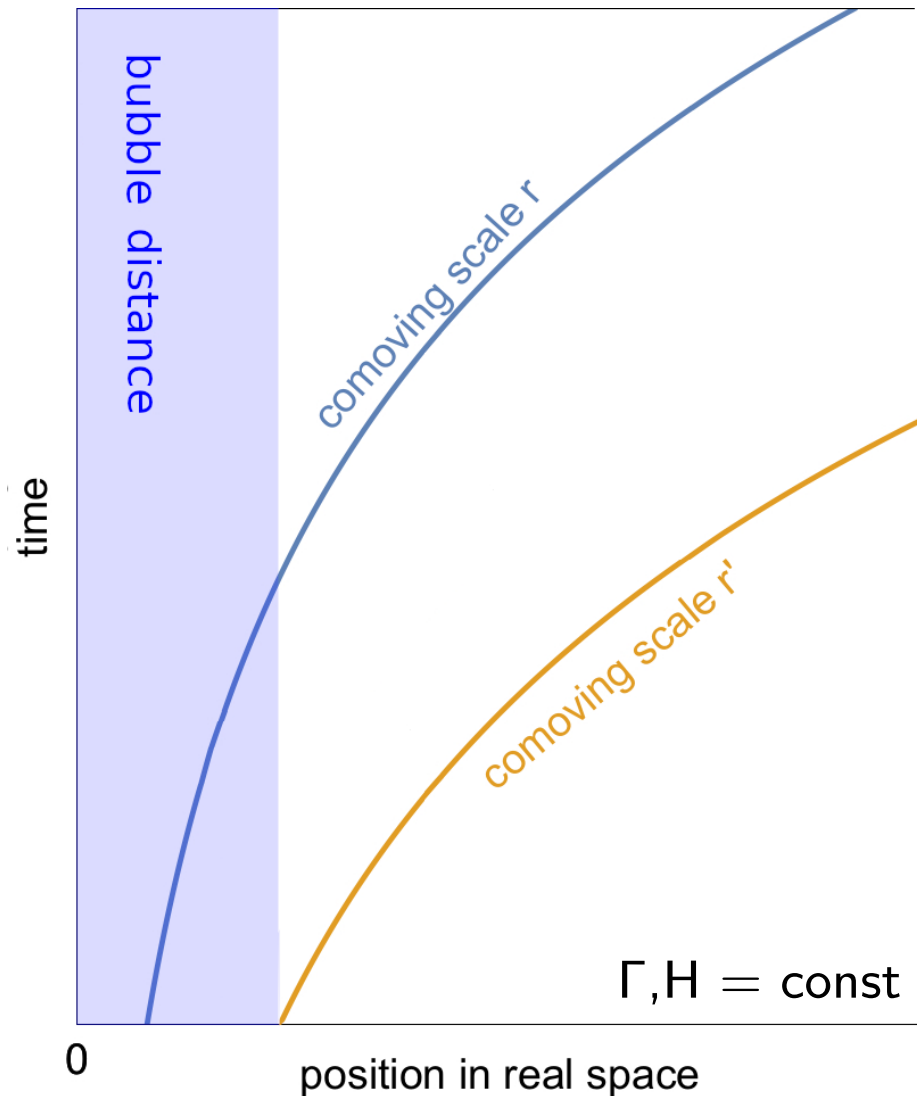
$$\Delta_{\mathcal{R}}^2 = \frac{H^2}{8\pi^2 \epsilon / \sqrt{3}}$$

$$\Delta_{\mathcal{R}}^2 = \frac{3}{4\pi} \frac{H^4}{\Gamma}$$

Watson et al. 2007, Feldstein, Tweedie 2007,
Huang 2007, Chialva, Danielsson 2008 & 2009,
Cline, Moore, Wang 2011

literature in vast
disagreement

Scalar Power Spectrum



two-point correlation:

$$\langle \delta\phi(r)\delta\phi(0) \rangle = \text{var}\phi(\Delta t) + \text{const}$$

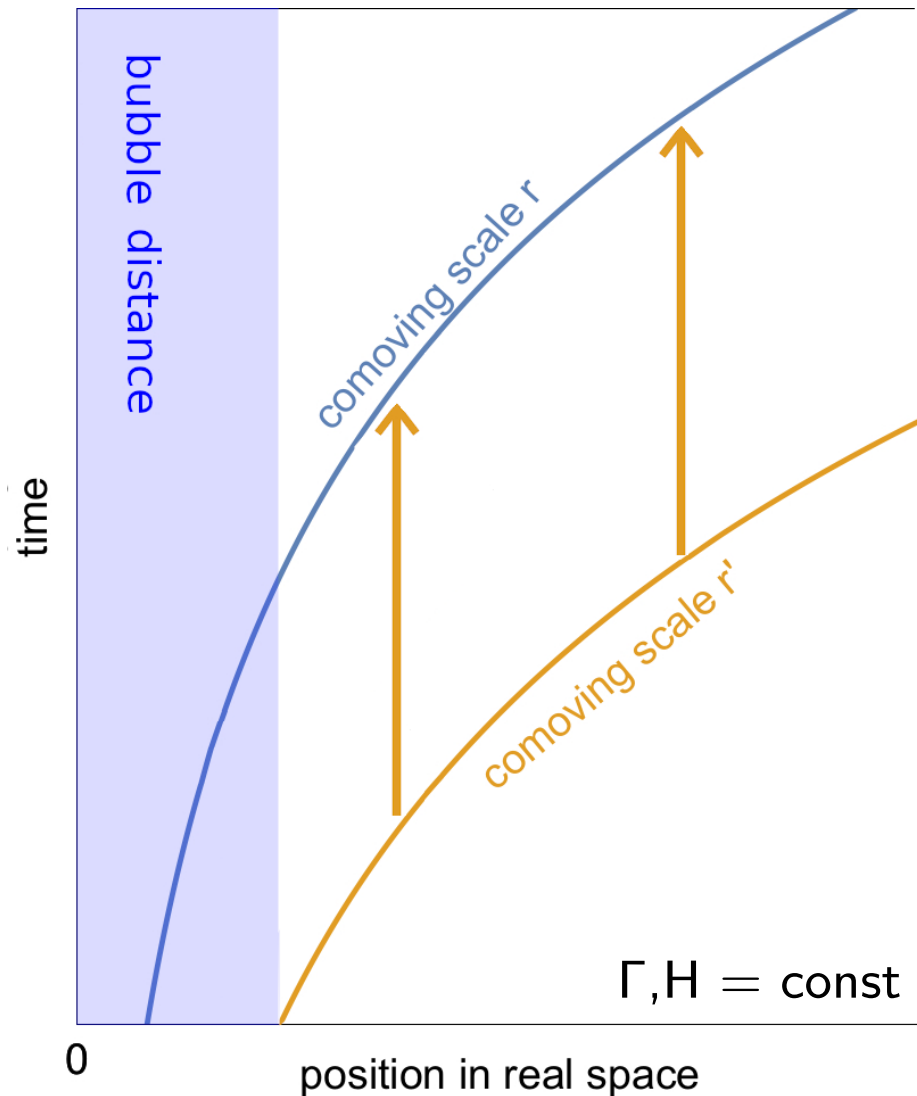
curvature perturbation

$$\mathcal{R} = H \left(\frac{d\langle\phi\rangle}{dt} \right)^{-1} \delta\phi$$

scalar power spectrum

$$\Delta_{\mathcal{R}}^2 = \left(\frac{d\langle\phi\rangle}{Hdt} \right)^{-2} \frac{d\text{var}\phi}{Hdt}$$

Scalar Power Spectrum



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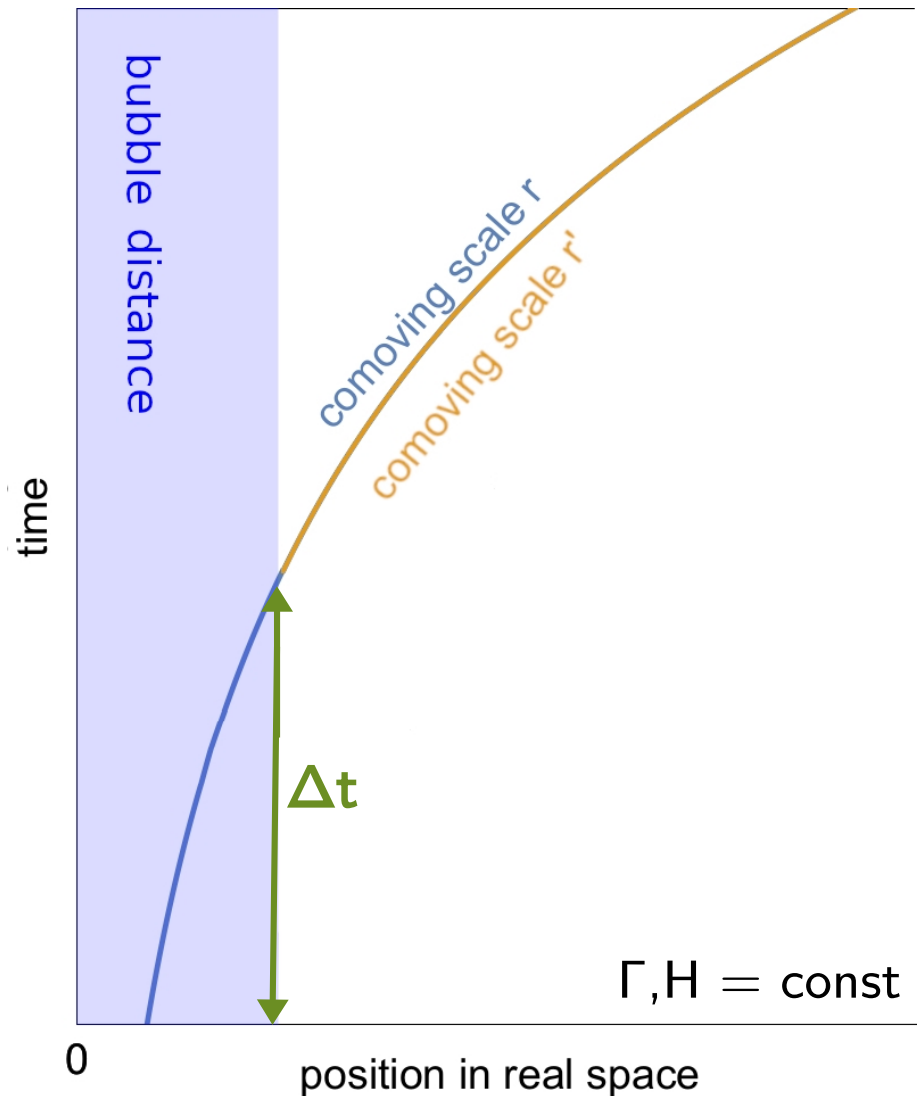
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Scalar Power Spectrum



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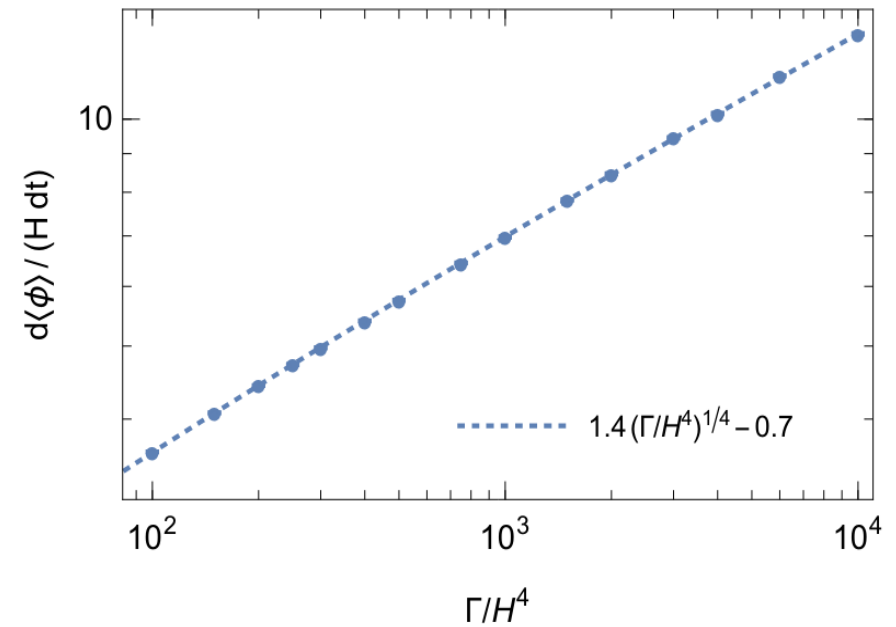
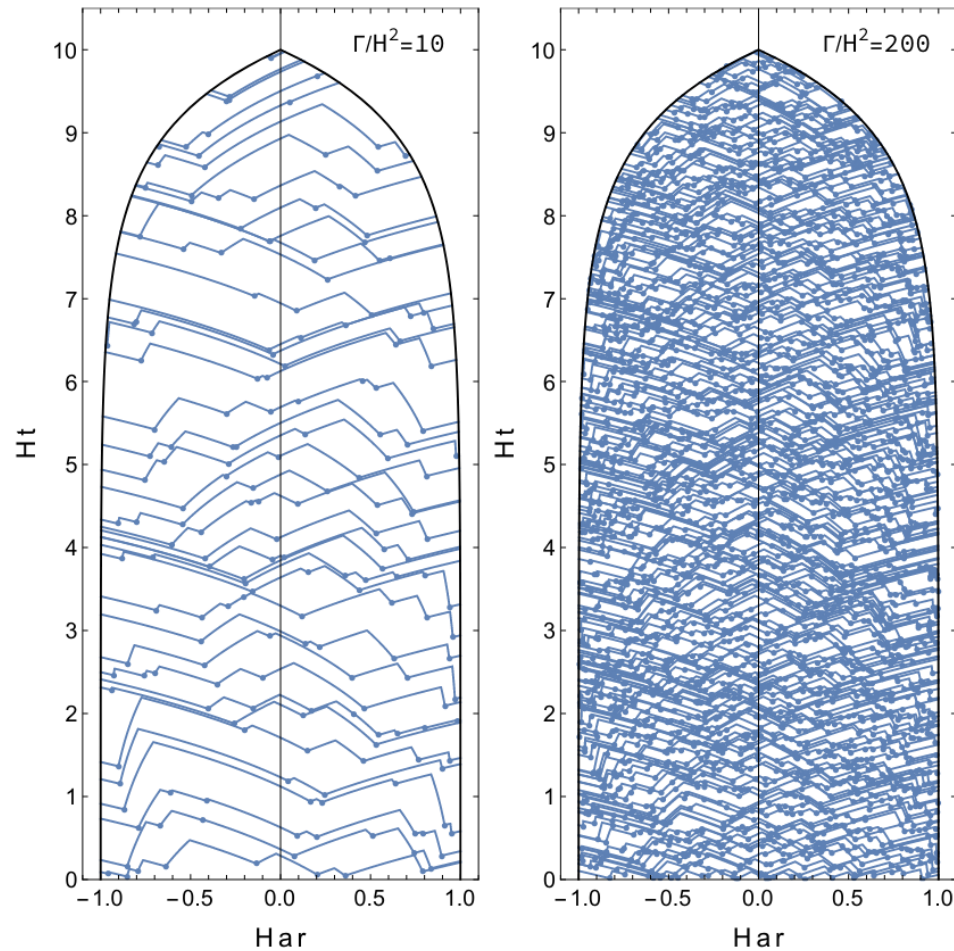
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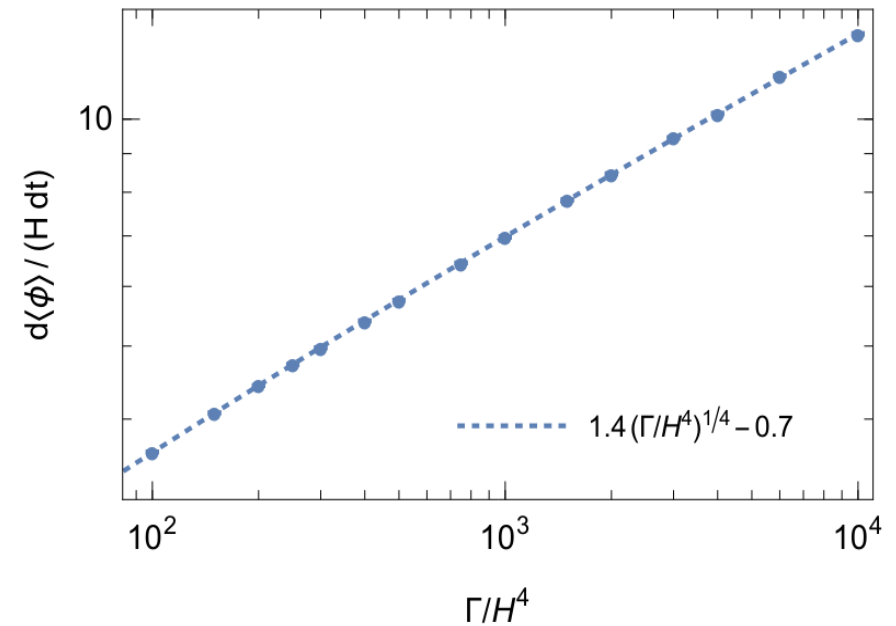
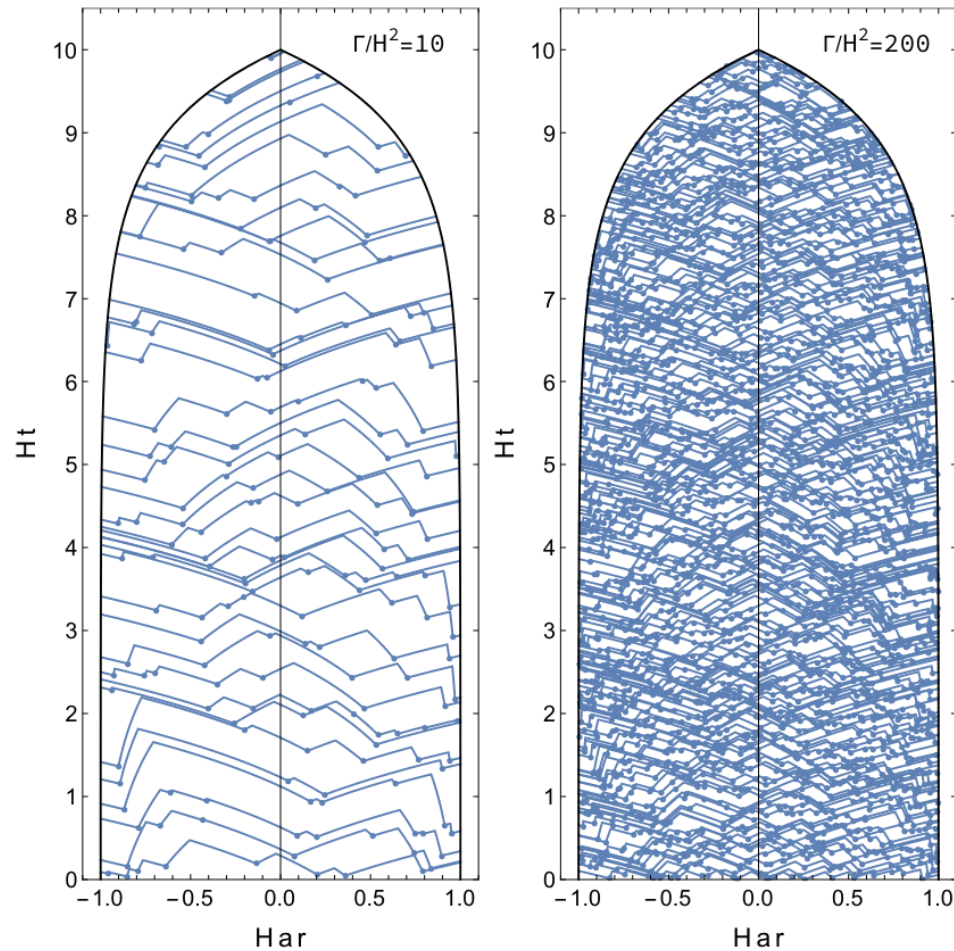
Simulations



$$\frac{d\langle\phi\rangle}{Hdt} \propto \left(\frac{\Gamma}{H^4}\right)^{1/4} \quad \frac{d\text{var}\phi}{Hdt} \propto \left(\frac{\Gamma}{H^4}\right)^{1/12}$$

$$\Delta_{\mathcal{R}}^2 \simeq 0.06 \left(\frac{\Gamma}{H^4}\right)^{-5/12}$$

Simulations



$$\frac{d\langle\phi\rangle}{Hdt} \propto \left(\frac{\Gamma}{H^4}\right)^{1/4} \quad \frac{d\text{var}\phi}{Hdt} \propto \left(\frac{\Gamma}{H^4}\right)^{1/12}$$

$$\Delta_{\mathcal{R}}^2 \simeq 0.06 \left(\frac{\Gamma}{H^4}\right)^{-5/12}$$

$$\Delta_{\mathcal{R}}^2 = (0.04 \pm 0.02) \left(\frac{\Gamma}{H^4}\right)^{-0.42 \pm 0.03}$$

Feldstein, Tweedie 2007

Comparison with CMB

- COBE normalization $\Delta_{\mathcal{R}}^2 = 2 \times 10^{-9}$ implies

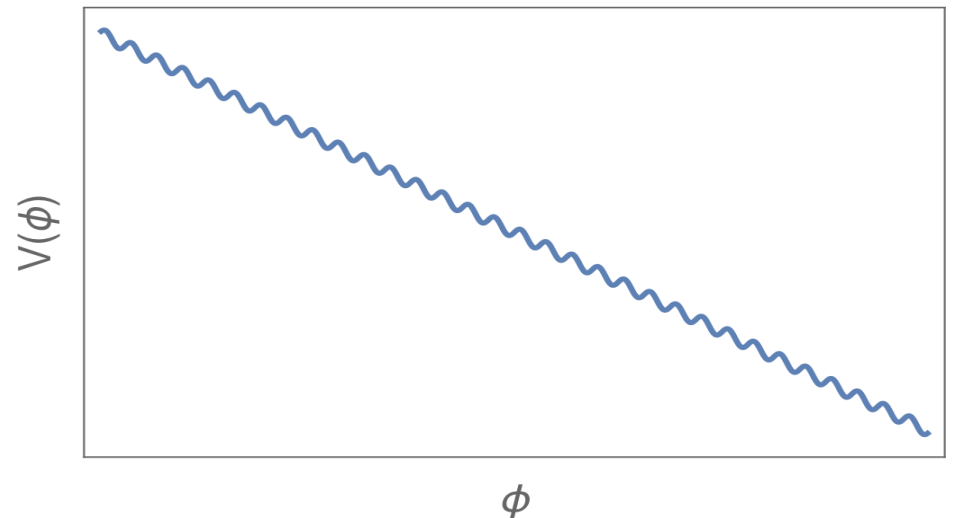
$$\frac{\Gamma}{H^4} = 10^{18} \implies \frac{\text{vacuum transitions}}{\text{e-fold of inflation}} \simeq \left(\frac{\Gamma}{H^4} \right)^{1/4} \simeq 4 \times 10^4$$

- scale-invariance of power spectrum broken by \dot{H} , $\dot{\Gamma}$

$$n_s = 1 + \frac{5}{12} \left(\frac{4\dot{H}}{H^2} - \frac{\dot{\Gamma}}{H\Gamma} \right)$$

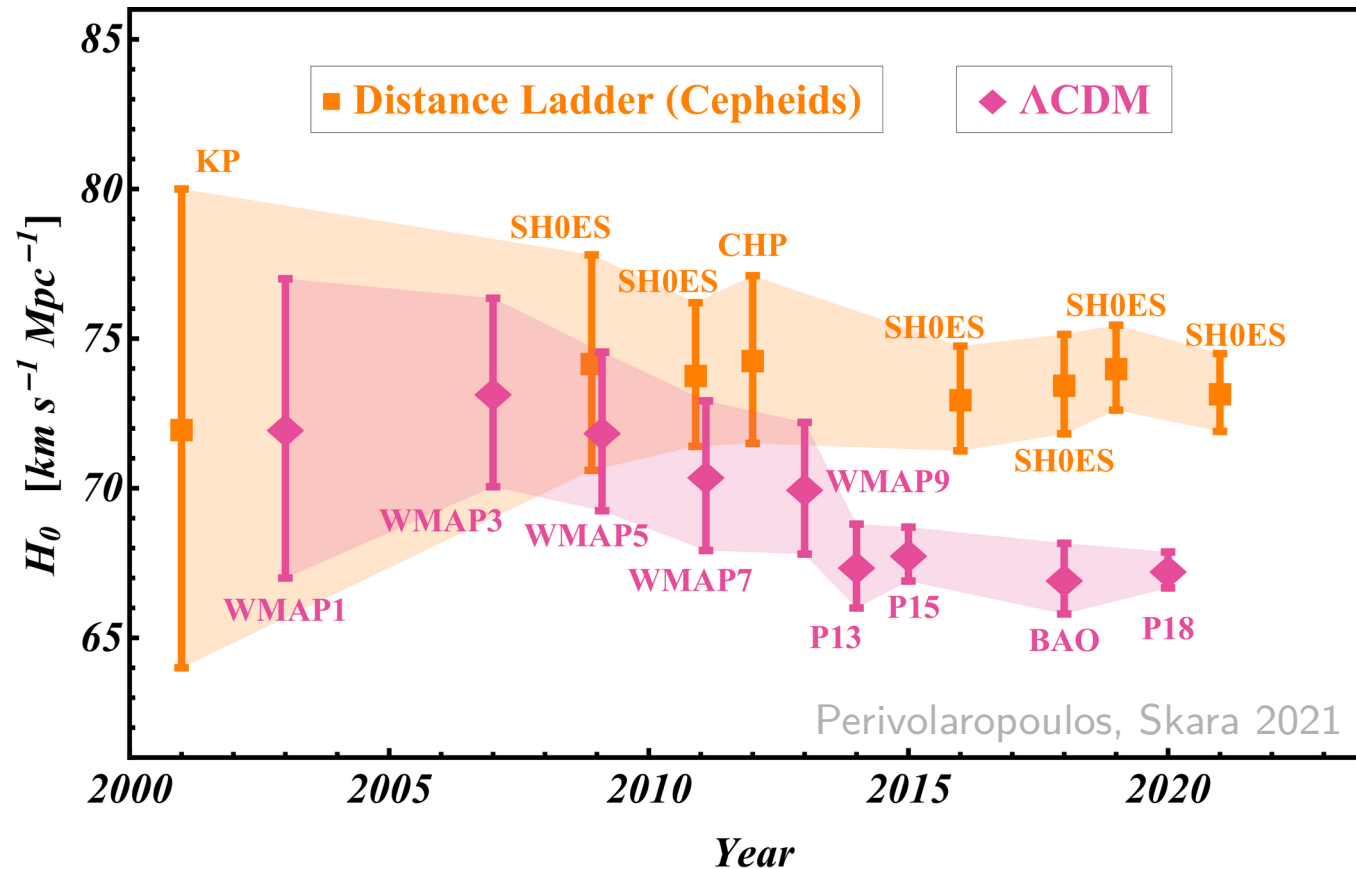
- simplest scenario with pure tilted cosine potential

$$n_s \simeq 1 - 0.03 \left(\frac{\Delta V}{10^{-6} V} \right)$$



Hubble Crisis

- H_0 disagrees between CMB and local measurements

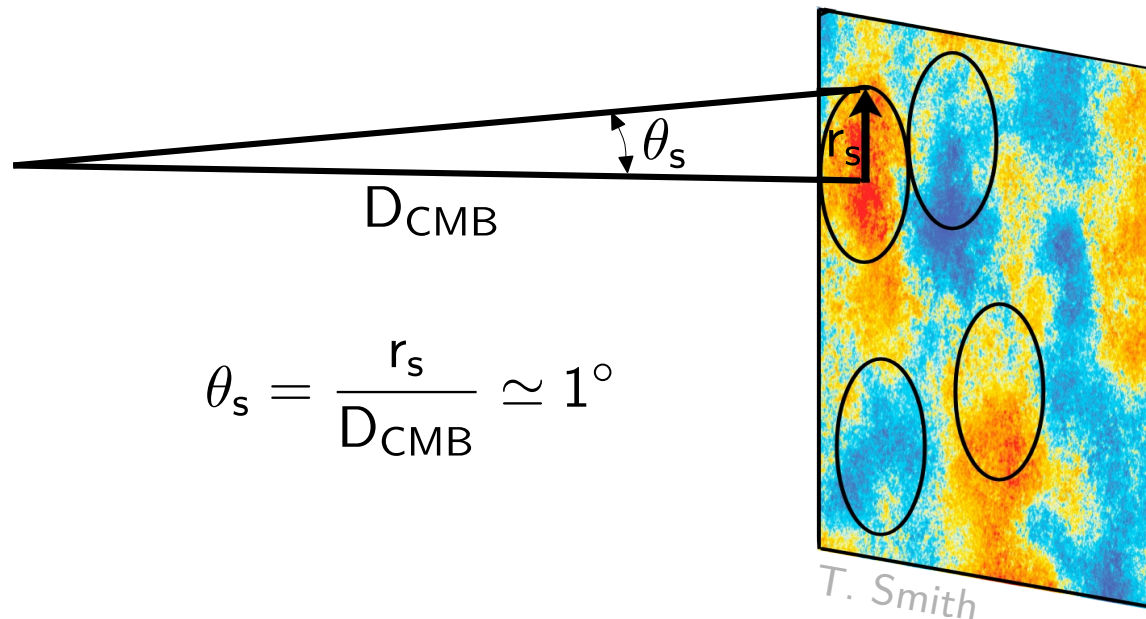


$$H_0 \left[\frac{\text{km}}{\text{s Mpc}} \right] = 73.2 \pm 1.3 \quad (\text{SH0ES}) \quad H_0 \left[\frac{\text{km}}{\text{s Mpc}} \right] = 67.3 \pm 0.6 \quad (\text{Planck 2018})$$

Riess et al. 2021

Early Time Solution

- CMB fixes angular size of sound horizon



$$\theta_s = \frac{r_s}{D_{\text{CMB}}} \simeq 1^\circ$$

$$r_s = \int_{z_{\text{CMB}}}^{\infty} \frac{c_s dz}{H(z)}$$

dominated by H_0

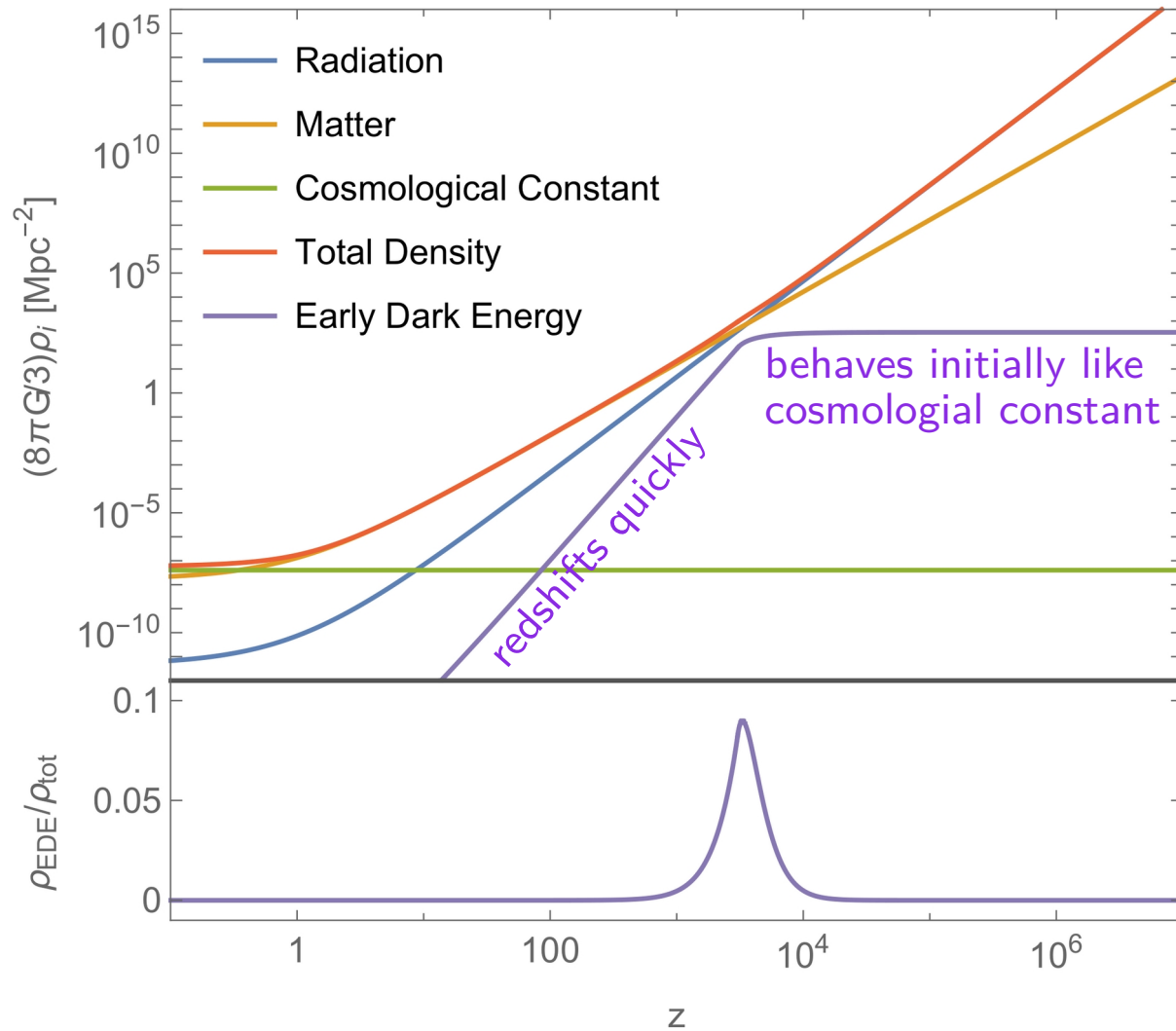
$$D_{\text{CMB}} = \int_0^{z_{\text{CMB}}} \frac{dz}{H(z)}$$

dominated by $H(z)$ around z_{CMB}

- additional energy density before recombination reduces sound horizon
- fixed θ_s then requires larger $D_{\text{CMB}} \longrightarrow H_0$ increases

Early Dark Energy

Karwal, Kamionkowski 2016, Poulin et al. 2018 & 2019



- cosmological data favor peaked energy injection around $z \sim 4000$

Smith et al. 2020

Datasets	Λ CDM	EDE
<i>Planck</i> high- ℓ	2446.66	2444
<i>Planck</i> low- ℓ	10496.65	10493.25
<i>Planck</i> lensing	10.37	10.24
BAO-low z	1.86	2.53
BAO-high z	1.84	2.1
Pantheon	1027.04	1027.11
SH0ES	16.80	1.68
Total χ^2_{\min}	14001.23	13980.94
$\Delta\chi^2_{\min}$	0	-20.29

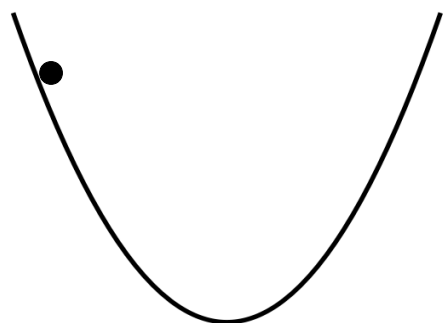
dark radiation only yields $\Delta\chi^2_{\min} \simeq -4$

Agrawal et al. 2019

Oscillating Scalar Field Models

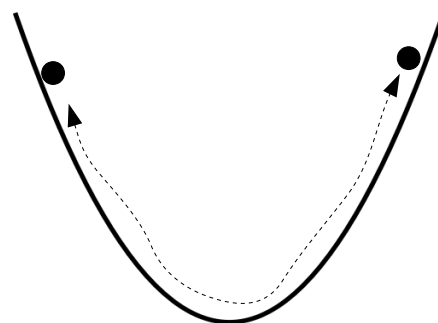
- scalar field in the dark sector displaced from its minimum

Smith et al. 2020



initially trapped
by Hubble friction

$$\rho_\phi = \text{const}$$



oscillations once

$$H < m_\phi$$

$$\rho_\phi \propto a^{-n}$$

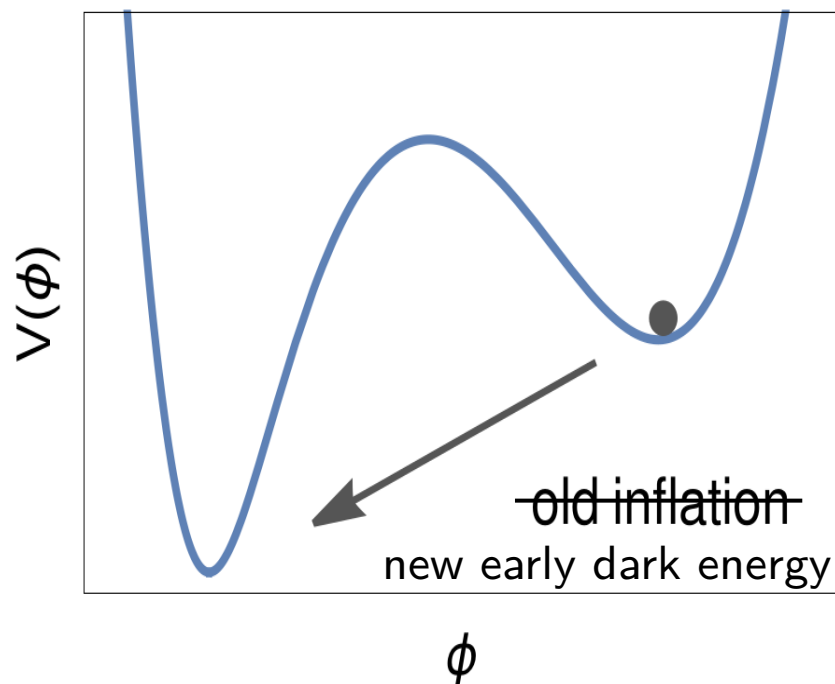
$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

- problem: standard potentials yield too slow redshift ($n \sim 3$), 'weird' potential required

like non-relativistic matter

$$V \sim \left(1 - \cos \frac{\phi}{f}\right)^3$$

Early Dark Energy via Phase Transition



- dark sector scalar field trapped in false vacuum
 $\rho = \text{const}$
- bubbles of true vacuum, energy stored in bubble walls

- upon collision wall energy is transferred to
 - (1) anisotropic stress $\rho \propto a^{-6}$ (?)
 - (2) gravity waves $\rho \propto a^{-4}$
 - (3) dark radiation $\rho \propto a^{-4}$
- } consistent with EDE

Niedermann, Sloth 2020 & 2021

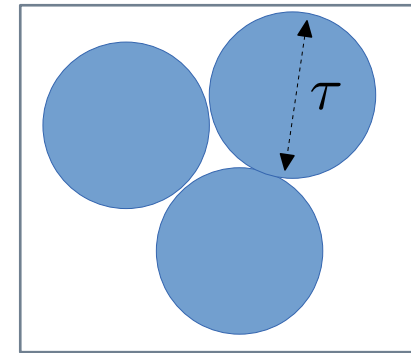
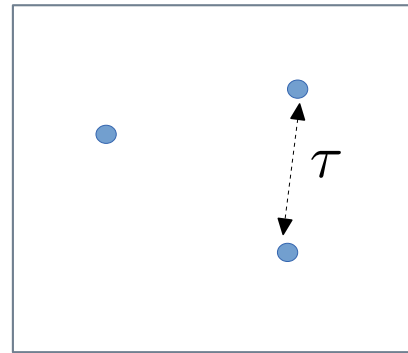
The Anisotropy Problem

- required lifetime of the universe in the false vacuum

$$\tau \sim 2.5 \times 10^4 \text{ yr} \times \left(\frac{5000}{z_*} \right)^2$$

EDE solution $z_* \lesssim 5000$

- anisotropies of size $\Gamma^{-1/4} \sim \tau$ are formed



- angular size of fluctuations at last scattering

$$\theta \simeq 0.1^\circ \times \frac{5000}{z_*}$$

CMB observations: $\theta \gtrsim 0.05$

Large Scale Structure: $\theta \simeq 0.002 - 0.2$

► CMB, LSS spoiled

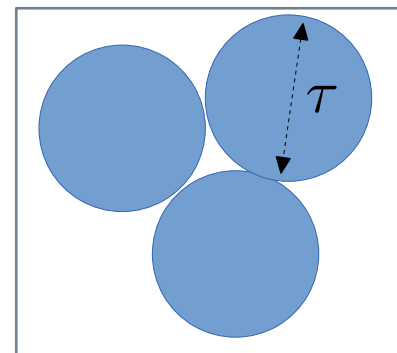
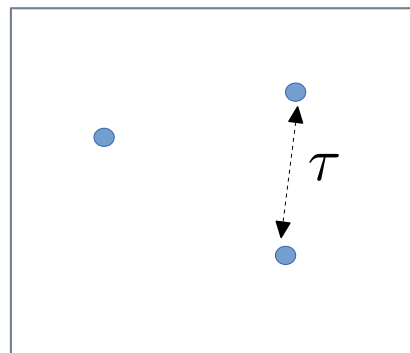
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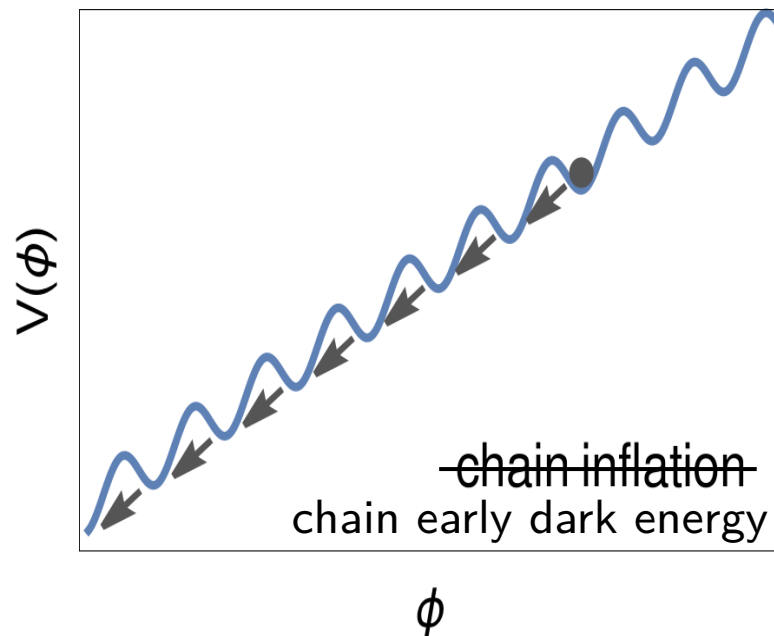
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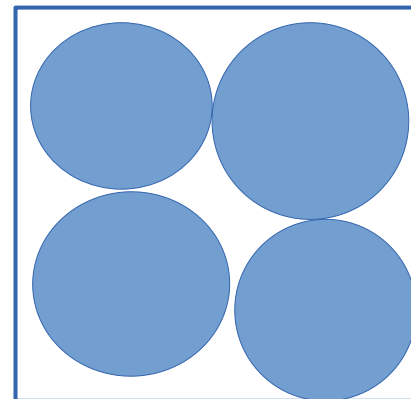
caveat: make Γ time-dependent in two-field system

Adams, Freese 1991, Niedermann, Sloth 2020 & 2021

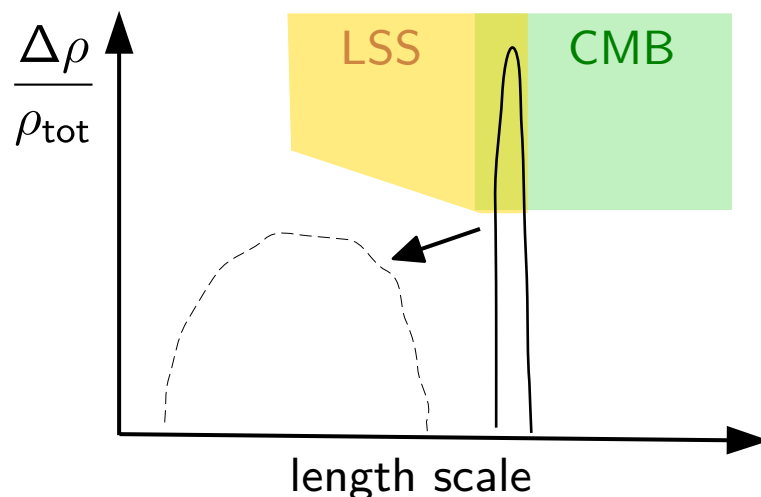
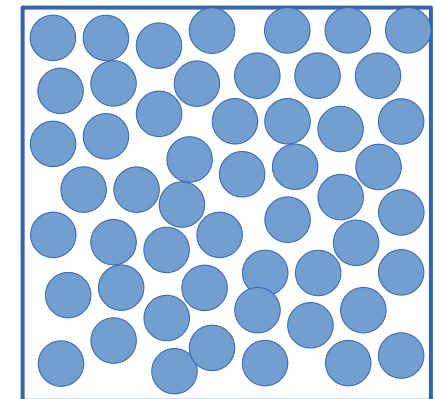
Chain Early Dark Energy



single phase transition



multiple phase transitions



- multiple phase transitions reduce size and amplitude of anisotropies
- constraints evaded for $N > 600$ transitions

Evolution of Energy Density

including energy from
bubble collisions:

$$z \frac{d\rho_\phi}{dz} \simeq \frac{1.4 \Delta V \Gamma^{1/4}}{H(z)}$$

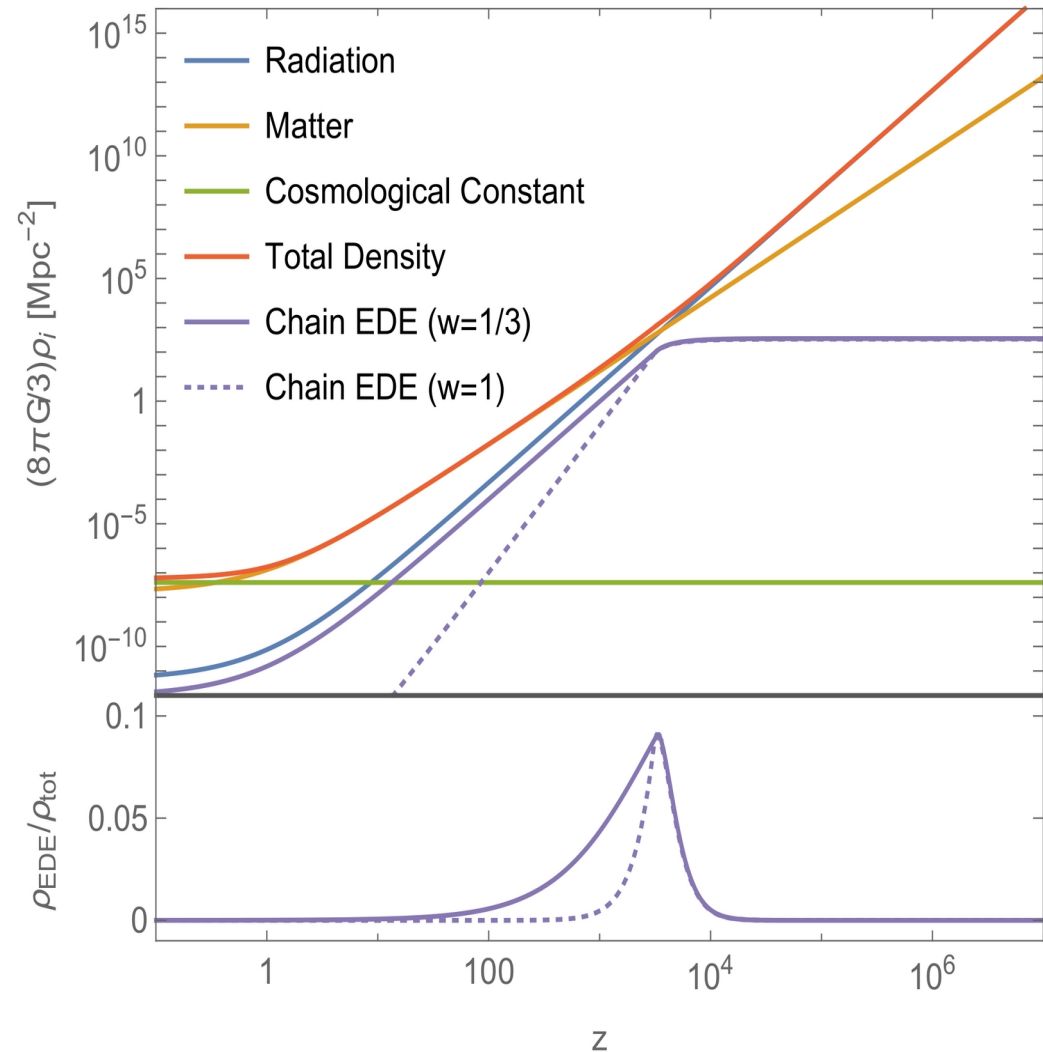
$$z \frac{d\rho_{\text{DS}}}{dz} \simeq -\frac{1.4 \Delta V \Gamma^{1/4}}{H(z)} + 3(1 + w) \rho_{\text{DS}}$$

$w = 1/3$ (dark radiation)

$w = 1$ (anisotropic stress)

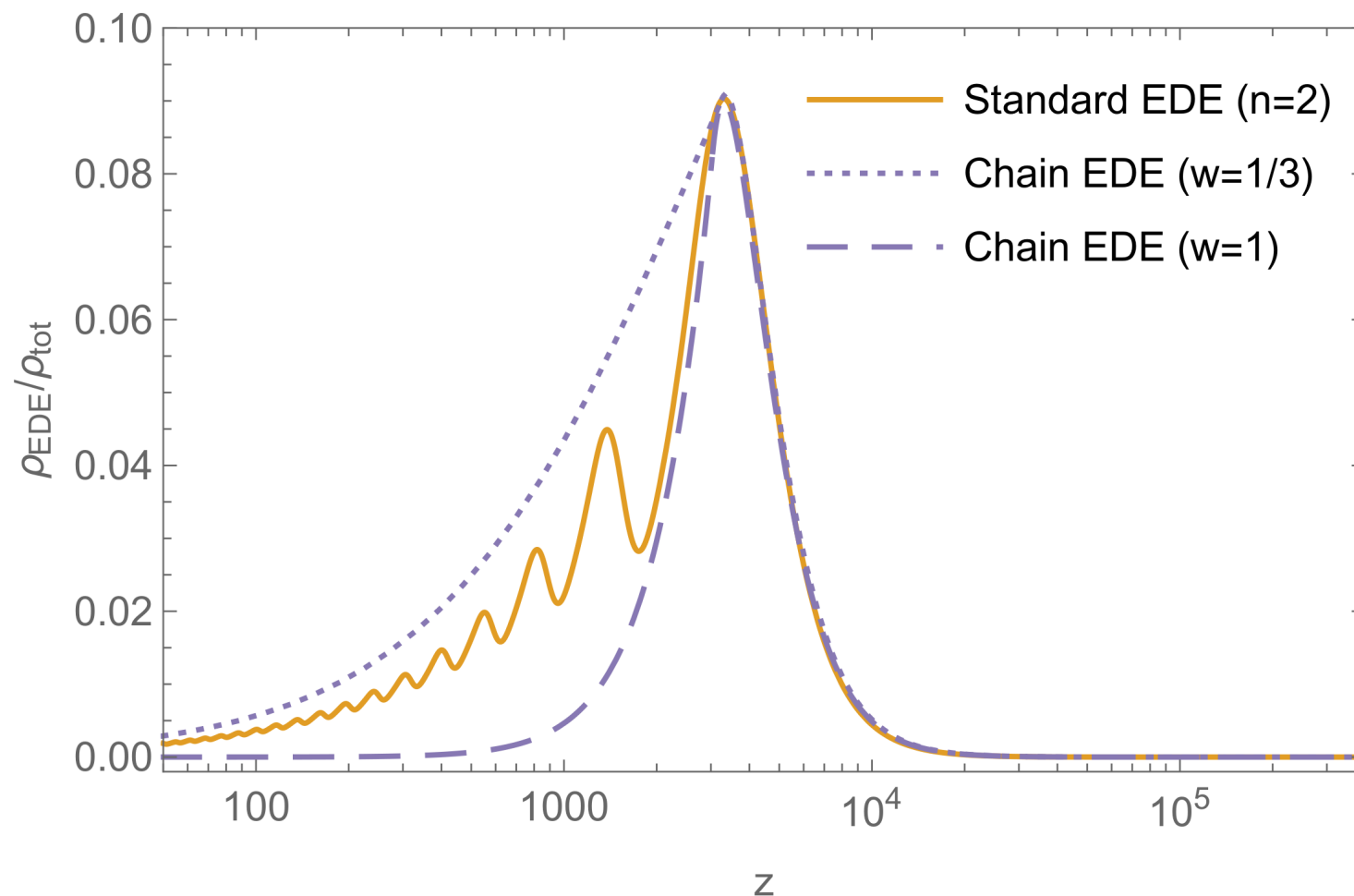
$\Gamma^{-1/4} \sim$ lifetime in single vacuum

ΔV = energy difference between vacua



$$V_0 = 0.25 \text{ eV}^4 \quad N \Gamma^{-1/4} = 8 \times 10^4 \text{ yr}$$

Comparison with Standard EDE

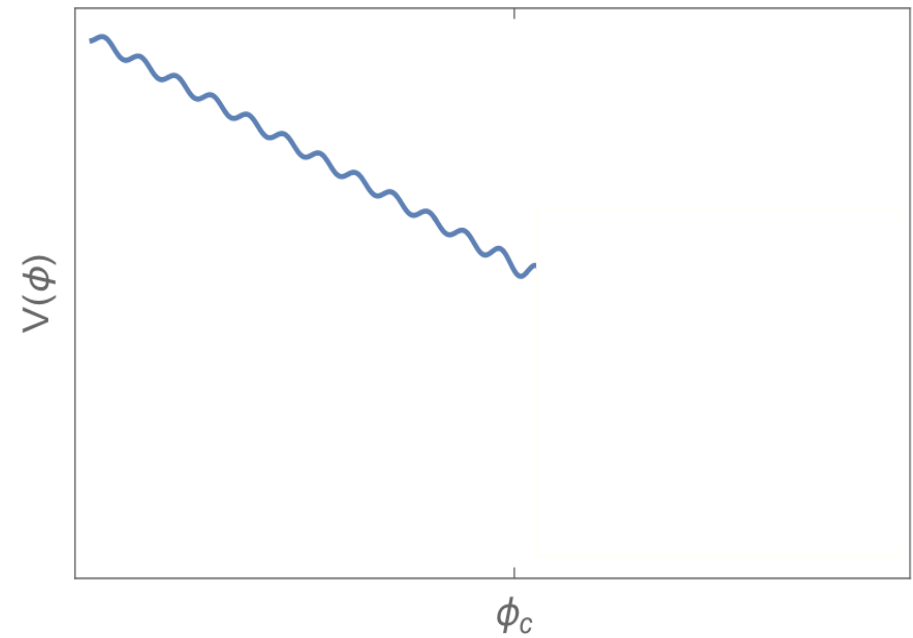


- evolution of ρ_{EDE} very similar to best-fit oscillating scalar field models ► **solution to Hubble tension**

Model Realization via Axions

- tilted cosine

$$V = -gM^3\phi + \Lambda_0^4 \cos \frac{\phi}{f}$$

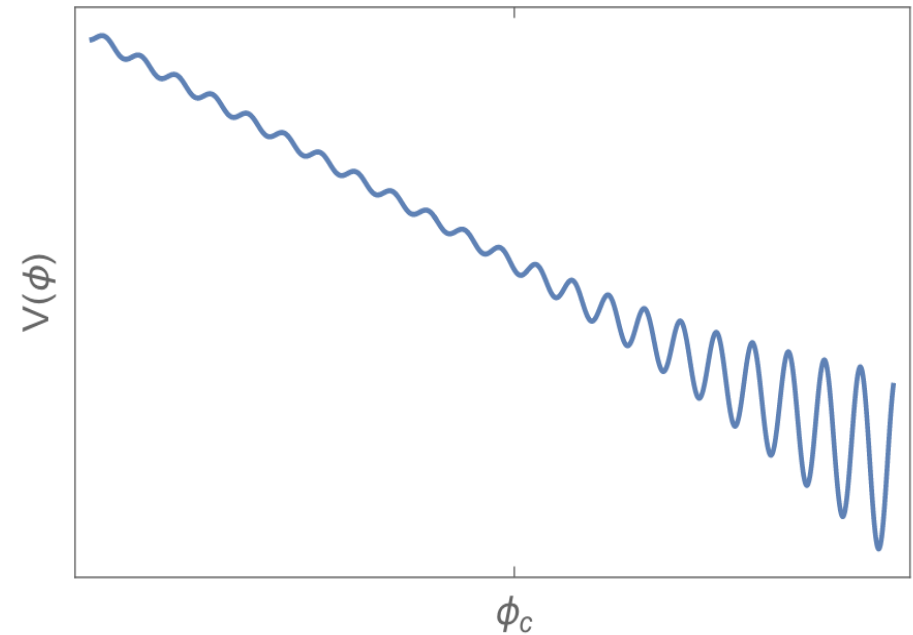


Model Realization via Axions

- tilted cosine + stopping

$$V = -gM^3\phi + (\Lambda_0^4 + \Lambda_1^2\chi^2) \cos \frac{\phi}{f} \\ + (M^2 - gM\phi)\chi^2 + \lambda\chi^4$$

Graham, Kaplan, Rajendran 2015

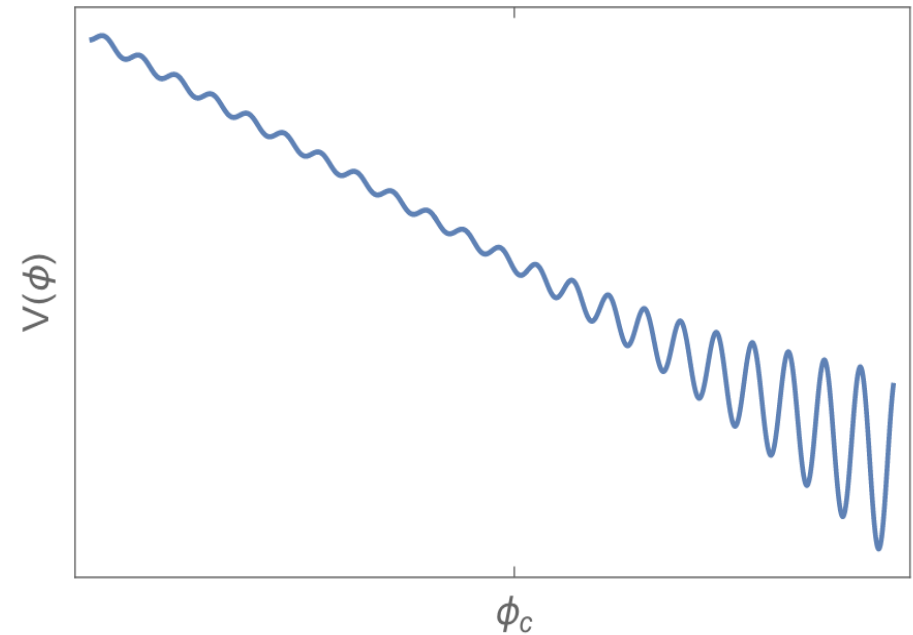
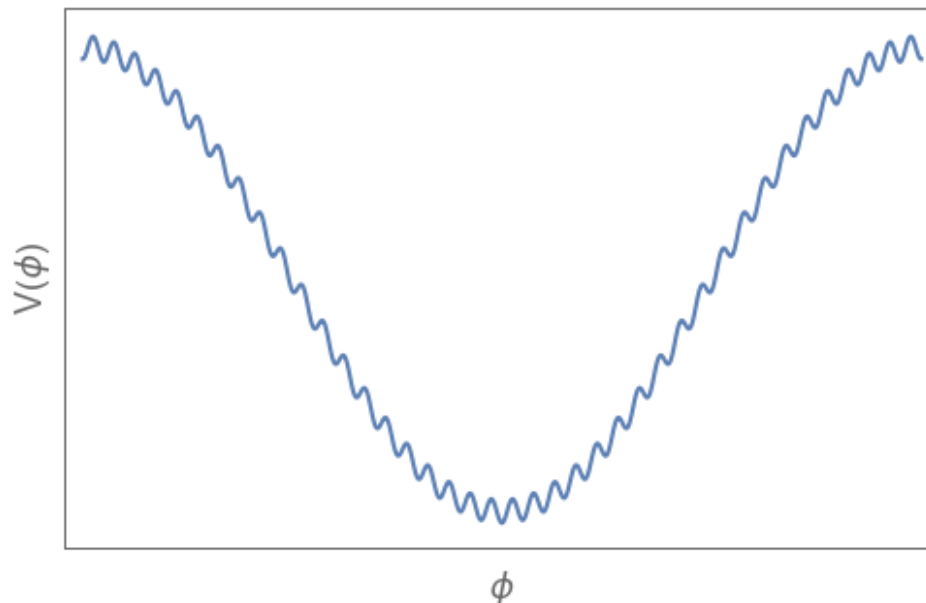


Model Realization via Axions

- tilted cosine + stopping

$$V = -gM^3\phi + (\Lambda_0^4 + \Lambda_1^2\chi^2) \cos \frac{\phi}{f} \\ + (M^2 - gM\phi)\chi^2 + \lambda\chi^4$$

Graham, Kaplan, Rajendran 2015



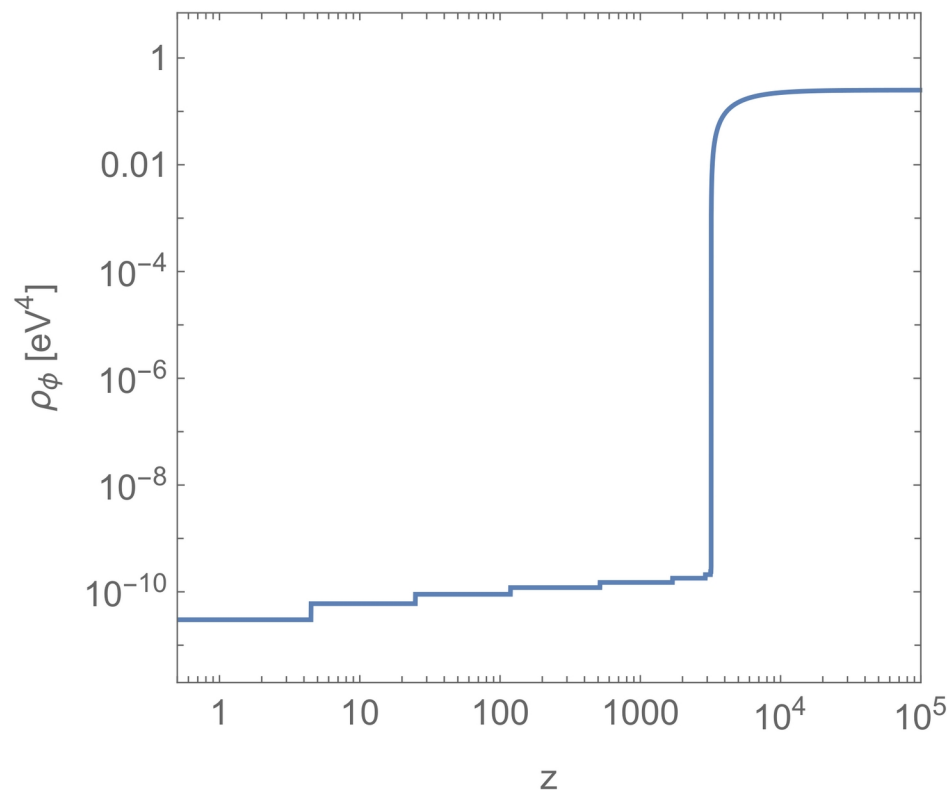
- double-periodic potentials, e.g. axion with leading and subleading instanton

Connection to Dark Energy

- energy difference between minima in chain EDE

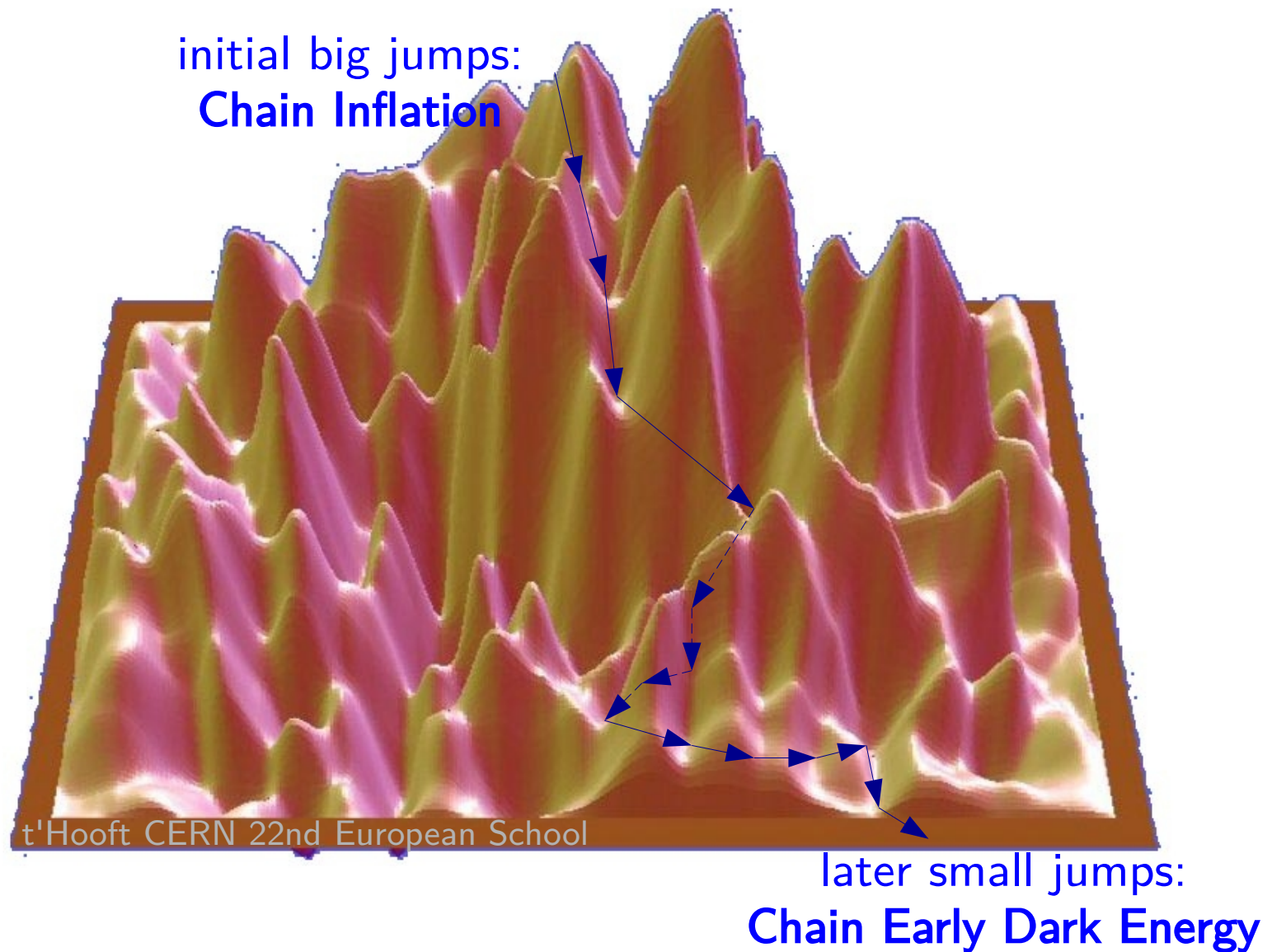
$$(\Delta V)^{1/4} \sim 2 \text{ meV} \times \frac{300}{N^{1/4}}$$

scale of today's
Dark Energy



- EDE field may get trapped in the lowest minimum with positive energy and account for today's Dark Energy

Recurrent Chain Dark Energy



see also: Freese, Liu, Spolyar 2006

Summary

- vacuum transitions can have played a major role in the history of the universe
- chain inflation is a serious competitor for slow roll inflation
- chain early dark energy provides a solution to the H_0 tension