Neutrino flavor conversion
in media :
challenges and novel pespectives

Cristina VOLPE
(AstroParticule et Cosmologie-APC, Paris)

Outline

- Introduction : open issues
- A novel perspective on neutrino flavor conversion in media
- Beyond the mean-field approximation
- A linearized approach to flavor instabilities, from many-body physics
- Perspectives

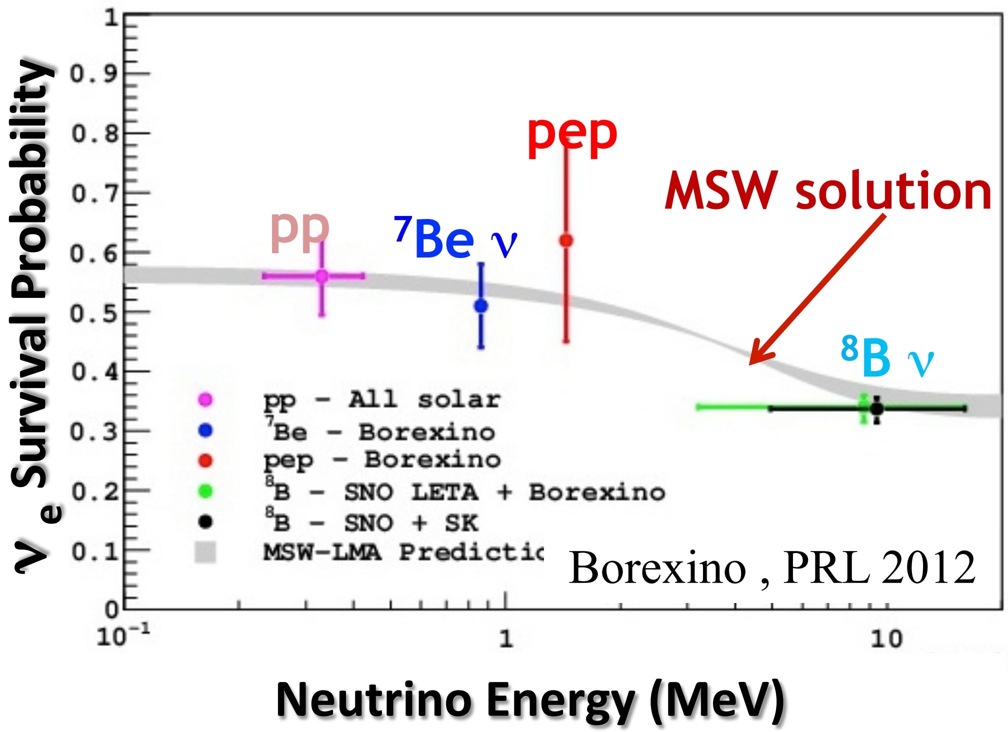
Neutrino flavor conversion in media

Important for observations in astrophysical and cosmological contexts

- ❑ solar neutrinos

  MSW effect : resonant flavor conversion due to ν−e interaction

  Wolfenstein PRD (1978)
  Mikheev and Smirnov, Sov. J. Nucl. Phys. (1985)



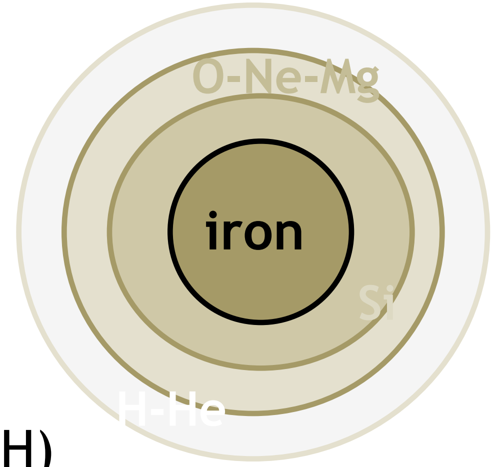
- ❑ Core-collapse supernovae
- ❑ Early universe - ^{4}He, D, ^{3}H, ^{7}Li primordial abundances
- ❑ Earth matter effects - solar, SN and atmospheric neutrinos

  Dziewonski, Anderson, Phys. Earth Planet. Interiors 25 (1981); Ermilova et al., JETP Lett. 43 (1986), Akhmedov, Yad. Fiz. 47 (1988)

Core-collapse supernovae

Core-collapse supernovae comprise :

- ❑ O-Ne-Mg supernovae
- ❑ Iron core-collapse supernovae
- ❑ Very massive
  -> accretion-disk black hole (AD-BH)



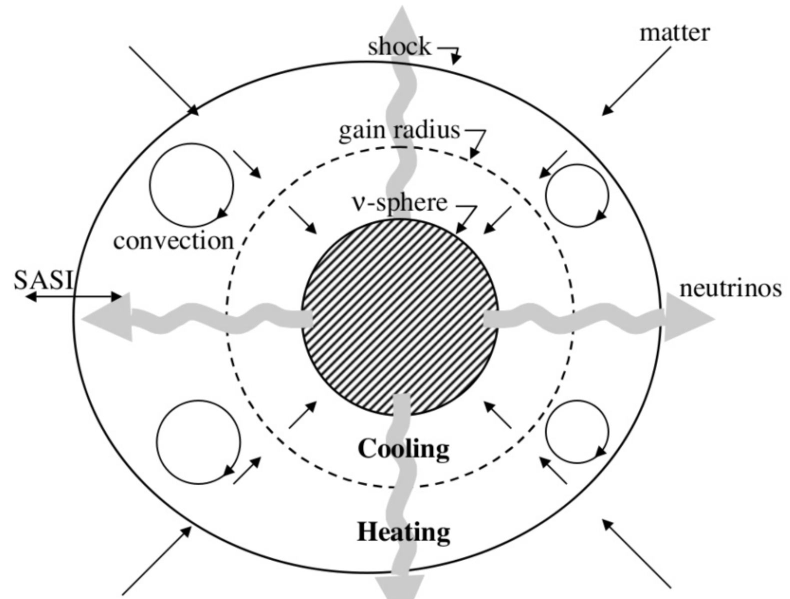
Two of the key open questions in astrophysics :

- How do very massive stars explode ?
- What is the site where heavy elements are made ?

The explosion

Current simulations :
multidimensional, realistic neutrino transport, convection and turbulence, hydrodynamical instabilities (SASI).

Neutrinos play a role in revitalizing the shock.

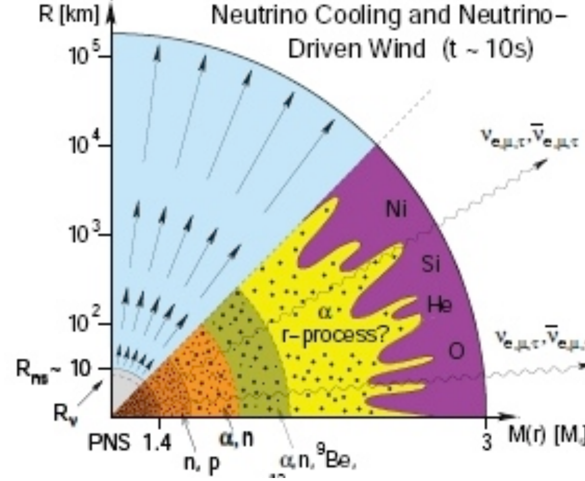


Bruenn et al, arXiv:1002.4909

first 3D : hints for explosion in 3D not enhanced compared to 2D

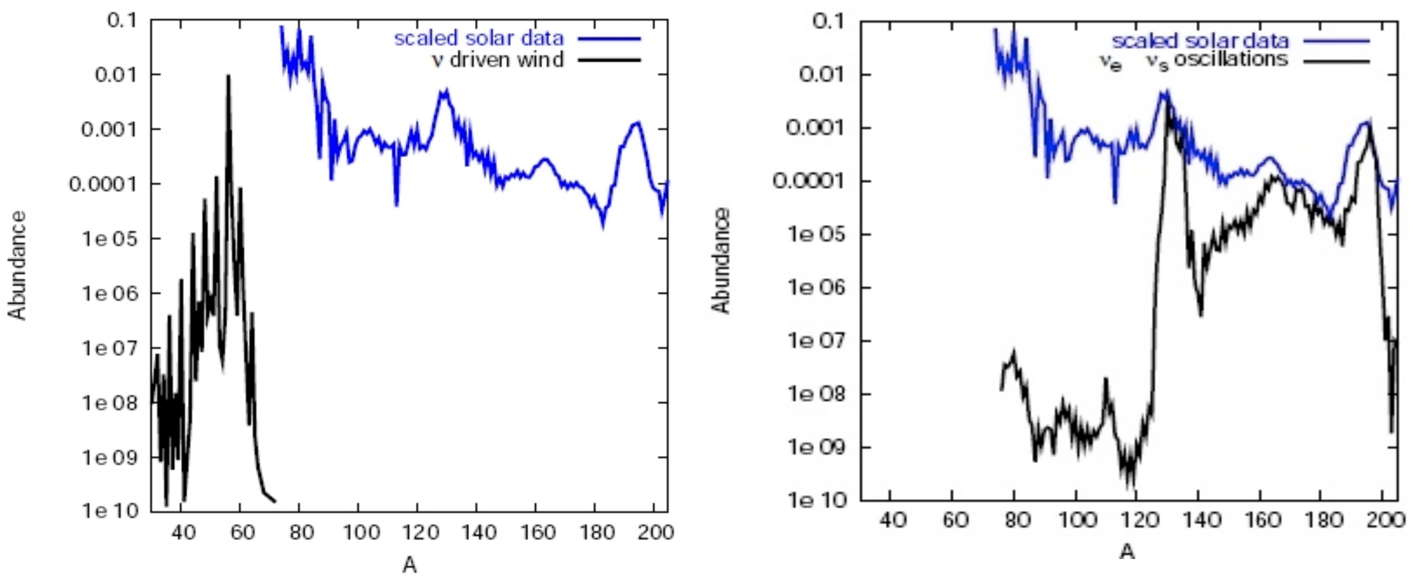
Hanke et al, arXiv:1303.6269

Nucleosynthesis



Supernovae, AD-BH, neutron star mergers :
sites for heavy elements nucleosynthesis.

Neutrinos determine if the site is neutron-rich
(r-process elements) or proton-rich (νp-process).



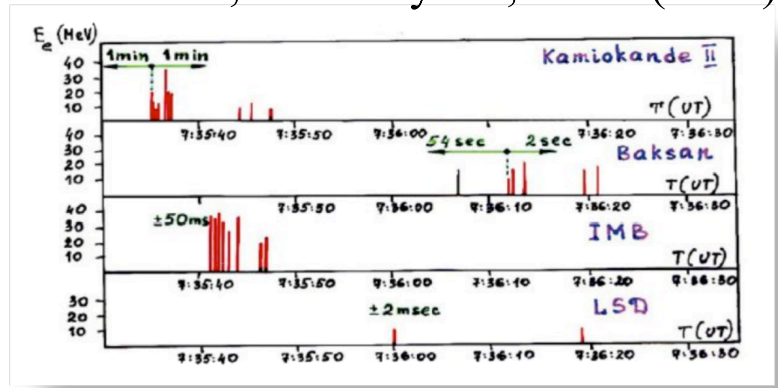
Beun, McLaughlin, Surman, Hix, PRD73 (2006).

ν flavor conversion impacts the nucleosynthetic outcomes.

Focus Issue on *«Nucleosynthesis and Neutrinos»,* Journal of Physics G (2014).

More observations

Suzuki, J. of Physics, Conf. (2008)



SN1987A events

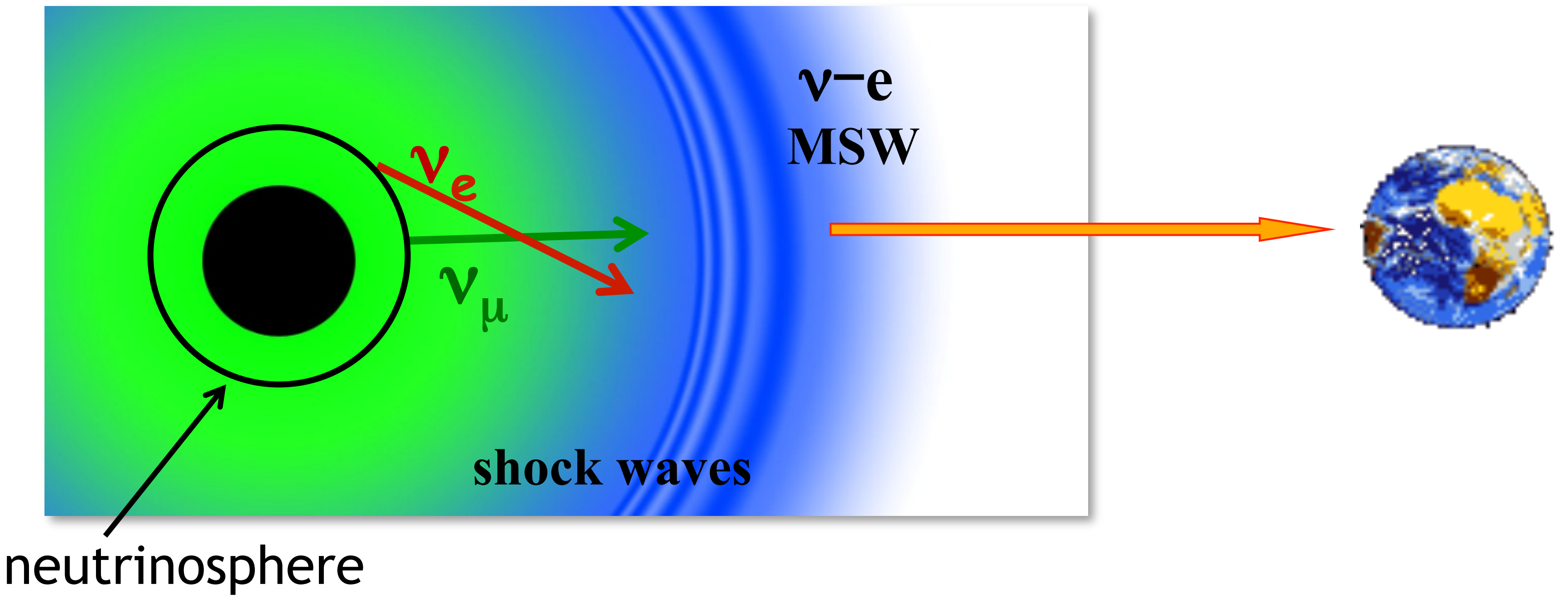
- **Time and energy signal** from a supernova explosion. In our galaxy, 1-3 events/century; one explosion/3years at 3 Mpc.

- **The Diffuse Supernova Neutrino Background :** The SN fluxes integrated over cosmological redshift.

	Events (10 y)	window	detector	
$\bar{\nu}_e$	90 (IH/NH)	9-25 MeV	50 kton	scintillator (LENA, JUNO)
$\bar{\nu}_e$	300	19-30 MeV	440 kton	water Cherenkov
ν_e	30	17-41 MeV	50 kton	liquid argon

Galais, Kneller, Volpe, Gava, PRD 81(2010) , arXiv :0906.5294.

ν flavour conversion in supernovae



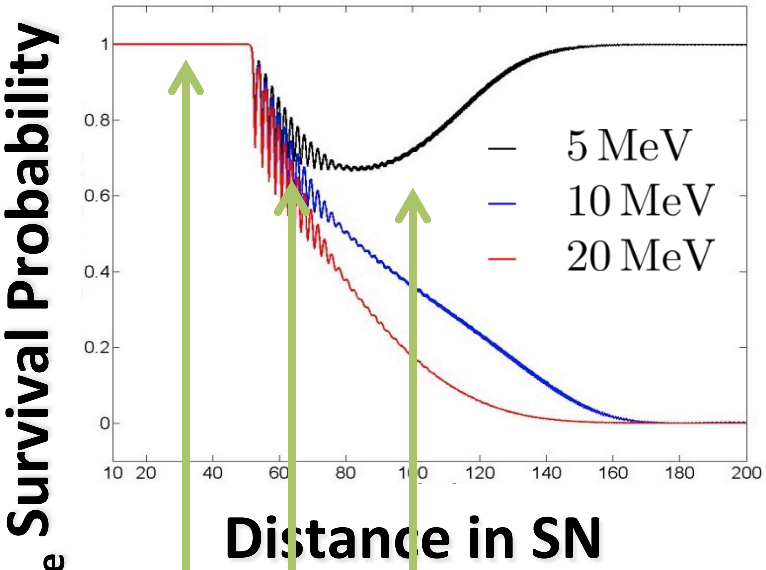
Numerous aspects are being investigated. The situation differs from the case of the sun. Conversion phenomena appear because of

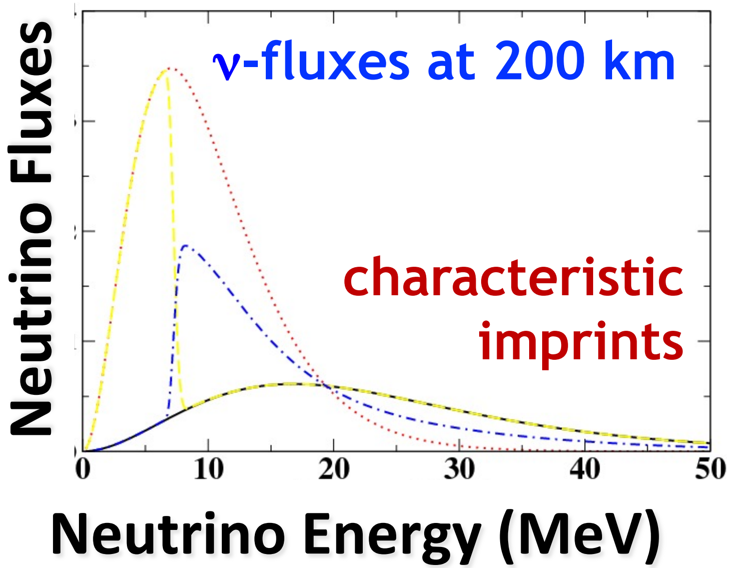
- ❑ the ν interaction with ν and with matter (MSW effect).
- ❑ dynamical aspects - shock waves and turbulence.

Novel phenomena intensively studied in the last years.

The effects of the $\nu\nu$ interaction

Pantaleone, PLB 287 (1992), Samuel,PRD 48 (1993), Sigl and Raffelt, NPB 406 (1993),





- synchronization mode : no flavor conversion occurs
- bipolar regime : occurrence of an instability in flavour space

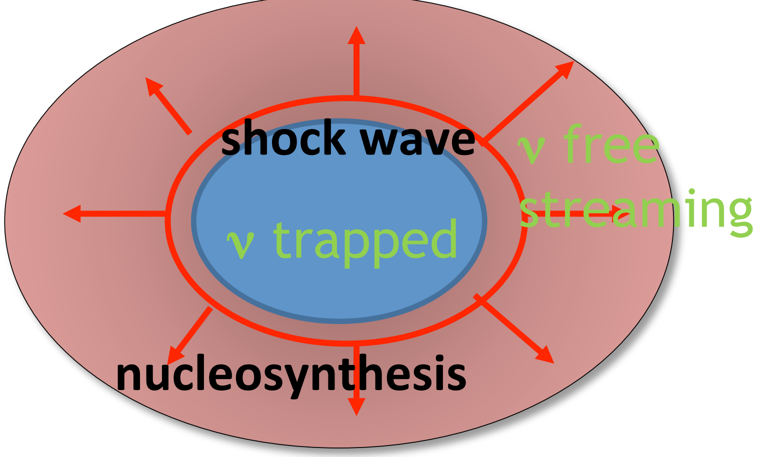
Duan,Fuller,Qian PRD74 (2006) 76 (2007), Hannestad, et al. PRD 74 (2006), Galais, Kneller, Volpe JPG 39 (2012)

- spectral split : full or no conversion depending on energy

Duan, Fuller, Qian, PRD76(2007); Raffelt, Smirnov PRD 76, PRL (2007) – MSW-like in the comoving frame; Galais and Volpe, PRD 84 (2011) – magnetic resonance-like phenomenon

Collective flavor conversion modes appear

Key open questions



Further work needed to finally assess :

- ❑ impact on the shock wave

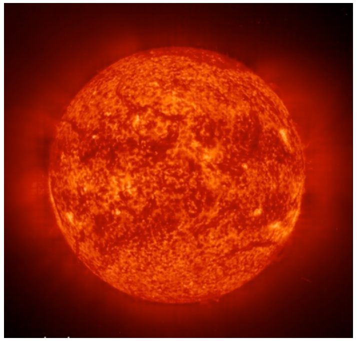
  Qian et al, PRL 71 (1993)
  Dasgupta, o'Connor, Ott, PRD 85 (2012)

- ❑ impact on (heavy elements) nucleosynthesis,
  see e.g. Duan, Friedland, McLaughlin, Surman, JPG 38 (2011)

- ❑ merging with realistic supernova simulations (eg. breaking the spherical or azymuthal symmetries)

- ❑ are current approximations sufficient ?

The mean-field approximation ?

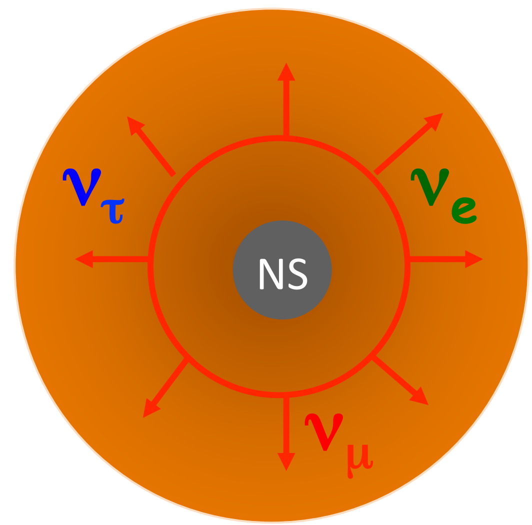


SOLAR NEUTRINOS

Yes

Supernovae and AD-BH

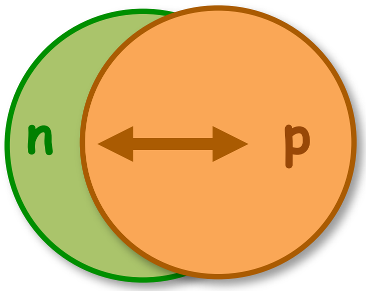
New flavour conversion phenomena ($\nu\nu$-interaction, shock-waves and turbulence) being uncovered



THE EARLY UNIVERSE - The Boltzmann approximation for particles with mixings is used.
The role of other two-body correlations?

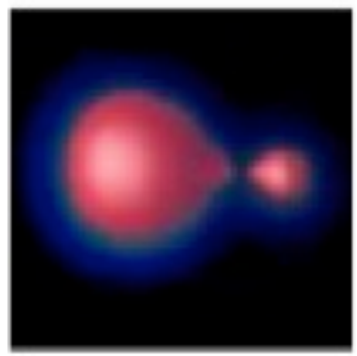
Looking across fields...

Establishing the connection between ν flavour conversion in media and other many-body systems such as



giant resonances

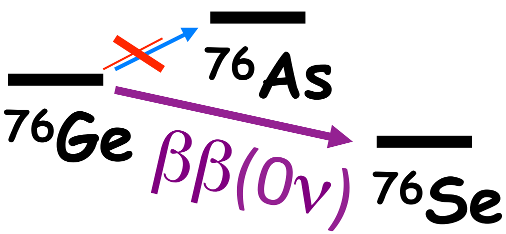
phonons



nuclear collision

metallic clusters

condensed matter

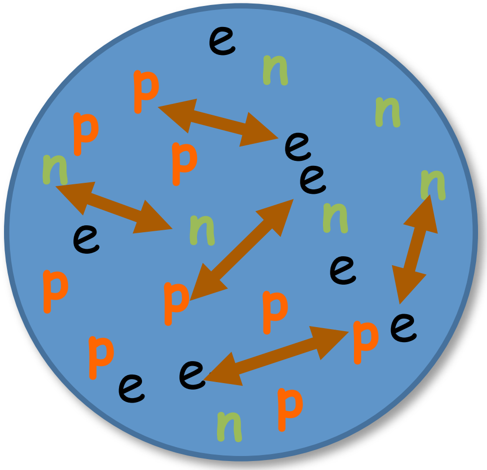


double-beta decay

The exact evolution

Let us consider a system of **N-particles** described by a Hamiltonian :

$$H = \sum_k H_0(k) + \sum_{k<k'} V(k,k')$$



The system evolution is determined by solving the Liouville Von-Neumann equation for D :

$$i\frac{dD}{dt} = [H, D] \qquad D = |\psi(t)\rangle\langle\psi(t)|$$

N-body density matrix

Born-Bogoliubov-Green-Kirkwood-Yvon -BBGKY- hierarchy

Kirkwood, J. Chem. Phys. 3 (1935), Yvon, Actual. Sci. Ind. 203 (1935),
Born and Green Proc. R. Soc. A 188 (1946), Bogoliubov, J. Phys. 10 (1946)

The s-reduced density matrix : $\rho_{1\ldots s} = \langle \psi(t) | a_s^\dagger \ldots a_1^\dagger a_1 \ldots a_s | \psi(t) \rangle$

one-body density

$$\rho_1 = \langle \psi(t) | a_1^\dagger a_1 | \psi(t) \rangle$$

two-body density

$$\rho_{12} = \langle \psi(t) | a_2^\dagger a_1^\dagger a_1 a_2 | \psi(t) \rangle$$

Solving exactly the many-body problem is equivalent to

$$i\dot{\rho}_1 = [t_1, \rho_1] + \mathrm{Tr}_{(2)} \{[v_{12}, \rho_{12}]\}$$

$$i\dot{\rho}_{12} = [t_1 + t_2 + v_{12}, \rho_{12}] + \mathrm{Tr}_{(3)} \{[v_{13} + v_{23}, \rho_{123}]\}$$

$$\vdots$$

$$i\dot{\rho}_{1\cdots n} = \left[\sum_{i=1}^{n} t_i + \sum_{j>i=1}^{n} v_{ij}, \rho_{1\cdots n}\right] + \sum_{i=1}^{n} \mathrm{Tr}_{(n+1)} \left\{\left[v_{i(n+1)}, \rho_{1\cdots(n+1)}\right]\right\}$$

BBGKY : a hierarchy of equations for s-reduced density matrices

A novel perspective to ν conversion

Volpe, Väänänen, Espinoza, PRD87 (2013), arXiv: 1302.2374

BBGKY for neutrinos :

- a system of particles and anti-particles
- particles with mixings

occupation number op.

ν

$$\rho_\nu = \begin{pmatrix} \langle a^\dagger_{\nu_\alpha,i} a_{\nu_\alpha,i} \rangle & \langle a^\dagger_{\nu_\beta,j} a_{\nu_\alpha,i} \rangle \\ \langle a^\dagger_{\nu_\alpha,i} a_{\nu_\beta,j} \rangle & \langle a^\dagger_{\nu_\beta,j} a_{\nu_\beta,j} \rangle \end{pmatrix}$$

anti-ν

$$\bar{\rho}_\nu = \begin{pmatrix} \langle b^\dagger_{\nu_\alpha,i} b_{\nu_\alpha,i} \rangle & \langle b^\dagger_{\nu_\beta,j} b_{\nu_\alpha,i} \rangle \\ \langle b^\dagger_{\nu_\alpha,i} b_{\nu_\beta,j} \rangle & \langle b^\dagger_{\nu_\beta,j} b_{\nu_\beta,j} \rangle \end{pmatrix}$$

decoherence or mixing terms

The BBGKY is a rigorous theoretical framework :

- ✓ to go from the N-body to the 1-body description
- ✓ that is very general, equivalent the Green's function formalism (equal-time limit)

UNIFIED APPROACH for ASTROPHYSICAL and COSMOLOGICAL APPLICATIONS

that allows to go beyond current approximations

The first BBGKY equation

$$\rho_1 = \langle\psi(t)|a_1^\dagger a_1|\psi(t)\rangle \qquad \rho_{12} = \langle\psi(t)|a_2^\dagger a_1^\dagger a_1 a_2|\psi(t)\rangle$$

one-body density **two-body density**

The two-body density matrix can be written as :

$$\rho_{12} = \rho_1\rho_2 + \cancel{C_{12}}$$

two-body correlation function

The first BBGKY equations gives for the mean-field evolution equations

$$i\dot{\rho}_1 = [t_1, \rho_1] + \mathrm{Tr}_{(2)}\{[v_{12}, \rho_{12}]\}$$

$$\cancel{i\dot{\rho}_{12} = [t_1 + t_2 + v_{12}, \rho_{12}] + \mathrm{Tr}_{(3)}\{[v_{13} + v_{23}, \rho_{123}]\}}$$

the mean-field approximation

The first BBGKY equation

$$\rho_1 = \langle \psi(t) | a_1^\dagger a_1 | \psi(t) \rangle \qquad \rho_{12} = \langle \psi(t) | a_2^\dagger a_1^\dagger a_1 a_2 | \psi(t) \rangle$$

one-body density   two-body density

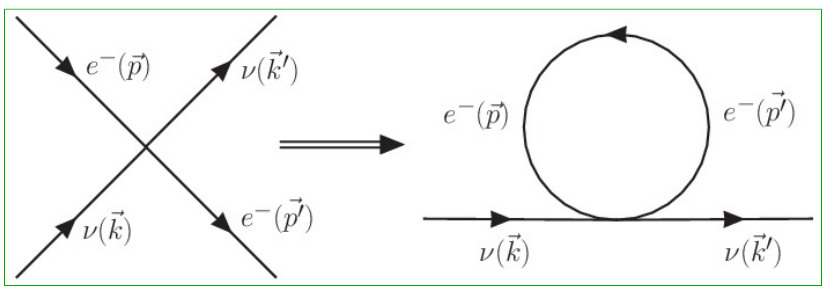
The two-body density matrix can be written as :

$$\rho_{12} = \rho_1 \rho_2 + \cancel{c_{12}}$$

two-body correlation function

The first BBGKY equations gives for the mean-field evolution equations

$$i\dot{\rho}_1 = [h_1(\rho), \rho_1]$$ with $$h_1(\rho) = H_0(1) + \Gamma_1(\rho)$$



$$\Gamma_{1,ij}(\rho) = \sum_{mn} v_{(im,jn)} \rho_{2,nm}$$

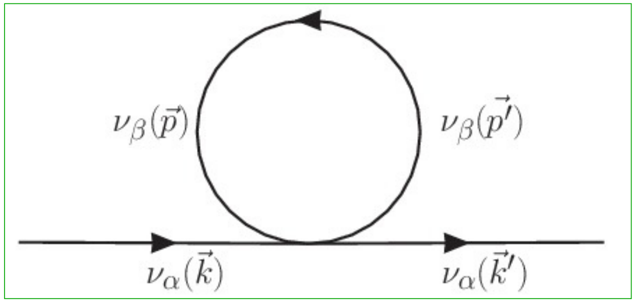
MEAN-FIELD

the mean-field approximation

BBGKY for neutrinos : mean-field equations

Deriving the ν evolution equations in presence of

✓ a ν and $\bar{\nu}$ backgrounds --- early Universe, supernovae, AD-BH



- $$i\dot{\rho} = [h(\rho), \rho]$$
  $$\Gamma_{1,ij}(\rho) = \sum_{mn} v_{(im,jn)}\rho_{2,nm}$$
- $$H_{NC} = \frac{G_F}{2\sqrt{2}} \int d^3\vec{x} [\bar{\phi}_{\nu_e}\gamma_\mu(1-\gamma_5)\phi_{\nu_e}], [\bar{\phi}_{\nu_y}\gamma^\mu(1-\gamma_5)\phi_{\nu_y}]$$ interaction H
- $$\phi(\vec{x}) = \sum_h \int \frac{d^3\vec{p}}{(2\pi)^3 2E_p} [a(\vec{p},h)u_{\vec{p},h}e^{i\vec{p}\cdot\vec{x}} + b^\dagger(\vec{p},h)v_{\vec{p},h}e^{-i\vec{p}\cdot\vec{x}}]$$ field
- $$|m\rangle = a_m^\dagger|\rangle$$ single particle states
- $$\Gamma_{\nu_\alpha,\nu_\beta}(\rho_\nu) = \frac{G_F}{2\sqrt{2}} \int \frac{d^3\vec{p}}{(2\pi)^3 2E_p} \int \frac{d^3\vec{p}'}{(2\pi)^3 2E_{p'}} (2\pi)^3\delta^3(\vec{p}+\vec{k}-\vec{p}'-\vec{k}')$$

  x $[\bar{u}_{\nu_\beta}(\vec{k},h_\beta)\gamma_\mu(1-\gamma_5)u_{\nu_\alpha}(\vec{k}',h'_\alpha)]\ [\bar{u}_{\nu_\alpha}(\vec{p},h_\alpha)\gamma^\mu(1-\gamma_5)u_{\nu_\beta}(\vec{p}',h'_\beta)]$

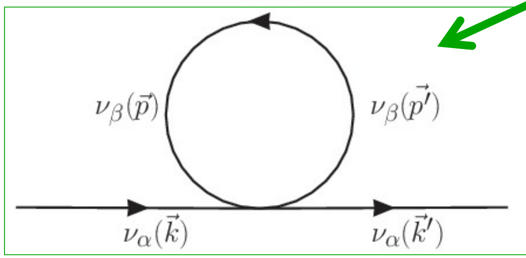
  x $\langle a^\dagger_{\nu_\alpha}(\vec{p},h_\alpha)a_{\nu_\beta}(\vec{p}',h'_\beta)\rangle$ mean-field

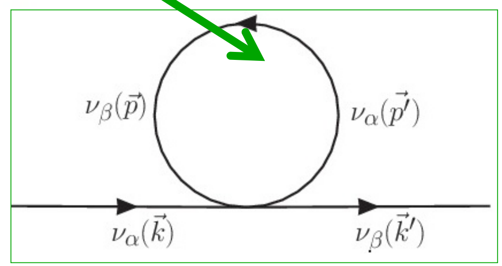
BBGKY for neutrinos : mean-field equations

- $\rho^{\nu_\beta,\nu_\alpha}_{\vec{p}'h',\vec{p}h} \equiv \langle a^\dagger_{\nu_\alpha}(\vec{p},h_\alpha) a_{\nu_\beta}(\vec{p}',h'_\beta)\rangle$ (normal) density matrix
- $\rho^{\nu_\beta,\nu_\alpha}_{\vec{p}'h',\vec{p}h} = (2\pi)^3 2E_p \delta_{hh'}\delta^3(\vec{p}-\vec{p}')\rho^{\nu_\beta,\nu_\alpha}_{\vec{p}}$ homogeneous background
- $\Gamma(\rho_\nu,\bar{\rho}_\nu) = \sqrt{2}G_F \sum_{\underline{\nu_\alpha}} \int \frac{d^3\vec{p}}{(2\pi)^3 2E_p} (\rho_{\underline{\nu_\alpha},p} - \bar{\rho}^*_{\underline{\nu_\alpha},p}) \left(1 - \hat{\vec{p}}\cdot\hat{\vec{k}}\right)$

ν mean-field

diagonal contribution



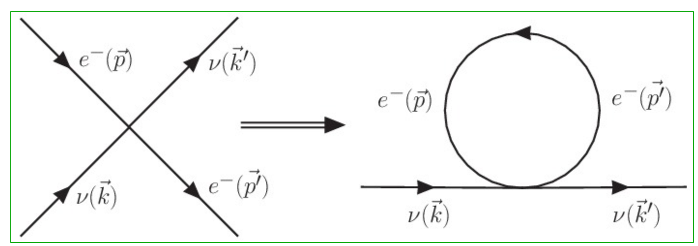


off-diagonal contribution

✓ a matter background - MSW effect

- $\Gamma_{\nu_e}(\rho_e) = \sqrt{2}G_F\ \rho_e$

electron mean-field



MEAN-FIELD EQUATIONS - MSW and νν – reDERIVED

consistent with previous derivations

Beyond the mean-field approximation

$$\rho_{12} = \rho_1 \rho_2 + c_{12}$$ **two-body correlations**

The evolution equation for the two-body correlation function :

$$
\begin{aligned}
i\dot{c}_{12} = {} & [h_1 + h_2, c_{12}] \\
& + (1-\rho_1)(1-\rho_2) v_{12} \rho_1 \rho_2 - v_{12} \rho_1 \rho_2 (1-\rho_1)(1-\rho_2) \\
& \cancel{+ (1-\rho_1-\rho_2) v_{12} c_{12} - c_{12} v_{12} (1-\rho_1-\rho_2)} \\
& \cancel{+ \mathrm{Tr}_{(3)} \{[v_{13}, (1-P_{13}) \rho_1 c_{23} (1-P_{12})]\}} \\
& \cancel{+ \mathrm{Tr}_{(3)} \{[v_{23}, (1-P_{23}) \rho_2 c_{13} (1-P_{12})]\}} \\
& \cancel{+ \mathrm{Tr}_{(3)} \{[v_{13} + v_{23}, c_{123}]\}}
\end{aligned}
$$

Including the collision term one obtains the Boltzmann equation for ν.

different approach in Sigl and Raffelt, Nucl. Phys. B406 (1993)

Beyond the mean-field approximation

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& + (1-\rho_1-\rho_2) v_{12} c_{12} - c_{12} v_{12} (1-\rho_1-\rho_2) \\
& \cancel{+ \mathrm{Tr}_{(3)} \{ [v_{13}, (1-P_{13}) \rho_1 c_{23} (1-P_{12})] \}} \\
& \cancel{+ \mathrm{Tr}_{(3)} \{ [v_{23}, (1-P_{23}) \rho_2 c_{13} (1-P_{12})] \}} \\
& \cancel{+ \mathrm{Tr}_{(3)} \{ [v_{13} + v_{23}, c_{123}] \}}
\end{aligned}
$$

Beyond the mean-field approximation

The inclusion of two-body correlations :

$$c_{(ik,jl)} \approx \kappa^{*}_{jl}\kappa_{ik} \qquad \kappa^{*}_{jl} \equiv \langle a^{\dagger}_{j} b^{\dagger}_{l} \rangle \qquad \kappa_{ik} \equiv \langle b_{k} a_{j} \rangle$$

ν-$\bar{\nu}$ pairing correlations

abnormal density

The evolution equation for the two-body correlation function :

$$
\begin{aligned}
i\dot{c}_{12} = {} & [h_1 + h_2, c_{12}] \\
& \cancel{+ (1 - \rho_1)(1 - \rho_2) v_{12} \rho_1 \rho_2 - v_{12} \rho_1 \rho_2 (1 - \rho_1)(1 - \rho_2)} \\
& + (1 - \rho_1 - \rho_2) v_{12} c_{12} - c_{12} v_{12} (1 - \rho_1 - \rho_2) \\
& \cancel{+ \mathrm{Tr}_{(3)} \{[v_{13}, (1 - P_{13}) \rho_1 c_{23} (1 - P_{12})]\}} \\
& \cancel{+ \mathrm{Tr}_{(3)} \{[v_{23}, (1 - P_{23}) \rho_2 c_{13} (1 - P_{12})]\}} \\
& \cancel{+ \mathrm{Tr}_{(3)} \{[v_{13} + v_{23}, c_{123}]\}}
\end{aligned}
$$

$\nu\bar{\nu}$ pairing correlations : new contributions at mean-field level

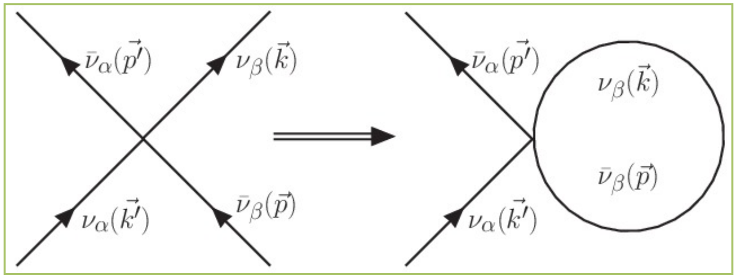
Extended ν equations in media

As the normal densities, the abnormal ones have off-diagonal terms :

$$\kappa = \begin{pmatrix} \langle b_{\nu_e} a_{\bar{\nu}_e} \rangle & \langle b_{\nu_x} a_{\bar{\nu}_e} \rangle \\ \langle b_{\nu_e} a_{\bar{\nu}_x} \rangle & \langle b^\dagger_{\nu_x} a_{\bar{\nu}_x} \rangle \end{pmatrix} \qquad \kappa^* = \begin{pmatrix} \langle a^\dagger_{\bar{\nu}_e} b^\dagger_{\nu_e} \rangle & \langle a^\dagger_{\bar{\nu}_e} b^\dagger_{\nu_x} \rangle \\ \langle a^\dagger_{\bar{\nu}_x} b^\dagger_{\nu_e} \rangle & \langle a^\dagger_{\bar{\nu}_x} b^\dagger_{\nu_x} \rangle \end{pmatrix}$$

PAIRING MEAN-FIELD

$$\Delta_{ij} \equiv \sum_{kl} v_{(ij,kl)} \kappa_{kl}$$



By using the same procedure as before and assuming an homogeneous background, i.e. $\kappa^{\nu_\alpha \bar{\nu}_\beta *}_{\vec{k},\vec{p}} = (2\pi)^3 2 E_k \delta^3(\vec{p} + \vec{k}) \kappa^{\nu_\alpha \bar{\nu}_\beta *}_{\vec{p}}$, the abnormal field

$$\Delta_{\mathbf{p}} = -\sqrt{2} G_F \int \frac{d^3\mathbf{q}}{(2\pi)^3} D(\mathbf{p}, \mathbf{q}) \, \kappa_{\mathbf{q}}$$

$$\Delta^*_{\nu_\alpha, \bar{\nu}_\beta}(\kappa^*_\nu) = -N(\vec{k}', \vec{p}') \sqrt{2} G_F \int \frac{d\cos\theta dp}{(2\pi)^2} \sin\theta \sin\theta' p^2 \kappa^{\nu_\alpha \bar{\nu}_\beta *}_{\vec{p}}$$

if cylindrical symmetry

Extended equations in media

The extended mean-field ν equations in matrix form

$$\mathcal{R} = \begin{pmatrix} \rho & \kappa \\ \kappa^\dagger & 1-\bar{\rho}^* \end{pmatrix}$$

$$i\dot{\mathcal{R}} = [\mathcal{H}, \mathcal{R}]$$

$$\mathcal{H} = \begin{pmatrix} h & \Delta \\ \Delta^\dagger & -\bar{h}^* \end{pmatrix}$$

generalised density

generalised Hamiltonian

Volpe, Väänänen, Espinoza, PRD87 (2013), arXiv: 1302.2374

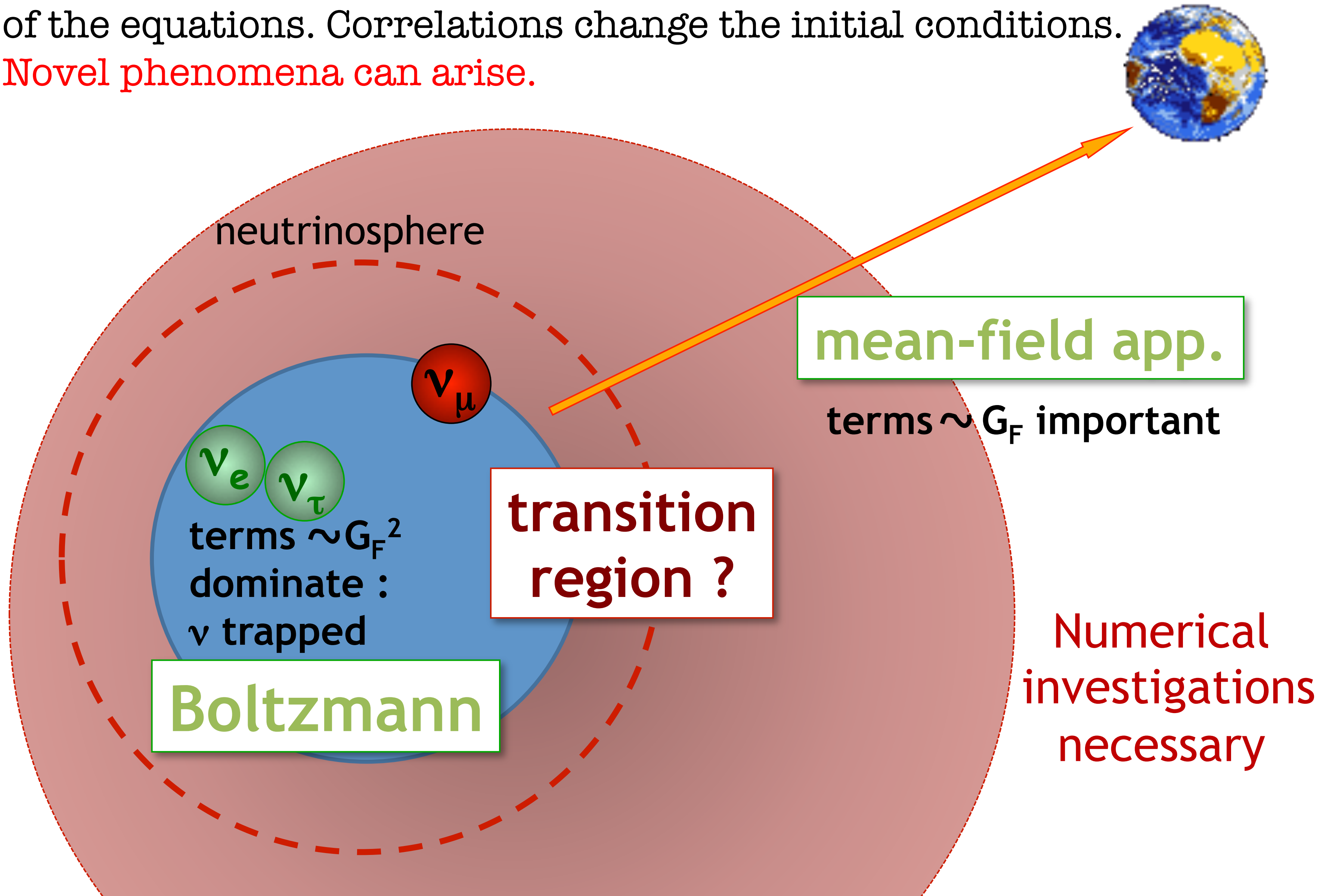
I am deriving, with J. Serreau, the most general mean-field equations using a different method. The results confirm these equations with extra terms.

Serreau and Volpe, to be submitted soon

the role of neutrino-antineutrino correlations?
more generally, of two-body correlations (collisions)?

SN : the transition region ?

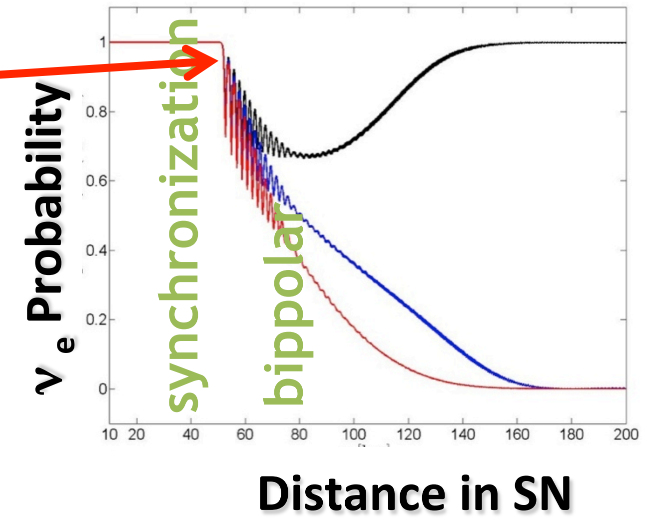
Small contributions can be amplified by the non-linearity of the equations. Correlations change the initial conditions.
Novel phenomena can arise.

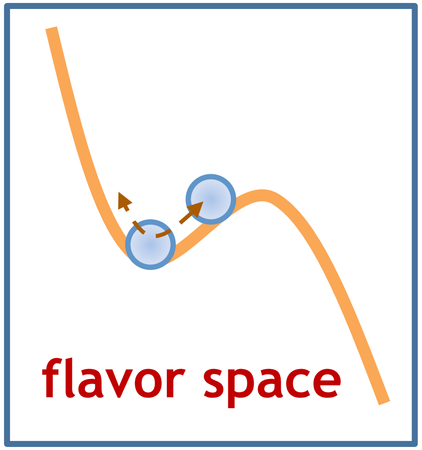


Flavor collective modes and instabilities

An analitycal condition for the occurrence of the bipolar instability in the two-flavor case. We have introduced generalized adiabaticity parameters in presence of $\nu\nu$ interaction for N flavors also given.

Galais, Kneller, Volpe, PRD87 (2013), arXiv: 1302.2374





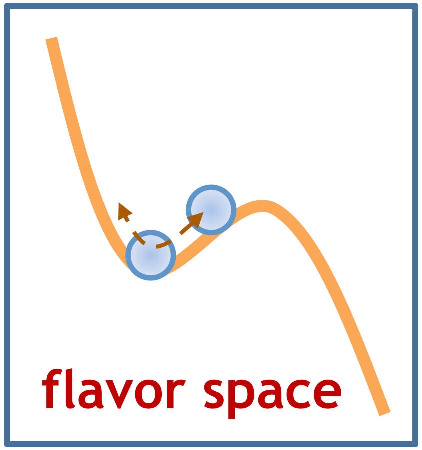
Small amplitude motion

The presence of collective modes and of instabilities can be studied in a small amplitude approximation (linearization).

Advantages : more economical numerically

Disadvantages : location and type of instability

General Linearization approach



Väänänen, Volpe, PRD88 (2013), arXiv: 1306.6372

A linearisation methods from many-body microscopic approaches to obtain eigenvalue equations and a stability matrix, valid in the small amplitude approximation

- $i\dot{\rho} = [h(\rho), \rho] \quad [h^0, \rho^0] = 0$
- $\delta\rho = \rho_0 + \delta\rho(t) = \rho^0 + \rho' e^{-i\omega t} + \rho'^{\dagger} e^{i\omega^* t}$. $\rho'_{im} = \langle a^{\dagger}_m a_i \rangle$
- $h(\rho) = h^0 + \frac{\delta h}{\delta\rho}\Big|_{\rho^0} \delta\rho + \ldots$
- $\omega\rho' e^{-i\omega t} - \omega^* \rho'^* e^{i\omega^* t} = [h^0, \rho^0 + \delta\rho] + \left[\frac{\delta h}{\delta\rho}\delta\rho, \rho^0\right]$.
- $\omega\rho'_{ij} = \sum_{k<l} \{(\tilde{k}_k - \tilde{k}_l)\delta_{ki}\delta_{jl}\rho'_{kl} + (\rho^0_j - \rho^0_i)\frac{\delta h_{ij}}{\delta\rho'_{kl}}\Big|_{\rho^0}\rho'_{kl}\Big]$

Stability Matrix

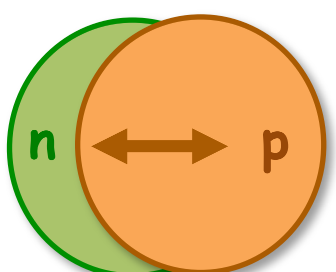
- We have applied this procedure to a system of ν and anti-ν.
- The linearized eigenvalue equations can be cast in a matrix form, our results are very general (and go beyond assumptions made). A and B matrices depend on the specific system and geometry.

Stability matrix

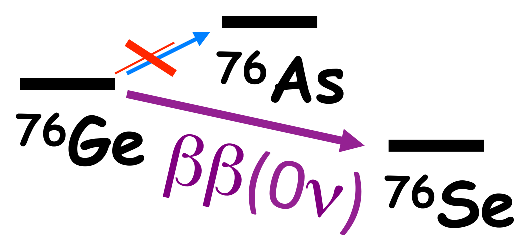
$$\boxed{\begin{pmatrix} A & B \\ \bar{B} & \bar{A} \end{pmatrix}} \begin{pmatrix} \rho' \\ \bar{\rho}' \end{pmatrix} = \omega \begin{pmatrix} \rho' \\ \bar{\rho}' \end{pmatrix}$$

S eigenvalues :
-> real : stable collective
-> imaginary : instabilities

It establishes the connection to collective modes in other many-body systems (atomic nuclei, metallic clusters, ...)



nuclear resonances



Quasi-particles

- Linearized equations and a stability matrix obtained also for the extended Hamiltonian, including ν-antiν correlations.

$$i\dot{\mathcal{R}} = [\mathcal{H}, \mathcal{R}] \qquad [\mathcal{H}^0, \mathcal{R}^0] = 0 \qquad \delta\mathcal{R} = \mathcal{R}^0 + \delta\mathcal{R}'$$

$$\mathcal{H}(\mathcal{R}) = \mathcal{H}^0 + \left.\frac{\delta\mathcal{H}}{\delta\mathcal{R}}\right|_{\mathcal{R}^0} \delta\mathcal{R} + \dots$$

- By introducing a generalized Bogoliubov-Valatin transformation,

$$\begin{cases} \alpha_k^\dagger &= \sum_l Z_{lk} a_l^\dagger + W_{lk} b_{\bar{l}} \\ \alpha_{\bar{k}} &= \sum_l V_{lk}^* a_l^\dagger + U_{lk}^* b_{\bar{l}} \end{cases} \qquad \begin{cases} \alpha_{k'} &= \sum_{l'} Z_{l'k'}^* a_{l'} + W_{l'k'}^* b_{\bar{l}'}^\dagger \\ \alpha_{\bar{k}'}^\dagger &= \sum_{l'} V_{l'k'} a_{l'} + U_{l'k'} b_{\bar{l}'}^\dagger \end{cases}$$

this Hamiltonian can be put in a diagonal form and describe a system of independent quasi-particles.

Recent theoretical works related to the issues discussed today :

- beyond the mean-field :

  Balantekin and Pehlivan, JPG 34 (2007)

  Cherry et al., PRL108 (2012) (back-scattered neutrinos)

  Vlasenko, Fuller, Cirigliano, 1309.2628 (collisions)

- connection between $\nu\nu$ interaction and the BCS reduced Hamiltonian :

  Pehlivan, Balantekin, Suzuki, Kajino, PRD 84 (2011)

- linearization in neutrino evolution :

  Sawyer PRD79 (2009),

  Banerjee, Dighe, Raffelt, PRD 84 (2011)

Unknown ν-properties

Interesting information from astrophysical or cosmological ν on

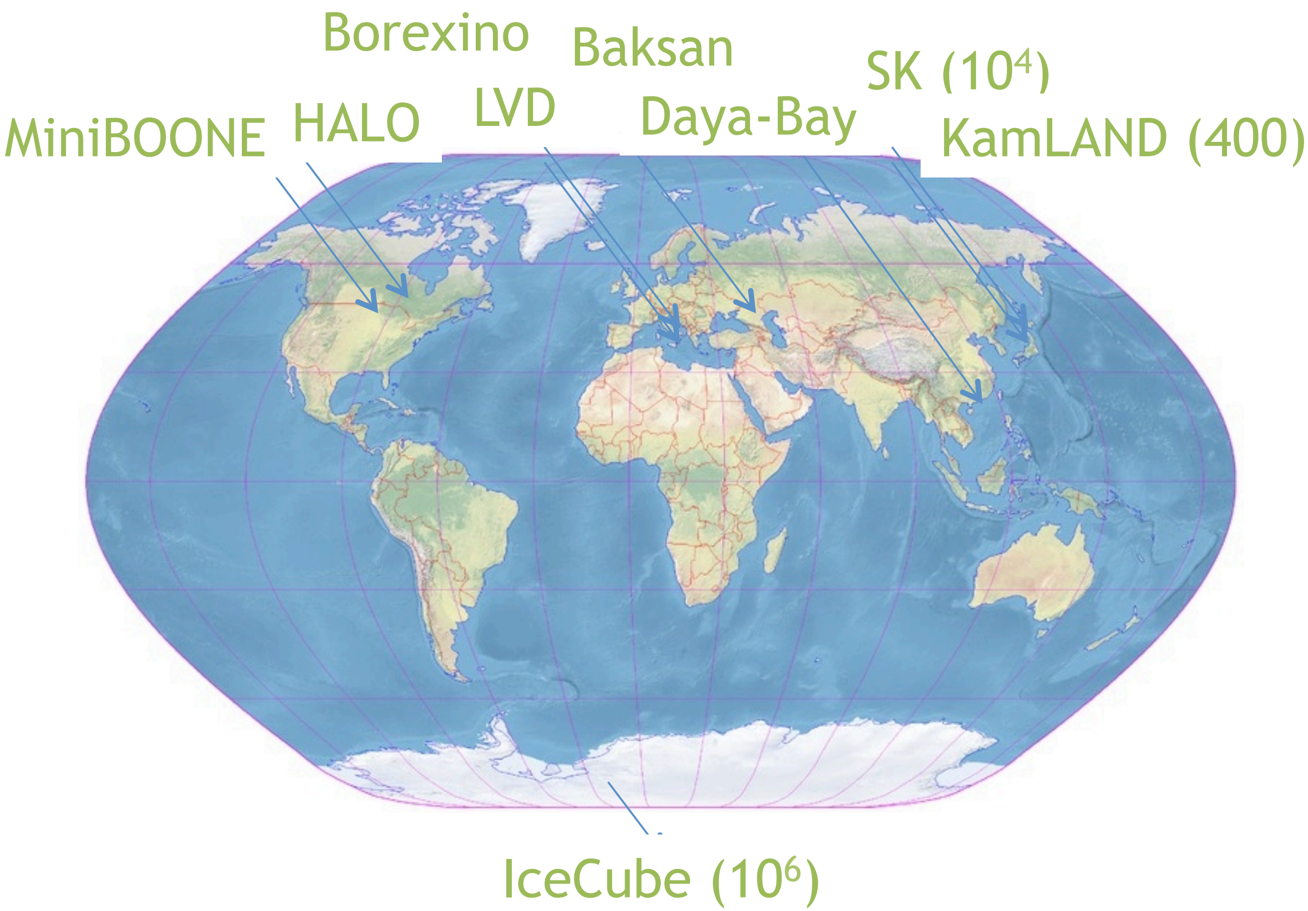
- the mass ordering
- the neutrino absolute mass
- sterile neutrinos
- CP violation – effects exist in supernovae

  Balantekin, Gava, Volpe, PLB662, (2008), arXiv:0710.3112
  Gava, Volpe, Phys. Rev. D78 (2008), arXiv:0807.3418

  - at BBN epoch (primordial abundances)

  Gava, Volpe, Phys. Lett. B 837 (2010) 50, arXiv: 1002.0981

Current SN observatories



Different detection channels available :
scattering of anti-ν_e with p, ν_e with nuclei, ν_x with e, p

Large scale detectors coming --LENA, JUNO, GLACIER, Hyper-K,…

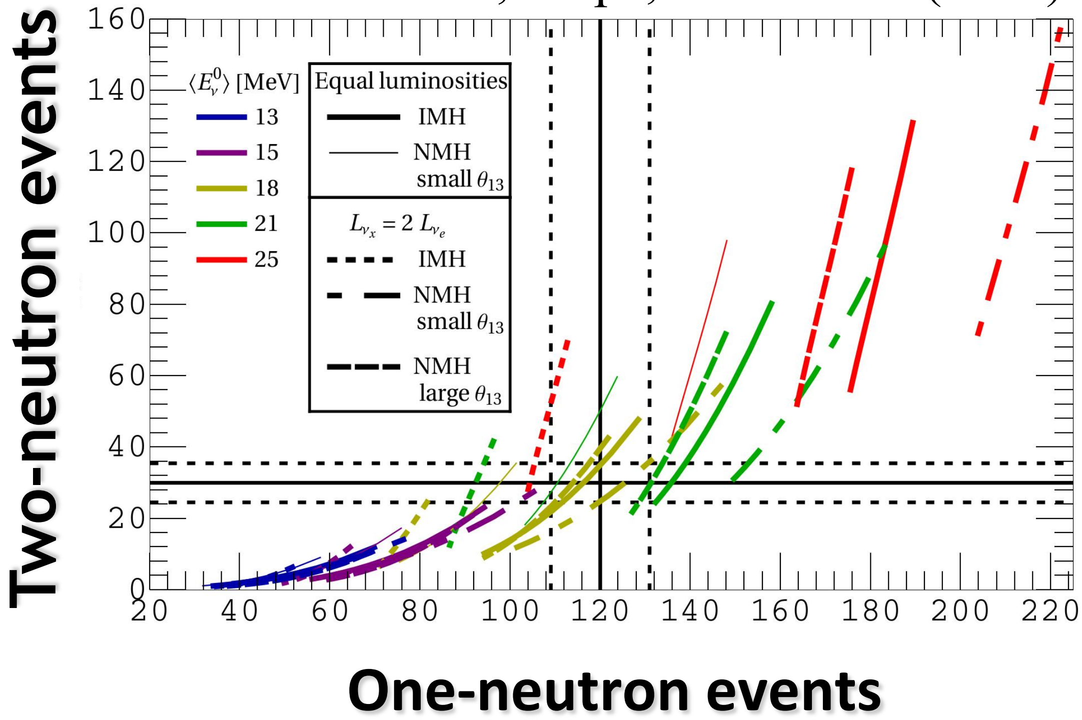
Reducing the degeneracies

CC+ NC events predicted in a lead-based detector for a SN at 10

HALO-2, 1 kton lead

Predictions include
$\nu\nu$ and ν-matter,
flux uncertainties
at neutrinosphere

Vaeaenaenen, Volpe, JCAP 1110 (2011)



Combining detection channels with
different energy thresholds

Conclusions and perspectives

- Supernova neutrinos and ν in media are connected with key open issues.

- Steady progress in our understanding of ν propagation in dense media. Important questions needs to be investigated to put it on a solid ground.

- Need to study the role of two-body correlations in the transition region.

- BBGKY for neutrinos, a novel approach to :
  - to derive from first principles of mean-field equations employed in the whole literature ;
  - to provide extended mean-field equations including two-body correlations from $\nu\bar{\nu}$ pairing

Conclusions and perspectives

- Used methods from many-body physics (RPA, QRPA) :
  - to give general linearized equations and a stability matrix for the ν motion in the small amplitude approximation.
  - to describe static case in terms of quasi-particles.

- Numerical investigations necessary.

- Also need to explore breaking of other assumption or symmetries (geometrical, homogeneity, ...).

Established the connection between a gas of sn ν and the dynamics, collective modes and instabilities of many-body systems in other domains.

Thank you