

Licia Verde

ICREA & ICC-UB-IEEC BARCELONA

Beyond precision cosmology, some examples

http://icc.ub.edu/~liciaverde





Institut de Ciències del Cosmos





First decade or so of 2000: Precision cosmology (WMAP, Boomerang, Acbar, ...SDSS, 2dF, Supernovae etc..) ΛCDM: The standard cosmological model Just 6 numbers..... describe the Universe composition and evolution

Homogenous background







 $\Omega_b, \Omega_c, \Omega_\Lambda, H_0, au$

atoms 4%
cold dark matter 23%
dark energy 73%

 $\Lambda? \quad \text{CDM?}$

 A_s, n_s

nearly scale-invariant
adiabatic
Gaussian

ORIGIN??

Context and overview

- Cosmology over the past 20 years has made the transition to *precision cosmology*
- Cosmology has now a standard model. The standard cosmological model only needs few parameters to describe origin composition and evolution of the Universe
- Big difference between modeling and understanding
- Implies Challenges and opportunities
- Comology is special: we can't make experiments, only observations. We have to use the entire Universe as a detector: the detector is given, we can't tinker with it.

Can now do precision tests of fundamental physics with cosmological data Wonderful agreement of new data with the ΛCDM model

* With some notable exceptions which are still up for discussion.

Is this the whole story?

- What is the 96% of the Universe?
- What set out the primordial perturbations?
- The model is "preposterous", how can it be correct?
- Challenges and opportunities





It goes deeper than that

"We can't live in a state of perpetual doubt, so we make up the best story possible and we live as if the story were true."

Daniel Kahneman about theories

GR, big bang, choice of metric, nucleosynthesis, etc etc...

Cosmology tends to rely heavily on models (both for "signal" and "noise")

Essentially, all models are wrong , but some are useful (Box and Draper 1987)

With ~1% precision, systematics become the name of the game

Systematics in the data Systematics in the model (analysis) Is it possible to be model independent? At what price?

In its infancy...

here's some exploratory examples

The trouble with Ho; Bernal, LV, Riess, JCAP, 2016

The length of the local standard ruler; Verde, Bernal, Heavens, Jimenez 2017, MNRAS

Early cosmology Constrained; Verde, Bellini et al. 2016(arxiv) 2017 JCAP

The trouble with H₀

JL Bernal, LV, A Riess, JACP 2016

- Direct measurement: 73.24 ± 1.74 km/s/Mpc (Riess et al 2016; verified with GAIA parallaxes)
- Planck (ΛCDM): 67.8± 0.9(66.9±0.6) km/s/Mpc
- Formally 3.4 σ , maybe we should pay attention
- Possibly worst with Planck low I polarization reanalysis



Three avenues

Without invoking systematics (beside those declared by the authors themselves)

- Allow early cosmology to deviate from ΛCDM (unaltered late-time cosmology)
- Give freedom to late cosmology (unaltered early time physics*)
- Model-independent

Data

- CMB: Planck TT, lowP, TEEE*, Lensing (even l>1000 or l<1000 for TT)#
- BAO: compilation 6dF, SDSS MGS, LOWZ, CMASS, WiggleZ
- SNe: JLA compilation
- Riess 2016 H_o measurement

Addison et al. 2016, response of Planck team (Planck 2016)* Planck high | polarization "ok for LCDM not ok beyond LCDM"

SNe : standard candles



Since L is poorly known this is "uncalibrated", usually calibrated using measurements of H0

DA: function of geomerty and integral of 1/H

BAO







Baryon acoustic oscillations (BAO) Standard ruler

- Physics: sound waves in early Universe propagate until radiation and matter decouple
- Imprints a scale standard ruler
- Key Observable.
- Useful for:
 - geometry of Universe (Dark Energy equation of state, or modifications to GR)
 - early Universe physics (well known) sets it



CMB and early universe physics in LCDM constrain the standard ruler length to 0.2%

Standard candles & Standard rulers

Type-Ia SNe measure relative distances, since there is large uncertainty on the absolute magnitude M of a fiducial SN

NASA/JPL-Caltech

BAOs measure absolute distances, but depend on the value of sound horizon rdrag

Visually



μ(Z)

 $= 25 + 5 \log_{10} D_L(z)$

 $D_V(z) = \{D_A(z)^2 \ c \ z \ /H(z)\}^{1/3}$

At glance: direct and inverse distance ladder



H0: Direct measurement: 73.24 ± 1.74 km/s/Mpc (Riess et al 2016)

rd, high z anchor

Low z anchor

Cuesta et al 2014, Bernal et al 2016

Modify Early Universe physics: primordial helium fraction

Changes recombination history, affects the diffusion damping and redshift of last scattering



metal-poor HII regions

Aver et al 2013, MS dwarfs Casagrande et al 2003 Izotov et al 2014

 2σ upper limit, initial Solar helium abundance Serenelli Basu 2010



Modify Early Universe physics: early time expansion history, N_{eff}



Modify Early Universe physics: change also the perturbations

• N_{eff}, c_s c_{vis}

If all species have the same cs cvis, not much change No evidence for relativistic species not being neutrinos

• What if you have three neutrinos + something else with different effective parameters (dark radiation)?

	$\Delta N_{ m eff}$	c_s^2	$c_{ m vis}^2$	H_0	$\left(\Omega_{ m M}/0.3 ight)^{0.5}\sigma_{8}$
lowP+ TTTEEE	< 0.36	$0.25\substack{+0.07 \\ -0.15}$	$0.12\substack{+0.58\\-0.11}$	$68.9^{+1.1}_{-0.9}$	$0.85\substack{+0.02 \\ -0.02}$
lowP+ TTTEEE+CMB lensing	< 0.34	$0.24\substack{+0.10 \\ -0.13}$	0.49 ± 0.33	$68.9^{+1.2}_{-0.9}$	$0.83\substack{+0.02 \\ -0.01}$
lowP+ TTTEEE+CMB lensing+BAO	< 0.28	$0.26\substack{+0.09\\-0.16}$	$0.28\substack{+0.45 \\ -0.26}$	$68.7^{+0.6}_{-0.7}$	$0.84\substack{+0.01 \\ -0.02}$
lowP + TT	< 0.76	$0.25\substack{+0.08 \\ -0.10}$	$0.84\substack{+0.20 \\ -0.51}$	$70.6^{+2.6}_{-2.0}$	$0.84\substack{+0.03 \\ -0.04}$
lowP+ TT+CMB lensing	< 0.77	$0.27\substack{+0.07 \\ -0.11}$	$0.81\substack{+0.19 \\ -0.50}$	$71.3^{+1.9}_{-2.2}$	$0.82\substack{+0.02 \\ -0.01}$
lowP+TT+CMB lensing+BAO	< 0.44	$0.29\substack{+0.20\\-0.16}$	$0.9\substack{+0.1 \\ -0.7}$	$69.0\substack{+0.9 \\ -0.8}$	$0.84\substack{+0.01 \\ -0.02}$

What's that?



Fig courtesy of J. Lesgourgues

Modify Early Universe physics: change also the perturbations

- N_{eff}, c_s c_{vis}
- What if you have three neutrinos + something else with different effective parameters (dark radiation)?



Spline reconstruction of the expansion history
 H(z) with 4 (5 with SNe) knots.

Direct and inverse cosmic distance ladder (Cuesta et al 2015)



Spline reconstruction of the expansion history
 H(z) with 4 (5 with SNe) knots.

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Direct cosmic distance ladder

Spline reconstruction of the expansion history
 H(z) with 4 (5 with SNe) knots.

Direct and inverse cosmic distance ladder (Cuesta et al 2015)



Spline reconstruction of the expansion history
 H(z) with 4 (5 with SNe) knots.

Direct and inverse cosmic distance ladder (Cuesta et al 2015)



Direct cosmic distance ladder

r_s from CMB independent from late time physics?

aside

Early cosmology constrained (Verde, Bellini, Pigozzo et al 2016.) Based on Audren et al 2012

Late time effects in the CMB, combined with early time effects



The answer is yes: 147.0pm 0.34 Mpc (assume standard early time physics)

Towards model-independent

Ho+BAO+rs



Late time increased acceleration?

Towards model-independent

Ho+SNe



Late time increased acceleration? nope

Towards model-independent

Ho+BAO+ SNe +rs



Ho+BAO+ Sne



The SHAPE of expansion history is well constrained



The issue is with the normalization

The H_0 problem as a r_s problem



In a nutshell

The H0 problem is an anchor problem (either at z=0 or at z=1100), which cannot be solved by tweaking (or bending) the expansion history

It can be solved by changing rs (thus early time physics) but polarization data impose some tight limits.
Why so much interest in Neff...

With high I polarization

w/o high I polarization



 $\Delta Neff ~0.4$ fixes everything but is disfavored by high I Planck polarization

The trouble with H₀

The low redshift standard ruler

Verde, Bernal et al 2016



The low redshift standar ruler



BAO Physics

- Theoretical prediction depends on physics at z ~ 1000 (recombination)
- Measured size of BAO ruler depends on cosmology at z < a few
- Some aspects of recombination physics are set at z ~ 10⁹. (Helium abundance)
- We can test a complete model with data from all of these 3 very different epochs (1 minute, 400,000 yr, 10¹⁰ years)
- If the model is wrong at one epoch, parameter inference (relevant to physics at another epoch) may be incorrect

How long is the standard ruler?

Normally model-dependent

$$r_s = \int_0^{t_*} \frac{c_s(t)}{1+z} dt = \int_{z_*}^\infty \frac{c_s(z)}{H(z)} dz$$

"Only" measure angles

$$\theta = r_d/D_A$$



The geometry can be measured with a standard ruler whose length is not necessarily important

Can we measure how long it is in a model independent way?

(so we can use it to test models)

A. Heavens, R. Jimenez, LV, PRL 2014, arXiv:1409.6217 PS similar philosophy of Sutherland 2012 MNRAS (model-dependent way)

How long is the standard ruler?

$$\theta = r_d / D_A$$

From the Robertson-Walker metric:

 $H(z) = \dot{R}/R$

$$ds^2 = c^2 dt^2 - R^2(t) \{ dr^2 + S_k^2(r) [d\theta^2 + \sin^2 \theta d\phi^2] \}$$

 $S_k(r) = \sin(r), r, \sinh(r) \text{ for } k = 1, 0, -1$

$$D_A(z) = (1+z)^{-1} R_0 S_k \left(\frac{\chi(z)}{R_0}\right) \qquad \chi(z) = c \int_0^z \frac{dz'}{H(z')}$$

No GR; just symmetry (isotropy, homogeneity)

And the existence of a standard ruler, of course (standard candles etc..)

No need to use a model for the nearby Universe - use data

Type IA supernovae: standard candles. Measure 'Luminosity Distance'

$$f \equiv \frac{L}{4\pi D_L^2}$$

Any (Riemannian) metric theory of gravity implies

$$D_L(z) = (1+z)^2 D_A(z)$$

JLA sample, Betoule et al 2014





Probes of the expansion history

Standard clocks

Old elliptical galaxies have stellar populations well described by a single age.

Differential ages give

$$\delta t(z) \simeq rac{\delta z}{H(z)(1+z)}$$

(ignores spread in formation time in comparison with Hubble time)

Simon et al. 2005, Stern et al. 2010, Moresco et al. 2012, 2015, 2016

And, of course, H0

(Riess et al. 2016)

Basic model parameters and assumptions

Assume only:

Cosmological Principle (Homogeneity, Isotropy)

A metric theory of gravity

Smooth expansion history

Equation of state (of dark energy etc) is not assumed

Independent of Cosmological Model

Late-time data , z<1.3(1.8) tells us the length of the standard ruler.

Physics of the early Universe sets it

The low redshift standar ruler

Parameters (8 or 10)

Ruler length

Supernova magnitude (log[luminosity])

Inverse curvature radius of Universe, $c/(R_0 H_0)$ (i.e., in units of $H_0/c \sim 1/size$ of observable Universe)

1/H(z) at 5 (7) different redshifts from 0 to 1.3 (1.8) (linearly interpolate in between)

8(10) parameters. Improper uniform priors. Monte Carlo Markov Chain with ~10⁷ points. Gelman-Rubin convergence: R = 1+O(0.0001)The low redshift standar ruler

Results

Two ways to report the low redshift standard ruler

If only SNe and BAO: r_sh

If add Ho (or H(z) e.g., Clocks): r_s

Results

Data	$r^h_s[h^{-1}{ m Mpc}]$	$r_{ m s}[{ m Mpc}]$	H_0	ΔM	$\Omega_k = k(c/H_0R_0)^2$
SBH	$102.0 \pm 2.5(^{+2.2}_{-2.8})$	140.8 ± 4.9	72.8 ± 1.8	0.079 ± 0.083	$-0.49 \pm 0.64 \left(-0.99^{+0.86}_{-0.26}\right)$
BH	107.2 ± 7.2	147 ± 10	73.0 ± 1.8	N/A	unconstrained
SB	101.0 ± 2.3	unconstrained	unconstrained	unconstrained	0.07 ± 0.61
CB	103.9 ± 5.6	149.5 ± 4.3	69.6 ± 4.2	N/A	unconstrained
CSB	100.5 ± 1.9	150.0 ± 4.7	67.0 ± 2.5	-0.090 ± 0.079	0.36 ± 0.41
CBH	107.2 ± 3.4	148.0 ± 3.9	72.5 ± 1.7	N/A	unconstrained
CSBH	102.3 ± 1.8	143.9 ± 3.1	71.1 ± 1.5	0.028 ± 0.047	$-0.03 \pm 0.31 \left(-0.08^{+0.32}_{-0.28}\right)$
SBH	100.7 ± 1.8	138.5 ± 4.3	72.8 ± 1.8	0.083 ± 0.061	flat
BH	107.1 ± 7.2	147 ± 10	73.0 ± 1.8	N/A	flat
SB	101.2 ± 1.8	unconstrained	unconstrained	unconstrained	flat
CB	103.7 ± 5.5	149.8 ± 4.2	69.2 ± 4.0	N/A	flat
CSB	101.4 ± 1.7	148.3 ± 4.3	68.5 ± 2.1	-0.047 ± 0.064	flat
CBH	107.4 ± 3.4	148.0 ± 3.6	72.6 ± 1.7	N/A	flat
CSBH	102.3 ± 1.6	143.9 ± 3.1	71.1 ± 1.4	0.026 ± 0.043	flat

2% measure

Planck, LCDM: 147.27 ± 0.31 Mpc

The low redshift standar ruler

Results



Without H_0 or clocks only r_s h is constrained

The low redshift standar ruler

In a nutshell

It is possible to measure the standard ruler from low redshift observations in a model-independent way without making any assumptions about gravity, GR, dark energy, or early time physics etc.

Value so measured is in agreement with CMB-LCDM except for SBH combination

The error-bars are still factor 6-7 larger than that of CMB-inferred value in LCDM The error bars are however (slightly) smaller that CMB-inferred ones for NeffLCDM model. Can be used to constrain Neff

Error on rs dominated by error on H0

Parameters, physics

Early cosmology constrained (Verde, Bellini, Pigozzo et al.)

Based on Audren et al 2012



Late time effects in the CMB, combined with early time effects

Early cosmology constrained (Verde, Bellini, Pigozzo et al.) Based on Audren et al 2012



Recipe to remove Late time effects in the CMB

Based on Audren et al 2012



ISW

Early cosmology constrained (Verde, Bellini, Pigozzo et al.) Based on Audren et al 2012

Late time effects in the CMB

Lensing potential



Dark energy "hardened" constraints



More powerful than r_s only

now analise data so that the late-time information is removed*

We consider:

LCDM: standard 6-parameters model

DE fld: dark energy fluid.perfect fluid, w<0 and $c_s=1$

K: LCDM but non flat (recall that CMB constraints on flatness come from late-time effects)

nu:3 neutrinos but c_{eff} and c_{vis} are free parameters

Matteriation: extra fluid with 0<w<1/3, no perturbations effect only on the background

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*dark-energy "hardened" CMB prior
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	LCDM	DE fld	k	nu	matteriation
$10\Omega_b^{ m rec}$	$1.187\substack{+0.034\\-0.034}$	$1.186\substack{+0.034\\-0.034}$	$1.188\substack{+0.036\\-0.034}$	$1.205\substack{+0.047\\-0.047}$	$1.185\substack{+0.035\\-0.033}$
$10\Omega^{rec}_{ m cdm}$	$6.378\substack{+0.071\\-0.070}$	$6.379\substack{+0.071\\-0.070}$	$6.376\substack{+0.071\\-0.075}$	$6.378\substack{+0.071\\-0.072}$	$6.351\substack{+0.082\\-0.085}$
$H_{\rm rec} \; [{ m Mpc}^{-1}]$	$5.197\substack{+0.045\\-0.045}$	$5.198\substack{+0.046\\-0.045}$	$5.196\substack{+0.046\\-0.047}$	$5.12\substack{+0.14 \\ -0.14}$	$5.193\substack{+0.046 \\ -0.048}$
n_s	$0.964\substack{+0.010\\-0.010}$	$0.964\substack{+0.010\\-0.010}$	$0.964\substack{+0.011\\-0.011}$	$0.956\substack{+0.023\\-0.023}$	$0.966\substack{+0.011\\-0.011}$
$e^{-2\tau}A_s$	$1.879\substack{+0.024\\-0.025}$	$1.879\substack{+0.026\\-0.026}$	$1.876\substack{+0.025\\-0.026}$	$1.871\substack{+0.072\\-0.070}$	$1.875\substack{+0.026\\-0.026}$
$10^9 \Omega^{rec}_{\Lambda}$	$1.29\substack{+0.12\\-0.12}$	-	$1.66^{+1.38}_{-1.46}$	$1.24\substack{+0.17\\-0.17}$	$1.31\substack{+0.13\\-0.12}$
$10^4\Omega_k^{ m rec}$	-	-	$0.27\substack{+1.17 \\ -1.24}$	-	-
$10^3 \Omega_{ m fld}^{ m rec}$	-	$0.039\substack{+0.197\\-0.039}$	-	-	$2.50\substack{+3.40 \\ -2.50}$
$w_{ m fld}^0$	-	$-1.15\substack{+0.44\\-0.73}$	-	-	$0.05\substack{+0.10 \\ -0.05}$
$N_{ m eff}$	-	-	-	$2.77\substack{+0.47 \\ -0.46}$	-
$c_{ m eff}^2$	-	-	-	$0.33\substack{+0.01\\-0.01}$	-
$c_{ m vis}^2$	-	-	-	$0.34\substack{+0.10\\-0.10}$	-
$z_{ m rec}$	$1089.0\substack{+0.5\\-0.5}$	$1089.0\substack{+0.5\\-0.5}$	$1089.0\substack{+0.5\\-0.6}$	$1088.7\substack{+0.6\\-0.6}$	$1089.0\substack{+0.5\\-0.5}$
r_d^s	$147.35\substack{+0.66\\-0.66}$	$147.34\substack{+0.66\\-0.66}$	$147.37\substack{+0.68\\-0.69}$	$150.0\substack{+4.8\\-4.7}$	$147.47\substack{+0.69\\-0.71}$
χ^2_{min}	2456	2454	2456	2454	2455

1%

Bounds are 95% C.L.

Note H at recombination...

The H_0 - r_s relation



The cosmic triangle (last millenium)



Bachall et al 1999

The (new) cosmic triangle



 1σ

2σ

In summary

- The ``Ho trouble": It is a mis-match of anchors no evidence for strange expansion history.
- Can shift Ho or r_s. (keep this in mind if looking for new physics)
- ΔNeff~0.4 would do (if no Planck polarization)
- More better data are coming in

Void (underdensity by -0.57 out to 300 Mpc) Romano 2017, see also Wotjak et al 2013

A Void?



Standard sirens to the rescue (with some patience)



LIGO Nature paper last week

70(+12-8) Km/s/Mpc

14% error

Standard sirens to the rescue (with some patience)



LIGO Nature paper last week 70(+12-8) Km/s/Mpc

Final conclusions

- Systematic errors are not only in the measurement but also in the interpretation (the adopted underlying model). Cosmology is special.
- It would be good to have a principled framework to deal with systematics errors (for another time)
- Concentrated on difference between "constraining model's parameters" and "understanding"
- Attempt to be model-independent. Some examples of possible approaches.





Example: an early dark energy model

Doran, Robers 2006



 Ω m,0 = 0.3, Ω d,0 \approx 0.7, Ω d,e = 0.001 and zeq = 3570.

Effect of dark energy perturbations

CAMB (includes perturbations*) CLASS (does not, Background only)



*sound speed c2s = 1 and no anisotropic stress.

Effect on cosmological parameters



Effect on cosmological parameters



Effect on cosmological parameters

Parameter	TT	TT_lens	TTTEEE	TTTEEE_lens	Code
	< 9.240	< 9.80	< 5.53	< 6.25	CAMB
$10^3 \Omega_{ m d,e}$	_	$0.9\substack{+5.4 \\ -5.1}$	$0.2\substack{+4.4\\-4.2}$	$0.4\substack{+4.4 \\ -4.3}$	CLASS
	< -0.69	< -0.68	< -0.72	< -0.68	CAMB
w_0	_	$-0.82\substack{+0.36\\-0.36}$	$-0.85\substack{+0.34 \\ -0.37}$	$-0.82\substack{+0.36\\-0.36}$	CLASS
	$0.786\substack{+0.057\\-0.065}$	$0.778\substack{+0.046\\-0.055}$	$0.795\substack{+0.051\\-0.060}$	$0.778\substack{+0.046\\-0.055}$	CAMB
σ_8	_	$0.790\substack{+0.067\\-0.063}$	$0.805\substack{+0.069\\-0.067}$	$0.788\substack{+0.065\\-0.060}$	CLASS
	$146.9^{+1.2}_{-1.0}$	$147.0^{+1.2}_{-1.3}$	$146.96\substack{+0.73 \\ -0.84}$	$147.01\substack{+0.80\\-0.88}$	CAMB
$r_{ m d} \; [{ m Mpc}]$	_	$147.55\substack{+0.93\\-0.92}$	$147.26\substack{+0.63\\-0.64}$	$147.36\substack{+0.60\\-0.59}$	CLASS
	$63.5^{+4.9}_{-6.1}$	$63.6\substack{+5.0 \\ -6.2}$	$63.7^{+4.3}_{-5.6}$	$63.4\substack{+4.8 \\ -6.0}$	CAMB
$H_0 \; [\mathrm{km \; s^{-1} \; Mpc^{-1}}]$	_	$62.8^{+11.1}_{-10.8}$	$63.1\substack{+10.6 \\ -10.3}$	$62.3\substack{+10.7 \\ -10.4}$	CLASS

Separating late-early effects...

ISW



Hi-class: Zumalacrregui et al 2016; Bellini, Sawicki 2014

LENSING
Separating early-late effects

Follow: Vonlanthen et al 2010, and Audren et al 2013



Separating early-late effects

REIONIZATION



Separating early-late effects

Distance to last scattering or geometry or expansion history...



And now we go wild...

Several models:

- LCDM
- DE fld : Lambda replaced by standard perfect fluid, c_s²=1, w<0, constant
- k: allow curvature
- nu: a LCDM with non standard neutrinos that have free $c_s^2 c_{vis}^2$
- matteriation: there is a (dark) fluid with 0<w<1/3

Warning: for values of $~\Omega_{\Lambda,0}>0.8~$ The ISW removal procedure is not sufficent

We should cut more low multiploes but constraints degrade rapidly

But relevant parameters do not show degeneracy with $\Omega_{\Lambda,0}$

So constraints are not affected

Results: LCDM



Results: DEfld

DE fld : Lambda replaced by standard perfect fluid, c_s²=1, w<0, constant



Results: k



Results: nu



Results: matteriation



Results: summary

	LCDM	DE fld	k	nu	matteriation
$10\Omega_b^{ m rec}$	$1.187\substack{+0.034\\-0.034}$	$1.186\substack{+0.034\\-0.034}$	$1.188\substack{+0.036\\-0.034}$	$1.205\substack{+0.047\\-0.047}$	$1.185\substack{+0.035\\-0.033}$
$10\Omega_{ m cdm}^{rec}$	$6.378\substack{+0.071\\-0.070}$	$6.379\substack{+0.071\\-0.070}$	$6.376\substack{+0.071 \\ -0.075}$	$6.378\substack{+0.071\\-0.072}$	$6.351\substack{+0.082\\-0.085}$
$H_{\rm rec} \ [{ m Mpc}^{-1}]$	$5.197\substack{+0.045\\-0.045}$	$5.198\substack{+0.046\\-0.045}$	$5.196\substack{+0.046\\-0.047}$	$5.12\substack{+0.14\\-0.14}$	$5.193\substack{+0.046\\-0.048}$
$n_{ m s}$	$0.964\substack{+0.010\\-0.010}$	$0.964\substack{+0.010\\-0.010}$	$0.964\substack{+0.011\\-0.011}$	$0.956\substack{+0.023\\-0.023}$	$0.966\substack{+0.011\\-0.011}$
$e^{-2 au}A_{ m s}$	$1.879\substack{+0.024\\-0.025}$	$1.879\substack{+0.026\\-0.026}$	$1.876\substack{+0.025\\-0.026}$	$1.871\substack{+0.072\\-0.070}$	$1.875_{-0.026}^{+0.026}$
$10^4 \Omega_{ m k}^{ m rec}$	-	-	$0.27^{+1.17}_{-1.24}$	-	-
$10^3 \Omega_{ m fld}^{ m rec}$	-	$0.039\substack{+0.197\\-0.039}$	-	-	$2.50\substack{+3.40 \\ -2.50}$
$w_{ m fld}^0$	-	$-1.15\substack{+0.44\\-0.73}$	-	-	$0.05\substack{+0.10\\-0.05}$
$N_{ m eff}$	-	-	-	$2.77\substack{+0.47 \\ -0.46}$	-
$c_{ m eff}^2$	-	-	-	$0.33\substack{+0.01\\-0.01}$	-
$c_{ m vis}^2$	-	-	-	$0.34\substack{+0.10\\-0.10}$	-
$z_{ m rec}$	$1089.0\substack{+0.5\\-0.5}$	$1089.0\substack{+0.5\\-0.5}$	$1089.0\substack{+0.5\\-0.6}$	$1088.7\substack{+0.6\\-0.6}$	$1089.0\substack{+0.5\\-0.5}$
r_d^s	$147.35\substack{+0.66\\-0.66}$	$147.34\substack{+0.66\\-0.66}$	$147.37\substack{+0.68\\-0.69}$	$150.0\substack{+4.8\\-4.7}$	$147.47\substack{+0.69\\-0.71}$
χ^2_{min}	2456	2454	2456	2454	2455

1%