Geometrical CP violation with a complete fermion sector

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Outline

Spontaneous CP violation

Complex VEVs not sufficient. CP conserved if:

$$H_i \longrightarrow H_i' = U_{ij} H_j \,,$$

$$U_{ij} \langle H_j \rangle^* = \langle H_i \rangle \,,$$

while U leaves the Lagrangian invariant.

Calculable phases

Branco, Gérard, Grimus (1984) PLB

- Phases have geometrical values independent of couplings.
- >2 Higgs doublets and non-Abelian symmetries.
- Interesting $\Delta(27)$ example found.

$\Delta(27)$ calculable phase

$\Delta(27)$: Det $= 1$; cyclic generator; phase generator ($e^{i2\pi/3}$)

$$V(H) \sim \lambda_3 (H_1^\dagger H_2 H_1^\dagger H_3 + H_2^\dagger H_3 H_2^\dagger H_1 + H_3^\dagger H_1 H_3^\dagger H_2) + \text{H.c.}$$

$$V \propto \lambda_3 v^4 (e^{iA_1} + e^{iA_2} + e^{iA_3} + H.c.)\,, \quad A_i = -2\alpha_i + \alpha_j + \alpha_k$$

$$(\sum A_i = 0)$$

Geometrical (complex) VEVs

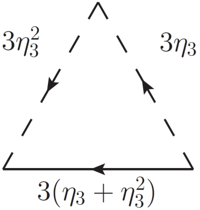
Phases depend on the sign of λ_3:
$V \propto \lambda_3 v^4 (e^{iA_1} + e^{iA_2} + e^{iA_3} + H.c.)\,, \quad A_i = -2\alpha_i + \alpha_j + \alpha_k$

$$\langle H \rangle = \frac{v}{\sqrt{3}} (1, \omega, \omega^2)\,, A_i = 0\,, \tag{1a}$$

$$\langle H \rangle = \frac{v}{\sqrt{3}} (\omega, 1, 1)\,, A_i = 2\pi/3 \tag{1b}$$

Triangle

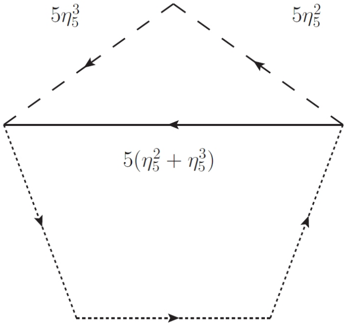
$$V \propto \lambda_3 v^4 (e^{iA_1} + e^{iA_2} + e^{iA_3} + H.c.)\,, \quad A_i = -2\alpha_i + \alpha_j + \alpha_k$$



$$(\eta_3 = \omega = e^{i2\pi/3}, \omega^3 = 1)$$

Geometrical (complex) VEVs - see also

IdMV (2012) JHEP 1205.3780
Holthausen, Lindner, Schmidt (2012) JHEP 1211.6953
Ivanov, Lavoura (2013) EPJ 1302.3656
IdMV (2013) Discrete 2012 Proceedings 1302.3991



Fermions: $\Delta(27)$

IdMV, Emmanuel-Costa (2011) PLB 1106.5477

$QH^\dagger u^c$, QHd^c and Q_i as...

- triplet: 1 sector $\mathbf{3_{0i}} \times \mathbf{3_{0i}} \times \mathbf{3_{0i}}$.
  Already pointed out as not viable.
- singlets: Both sectors $\mathbf{1_{rs}} \times \mathbf{3_{01}} \times \mathbf{3_{02}}$
  can get:
  (not shown) rank 1 mass matrices or
  (M_d) one generation decoupled or
  ($\tilde{M}_d$) “diagonal” matrices with three distinct eigenvalues.

Singlets on the left

$$(Hd^c) \to 1_{rs}\text{: } H_1 d^{c1} + c.p.\ (1_{00})$$
$$H_1 d^{c2} + c.p. \text{ or } H_1 d^{c3} + c.p.\ (1_{0i})$$

Distinct singlets for each Q_i:

$$\tilde{M}_d = v \begin{pmatrix} y_1\omega & y_1 & y_1 \\ y_2 & y_2\omega & y_2 \\ y_3 & y_3 & y_3\omega \end{pmatrix}$$

Repeated singlet for Q_1 and Q_2:

$$M_d = v \begin{pmatrix} y_1\omega & y_1 & y_1 \\ y_2\omega & y_2 & y_2 \\ y_3 & y_3 & y_3\omega \end{pmatrix}$$

No CKM

$$\tilde{M}_d \tilde{M}_d^\dagger = 3v^2 \begin{pmatrix} y_1^2 & 0 & 0 \\ 0 & y_2^2 & 0 \\ 0 & 0 & y_3^2 \end{pmatrix}$$

$$M_d M_d^\dagger = 3v^2 \begin{pmatrix} y_1^2 & y_1 y_2 & 0 \\ y_1 y_2 & y_2^2 & 0 \\ 0 & 0 & y_3^2 \end{pmatrix}$$

$$(1 + \omega + \omega^2 = 0)$$

Off-diagonal entries

Bhattacharyya, IdMV, Leser (2012) PRL 1210.0545
Add θ, in the irrep $\mathbf{1}_{01}$. Get $QH_i d^{cj}\theta$

$$M_\theta = v \begin{pmatrix} y_{\theta 1} & y_{\theta 1}\omega & y_{\theta 1} \\ y_{\theta 2} & y_{\theta 2}\omega & y_{\theta 2} \\ y_{\theta 3}\omega & y_{\theta 3} & y_{\theta 3} \end{pmatrix}$$

Obtain the required off-diagonal entries...
But no complex phase.

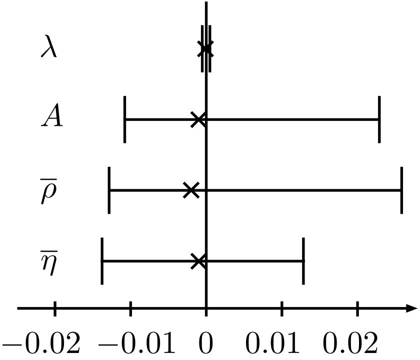
Complex phase

Use $QH_i d^{cj}(H_k H^{\dagger l})$

$$M_H = v \begin{pmatrix} y_{H1} & y_{H1}\omega^2 & y_{H1}\omega^2 \\ y_{H2} & y_{H2}\omega^2 & y_{H2}\omega^2 \\ y_{H3}\omega^2 & y_{H3}\omega^2 & y_{H3} \end{pmatrix}$$

$M_d + M_\theta + M_H$: everything needed to account for observations!

Observations



Lagrangian and results

$$\mathcal{L} = Q\left(H^{\dagger i} u^c_j + H_i d^{cj} + H_i d^{cj}\theta + H_i d^{cj}(H_k H^{\dagger l})\right) .$$

$$\lambda^{\text{exp}} = 0.22535 \pm 0.00065 \quad \lambda = 0.22534,$$

$$A^{\text{exp}} = 0.811^{+0.022}_{-0.012} \quad A = 0.810,$$

$$\bar{\rho}^{\,\text{exp}} = 0.131^{+0.026}_{-0.013} \quad \bar{\rho} = 0.129,$$

$$\bar{\eta}^{\,\text{exp}} = 0.345^{+0.013}_{-0.014} \quad \bar{\eta} = 0.344.$$

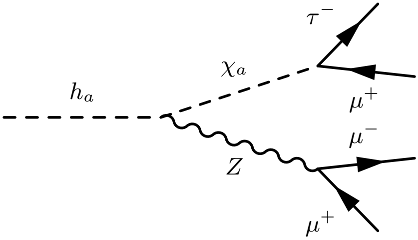
Potential

$$V(H,\theta) = m_1^2 \left[H_1 H_1^\dagger\right] + m_2^2 \theta\theta^\dagger + m_3(\theta^3 + \text{h.c.})$$
$$+ \lambda_1 \left[(H_1 H_1^\dagger)^2\right] + \lambda_2 \left[H_1 H_1^\dagger H_2 H_2^\dagger\right] + \lambda_3 \left[H_1 H_2^\dagger H_1 H_3^\dagger + \text{h.c.}\right]$$
$$+ \lambda_4 (\theta\theta^\dagger)^2 + \lambda_5 \left[\theta(H_1 H_2^\dagger) + \text{h.c.}\right] + \lambda_6 \left[\theta\theta(H_1 H_3^\dagger) + \text{h.c.}\right] ,$$

λ_5, λ_6 must be suppressed

LHC tests

In the θ decoupling limit, similar to S_3 case
Bhattacharyya, Leser, Paes (2012) PRD 1206.4202
CP even state $h_a \sim H_2 - H_3$
No $h_a VV$; h_a (χ_a) couples off-diagonal to fermions ($h_a uc$, $h_a ct$)



What next?

Explaining the hierarchies?
Safeguard potential?
Leptons?

Potential and Hierarchies

IdMV, Pidt (2013) JPG, 1307.0711
Solve simultaneously, single symmetry

Potential

Charge 1_{0i} scalars, force $\lambda_5 = \lambda_6 = 0$
Must keep the 1_{0i} in the Yukawa sector!

Hierarchies

Solution: 1_{0i} as FN field(s).

Need extra FN scalar; replace the M structures
Kept the $MM^\dagger$

$U(1)_F$

	Q_1	Q_2	Q_3	u^c	d^c	H	φ	θ
$\Delta(27)$	1_{00}	1_{00}	1_{02}	3_{01}	3_{02}	3_{01}	1_{00}	1_{02}
$U(1)_F$	3	2	0	0	0	0	-1	-2

FN terms

$$L_d = y_3 Q_3 (Hd^c)_{01} + y_2 Q_2 (Hd^c)_{00} \varphi^2 + y_1 Q_1 (Hd^c)_{00} \varphi^3$$
$$+ p_2 Q_2 (Hd^c)_{01} \theta + p_1 Q_1 (Hd^c)_{01} \varphi \theta$$
$$+ h_3 (HH^\dagger) Q_3 (Hd^c)_{01} + h_2 (HH^\dagger) Q_2 (Hd^c)_{00} \varphi^2 + h_1 (HH^\dagger) Q_1 (Hd^c)_{00} \varphi^3 .$$

FN diagrams

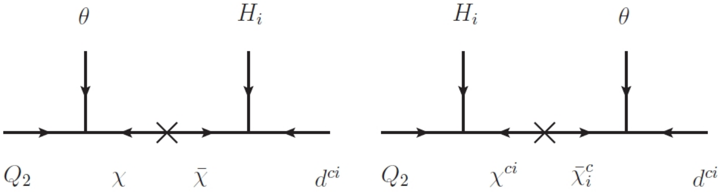


Figure : χ,$\bar{\chi}$ are $SU(2)$ doublets, χ^{ci}, $\bar{\chi}_i^c$ are $\Delta(27)$ triplets.

Replaced M

$$M = M_d + M_p + M_h$$

$$M_d = v \begin{pmatrix} y_1\omega & y_1 & y_1 \\ y_2\omega & y_2 & y_2 \\ y_3 & y_3 & y_3\omega \end{pmatrix} ,$$

$$M_p = v \begin{pmatrix} p_1 & p_1 & p_1\omega \\ p_2 & p_2 & p_2\omega \\ 0 & 0 & 0 \end{pmatrix} ,$$

$$M_h = v^3 \begin{pmatrix} h_1 & h_1\omega^2 & h_1\omega^2 \\ h_2 & h_2\omega^2 & h_2\omega^2 \\ h_3\omega^2 & h_3\omega^2 & h_3 \, . \end{pmatrix}$$

GCPV safeguarded

$$
\begin{aligned}
V(H,\varphi,\theta) = & m_\varphi^2 \varphi\varphi^\dagger + m_\theta^2 \theta\theta^\dagger + \lambda_\varphi (\varphi\varphi^\dagger)^2 + \lambda_\theta (\theta\theta^\dagger)^2 + m_H^2 \left[H_i H_i^\dagger \right] \\
& + \lambda_1 \left[(H_i H_i^\dagger)^2 \right] + \lambda_2 \left[H_1 H_1^\dagger H_2 H_2^\dagger \right] \\
& + \lambda_3 \left[H_1 H_2^\dagger H_1 H_3^\dagger + \text{h.c.} \right] \\
& + \left(\lambda_{\varphi H} \varphi\varphi^\dagger + \lambda_{\theta H} \theta\theta^\dagger \right) \left[H_i H_i^\dagger \right] ,
\end{aligned}
$$

Leptons

IdMV, Pidt (2013) JHEP, 1307.6545

If we have $LLH^\dagger H^\dagger$ invariants and H as 3_{0i}
LHl^c is $\mathbf{3_{0i}} \times \mathbf{3_{0i}} \times \mathbf{3_{0i}}$.

Our solution: separate ϕ ($\Delta(27)$ triplet with GCPV) and H.*
Quarks: FN charges, φ, θ remain

*See also: Ma (2013) 1304.1603 (additional $SU(2)$ doublets)

Complete fermion sector

Leptons:

$$H\left(y_3(L\phi^\dagger)_{01}\tau^c + y_2(L\phi^\dagger)_{02}\mu^c(\theta^2) + y_1(L\phi^\dagger)_{00}e^c(\theta^3)\right).$$

$$V_l = \frac{1}{\sqrt{3}}\begin{pmatrix}\omega^2 & \omega & 1 \\ 1 & \omega & \omega^2 \\ 1 & 1 & 1\end{pmatrix}.$$

No free parameters, y_i set to get masses.

Neutrino structures

$$H^\dagger H^\dagger \xi \left(z_1 (L_i L_i \phi_i)_{00} + z_4 (L_i L_j \phi_k)_{00} \right)$$
$$H^\dagger H^\dagger \xi' \left(z_2 (L_i L_i \phi_i)_{02} + z_5 (L_i L_j \phi_k)_{02} \right)$$
$$H^\dagger H^\dagger \xi'' \left(z_3 (L_i L_i \phi_i)_{01} + z_6 (L_i L_j \phi_k)_{01} \right) ,$$

ξ, ξ', ξ'' are 1_{00}, 1_{01}, 1_{02} spurion fields

$$H^\dagger H^\dagger \left(A (L\phi^\dagger)_{00} (L\phi^\dagger)_{00} + B (L\phi^\dagger)_{01} (L\phi^\dagger)_{02} \right)$$

Model types

Spurions get identified as combinations of φ, θ
We have one type of model with $\xi \sim \theta^3$ and $\xi' \sim \theta^2\varphi^2$

$$\begin{aligned} & H^\dagger H^\dagger (\theta^3)^\dagger \left(A(L\phi^\dagger)_{00}(L\phi^\dagger)_{00} + B(L\phi^\dagger)_{01}(L\phi^\dagger)_{02} \right) \\ & + H^\dagger H^\dagger \theta^3 \left(z_1 (L_i L_i \phi_i)_{00} + z_4 (L_i L_j \phi_k)_{00} \right) \\ & + H^\dagger H^\dagger \theta^2 \varphi^2 \left(z_2 (L_i L_i \phi_i)_{02} + z_5 (L_i L_j \phi_k)_{02} \right) . \end{aligned}$$

Model types

Another type of model has the h.c. assignments:

$$H^\dagger H^\dagger \theta^3 \left(A(L\phi^\dagger)_{00}(L\phi^\dagger)_{00} + B(L\phi^\dagger)_{01}(L\phi^\dagger)_{02} \right)$$
$$+ H^\dagger H^\dagger (\theta^3)^\dagger \left(z_1 (L_i L_i \phi_i)_{00} + z_4 (L_i L_j \phi_k)_{00} \right)$$
$$+ H^\dagger H^\dagger (\theta^2 \varphi^2)^\dagger \left(z_3 (L_i L_i \phi_i)_{01} + z_6 (L_i L_j \phi_k)_{01} \right) .$$

Complete potential

$$
\begin{aligned}
V(H,\phi,\varphi,\theta) &= m_H^2 HH^\dagger + m_\varphi^2 \varphi\varphi^\dagger + m_\theta^2 \theta\theta^\dagger \\
&+ \lambda_H (HH^\dagger)^2 + \lambda_\varphi(\varphi\varphi^\dagger)^2 + \lambda_\theta(\theta\theta^\dagger)^2 + \lambda_{\varphi\theta}(\varphi\varphi^\dagger)(\theta\theta^\dagger) \\
&+ \left(\lambda_{\varphi H}\varphi\varphi^\dagger + \lambda_{\theta H}\theta\theta^\dagger\right)\left(HH^\dagger\right) \\
&+ m_\phi^2 \left[\phi_i\phi_i^\dagger\right] + \lambda_1\left[(\phi_i\phi_i^\dagger)^2\right] + \lambda_2\left[\phi_1\phi_1^\dagger\phi_2\phi_2^\dagger\right] \\
&+ \lambda_3\left[\phi_1\phi_2^\dagger\phi_1\phi_3^\dagger + \text{h.c.}\right] \\
&+ \left(\lambda_{H\phi}HH^\dagger + \lambda_{\varphi\phi}\varphi\varphi^\dagger + \lambda_{\theta\phi}\theta\theta^\dagger\right)\left[\phi_i\phi_i^\dagger\right] .
\end{aligned}
$$

Geometrical CP violation with complete fermion sector

Conclusions

- First time GCPV with viable fermions.
- Precision data restricts viable irrep. choices.
- Additional symmetry safeguards potential.
- Same symmetry alleviates mass hierarchies.
- Found compatible lepton models.