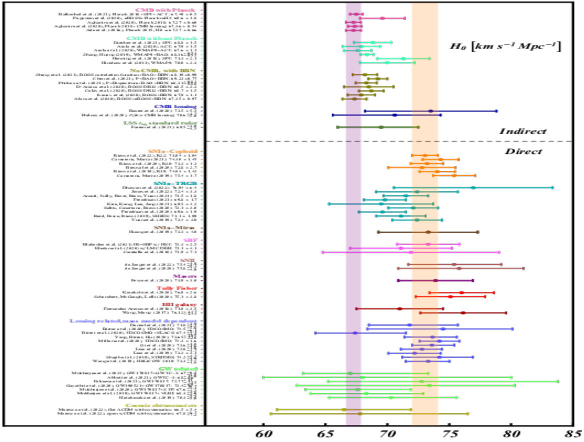
Obtaining smooth distances from a lumpy universe

Obinna Umeh

Arxiv: 2201.11089, 2202.08237, 2202.08237, 2205.......

May 16, 2022

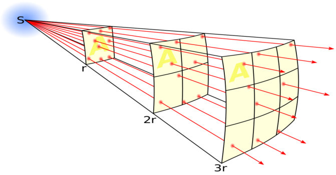
Motivation:the cosmological tension



- FLRW(smooth) + dark stuff
- Change metric ansatz (eg. LTB, Bianchi spacetimes)
- Modify gravity
- Lumpy to smooth

What is actually measured: Flux, angles and redshift

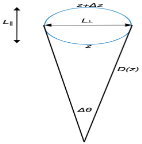
Standard candle



$$m - M = -2.5 \log \left[\frac{F_{d_L}}{F_{D_F}} \right] = 5 \log \left[\frac{d_L}{D_F} \right]$$

- Reference flux $D_F = 1$ mpc for cosmology

Standard ruler (*R. Maartens, Phil. Trans. Roy. Soc. Lond. A* **369** *(2011), 5115-5137*)



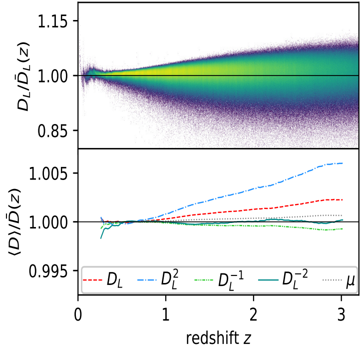
$$L_\parallel = \frac{\Delta z}{(1+z)H_\parallel(z)}$$

$$L_\perp = d_A(z)\Delta\theta$$

Alcock-Paczynski parameters

$$\alpha_\parallel = \frac{H^{\rm fid}}{H}, \qquad \alpha_\perp = \frac{d_A}{d_A^{\rm fid}}.$$

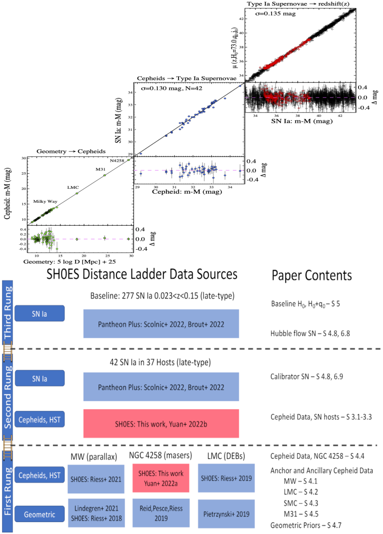
Earlier studies and their limitations



Adamek+Phys. Rev. D **100** *(2019) no.2, 021301*

[arXiv:1812.04336 [astro-ph.CO]]

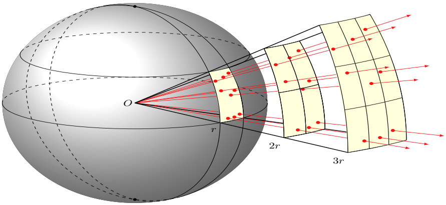
- Cut-off redshift $0.023 \leq z \leq 1100$.
- Weinberg photon number conservation/FLRW space conjecture



Riess+,[arXiv:2112.04510 [astro-ph.CO]].

Observed flux density (*Weinberg S., 1976, ApJL, 208, L1*)

- The total flux, F, from a source(sn1a event) is related to the area according to $F = L/4\pi d_L^2$



- Number count of photons in an FLRW spacetime

$$\frac{A_{\rm FLRW}}{A_{\rm tel}} = \frac{N_{\rm tot}}{\bar{n}_{\rm FLRW}}$$

- Number count of photons in an inhomogeneous spacetime

$$\frac{A_{\rm Inh}}{A_{\rm tel}} = \frac{N_{\rm tot}}{n_{\rm Inh}}$$

Weinberg conjecture and the FLRW spacetime

- Assumption: law of large numbers gives the mean

$$\lim_{m\to\infty}\sum_{m=1}^{\mathrm{m}}\frac{n_{\mathrm{Inh,m}}}{m}=\bar{n}_{\mathrm{FLRW}}$$

- Criticism: there is no evidenece that the mean is described by the FLRW spacetime *N. Mustapha, B. A. Bassett, C. Hellaby and G. F. R. Ellis, Class. Quant. Grav.* **15** *(1998), 2363-2379 [arXiv:gr-qc/9708043 [gr-qc]].*
- Assumption: No focal point $\det[N]\neq 0$

$$\int \mathrm{d}A_{\mathrm{Inh}}=\int \mathrm{d}\Omega\sqrt{\det[\mathrm{N}]}=\bar{A}_{\mathrm{FLRW}}$$

- Criticism: There are caustics/strong lensing in the real universe (*Ellis, Bassett and Dunsby, Class. Quant. Grav.* **15** *(1998), 2345-2361 [arXiv:gr-qc/9801092 [gr-qc]].*)
- Kaiser and Peacock criticism (*N. Kaiser and J. A. Peacock, Mon. Not. Roy. Astron. Soc.* **455** *(2016) no.4, 4518-4547 [arXiv:1503.08506].*). Only effects neglected is the impact of gravity on clocks and rulers and it is small.

Radial lensing (Swiss-Cheese construction) *Ellis and Solomons, Class. Quant. Grav.* **15** *(1998), 2381-2396 [arXiv:gr-qc/9802005 [gr-qc]].*

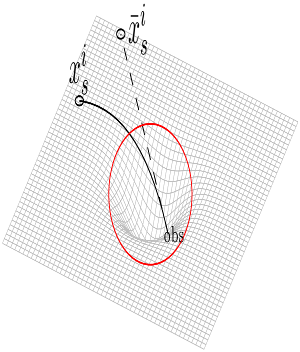
- We live in a gravitational bound over-density(Local group) which distorts the spacetime around us.
- Infinitesimally close geodesics travel longer distance
- Determine the deviation vector ξ_b from

$$\frac{\xi^i(\lambda_s,\boldsymbol{n})}{\bar{d}_A} \equiv \frac{x^i_{\rm pert}(\lambda_s,\boldsymbol{n})-\bar{x}^i_{\rm FLRW}(\lambda_s)}{\bar{d}_A}$$

- The area is given

$$d_A^2 = \sqrt{\det[\mathbf{N}]} = \det[\mathcal{J}_{\perp A}{}^B]$$

- where the Jacobian is given by $\mathcal{J}^a{}_b = \delta_{ab} + \partial_a \xi_b$.



Radial lensing contribution[*F. Schmidt and D. Jeong, Phys. Rev. D* **86** *(2012), 083527*]

- The Jacobian is give by

$$\mathcal{J}_{AB} = \begin{pmatrix} 1-\kappa_S-\gamma_1 & \gamma_2+\omega \\ -\gamma_2-\omega & 1-\kappa_S+\gamma_1 \end{pmatrix}$$

- where $\gamma_{AB} = \gamma_1 + i\gamma_2$

- $\kappa_S = -\left(\xi_{||}/r(\textit{radial lensing}) - \kappa(\textit{tan. conv.})\right)$.
- Decompose the deviation vector $\xi^i \approx \xi_{\parallel} n^i + \xi^i_{\perp}$
- Decomposition of spatial derivative of the deviation vector

$$\begin{aligned} \partial_i \xi_j &= \partial_j \left[n_i \xi_{\parallel} + \xi_{\perp i} \right] \\ &= n_i n_j \nabla_{\parallel} \xi_{\parallel} + 2n_{(i}\partial_{\perp j)}\xi_{\parallel} + \frac{1}{r} N_{ij} \xi_{\parallel} + \nabla_{\perp i} \xi_{\perp j} \end{aligned}$$

- Trace of the spatial derivative of the LOS direction vector

$$\partial_i n_j = \frac{1}{r} N_{ij}$$

Area and the area distance

- The determinant gives the area

$$d_A^2 = \bar{d}_A^2\Bigg[1 + 2\left(\frac{\xi_\parallel}{\bar{d}_A} - \kappa\right) + \left(\frac{{\xi_\parallel}^2}{\bar{d}_A^2} - 2\frac{\xi_\parallel\kappa}{\bar{d}_A} + \kappa^2\right) -\frac{1}{2}\left(\gamma_{AB}\gamma^{AB} - \omega_{AB}\omega^{AB}\right) + \mathcal{O}(\epsilon^3)\Bigg].$$

- The area distance becomes

$$d_A = \bar{d}_A\left[1 + \frac{\xi_\parallel}{\bar{d}_A} - \kappa - \frac{1}{4}\left(\gamma_{ij}\gamma^{ij} - \omega_{ij}\omega^{ij}\right) + \mathcal{O}(\epsilon^3)\right].$$

- The radial lensing in general perturbed spacetime is given by

$$\frac{\xi_\parallel}{\bar{d}_A} \approx -\left(1 - \frac{1}{r_s\mathcal{H}_s}\right)\left(\delta\lambda - \Delta x_\parallel\partial_\parallel\delta\lambda - \delta x_\perp^j\nabla_{\perp j}\delta\lambda\right)\mathcal{H}_s$$

Redshift corrections in a lumpy universe

- The affine parameter is not observable but redshift is

$$(1+z_{\rm obs}) = (1+\bar{z})\left(1+z^{\rm o}_{\rm pec}\right)\left(1+z^{\rm SN}_{\rm pec}\right)$$

- Redshifts and Peculiar Velocities relation

$$(1+z^{X}_{\rm pec)} = \sqrt{\frac{1+v^{X}_{\parallel}}{1-v^{X}_{\parallel}}}$$

- Peculiar velocity and local groud barycentric observer

$$v_{\rm sun-CRF} = \overbrace{v_{\rm sun-LSR} + v_{\rm LSR-GSR}}^{\rm Local} + \overbrace{v_{\rm GSR-LG} + v_{\rm LG-CRF}}^{\rm External},$$

- In perturbation theory $(1+z_{\rm obs}) = (-k_a u^a)_s/(-k_b u^b)_o$

$$\frac{1}{(1+z_{\rm obs})} = \frac{1}{(1+\bar{z})}\left[1+\cancelto{0}{(-\mathcal{H}\delta\lambda-\delta z)}\right]$$

- Constant redshift surface $(\delta\lambda - \delta z/\mathcal{H}) = 0$

Monopole in a lumpy universe at fixed redshift

- The radial lensing component

$$\frac{\xi_{\|}(z,\boldsymbol{n})}{\bar{d}_A} \approx -\left(1-\frac{1}{r_s\mathcal{H}_s}\right)\left[(\partial_{\|}v_s-\partial_{\|}v_o) + \frac{1}{\mathcal{H}_s}(\partial_{\|}v_s-\partial_{\|}v_o)\partial_{\|}^2 v_s+\cdots\right]$$

- Monopole of the area

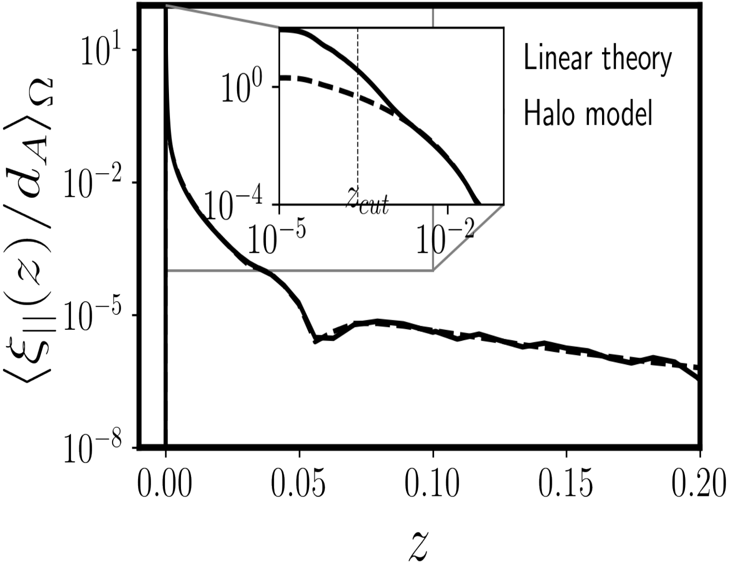
$$\langle d_A^2\rangle_\Omega = \bar{d}_A^2\left[1+2\left\langle\frac{\xi_{\|}}{\bar{d}_A}\right\rangle_\Omega+\left\langle\frac{\xi_{\|}{}^2}{\bar{d}_A^2}\right\rangle_\Omega-2\left\langle\frac{\xi_{\|}\kappa}{\bar{d}_A}\right\rangle_\Omega\right].$$

- Monopole of the area distance

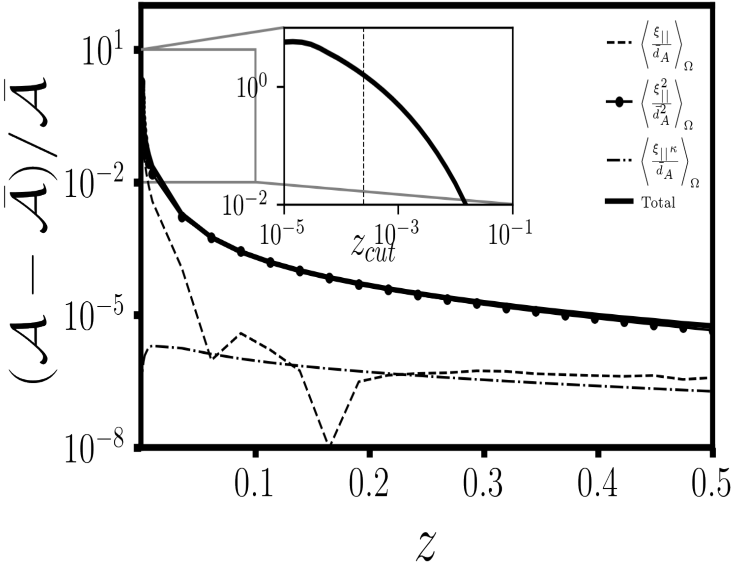
$$\langle d_A(z,\boldsymbol{n})\rangle_\Omega \approx \bar{d}_A(z)\left[1+\left\langle\frac{\xi_{\|}}{\bar{d}_A}\right\rangle_\Omega+\cdots\right]$$

- Radial lensing− >Doppler lensing(*Bacon+, Mon. Not. Roy. Astron. Soc.* **443** *(2014) no.3, 1900-1915 doi:10.1093/mnras/stu1270 [arXiv:1401.3694 [astro-ph.CO]].*)
- Doppler lensing: a source in the Hubble flow will appear dimmer because it moving away from us.

Correction to the monopole area distance



Correction to the monopole of the area or average flux



What does this mean for the cosmological tensions?

Cosmological fitting problem *(G. F. R. Ellis and W. Stoeger, Class. Quant. Grav.* **4** *(1987), 1697-1729)*

- Fiducial model(FLRW spacetime with free paremeters: H_0, Ω_m etc)

$$\bar{U} = \left\{ \bar{g}_{ab}, \bar{u}^a, \bar{N}, \bar{m}, \bar{\alpha}_{\parallel}, \bar{\alpha}_{\perp} \right\} ,$$

- Observational representative model specified on length scale $|\boldsymbol{x}_1 - \boldsymbol{x}_2| < R$

$$U = \left\{ g_{ab}, u^a, m, \alpha_{\parallel}, \alpha_{\perp} \right\} .$$

- Apparent magnitude or the Alcock-Paczynski parameters

$$\langle X(z, \boldsymbol{n}) \rangle_\Omega = \frac{1}{4\pi} \int \mathrm{d}^2\Omega X(z, \boldsymbol{n}) = \int \mathrm{d}\boldsymbol{n}\, X(z, \boldsymbol{n}) ,$$

- Schematically, this is what we do for any observable X:

$$\chi^2_X = \sum_i \left[\frac{\langle X_i \rangle_\Omega - \bar{X}\left(z_i | H_0^X, \Omega_m, M_b\right)}{\sigma_i^X} \right]^2 ,$$

ShoES calibration and distance anchor (zero-point)

- Distance modulus

$$m - M_b \;=\; 5\log\left[\frac{d_L}{[\mathrm{Mpc}]}\right] + 25\,,$$

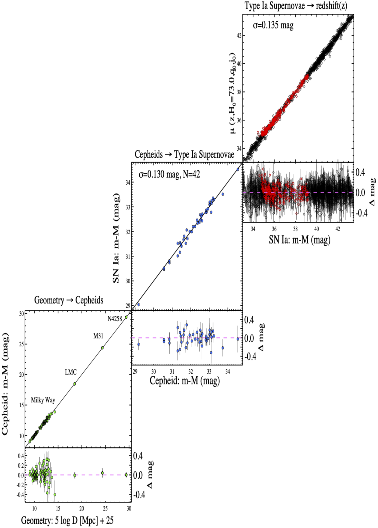
- Calibrate M_b using cepheids

$$M_b = m_{0,sn1a} - \mu_{0,ceph}$$

- $\mu_{0,ceph}$ is obtained from

$$\begin{aligned} M_b \;&=\; m_{0,\mathrm{sn1a}} - m_{0,\mathrm{ceph}} + M_{H,\mathrm{cep}} \\ &=\; -2.5\log_{10}[F_{0,\mathrm{Sn1a}}/F_{0,\mathrm{ceph}}] \\ &\quad + M_{H,\mathrm{ceph}} \end{aligned}$$

- $M_{H,\mathrm{cephj}} = m_{\mathrm{NGC,j}} - \mu_{\mathrm{NGC}}$
- and $\mu_{\mathrm{NGC}} = \log[D] + 25$



L. Lombriser, Phys. Lett. B **803** *(2020), 135303*

Riess+, [arXiv:2112.04510 [astro-ph.CO]].

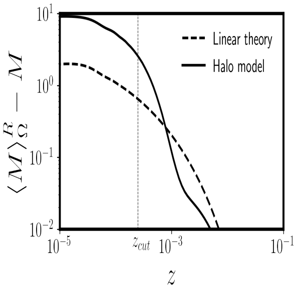
Supernova absolute magnitude tension

- Distance modulus

$$\langle m\rangle_\Omega - \langle M\rangle_\Omega^R - 25 = 5\log_{10}\left[\frac{cz}{H_0}\left(1 + \frac{1}{2}(1-q_0)z + \mathcal{O}(z^2)\right]\right.$$

- Renormalised absolute magnitude

$$\langle M\rangle_\Omega^R - M = \log_{10}\left[1 + \langle \xi_\parallel/\bar{d}_A\rangle_\Omega - \frac{1}{2}\langle {\xi^2}_\parallel/\bar{d}_A^2\rangle_\Omega\right].$$



- The same amount required to resolve the tension(*G. Efstathiou, Mon. Not. Roy. Astron. Soc.* **505** *(2021) no.3, 3866-3872 doi:10.1093/mnras/stab1588 [arXiv:2103.08723 [astro-ph.CO]].*)

Hubble tension in the BAO

- Alcock-Paczynski parameters assuming FLRW spacetime

$$\bar{\alpha}_{\parallel} = \frac{H^{\rm fid}}{H}\,, \qquad \bar{\alpha}_{\perp} = \frac{\bar{d}_A}{d_A^{\rm fid}}\,.$$

- Alcock-Paczynski parameters in a perturbed FLRW spacetime

$$\begin{aligned} \alpha_{\parallel} &= \frac{\partial r_{\parallel}}{\partial r_{\parallel}^{\rm fid}} = \frac{\partial r_{\parallel}}{\partial z}\frac{\partial z}{\partial r_{\parallel}^{\rm fid}} \approx \frac{H^{\rm fid}}{H}\left[1 + \frac{1}{H}\frac{\partial \delta z}{\partial r} + \mathcal{O}(\delta z)\right]\,, \\ \alpha_{\perp} &= \frac{\partial r_{\perp}^{A}}{\partial \theta}\frac{\partial \theta}{\partial^{B} r_{\perp}^{\rm fid}} \approx \frac{d_A}{d_A^{\rm fid}}\,. \end{aligned}$$

- Using the perturbed spacetime as data

$$\left\langle \alpha_{\parallel} \right\rangle_{\Omega} = \bar{\alpha}_{\parallel} \qquad \text{and} \qquad \langle \alpha_{\perp} \rangle_{\Omega} = \bar{\alpha}_{\perp}\,.$$

- Inferred Hubble rate becomes

$$H^{\rm eff} = H\left[1 - \frac{2}{15}\left\langle \frac{\sigma_{ij}\sigma^{ij}}{H^2}\right\rangle_{\Omega}\right] \to \frac{H_0^{\rm eff} - H_0}{H_0} =\sim (8-12)\%\,.$$

Conclusion

- The background FLRW spacetime does not give the correct distance estimates in the universe at all time.
- We claim/show that the early/late universe tension is due to this.
- Effect of our proper motion is yet to be taken in account.

Extra slides

The zero-velocity surface (time-like curves)

- The Jacobi equation for the timelike curves

$$\frac{\mathrm{d}^2 V_{ab}}{\mathrm{d}\tau^2} = -R_{aced} u^c u^d V^e{}_b$$

- with initial conditions

$$V^a{}_b|_{\tau_{\mathrm{ini}}} = 0 \,, \qquad \left.\frac{\mathrm{d} V^a{}_b}{\mathrm{d}\tau}\right|_{\tau_{\mathrm{ini}}} = \delta^a{}_b$$

- Take the trace (Focusing equation for the volume)

$$\frac{1}{\det[\mathrm{V}]}\frac{\mathrm{d}\det[\mathrm{V}]}{\mathrm{d}\tau} = \Theta$$

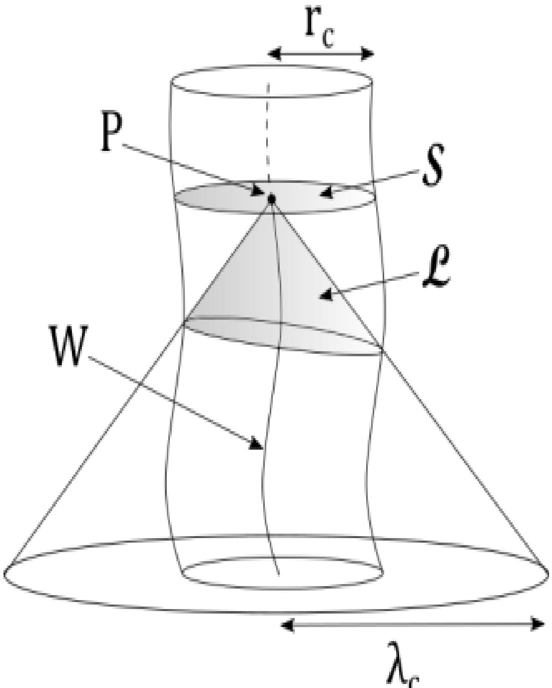
- $\Theta = \Theta_H + \Theta_L$, where Θ_H indicates the global expansion part. It is always positive, $\Theta_H > 0$. And Θ_L indicates the local component of the expansion . In Newtonian gauge $\Theta_L = \mathrm{D}_a v^a$
- Zero-velocity surface corresponds to $\Theta = 0$ or $\Theta_H = -\Theta_L$, $V(x) = V(\tau_{\mathrm{ini}}, x)$ which implies $V(R) = 0$

Zero-velocity surface

- Constant redshift surface or the expanding cordinate system cannot be extrapolated down to the observer.

$$\frac{1}{\det[\mathrm{V}]}\frac{\mathrm{d}\det[\mathrm{V}]}{\mathrm{d}\tau} = \Theta \approx 3H_0 + \frac{c}{r}\frac{\mathrm{d}\ln\rho}{\mathrm{d}\ln r} = 0$$

- There is a finite value of r where $\Theta = 0$.
- There are observational constraints for our local group which falls within the following range $r \sim R \sim (0.95 - 1.05)\mathrm{Mpc}$ (*Karachentsev+, Mon. Not. Roy. Astron. Soc.* **393** *(2009), 1265 doi:10.1111/j.1365-2966.2008.14300.x [arXiv:0811.4610 [astro-ph]].*)
- This implies that one-parameter family of non-spacelike geodesics in global coordinate do not cover the entire universe


r_c
P
$\mathcal{S}$
$\mathcal{L}$
W
λ_c