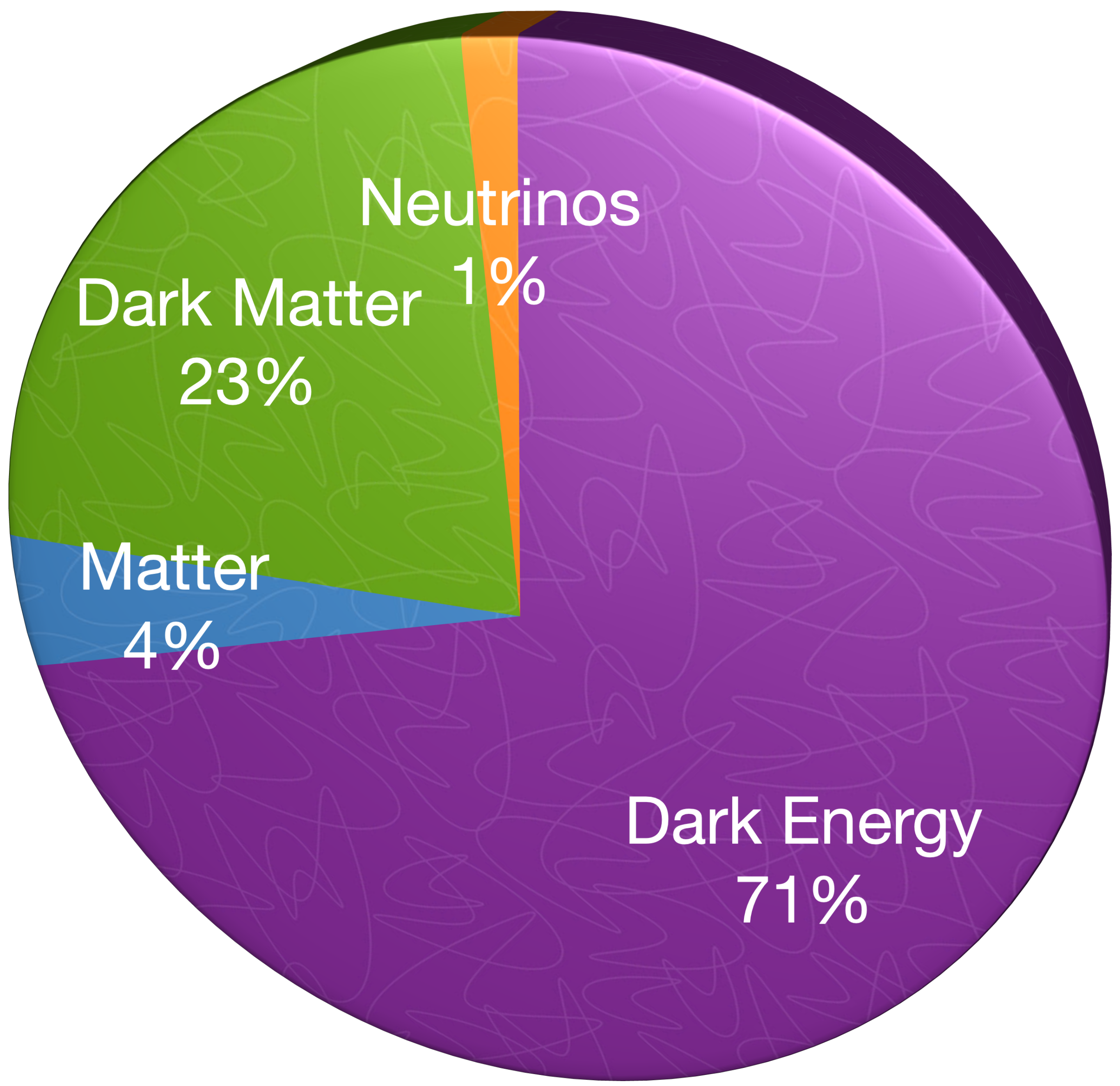
Assessing the tension between a black hole dominated early universe and leptogenesis

Jessica Turner

Institute for Particle Physics Phenomenology, Durham University

Particle and Astroparticle Theory Seminar
6 December 2021

Universe's Energy Budget



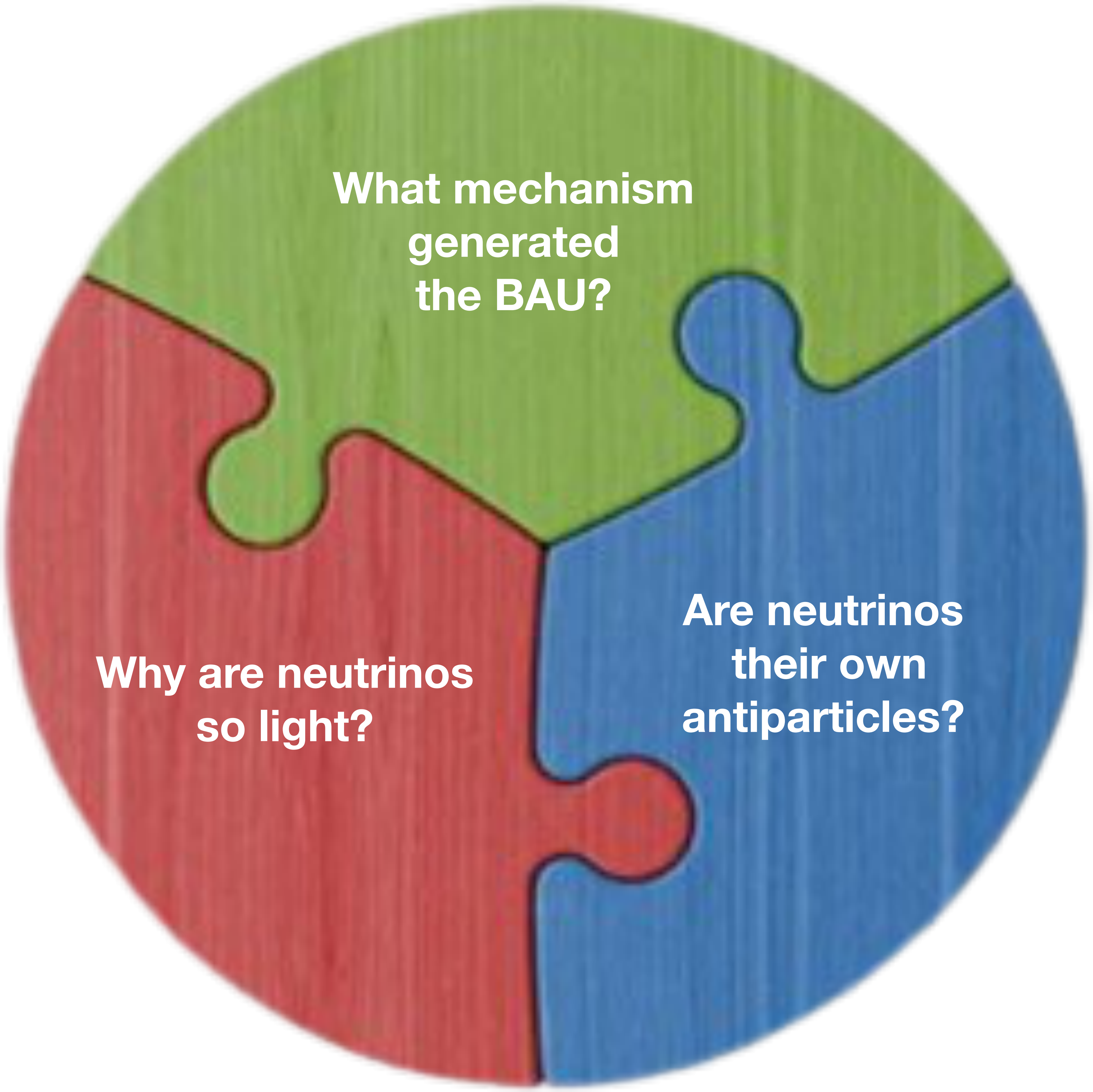
$$\eta_B = (6.02 - 6.18) \times 10^{-10}$$

Planck 1807.06209 (2018)

Sakharov's Conditions

- ☑ Baryon number violation — Kuzmin, Rubakov & Shaposhnikov *Phys.Lett.B 155* (1985)
- ☒ C & CP-violation — Gavela, Hernandez, Orloff & Pene *Mod.Phys.Lett. A9 795-810* (1994) Huet & Sather *Phys.Rev. D51 379-394* (1994)
- ☒ Departure from thermal equilibrium — Kajantie, Laine, Rummukainen & Shaposhnikov *Phys.Rev.Lett. 77 2887-2890* (1996)

Motivation for Leptogenesis

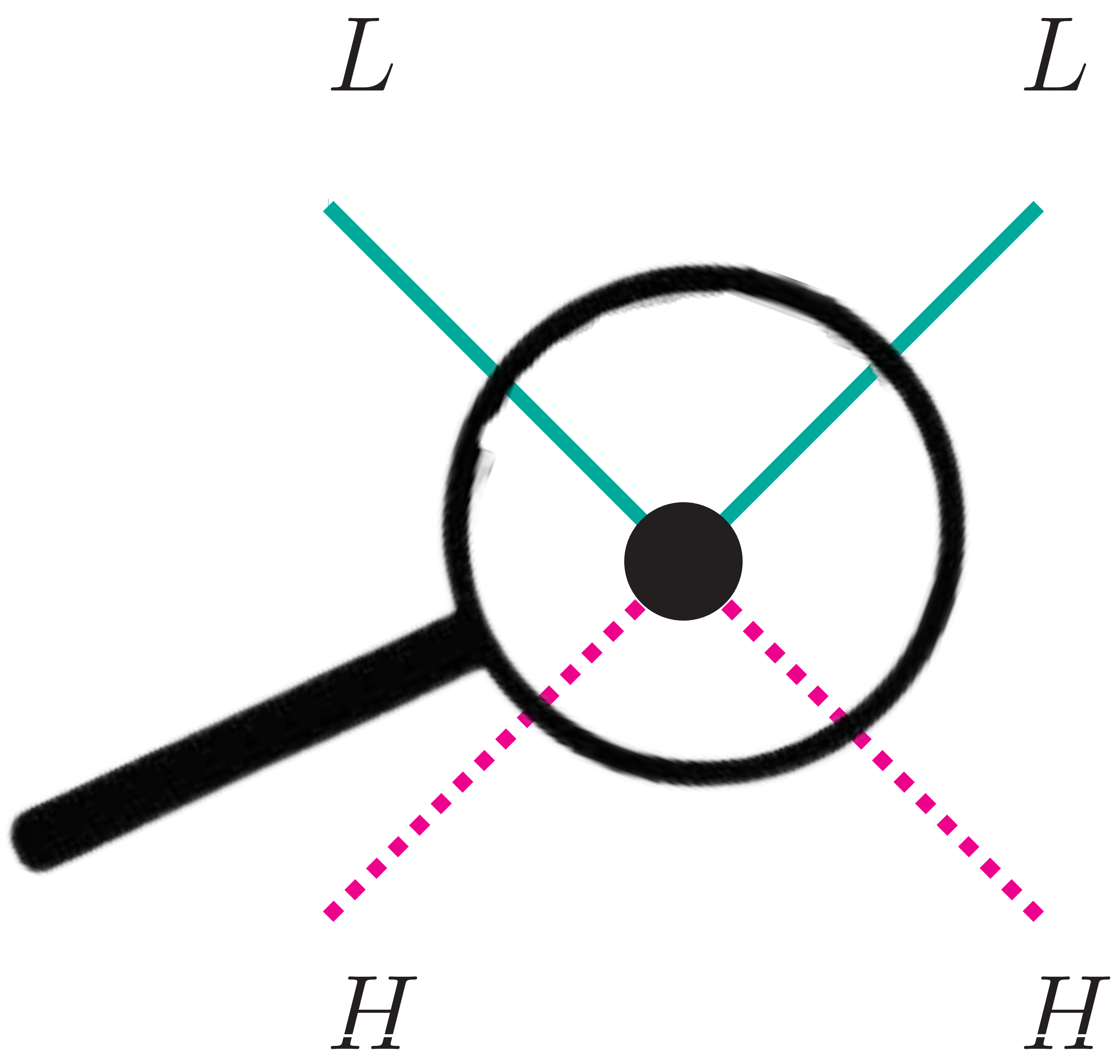


Seesaw Mechanism

Standard Model is an effective theory which contains non-renormalisable operators

Weinberg, *Phys.Rev.Lett. 43* (1979)

$$\mathcal{L} \supset -Y_{ij}\frac{L^i H L^j H}{2M} + \mathcal{O}\left(\frac{1}{M^2}\right) + \mathrm{h.c}$$



Seesaw Mechanism

After SSB a Majorana mass is produced for the active neutrinos

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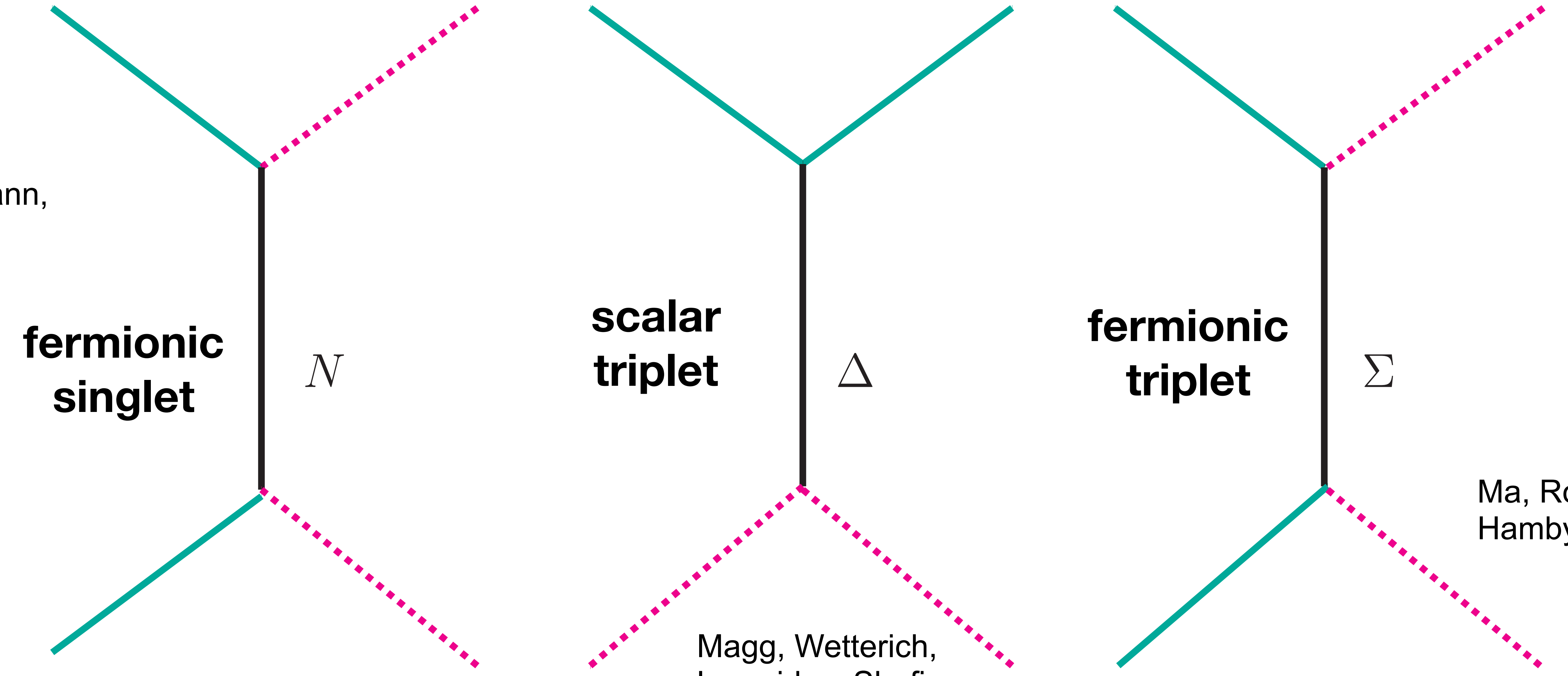
Minkowski, Yanagida, Glashow, Gell-Mann,
Ramond, Slansky,
Mohapatra, Senjanovic



Ma, Roy, Senjanovic,
Hambye

Magg, Wetterich,
Lazarides, Shafi.
Mohapatra,
Senjanovic,
Schecter, Valle

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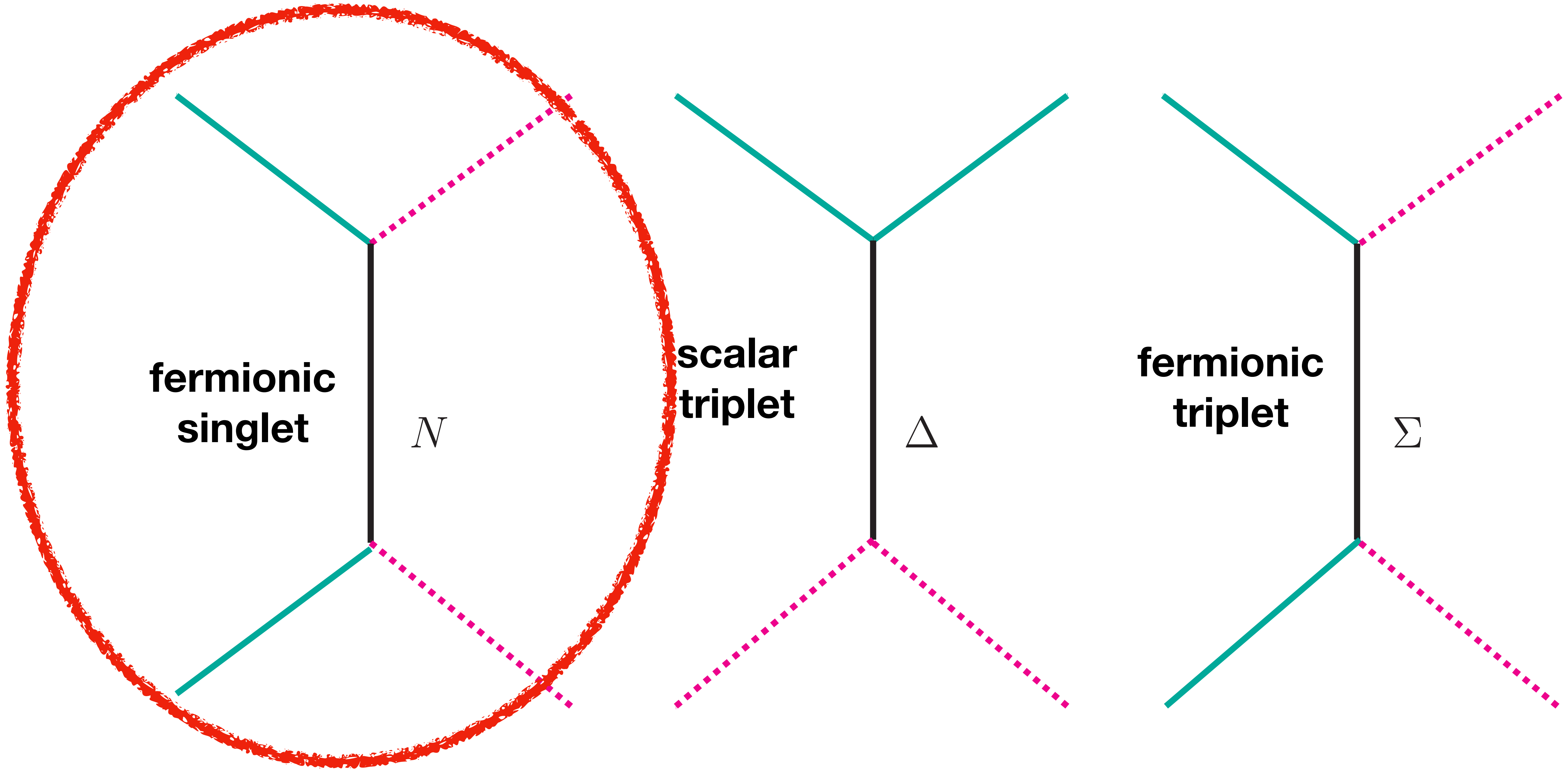

Seesaw Mechanism

$$\mathcal{L} \supset -\overline{L_\alpha} Y_{\alpha i} N_i \tilde{H} - \frac{1}{2}\overline{N_i^C} M_{N_i} N_i + \text{h.c.}$$

After diagonalising the mass matrix

$$m_\nu \approx \frac{m_D m_D^T}{M_N} = \frac{Y^2 v^2}{M_N}$$

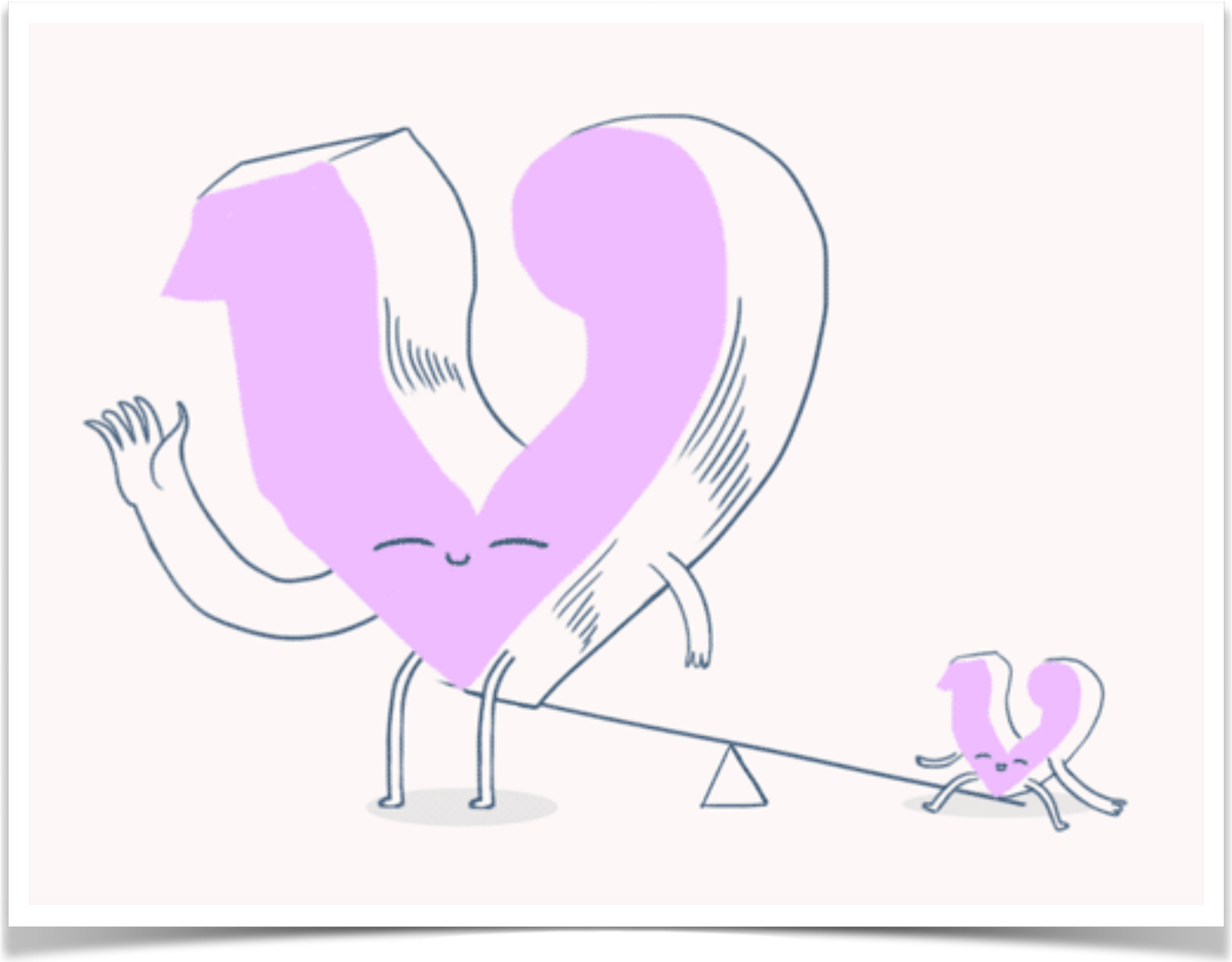

Image courtesy of Symmetry Magazine

Sakharov's Conditions

- ☑ Baryon number violation
- ☑ C & CP-violation
- ☑ Departure from thermal equilibrium

Mass RHN

$\mathcal{O}(10^{12})$ GeV

Fukugida & Yanagida *Phys.Lett. B17 45-47* (1986) Buchmuller, Di Bari & Plumacher *New J.Phys. 6 105* (2004)
Barbieri, Creminelli, Strumia & Tetradis *Nucl.Phys. B575 61-77* (2000)

high-scale leptogenesis

Leptogenesis: dynamical generation of lepton asymmetry. Electroweak sphaleron: lepton →baryon asymmetry

$\mathcal{O}(10^{6})$ GeV

Racker, Rius & Pena *JCAP 1207 030* (2013) Moffat, Petcov, Pascoli, Schulz & Turner *Phys.Rev. D98 no.1, 015036* (2018)

intermediate scale leptogenesis

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Pilaftis & Underwood *Nucl.Phys. B692 303-345* (2004) Abada, Aissaoui, Losada *Nucl.Phys. B728 55-66* (2005)

resonant leptogenesis

Need to Boltzmann equations which track the time evolution of the RHN and lepton asymmetry

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Akhmedov, Rubakov & Smirnov *Phys.Rev.Lett. 81 1359-1362* (1998) Asaka & Shaposhnikov *Phys.Lett. B620 17-26* (2005)
Asaka, Eijima & Ishida *JHEP 1104 011*(2011)

leptogenesis via oscillations

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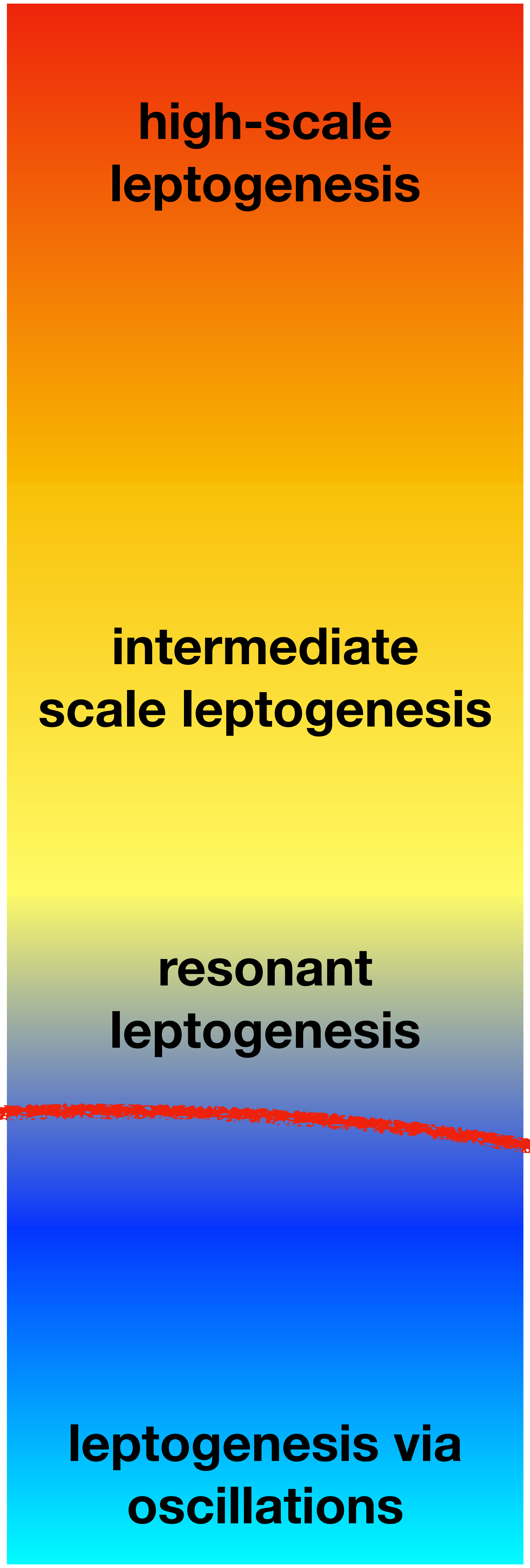
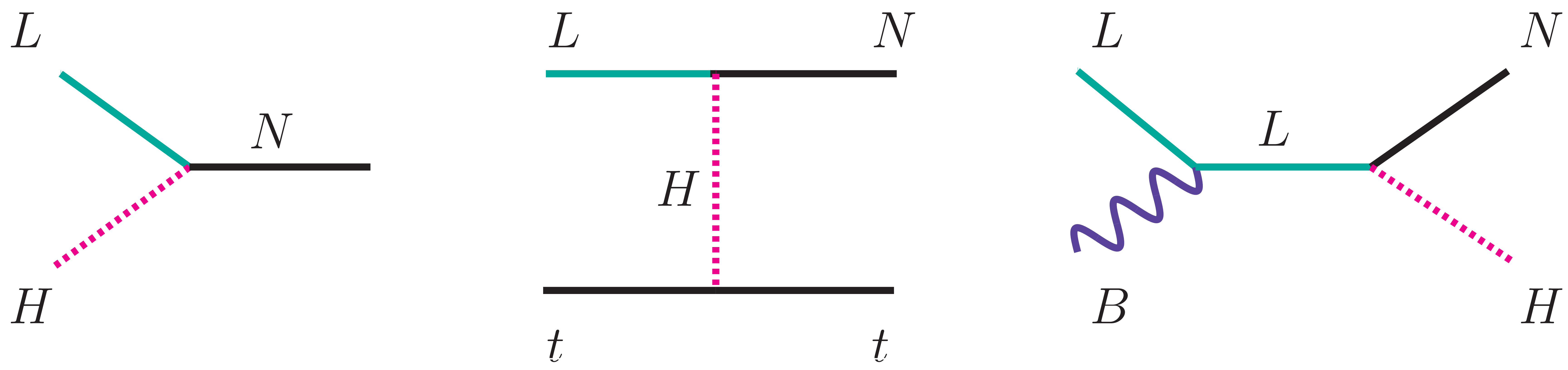


Leptogenesis: dynamical generation of lepton asymmetry. Electroweak sphaleron: lepton →baryon asymmetry

Need to Boltzmann equations which track the time evolution of the RHN and lepton asymmetry

Leptogenesis via oscillations

- highly degenerate RHNs produced via scattering at $T > T_{EW}$



Akhmedov, Rubakov & Smirnov (1998)

- small Yukawa couplings → RHNs may not have equilibrated by the EWPT

- RHNs CP-violating oscillations → source of lepton number and flavour asymmetry.

Leptogenesis via oscillations 2RHN

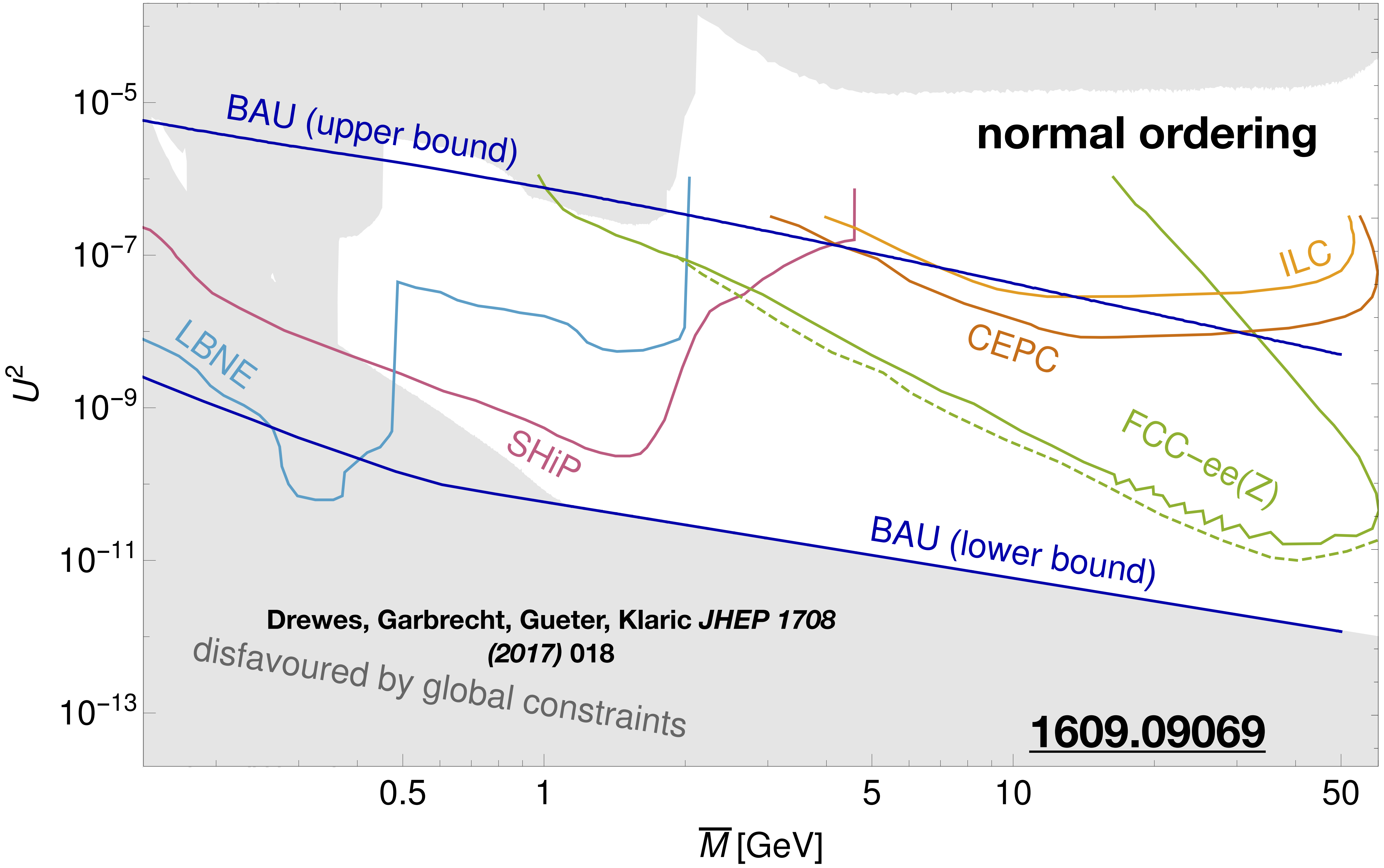
- GeV-scale RHNs → rich phenomenology

Casas & Ibarra, *Nucl.Phys. B618 (2001) 171-204*

$$Y = \frac{1}{v} U \sqrt{m} R^T \sqrt{M}$$

4 masses, 4 angles, 3 phases (2 masses + 3 angles measured)

$$\nu_\alpha = U_{\alpha i}\nu_i + \Theta_{\alpha I} N_I^c \qquad |U|^2 = \sum_{\alpha I} |\Theta_{\alpha I}|^2 \qquad \overline{M} = \frac{M_1 + M_2}{2}$$



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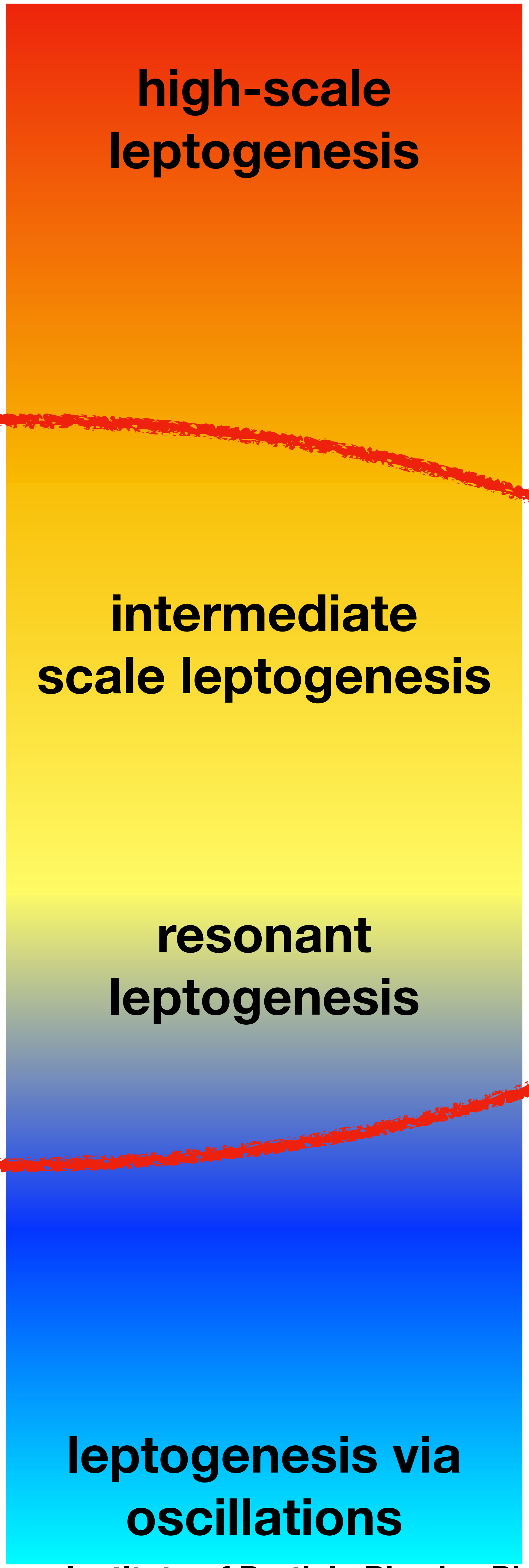
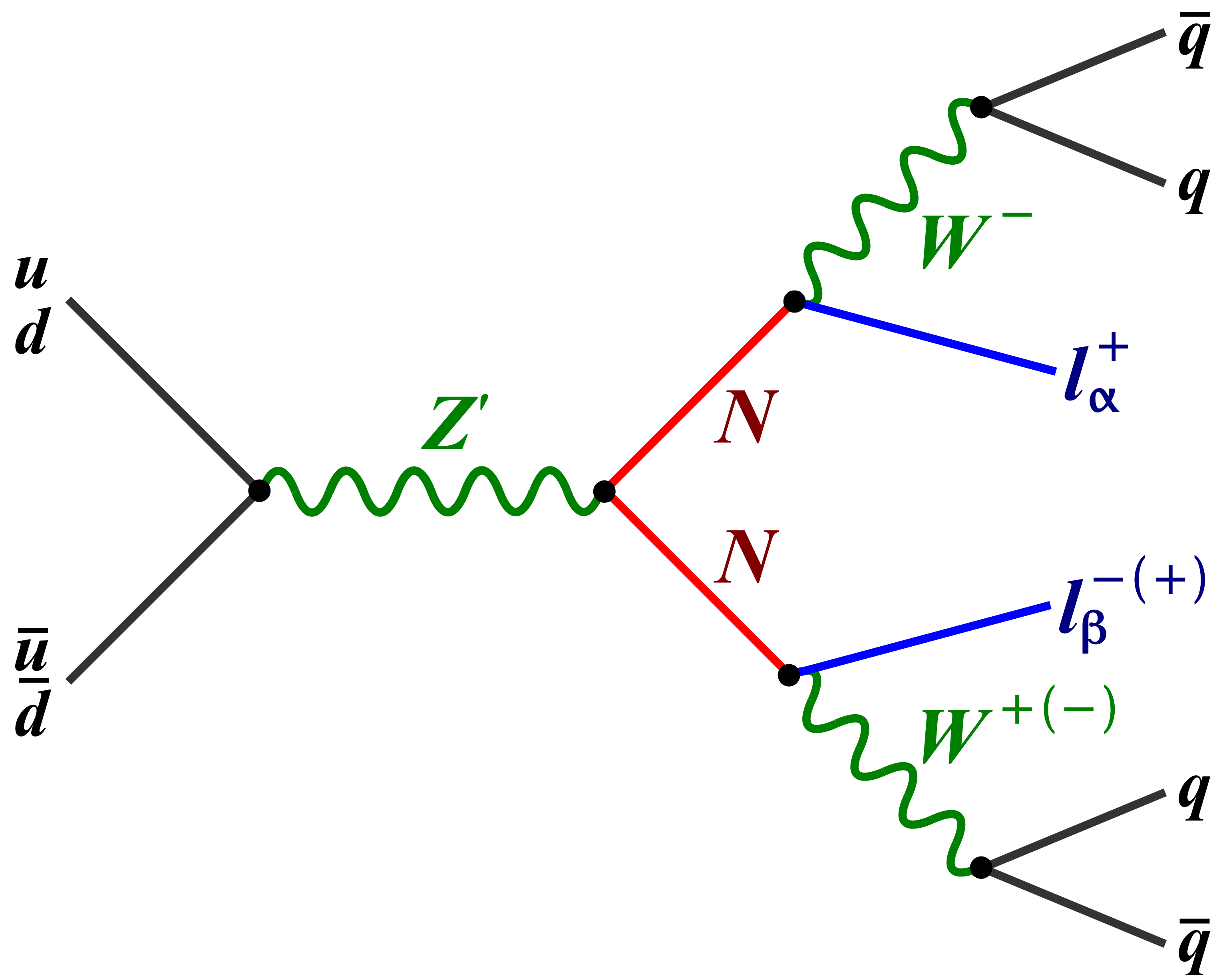
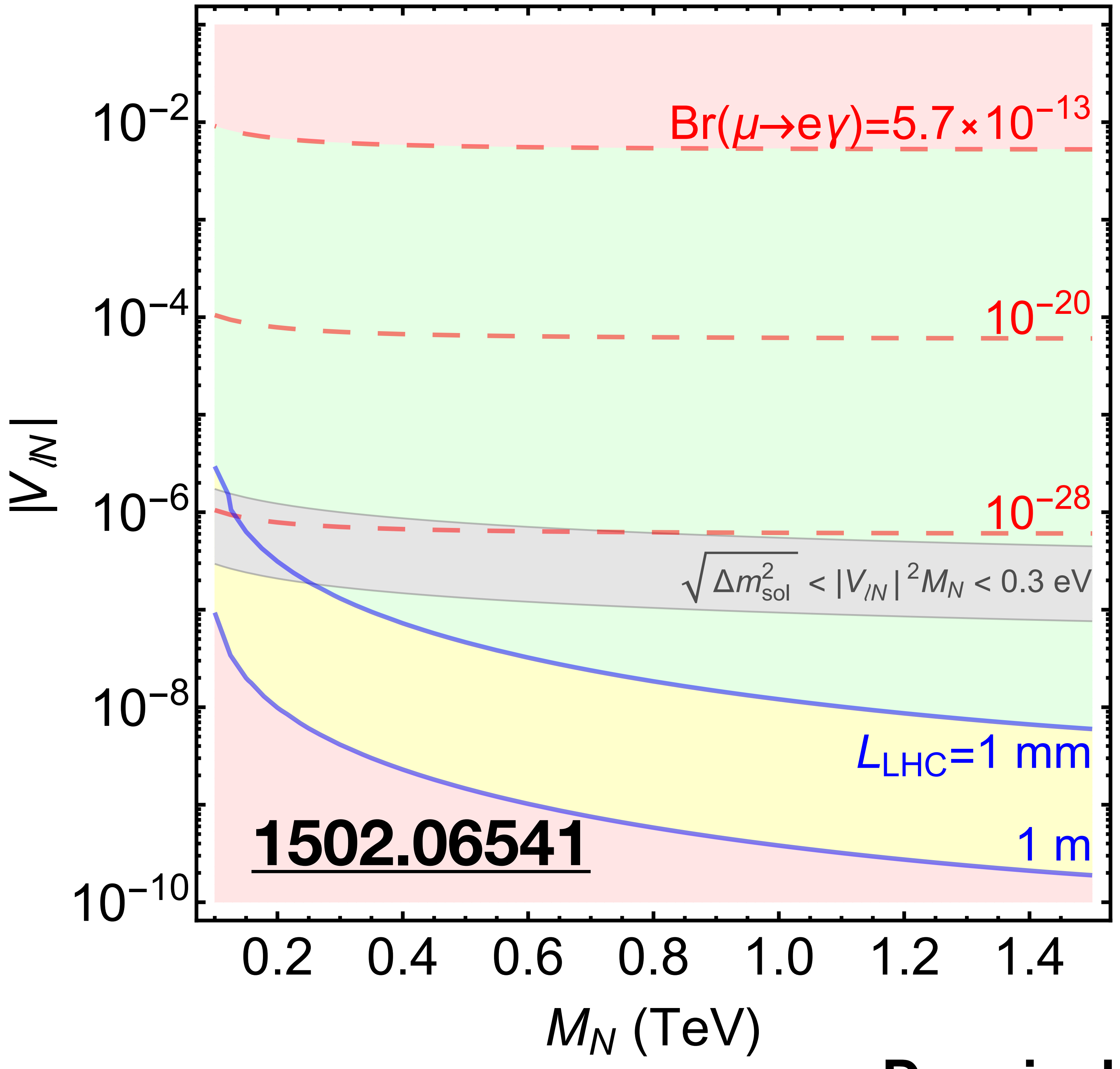


Resonant Leptogenesis

Pilaftis & Underwood *Nucl.Phys. B692 303-345*(2004) Abada, Aissaoui, Losada *Nucl.Phys. B728 55-66* (2005)

- RHNs decay width similar to their mass differences. Mass range ~ TeV
- RHN masses explained by additional $U(1)_{B-L}$ symmetry and can be sufficiently long-lived → displaced-vertex signature searched for at LHC, MATHUSLA or SHiP.





Deppisch, Dev & Pilaftsis *New J.Phys. 17 no.7, 075019* (2015)
Helo, Kovalenko & Hirsch *Phys.Rev. D89 073005* (2014)
Gago, Hernández, Jones-Pérez, Losada & Briceño *Nucl.Part.Phys.Proc. 273-275 2693-2695* (2016)
Antusch, Cazzato & Fischer *JHEP 1612 007* (2016)

Deppisch, Dev, Pilaftsis, New J.Phys. 17 (2015) no.7, 075019

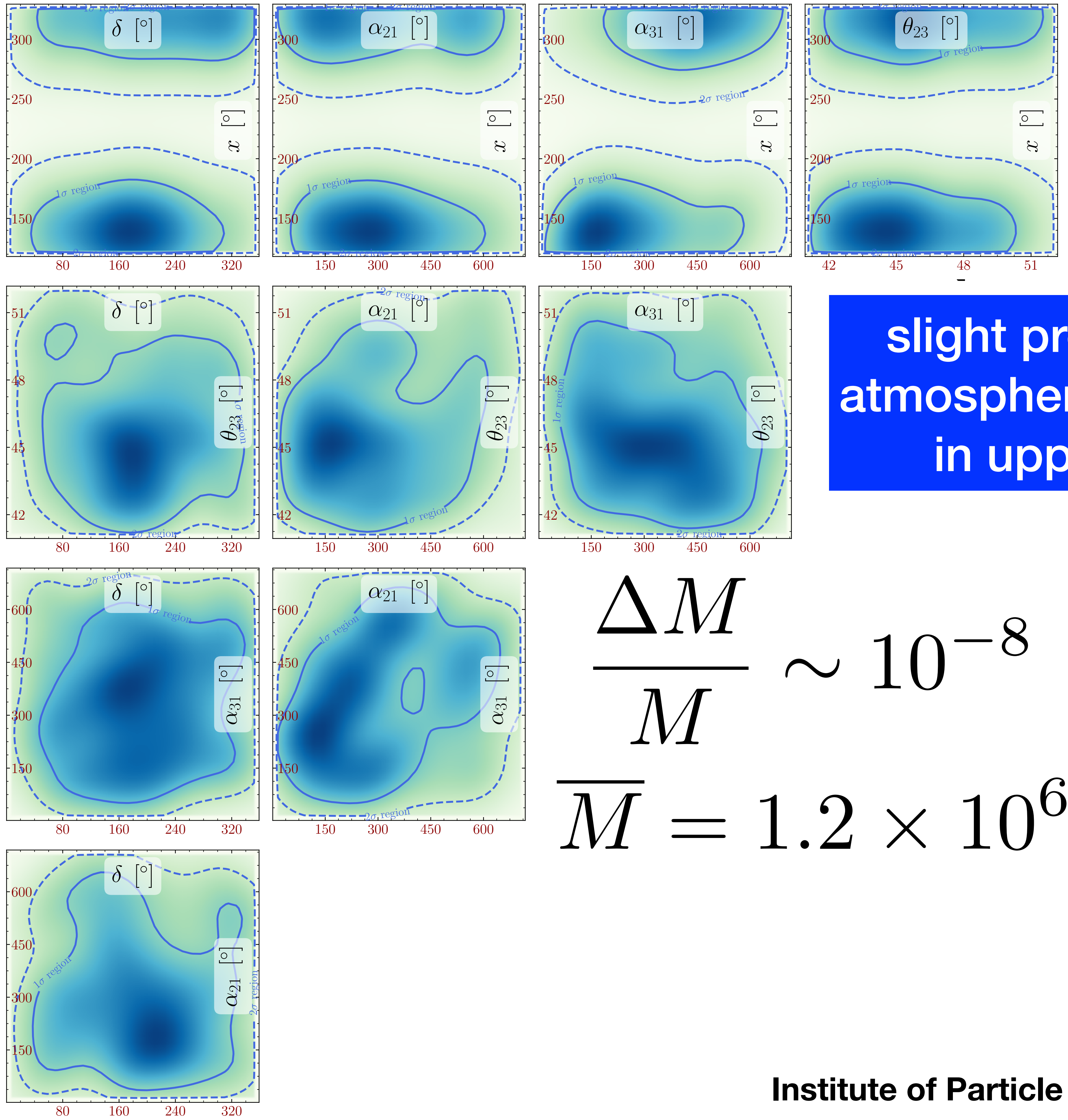
Resonant Leptogenesis in the Neutrino Option

- Assume Higgs potential vanishes at M
- Integrate out TeV RHN and RG evolve: Higgs potential produced for M ~ 10^3 TeV

Brdar, Hemboldt, Iwamoto, Schmitz *Phys.Rev. D100 075029* (2019)
Brivio, Moffat, Pascoli, Petcov, Turner *JHEP 1910 059* (2019)

Brivio et al
1905.12642

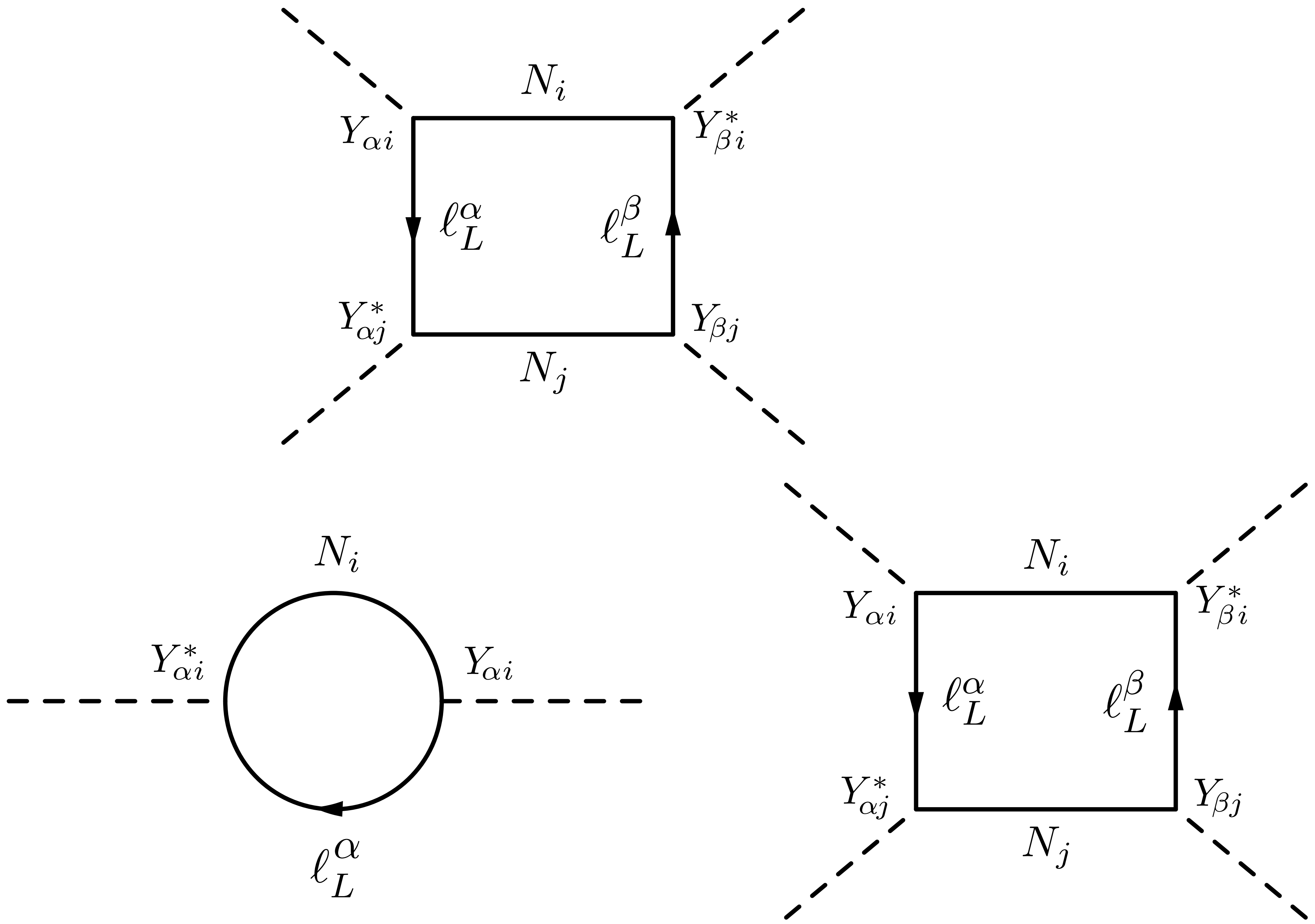
Normal Ordering



slight preference for atmospheric angle to be in upper octant.

$$\frac{\Delta M}{M} \sim 10^{-8}$$

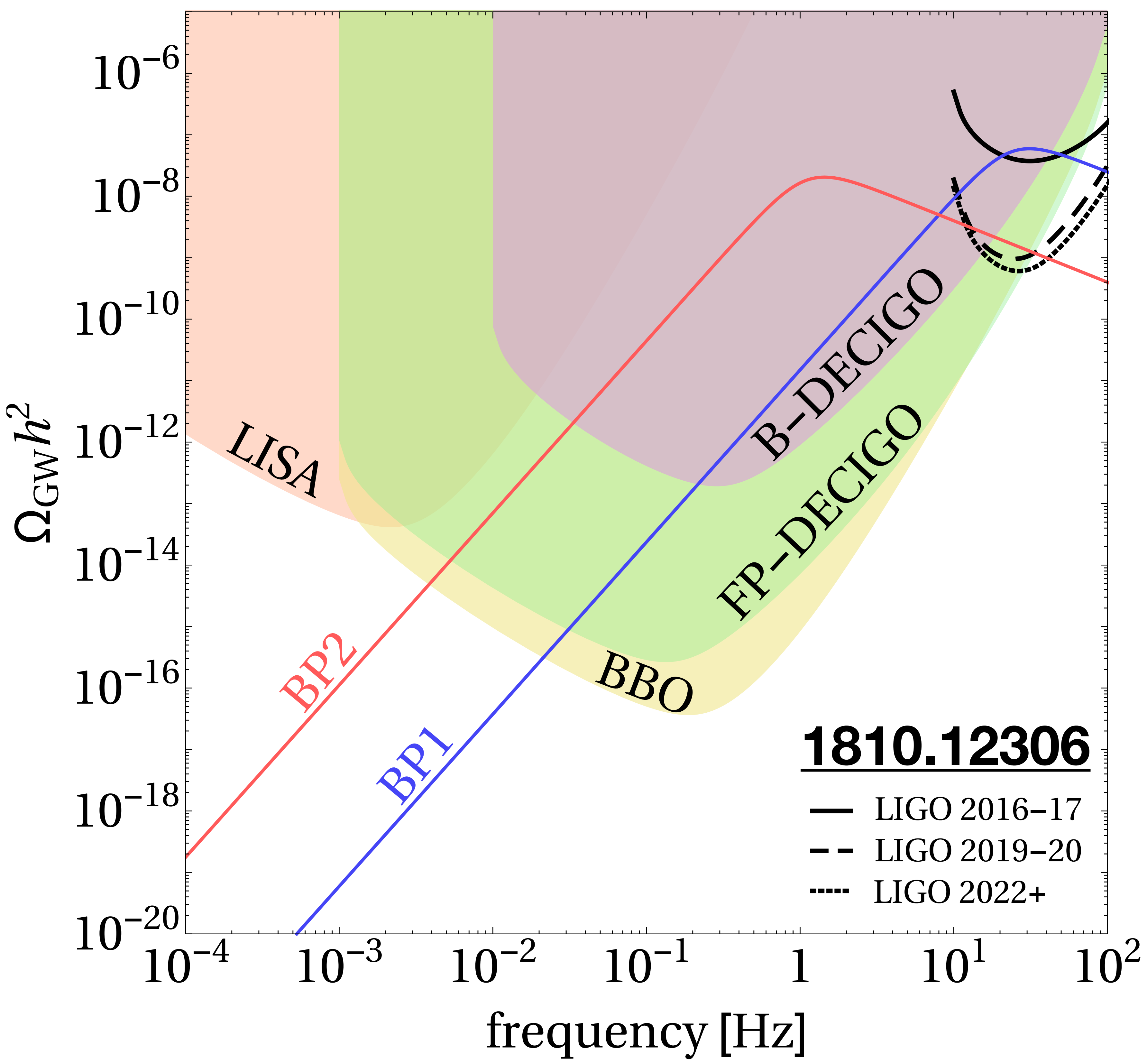
$$\overline{M} = 1.2 \times 10^{6}\ \mathrm{GeV}$$



Scale invariance broken at quantum level

Resonant Leptogenesis in the Neutrino Option

- UV-completion of Neutrino Option (Brdar, Emonds, Helmboldt, Lindner) minimal renormalisable model based on classical scale invariance
- New scalar breaks scale-invariance → generates mass for RHNs and strong first order phase transition



Brdar, Emonds, Helmboldt, Lindner
Phys.Rev. D99 (2019) no.5, 055014

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intermediate scale leptogenesis

Difficult to test as RHNs very heavy however gravitational waves offer an additional telescope on high-scale leptogenesis

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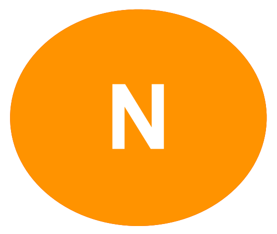
leptogenesis via oscillations

Thermal leptogenesis

Fukugida, Yanagida



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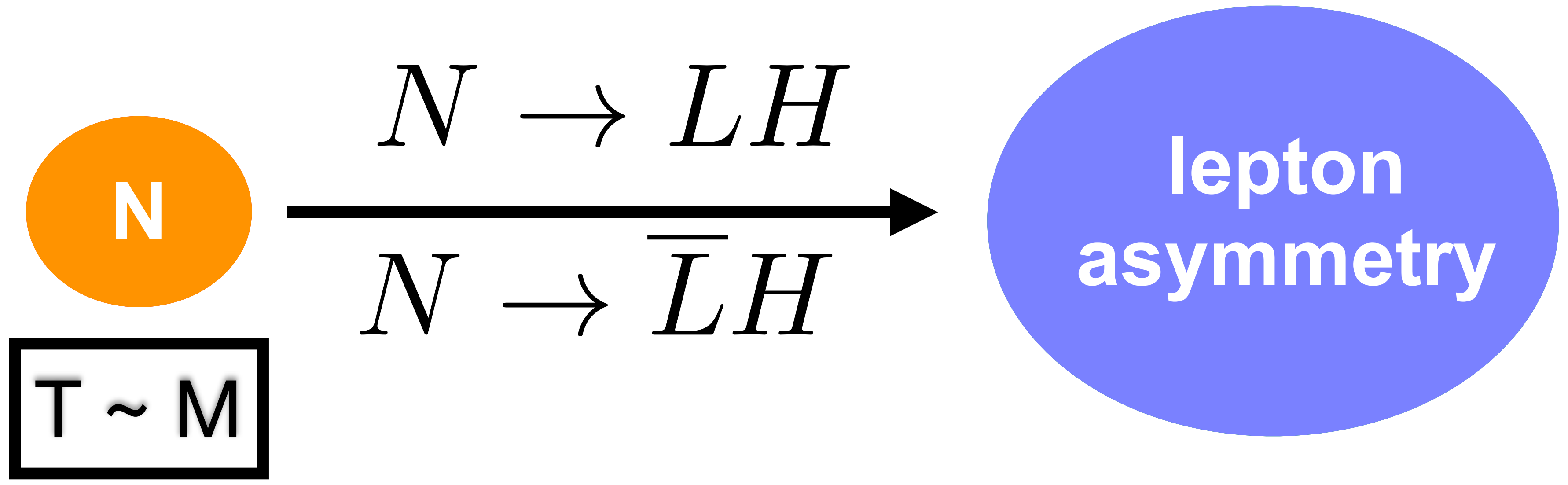

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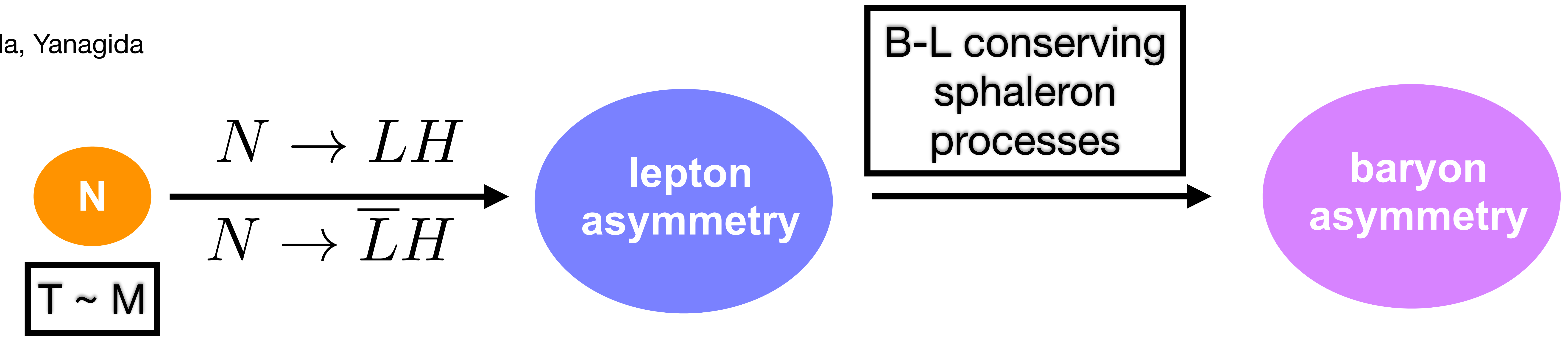

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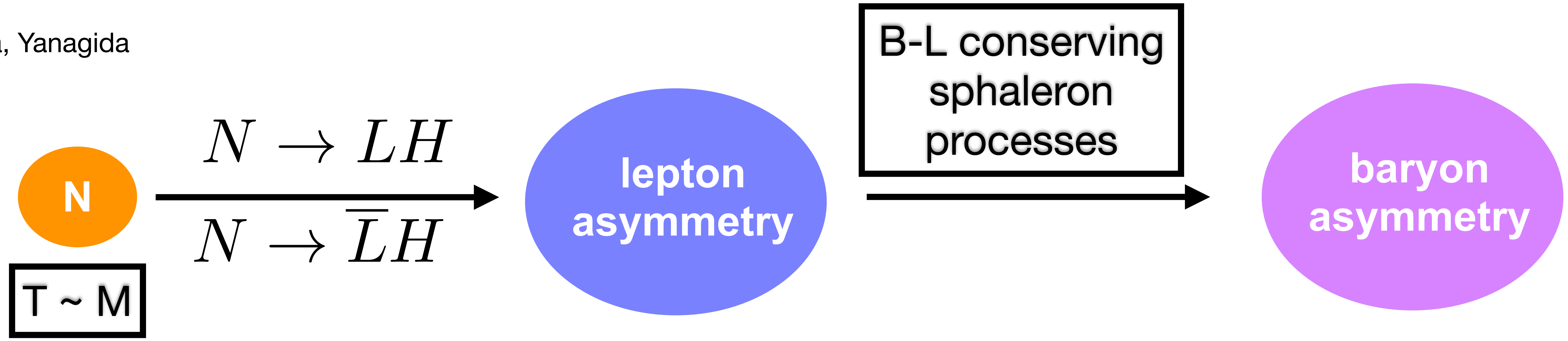
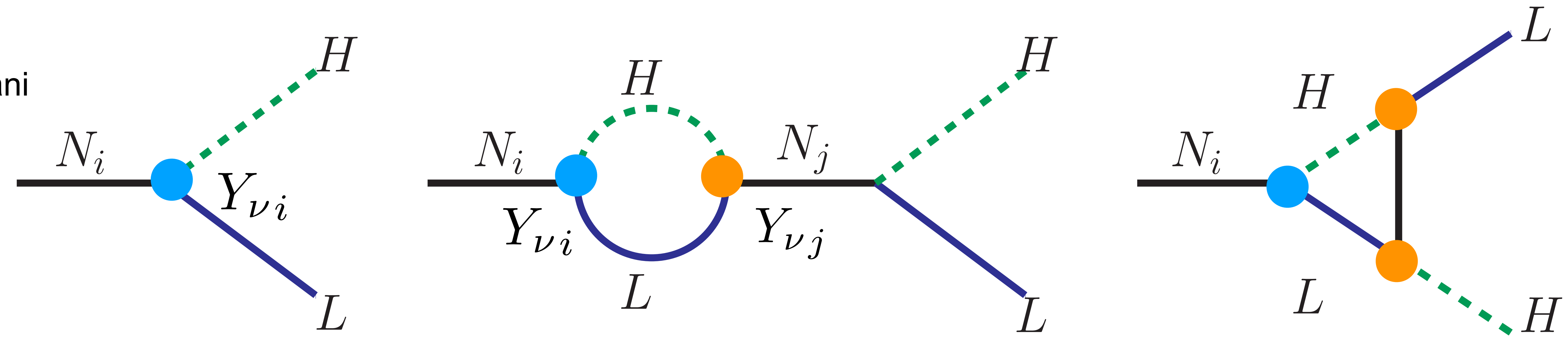


Decay asymmetry from interference between tree and loop level diagrams

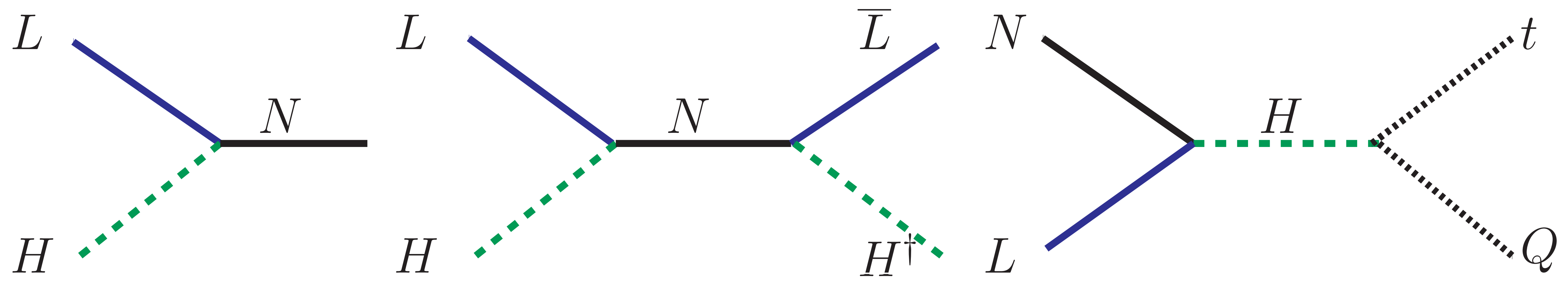
Covi, Roulet, Vissani



$$\epsilon_i = \frac{\Gamma_i - \overline{\Gamma}_i}{\Gamma_i + \overline{\Gamma}_i}$$

Thermal leptogenesis

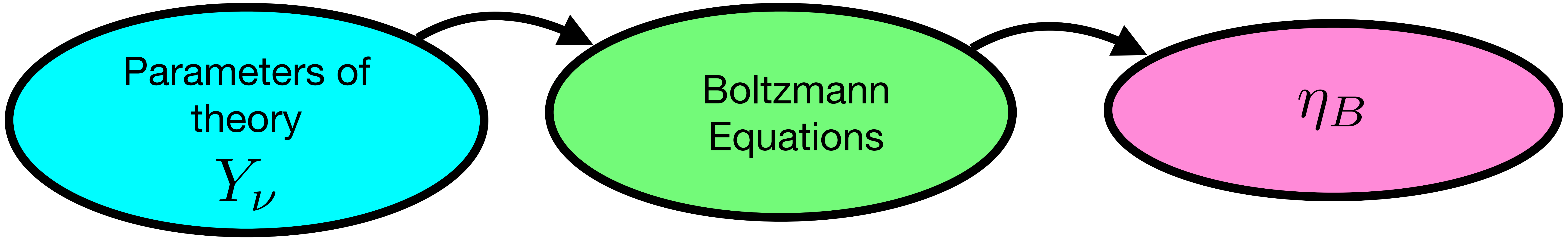
Washout and scattering processes



inverse Decays

ΔL=2

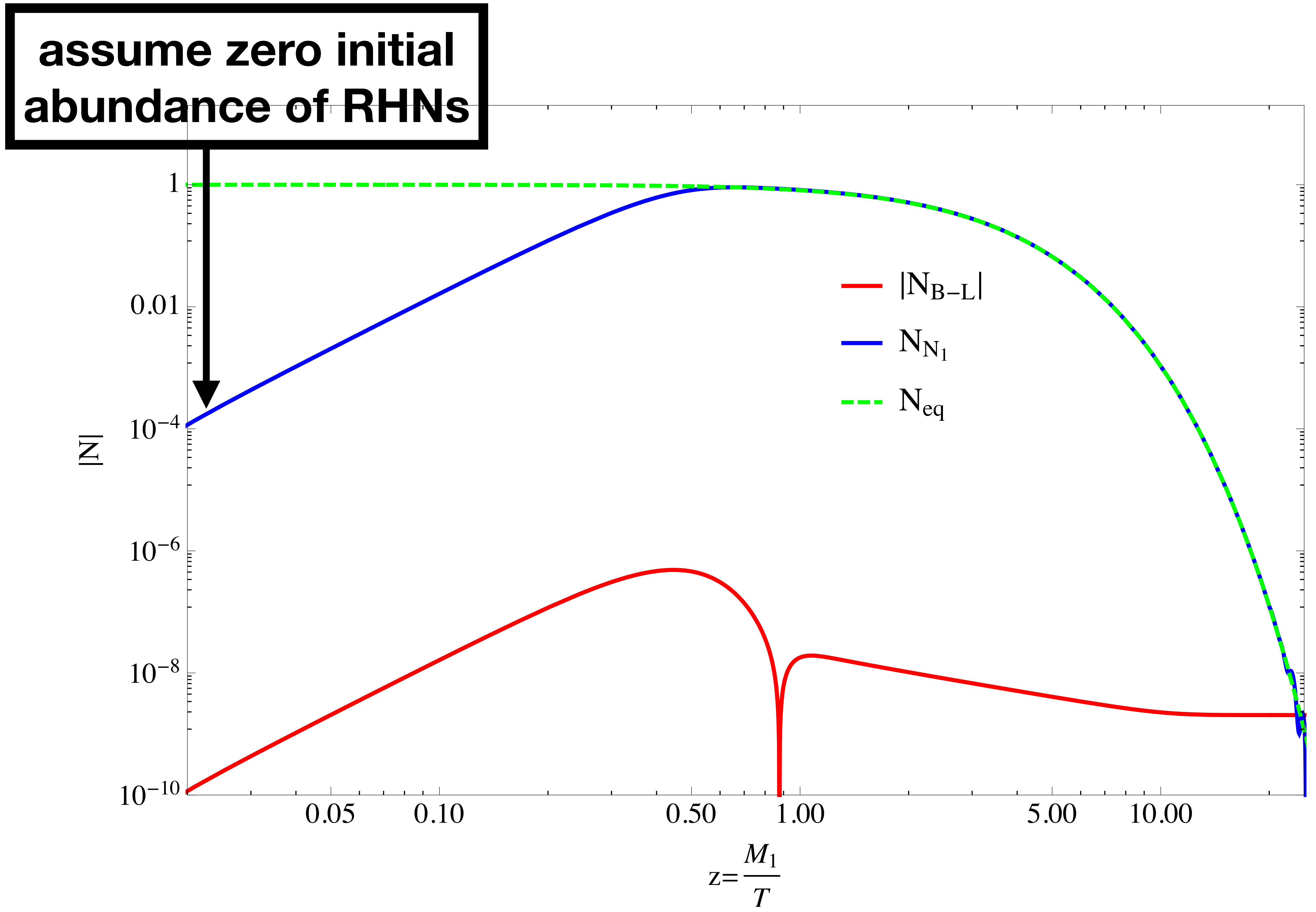
ΔL=1

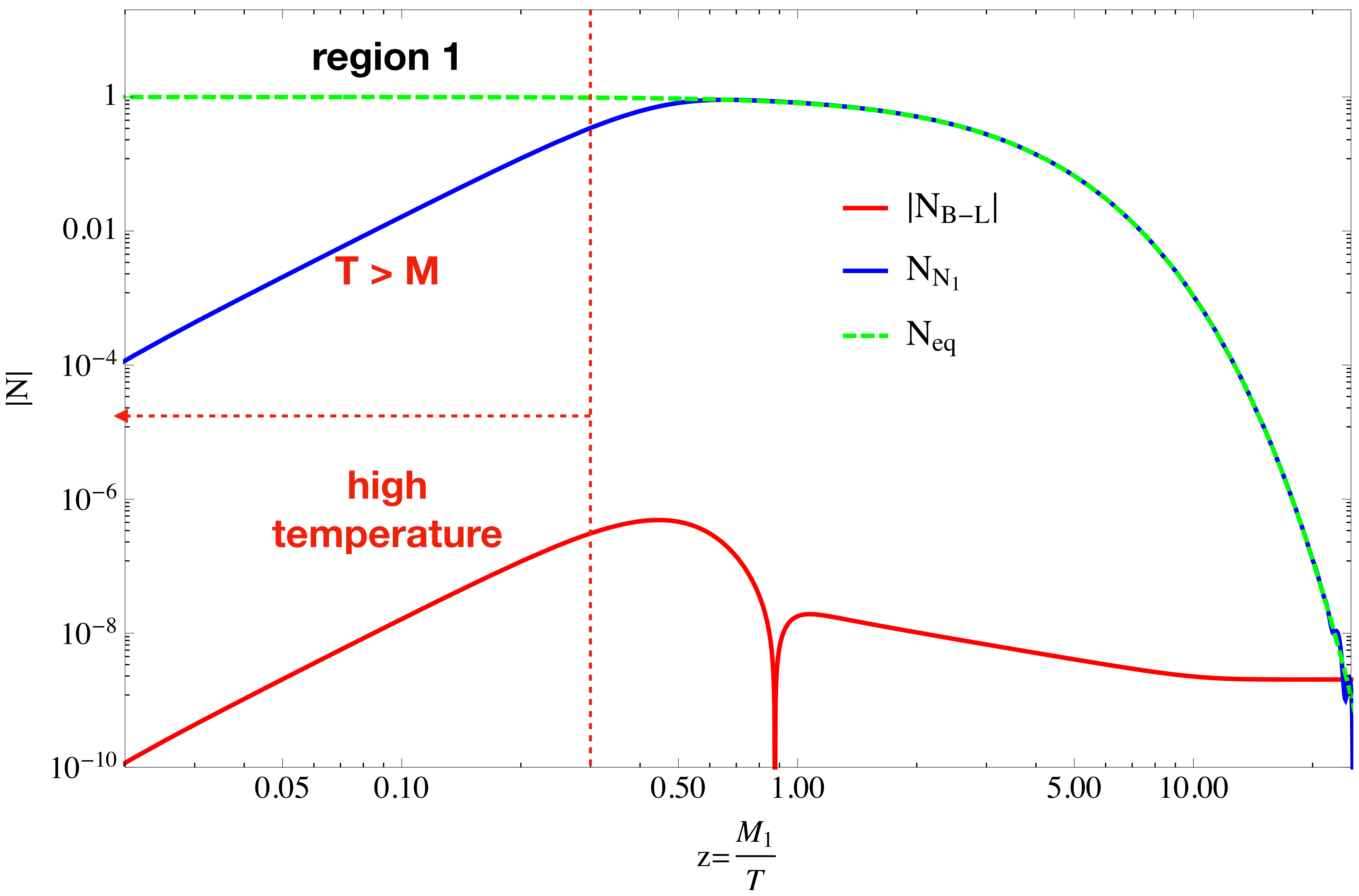


$$\frac{dn_{N_i}}{dz} = -\,D_i(n_{N_i} - n_{N_i}^{\rm eq}),$$

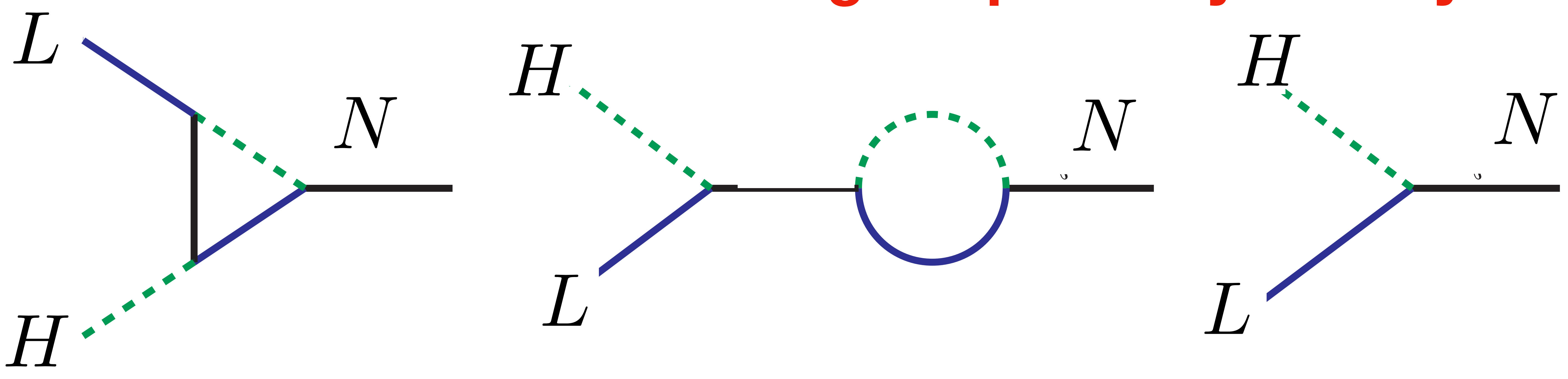
source **sink**

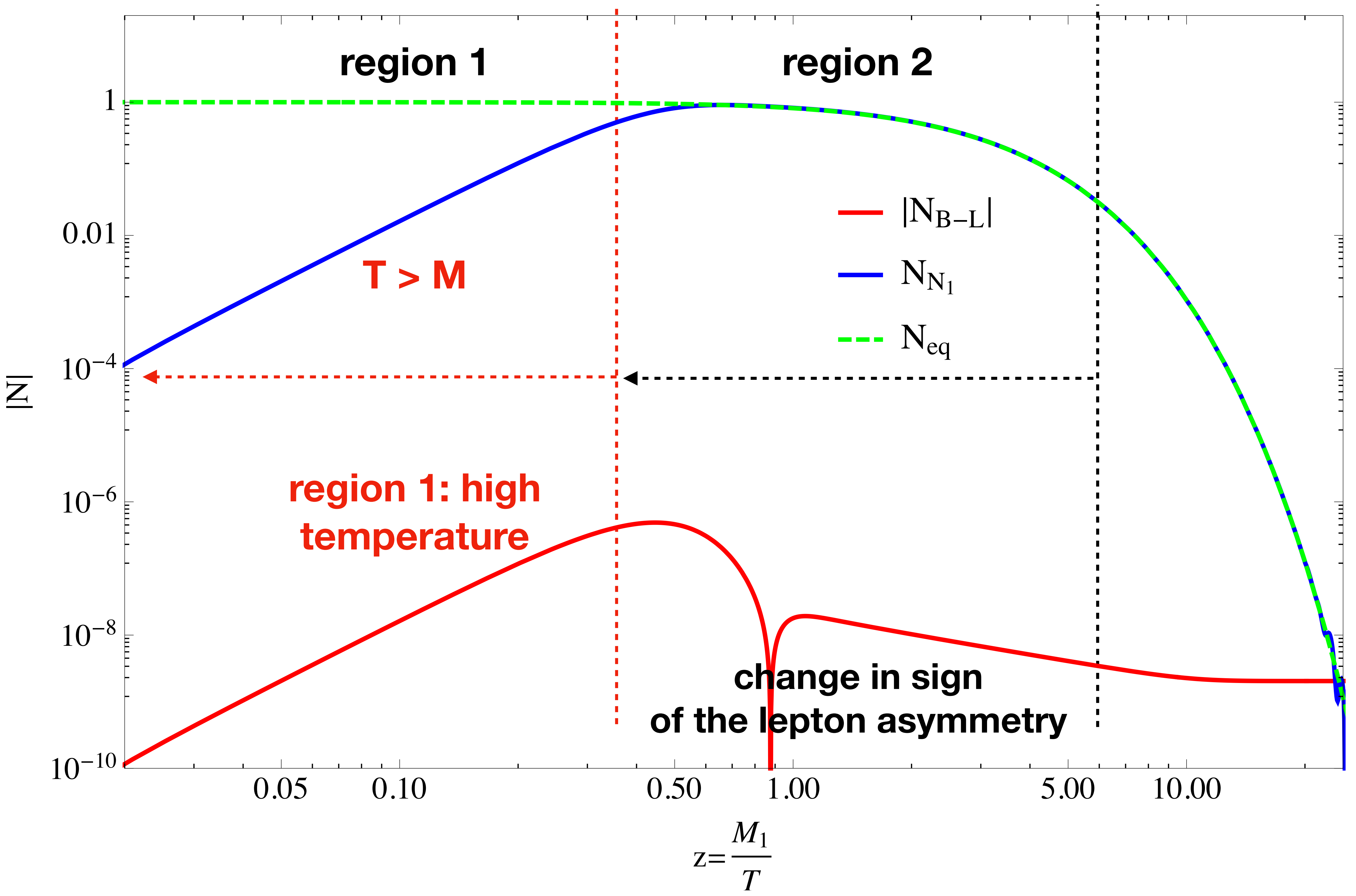
$$\frac{dn_{B-L}}{dz} = \sum_{i=1}^{3}\left(\epsilon^{(i)} D_i(n_{N_i} - n_{N_i}^{\rm eq}) - W_i n_{B-L}\right).$$

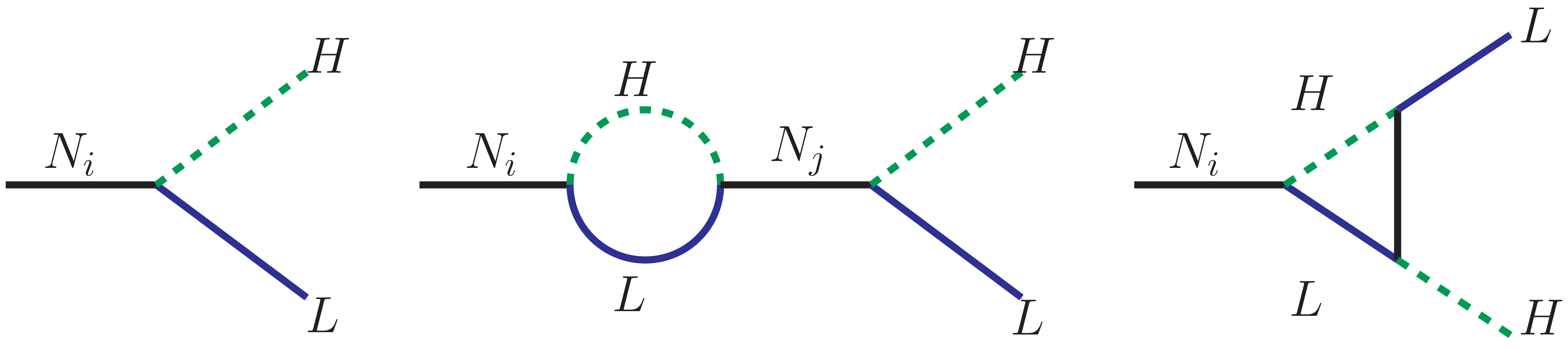

assume zero initial abundance of RHNs
1
0.01
10^{-4}
10^{-6}
10^{-8}
10^{-10}
|N|
0.05
0.10
0.50
1.00
5.00
10.00
$z=\frac{M_1}{T}$
$|N_{B-L}|$
N_{N_1}
N_{eq}

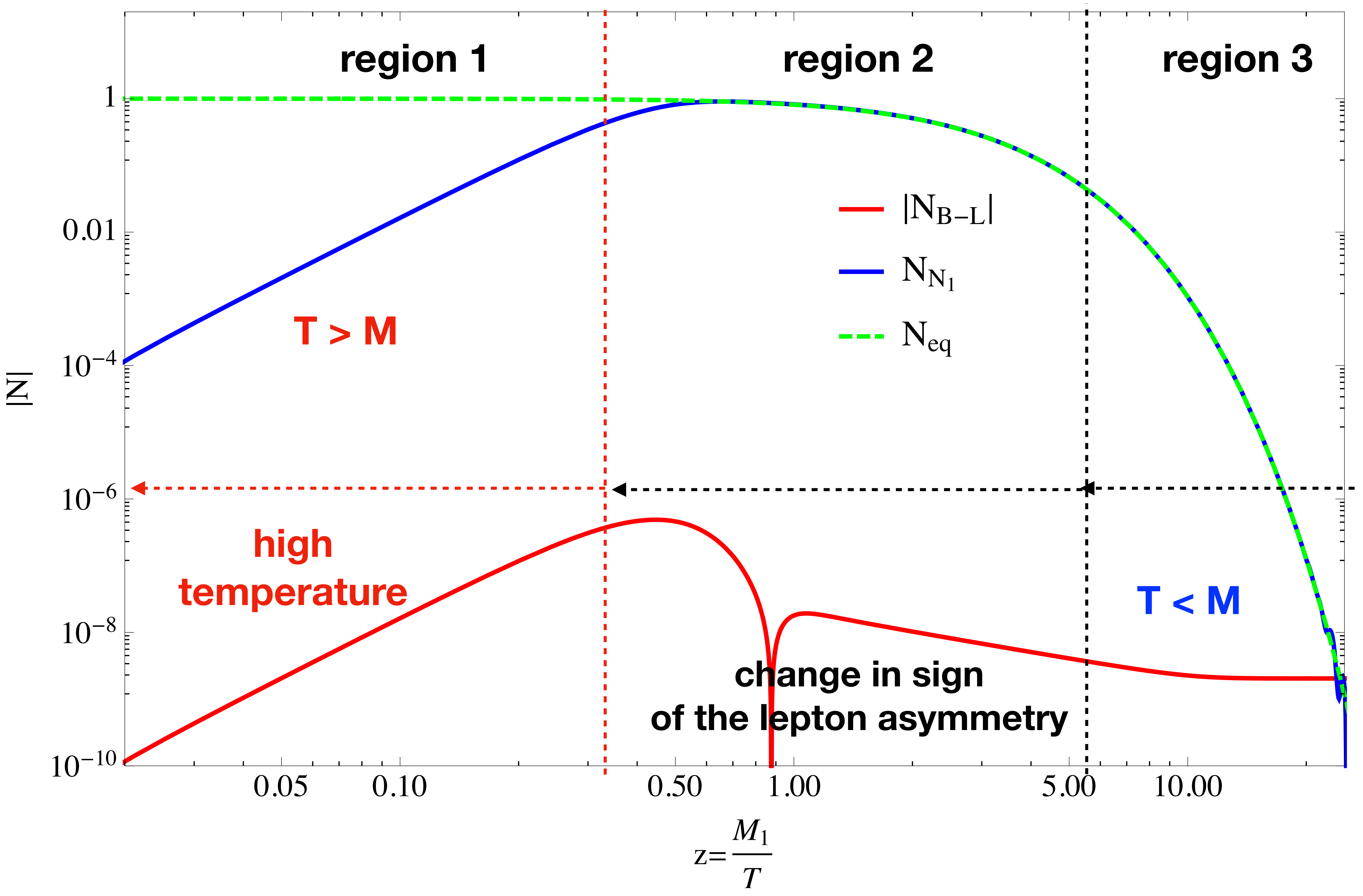


Region 1: leptons and Higgs have enough energy to inverse decay creating a lepton asymmetry




region 1
region 2
1
0.01
10^{-4}
10^{-6}
10^{-8}
10^{-10}
|N|
T > M
$|N_{B-L}|$
N_{N_1}
N_{eq}
region 1: high temperature
change in sign of the lepton asymmetry
0.05
0.10
0.50
1.00
5.00
10.00
$z=\frac{M_1}{T}$


N_i
H
L
N_i
H
L
N_j
H
L
N_i
H
L
L
H



<u>Region 3:</u> At T < M, RHN abundance is depleted. Lepton asymmetry freezes out.

Primordial Black holes induced leptogenesis

Work in collaboration with **Yuber Perez Gonzalez**: 2010.03565

Astrophysical BHs require $M > 3M_\odot$

For smaller BH mass (between Planck and solar mass scale) require large perturbations in the early Universe : **bubble collision, collapse of density perturbations…**

Carr et al, 0912.5297

Hawking, 1975

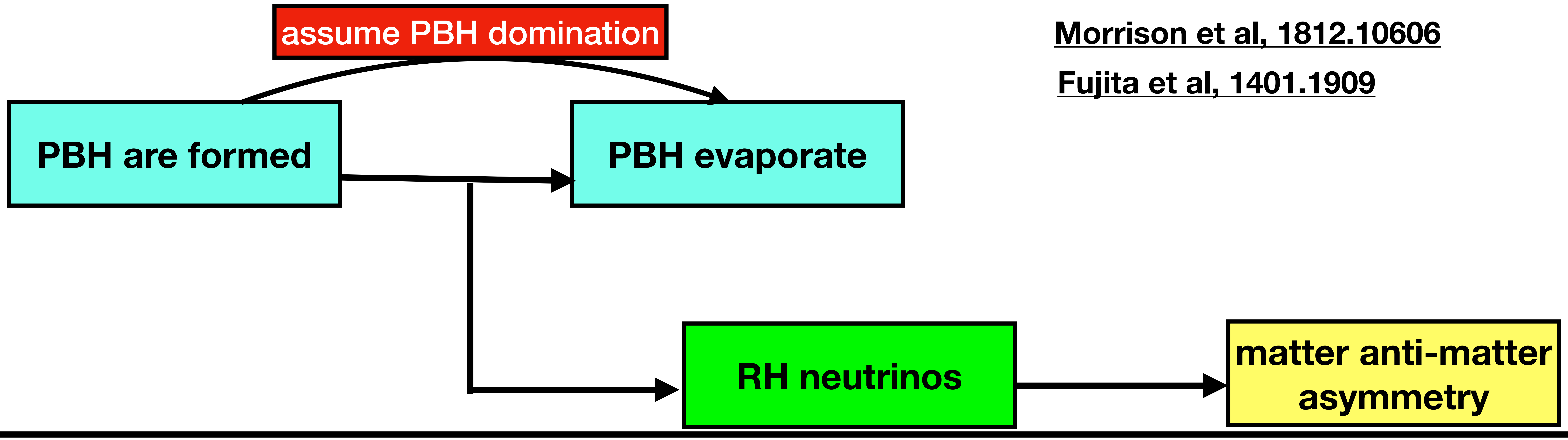
$r_S \sim \lambda_C \longrightarrow$ PBHs evaporate by emitting particles

$$\frac{dM}{dt} = -\sum_a \frac{g_a}{2\pi^2}\int_0^\infty \frac{\sigma_{\rm abs}^{s_a}(GMp)\, p^3\, dp}{\exp[E_a(p)/T_{\rm BH}] - (-1)^{2s_a}}$$

$$T_{\rm BH} = \frac{1}{8\pi GM} \approx 1.06 \left(\frac{10^{13}\ {\rm g}}{M}\right)\ {\rm GeV}\,.$$

PBHs are totally indiscriminate in their particle production: just need T_{BH} to be close to particle mass

Primordial Black holes induced leptogenesis



Morrison et al, 1812.10606

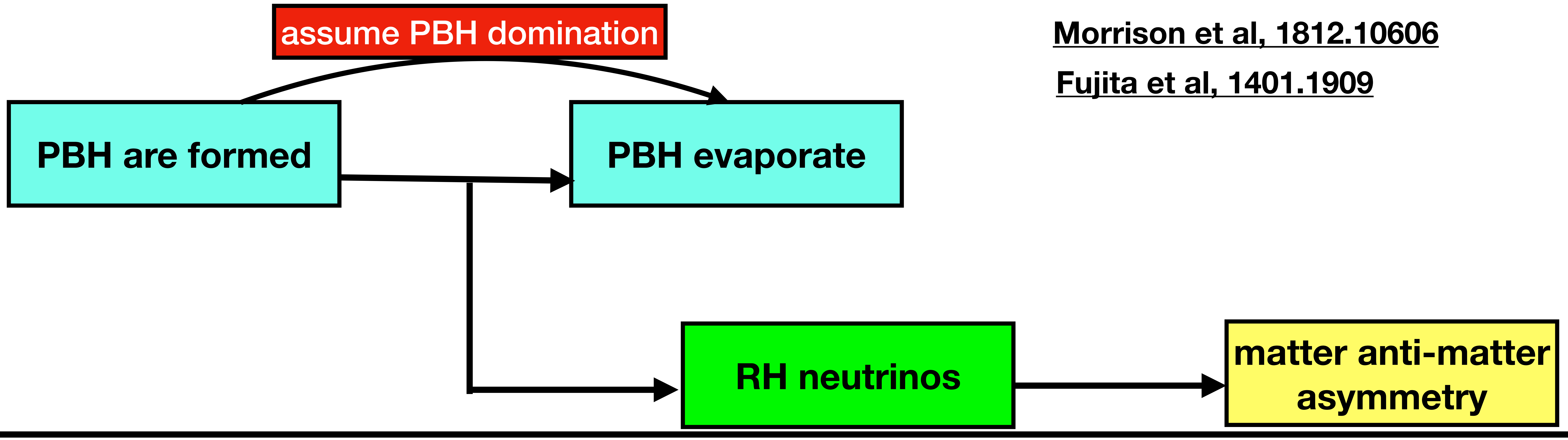
Fujita et al, 1401.1909

$$z_{\mathrm{BH}} = \frac{M_N}{T_{\mathrm{BH}}}$$

$$aH\frac{dn_{N_1}}{da} = -(n_{N_1} - n_{N_1}^{\mathrm{eq}})\Gamma_{N_1}^{T} + n_{\mathrm{BH}}\widetilde{\Gamma}_{N_1}^{\mathrm{BH}} \,,$$

$$aH\frac{dn_{\alpha\beta}^{\mathrm{B-L}}}{da} = \epsilon_{\alpha\beta}^{(1)}\left[\left(n_{N_1} - n_{N_1}^{\mathrm{eq}}\right)\Gamma_{N_1}^{T} + n_{\mathrm{BH}}\widetilde{\Gamma}_{N_1}^{\mathrm{BH}}\right] - \mathcal{W}_{\alpha\beta}$$

Primordial Black holes induced leptogenesis



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Fujita et al, 1401.1909

$$z_{\rm BH} = \frac{M_N}{T_{\rm BH}}$$

contribution to RHN population from thermal plasma

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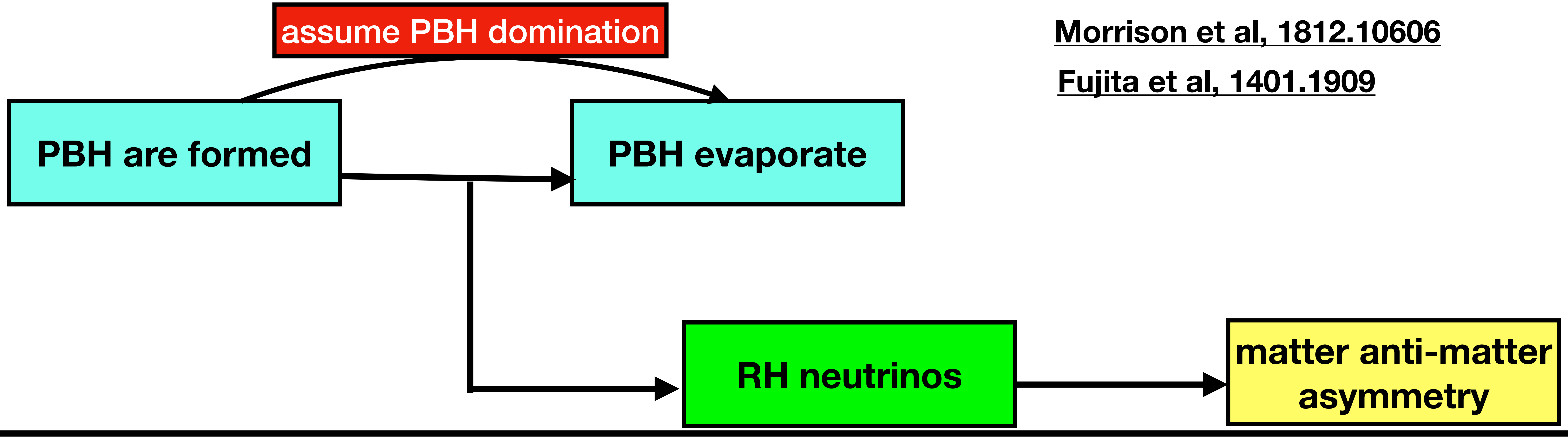
$$aH \frac{dn_{\alpha\beta}^{\rm B-L}}{da} = \epsilon_{\alpha\beta}^{(1)} \left[\left(n_{N_1} - n_{N_1}^{\rm eq} \right) \Gamma_{N_1}^T + n_{\rm BH}\widetilde{\Gamma}_{N_1}^{\rm BH} \right] - \mathcal{W}_{\alpha\beta}$$

Primordial Black holes induced leptogenesis



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contribution to RHN population from PBH evaporation

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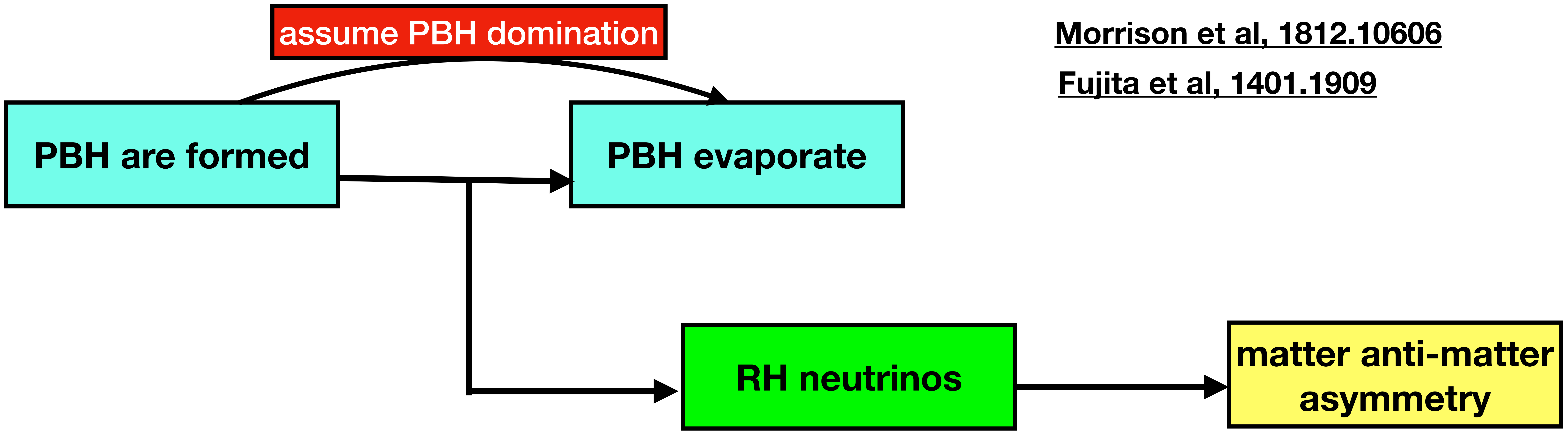
Primordial Black holes induced leptogenesis



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contribution to RHN population from PBH evaporation

lepton asymmetry production from RHN decays and inverse decays

$$aH\frac{dn_{\alpha\beta}^{\rm B-L}}{da} = \epsilon_{\alpha\beta}^{(1)}\left[\left(n_{N_1} - n_{N_1}^{\rm eq}\right)\Gamma_{N_1}^{T} + n_{\rm BH}\widetilde{\Gamma}_{N_1}^{\rm BH}\right] - \mathcal{W}_{\alpha\beta}$$

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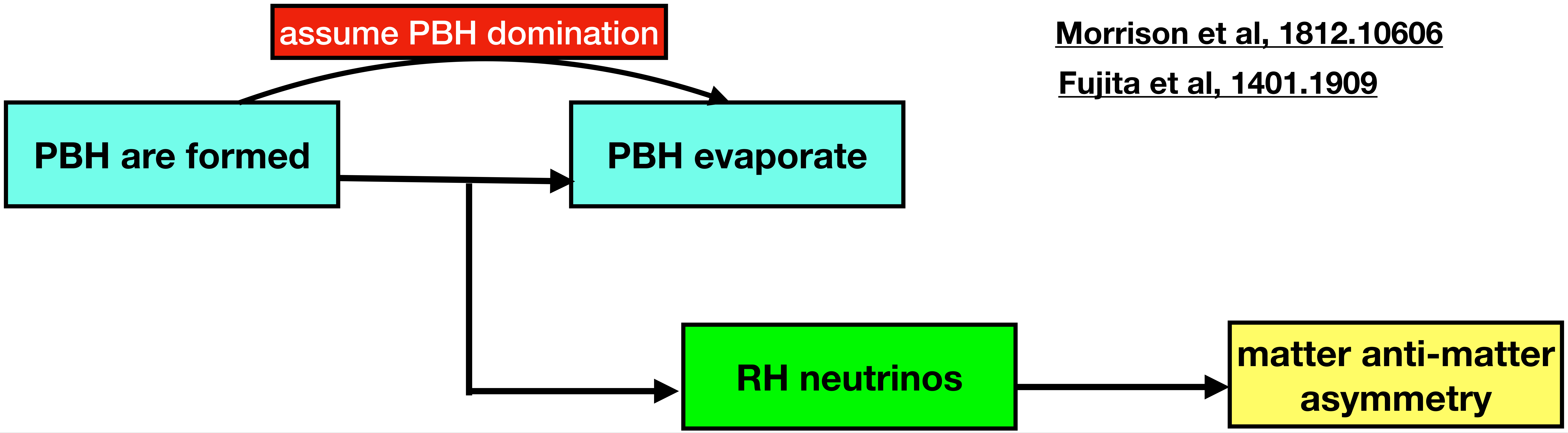

Morrison et al, 1812.10606

Fujita et al, 1401.1909

$$z_{\rm BH} = \frac{M_N}{T_{\rm BH}}$$

contribution to RHN population from thermal plasma

$$aH\frac{dn_{N_1}}{da} = -(n_{N_1} - n_{N_1}^{\rm eq})\Gamma_{N_1}^{T} + n_{\rm BH}\widetilde{\Gamma}_{N_1}^{\rm BH},$$

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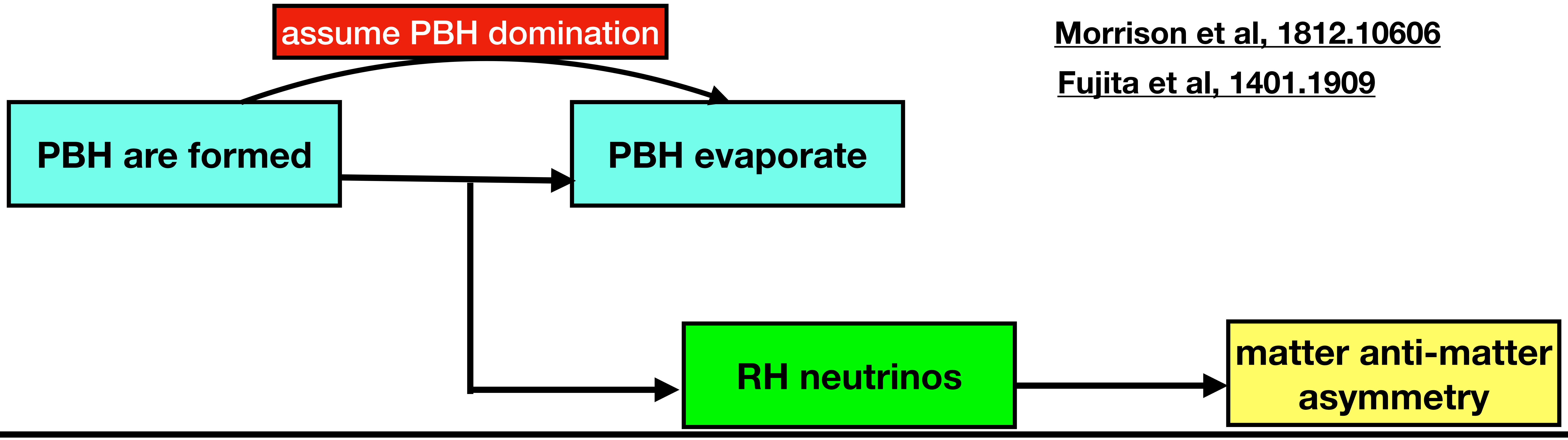
lepton asymmetry production from RHN decays and inverse decays

$$aH\frac{dn_{\alpha\beta}^{\rm B-L}}{da} = \epsilon_{\alpha\beta}^{(1)}\left[\left(n_{N_1} - n_{N_1}^{\rm eq}\right)\Gamma_{N_1}^{T} + n_{\rm BH}\widetilde{\Gamma}_{N_1}^{\rm BH}\right] - \mathcal{W}_{\alpha\beta}$$

thermal averaged decay of RHN from PBH evaporation (function of $z_{\rm BH}$)

Primordial Black holes induced leptogenesis



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contribution to RHN population from thermal plasma

lepton asymmetry production from RHN decays and inverse decays

Washout effects only sensitive to neutrino parameters

$$aH\frac{dn_{N_1}}{da} = -(n_{N_1} - n_{N_1}^{\rm eq})\Gamma_{N_1}^{T} + n_{\rm BH}\widetilde{\Gamma}_{N_1}^{\rm BH},$$

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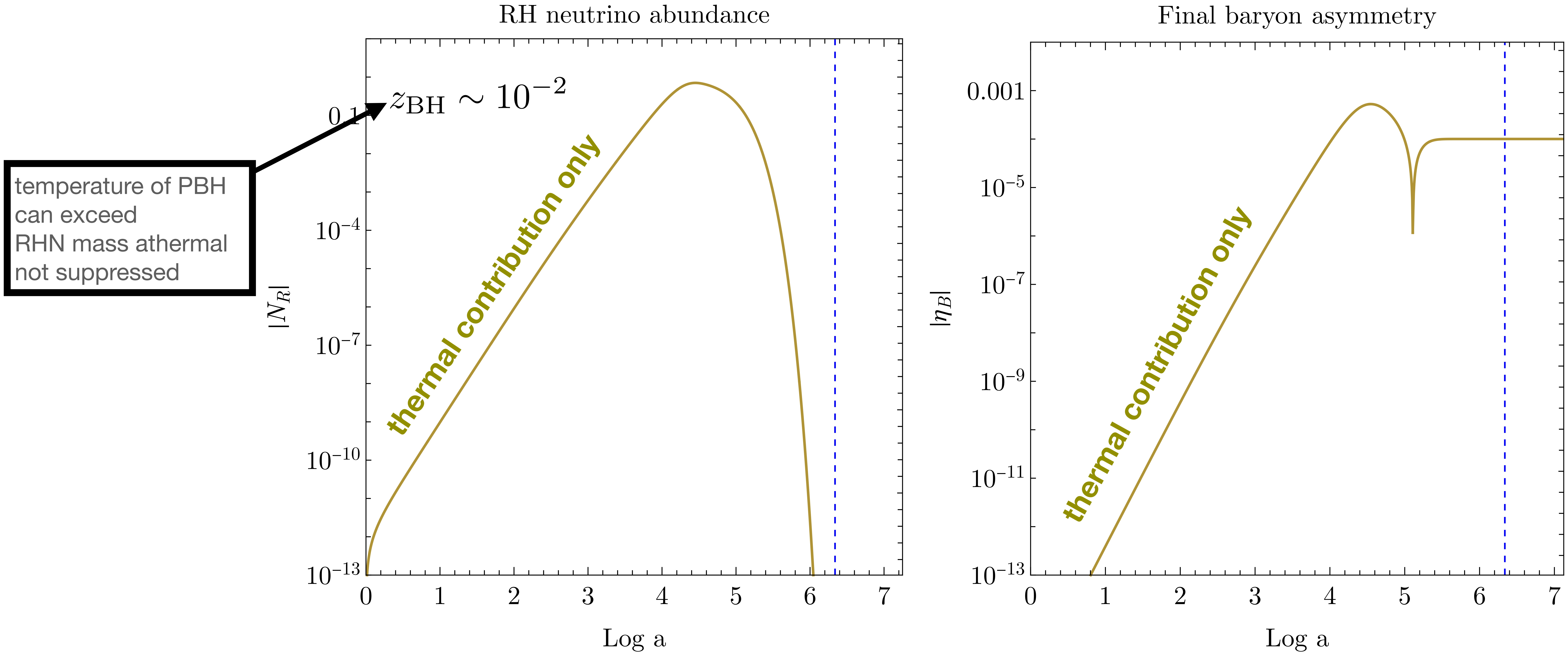
Primordial Black holes induced leptogenesis

A. PBH evaporate **before/during** RHNs are thermally produced from plasma → PBH evaporation creates an initial condition which gets erased by fast interactions in the plasma

B. PBH evaporation happens **shortly after** thermal leptogenesis $M_i = 1.7\,\mathrm{g} \quad \beta_i = 10^{-3} \quad M_N = 10^{11}\,\mathrm{GeV}$



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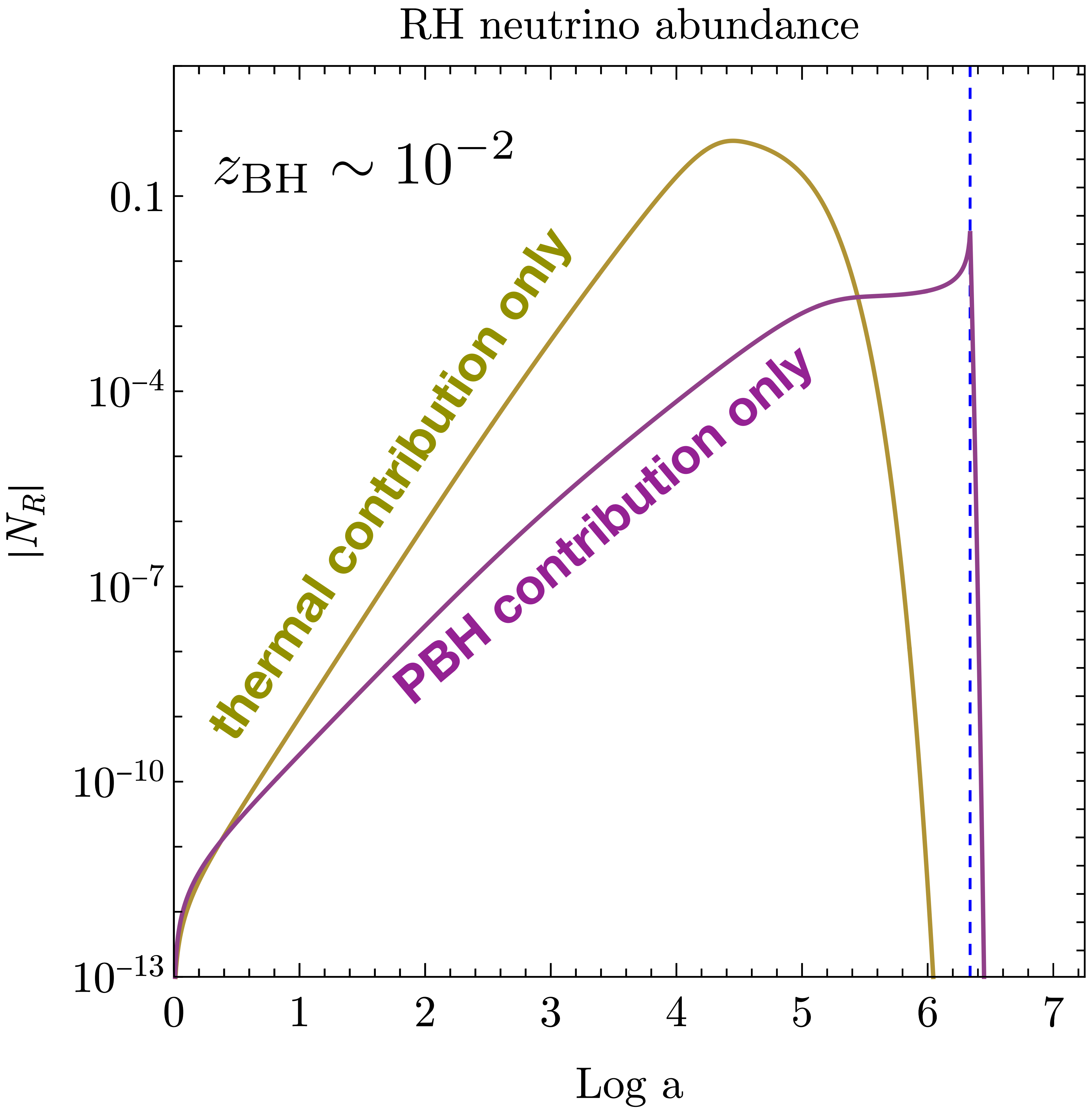

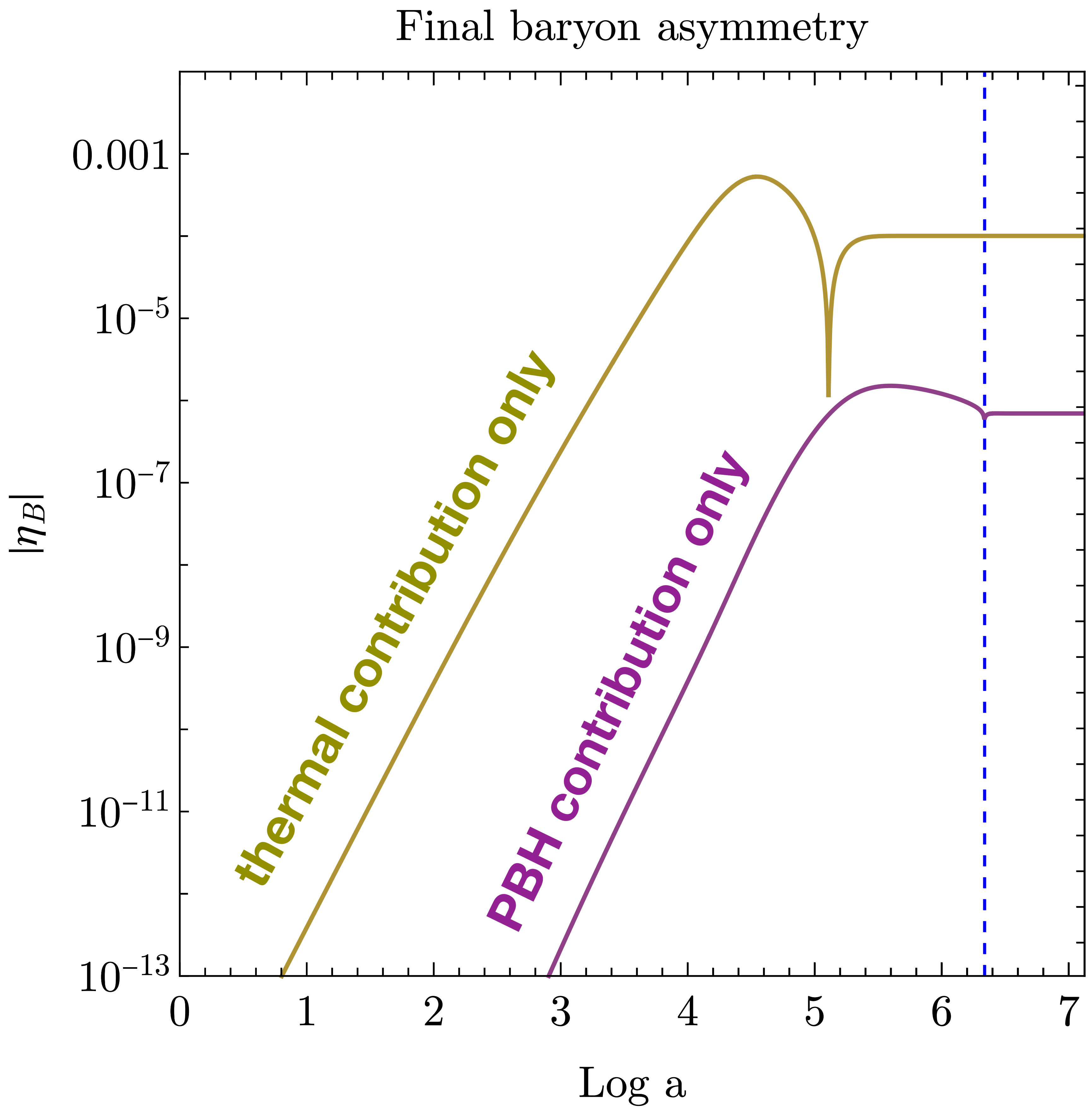

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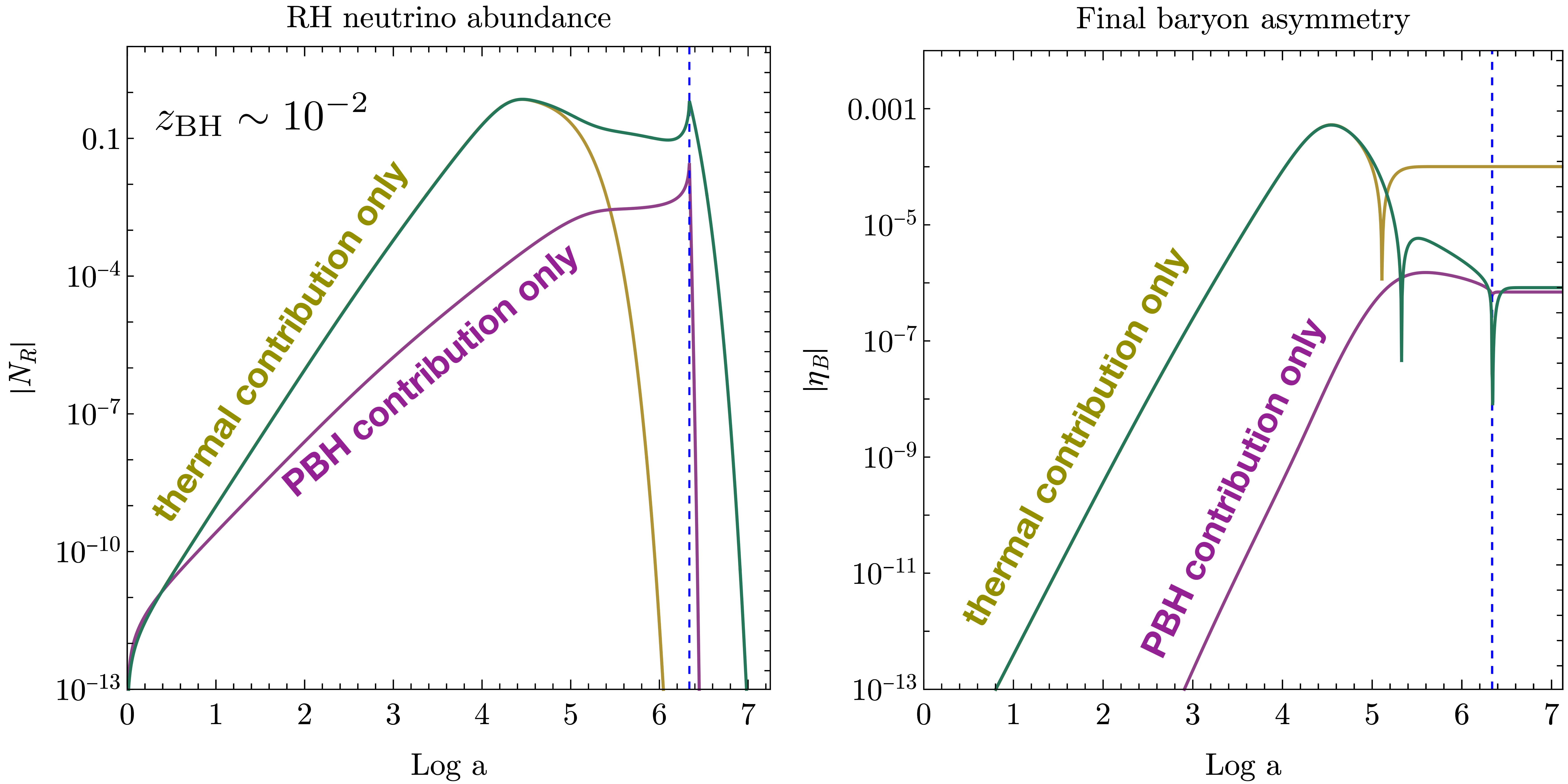
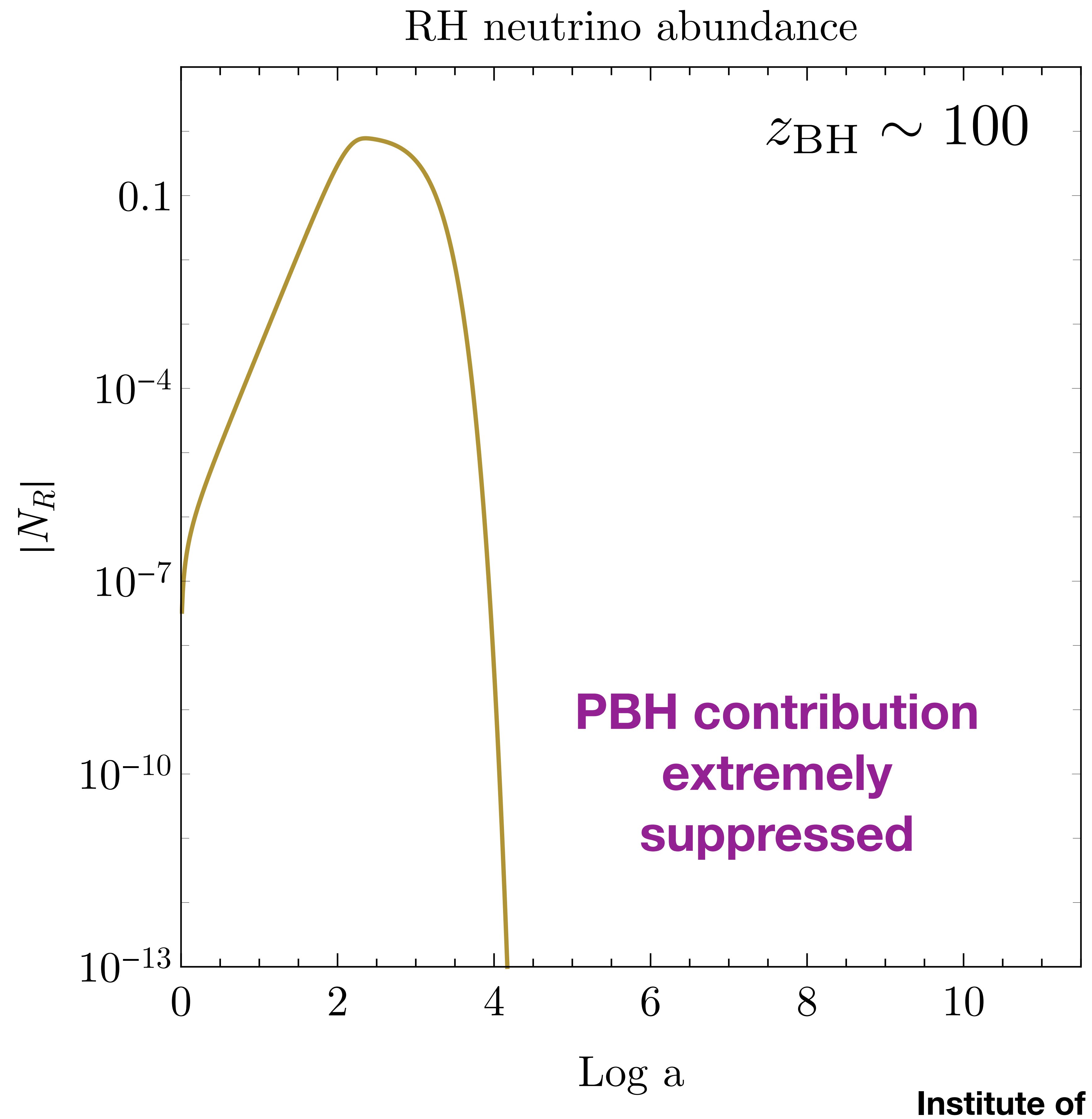
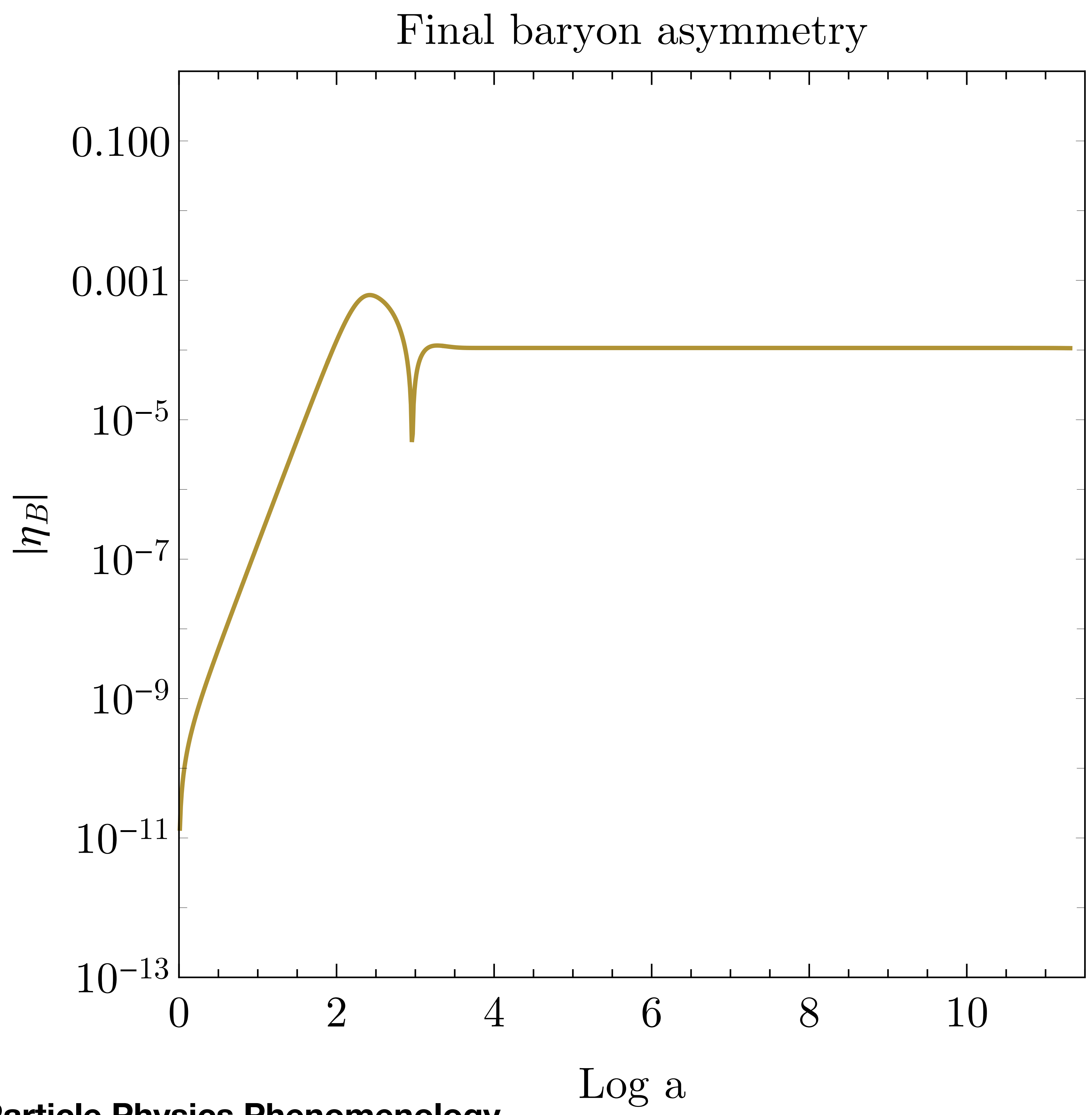


Primordial Black holes induced leptogenesis

D. PBH evaporation occurs **way after** thermal leptogenesis era

$$M_i = 10^4\,\mathrm{g} \quad \beta_i = 10^{-3} \quad M_N = 10^{11}\,\mathrm{GeV}$$





Primordial Black holes induced leptogenesis

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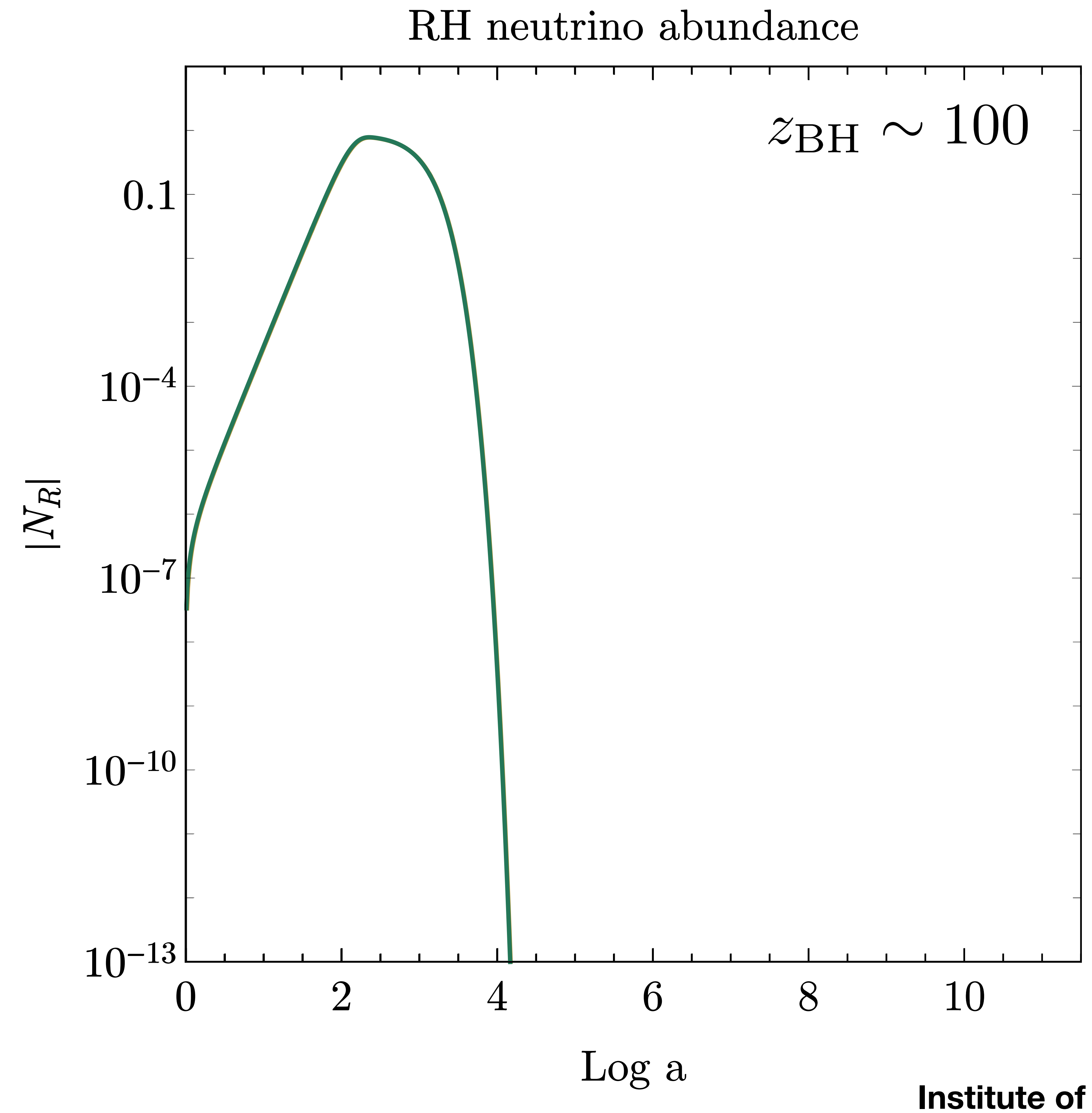
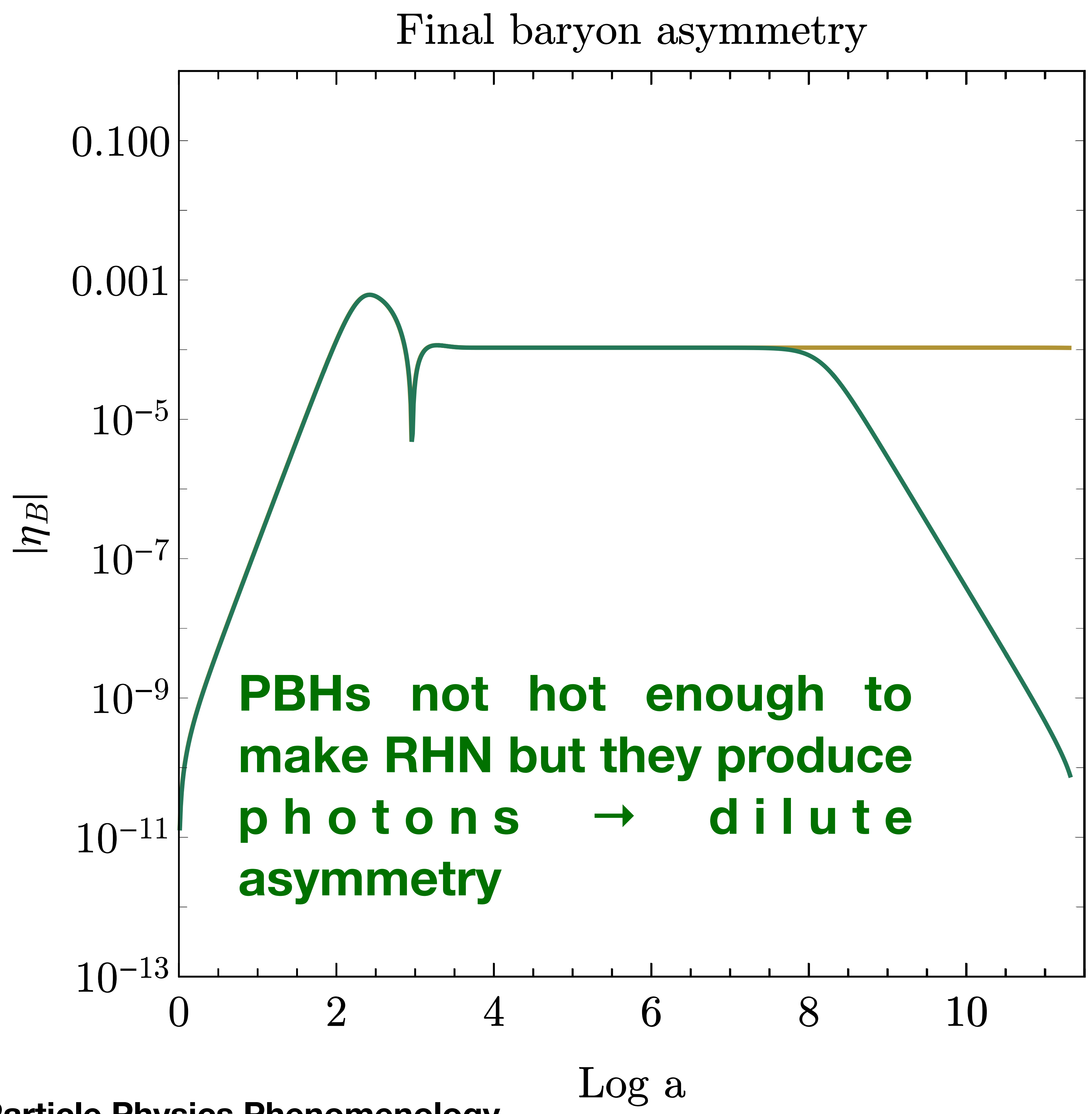
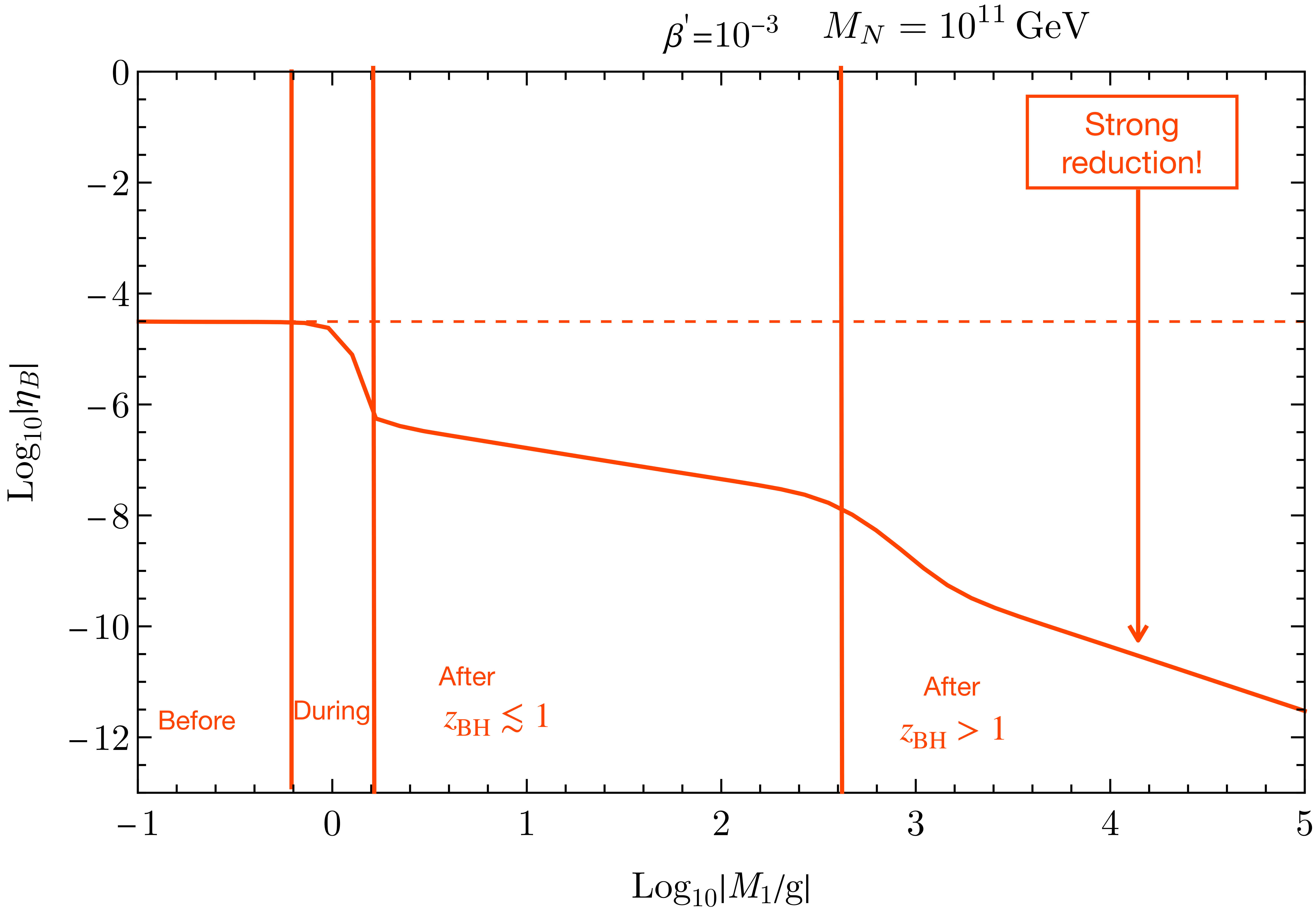


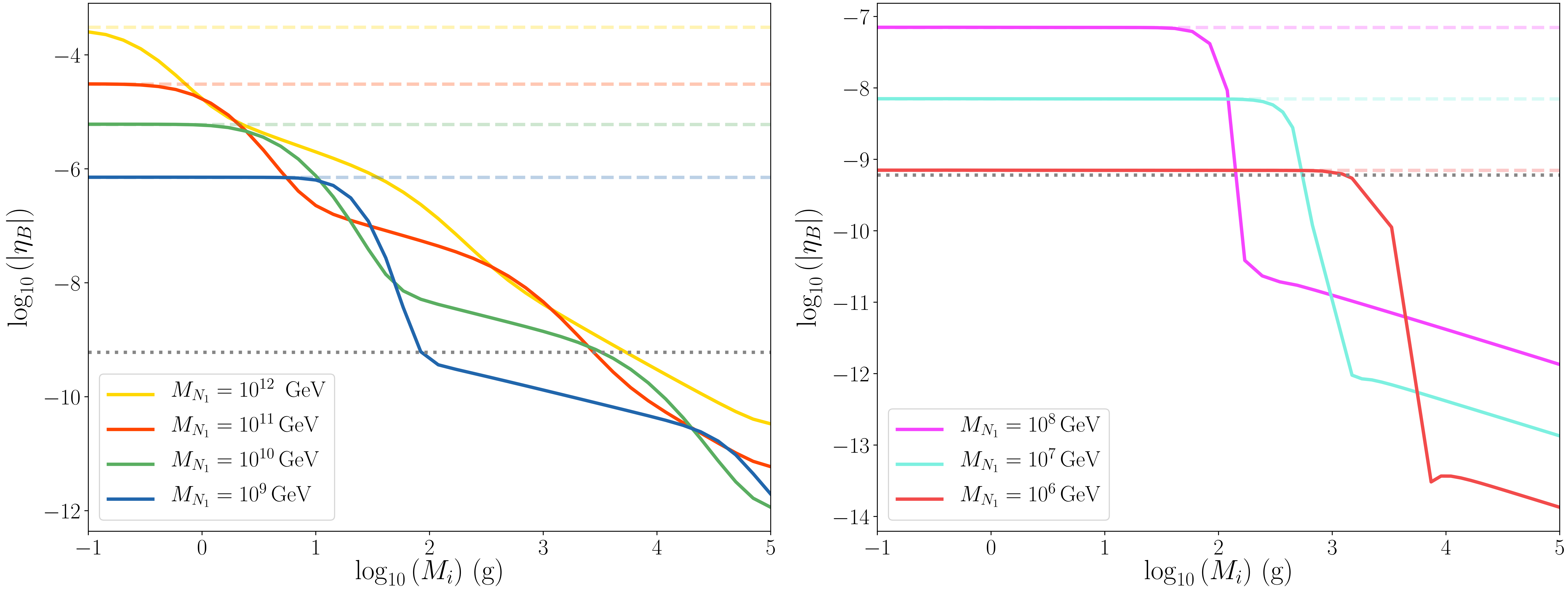


Primordial Black holes induced leptogenesis



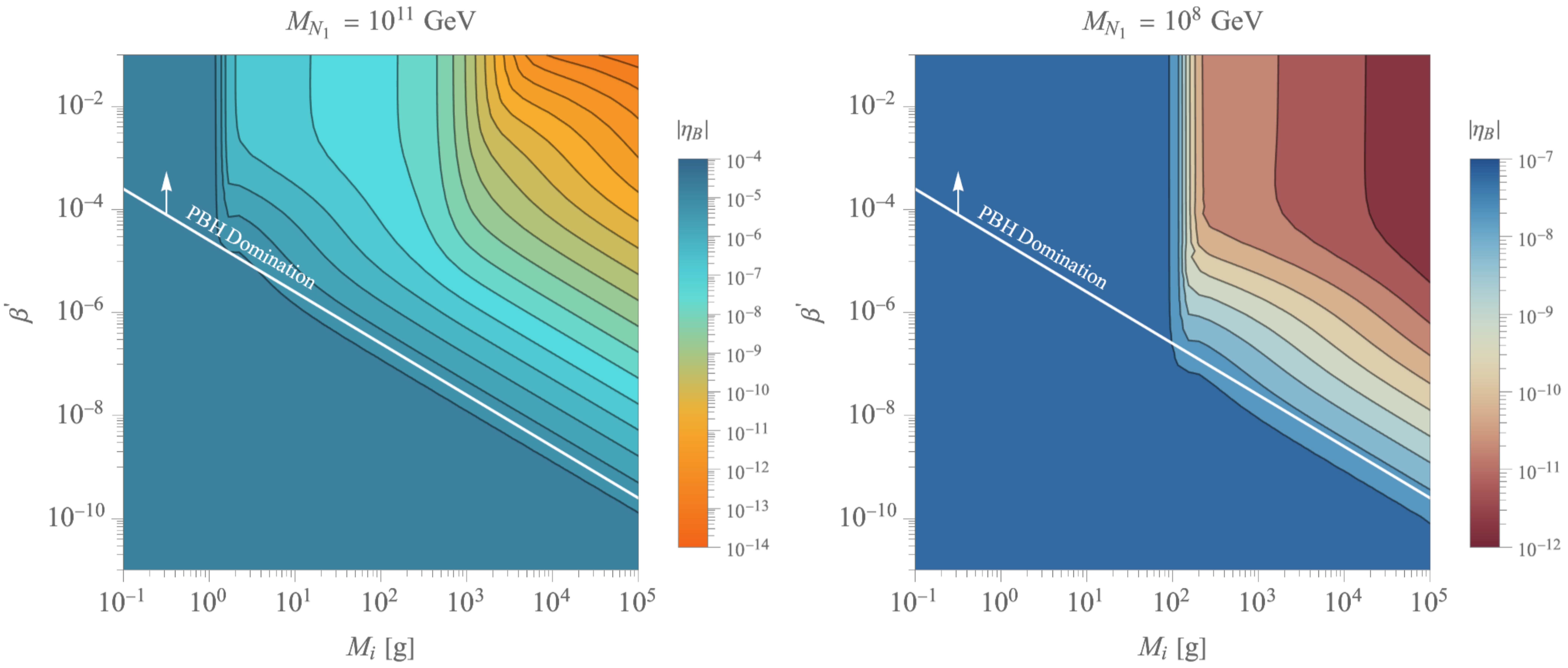
Thermal leptogenesis and primordial black holes

$\beta' = 10^{-3}$



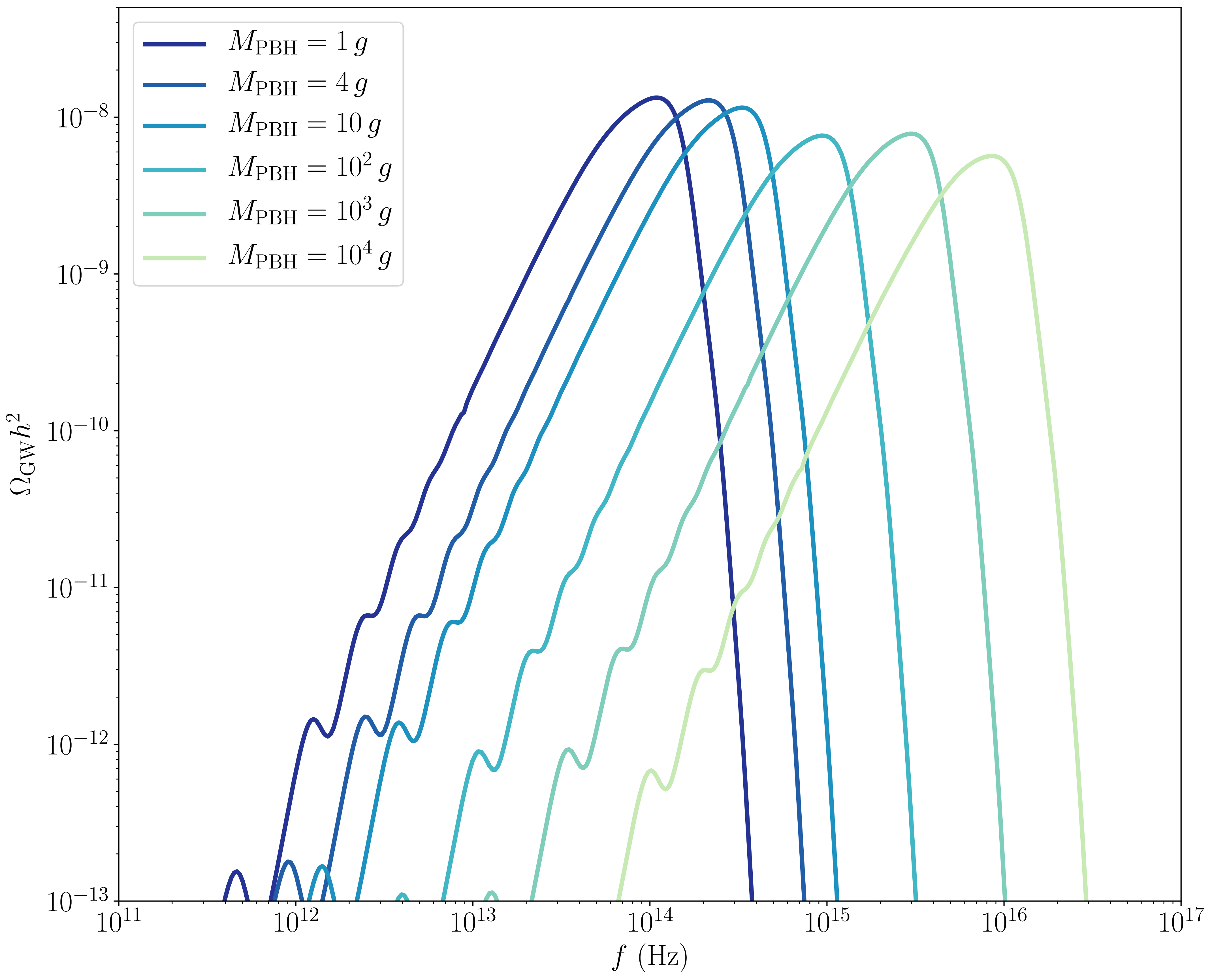
Chose Yukawa matrix for maximal baryon asymmetry

Thermal leptogenesis and primordial black holes



Dilution effect present as long as there is PBH domination

Thermal leptogenesis and primordial black holes



GW spectrum produced **directly** from PBHs very high frequency.

Smaller PBH evaporate earlier and experience more redshift

We could potentially observe GWs from isocurvature perturbations (Papanikolaou et al, 2010.11573, Domenech et al 2012.08151)

ULYSSES: Universal LeptogeneSiS Equation Solver

- Thermal and resonant leptogenesis
- Easy parallelisation
- rapid evaluation
- python package

In collaboration with Granelli, Perez-Gonzalez, Moffat & Schulz. Happy for people to add their own plugins

Thermal leptogenesis and primordial black holes

- Leptogenesis is one of the leading explanations of the matter anti-matter asymmetry. Added bonus is that light neutrino masses are also explained.

- It is entirely feasible the Universe underwent some non-standard cosmology such as PBH domination

- Due to the democratic nature of PBH, all particle degrees of freedoms are produced if the PBH is sufficiently hot.

- Non-trivial interplay between leptogenesis era and PBH evaporation. In some regions of the PS there is significant enhancement while in the low mass right-handed neutrino regime, heavier PBHs produce a giant entropy dump which dilutes the matter anti-matter asymmetry.

- While thermal leptogenesis is a very scale mechanism and therefore difficult to test, future probes of ultrahigh frequency GWs could falsify the intermediate scale leptogenesis.


Thank you!

$$\frac{dM}{dt} = -\sum_a \frac{g_a}{2\pi^2} \int_0^\infty \frac{\sigma_{\rm abs}^{s_a}(GMp)\, p^3\, dp}{\exp[E_a(p)/T_{\rm BH}] - (-1)^{2s_a}}$$

$$= -\kappa\, \varepsilon(M) \left(\frac{1\ \mathrm{g}}{M}\right)^2$$

$$\varepsilon(M) = \varepsilon_{\rm SM}(M) + \varepsilon_N(M)$$

$$\varepsilon_N(M) \approx 2\, n_{N_i} f^0_{1/2} \sum_{i=1}^{n_{N_i}} \exp\left[-\frac{8\pi G M M_{N_i}}{4.53}\right]$$

System of equations

$$
\begin{aligned}
\frac{dM}{dt} &= -\sum_a \frac{g_a}{2\pi^2} \int_0^\infty \frac{\sigma_{\rm abs}^{s_a}(GMp)\, p^3\, dp}{\exp[E_a(p)/T_{\rm BH}] - (-1)^{2s_a}} \\
\frac{d\varrho_{\rm R}}{da} &= \frac{1}{a\Delta}\left[4(\Delta-1) - \Sigma\right] - \frac{\varepsilon_{\rm SM}(M)}{\varepsilon(M)} \frac{1}{M}\frac{dM}{da} a\varrho_{\rm BH} \\
\frac{d\varrho_{\rm BH}}{da} &= \frac{1}{M}\frac{dM}{da}\varrho_{\rm BH}\,, \\
H^2 &= \frac{8\pi G}{3}\left(\varrho_{\rm BH} a^{-3} + \varrho_{\rm R} a^{-4}\right) \\
\frac{dT}{da} &= -\frac{T}{\Delta}\left\{\frac{1}{a} + \frac{\varepsilon_{\rm SM}(M)}{\varepsilon(M)}\frac{1}{M}\frac{dM}{da}\frac{g_*(T)}{g_{*S}(T)}\frac{a\varrho_{\rm BH}}{4\varrho_{\rm R}}\right\} \\
aH\frac{dn_{N_1}^{\rm TH}}{da} &= -(n_{N_1}^{\rm TH} - n_{N_1}^{\rm eq})\Gamma_{N_1}^{T}\,, \\
aH\frac{dn_{N_1}^{\rm BH}}{da} &= -n_{N_1}^{\rm BH}\Gamma_{N_1}^{\rm BH} + n_{\rm BH}\Gamma_{\rm BH\to N_1} \\
aH\frac{dn_{\alpha\beta}^{\rm B-L}}{da} &= \epsilon_{\alpha\beta}^{(1)}\left[(n_{N_1}^{\rm TH} - n_{N_1}^{\rm eq})\Gamma_{N_1}^{T} + n_{N_1}^{\rm BH}\Gamma_{N_1}^{\rm BH}\right] + \mathcal{W}_{\alpha\beta}
\end{aligned}
$$

Code can be found at ULYSSES and plugin is etabPBH_vf.py