CP violation

as a consequence of another

Symmetry

Andreas Trautner

based on:

NPB883 (2014) 267-305	1402.0507	w/ M.-C. Chen, M. Fallbacher, K.T. Mahanthappa and M. Ratz.
NPB894 (2015) 136-160	1502.01829	w/ M. Fallbacher.
JHEP 1702 (2017) 103	1612.08984	w/ M. Ratz.
PLB 786 (2018) 283-287	1808.07060	w/ H.P. Nilles, M. Ratz., P. Vaudrevange

Teilchentee Heidelberg
25.10.18

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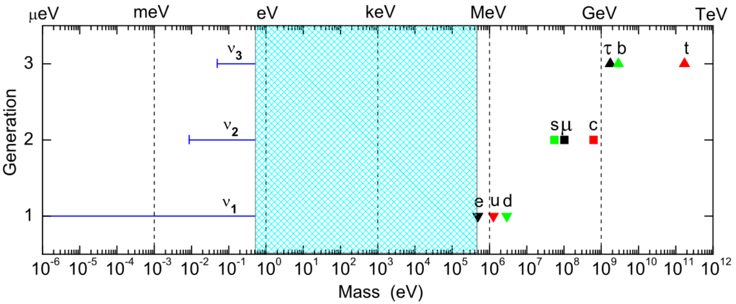
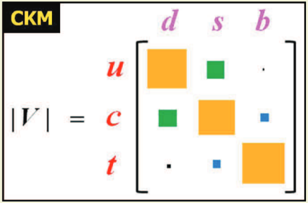
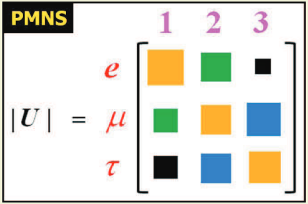
Motivation

- Standard Model flavor puzzle. Observed patterns:



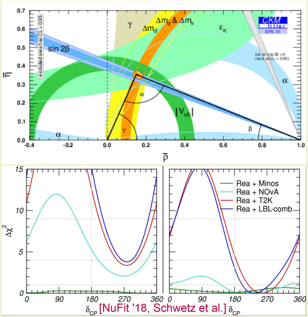




[Xing '14]

Motivation

- Standard Model flavor puzzle.
  4x 3 masses, 2x 3 angles, 2x 1 CP violating phases(+2).
- Origin of CP violation?
  - CP violation established in quark sector, consistent with SM (CKM). ✓
  - open question: CP violation in lepton sector ?
  - open question: Why $\overline{\theta} = (\theta + \arg\det y_u y_d) < 10^{-10}$? Why CPV *only* in FV processes?



- Flavor and CP are intertwined.

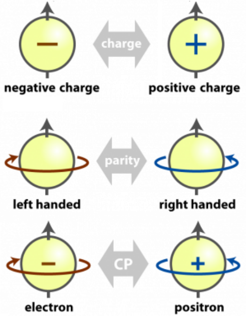
↷ *The* theory of flavor should also be a theory of CPV.

Goal: Understand origin of CPV ⇒ hints for origin of flavor.

Outline

- CP(V) in the Standard Model
- What is an outer automorphism?
- CP as a special outer automorphism
- Fun with outer automorphisms
  - CPV as a consequence of other symmetries
  - Outer automorphisms beyond CP
- Example (toy-)models:
  - 3HDM with $\Delta(54)$ symmetry
  - $\mathrm{SU}(3) \to \mathrm{T}_7$ with CPV and $\overline{\theta} = 0$

Textbook CP



Dirac theory:

$$\Psi(t,\vec{x}) \xmapsto{\mathcal{CP}} \mathrm{i}\gamma_2\gamma_0\,\Psi^*(t,-\vec{x})$$

$$A_\mu(t,\vec{x}) \xmapsto{\mathcal{CP}} -\mathcal{P}_\mu^{\ \nu}\,A_\nu(t,-\vec{x})$$

$$\widehat{\boldsymbol{a}}_s(\vec{p}) \xmapsto{\mathcal{CP}} \mathrm{e}^{\mathrm{i}\,\varphi}\,\widehat{\boldsymbol{b}}_s(-\vec{p})$$

$$\widehat{\boldsymbol{b}}_s^\dagger(\vec{p}) \xmapsto{\mathcal{CP}} \mathrm{e}^{\mathrm{i}\,\varphi}\,\widehat{\boldsymbol{a}}_s^\dagger(-\vec{p})\ .$$

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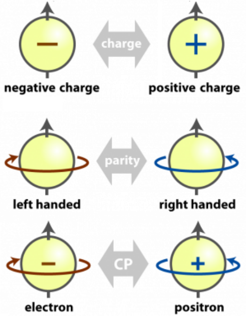

Dirac theory

$\mathcal{P} = \mathrm{diag}\,(1,-1,-1,-1)$

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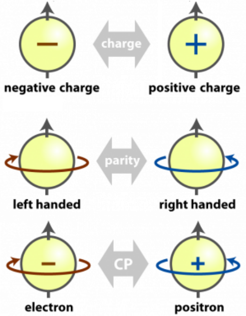

Dirac theory:

$$\Psi(t,\vec{x}) \xmapsto{\mathcal{CP}} \color{red}{\mathrm{i}\gamma_2\gamma_0}\color{black}\,\Psi^*(t,-\vec{x})$$

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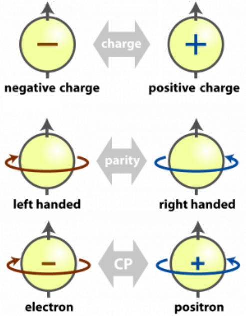

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$$\begin{tikzcd}
\underset{(\mathrm{L},-)}{\chi} \arrow[rr, leftrightarrow, "\mathrm{P}"] \arrow[d, leftrightarrow, "\mathrm{CP}"] \arrow[drr, leftrightarrow, "\mathrm{C}"] & & \underset{(\mathrm{R},-)}{\xi^\dagger} \arrow[d, leftrightarrow, "\mathrm{CP}"] \arrow[dll, leftrightarrow, "\mathrm{C}"'] \\
\underset{(\mathrm{R},+)}{\chi^\dagger} \arrow[rr, leftrightarrow, "\mathrm{P}"] & & \underset{(\mathrm{L},+)}{\xi}
\end{tikzcd}$$

CP as a special outer automorphism

One generation of (chiral) fermion fields with gauge symmetry $[T_a, T_b] = \mathrm{i} f_{abc}\, T_c$

$$\mathscr{L} \;=\; \mathrm{i}\,\overline{\Psi}\,\gamma^\mu \left(\partial_\mu - \mathrm{i}\, g\, T_a\, W_\mu^a\right) \Psi - \frac{1}{4}\, G_{\mu\nu}^a\, G^{\mu\nu,a}\;.$$

The most general possible CP transformation:

$$\begin{aligned} W_\mu^a(x) \;&\mapsto\; R^{ab}\, \mathcal{P}_\mu^{\ \nu}\, W_\nu^b(\mathcal{P}\,x)\;, \\ \Psi_\alpha^i(x) \;&\mapsto\; \eta_{\mathsf{CP}}\, U^{ij}\, \mathcal{C}_{\alpha\beta}\, \Psi^{*\,j}_{\ \beta}(\mathcal{P}\,x)\;. \end{aligned}$$

[Grimus, Rebelo,'95]

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This is (can be) a conserved symmetry of the *action* iff

$$\begin{aligned} &\text{(i)} && R_{aa'}\,R_{bb'}\,f_{a'b'c} \;=\; f_{abc'}\,R_{c'c}\;,\\ &\text{(ii)} && U\,(-T_a^{\mathrm{T}})\,U^{-1} \;=\; R_{ab}\,T_b\;,\\ &\text{(iii)} && \mathcal{C}\,(-\gamma^{\mu\,\mathrm{T}})\,\mathcal{C}^{-1} \;=\; \gamma^\mu\;. \end{aligned}$$

This implies:

(i) CP is an automorphism of the gauge group.

(ii) CP maps representations to their complex conjugate representations. $\left(T_a \mapsto -T_a^{\mathrm{T}}\right)$

(iii) CP is an automorphism of the Lorentz group which maps representations to their complex conjugate representation. $\left(\chi_{\mathrm{L}} \mapsto (\chi_{\mathrm{L}})^\dagger\right)$

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$$\Rightarrow \mathcal{C} = \mathrm{e}^{\mathrm{i}\eta}\, \gamma_2\gamma_0$$

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What is an outer automorphism?

Example: $\mathbb{Z}_3$ symmetry, generated by $\mathsf{a}^3 = \mathsf{id}$.

$\mathbb{Z}_3$	id	a	a^2
$\mathbf{1}$	1	1	1
$\mathbf{1}'$	1	ω	ω^2
$\mathbf{1}''$	1	ω^2	ω

$(\omega := e^{2\pi\,\mathrm{i}/3})$

- All elements of $\mathbb{Z}_3$: $\{\mathsf{id}, \mathsf{a}, \mathsf{a}^2\}$.
- Outer automorphism group ("Out") of $\mathbb{Z}_3$: generated by

$$u(\mathsf{a}) : \mathsf{a} \mapsto \mathsf{a}^2. \quad \big(\text{think: } \mathsf{u\,a\,u}^{-1} \,=\, \mathsf{a}^2\big)$$

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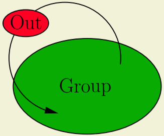

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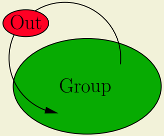

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In words: Out is a **"symmetry of the symmetry"**.

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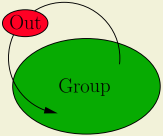

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Concrete: Out is a 1:1 mapping of representations $\boldsymbol{r} \mapsto \boldsymbol{r}'$.
Comes with a transformation matrix U, which is given by

$$\boxed{U\rho_{\boldsymbol{r}'}(\mathrm{g})U^{-1} = \rho_{\boldsymbol{r}}(u(\mathrm{g}))\ , \qquad \forall \mathrm{g} \in G\ .}$$

(consistency condition)

[Holthausen, Lindner, Schmidt, '13]
[Fallbacher, AT, '15]

- $\rho_{\boldsymbol{r}}(g)$: representation matrix for group element $g \in G$
- $u : g \mapsto u(g)$: **outer** automorphism

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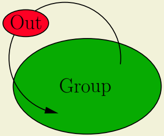

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Recognize eq. (i)-(iii)
as special case of this!

[...chmidt, '13]
[...cher, AT '15]

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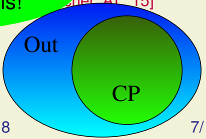

Note: Physical CP trafo $\boldsymbol{r}' = \boldsymbol{r}^*$ is a special case of this.

CP transformation in Standard Model

In the Standard Model

$$\mathrm{SU}(3)\otimes\mathrm{SU}(2)\otimes\mathrm{U}(1) \qquad \text{and} \qquad \mathrm{SO}(3,1)\;,$$

physical CP is described by a *simultaneous* outer automorphism transformation of all symmetries that maps

$$\boldsymbol{r} \;\longleftrightarrow\; \boldsymbol{r}^*\;,$$

$$\left(\text{e.g. } (\mathbf{3},\mathbf{2})^{\mathrm{L}}_{1/6} \;\longleftrightarrow\; \left(\overline{\mathbf{3}},\overline{\mathbf{2}}\right)^{\mathrm{R}}_{-1/6}\right)\;,$$

for the representations of *all* symmetries.

Conservation of such a transformation warrants $\overline{\theta},\ \delta_{\cancel{\mathrm{CP}}}=0$.

Violation of such a transformation is implied by experiment, and necessary requirement for baryogenesis. [Sakharov '67]

$$J \;=\; \det\left[M_u M_u^\dagger, M_d M_d^\dagger\right] \;\neq\; 0 \;\Longleftrightarrow\; \varepsilon_{i\to f} \;=\; \frac{\left|\Gamma_{i\to f}\right|^2 - \left|\Gamma_{\bar{\imath}\to\bar{f}}\right|^2}{\left|\Gamma_{i\to f}\right|^2 + \left|\Gamma_{\bar{\imath}\to\bar{f}}\right|^2} \;\neq\; 0$$

see also [Bernabéu, Branco, Gronau '86], [Botella, Silva '94]

Physical CP transformation

We *extrapolate* from the SM to possible symmetries in BSM.

$\Rightarrow$ "Definition" of CP in words:

CP is **a** special outer automorphism transformation which maps *all present* symmetry representations (global, local, space-time) to their complex conjugates.

[AT '16]

This definition is consistent with the definitions in [Buchbinder et al. '01] & [Grimus, Rebelo '95]

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Any such transformation:

- warrants *physical* CP conservation (if conserved),

⇒ must be broken (by observation).

Note that a physical CP transformation:

- does not have to be unique,
- does not have to be of order 2, e.g. [Grimus et al. '87],[Weinberg '05],[Ivanov, Silva '15] [Chen, Fallbacher, Mahanthappa, Ratz, AT '14]
- is, in general, not guaranteed to exist for a given symmetry group. (It *does* exist for G_{SM}!)

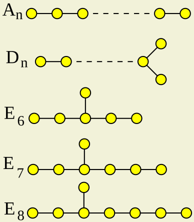
Outer automorphisms of groups

Outer automorphisms exist for continuous & discrete groups.

There are easy ways to depict this:

Continuous groups:

Outer automorphisms of a simple Lie algebra are the symmetries of the corresponding Dynkin diagram.



	Lie Group	Out	Action on reps
$A_{n>1}$	$\mathrm{SU}(N)$	$\mathbb{Z}_2$	$\boldsymbol{r} \rightarrow \boldsymbol{r}^*$
$D_{n>4}$	$\mathrm{SO}(2N)$	$\mathbb{Z}_2$	$\boldsymbol{r} \rightarrow \boldsymbol{r}^*$
E_6	E_6	$\mathbb{Z}_2$	$\boldsymbol{r} \rightarrow \boldsymbol{r}^*$
$D_{n=4}$	$\mathrm{SO}(8)$	S_3	$\boldsymbol{r}_i \rightarrow \boldsymbol{r}_j$
all others		/	/

Outer automorphisms of groups

Discrete groups:

Outer automorphisms of a discrete group are symmetries of the character table (not 1:1).

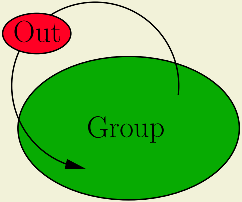
T_7	C_{1a}	C_{3a}	C_{3b}	C_{7a}	C_{7b}
$\mathbf{1}_0$	1	1	1	1	1
$\mathbf{1}_1$	1	ω	ω^2	1	1
$\overline{\mathbf{1}}_1$	1	ω^2	ω	1	1
$\mathbf{3}_1$	3	0	0	η	η^*
$\overline{\mathbf{3}}_1$	3	0	0	η^*	η

$\Delta(54)$	C_{1a}	C_{3a}	C_{3b}	C_{3c}	C_{3d}	C_{2a}	C_{6a}	C_{6b}	C_{3e}	C_{3f}
$\mathbf{1}_0$	1	1	1	1	1	1	1	1	1	1
$\mathbf{1}_1$	1	1	1	1	1	-1	-1	-1	1	1
$\mathbf{2}_1$	2	2	-1	-1	-1	0	0	0	2	2
$\mathbf{2}_2$	2	-1	2	-1	-1	0	0	0	2	2
$\mathbf{2}_3$	2	-1	-1	2	-1	0	0	0	2	2
$\mathbf{2}_4$	2	-1	-1	-1	2	0	0	0	2	2
$\mathbf{3}_1$	3	0	0	0	0	1	ω^2	ω	3ω	$3\omega^2$
$\overline{\mathbf{3}}_1$	3	0	0	0	0	1	ω	ω^2	$3\omega^2$	3ω
$\mathbf{3}_2$	3	0	0	0	0	-1	$-\omega^2$	$-\omega$	3ω	$3\omega^2$
$\overline{\mathbf{3}}_2$	3	0	0	0	0	-1	$-\omega$	$-\omega^2$	$3\omega^2$	3ω

The outer automorphisms group of any ("small") discrete group can easily be found with `GAP` [GAP] .

Group	Out	Action on reps
$\mathbb{Z}_3$	$\mathbb{Z}_2$	$\boldsymbol{r} \to \boldsymbol{r}^*$
$\mathrm{A}_{n\neq 6}$	$\mathbb{Z}_2$	$\boldsymbol{r} \to \boldsymbol{r}^*$
$\mathrm{S}_{n\neq 6}$	/	/
$\Delta(27)$	$\mathrm{GL}(2,3)$	$\boldsymbol{r}_i \to \boldsymbol{r}_j$
$\Delta(54)$	S_4	$\boldsymbol{r}_i \to \boldsymbol{r}_j$
...		

Two types of groups (without mathematical rigor)



List of representations: $\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_k, {\boldsymbol{r}_k}^*, \ldots$

Out in general : $\quad \boldsymbol{r}_i \mapsto \boldsymbol{r}_j \quad \forall$ irreps i,j $(1:1)$

Criterion:

Is there an (outer) automorphism transformation that maps

$$\boldsymbol{r}_i \mapsto {\boldsymbol{r}_i}^* \qquad \text{for all irreps } i\,?$$

No	Yes
⇒ Group of "**type I**"	⇒ Group of "**type II**"

This tells us whether a CP transformation is possible, or not!

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General answer: **No.**

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For example: Discrete groups of **type I**:

G	$\mathbb{Z}_5 \rtimes \mathbb{Z}_4$	T_7	$\Delta(27)$	$\mathbb{Z}_9 \rtimes \mathbb{Z}_3$	$\ldots$
SG id	$(20,3)$	$(21,1)$	$(27,3)$	$(27,4)$	

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SG id	$(20,3)$	$(21,1)$	$(27,3)$	$(27,4)$	

- These are **inconsistent** with the trafo $\boldsymbol{r}_i \mapsto \boldsymbol{r}_i^* \; \forall\, i$.

$\Rightarrow$ CP transformation is inconsistent with a type I symmetry.
(assuming sufficient # of irreps are in the model)

There are models in which CP is violated *as a consequence* of another symmetry.

[Chen, Fallbacher, Mahanthappa, Ratz, AT '14]

The corresponding CPV phases are calculable and quantized (e.g. $\delta_{\cancel{CP}} = 2\pi/3, \ldots$) stemming from the necessarily complex Clebsch-Gordan coefficients of the "type I" group. This has been termed "explicit geometrical" CP violation.

[Chen, Fallbacher, Mahanthappa, Ratz, AT '14]
[Branco, '15], [de Medeiros Varzielas, '15]

Do CP transformations exist for all symmetries?

On the contrary

Semi-simple Lie groups: they are all of type II !

- There always exists an (outer) automorphism transformation that maps all $\boldsymbol{r} \mapsto \boldsymbol{r}^*$ simultaneously.
  [Grimus, Rebelo '95]

$\Rightarrow$ CP can only be violated (explicitly) if the number of rephasing degrees of freedom is less than the number of complex parameters. cf. e.g. [Haber, Surujon '12]

This is the case in the Standard Model.

☹ This just parametrizes CPV, there is no way to predict $\delta_{\not{CP}}$.

Aside: There are models with higher-order CP transformations which allow for complex couplings, yet conserve CP (groups of type II B).
[Chen, Fallbacher, Mahanthappa, Ratz, AT '14], [Ivanov, Silva '15]

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  - ✗ no type I subgroups of $\mathrm{SU}(2)$.
  - ✗ no type I subgroups of the Lorentz group.
    (Open question: Type I "spacetime crystals"? [Wilczek '12]).
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- Discrete flavor symmetries?
  - Many models with type I groups:

$$\mathrm{T}_7, \Delta(27), \Delta(54), \mathcal{PSL}_2(7), ...$$

    e.g. [Björkeroth, Branco, Ding, de Anda, Ishimori, King, Medeiros Varzielas, Neder, Stuart et al. '15-'18] [Chen, Pérez, Ramond '14], [Krishnan, Harrison, Scott '18]
  - These can originate from extra dimensions, e.g. in string theory. [Kobayashi et al. '06], [Nilles, Ratz, Vaudrevange '12]
  - Semi-realistic heterotic orbifold model with $\Delta(54)$ flavor symmetry and geometrical CP violation. [Nilles, Ratz, AT, Vaudrevange '18]

Outer automorphisms
beyond C,P

Physics of outer automorphisms

Outs map representations to other representations, $\boldsymbol{r}_i \mapsto \boldsymbol{r}_j$.
In general **two logical possibilities**:

(A) Both, $\boldsymbol{r}_i$ and $\boldsymbol{r}_j$, are included in model.

(B) $\boldsymbol{r}_j$ is not part of the model.

$\boxed{\text{This is more general than C and/or P transformations.}}$

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Physical consequences:

(A) Maps operators to other operators.

$$\mathscr{L} \supset c_1 \, \mathcal{O}_1(x) + c_2 \, \mathcal{O}_2(x) + \ldots$$

Only possible effect: Transformation of the couplings

$$S\left[\mathcal{L}\left(\phi, c\right)\right] \;\longrightarrow\; S\left[\mathcal{L}\left(U\phi, c\right)\right] \;=\; S\left[\mathcal{L}\left(\phi, \tilde{U}c\right)\right] \;.$$

$c \neq \tilde{U}c \Rightarrow$ looks like an explicitly broken symmetry.
(Actually, this points to redundancies in the parameter–space of a theory).

(B) Transformation is simply not possible with the given field content.
$\curvearrowright$ **Explicitly** and **maximally** broken transformation.

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Examples:

(A) CP transformation in the SM: $\boldsymbol{r} \to \boldsymbol{r}^*$ for *all* representations

$$\text{e.g. } \mathrm{Q_L}: \; (\mathbf{3},\mathbf{2})^{\mathrm{L}}_{1/6} \; \longrightarrow \; (\overline{\mathbf{3}},\overline{\mathbf{2}})^{\mathrm{R}}_{-1/6} \; .$$

Maps $V_{\mathrm{CKM}} \to (V_{\mathrm{CKM}})^*$. Symmetry? No, since $\mathrm{e}^{\mathrm{i}\delta_{\mathrm{CKM}}} \neq \left[\mathrm{e}^{\mathrm{i}\delta_{\mathrm{CKM}}}\right]^*$.
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(B) P transformation in the Standard Model: $\boldsymbol{r}_{\mathrm{L}} \to \boldsymbol{r}_{\mathrm{R}}$.

$$\text{e.g. } \mathrm{Q_L}: \ (\mathbf{3},\mathbf{2})^{\mathrm{L}}_{1/6} \ \longrightarrow \ (\mathbf{3},\mathbf{2})^{\mathrm{R}}_{1/6} \ .$$

$\mathrm{Q_R}$ is not part of the model $\Rightarrow$ this transformation is **not** possible.
Transformation is broken **explicitly** and **maximally** (=by representation content).

Note:

Since $\mathcal{L}$ is real it always contains $\boldsymbol{r}$ and $\boldsymbol{r}^*$. Therefore,
CP **cannot** be broken maximally!

Fun with
outer automorphisms I

Outer automorphisms beyond C,P

Explicit example: 3HDM model with $\Delta(54)$ symmetry.

[Fallbacher, AT, '15]

(This is the original "geometrical T violation" model of Branco, Gerard, and Grimus.) [Branco, Gerard, Grimus, '83]

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$$V(H,\vec{a}) = -m^2\, H_i^\dagger H_i + a_0\, I_0(H^\dagger, H) \begin{array}{l} + a_1\, \boxed{I_1(H^\dagger,H)} + a_2\, \boxed{I_2(H^\dagger,H)} \\[2em] + a_3\, \boxed{I_3(H^\dagger,H)} + a_4\, \boxed{I_4(H^\dagger,H)}\,, \end{array}$$

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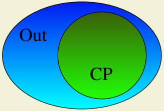

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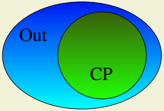

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This 3HDM model is an example for some very general statements on outer automorphisms: [Fallbacher, AT, '15], [AT '16]

- Outs allow to identify physical redundancies in the parameter space.
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$$J \;=\; -9\sqrt{3}(a_1-a_2)(a_1-a_3)(a_1-a_4)(a_2-a_3)(a_2-a_4)(a_3-a_4)\;.$$ [Nishi]

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  $\Rightarrow$ direction of the **VEVs are calculable**
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- Outs can give rise to emergent symmetries.
  here: $U\,\langle H\rangle_{\mathrm{I}} \;=\; \langle H\rangle_{\mathrm{I}}$, where $U \;\in\; \mathrm{Out}(G)$.

  $\Rightarrow$ VEV has higher symmetry than potential.

In this particular 3HDM, this explains the origin of spontaneous ("geometrical") CP violation with calculable phases. [Fallbacher, AT, '15]

Fun with outer automorphisms II

An interesting observation

Observation:

Type I groups can arise as subgroups of type II groups.

For example: small finite subgroups of simple Lie groups.

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Structure of outer automorphisms:

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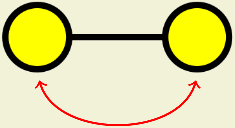

$$\boldsymbol{r}_i \leftrightarrow \boldsymbol{r}_i^{\,*} \quad \forall i \checkmark$$

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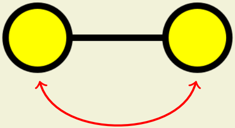

$$\boldsymbol{r}_i \ \leftrightarrow \ {\boldsymbol{r}_i}^* \quad \forall i \ ✓$$

$$\mathrm{Out}(\,\mathrm{T}_7\,) \ \cong \ \mathbb{Z}_2$$

T_7	C_{1a}	C_{3a}	C_{3b}	C_{7a}	C_{7b}
$\mathbf{1}_0$	1	1	1	1	1
$\mathbf{1}_1$	1	ω	ω^2	1	1
$\overline{\mathbf{1}}_1$	1	ω^2	ω	1	1
$\mathbf{3}_1$	3	0	0	η	η^*
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$$\boldsymbol{r}_i \ \not\leftrightarrow \ {\boldsymbol{r}_i}^* \quad \forall i \ ✗$$

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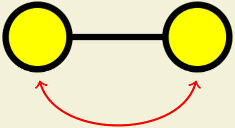

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$\boldsymbol{r}_i \leftrightarrow \boldsymbol{r}_i{}^* \quad \forall i$ ✓ $\qquad\qquad$ $\boldsymbol{r}_i \cancel{\leftrightarrow} \boldsymbol{r}_i{}^* \quad \forall i$ ✗

Note: $\mathrm{Out}(\,\mathfrak{su}(3)\,)$ acts on the $\mathrm{T}_7 \subset \mathrm{SU}(3)$ subgroup as $\mathrm{Out}(\,\mathrm{T}_7\,)$!

Toy model overview

Facts:

- $\mathrm{SU}(3)$ is **consistent** with a physical CP transformation.
- The T_7 subgroup of $\mathrm{SU}(3)$ is **inconsistent** with a physical CP transformation.

Question: How is CP violated in a breaking $\mathrm{SU}(3) \to \mathrm{T}_7$?

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$$\mathscr{L} \;=\; (D_\mu \phi)^\dagger \, (D^\mu \, \phi) - \frac{1}{4} \, G^a_{\mu\nu} \, G^{\mu\nu,a} - V(\phi) \;,$$

$$V(\phi) \;=\; -\,\mu^2 \phi^\dagger \phi + \sum_{i=1}^{5} \lambda_i \, \mathcal{I}_i^{(4)}(\phi) \;. \qquad \text{with } \lambda_i \in \mathbb{R}$$

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- VEV of the $\mathbf{15}$-plet $\langle\phi\rangle$ breaks $\mathrm{SU}(3) \to \mathrm{T}_7$. [Luhn, '11], [Merle, Zwicky '11]
- $\mathrm{Out}(\,\mathfrak{su}(3)\,) \cong \mathbb{Z}_2 \;\to\; \mathrm{Out}(\,\mathrm{T}_7\,) \cong \mathbb{Z}_2$; Out **unbroken** by VEV.

$$\mathrm{SU}(3) \rtimes \mathbb{Z}_2 \xrightarrow{\langle\phi\rangle} \mathrm{T}_7 \rtimes \mathbb{Z}_2;.$$

CP violation in $\mathbf{SU(3)} \rightarrow \mathbf{T_7}$ toy model

[Ratz, AT '16]

Name	$\mathrm{SU}(3)$	$\xrightarrow{\langle\phi\rangle}$ Name	T_7	mass
A_μ	$\mathbf{8}$	Z_μ	$\mathbf{1_1}$	$m_Z^2 = 7/3\, g^2\, v^2$
		W_μ	$\mathbf{3}$	$m_W^2 = g^2\, v^2$
ϕ	$\mathbf{15}$	$\mathrm{Re}\,\sigma_0$	$\mathbf{1_0}$	$m^2_{\mathrm{Re}\,\sigma_0} = 2\,\mu^2$
		$\mathrm{Im}\,\sigma_0$	$\mathbf{1_0}$	$m^2_{\mathrm{Im}\,\sigma_0} = 0$
		σ_1	$\mathbf{1_1}$	$m^2_{\sigma_1} = -\mu^2 + \sqrt{15}\,\lambda_5\, v^2$
		τ_1	$\mathbf{3}$	$m^2_{\tau_1} = m^2_{\tau_1}(\mu, \lambda_i)$
		τ_2	$\mathbf{3}$	$m^2_{\tau_2} = m^2_{\tau_2}(\mu, \lambda_i)$
		τ_3	$\mathbf{3}$	$m^2_{\tau_3} = m^2_{\tau_3}(\mu, \lambda_i)$

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The action is invariant under the $\mathbb{Z}_2 - \mathrm{Out}$ transformation:

SU(3)	T_7
$A^a_\mu(x) \mapsto R^{ab}\,\mathcal{P}_\mu^{\ \nu}\,A^b_\nu(\mathcal{P}\,x)\,,$ $\phi_i(x) \mapsto U_{ij}\,\phi^*_j(\mathcal{P}\,x)\,.$	$W_\mu(x) \mapsto \mathcal{P}_\mu^{\ \nu}\,W^*_\nu(\mathcal{P}\,x)\,,$ $\sigma_0(x) \mapsto \sigma_0(\mathcal{P}\,x)\,,$ $\tau_i(x) \mapsto \tau^*_i(\mathcal{P}\,x)\,,$ $Z_\mu(x) \mapsto -\mathcal{P}_\mu^{\ \nu}\,Z_\nu(\mathcal{P}\,x)\,,$ $\sigma_1(x) \mapsto \sigma_1(\mathcal{P}\,x)\,.$
physical CP ✓	physical CP ✗

CP violation in $\mathbf{SU(3)} \rightarrow \mathbf{T_7}$ toy model

- The VEV does **not** break the CP transformation, $U\langle\phi\rangle^* = \langle\phi\rangle$.
- However, at the level of T_7, the $\mathrm{SU}(3)$-CP transformation merges to $\mathrm{Out}(\,\mathrm{T}_7\,)$:

$$
\begin{array}{lccccccccccccccccc}
 & \mathbf{15} & \rightarrow & \mathbf{1_0} & \oplus & \mathbf{1_1} & \oplus & \overline{\mathbf{1}}_\mathbf{1} & \oplus & \mathbf{3} & \oplus & \mathbf{3} & \oplus & \overline{\mathbf{3}} & \oplus & \overline{\mathbf{3}} \\
\mathbb{Z}_2 - \mathrm{Out}: & \downarrow & & & & & & & & & & & & & & \\
 & \overline{\mathbf{15}} & \rightarrow & \mathbf{1_0} & \oplus & \overline{\mathbf{1}}_\mathbf{1} & \oplus & \mathbf{1_1} & \oplus & \overline{\mathbf{3}} & \oplus & \overline{\mathbf{3}} & \oplus & \mathbf{3} & \oplus & \mathbf{3}
\end{array}
$$

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$$\mathbb{Z}_2 - \mathrm{Out}: \quad \begin{tikzcd} \mathbf{15} \arrow[d] & \to & \mathbf{1_0} \arrow[d] & \oplus & \mathbf{1_1} \arrow[drr] & \oplus & \overline{\mathbf{1}}_\mathbf{1} \arrow[dll] & \oplus & \mathbf{3} \arrow[d] & \oplus & \mathbf{3} \arrow[d] & \oplus & \overline{\mathbf{3}} \arrow[d] & \oplus & \overline{\mathbf{3}} \arrow[d] \\ \overline{\mathbf{15}} & \to & \mathbf{1_0} & \oplus & \overline{\mathbf{1}}_\mathbf{1} & \oplus & \mathbf{1_1} & \oplus & \overline{\mathbf{3}} & \oplus & \overline{\mathbf{3}} & \oplus & \mathbf{3} & \oplus & \mathbf{3} \end{tikzcd}$$

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\downarrow & & \downarrow & & \searrow & & \swarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\
\overline{\mathbf{15}} & \to & \mathbf{1_0} & \oplus & \overline{\mathbf{1}}_\mathbf{1} & \oplus & \mathbf{1_1} & \oplus & \overline{\mathbf{3}} & \oplus & \overline{\mathbf{3}} & \oplus & \mathbf{3} & \oplus & \mathbf{3}
\end{array}
$$

$\Rightarrow$ The $\mathbb{Z}_2$-Out is conserved at the level of T_7, but it is **not** interpreted as a physical CP trafo,

$$\mathrm{SU}(3) \rtimes \mathbb{Z}_2^{(\mathrm{CP})} \xrightarrow{\langle\phi\rangle} \mathrm{T}_7 \rtimes \mathbb{Z}_2^{\cancel{(\mathrm{CP})}}\,.$$

- There is no other possible allowed CP transformation at the level of T_7 (type I).
- Imposing a transformation $\boldsymbol{r}_{\mathrm{T}_7,i} \leftrightarrow \boldsymbol{r}_{\mathrm{T}_7,i}{}^*$ enforces decoupling, $g = \lambda_i = 0$.

CP violation in $\mathbf{SU(3)} \to \mathbf{T_7}$ toy model

Explicit crosscheck: compute decay asymmetry.

$$\varepsilon_{\sigma_1 \to W\,W^*} \;:=\; \frac{\left|\mathscr{M}(\sigma_1 \to W\,W^*)\right|^2 - \left|\mathscr{M}(\sigma_1^* \to W\,W^*)\right|^2}{\left|\mathscr{M}(\sigma_1 \to W\,W^*)\right|^2 + \left|\mathscr{M}(\sigma_1^* \to W\,W^*)\right|^2}\;.$$

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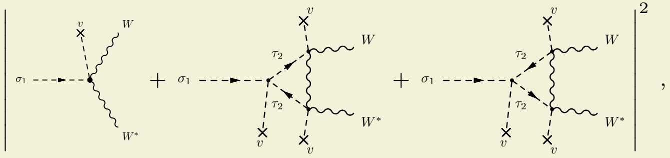
Contribution to $\varepsilon_{\sigma_1 \to W\,W^*}$ from interference terms, e.g.



corresponding to non-vanishing CP-odd basis invariants

$$\mathcal{I}_1 \;=\; \left[Y^\dagger_{\sigma_1 W W^*}\right]_{k\ell} \left[Y_{\sigma_1 \tau_2 \tau_2^*}\right]_{ij} \left[Y_{\tau_2^* W W^*}\right]_{imk} \left[\left(Y_{\tau_2^* W W^*}\right)^*\right]_{jm\ell}\,,$$
$$\mathcal{I}_2 \;=\; \left[Y^\dagger_{\sigma_1 W W^*}\right]_{k\ell} \left[Y_{\sigma_1 \tau_2 \tau_2^*}\right]_{ij} \left[Y_{\tau_2^* W W^*}\right]_{i\ell m} \left[\left(Y_{\tau_2^* W W^*}\right)^*\right]_{jkm}\,.$$

- ✓ Contribution to $\varepsilon_{\sigma_1 \to W\,W^*}$ is proportional to $\operatorname{Im}\mathcal{I}_{1,2} \neq 0$.
- ✓ All CP odd phases are geometrical, $\mathcal{I}_1 = \mathrm{e}^{2\,\pi\,\mathrm{i}/3}\,\mathcal{I}_2$.
- ✓ $\left(\varepsilon_{\sigma_1 \to W\,W^*}\right) \to 0$ for $v \to 0$, i.e. CP is restored in limit of vanishing VEV.

Natural protection of $\theta = 0$

Topological vacuum term of the gauge group

$$\mathscr{L}_\theta \;=\; \theta\,\frac{g^2}{32\pi^2}\,G^a_{\mu\nu}\,\widetilde{G}^{\mu\nu,a}\;,$$

is forbidden by $\mathbb{Z}_2 - \mathrm{Out}$ (the $\mathrm{SU}(3)$-CP transformation).

The unbroken Out

$$\mathbb{Z}_2 - \mathrm{Out} \;:\; W_\mu(x) \;\mapsto\; \mathcal{P}_\mu^{\;\nu}\,W^*_\nu(\mathcal{P}\,x)\;, \quad Z_\mu(x) \;\mapsto\; -\,\mathcal{P}_\mu^{\;\nu}\,Z_\nu(\mathcal{P}\,x)\;,$$

still enforces $\theta = 0$ even though CP is violated for the physical T_7 states.

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Physical scalars (T_7 singlets and triplets):

$$\begin{aligned}\mathrm{Re}\,\sigma_0 \;&=\; \frac{1}{\sqrt{2}}\,(\phi_1+\phi_1^*)\;, \qquad\qquad \mathrm{Im}\,\sigma_0 \;=\; -\,\frac{\mathrm{i}}{\sqrt{2}}\,(\phi_1-\phi_1^*)\;,\\ \sigma_1 \;&=\; \phi_2\;,\end{aligned}$$

$$\begin{pmatrix}\tau_1\\ \tau_2\\ \tau_3\end{pmatrix} \;=\; \begin{pmatrix}V_{11} & V_{12} & V_{13}\\ V_{21} & V_{22} & V_{23}\\ V_{31} & V_{32} & V_{33}\end{pmatrix}\begin{pmatrix}T_2\\ \overline{T}^*_3\\ T_1\end{pmatrix}\;.$$

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Possible application to strong CP problem?

- Starting point: CP conserving theory based on

$$[G_{\mathrm{SM}} \times G_{\mathrm{F}}] \rtimes \mathrm{CP}\;.$$

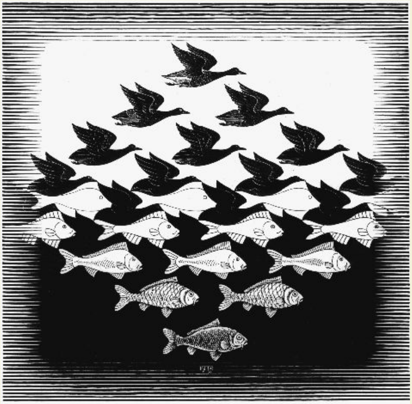
- break $G_{\mathrm{F}} \rtimes \mathrm{CP} \;\longrightarrow\; \mathrm{Type\,I} \rtimes \mathrm{Out}.$

↷ CP broken in flavor sector but not in strong interactions.

- Main problem: finding realistic model based on Type I group allowing for outer automorphism.

Summary

- Outer automorphisms are **symmetries of symetries** ($\rightarrow$ think of them as mappings among the irreps).
- CP is **a** special outer automorphism which maps *all* present representations to their complex conjugate representation.
- There are **"type I"** groups which do not allow for CP trafos $\Rightarrow$ CPV (explicit/spontaneous) with quantized phases.
- Concept of Outs is more general than C and/or P; Outs give very profound insights about models (permutation of invariants, calculability of VEVs, emergent symmetries...).
- Physical interpretation of one and the same transformation (namely the $\mathbb{Z}_2$-Out) can change depending on the symmetries of the ground state of a model.
- Explicit toy model example, $\mathrm{SU}(3) \rightarrow \mathrm{T}_7$, in which CP is spontaneously violated for the physical states of the theory (with quantized phases) while an unbroken outer automorphism protects $\theta = 0$.



Thank You

Backup slides

Basically two types of discrete groups

- Groups which **do not** allow for CP transformation: **Type I**
  Fine print: assuming sufficient # of irreps are there

G	$\mathbb{Z}_5 \rtimes \mathbb{Z}_4$	T_7	$\Delta(27)$	$\mathbb{Z}_9 \rtimes \mathbb{Z}_3$	$\ldots$
SG id	$(20,3)$	$(21,1)$	$(27,3)$	$(27,4)$	

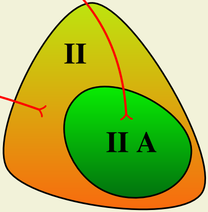
- Groups which **do** allow for CP transformation: **Type II**

Among those: all groups which allow for real CG's: Type II **A**

G	S_3	A_4	T'	S_4	A_5
SG id	$(6,1)$	$(12,3)$	$(24,3)$	$(24,12)$	$(60,5)$

But also: CP trafo w/o real CG's: Type II **B**

G	$\Sigma(72)$	$((\mathbb{Z}_3 \times \mathbb{Z}_3) \rtimes \mathbb{Z}_4) \rtimes \mathbb{Z}_4$
SG id	$(72,41)$	$(144,120)$



Type II A groups: CP violation completely analogue to well–known case: $\mathbf{SU}(N)$
(i.e. it depends on # of rephasing d.o.f.'s vs # complex couplings)
Type II B groups: CP violation tied to certain operators

The model

3HDM model with $[\boldsymbol{\Delta(27)} \Rightarrow]$ $\boldsymbol{\Delta(54)}$ symmetry.

(This is the original "geometrical T violation" model of Branco, Gerard, and Grimus.) [Branco, Gerard, Grimus, '83]

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- Triplet $H := (H_1, H_2, H_3)$ of Higgs doublets H_i, each transforming as $(\mathbf{1}, \mathbf{2})_{1/2}$ under G_{SM}.

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$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, C = \pm \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} .$$

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- “Traditional” way to write the potential: $(i, j = 1, .., 3;\ i \neq j)$

$$\begin{aligned} V \;=\; & -m^2\, H_i^\dagger H_i + \lambda_1 \left(H_i^\dagger H_i\right)^2 + \lambda_2 \left(H_i^\dagger H_i\right)\left(H_j^\dagger H_j\right) + \lambda_3 \left(H_i^\dagger H_j\right)\left(H_j^\dagger H_i\right) \\ & + \mathrm{e}^{\mathrm{i}\,\Omega} \lambda_4 \left[\left(H_1^\dagger H_2\right)\left(H_1^\dagger H_3\right) + \text{cyclic}\right] + \mathrm{h.c.}\;. \end{aligned}$$

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$$+\, \mathrm{e}^{\mathrm{i}\,\Omega}\lambda_4 \left[\left(H_1^\dagger H_2\right)\left(H_1^\dagger H_3\right) + \text{cyclic}\right] + \mathrm{h.c.}\,.$$

- Notation:

$$\langle 0|\, H_i\, |0\rangle \equiv \langle H_i\rangle := \begin{pmatrix} 0 \\ \mathrm{v}_i\, \mathrm{e}^{\mathrm{i}\,\varphi_i} \end{pmatrix} \quad \text{for } i = 1,..,3$$
$$\langle H\rangle = (\mathrm{v}_1\,\mathrm{e}^{\mathrm{i}\,\varphi_1}, \mathrm{v}_2\,\mathrm{e}^{\mathrm{i}\,\varphi_2}, \mathrm{v}_3\,\mathrm{e}^{\mathrm{i}\,\varphi_3})^{\mathrm{T}}\,.$$

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- Potential gives rise to four classes of VEVs: $v_i = \dfrac{m}{\sqrt{2(a_0 + a_i)}}$, $\omega := \mathrm{e}^{2\pi\,\mathrm{i}/3}$

$$\langle H\rangle_{\mathrm{I}} = v_1 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \langle H\rangle_{\mathrm{II}} = v_2 \begin{pmatrix} \omega \\ 1 \\ 1 \end{pmatrix}, \langle H\rangle_{\mathrm{III}} = v_3 \begin{pmatrix} \omega^2 \\ 1 \\ 1 \end{pmatrix}, \langle H\rangle_{\mathrm{IV}} = v_4 \begin{pmatrix} \sqrt{3} \\ 0 \\ 0 \end{pmatrix} .$$

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Spontaneous geometrical CP violation

If a CP transformation $H \mapsto U H^*$ is a symmetry of the Lagrangian, then

$$\langle H \rangle \neq U \langle H \rangle^*$$

must hold in order for this CP transformation to be spontaneously violated. [Branco et al. '83]

- For example: $U = \mathbb{1}$ is a CP symmetry if $\Omega = 0, \pi$.
  It is broken by VEVs of type II and III.
  $\curvearrowright$ there appears a physical CPV phase: $\omega = \mathrm{e}^{2\pi\,\mathrm{i}/3}$.
- All possible forms of U are given by solutions to the "consistency condition" (for various u's)

$$U\, \rho_{\mathbf{3}^*}(g)\, U^\dagger \;=\; \rho_{\mathbf{3}}(u(g)).$$

[Holthausen, Lindner, Schmidt, '13; Feruglio, Hagedorn, Ziegler, '13]

$\Rightarrow$ Actually: CP transformations are special outer automorphism transformations of all present symmetries (in particular $\boldsymbol{\Delta(54)}$).

3HDM dictionary

A more "natural" way to write the potential:

$$\begin{aligned}\left[\left(H^\dagger_{\bar{\mathbf{3}}}\otimes H_{\mathbf{3}}\right)\otimes\left(H^\dagger_{\bar{\mathbf{3}}}\otimes H_{\mathbf{3}}\right)\right]_{\mathbf{1}_0} &= a_0\left[\left(H^\dagger\otimes H\right)_{\mathbf{1}_0}\otimes\left(H^\dagger\otimes H\right)_{\mathbf{1}_0}\right]\\ &+\frac{a_1}{\sqrt{2}}\left[\left(H^\dagger\otimes H\right)_{\mathbf{2}_1}\otimes\left(H^\dagger\otimes H\right)_{\mathbf{2}_1}\right]_{\mathbf{1}_0}+\frac{a_2}{\sqrt{2}}\left[\left(H^\dagger\otimes H\right)_{\mathbf{2}_3}\otimes\left(H^\dagger\otimes H\right)_{\mathbf{2}_3}\right]_{\mathbf{1}_0}\\ &+\frac{a_3}{\sqrt{2}}\left[\left(H^\dagger\otimes H\right)_{\mathbf{2}_4}\otimes\left(H^\dagger\otimes H\right)_{\mathbf{2}_4}\right]_{\mathbf{1}_0}+\frac{a_4}{\sqrt{2}}\left[\left(H^\dagger\otimes H\right)_{\mathbf{2}_2}\otimes\left(H^\dagger\otimes H\right)_{\mathbf{2}_2}\right]_{\mathbf{1}_0}\;.\end{aligned}$$

Relations between the two different bases:

$$\begin{aligned} 3\,\lambda_1 &= a_0+a_4\;,\quad 3\,\lambda_2 = 2a_0-a_4\;,\quad 3\,\lambda_3 = a_1+a_2+a_3\;,\\ 3\,\lambda_4 &= \left|a_1+\omega^2\,a_2+\omega\,a_3\right|\;,\quad \text{and}\quad \Omega = \arg\left(a_1+\omega^2\,a_2+\omega\,a_3\right)\;.\end{aligned}$$

Bounded–below criterions:

$$0<\lambda_1\qquad\text{and}\qquad 0<\lambda_1+\lambda_{23}+2\,\lambda_4\,\cos\left[2\pi/3+(\Omega \mod 2\pi/3)\right]\;,$$

vs.

$$0<a_0+a_\ell\;,\qquad\text{for}\;\;\ell=1,..,4\;.$$

Invariants spelled out:

$$
\begin{aligned}
I_0(H^\dagger, H) &= \frac{1}{3}\left(H_1^\dagger H_1 + H_2^\dagger H_2 + H_3^\dagger H_3\right)^2 \ , \\
I_1(H^\dagger, H) &= \frac{\sqrt{2}}{3}\Big[\left(H_1^\dagger H_2 H_1^\dagger H_3 + H_2^\dagger H_1 H_2^\dagger H_3 + H_3^\dagger H_1 H_3^\dagger H_2 + \text{h.c.}\right) + \\
&\qquad H_1^\dagger H_2 H_2^\dagger H_1 + H_1^\dagger H_3 H_3^\dagger H_1 + H_2^\dagger H_3 H_3^\dagger H_2\Big] \ , \\
I_2(H^\dagger, H) &= \frac{\sqrt{2}}{3}\Big[H_1^\dagger H_1 H_1^\dagger H_1 + H_2^\dagger H_2 H_2^\dagger H_2 + H_3^\dagger H_3 H_3^\dagger H_3 \\
&\qquad -H_1^\dagger H_1 H_2^\dagger H_2 + -H_1^\dagger H_1 H_3^\dagger H_3 + -H_2^\dagger H_2 H_3^\dagger H_3\Big] \ , \\
I_3(H^\dagger, H) &= \frac{\sqrt{2}}{3}\Big[\left(\omega^2 H_1^\dagger H_2 H_1^\dagger H_3 + \omega^2 H_2^\dagger H_1 H_2^\dagger H_3 + \omega^2 H_3^\dagger H_1 H_3^\dagger H_2 + \text{h.c.}\right) + \\
&\qquad H_1^\dagger H_2 H_2^\dagger H_1 + H_1^\dagger H_3 H_3^\dagger H_1 + H_2^\dagger H_3 H_3^\dagger H_2\Big] \ , \\
I_4(H^\dagger, H) &= \frac{\sqrt{2}}{3}\Big[\left(\omega H_1^\dagger H_2 H_1^\dagger H_3 + \omega H_2^\dagger H_1 H_2^\dagger H_3 + \omega H_3^\dagger H_1 H_3^\dagger H_2 + \text{h.c.}\right) + \\
&\qquad H_1^\dagger H_2 H_2^\dagger H_1 + H_1^\dagger H_3 H_3^\dagger H_1 + H_2^\dagger H_3 H_3^\dagger H_2\Big] \ .
\end{aligned}
$$

“Physical” CP transformation

Recall: e.g. complex scalar field σ, with field operator

$$\widehat{\boldsymbol{\sigma}}(x) \;=\; \int \widetilde{\mathrm{d}p}\, \left\{ \widehat{\boldsymbol{a}}(\vec{p})\, \mathrm{e}^{-\mathrm{i}\,p\,x} + \widehat{\boldsymbol{b}}^{\dagger}(\vec{p})\, \mathrm{e}^{\mathrm{i}\,p\,x} \right\} \;.$$

Physical CP transformation of the complex scalar field

$$\mathrm{CP} \;:\quad \sigma(x) \;\mapsto\; \mathrm{e}^{\mathrm{i}\,\varphi}\, \sigma^{*}(\mathcal{P}\,x)\;,$$

corresponds to

$$\mathrm{CP} \;:\quad \widehat{\boldsymbol{a}}(\vec{p}) \;\mapsto\; \mathrm{e}^{\mathrm{i}\,\varphi}\, \widehat{\boldsymbol{b}}(-\vec{p}) \qquad \text{and} \qquad \widehat{\boldsymbol{b}}^{\dagger}(\vec{p}) \;\mapsto\; \mathrm{e}^{\mathrm{i}\,\varphi}\, \widehat{\boldsymbol{a}}^{\dagger}(-\vec{p})\;.$$

Note:

“matter”: $\widehat{\boldsymbol{a}}^{(\dagger)}$ “anti-matter”: $\widehat{\boldsymbol{b}}^{(\dagger)}$.

Toy model details

Complex scalar ϕ in T_7-diagonal basis of $\mathrm{SU}(3)$: (in unitary gauge)

$$\phi \;=\; \left(v\,+\,\phi_1,\,\frac{\phi_2}{\sqrt{2}},\,\frac{\phi_2^*}{\sqrt{2}},\,\phi_4,\,\phi_5,\,\phi_6,\,\frac{\phi_7}{\sqrt{2}},\,\frac{\phi_8}{\sqrt{2}},\,\frac{\phi_9}{\sqrt{2}},\,\phi_{10},\,\phi_{11},\,\phi_{12},\,\frac{\phi_7^*}{\sqrt{2}},\,\frac{\phi_8^*}{\sqrt{2}},\,\frac{\phi_9^*}{\sqrt{2}}\right)\;.$$

T_7 representations of the components:

$$\begin{aligned}
\phi_1 \;&\widehat{=}\; \mathbf{1}_0\;, & \phi_2 \;&\widehat{=}\; \mathbf{1}_1\;,\\
T_1 \;&:=\; (\phi_4,\,\phi_5,\,\phi_6) \;\widehat{=}\; \mathbf{3}\;, & T_2 \;&:=\; (\phi_7,\,\phi_8,\,\phi_9) \;\widehat{=}\; \mathbf{3}\;,\\
\overline{T}_3 \;&:=\; (\phi_{10},\,\phi_{11},\,\phi_{12}) \;\widehat{=}\; \overline{\mathbf{3}}\;.
\end{aligned}$$

The physical scalars are

$$\begin{aligned}
\mathrm{Re}\,\sigma_0 \;&=\; \frac{1}{\sqrt{2}}\left(\phi_1+\phi_1^*\right)\;, & \mathrm{Im}\,\sigma_0 \;&=\; -\,\frac{\mathrm{i}}{\sqrt{2}}\left(\phi_1-\phi_1^*\right)\;,\\
\sigma_1 \;&=\; \phi_2\;,
\end{aligned}$$

$$\begin{pmatrix}\tau_1\\ \tau_2\\ \tau_3\end{pmatrix} \;=\; \begin{pmatrix}V_{11} & V_{12} & V_{13}\\ V_{21} & V_{22} & V_{23}\\ V_{31} & V_{32} & V_{33}\end{pmatrix}\begin{pmatrix}T_2\\ \overline{T}_3^*\\ T_1\end{pmatrix}\;.$$

The physical vectors are

$$\begin{aligned}
Z^\mu \;&=\; \frac{1}{\sqrt{2}}\left(A_7^\mu-\mathrm{i}\,A_8^\mu\right)\;, & W_1^\mu \;&=\; \frac{1}{\sqrt{2}}\left(A_4^\mu-\mathrm{i}\,A_1^\mu\right)\;,\\
W_2^\mu \;&=\; \frac{1}{\sqrt{2}}\left(A_5^\mu-\mathrm{i}\,A_2^\mu\right)\;, & W_3^\mu \;&=\; \frac{\mathrm{i}}{\sqrt{2}}\left(A_6^\mu-\mathrm{i}\,A_3^\mu\right)\;.
\end{aligned}$$

Toy model details

The VEV in this basis is simply

$$\langle\phi\rangle_1 \;=\; v \quad \text{and} \quad \langle\phi\rangle_i \;=\; 0 \quad \text{for} \quad i=2,\dots,15\;,$$

where

$$|v| \;=\; \mu\times 3\sqrt{\frac{7}{2}}\left(-7\sqrt{15}\,\lambda_1+14\sqrt{15}\,\lambda_2+20\sqrt{6}\,\lambda_4+13\sqrt{15}\,\lambda_5\right)^{-1/2}\;.$$

The masses of the physical states are

$$m_Z^2 \;=\; \frac{7}{3}\,g^2\,v^2 \quad \text{and} \quad m_W^2 \;=\; g^2\,v^2\;.$$

$$\begin{aligned} m^2_{\mathrm{Re}\,\sigma_0} \;&=\; 2\,\mu^2\;, \qquad m^2_{\mathrm{Im}\,\sigma_0} \;=\; 0\;,\\ m^2_{\sigma_1} \;&=\; -\,\mu^2+\sqrt{15}\,\lambda_5\,v^2\;. \end{aligned}$$

The massless mode is the goldstone boson of an additional $\mathrm{U}(1)$ symmetry of the potential. It can be avoided by either

- gauging the additional $\mathrm{U}(1)$,
- or breaking it softly by a cubic coupling of ϕ.

Toy model details

T_7 invariant couplings ($\omega := \mathrm{e}^{2\pi\mathrm{i}/3}$)

$$Y_{\sigma_1 W W^*} \;=\; \frac{v\,g^2}{\sqrt{6}}\,\mathrm{e}^{-\pi\,\mathrm{i}/6}\,\mathrm{diag}(1,\,\omega,\,\omega^2)\;,\quad Y_{\sigma_1\tau_2\tau_2^*} \;=\; v\,y_{\sigma_1\tau_2\tau_2^*}\,\mathrm{diag}(1,\,\omega,\,\omega^2)\;,$$

$$\begin{aligned}\left[Y_{\tau_2^* W W^*}\right]_{121} &= \left[Y_{\tau_2^* W W^*}\right]_{232} = \left[Y_{\tau_2^* W W^*}\right]_{313} = v\,g^2\,y_{\tau_2^* W W^*}\;,\\ \left[Y_{\tau_2^* W W^*}\right]_{ijk} &= 0 \qquad \text{(else)}\;.\end{aligned}$$

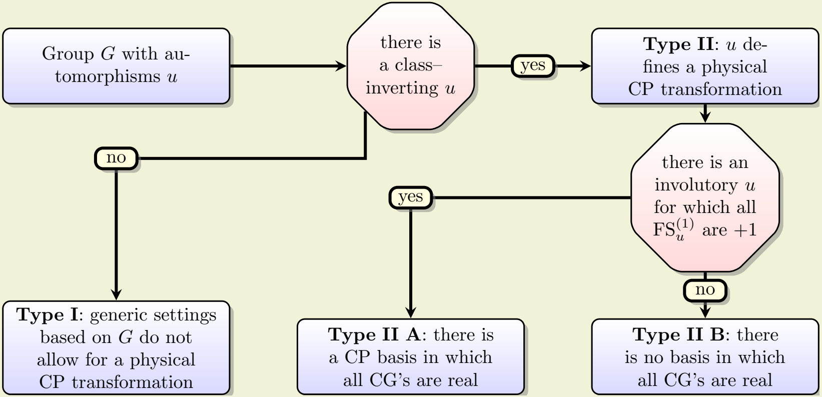
Toy model details

$$
\begin{aligned}
y_{\sigma_1 \tau_2 \tau_2^*} \;=\; \frac{1}{504\sqrt{3}} \Big\{ & V_{21}^2 \, \Big[-14\sqrt{10}\left(17+5\sqrt{3}\,\mathrm{i}\right)\lambda_1 + 84\sqrt{30}\left(\sqrt{3}-\mathrm{i}\right)\lambda_2 \\
& \quad -240\left(1+\sqrt{3}\,\mathrm{i}\right)\lambda_4 - \sqrt{10}\left(197-55\sqrt{3}\,\mathrm{i}\right)\lambda_5\Big] \\
& + 8\,V_{22}^2 \, \Big[28\sqrt{10}\left(1-\sqrt{3}\,\mathrm{i}\right)\lambda_1 - 14\sqrt{30}\,\mathrm{i}\,\lambda_2 + 112\sqrt{3}\,\mathrm{i}\,\lambda_3 \\
& \quad -\left(30-26\sqrt{3}\,\mathrm{i}\right)\lambda_4 + \sqrt{10}\left(20-\sqrt{3}\,\mathrm{i}\right)\lambda_5\Big] \\
& + 8\,V_{23}^2 \, \Big[28\sqrt{10}\left(1+\sqrt{3}\,\mathrm{i}\right)\lambda_1 - 14\sqrt{30}\,\mathrm{i}\,\lambda_2 - 168\,\lambda_3 \\
& \quad +\left(6+65\sqrt{3}\,\mathrm{i}\right)\lambda_4 - 4\sqrt{10}\left(1-2\sqrt{3}\,\mathrm{i}\right)\lambda_5\Big] \\
& + 8\,V_{21}\,V_{22} \, \Big[-35\sqrt{10}\left(1-\sqrt{3}\,\mathrm{i}\right)\lambda_1 + 21\sqrt{30}\left(\sqrt{3}+\mathrm{i}\right)\lambda_2 \\
& \quad -56\left(3+\sqrt{3}\,\mathrm{i}\right)\lambda_3 + 6\left(1+17\sqrt{3}\,\mathrm{i}\right)\lambda_4 - \sqrt{10}\left(67+19\sqrt{3}\,\mathrm{i}\right)\lambda_5\Big] \\
& + 4\,V_{21}\,V_{23} \, \Big[-28\sqrt{10}\left(2+\sqrt{3}\,\mathrm{i}\right)\lambda_1 - 42\sqrt{30}\left(\sqrt{3}+\mathrm{i}\right)\lambda_2 \\
& \quad +30\left(11+3\sqrt{3}\,\mathrm{i}\right)\lambda_4 - \sqrt{10}\left(31+11\sqrt{3}\,\mathrm{i}\right)\lambda_5\Big] \\
& - 8\,V_{22}\,V_{23} \, \Big[14\sqrt{10}\,\lambda_1 - 14\sqrt{30}\,\mathrm{i}\,\lambda_2 \\
& \quad +10\left(3+5\sqrt{3}\,\mathrm{i}\right)\lambda_4 + \sqrt{10}\left(1-3\sqrt{3}\,\mathrm{i}\right)\lambda_5\Big]\Big\}
\end{aligned}
$$

and

$$
y_{\tau_2^* W W^*} \;=\; -\frac{\sqrt{2}}{3}\left(2\,V_{21} + V_{22} + 2\,V_{23}\right) .
$$

CP symmetries in settings with discrete G



(For details see [Chen, Fallbacher, Mahanthappa, Ratz, AT, '14])

Mathematical tool to decide: Twisted Frobenius-Schur indicator FS_u(Backup slides)

Twisted Frobenius–Schur indicator

Criterion to decide: existence of a CP outer automorphism.

$\curvearrowright$ can be probed by computing the

"twisted Frobenius–Schur indicator" $\mathbf{FS}_{\boldsymbol{u}}$

$$\mathrm{FS}_u(\boldsymbol{r}_i) \;:=\; \frac{1}{|G|}\sum_{g\in G}\chi_{\boldsymbol{r}_i}(g\,u(g))$$

($\chi_{\boldsymbol{r}_i(g)}$: Character)

[Chen, Fallbacher, Mahanthappa, Ratz, AT, 2014]

$$\mathrm{FS}_u(\boldsymbol{r}_i) \;=\; \begin{cases} +1 \text{ or } -1 \quad \forall\, i, & \Rightarrow\; u \text{ is good for CP,} \\ \text{different from } \pm 1, & \Rightarrow\; u \text{ is no good for CP.} \end{cases}$$

In analogy to the Frobenius–Schur indicator

$\mathrm{FS}_{\cancel{u}}(\boldsymbol{r}_i) = +1, -1, 0$ for real / pseudo–real / complex irrep.

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