Thermalisation of Sterile Neutrinos

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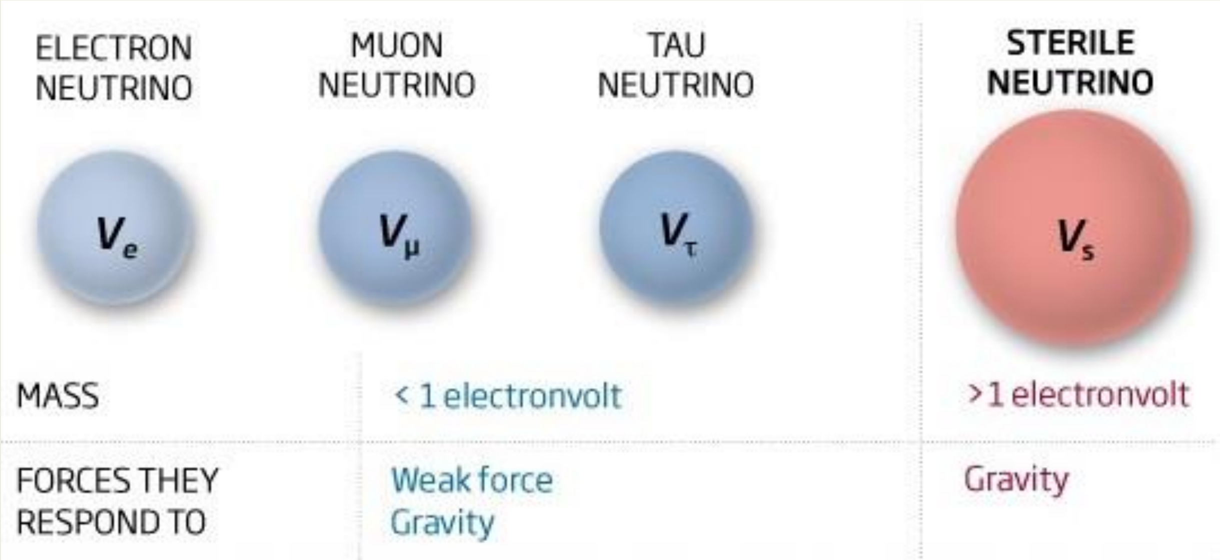
Outline

- Introduction to eV sterile neutrinos.
- Bounds from Cosmology.
- Standard sterile neutrino thermalisation.
- Thermalisation suppression by a large lepton asymmetry.
- Thermalisation suppression by a secret sterile neutrino interaction.
- Conclusions.

What is a sterile neutrino?

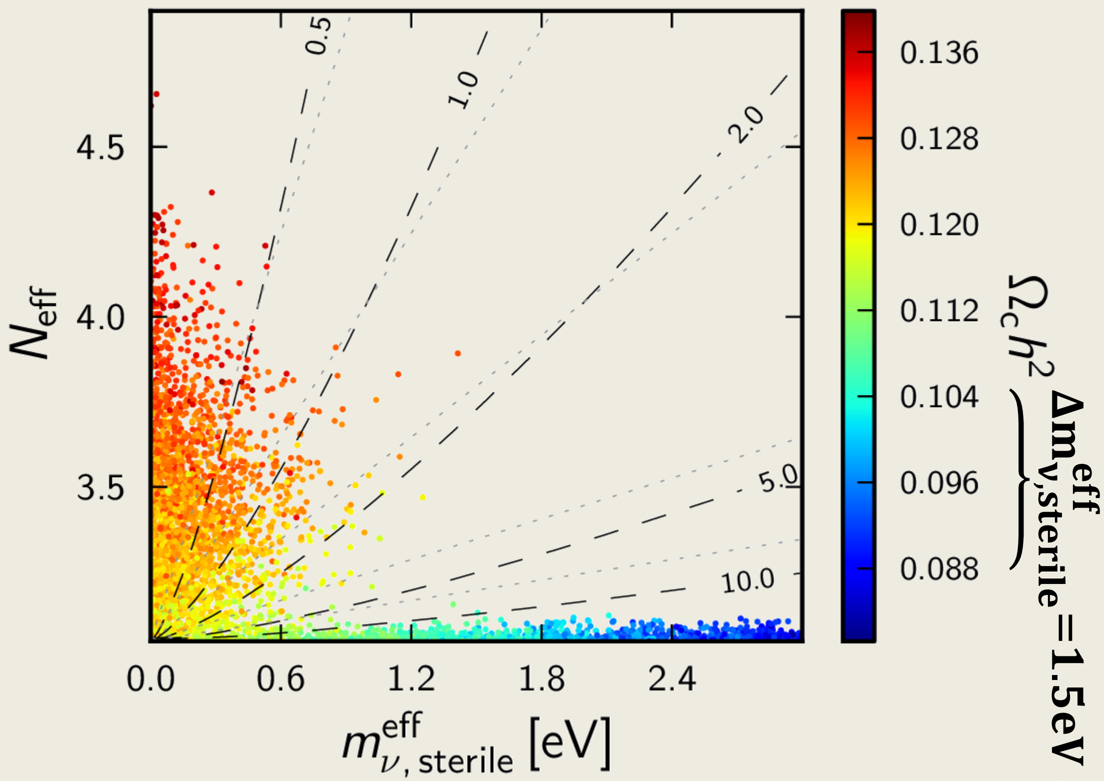


- Not charged under the SM gauge group
- Non-zero mixing angle with active neutrinos

Motivation: Neutrino anomalies

Experiment	What do they measure?	Estimate of significance
Nuclear reactors (ILL, Bugey, Gösgen...)	A small deficit in the $\bar{\nu}_e$ flux from ^{235}U, ^{238}U, ^{239}Pu and ^{241}Pu fission.	$\mathbf{2.5\sigma}$? 1309.4146 1101.2755
Galium detectors (SAGE and GALLEX)	A small deficit in the ν_e-flux from ^{51}Cr and ^{37}Ar decay.	$\mathbf{3.0\sigma}$ 1006.3244
Short baseline oscillation experiments (LSND and MiniBooNE)	$\bar{\nu}_\mu - \bar{\nu}_e$ and $\nu_\mu - \nu_e$ oscillations.	$\mathbf{3.8\sigma + 0\sigma + 3.0\sigma}$ (1007.1150 + ...)
Big Bang Nucleosynthesis (BBN)	Amount of radiation at T=1 MeV	**Consistent** with 0 or 1 fully thermalised neutrino.
Cosmic Microwave Background (WMAP + ACT/SPT + BAO + H_0)	Amount of radiation at recombination + other effects	$\mathbf{1.5\sigma - 2.5\sigma}$ ÷!? Planck (1009.0866 + ...)

Planck bounds



- Likelihood represented by density of points.
- Non-trivial that DW and thermal are equivalent.
- Large Neff models corresponds to more CDM.

$$\mathrm{m}_{\nu,\mathrm{sterile}}^{\mathrm{eff}} \equiv \begin{cases} (\Delta N_{\mathrm{eff}})^{3/4} m_{\mathrm{sterile}}^{\mathrm{thermal}} \\ \Delta N_{\mathrm{eff}} m_{\mathrm{sterile}}^{\mathrm{DW}} \end{cases}$$

$$\Omega_\nu h^2 = \frac{\mathrm{m}_{\nu,\mathrm{sterile}}^{\mathrm{eff}}}{94.1\mathrm{eV}}$$

Planck (neutrino?) anomalies

Planck+WP for ΛCDM model

Parameter	68% limits
Ω_m	0.316 ± 0.017
σ_8	0.829 ± 0.012
H_0 in $\frac{\text{km}}{\text{s}\cdot\text{Mpc}}$	$\mathbf{67.3 \pm 1.2}$
$\left(\frac{\sigma_8 \Omega_m}{0.27}\right)^{0.3}$	$\mathbf{0.869 \pm 0.023}$
$\left(\frac{\sigma_8 \Omega_m}{0.27}\right)^{0.46}$	$\mathbf{0.891 \pm 0.031}$

Other datasets

- Local H_0 measurement:

$$H_0 = \mathbf{73.8 \pm 2.4} \frac{\text{km}}{\text{s}\cdot\text{Mpc}}$$

- Cluster counts:

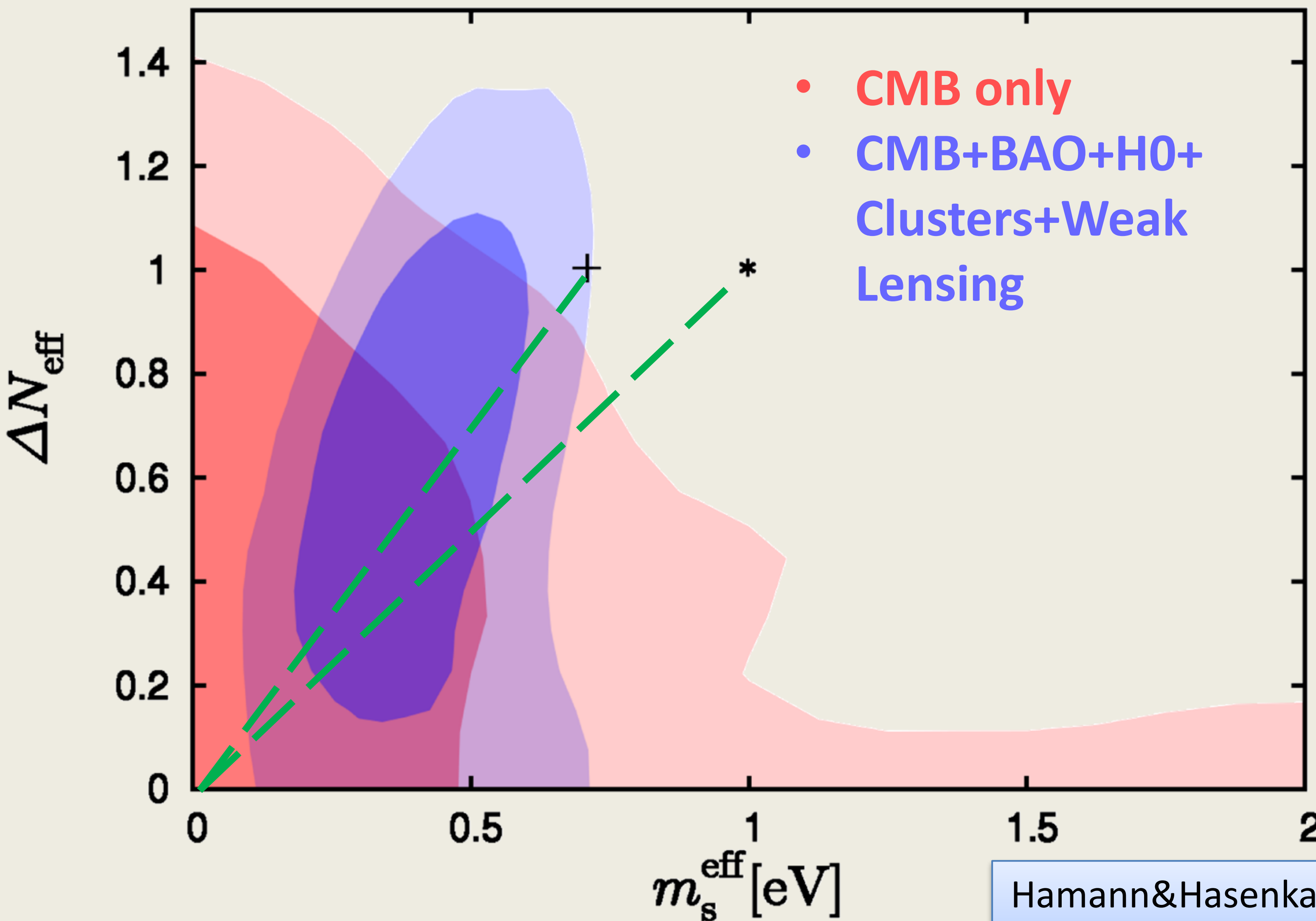
$$\left(\frac{\sigma_8 \Omega_m}{0.27}\right)^{0.3} = \mathbf{0.782 \pm 0.010}$$

- Weak lensing:

$$\left(\frac{\sigma_8 \Omega_m}{0.27}\right)^{0.46} = \mathbf{0.774 \pm 0.04}$$

Hamann&Hasenkamp
arXiv:1308.3255

Sterile neutrino resurrection?

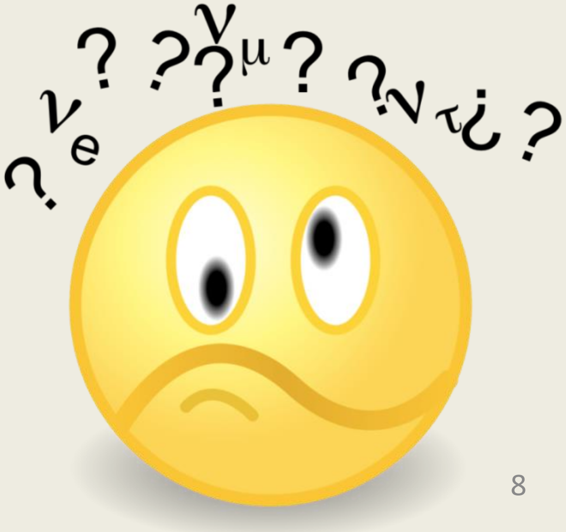


- CMB only
- CMB+BAO+H0+ Clusters+Weak Lensing

Hamann&Hasenkamp arXiv:1308.3255

Producing sterile neutrinos

- Freeze-Out at high temperature:
  - Initial population is completely diluted.
- Propagation and measurement!
  - Rate: $\Gamma_t \sim \sin^2(2\theta_m)\,\Gamma_\nu$
- Mixing angle depends on medium
- Quantum Zeno effect



1+1 approximation

- 2 level system ⇒ 2x2 density matrix

$$\rho = \begin{bmatrix} \text{active } \nu & \text{entanglement} \\ \text{entanglement} & \text{sterile } \nu \end{bmatrix}$$

- Hermitian and unitary ⇒ Expansion in Pauli matrices:

$$\rho = \frac{1}{2} f_0 (P_0 \mathbb{I} + \boldsymbol{P} \cdot \sigma)$$

$$\bar{\rho} = \frac{1}{2} f_0 (\overline{P_0} \mathbb{I} + \bar{\boldsymbol{P}} \cdot \sigma)$$

$$\mathbb{I} \equiv \begin{bmatrix} 1 & \\ & 1 \end{bmatrix}$$

$$\sigma_x \equiv \begin{bmatrix} & 1 \\ 1 & \end{bmatrix}$$

$$\sigma_y \equiv \begin{bmatrix} & -\mathbb{i} \\ \mathbb{i} & \end{bmatrix}$$

$$\sigma_z \equiv \begin{bmatrix} 1 & \\ & -1 \end{bmatrix}$$

$$\rho = \begin{bmatrix} \nu_a & \text{ent.} \\ \text{ent.} & \nu_s \end{bmatrix}$$

Change of variables

We do the change of variables:

$$P_a^{\pm} = P_0 + P_z \pm (\bar{P}_0 + \bar{P}_z)$$
$$P_s^{\pm} = P_0 - P_z \pm (\bar{P}_0 - \bar{P}_z)$$
$$P_x^{\pm} = P_x \pm \bar{P}_x$$
$$P_y^{\pm} = P_y \pm \bar{P}_y$$

Note that:

$$P_a^{\pm} = \frac{2}{f_0}(\rho_{aa} \pm \bar{\rho}_{aa})$$
$$P_s^{\pm} = \frac{2}{f_0}(\rho_{ss} \pm \bar{\rho}_{ss})$$

$$\mathbb{I} \equiv \begin{bmatrix} 1 & \\ & 1 \end{bmatrix}$$
$$\sigma_x \equiv \begin{bmatrix} & 1 \\ 1 & \end{bmatrix}$$
$$\sigma_y \equiv \begin{bmatrix} & -\mathbb{i} \\ \mathbb{i} & \end{bmatrix}$$
$$\sigma_z \equiv \begin{bmatrix} 1 & \\ & -1 \end{bmatrix}$$

$$\rho = \begin{bmatrix} \nu_a & \text{ent.} \\ \text{ent.} & \nu_s \end{bmatrix}$$

Quantum Kinetic Equations

The equations of motion become:

$$\dot{P}_a^{\pm} = V_x P_y^{\pm} + \Gamma_a \left[2\, f_{a,\mathrm{eq}}^{\pm}/f_0 - P_a^{\pm}\right]$$
$$\dot{P}_s^{\pm} = -V_x P_y^{\pm} + \Gamma_s \left[2\, f_{s,\mathrm{eq}}^{\pm}/f_0 - P_s^{\pm}\right]$$
$$\dot{P}_x^{\pm} = -\left(V_z P_y^{\pm} + V_L P_y^{\mp}\right) - D P_x^{\pm}$$
$$\dot{P}_y^{\pm} = +\left(V_z P_x^{\pm} + V_L P_x^{\mp}\right) - D P_y^{\pm}$$
$$- \tfrac{1}{2} V_x \left(P_a^{\pm} - P_s^{\pm}\right)$$

$$\dot{P}_a^{\pm} = V_x P_y^{\pm} + \Gamma_a\left[2 f_{a,\mathrm{eq}}^{\pm}/f_0 - P_a^{\pm}\right]$$

$$\dot{P}_s^{\pm} = -V_x P_y^{\pm} + \Gamma_s\left[2 f_{s,\mathrm{eq}}^{\pm}/f_0 - P_s^{\pm}\right]$$

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$$\rho = \begin{bmatrix} \nu_a & \text{ent.} \\ \text{ent.} & \nu_s \end{bmatrix}$$

- Effective scattering terms $\Gamma_a[\ldots]$ and $\Gamma_s[\ldots]$
- Coherrence damping $D \simeq \frac{1}{2}[\Gamma_a + \Gamma_s]$
- Vacuum and matter potentials:

$$V_x = \frac{\delta m_s^2}{2p_\nu} \sin 2\theta_s,\ V_L \sim G_F T^3 L,\ V_z = V_a + V_s - V_x$$

$$V_a \sim \frac{G_F}{M_Z^2} p_\nu T^4 n_{\nu_a}^{+},\ V_s \sim \frac{G_X}{M_X^2} p_\nu u_{\nu_s}^{+}$$

the code: LASAGNA

- Solves the QKEs numerically.
- Adaptive grid follows N resonances.
- Written in C, data analysis in MATLAB.
- Stiff ODE solvers: RADAU5 and ndf15.
- Linear Algebra solvers: dense, sparse, SuperLU.
- http://users-phys.au.dk/steen/codes.html

$$\rho = \begin{bmatrix} \nu_a & \text{ent.} \\ \text{ent.} & \nu_s \end{bmatrix}$$

Scenario 1: Lepton asymmetry

The equations of motion become:

$$\dot{P}_a^{\pm} = V_x P_y^{\pm} + \Gamma_a \left[2 f_{a,\text{eq}}^{\pm}/f_0 - P_a^{\pm}\right]$$

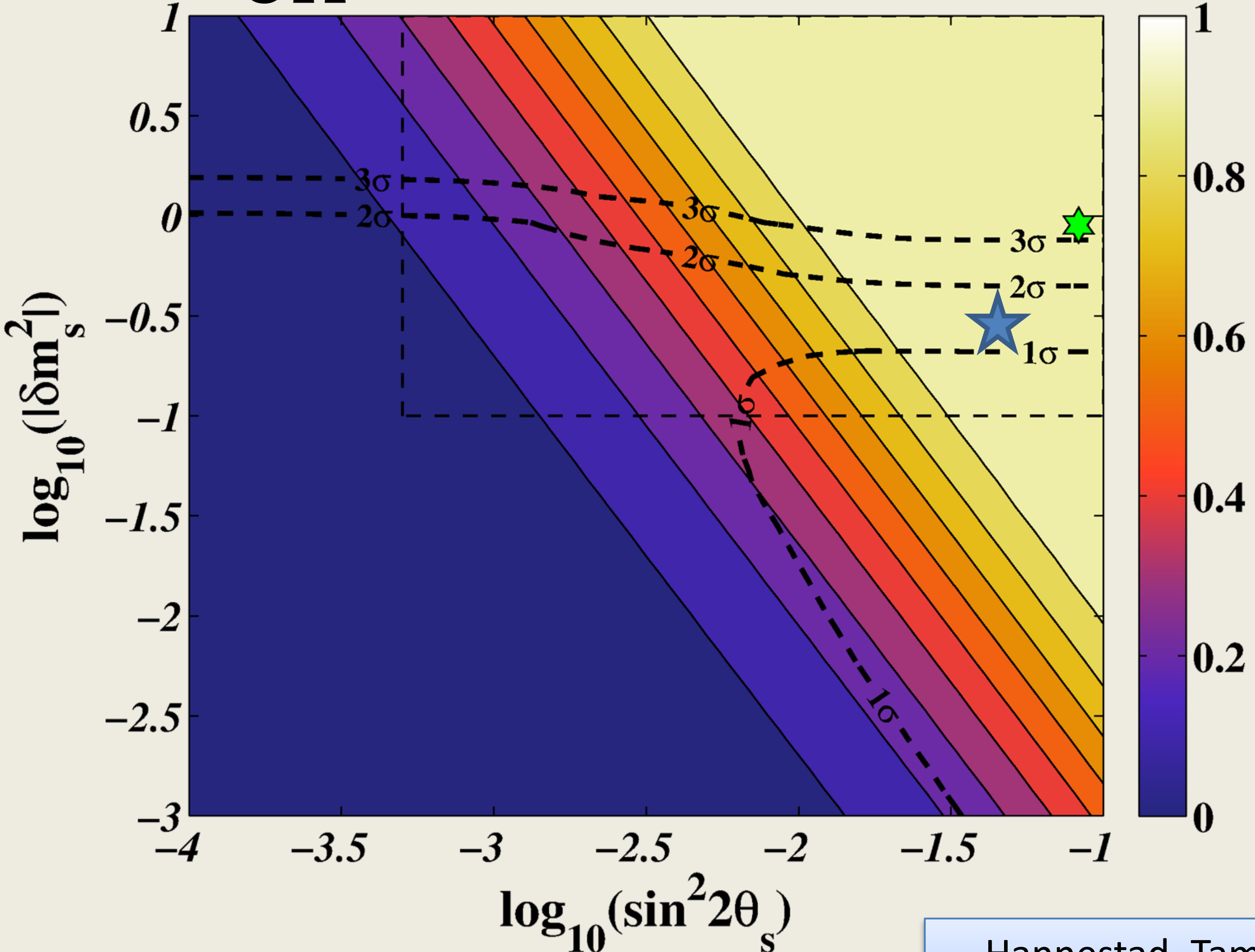
$$\dot{P}_s^{\pm} = -V_x P_y^{\pm} + \Gamma_s \left[2 f_{s,\text{eq}}^{\pm}/f_0 - P_s^{\pm}\right]$$

$$\dot{P}_x^{\pm} = -\left(V_z P_y^{\pm} + V_L P_y^{\mp}\right) - D P_x^{\pm}$$

$$\dot{P}_y^{\pm} = +\left(V_z P_x^{\pm} + V_L P_x^{\mp}\right) - D P_y^{\pm} - \tfrac{1}{2} V_x \left(P_a^{\pm} - P_s^{\pm}\right)$$

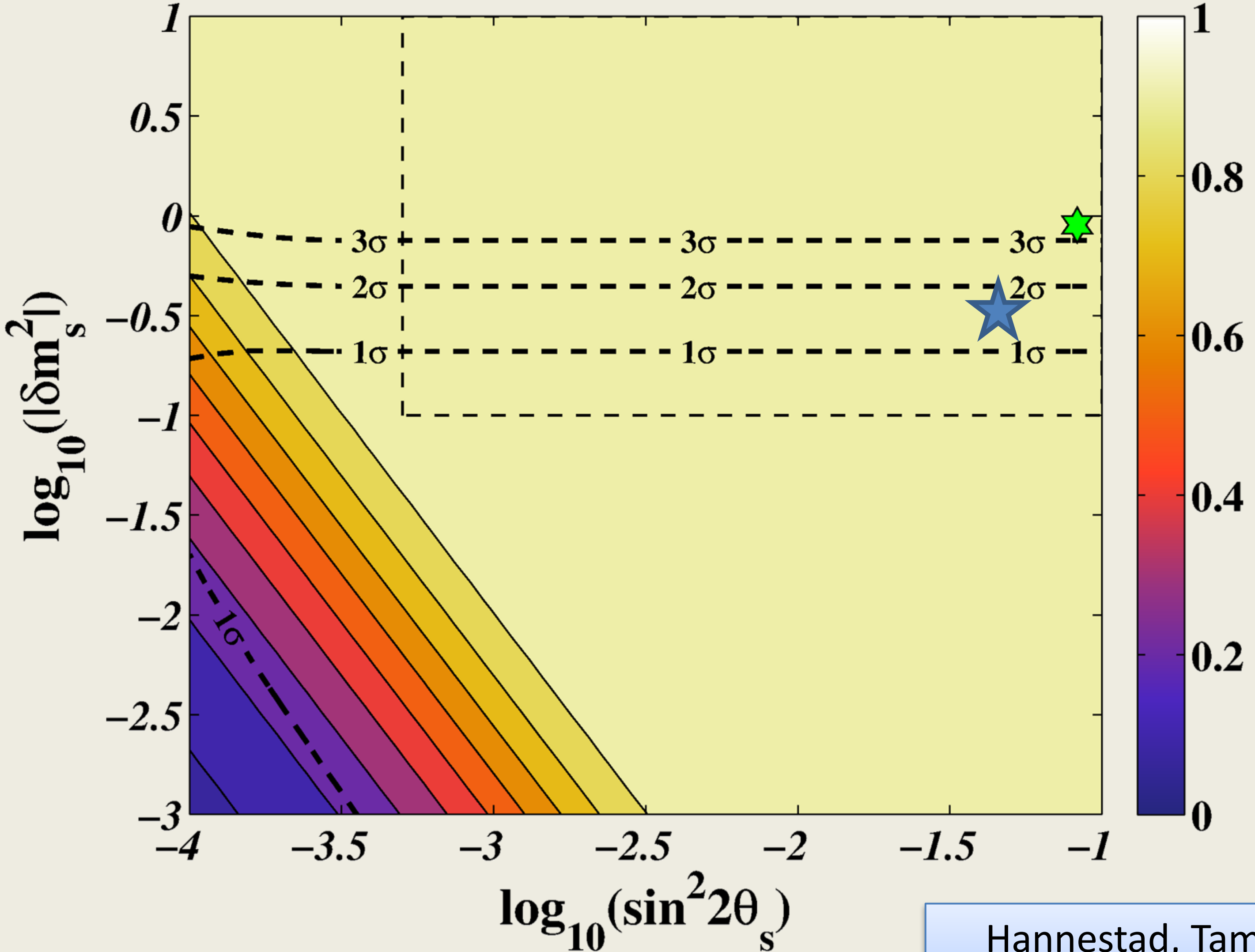
$$V_z = V_a + V_s - V_x$$

$\delta \mathrm{N_{eff}}$ for $\delta m_s^2 > 0, L = 0$



Hannestad, Tamborra, TT
arXiv:1204.5861

δN_{eff} for $\delta m_s^2 < 0, L = 0$



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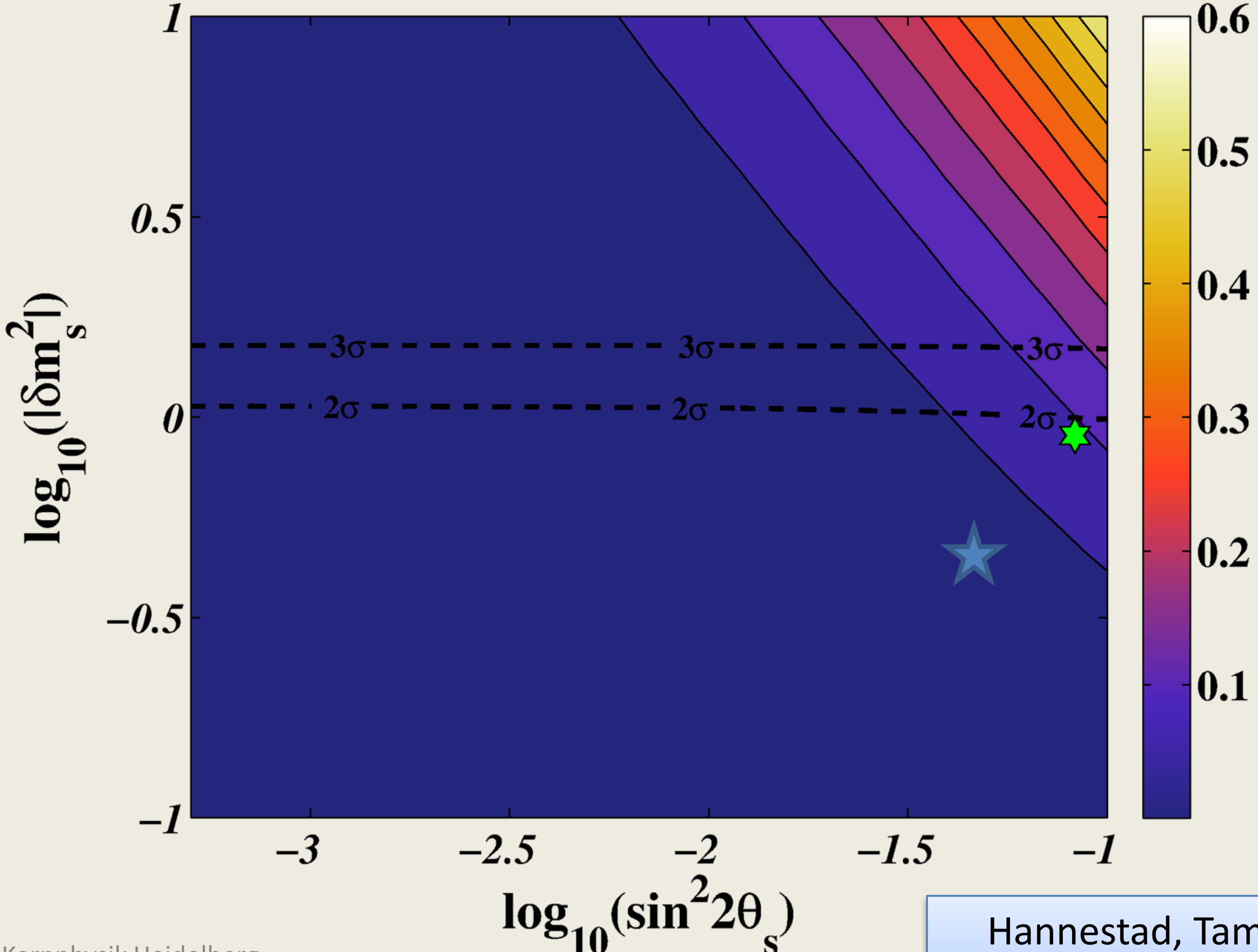
δN_{eff} for $\delta m_s^2 > 0, L = 10^{-2}$



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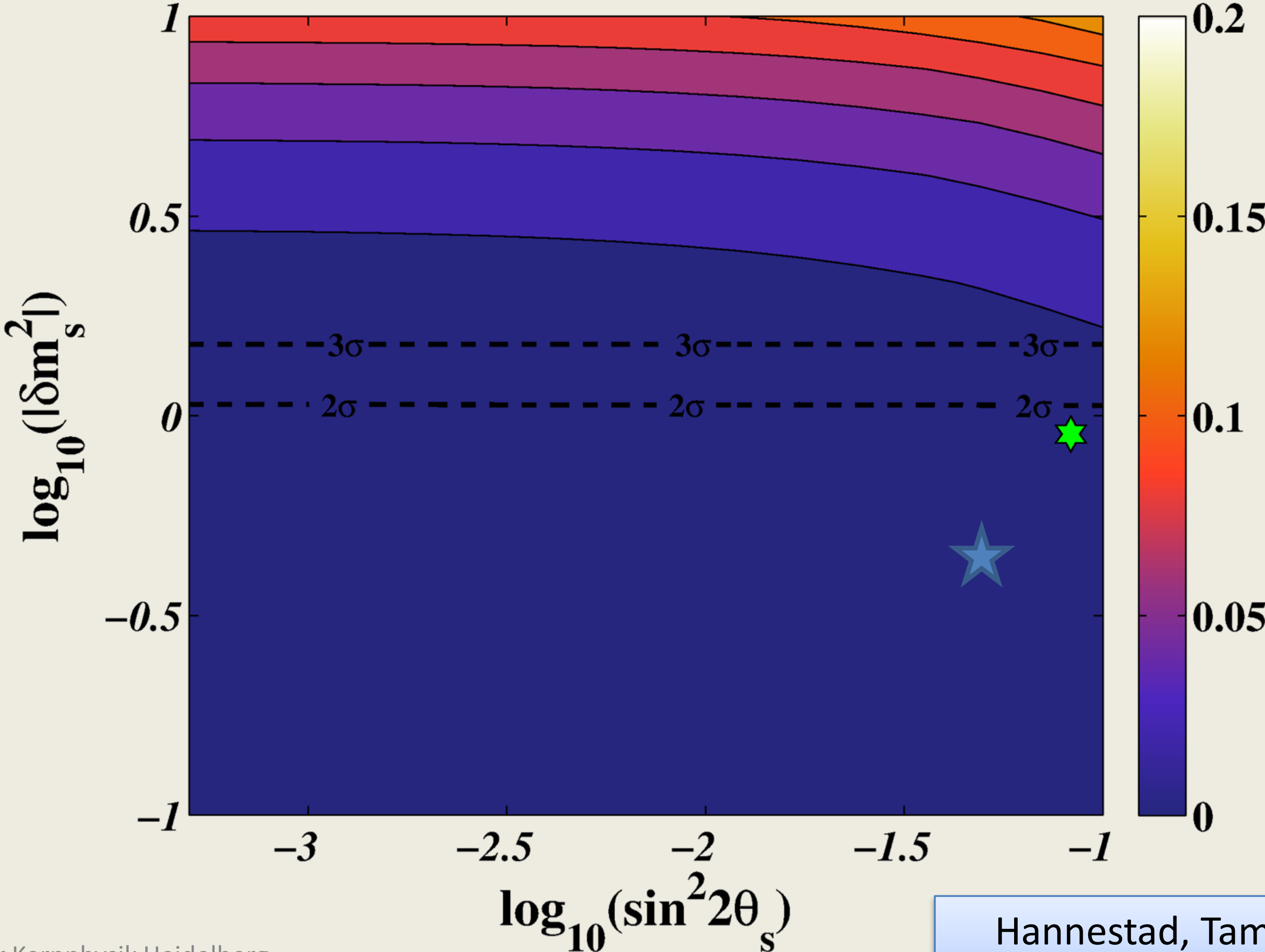
δN_{eff} for $\delta m_s^2 < 0, L = 10^{-2}$



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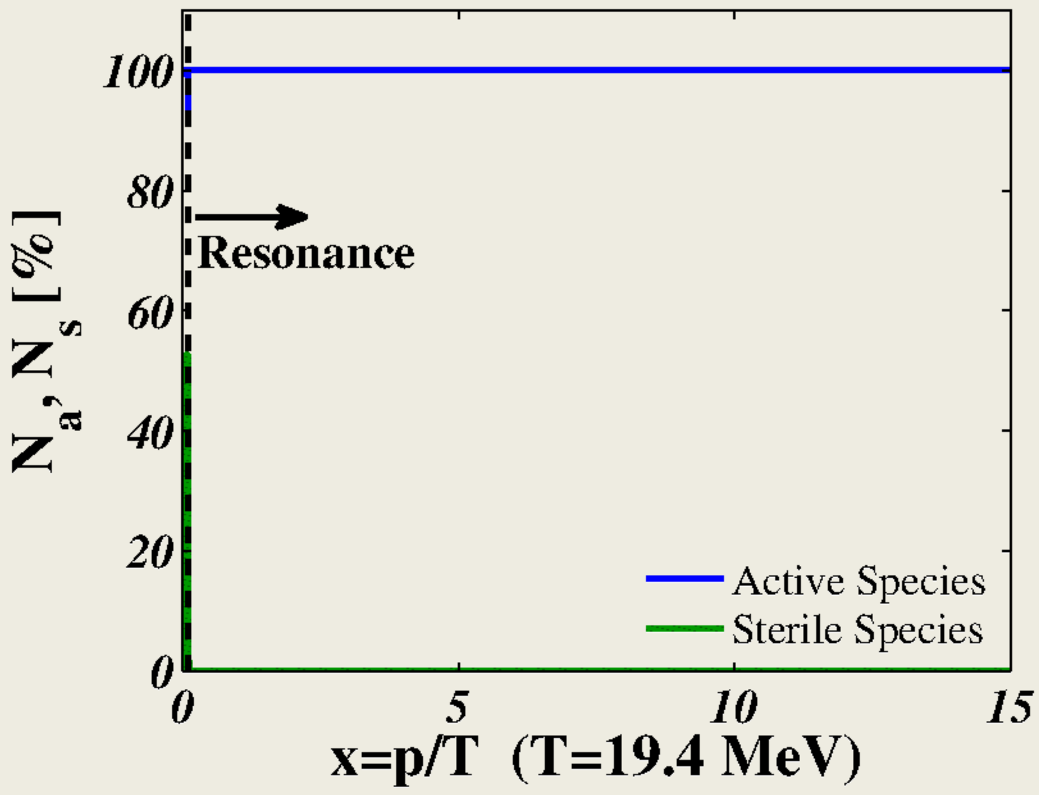
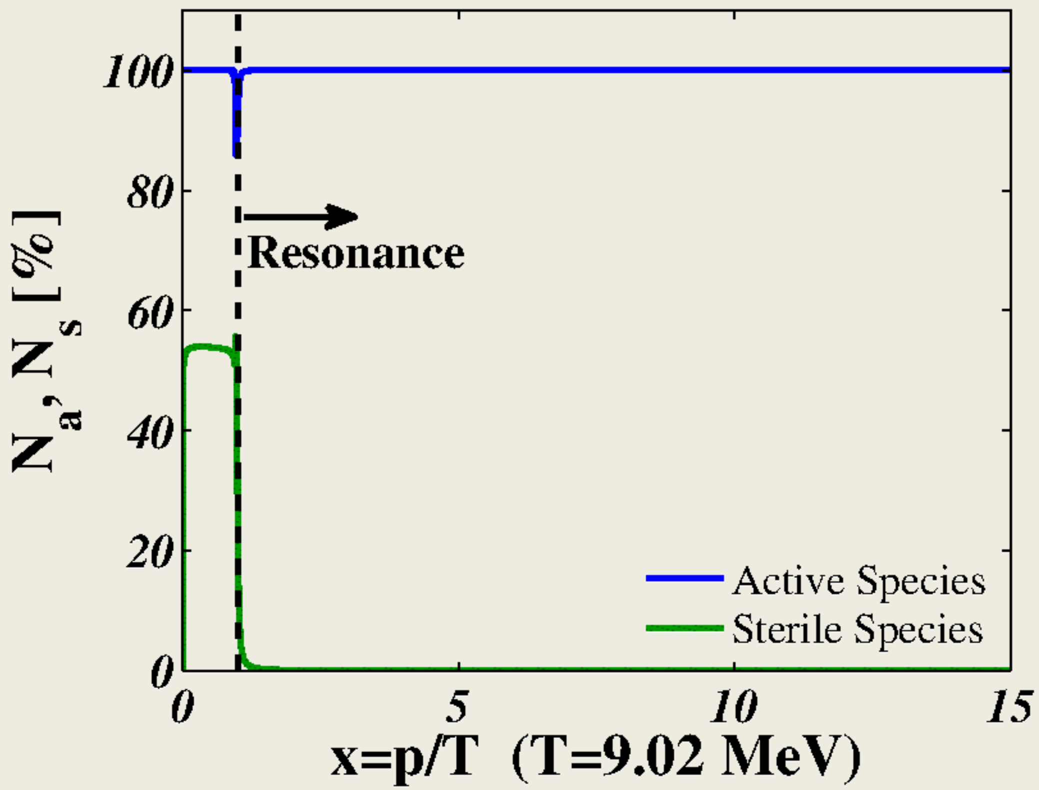
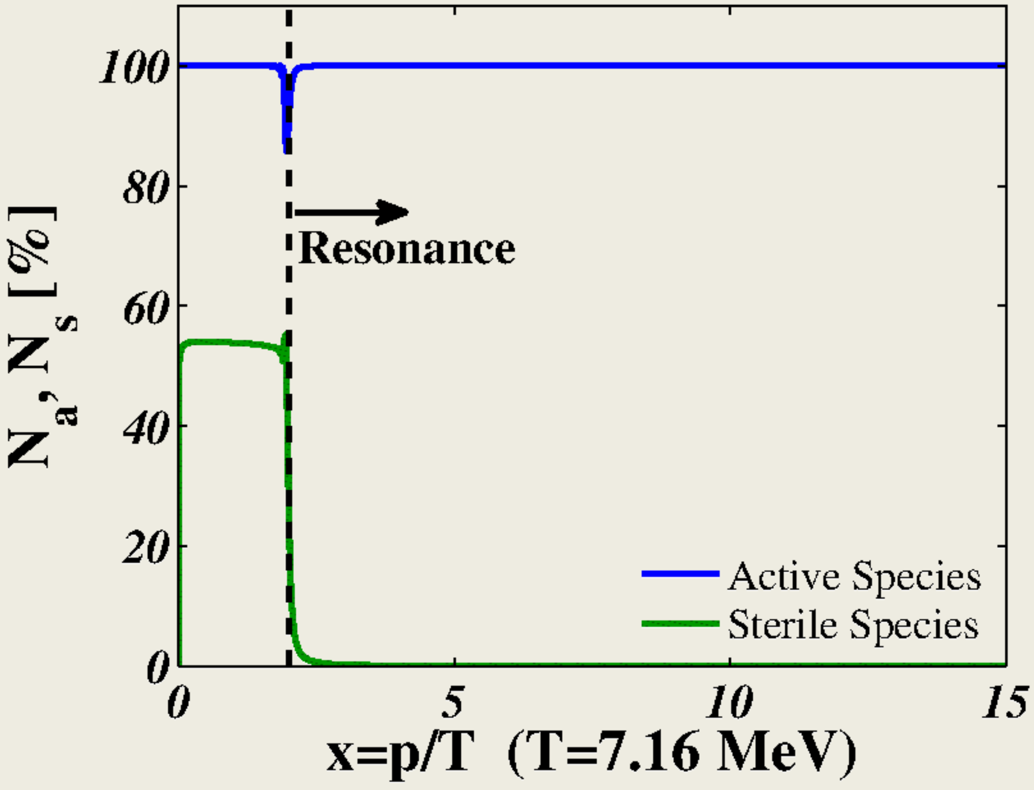
Thermalisation cartoon



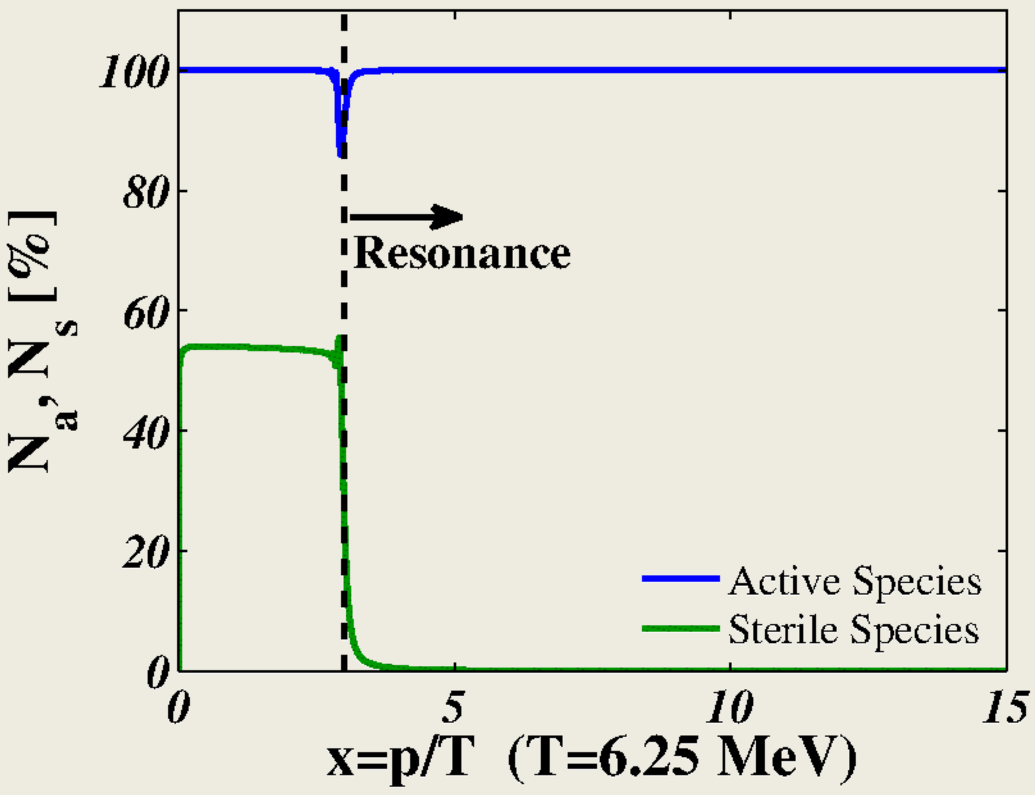
Thermalisation cartoon



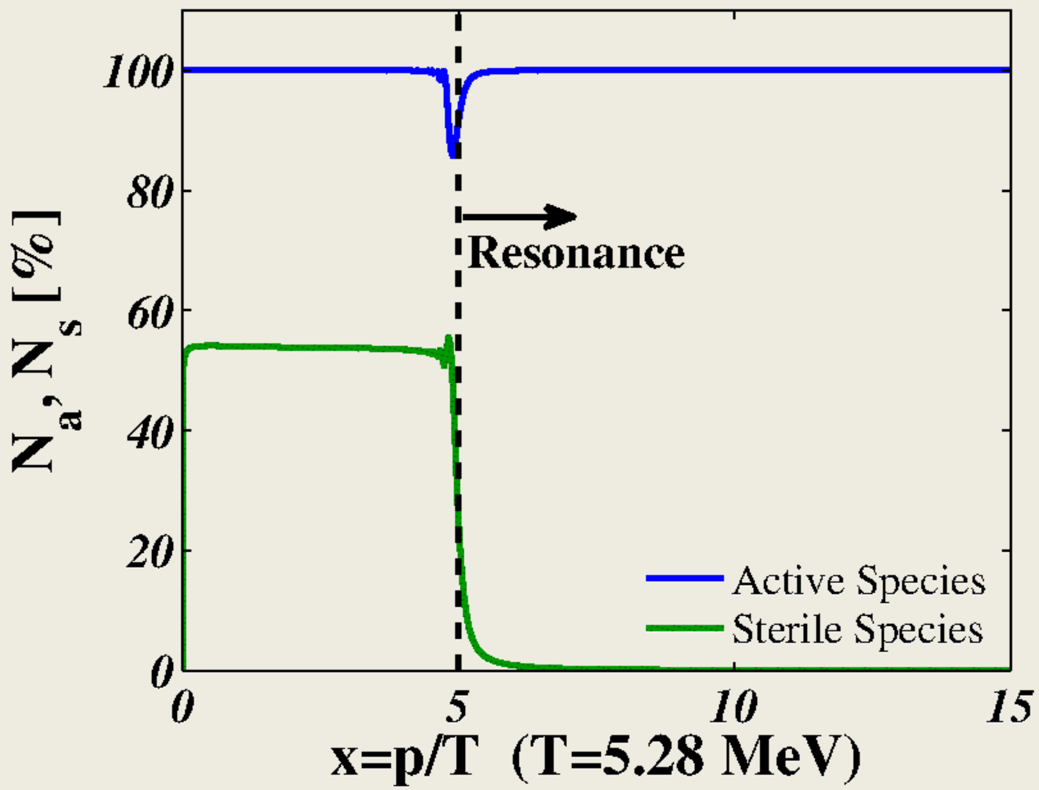
Thermalisation cartoon



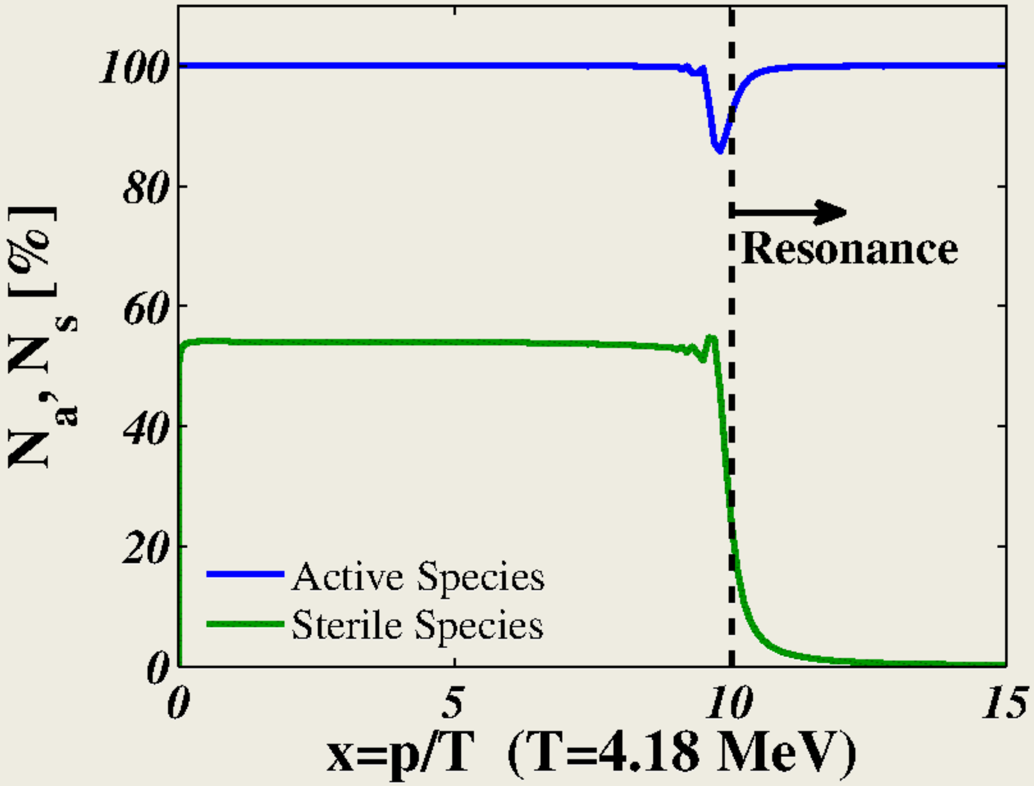
Thermalisation cartoon



Thermalisation cartoon



Thermalisation cartoon



Scenario 1 discussion

- ✓ It is possible to suppress thermalisation for interesting values of the mixing parameters.
- ✓ A lepton asymmetry of this size is not ruled out by any data.
- ➢ Generating a sufficiently large lepton asymmetry is non-trivial.
- ➢ Partial thermalisation is not natural.

Scenario 2: Secret ν_s interactions

- Basic idea is the quantum Zeno effect: Rapid scatterings keep the density matrix diagonal.
- We assume a new massive gauge boson in the sterile sector and integrate it out.

Scenario 2: Secret ν_s interactions

The equations of motion become:

$$\dot{P}_a^{\pm} = V_x P_y^{\pm} + \Gamma_a\left[2\, f_{a,\mathrm{eq}}^{\pm}/f_0 - P_a^{\pm}\right]$$

$$\dot{P}_s^{\pm} = -V_x P_y^{\pm} + \Gamma_s\left[2\, f_{s,\mathrm{eq}}^{\pm}/f_0 - P_s^{\pm}\right]$$

$$\dot{P}_x^{\pm} = -\left(V_z P_y^{\pm} + V_L P_y^{\mp}\right) - D P_x^{\pm}$$

$$\dot{P}_y^{\pm} = +\left(V_z P_x^{\pm} + V_L P_x^{\mp}\right) - D P_y^{\pm} - \frac{1}{2} V_x\left(P_a^{\pm} - P_s^{\pm}\right)$$

$$V_z = V_a + V_s - V_x$$

$L = 0 \Rightarrow P^+$ and P^- decouple.

$$\rho = \begin{bmatrix} \nu_a & \text{ent.} \\ \text{ent.} & \nu_s \end{bmatrix}$$

Scenario 2: Secret ν_s interactions

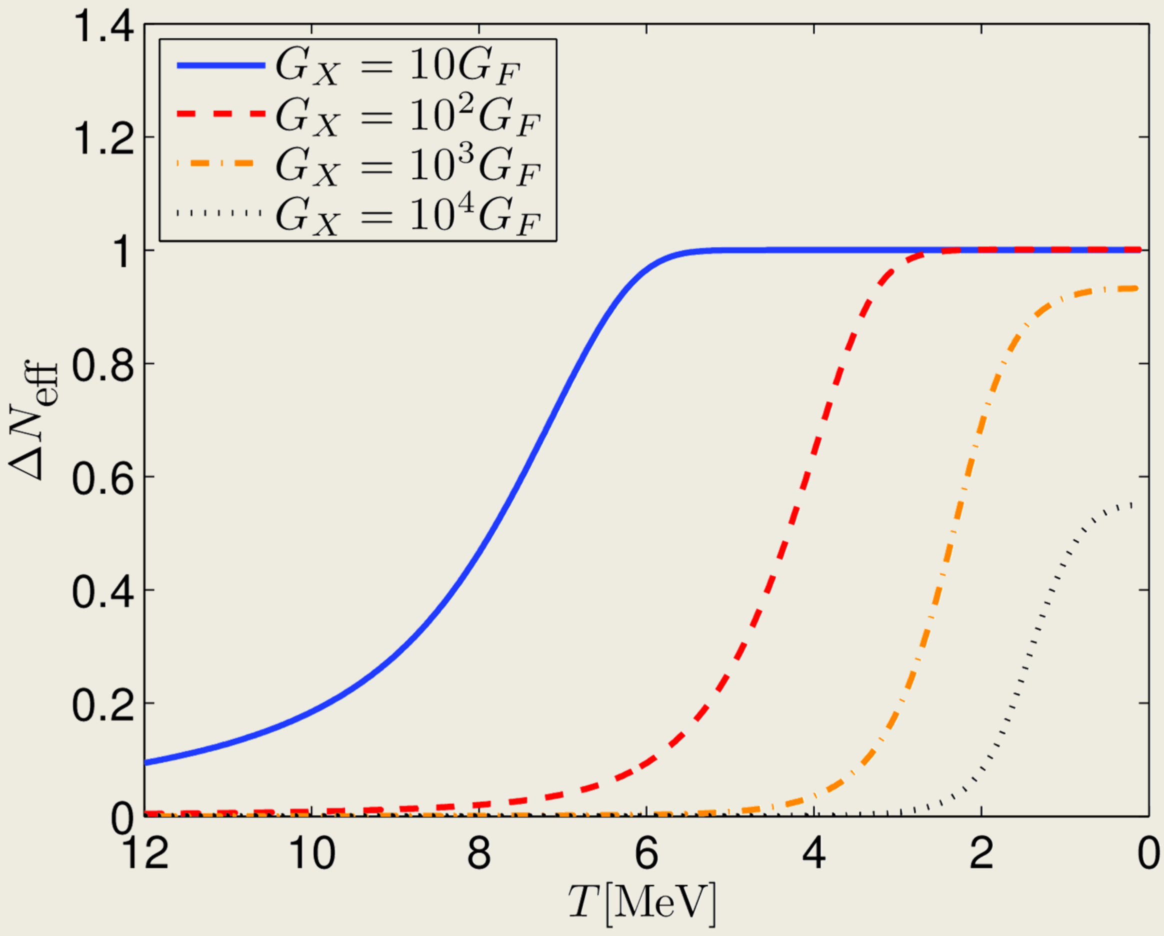
Assuming $L = 0$ leads to

$$\dot{P}_a^+ = V_x P_y^+ + \Gamma_a \left[2\, f_{a,\mathrm{eq}}^+ / f_0 - P_a^+\right]$$
$$\dot{P}_s^+ = -V_x P_y^+ + \Gamma_s \left[2\, f_{s,\mathrm{eq}}^+ / f_0 - P_s^+\right]$$
$$\dot{P}_x^+ = -V_z P_y^+ - D\, P_x^+$$
$$\dot{P}_y^+ = +V_z P_x^+ - D\, P_y^+ - \tfrac{1}{2} V_x (P_a^+ - P_s^+)$$
$$V_z = V_a + V_s - V_x$$

$$\rho = \begin{bmatrix} \nu_a & \mathrm{ent.} \\ \mathrm{ent.} & \nu_s \end{bmatrix}$$

ΔN_{eff} evolution

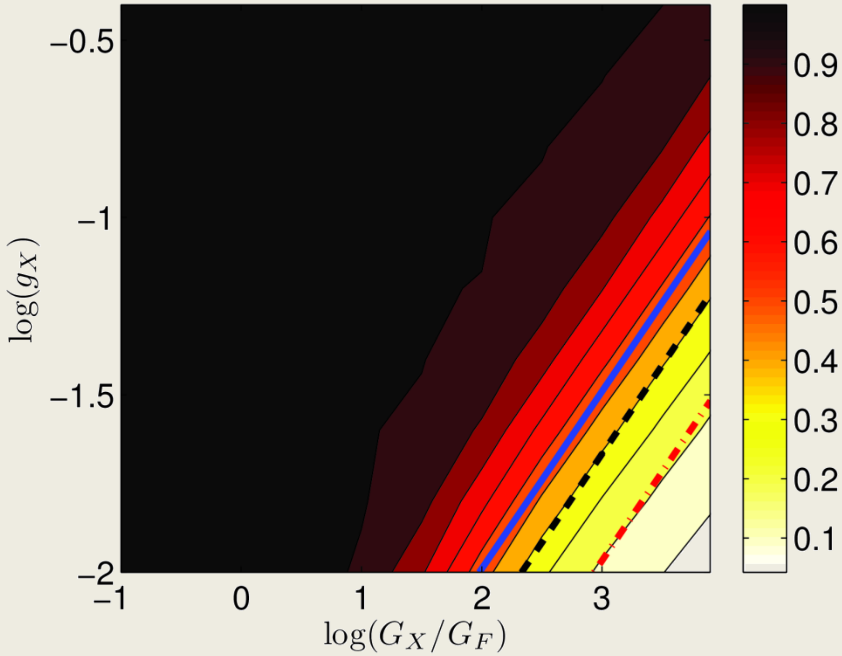
$$G_x = \frac{g_x^2}{M_x^2}$$



Hannestad, Hansen, TT
arXiv:1310.5926

Thermalisation

$$G_x = \frac{g_x^2}{M_x^2}$$



$M_x = 100\text{MeV}$ (blue solid line)
$M_x = 200\text{MeV}$ (black dashed line)
$M_x = 300\text{MeV}$ (red dotted line)

- Oscillation parameters

$$\delta m_s^2 = 1\text{eV}^2$$
$$\sin^2 2\theta_s = 0.05$$

- Thermalisation depends almost entirely on M_x:

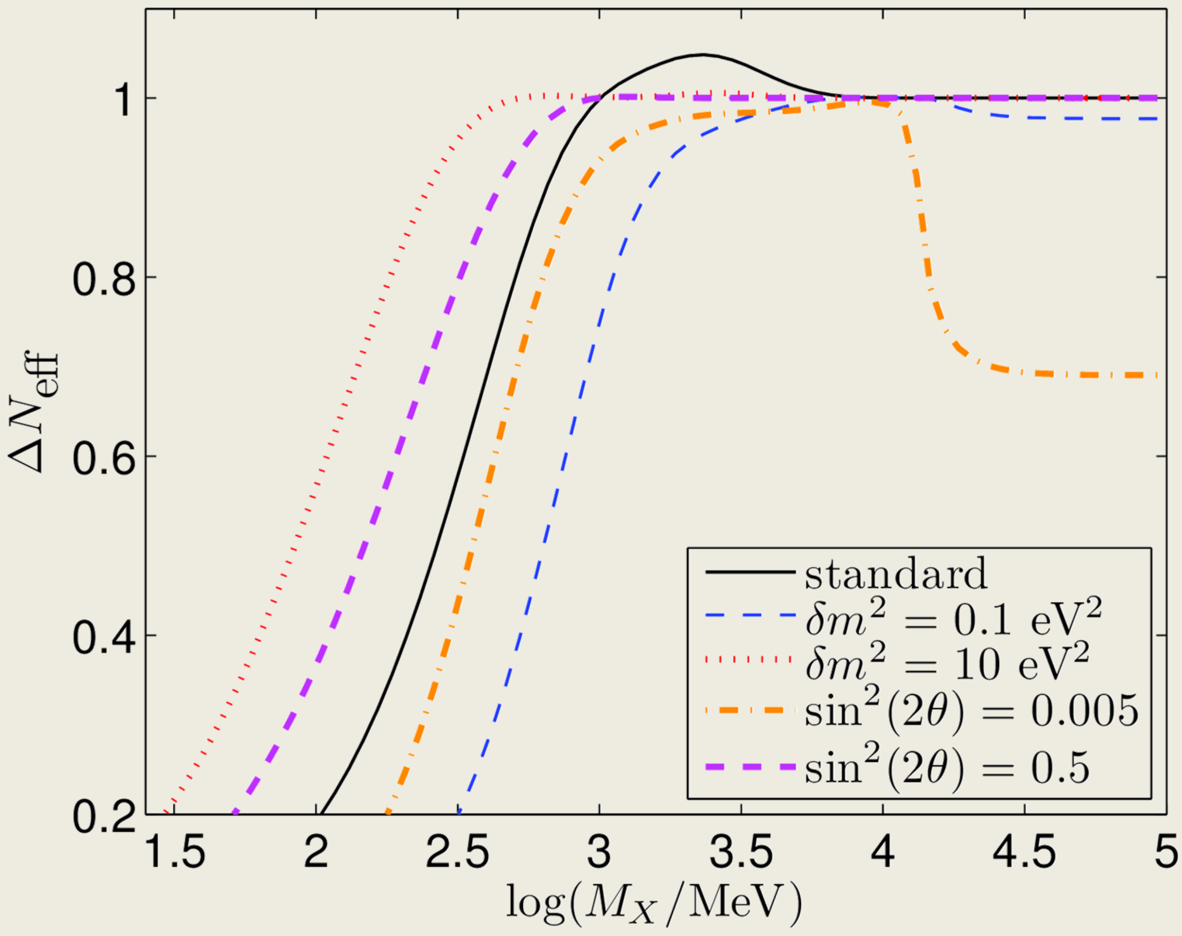
$\Gamma \propto G_x^2$

$$\sin^2 2\theta_m \propto \frac{1}{V_s^2} \propto \frac{M_x^4}{G_x^2}$$

$$\Gamma_t \sim \Gamma \sin^2 2\theta_m \propto M_x^4$$

Hannestad, Hansen, TT
arXiv:1310.5926

ΔN_{eff} as a function of M_x



Hannestad, Hansen, TT
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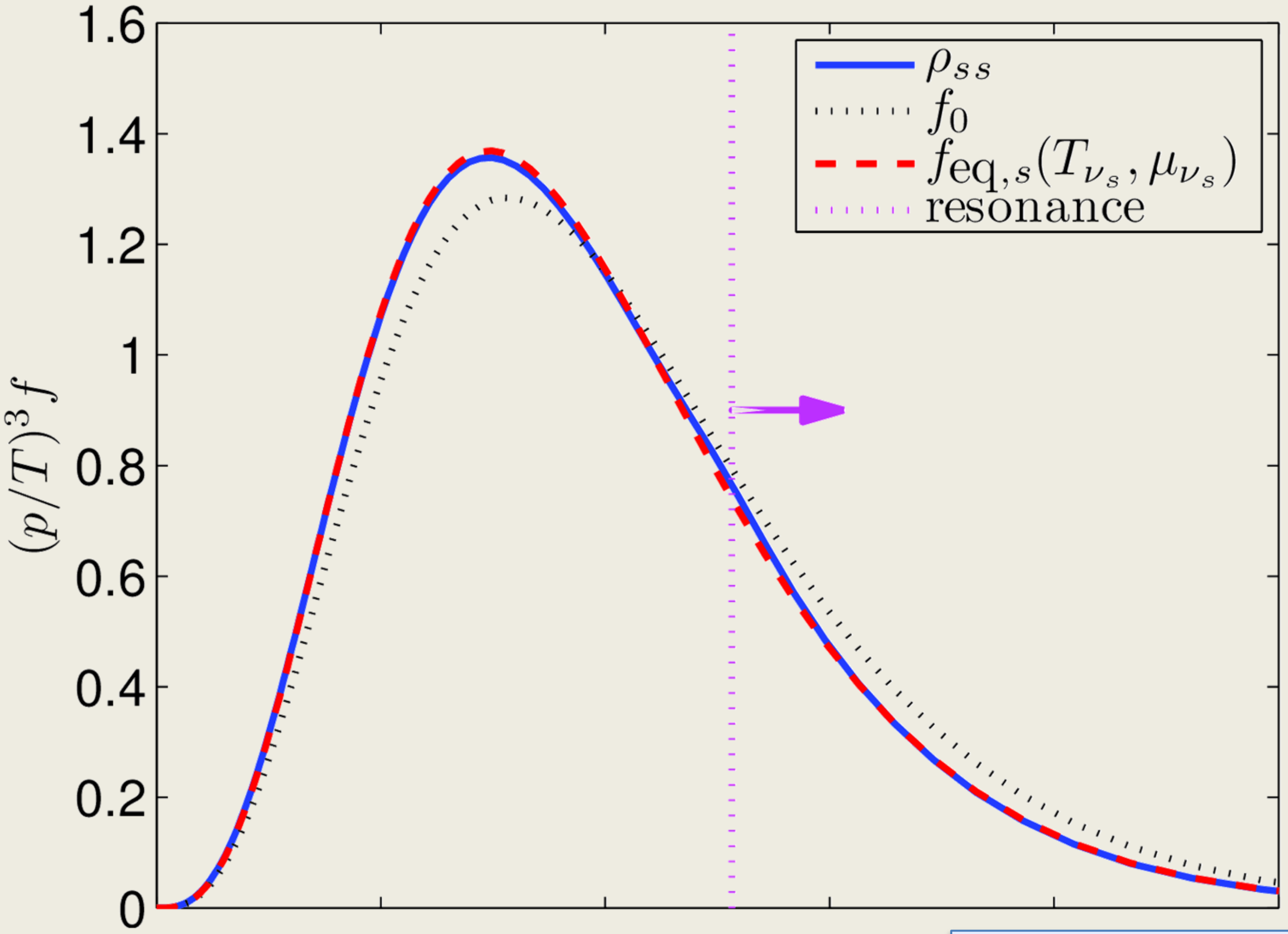
$\Delta N_{\mathrm{eff}} > 1$?



Hannestad, Hansen, TT
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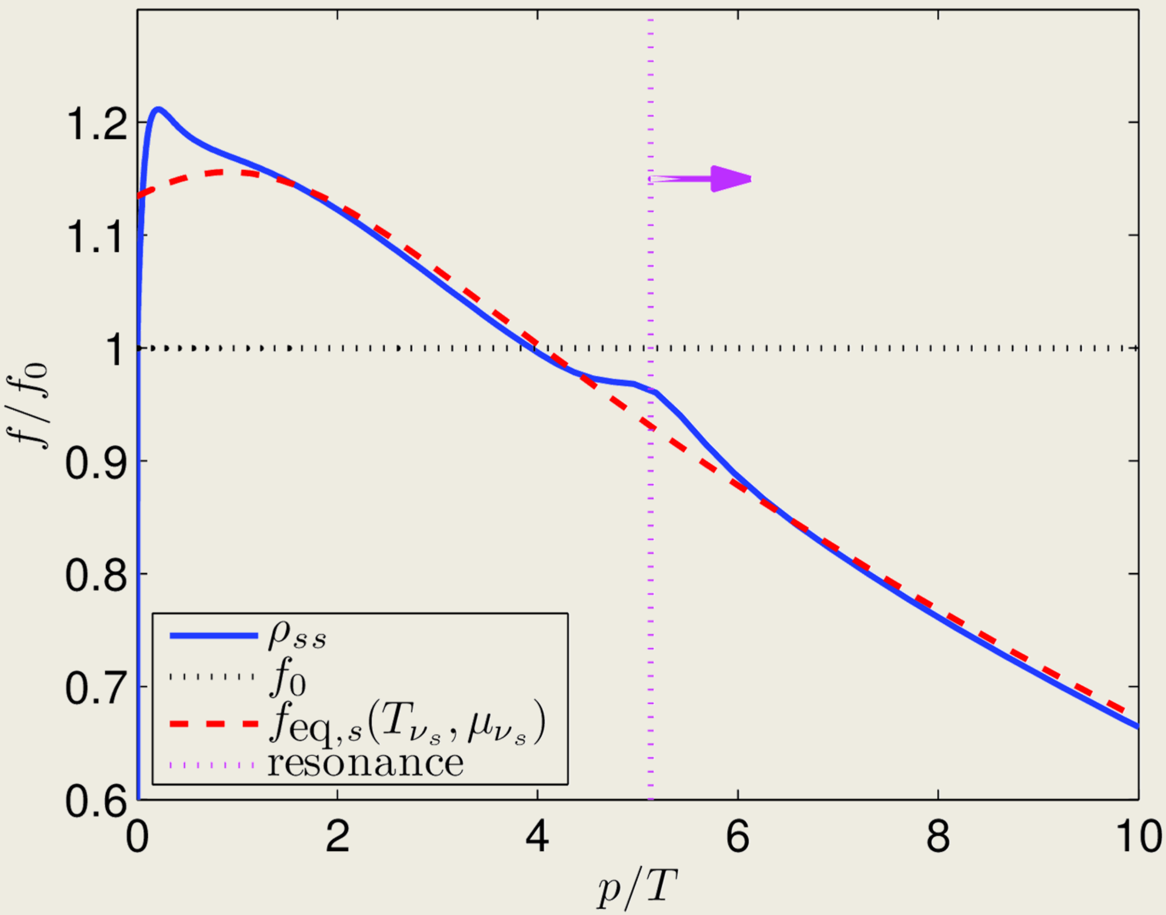
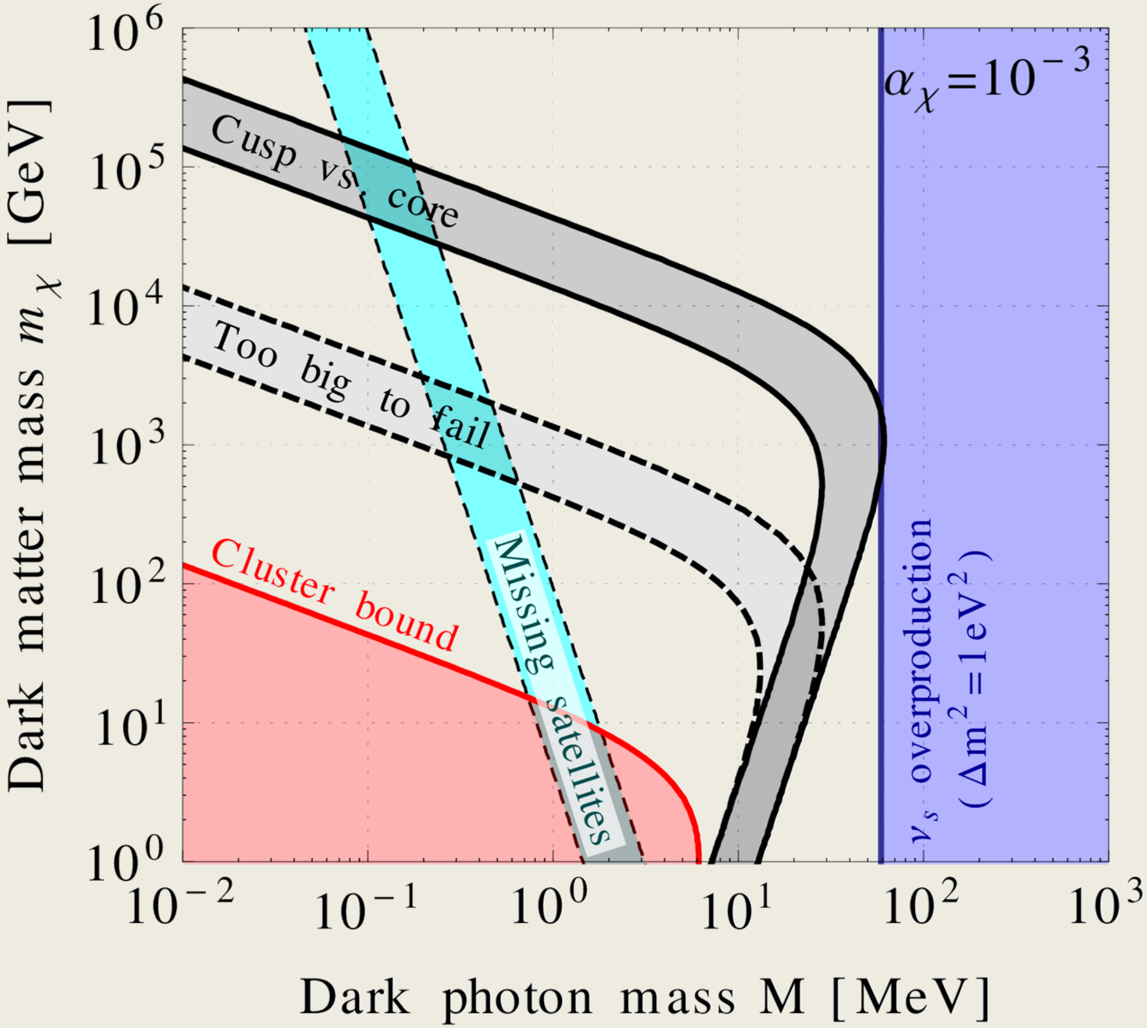
$\Delta N_{\rm eff} > 1$?



Hannestad, Hansen, TT
arXiv:1310.5926

Extension: Self-Interacting DM



- Missing satellites problem
- Cusp vs. core problem
- Too big to fail

Dasgupta&Kopp
arXiv:1310.6337

Conclusions

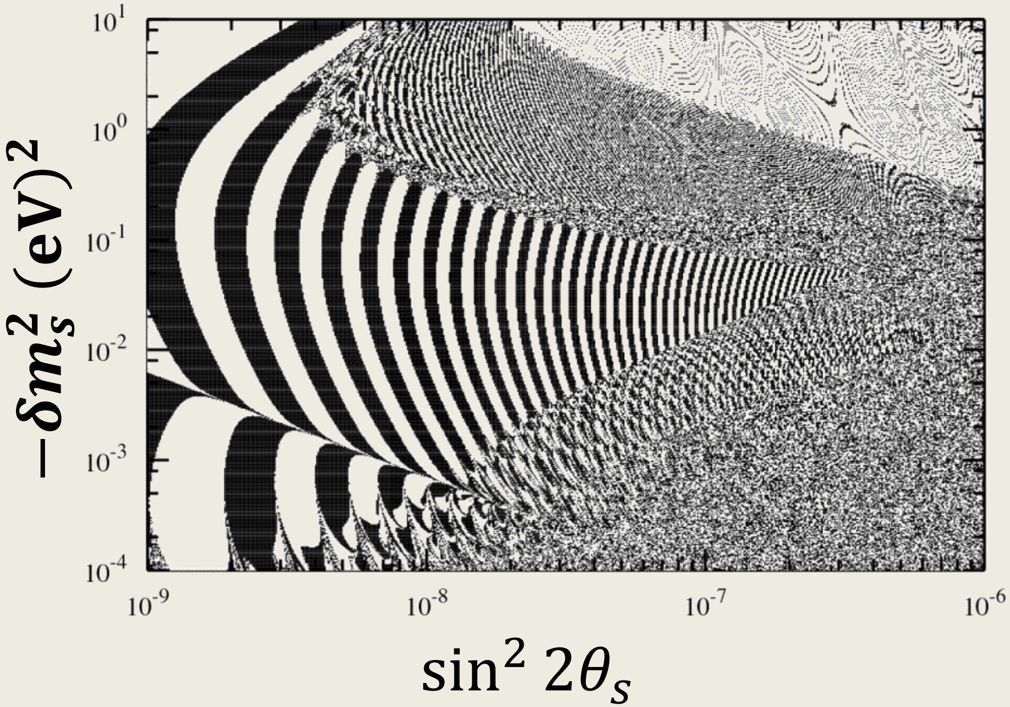
- eV-scale sterile neutrinos in conflict with LSS and Planck if fully thermalised.
  - But viable/preferred if partly thermalised
- Thermalisation can be supressed by
  - Potential from large lepton asymmetry
  - Potential from sterile neutrino self-interactions
- Second scenario can be naturally extended to self interacting Dark Matter.

Small bonus: Open questions

- Assuming that terrestrial evidence grows significantly!
- How is BBN (through ν_e-distribution) affected?
- What happens after decoupling? How much equilibration between active and sterile?
- Extending LASAGNA:
  - More than 2 species
  - Actual SM scattering kernels

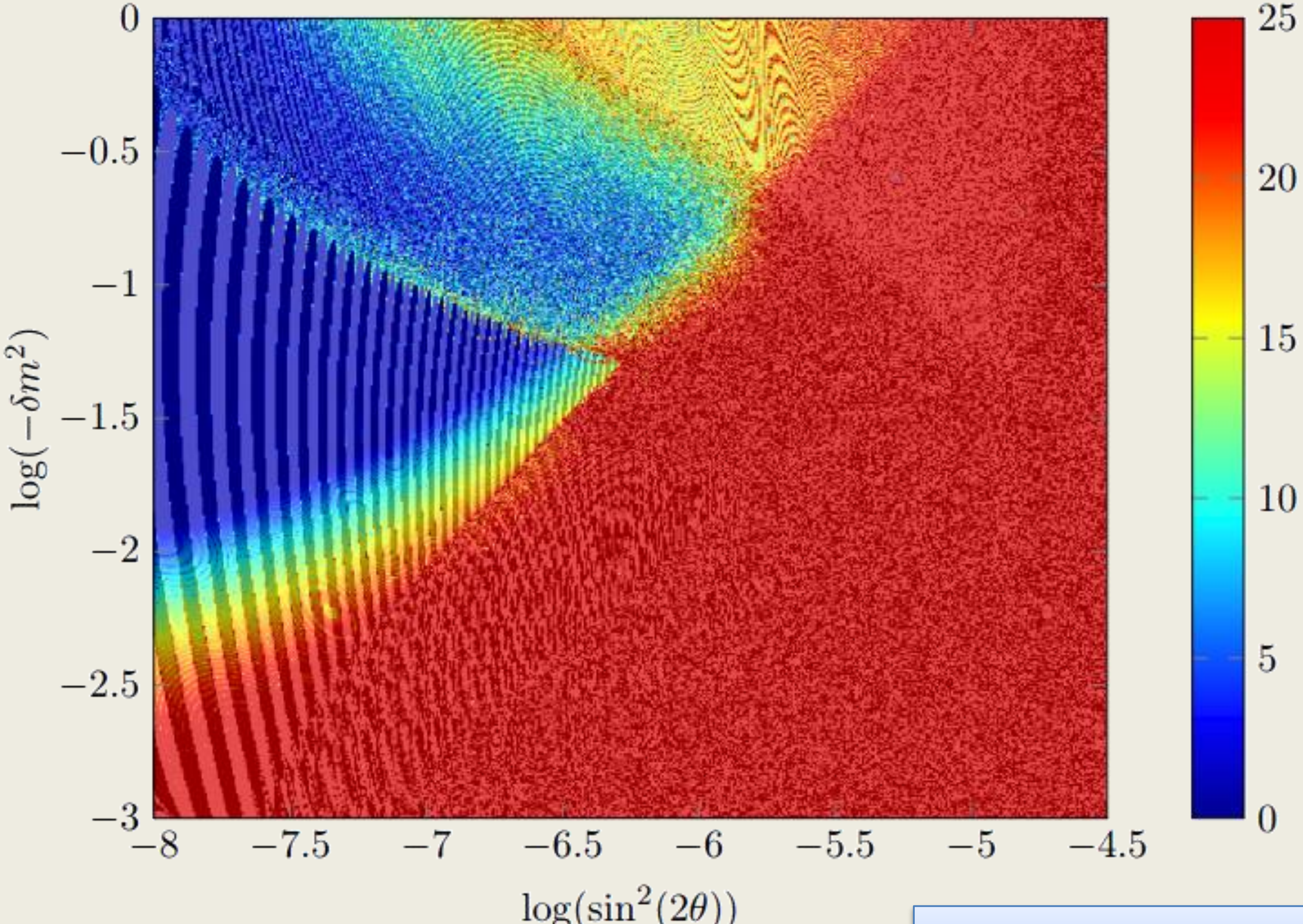
Large bonus: Chaoticity

- In the inverted hierarchy $\delta m_s^2 < 0$, a small initial lepton asymmetry can grow exponentially
- The final sign of the lepton asymmetry depends chaotically on the initial sign in QRE.

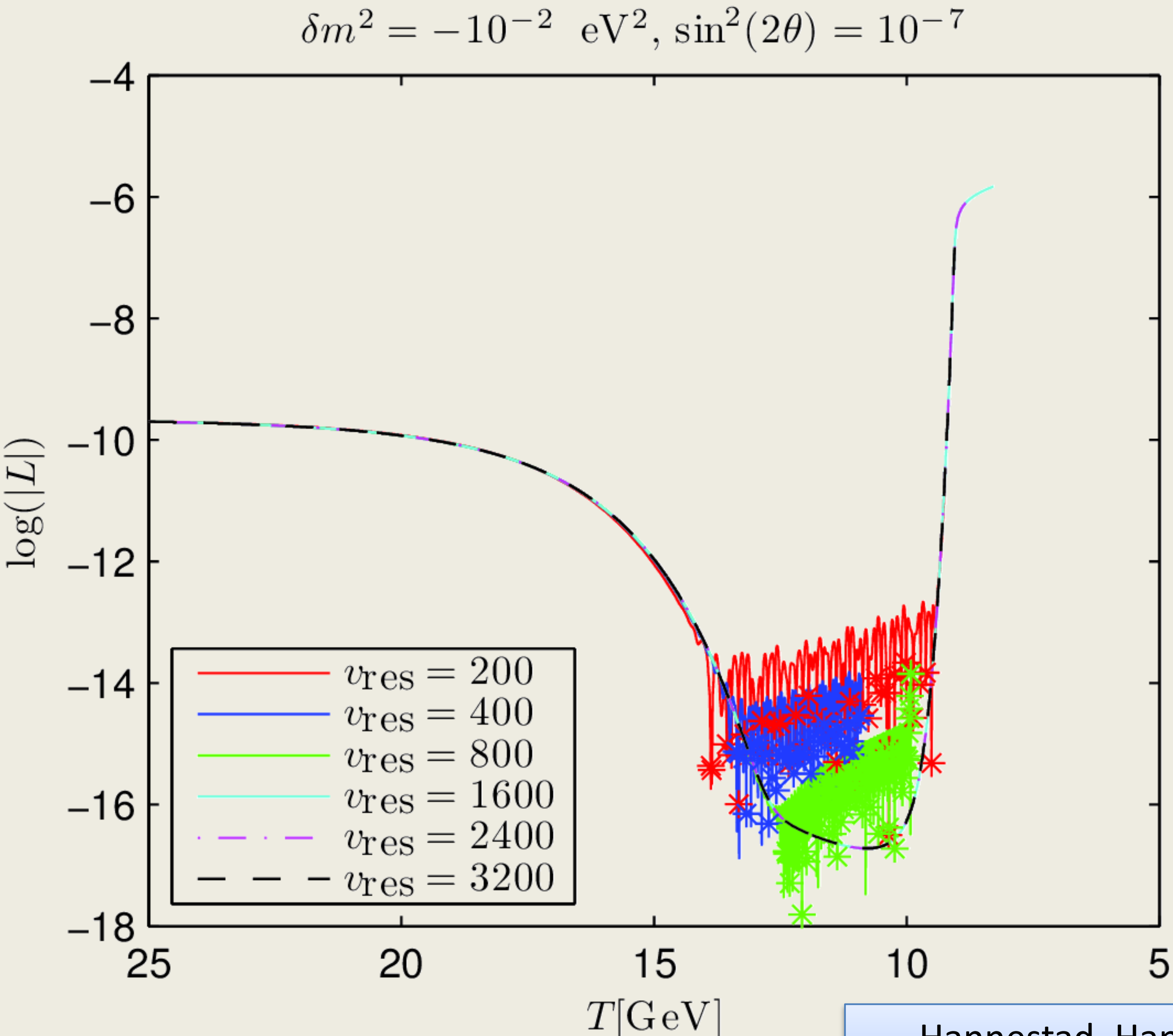


Enqvist, Kainulainen&Sorri
hep-ph/9906452
Abazajian&Agrawal
arXiv:0807.0456

Chaos in QRE confirmed



Hannestad, Hansen, TT
arXiv:1302.7279



Hannestad, Hansen, TT
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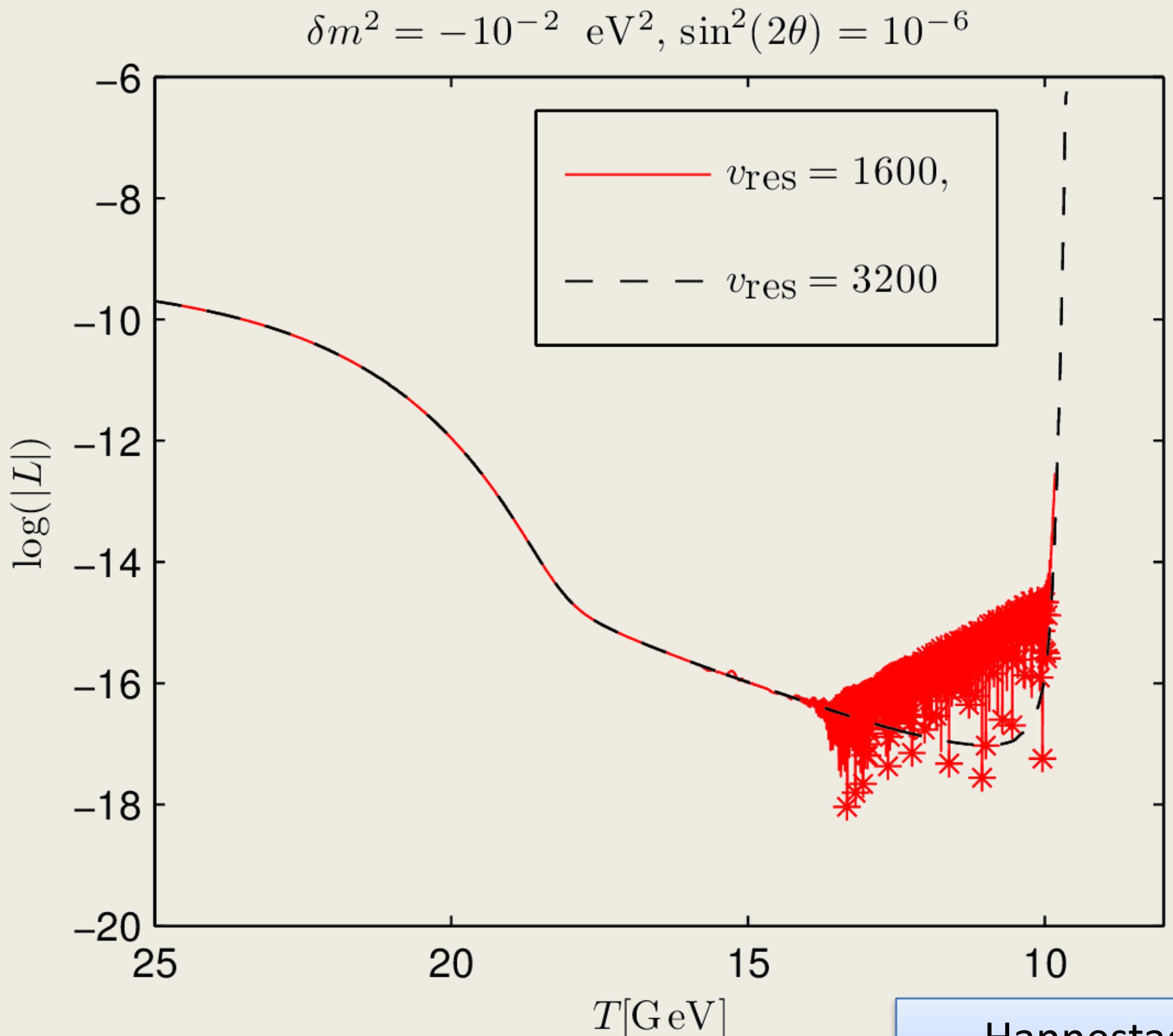
Chaos disappears in QKE!



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