

Flavor Hierarchies from a Minimal $U(2)$ Symmetry

Anders Eller Thomsen

Based on work with S. Antusch, A. Greljo, and B. Stefanek
[2309.11547] and [2311.09288]

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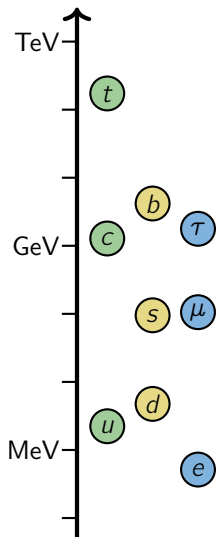


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The Flavor Puzzle

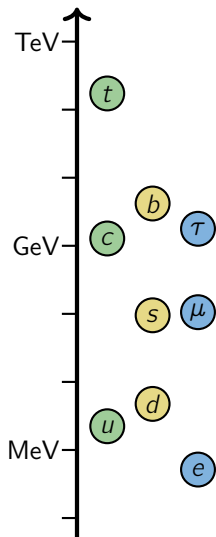
What can approximate symmetries tell us?

The Flavor Puzzle



$$V_{\text{CKM}} \sim \begin{pmatrix} \text{dark blue} & \text{light blue} & \text{white} \\ \text{light blue} & \text{dark blue} & \text{light blue} \\ \text{white} & \text{light blue} & \text{dark blue} \end{pmatrix}$$

The Flavor Puzzle

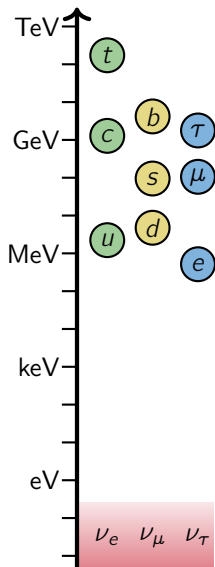


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$$\mathcal{L}_{\text{SM}} \supset -Y_u^{pr} \bar{q}^p \tilde{H} u^r - Y_d^{pr} \bar{q}^p H d^r - Y_e^{pr} \bar{\ell}^p H e^r$$

- Small Yukawas are natural in the sense of 't Hooft
- Why are they hierarchical if they all enter at the same level?

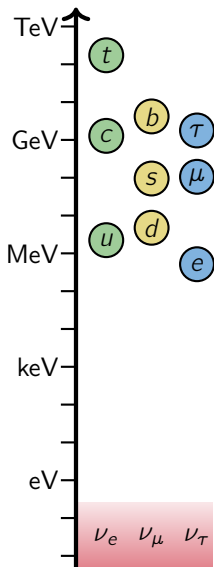
The Flavor Puzzle



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The Flavor Puzzle



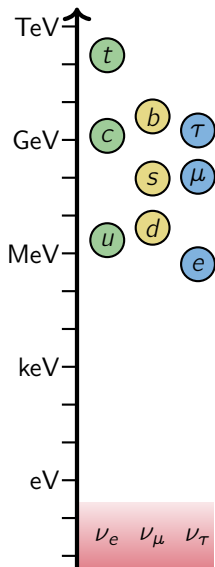
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$$\mathcal{L}_{\text{SMEFT}} \supset -\frac{C^{pr}}{\Lambda} \bar{\ell}_p \tilde{H} \tilde{H}^\tau \ell_r^c$$

- Smallness of the neutrino masses? $\rightarrow$ Heavy NP
- Why is the PMNS so radically different from the CKM?

The Flavor Puzzle



$$V_{\text{CKM}} \sim \begin{pmatrix} \text{dark blue} & \text{light blue} & \\ \text{light blue} & \text{dark blue} & \\ & & \text{dark blue} \end{pmatrix}$$

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What about explanations?

- Many different BSM mechanisms: Froggatt–Nielsen, (gauged) flavor symmetries, accidental flavor, radiative mass models, multi-Higgs models, multi-scale flavor, warped compactification, clockwork, modular symmetries...
- UV completions of the models are often complex

Flavor symmetries

It is instructive to consider flavor in terms of symmetries:

SM Kinetic terms: $\mathcal{L} \supset i\bar{q}^p \not{D} q^p + i\bar{u}^p \not{D} u^p + i\bar{d}^p \not{D} d^p + i\bar{\ell}^p \not{D} \ell^p + i\bar{e}^p \not{D} e^p$

$\hookrightarrow$ Flavor symmetry: $U(3)^5 = U(3)_q \times U(3)_u \times U(3)_d \times U(3)_\ell \times U(3)_e$

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Minimal Flavor Violation (MFV):

D'Ambrosio *et al.* [hep-ph/0207036]

the only breaking of $U(3)^5$ is due to the spurions

$$Y_u \sim (\mathbf{3}, \bar{\mathbf{3}}, \mathbf{1}, \mathbf{1}, \mathbf{1}), \quad Y_d \sim (\mathbf{3}, \mathbf{1}, \bar{\mathbf{3}}, \mathbf{1}, \mathbf{1}), \quad Y_e \sim (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{3}, \bar{\mathbf{3}})$$

✗ The symmetry does not explain flavor! (*nor was it meant too*)

✗ $y_t \sim 1$ is large: there is no order parameter

✓ Selection rules allow for multi-TeV scale NP

$$Y_u \sim \begin{pmatrix} \begin{array}{ccc} \text{light} & \text{light} & \text{light} \\ \text{light} & \text{medium} & \text{medium} \\ \text{medium} & \text{medium} & \text{dark} \end{array} \end{pmatrix}$$

An approximate symmetry of the SM: $U(2)^5$

The leading violation of $U(3)^5$:

$$Y_u \sim \begin{pmatrix} \text{lightest} & \text{light} & \text{medium} \\ \text{light} & \text{medium} & \text{dark} \\ \text{medium} & \text{dark} & \text{darkest} \end{pmatrix}$$

An approximate symmetry of the SM: $U(2)^5$

The leading violation of $U(3)^5$:

$$Y_u \sim \underbrace{\begin{pmatrix} & & \\ & & \\ & & \blacksquare \end{pmatrix}}_{U(2)_u} \Bigg\}^{U(2)_q}$$

An approximate symmetry of the SM: $U(2)^5$

The leading violation of $U(3)^5$:

$$Y_u \sim \left(\underbrace{\begin{pmatrix} & & \\ & & \\ & & \blacksquare \end{pmatrix}}_{U(2)_u} \right) \}^{U(2)_q}$$

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$U(2)^5$ symmetry: small spurions break $U(2)^5$ further

Kagan et al. [0903.1794]; *Barbieri et al.* [1105.2296]; *Blankenburg et al.* [1204.0688]; *Fuentes-Martín et al.* [1909.02519], ...

$$\Delta_u, \Delta_d, \Delta_e \ll V_q, V_\ell \ll 1$$

$$Y_u = \begin{pmatrix} \Delta_u & x_t V_q \\ & 1 \end{pmatrix}, \quad Y_d = y_b \begin{pmatrix} \Delta_d & x_b V_q \\ & 1 \end{pmatrix}, \quad Y_e = y_\tau \begin{pmatrix} \Delta_e & x_\tau \overset{\text{can be 0}}{V_\ell} \\ & 1 \end{pmatrix}$$

An approximate symmetry of the SM: $U(2)^5$

The leading violation of $U(3)^5$:

$$Y_u \sim \underbrace{\left(\begin{array}{c|c} & \\ \hline & \end{array} \right)}_{U(2)_u} \left. \vphantom{\begin{pmatrix} & \\ \hline & \end{pmatrix}} \right\} U(2)_q \quad Y_d \sim \underbrace{\left(\begin{array}{c|c} & \\ \hline & \end{array} \right)}_{U(2)_d} \left. \vphantom{\begin{pmatrix} & \\ \hline & \end{pmatrix}} \right\} U(2)_q \quad Y_e \sim \underbrace{\left(\begin{array}{c|c} & \\ \hline & \end{array} \right)}_{U(2)_e} \left. \vphantom{\begin{pmatrix} & \\ \hline & \end{pmatrix}} \right\} U(2)_\ell$$

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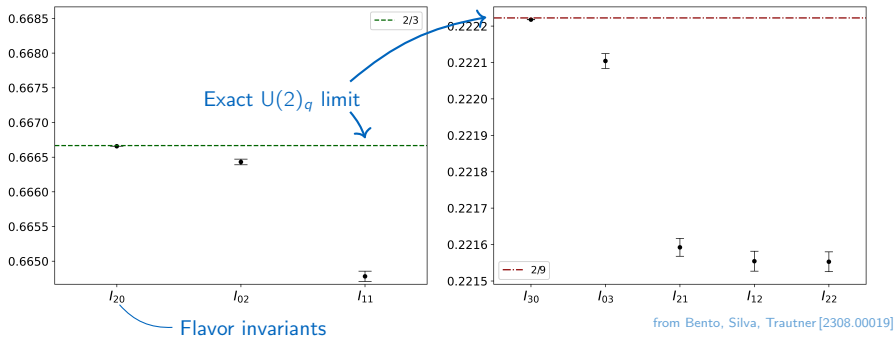
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- ✓ Hierarchical charged fermion masses + CKM matrix
- ✗ Complicated symmetry and breaking pattern
- ✗ Suggests a hierarchical PMNS matrix
- ✓ Selection rules allow for multi-TeV NP

How good is the $U(2)_q$ symmetry?

Flavor invariants, e.g., $I_{21} = \text{Tr}[H_u^2 H_d]/(y_t^4 y_b^2)$, $H_f = Y_f Y_f^\dagger - \frac{1}{3} \text{Tr}[Y_f Y_f^\dagger]$



Very good!

$U(2)$ is Right for Leptons and Left for Quarks

All flavor structure from a single $U(2)$

One $U(2)$ symmetry

Assume the existence of a $U(2)_q$ flavor symmetry:

$$Y_u \sim \left(\begin{array}{ccc} \text{---} & \text{---} & \text{---} \\ \text{■} & \text{■} & \text{■} \end{array} \right) \Bigg\}^{U(2)_q}$$

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Assume the existence of a $U(2)_q$ flavor symmetry:

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$U(2)_u$

One $U(2)$ symmetry

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$\underbrace{\hspace{10em}}_{U(2)_u}$

Causing accidental symmetries in the SM Yukawa couplings

$$U(2)_q \xrightarrow{\text{accidental}} U(2)_u \times U(2)_d$$

$$U(2)_e \xrightarrow{\text{accidental}} U(2)_\ell$$

One $U(2)$ symmetry

Assume the existence of a $U(2)_q$ flavor symmetry:

$$Y_u \sim \left(\begin{array}{ccc} & & \\ \text{■} & \text{■} & \text{■} \end{array} \right) \Bigg\}^{U(2)_q} \xrightarrow{U(3)_u \text{ rot.}} \left(\begin{array}{ccc} & & \\ & & \\ \text{■} & & \end{array} \right) \Bigg\}^{U(2)_q}$$

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Causing accidental symmetries in the SM Yukawa couplings

$$U(2)_{q+e} \xrightarrow{\text{accidental}} U(2)_u \times U(2)_d \times U(2)_\ell$$

No selection rules on Weinberg operator (or NP) from the accidental SM symmetries

One $U(2)$ symmetry

Assume the existence of a $U(2)_q$ flavor symmetry:

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No selection rules on Weinberg operator (or NP) from the accidental SM symmetries

Assuming a **$U(2)_{q+e}$ flavor symmetry**

- ✓ hierarchy between the 3rd and light generation charged fermions
- ✓ anarchic PMNS matrix

$U(2)_{q+e}$ symmetry

$$q^p = q^\alpha \oplus q^3 \sim \mathbf{2}_1 \oplus \mathbf{1}_0, \quad e^p = e^\alpha \oplus e^3 \sim \mathbf{2}_1 \oplus \mathbf{1}_0, \quad V_{1,2}^\alpha \sim \mathbf{2}_1$$

Spurions $V_{1,2}^\alpha$ break $U(2)_{q+e}$ giving light generation masses:

$$\begin{aligned} \mathcal{L} \supset & - \left(x_u^p \bar{q}^3 + y_u^p \bar{q}_\alpha V_2^\alpha + z_u^p \bar{q}_\alpha V_1^\alpha \right) \tilde{H} u^p \\ & - \left(x_d^p \bar{q}^3 + y_d^p \bar{q}_\alpha V_2^\alpha + z_d^p \bar{q}_\alpha V_1^\alpha \right) H d^p \\ & - \bar{\ell}^p H \left(x_e^p e^3 + y_e^p V_{2\alpha}^* e^\alpha + z_e^p V_{1\alpha}^* e^\alpha \right) \end{aligned}$$

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The effective SM Yukawa matrices take the form, e.g.,

$$Y_u = \begin{pmatrix} z_{u1}b & z_{u2}b & z_{u3}b \\ & y_{u2}a & y_{u3}a \\ & & x_{u3} \end{pmatrix} = L_u \hat{Y}_u R_u^\dagger, \quad \begin{cases} V_2^\alpha = (0, a) \\ V_1^\alpha = (b, 0) \\ 1 \gg a \gg b > 0 \end{cases}$$

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- ✓ hierarchy between the 1st and 2nd generation charged fermions
- ✓ hierarchical (close to identity) CKM matrix
- ✗ selection rules do not allow for TeV-scale NP *but the phenomenology is interesting*

Numerical benchmark

The down rotation dominates $V_{\text{CKM}} = L_u^\dagger L_d$ due to a compressed mass hierarchy:

$$L_d \simeq \begin{pmatrix} 1 & \frac{m_d}{m_s} \frac{z_{d2}}{z_{d1}} & \frac{m_d}{m_b} \frac{z_{d3}}{z_{d1}} \\ -\frac{m_d}{m_s} \frac{z_{d2}^*}{z_{d1}} & 1 & \frac{m_s}{m_b} \frac{y_{d3}}{y_{d2}} \\ \frac{m_d}{m_b} \left(\frac{y_{d3}^* z_{d2}^*}{y_{d2} z_{d1}} - \frac{z_{d3}^*}{z_{d1}} \right) & -\frac{m_s}{m_b} \frac{y_{d3}^*}{y_{d2}} & 1 \end{pmatrix}$$

$$L_u \simeq \begin{pmatrix} 1 & \frac{m_u}{m_c} \frac{z_{u2}}{z_{u1}} & \frac{m_u}{m_t} \frac{z_{u3}}{z_{u1}} \\ -\frac{m_u}{m_c} \frac{z_{u2}^*}{z_{u1}} & 1 & \frac{m_c}{m_t} \frac{y_{u3}}{y_{u2}} \\ \frac{m_u}{m_t} \left(\frac{y_{u3}^* z_{u2}^*}{y_{u2} z_{u1}} - \frac{z_{u3}^*}{z_{u1}} \right) & -\frac{m_c}{m_t} \frac{y_{u3}^*}{y_{u2}} & 1 \end{pmatrix}$$

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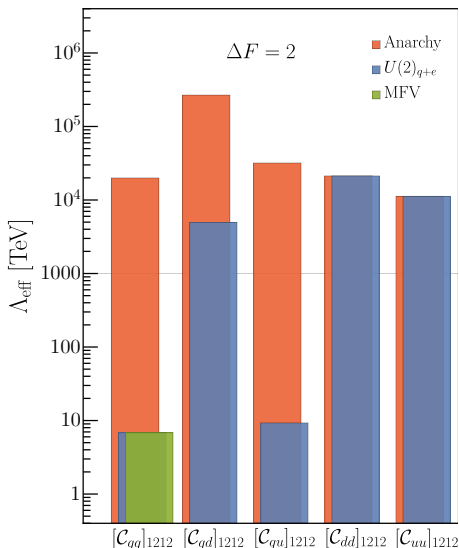
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A realistic benchmark $\mu = 1 \text{ PeV}$ with $(a, b) = (3 \cdot 10^{-3}, 5 \cdot 10^{-5})$ is

$z_{\ell 1} = 0.057$	$y_{\ell 2} = 0.20$	$x_{\ell 3} = 0.010$
$z_{u1} = 0.091$	$y_{u2} = 0.76$	$x_{u3} = 0.67$
$z_{d1} = 0.20$	$y_{d2} = 0.066$	$x_{d3} = 0.010$
$z_{d2} = 0.89e^{i\alpha}$	$z_{d3} = 0.72e^{i(\beta-1.2)}$	$y_{d3} = 0.13e^{i(\beta-\alpha)}$

* *The remaining parameters are largely unconstrained*

SMEFT bounds



EFT as a proxy for new physics:

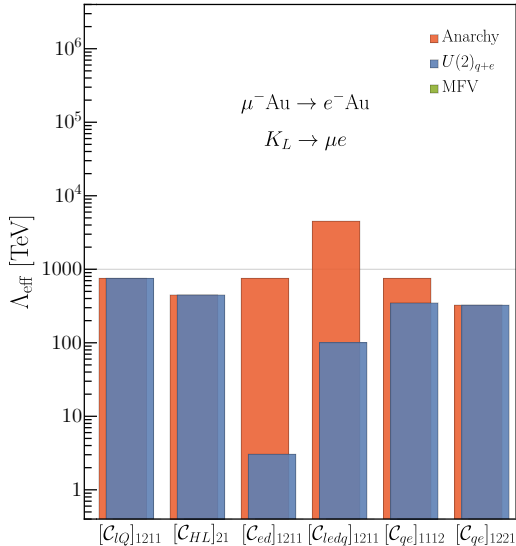
$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{1}{\Lambda_{\text{eff}}^2} \mathcal{O}_i + \dots$$

Interaction basis:

$$\mathcal{L}_{\text{SMEFT}} \supset -\frac{1}{\Lambda^2} (\bar{q}^3 \gamma_\mu q^3) (\bar{d}^1 \gamma_\mu d^2)$$

Down quark mass basis:

$$\mathcal{L}_{\text{SMEFT}} \supset -\frac{[L_d^*]_{31} [L_d]_{32}}{\Lambda^2} (\bar{q}^1 \gamma_\mu q^2) (\bar{d}^1 \gamma_\mu d^2)$$



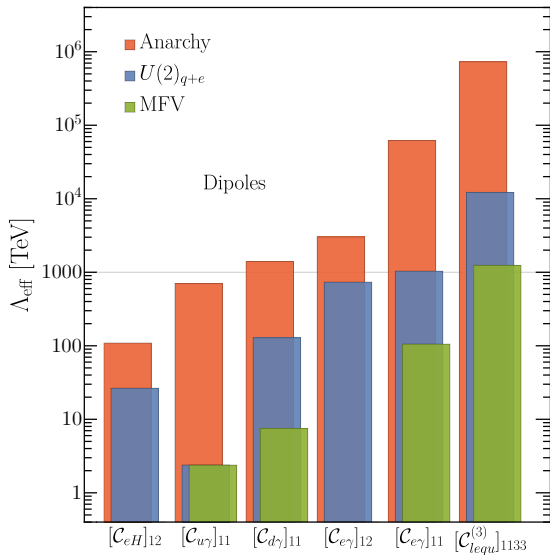
Interaction basis:

$$\mathcal{L}_{\text{SMEFT}} \supset -\frac{1}{\Lambda^2} (\bar{q}^\alpha \gamma_\mu q^\beta) (\bar{e}^\beta \gamma_\mu e^\alpha)$$

Mass basis:

$$\mathcal{L}_{\text{SMEFT}} \supset -\frac{1}{\Lambda^2} (\bar{q}^1 \gamma_\mu q^2) (\bar{e}^2 \gamma_\mu e^1)$$

SMEFT bounds



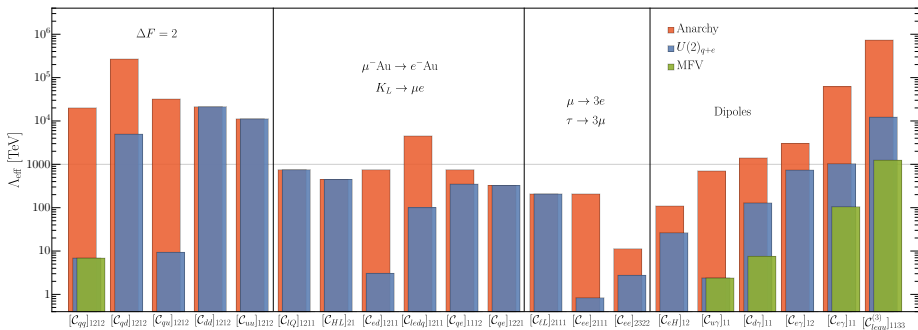
Interaction basis:

$$\mathcal{L}_{\text{SMEFT}} \supset \frac{1}{(16\pi^2)\Lambda^2} F^{\mu\nu} (\bar{\ell}^1 \sigma_{\mu\nu} e^3)$$

Mass basis:

$$\mathcal{L}_{\text{SMEFT}} \supset \frac{[R_e]_{31}}{(16\pi^2)\Lambda^2} F^{\mu\nu} (\bar{\ell}^1 \sigma_{\mu\nu} e^1)$$

SMEFT bounds



Antusch, Greljo, Stefaneke, AET [2311.09288]

- cLFV observables are more competitive than with $U(2)_{q+e}$ assumption
- Expect order-10 improvement in $\mu \rightarrow e$ conversion (Mu2e, COMET), electron EDM (ACME), and $\mu \rightarrow 3e$ (Mu3e)
- Gauged realizations of $SU(2)_{q+e}$ produce $[C_{qe}]_{1211} \sim \langle \Phi \rangle^{-2}$ (the breaking scale): this is a smoking gun

Rising Through the Ranks

A UV realization of the $U(2)$ idea

Aim for the UV realization

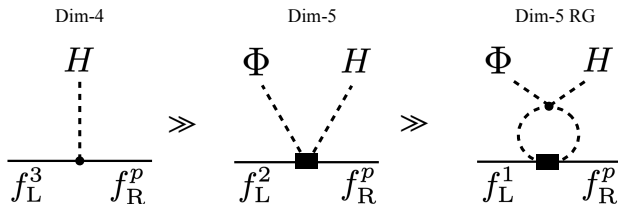
Rules of the “Game:”

- Explain hierarchy of charged fermion Yukawas, $Y_{u,d,e}$, and $V_{\text{CKM}} \sim \mathbb{1}$
- Marginal couplings are $\mathcal{O}(0.3)$ (couplings $\gtrsim 1$ tend to give Landau poles)
- Simple models are favored—no epicycles

Aim for the UV realization

Rules of the “Game:”

- Explain hierarchy of charged fermion Yukawas, $Y_{u,d,e}$, and $V_{\text{CKM}} \sim 1$
- Marginal couplings are $\mathcal{O}(0.3)$ (couplings $\gtrsim 1$ tend to give Landau poles)
- Simple models are favored—no epicycles



Starting point: gauged $SU(2)_{q+\ell}$!

Babu, Mohapatra '91

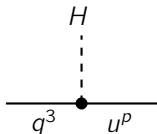
- $SU(2)_{q+\ell}$ is non-anomalous: no chiral BSM fermions are needed
- Accidental $U(2)_u \times U(2)_d \times U(2)_e$ ensures hierarchical Yukawas
- Neutrino masses left for model extension
- $\langle \Phi^\alpha \rangle = \begin{pmatrix} 0 \\ v_\Phi \end{pmatrix}$ breaks $SU(2)_{q+\ell}$, allowing light fermion masses

$$q^p = (q^\alpha, q^3) \sim (\mathbf{2}, \mathbf{1}), \quad \ell^p = (\ell^\alpha, \ell^3) \sim (\mathbf{2}, \mathbf{1}), \quad \Phi^\alpha \sim \mathbf{2}$$

- Heavy, flavored gauge bosons, Z' , of $U(2)_{q+\ell}$ have phenomenological implications

Mechanism for flavor hierarchies

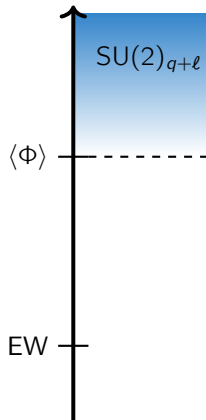
Rank-1 Yukawas: renormalizable interactions



$$Y_u = \begin{pmatrix} \blacksquare & \blacksquare & \blacksquare \end{pmatrix} \mathcal{O}(1)$$

$$m_t > 0, \quad m_c = m_u = 0$$

Field	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$SU(2)_{q+\ell}$
q_L^α	3	2	1/6	2
q_L^3	3	2	1/6	1
u_R^p	3	1	2/3	1
H	1	2	1/2	1
Φ	1	1	0	2

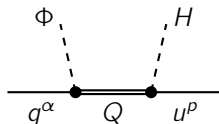


Greljo, AET [2309.11547]

A similar mechanism is behind masses for the down quarks and charged leptons

Mechanism for flavor hierarchies

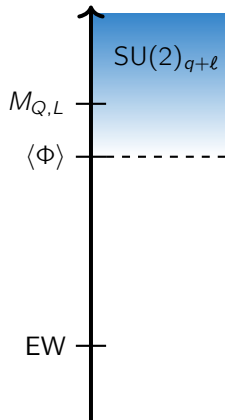
Rank-2 Yukawas: vector-like fermions



$$Y_u = \begin{pmatrix} \text{3 boxes} \\ \text{3 boxes} \end{pmatrix} \quad \frac{\langle \Phi \rangle}{M_Q} \sim \frac{1}{100}$$

$$m_t \gg m_c > 0, \quad m_u = 0$$

Field	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$SU(2)_{q+\ell}$
$Q_{L,R}$	3	2	1/6	1
$L_{L,R}$	1	2	-1/2	1

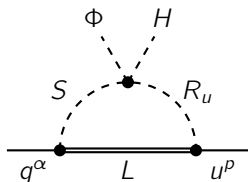


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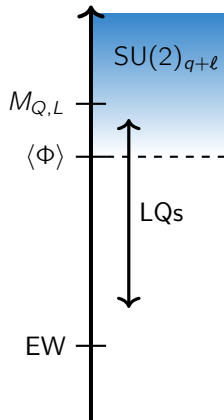
Rank-3 Yukawas: scalar leptoquarks



$$Y_u = \begin{pmatrix} \text{light blue} & \text{light blue} & \text{light blue} \\ \text{medium blue} & \text{medium blue} & \text{medium blue} \\ \text{dark blue} & \text{dark blue} & \text{dark blue} \end{pmatrix} \sim \frac{\langle \Phi \rangle}{M_L} \frac{\log(\mu/M_L)}{16\pi^2}$$

$$m_t \gg m_c \gg m_u > 0$$

Field	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$SU(2)_{q+l}$
R_u	3	2	7/6	1
R_d	3	2	1/6	1
S	3	1	2/3	2

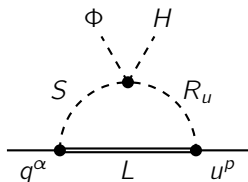


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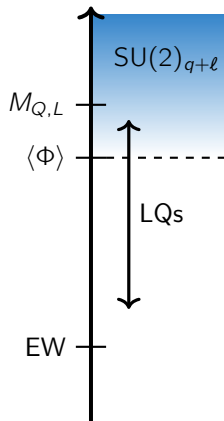
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Geljo, AET [2309.11547]

Benchmark at $\langle \Phi \rangle = M_{L,Q}/100 = 1 \text{ PeV}$:

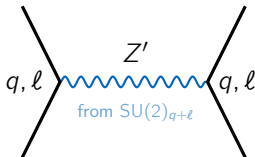
Most couplings in $[.1, 1]$ (exception $y_{b,\tau} = 0.01$)

Perhaps a 2HDM variation?

Phenomenology of Z'

There is a GIM-like mechanism suppressing 4-quark and 4-lepton FCNCs from

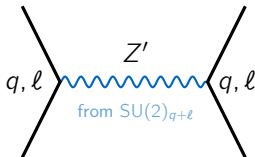
Darmé, Deandrea, Mahmoudi [2307.09595]



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Leading constraints from **q -to- ℓ “flavor transfer”**:

$$\text{BR}(K_L \rightarrow \mu^\pm e^\mp) = 5.9 \cdot 10^{-12} \cdot \left(\frac{300 \text{ TeV}}{\langle \Phi \rangle} \right)^4 (0.90 c_{2\ell} + 0.44 s_{2\ell})^2 < 4.7 \cdot 10^{-12}$$

Alignment of 2nd generation
leptons and quarks

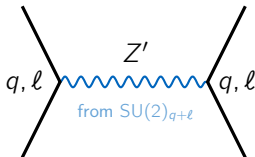
$$\text{CR}(\mu\text{Au} \rightarrow e\text{Au}) = 2 \cdot 10^{-11} \cdot \left(\frac{300 \text{ TeV}}{\langle \Phi \rangle} \right)^4 (1.01 s_{2\ell} - 0.25 c_{2\ell})^2 < 7 \cdot 10^{-13}$$

$$\hookrightarrow \langle \Phi \rangle \gtrsim 500 \text{ TeV}$$

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Future sensitivity:
 $\text{CR}(\mu\text{Al} \rightarrow e\text{Al}) \lesssim 10^{-16}$
Mu2e and COMET

$$\hookrightarrow \langle \Phi \rangle \gtrsim 500 \text{ TeV} \quad 5000 \text{ TeV} \text{ — future sensitivity}$$

Stability of the scalar potential admits two interesting scenarios:

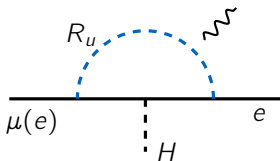
- I) $M_{R_u}, M_{R_d} \lesssim \langle \Phi \rangle \lesssim M_S \longrightarrow$ flavor physics
- II) $M_S \ll \langle \Phi \rangle \lesssim M_{R_u}, M_{R_d} \longrightarrow$ collider physics

Leptoquark phenomenology

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Light R_u ? Not likely

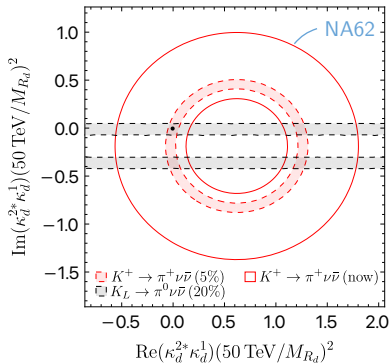


Lepton dipoles contributing to $\mu \rightarrow e \gamma$ and e EDM limits

$$M_{R_u} \gtrsim 500 \text{ TeV}$$

Phenomenology of R_d : Kaon physics

$$K \rightarrow \pi \nu \nu$$



R_d interacts mainly with 3-generation leptons:

$$\mathcal{L}_{UV} \supset -\kappa_d^p \bar{\ell}^3 \tilde{R}_d d^p + \text{H.c.}$$

Constraints from meson mixing (one loop)

$$|\text{Re}[(\kappa_d^{2*} \kappa_d^1)^2]| \left(\frac{50 \text{ TeV}}{M_{R_d}} \right)^2 \lesssim 1.0 \quad (\Delta m_K)$$

$$|\text{Im}[(\kappa_d^{2*} \kappa_d^1)^2]| \left(\frac{50 \text{ TeV}}{M_{R_d}} \right)^2 \lesssim 3 \cdot 10^{-3} \quad (\epsilon_K)$$

Low-scale scenario allowed:

$$M_{R_d} = 5 \text{ TeV} \quad |\kappa_d^2| \sim 0.3 \quad |\kappa_d^1| \lesssim 0.01$$

Phenomenology of R_d : B physics

$$\mathcal{L}_{UV} \supset -\kappa_d^p \bar{\ell}^3 \tilde{R}_d d^p + \text{H.c.}$$

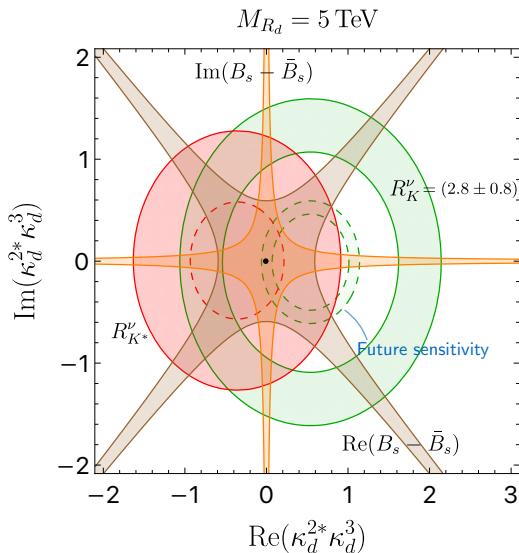
- Belle II 2023: $R_K^\nu = 2.8 \pm 0.8$

$$R_{K^{(*)}}^\nu = \frac{\text{BR}(B \rightarrow K^{(*)} \nu \nu)}{\text{BR}(B \rightarrow K^{(*)} \nu \nu)^{\text{SM}}}$$

- Consistency with meson mixing:

$$M_{R_d} \lesssim 5 \text{ TeV}$$

- Collider limits: $\gtrsim 1.5 \text{ TeV}$



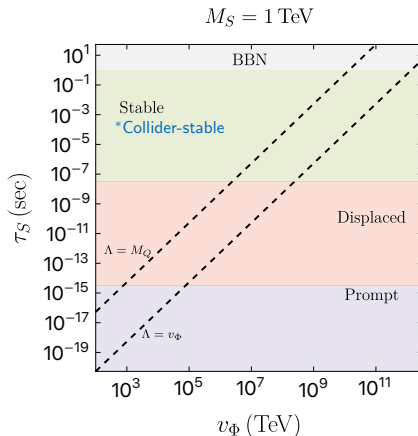
Long-lived particles—phenomenology of S

S as the lightest new particle:

- No renormalizable couplings to SM particles
- Decays suppressed by the UV scale

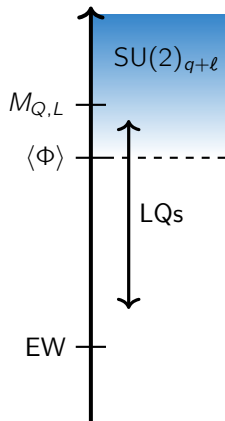
$$\frac{\Gamma_S}{M_S} \sim \frac{1}{16\pi^2} \frac{v_{EW}^2}{\Lambda^2}$$

- Interesting collider phenomenology for light S :
long-lived exotic hadrons
- Collider signature dependent on UV scale



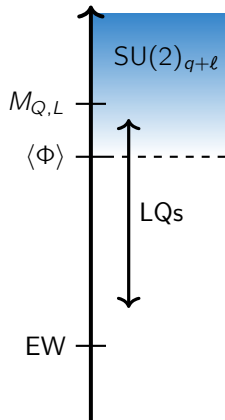
Summary

- A single $U(2)_{q+e(\ell)}$ can explain the entire flavor structure of the SM (– neutrinos)
- The simplicity of symmetry and breaking allows for simple(?) UV realizations of the mechanism
- There is interesting low-energy flavor phenomenology associated with its realization ($\mu \rightarrow e$ conversion in particular)
- Interesting variant in $SU(5)$, where $\mathbf{10}^\alpha$ is charged under $U(2)$ and $\bar{\mathbf{5}}$ is a singlet



Summary

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Thank You!