A Clockwork Tale

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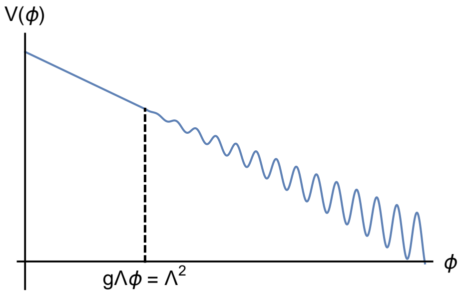
Prologue:
Making numbers small dynamically

Naturalness from a relaxion [Graham, Kaplan, Rajendran, '15]

- slow-rolling pseudo NG boson: **relaxion** ϕ, slope from $g\Lambda^3\phi$
- relaxion-dependent H mass term: $(\Lambda^2 - g\Lambda\phi)\, H^\dagger H$
- backreaction when H vev: $f_\pi^2\, m_\pi^2(v)\cos(\phi/f)$
- $\Longrightarrow\ v \ll \Lambda$ **dynamically selected**:



Clockworking the relaxion

- it requires **tiny** g, e.g. $g = O(v/\Lambda)^4 \approx 10^{-50}$
- technically natural, NG shift symmetry for $g \to 0$, but still ...
- it requires **trans-planckian** $\Delta\phi$
- Solution: [Choi, Im , '15; Kaplan, Rattazzi, '15]
- g from much larger period $F \gg f$:

$$-\mathcal{L} \supset \left[\Lambda^2 - \Lambda^2 \cos\left(\frac{\phi}{F} + \alpha\right)\right] H^\dagger H \;-\; \Lambda^4 \cos\left(\frac{\phi}{F} + \alpha\right) \;-\; m_{BR}^4(v) \cos\frac{\phi}{f}$$

$$\Longrightarrow g = \Lambda/F$$

- $F = 3^N f$ from **clockwork** chain:

$$-\mathcal{L} \supset \epsilon\left(\Phi_0^\dagger \Phi_1^3 + \Phi_1^\dagger \Phi_2^3 + \ldots + \Phi_{N-1}^\dagger \Phi_N^3\right) \;+\; \frac{\phi_1}{\hat{f}} G\widetilde{G} \;+\; \frac{\phi_N}{\hat{f}} G\widetilde{G}$$

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Get rid of the relaxion, keep the clockwork

- the last step itself is sufficient to **generate hierarchies**! [Giudice, McCullough, '16]
- **clockwork mechanism** $\to$ an elegant and economical way to generate **tiny numbers**/large hierarchies X with only $\mathcal{O}(1)$ **couplings** and $N \sim \log X$ **fields**
- a **framework** for model building: [Giudice, McCullough, '16; Giudice, McCullough, DT, in prep.]
  - low-scale invisible axions [Giudice, McCullough, '16; Farina, Pappadopulo, Rompineve, Tesi, '16]
  - **hierarchy problem** [Giudice, McCullough, '16]
  - neutrino physics [Hambye, DT, Tytgat, '16; Carena, Li, Machado $\times$ 2, Wagner, '17]
  - inflation [Kehagias, Riotto, '16]
  - SUSY [Giudice, McCullough, DT, in prep.] and SUGRA [Kehagias, Riotto, '17; Antoniadis, Delgado, Markou, Pokorski, '17]
  - . . . [not cited here for brevity]
  - **dark matter** [Hambye, DT, Tytgat, '16] (used in **this talk** to explain main features)
- dark matter cosmologically stable if decays by dim-5 ($\Lambda \gg M_{PL}$), dim-6 ($\Lambda \sim M_{GUT}$), tiny couplings $\Longrightarrow$ **all difficult to test**
- **clockwork** mechanism $\to$ dark matter cosmologically **stable** although it **decays into SM** via $\mathcal{O}(1)$ **interactions** with **TeV-scale** particles!
- large interactions $\Longrightarrow$ dark matter is a **thermal relic**, i.e. a **WIMP**

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Chapter 1:
How to do multiplications in QFT

The clockwork mechanism

Based on the simple observation that:

$1/2 \quad \times \quad 1/2 \quad \times \quad 1/2 \quad \times \quad 1/2 \quad \times \quad \ldots \quad \times \quad 1/2$ can **easily** be **tiny**

Use a **chain** of N fields

$$\phi_0 \;\underline{\;1/q\;}\; \phi_1 \;\underline{\;1/q\;}\; \phi_2 \;\underline{\;1/q\;}\; \phi_3 \;\underline{\;1/q\;}\; \ldots \;\underline{\;1/q\;}\; \phi_N \;\underline{\qquad}\; \mathbf{SM}$$

if clever **symmetry** $\longrightarrow \quad \phi_{light} \approx \phi_0 \quad \Longrightarrow \quad \phi_{light} - \mathbf{SM} \sim \mathbf{1/q^N} \quad (q > 1)$

For **fermions** use chiral symmetries

$$R_0 \;\underline{\;m\;}\; \underbrace{L_1 \quad R_1}_{qm} \;\underline{\;m\;}\; \underbrace{L_2 \quad R_2}_{qm} \;\underline{\;m\;}\; \underbrace{L_3 \quad R_3}_{qm} \;\underline{\;m\;}\; \ldots \;\underline{\;m\;}\; \underbrace{L_N \quad R_N}_{qm} \;\underline{\qquad}\; L_{SM}$$

light $N \approx R_0 \quad \Longrightarrow \quad N - L_{SM} \sim \mathbf{1/q^N}$

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Clockwork scalar

- For **scalars**, use a chain of $N+1$ symmetries: $U(1)_0 \times U(1)_1 \times \ldots \times U(1)_N$
- broken by N spurions $m_k^2 \equiv m^2$ with $\quad Q_k(m_k^2) = 1, \quad Q_{k+1}(m_k^2) = -q \quad (q > 1)$
- $\mathcal{L} = -\frac{f^2}{2} \sum_{k=0}^{N} |\partial U_k|^2 + \frac{m^2 f^2}{2} \sum_{k=0}^{N-1} \left(U_k^\dagger U_{k+1}^q + h.c. \right)$
- for the Goldstones ϕ_k, $U_k \propto e^{i\phi_k/f}$: $\quad -\mathcal{L} \supset \frac{m^2}{2} \sum_{k=0}^{N-1} \left(\phi_k - q\phi_{k+1} \right)^2$
- **unbroken** $U(1)$ with $\mathcal{Q} = \sum_k \frac{Q_k}{q^k} \quad \Longrightarrow \quad$ **massless** $\varphi_0 = \mathcal{N} \sum_k \frac{\phi_k}{q^k}$
- For instance, if $\mathcal{L} \supset \frac{\phi_N}{16\pi^2 f} G\widetilde{G} \quad \Longrightarrow \quad \frac{\varphi_0}{16\pi^2 F} G\widetilde{G}$ with $F = f\,\frac{q^N}{\mathcal{N}} \gg f$

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Clockwork dark matter [Hambye, DT, Tytgat, '16]

- chiral symmetry group:

  $U(1)_{R_0} \times U(1)_{L_1} \times U(1)_{R_1} \times \ldots \times U(1)_{L_N} \times U(1)_{R_N}$ with $U(1)_{R_N} \equiv U(1)_{L_{SM}}$

- **scalars**:

  $S_i \sim (-1, 1)$ under $U(1)_{R_i} \times U(1)_{L_{i+1}}$ $\quad C_i \sim (1, -1)$ under $U(1)_{L_i} \times U(1)_{R_i}$

- chain of fields:

$$\mathbf{R_0} \ \overset{S_1}{\text{——}} \ L_1 \ \overset{C_1}{\text{——}} \ R_1 \ \overset{S_2}{\text{——}} \ L_2 \ \overset{C_2}{\text{——}} \ \ldots \ \overset{C_N}{\text{——}} \ R_N \ \text{——} \ \mathbf{L_{SM}}$$

- **clockwork** mechanism when scalars acquire a **vev**:

$$m = y_S \langle S_i \rangle \qquad qm = y_C \langle C_i \rangle$$

- **Majorana mass** m_N for R_0, eigenstate $N \approx R_0$ is the **dark-matter** candidate

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Clockwork fermion

- the Lagrangian is

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm kinetic} - \sum_{i=1}^{N} \left(y_S S_i \bar{L}_i R_{i-1} - y_C C_i \bar{L}_i R_i + h.c. \right)$$
$$- \left(y \, \bar{L}_{SM} \widetilde{H} R_N + h.c. \right) - \frac{1}{2} \left(m_N \overline{R_0^c} \, R_0 + h.c. \right)$$

- after the scalars acquire vevs $m = y_S \langle S_i \rangle \,, \quad qm = y_C \langle C_i \rangle$:

$$\mathcal{L} \supset - m \sum_{i=1}^{N} \left(\bar{L}_i R_{i-1} - q \, \bar{L}_i R_i \right) - \frac{m_N}{2} \overline{R_0^c} \, R_0 + h.c.$$

- for $m_N = 0$, the “right-handed” mass matrix satisfies $M^\dagger M \equiv M^2_{scalar}$

- clockwork mechanism for $m_N \lessapprox qm \quad$ (for $q \gg 1$)

Chapter 2:
How clockwork matter became dark

The spectrum

Take $q \gg 1$ for simplicity

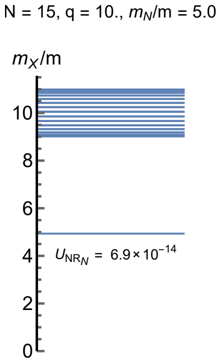
- the **dark-matter** Majorana fermion N with mass $\approx m_N$:

$$N \approx R_0 + \frac{1}{q^1}R_1 + \frac{1}{q^2}R_2 + \ldots + \frac{1}{q^N}R_N$$

- a **band** of N **pseudo-Dirac** ψ_i with mass $\approx qm$:

$$\psi_i \approx \frac{1}{\sqrt{N}} \sum_k \mathcal{O}(1) L_k + \mathcal{O}(1) R_k$$

- N scalars S_i and C_i expected in the same mass range (not necessarily dynamic, but not discussed here)



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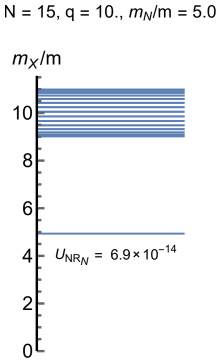

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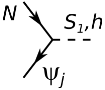

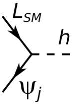

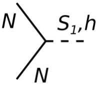

Cosmological (meta)stability of dark matter

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N can **decay**, e.g. $N \to \nu h, \nu Z, lW$, but

The coupling of **dark matter** to **SM fermions** is **clockwork suppressed**:

$$\mathcal{L} \supset -\frac{y}{q^N}\,\bar{L}_{SM}\widetilde{H}N_R$$

Dark matter cosmologically stable

The decay lifetime of N longer than the age of the Universe with $\mathcal{O}(1)$ **couplings** and $\lesssim$ **TeV-scale** states

- indirect detection $\Longrightarrow q^{2N} > 1.5 \times 10^{50}\,\left(\frac{m_N}{\mathrm{GeV}}\right) y^2$
  for example: $m_N \sim 100\,\mathrm{GeV}$, $y \sim 1$, $q \sim 10$, $N \sim 26$
- effect of **clockwork gears** ψ_j in loop diagrams also **clockwork-suppressed**

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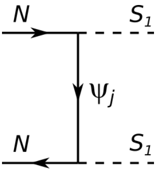
Scenario A: $m_S < m_N$

Dominant process:



from $N \sim R_0$, $\Psi_j \supset L_1$
and $y_S\, S_1 \overline{L}_1 R_0$

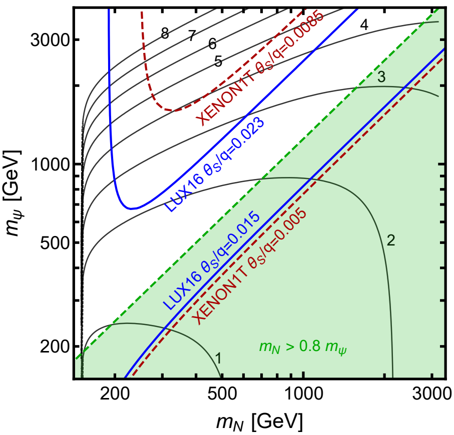
not clockwork-suppressed!

$\Longrightarrow$ **N is a WIMP**

perturbative $y_S < \sqrt{4\pi} \simeq 3.5$

$\Longrightarrow$ **N and ψ_j light enough**

m_{S_1} = 150 GeV



y_S needed for correct Ω_{DM}

Scenario B: $m_N < m_S$ and $2m_N < m_S + m_h$

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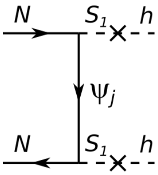
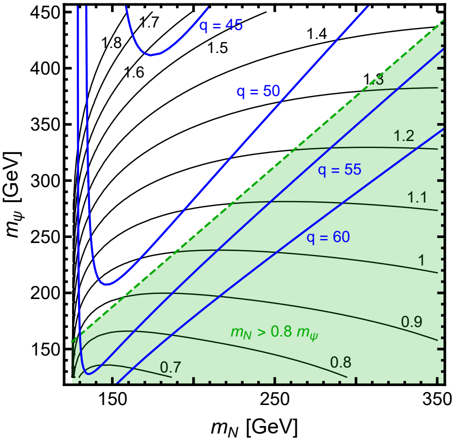


$\propto \quad (y_S\theta_S)^4$

$\theta_S \lesssim 0.4$ **from colliders**

y_S non-perturbative for universal θ_S:
$\theta_S \lesssim 0.4/\sqrt{N}$

it works also near the h and S resonances, for universal θ_S too



$y_S\theta_S$ needed for correct Ω_{DM}

Other limits and prospects

- Indirect detection: annihilation is p-wave, but decays $N \to h\nu$ monochromatic
- ψ_j in the hundreds of GeV range, coupled via $y \overline{L}_{SM} H R_N$ and $\psi_j \supset R_N$
  $\Longrightarrow$ pseudo-Dirac **RH neutrinos** in the **observable range**, **y sizeable**
  - EWPT: $|B_{l\psi}|^2 \equiv y^2 v^2/(2m_\psi^2) \lesssim 10^{-3}$
  - LFV: $BR(\mu \to e\gamma) \; \approx \; 8 \times 10^{-4} \, |B_{e\Psi}|^2 \, |B_{\mu\Psi}|^2 < 4.2 \times 10^{-13}$
  - direct L-conserving searches: up to $m_\psi \approx 200$ GeV with 300 fb^{-1} [Das, Dev, Okada, '14]
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- In scenario B S_1 **needs** to have **large mixing with h**, in A it can
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  - for $m_S < 500\,\text{GeV}$: $\theta_S < 0.3 - 0.4$ from direct searches
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Majorana neutrino masses [Hambye, DT, Tytgat, '16]

- SM leptons interact with TeV-scale ψ_i with large Yukawas $\Longrightarrow$ **huge m_ν**???
- Clockwork at work: if there were no $R_0 \Longrightarrow$ no chiral partner for νs
  but effect of R_0 has to go through the **whole clockwork chain**:

$$m_\nu \simeq \frac{m_D^2}{q^{2N} m_N}$$

- suppression here is smaller than for DM: $q = 10, m_N = 1\,\text{TeV} \Longrightarrow N \approx 7$
- ≥ 2 nonzero $m_\nu \Longrightarrow$ at least **2 clockwork chains**
- a **suggestive possibility**: 1 chain for dark matter, 2 chains for neutrino masses
- + resonant leptogenesis? Starting from 2 degenerate, a small mass splitting can be generated by interactions with the clockwork gears...
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Chapter 3:
One more dimension

Clockwork from a flat extra dimension [Hambye, DT, Tytgat, '16]

- the clockwork Lagrangian can come from a **discretized 5th dimension**
- **flat-spacetime** construction for fermion:
- 1 Dirac fermion with mass M in the 5D bulk $\to L_i, R_i$

$$\mathcal{L}_5 \supset \bar{\psi}(i\overleftrightarrow{\not{\partial}}_D - M)\psi = i\overline{\psi}\gamma^\mu\partial_\mu\psi + \left[\frac{1}{2}\left(\overline{L}\,\partial_Z R - (\partial_Z \overline{L})R\right) - M\overline{L}R + h.c.\right]$$

- + **Wilson term** $-\frac{a}{2}\,\partial_Z\overline{\psi}\,\partial_Z\psi = -\frac{a}{2}\,\partial_Z\overline{L}\,\partial_Z R$ removes 1 hopping direction
- to get light mode: orbifold with Dirichlet b.c. $L(0) = 0$
  (1 spare **chiral fermion** on one brane $\to R_0$)
- **SM chiral leptons** on the other brane $\to L_{SM}$
- discretized Lagrangian $\mathcal{L} \supset \sum_{i=0}^{N-1} \frac{1}{a}\overline{L}_{i+1}R_i - \sum_{i=1}^{N}\left(\frac{1}{a} + M\right)\overline{L}_i R_i$
- clockwork with $m = \frac{1}{a}, \quad qm = \frac{1}{a} + M, \quad q^N = \left(1 + \frac{\pi RM}{N}\right)^N \to e^{\pi RM}$

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Clockwork from the metric [Giudice, McCullough, '16]

- **curved-spacetime** construction for scalar:
- curved **metric** $ds^2 = X(|Z|)\,dx^2 + Y(|Z|)\,dZ^2$
- **massless** scalar in the 5D bulk:

$$\mathcal{S} = -2\int_0^R dZ \int d^4x\,\sqrt{-g}\,\frac{1}{2}\,g^{MN}\partial_M\phi\,\partial_N\phi = -\int_0^R dZ\int d^4x\,X^2 Y^{1/2}\left[\frac{(\partial_\mu\phi)^2}{X} + \frac{(\partial_Z\phi)^2}{Y}\right]$$

- discretized Lagrangian:

$$\mathcal{L} \supset \sum_{j=0}^{N-1} m_j^2(\phi_j - q_j\phi_{j+1})^2 \qquad \text{with } m_j^2 = \frac{X_j}{a^2 Y_j}, \quad q_j = \frac{X_j^{1/2}Y_j^{1/4}}{X_{j+1}^{1/2}Y_{j+1}^{1/4}}$$

- clockwork if $X_j \propto Y_j$
  finite for $N \to \infty$ if $X_j \propto Y_j \propto e^{-\frac{4}{3}k\,a\,j}$
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The clockwork metric

- in the **continuum**: $ds^2 = e^{\frac{4}{3}k|Z|}\left(dx^2 + dZ^2\right)$
- Kaluza-Klein modes for massless scalar

$$\psi_0(Z) \simeq \sqrt{k\pi R}\, e^{-k\pi R} \implies \frac{dP}{dZ} \propto e^{2\,k\,Z}$$

$$\psi_n(Z) = e^{-k\,Z} \times \text{oscillatory} \implies \frac{dP}{dZ} = \text{oscillatory}$$

- what about **Large Extra Dimension** or **Randall-Sundrum**?

	m_j	q_j
LED	$\frac{1}{a}$	1
RS	$\frac{1}{a}\, e^{-\hat{k}\,a\,j}$	$e^{\hat{k}\,a}$
clockwork	$\frac{1}{a}$	$e^{\hat{k}\,a}$

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Chapter 4:
Clockwork naturalness

Clockwork graviton

- discrete clockwork: $N+1$ **copies** of **4D gravity** $g_j^{\mu\nu}$
- linear approximation: $g_j^{\mu\nu} = \eta_j^{\mu\nu} + 2h_j^{\mu\nu}/M_j$
- clockwork Pauli-Fierz mass terms

$$\mathcal{L} = -\frac{m^2}{2}\sum_{j=0}^{N-1}\left([h_j^{\mu\nu} - q\, h_{j+1}^{\mu\nu}]^2 - [\eta_{\mu\nu}(h_j^{\mu\nu} - q\, h_{j+1}^{\mu\nu})]^2\right)$$

- invariant under $h_j^{\mu\nu} \to h_j^{\mu\nu} + \frac{1}{q^j}(\partial^\mu A^\nu + \partial^\nu A^\mu)$
- $\Longrightarrow$ **massless graviton** $\mathfrak{h}_0^{\mu\nu}$ **localized** at $j=0$:

$$\frac{1}{M_N} h_N^{\mu\nu} T_{\mu\nu} \longrightarrow \frac{1}{M_P}\mathfrak{h}_0^{\mu\nu} T_{\mu\nu} \quad \text{with} \quad M_P = \frac{q^N M_N}{\mathcal{N}}$$

- but... multi-gravity theories are dodgy $\to$ **continuum limit**

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The metric from the linear dilaton

- we want **massless 5D gravity** with a clockwork **metric** $ds^2 = e^{\frac{4}{3}k|Z|}(dx^2 + dZ^2)$
- clockwork gravity $\to$ metric should not be treated as a background
- can we **obtain** the metric?
- linear **dilaton** model (Jordan frame):

$$\mathcal{S} = \int d^4x\, dZ \sqrt{-g}\, \frac{M_5^3}{2}\, e^S (\mathcal{R} + g^{MN}\partial_M S\, \partial_N S + 4k^2) + \text{brane } \Lambda s$$

- k breaks global Weyl $g_{MN} \to e^{-2\alpha} g_{MN}, \quad S \to S + 3\alpha$
- go to Einstein frame, solve EoMs: $S = 2\,k\,|Z|\,, \quad ds^2 = e^{\frac{4}{3}k|Z|}(dx^2 + dZ^2)$
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A solution to the hierarchy problem

- effective **4D Planck mass**: $M_P^2 = 2M_5^2 \int_0^{\pi R} dZ\, e^{2kZ} = \frac{M_5^3}{k}(e^{2\pi kR} - 1)$
- 4D graviton fluctuations: $ds^2 \; = \; e^{\frac{4}{3}k|Z|}\left[\left(\eta_{\mu\nu} + \frac{2}{M_5^{3/2}} h_{\mu\nu}\right) dx^\mu dx^\nu \; + \; dZ^2\right]$
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- $\mathcal{L}_{SM}$ at $Z = 0$: $\frac{h^{\mu\nu}(x, Z=0)\, T_{\mu\nu}^{SM}(x)}{M_5^{3/2}} \longrightarrow \sum_n \frac{\mathfrak{h}_n^{\mu\nu}(x)\, T_{\mu\nu}^{SM}(x)}{\Lambda_n}$ with

$$\Lambda_0 = M_P\,, \qquad \Lambda_n^2 = M_5^3\, \pi R\,(1 + k^2R^2/n^2)$$

- the cutoff is $M_5 \implies m_h = O(M_5) \ll M_P \;\rightarrow$ **solution** to **hierarchy problem**
- for $k = 1$ TeV, $M_5 = 10$ TeV $\;\rightarrow\; k\,R \simeq 10$

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Phenomenology

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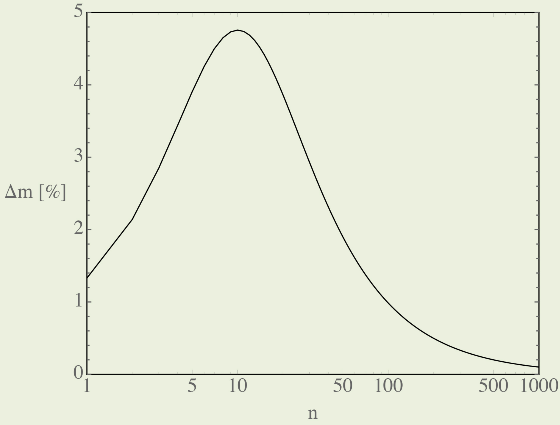
Clockwork mass splitting:

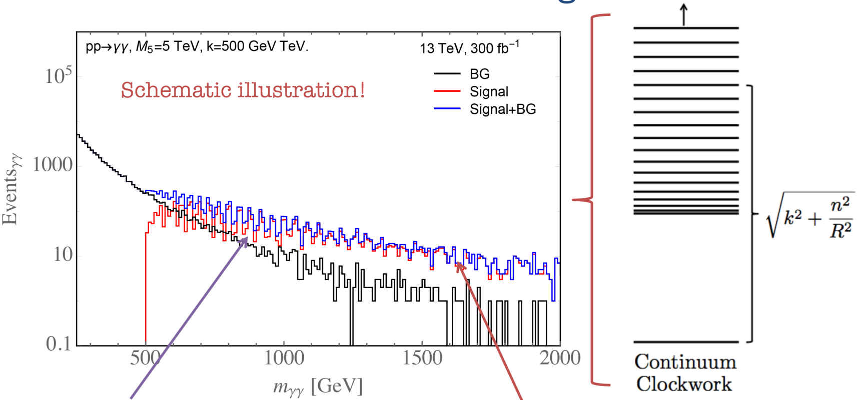


This … eory.

[slide by M. McCullough]

Phenomenology

At colliders would look something like:



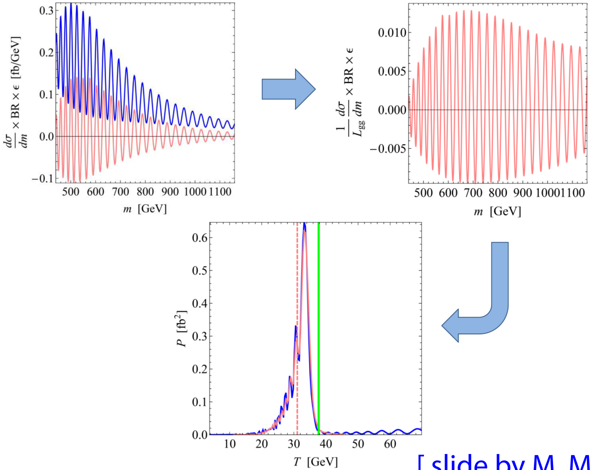
Most interestingly, due to splittings, signal appears to "oscillate". Thus get extra sensitivity by doing spectral analysis... The "power spectrum" of LHC data!

Can search for continuum spectrum at high energies.

[slide by M. McCullough]

Phenomenology

The fourier transform would then exhibit a peak near the inverse radius:



[slide by M. McCullough]

Epilogue: Scrambling the clockwork

Disassembling the clockwork? [Craig, Garcia Garcia, Sutherland, '17]

- **Disclaimer**: I'm simplifying the argument
- **Definition** of clockwork: *a theory with no exponential hierarchies in fundamental parameters that gives rise to exponentially suppressed* **site-dependent** *couplings to symmetry-protected zero mode*
- **Claim 1**: **no clockwork from geometry**
- for a scalar in curved clockwork metric: $\psi_0 = \text{const.} \equiv \mathcal{C}_0$
- coupling on a brane at $Z = Z_0$: $\frac{\phi}{16\pi^2 f_{5D}} G\widetilde{G} \Longrightarrow \frac{\varphi_0}{16\pi^2 F} G\widetilde{G}$

  with $F = f_{5D}^{3/2}/\mathcal{C}_0 = M_{PL}\left(\frac{f_{5D}}{M_5}\right)^{3/2}$ **independent on** $Z_0 \Longrightarrow$ **no clockwork**
- **Claim 1b**: **clockwork only for flat-spacetime** construction with bulk mass and boundary terms (they do it for scalar, but essentially a re-discovery of construction in [Hambye, DT, Tytgat, '16])

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  with $F = f_{5D}^{3/2}/\mathcal{C}_0 = M_{PL}\left(\dfrac{f_{5D}}{M_5}\right)^{3/2}$ **independent on Z_0** $\implies$ **no clockwork**
- **Claim 1b**: **clockwork only for flat-spacetime** construction with bulk mass and boundary terms (they do it for scalar, but essentially a re-discovery of construction in [Hambye, DT, Tytgat, '16])

Disassembling the clockwork? [Craig, Garcia Garcia, Sutherland, '17]

- **Disclaimer**: I'm simplifying the argument
- **Definition** of clockwork: *a theory with no exponential hierarchies in fundamental parameters that gives rise to exponentially suppressed* **site-dependent** *couplings to symmetry-protected zero mode*
- **Claim 1**: **no clockwork from geometry**
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Disassembling the clockwork? (continued)

- **Claim 2**: **no non-Abelian clockwork** (including gravity)
- non-Abelian Yang-Mills clockwork chain
- kinetic terms:

$$-\mathcal{L}_{kin} = \sum_{j=0}^{N} \frac{1}{4g_j^2} F_j F_j = \sum_{j=0}^{N} \frac{1}{4g_j^2} \left(F_j^{abelian} F_j^{abelian} + 4f\, A_j\, A_j\, \partial A_j + f\, f\, A_j\, A_j\, A_j\, A_j \right)$$

- if, in terms of zero mode $\mathcal{A}_0$, $A_j = c_j \mathcal{A}_0 + \ldots$

$$-\mathcal{L}_{kin} \supset \sum_{j=0}^{N} \frac{c_j^2}{4g_j^2} \mathcal{F}_0^{abelian} \mathcal{F}_0^{abelian} + \sum_{j=0}^{N} \frac{c_j^3}{4g_j^2} 4f\, \mathcal{A}_0\, \mathcal{A}_0\, \partial \mathcal{A}_0 + \sum_{j=0}^{N} \frac{c_j^4}{4g_j^2} f\, f\, \mathcal{A}_0\, \mathcal{A}_0\, \mathcal{A}_0\, \mathcal{A}_0$$

- gauge invariance for $\mathcal{A}_0$: $\sum_{j=0}^{N} \frac{c_j^2}{g_j^2} = \sum_{j=0}^{N} \frac{c_j^3}{g_j^2} = \sum_{j=0}^{N} \frac{c_j^4}{g_j^2} \equiv \frac{1}{g_{eff}^2}$
- $\implies \quad c_j \in \{0, 1\} \quad \rightarrow$ **no clockwork**
- $g_j =$ const. $\implies \quad g_{eff} \sim g_j$ no exponential suppression

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Reassembling the clockwork [Giudice, McCullough, '17]

- **Disclaimer**: I'm simplifying the argument
- **answer** to **Claim 2** (here for scalar, given also for gravity):
- $-\mathcal{L} \supset \frac{1}{2}\sum_{k=0}^{N} \partial_\mu \phi_k \, \partial^\mu \phi^k \; + \; \frac{m^2}{2}\sum_{k=0}^{N-1} \left(\phi_k - q\phi_{k+1}\right)^2$ same theory as
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- in this basis the unbroken symmetry is $\pi_k \to \pi_k + \alpha$, rather than $\phi_k \to \phi_k + q^{-k}\alpha$
  $\Longrightarrow \quad c_k = 1 \;\to$ argument for non-abelian **disappears**
- in this basis $g_k = g_0\, q^{-k} \neq$ const. $\Longrightarrow \quad g_{eff} \approx g_N = g_0\, q^{-N}$
- in this basis the theory **does not look** like clockwork, but it's the **same theory** (physics is the same)

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- **answer** to **Claim 1b**:
- the **curved** and **flat**-spacetime scalar constructions are the **same**! (related by field redefinition $\phi = e^{-kZ}\pi$)
- summary of the answer so far: 2 theories with same Lagrangian are the same
- **answer** to **Claim 1**:
- definition of [Craig, Garcia Garcia, Sutherland, '17] is (UV) **model dependent**, e.g.:

  $$S \supset \int d^5x\, \delta(Z - Z_0)\, e^{nS/2} \frac{\phi}{16\pi^2 f} G\widetilde{G}$$

  Z_0-profile of coupling depends on n, spurion charge of f under global Weyl
- for **model building** and **hierarchy problem**, relevant definition: *a theory with no exponential hierarchies in fundamental parameters that gives rise to exponentially suppressed* **ratios** *between the zero-mode and the clockwork-gears couplings*
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The End ?

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The relation between curved- and flat-spacetime constructions
opens a Pandora box...

[Giudice, McCullough, DT, in preparation]