Role of right-handed neutrinos in the electroweak symmetry breaking

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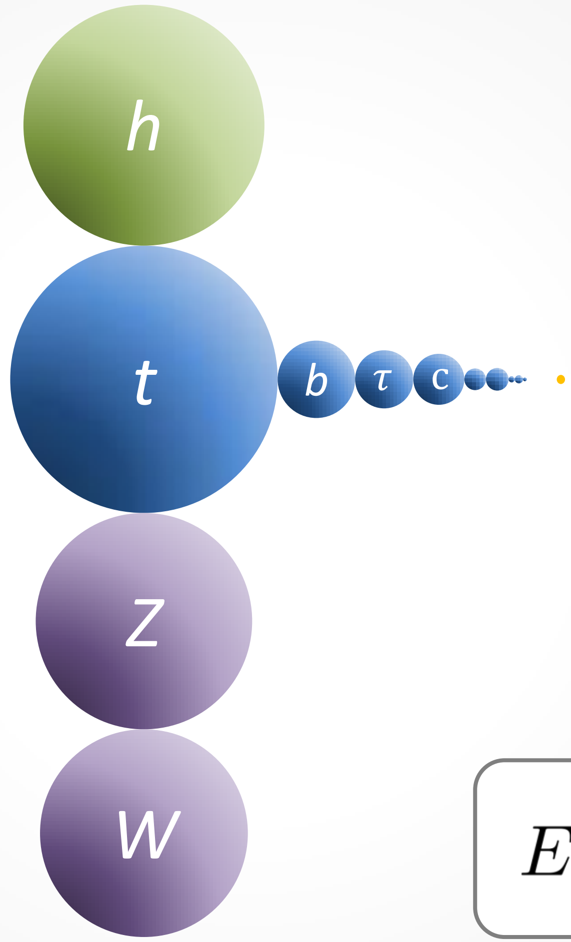
$M_h \doteq 125\,\mathrm{GeV}$

$m_t \doteq 172\,\mathrm{GeV}$

$v \doteq 246\,\mathrm{GeV}$



$$E = \sqrt{p^2 + m^2}$$

Standard Model

Electroweak symmetry breaking: W, Z bosons are massive.

$$\boxed{\mathrm{U}(1)_Y \times \mathrm{SU}(2)_L}$$

is spontaneously broken.

Standard Model describes it by complex Higgs field,

$$\mathcal{L} \quad \supset \quad \mu^2 H^\dagger H + \lambda (H^\dagger H)^2 + D^\mu H^\dagger D_\mu H$$

which develops nonzero v.e.v.

$$H^\dagger H = -\frac{\mu^2}{2\lambda} \quad \overset{\text{if}}{>} 0$$

Superconductivity

Meisner effect: photons are massive in the bulk of superconductor.

$$\boxed{\mathrm{U}(1)_{\mathrm{em}}}$$

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Ginzburg—Landau theory describes it by complex order parameter field,

$$\boxed{F \supset \alpha|\phi|^2 + \frac{\beta}{2}|\phi|^4 + \frac{1}{2m_e}|(-i\hbar\nabla - 2e\mathbf{A})\phi|^2}$$

which can develop nonzero value $|\phi|^2 = -\frac{\alpha}{\beta} \overset{\text{if}}{>} 0$

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PHENOMENOLOGICAL DESCRIPTION

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Bardeen—Cooper—Schrieffer theory: $\phi \sim \psi^e_{\mathbf{k}}\psi^e_{-\mathbf{k}}$

electrons acquire a gap

$$E = \sqrt{\epsilon^2_{\mathbf{k}} + |\Delta|^2}$$

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PHENOMENOLOGICAL DESCRIPTION?

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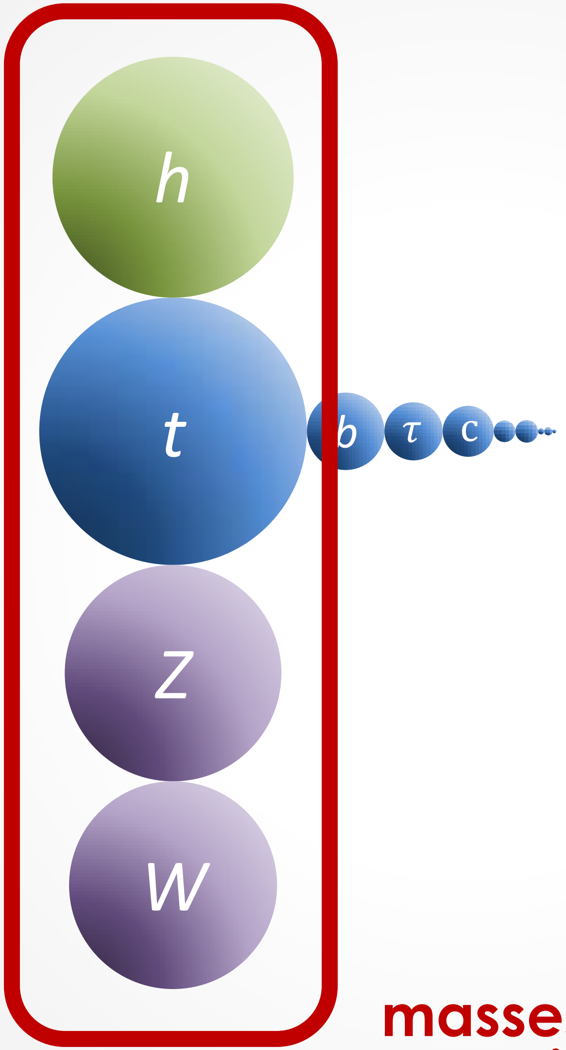
Dynamical EWSB: $H \sim \bar{\Psi}\Psi$

e.g. (E)TC

$$E = \sqrt{\mathbf{p}^2 + m^2}$$

Our approach



masses of the same order of magnitude

Our approach

primary is the dynamical origin of quark and lepton masses,

which is provided by their condensation:

$$m_t \propto \langle \bar{t}t \rangle\,, \quad m_b \propto \langle \bar{b}b \rangle\,, \quad m_c \propto \langle \bar{c}c \rangle\,,\ \dots\,, m_\tau \propto \langle \bar{\tau}\tau \rangle\,, \quad m_\mu \propto \langle \bar{\mu}\mu \rangle\,,\ \dots$$

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the electroweak symmetry breaking comes **automatically**

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the electroweak symmetry breaking comes **automatically**

bound states

W, Z gauge bosons

$$Z_L \sim (\bar{t}\gamma_5 t) + (\bar{b}\gamma_5 b) + (\bar{c}\gamma_5 c) + \ldots$$

Nambu—Goldstone bosons

Higgs boson

$$h \sim (\bar{t}t) + (\bar{b}b) + (\bar{c}c) + \ldots$$

unitarization of amplitudes

$$M_{W,Z} \sim v \sim m_t\,, m_b\,, m_c\,, \ldots$$

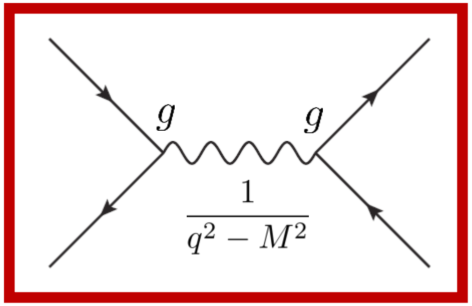
[Ho82] [KiMu85] [Na88] [MiYa89] [BaHiLi89]

in ***strict*** analogy with superconductivity

New dynamics among fermions

… substituting the Higgs sector of SM.

Renormalizable models:



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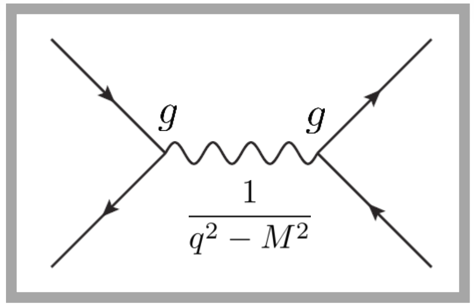
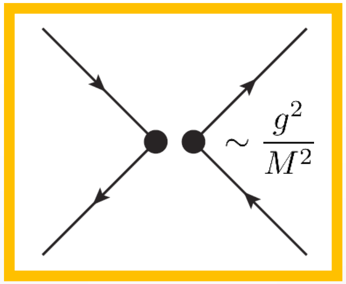


$$q^2 \ll M^2 \sim \Lambda^2 < \Lambda^2_{\mathrm{Planck}}$$

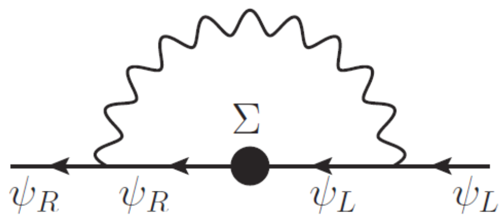
Simplification:
four-fermion interaction



Dynamical fermion mass generation

Full propagator: $S(p) = \dfrac{1}{\not{p} - \Sigma(p^2)}$

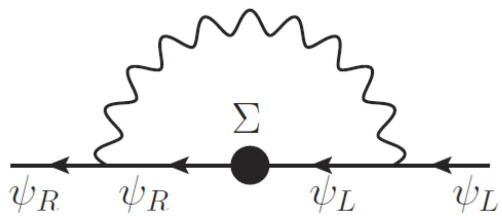
$\Sigma(p^2) \; =$



Mass is the pole in the propagator: $\det\left[p^2 - \Sigma^\dagger(p^2)\Sigma(p^2)\right] = 0$

Dynamical fermion mass

Full propagator: $S(p) = \dfrac{1}{\not p - \Sigma(p^2)}$

$\Sigma(p^2) \;=$ 

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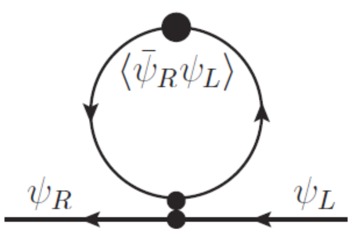
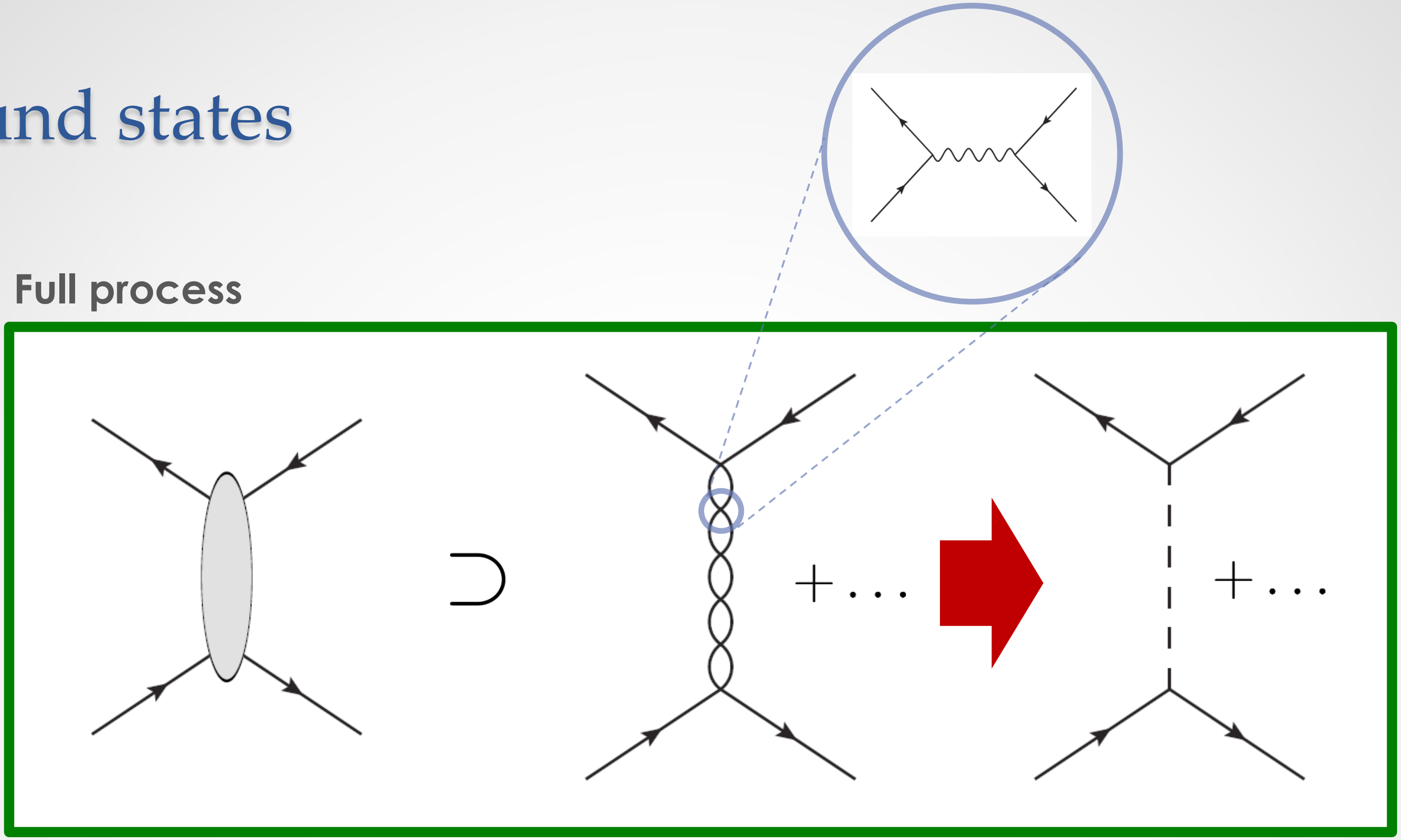
$\langle \bar{\psi}_R \psi_L \rangle \;=$ 

Bound states

Full process



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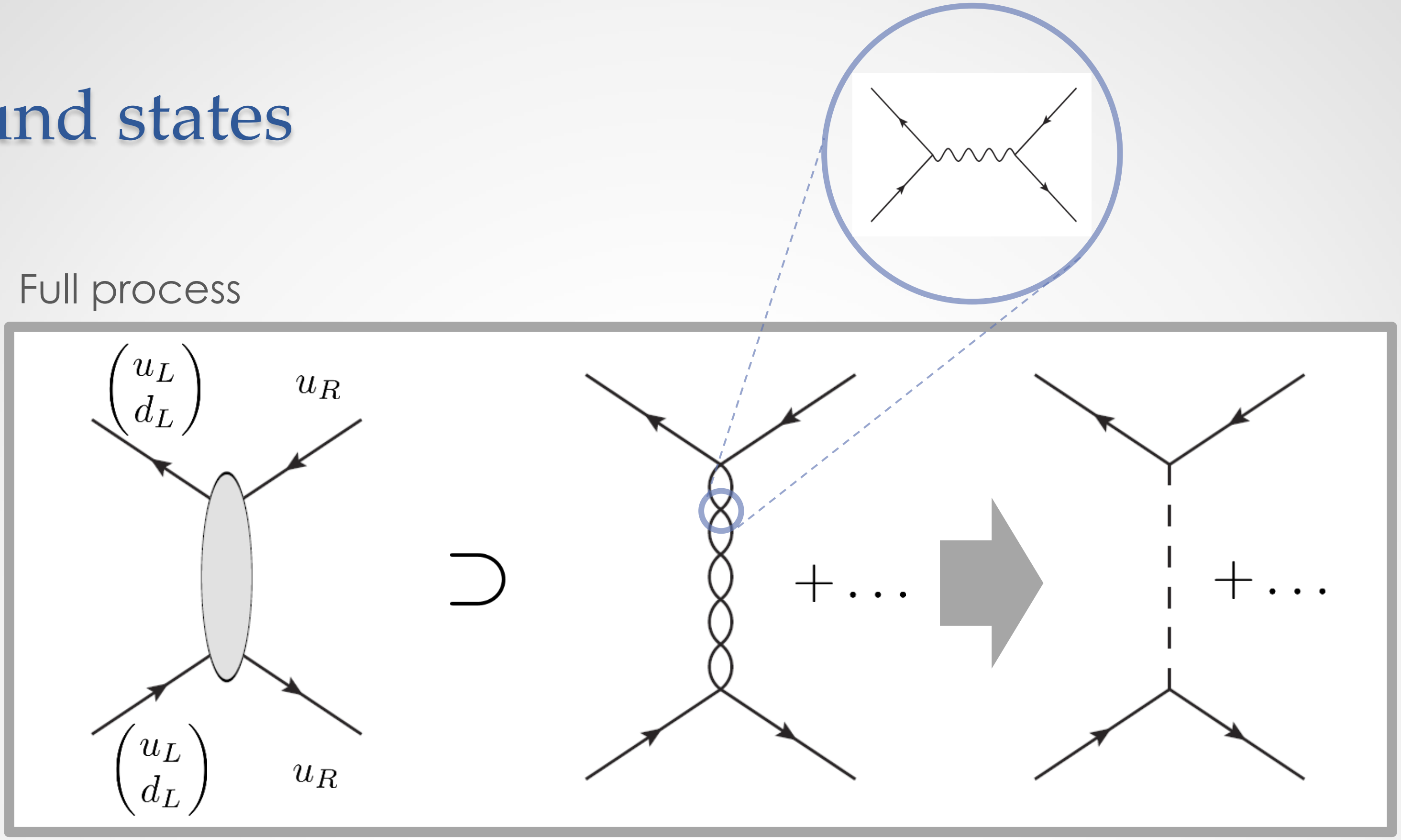
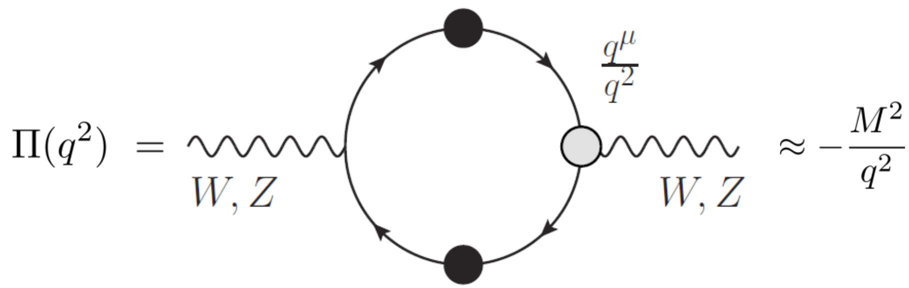


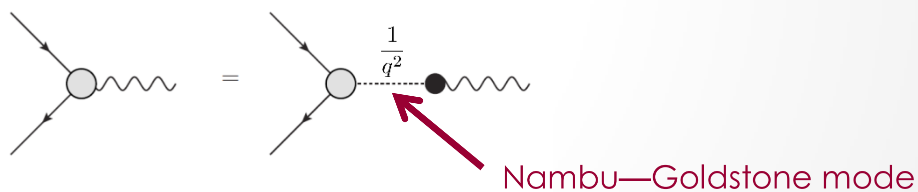
composite scalars:
Nambu—Goldstone bosons, Higgs boson

$$\bar{u}_R q_L = \bar{u}_R \begin{pmatrix} u_L \\ d_L \end{pmatrix} \Longrightarrow H_u \sim \begin{pmatrix} \bar{u}_R u_L \\ \bar{u}_R d_L \end{pmatrix}$$

Electroweak gauge boson mass generation

Pagels—Stokar formulae





$$\Delta \propto \frac{1}{q^2 + q^2\Pi(q^2)} \approx \frac{1}{q^2 - M^2}$$

Low-energy effective description

We expect that the low-energy effective description is the **multi-composite-Higgs-doublet model**.

$$\boxed{H_t,\ H_b,\ H_\tau,\ H_c,\ H_s,\ H_\mu,\ \ldots}$$

Example of the dynamics

P. Beneš, J. Hošek, A. S., arXiv:1101.3456
A. S., JHEP 1304, (2013) 139

Flavor gauge model

We postulate the **flavor SU(3)$_F$ gauge dynamics** which

- is chiral
- is asymptotically free
- does not confine
- self-breaks
- generates masses of quarks and leptons

$$\bar{h}^2(\Lambda_F^2) \sim 1$$

Flavor representations of fermions:

$$m_\psi \propto \langle \bar{\psi}_R(r_R)\psi_L(r_L)\rangle \; : \quad \bar{r}_R \times r_L$$

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$$m^{\mathbf{3}\times\mathbf{3}} \neq m^{\overline{\mathbf{3}}\times\mathbf{3}} \neq m^{\mathbf{3}\times\overline{\mathbf{3}}} \neq m^{\overline{\mathbf{3}}\times\overline{\mathbf{3}}}$$

$$\mathbf{3}\times\mathbf{3} = \mathbf{6} + \overline{\mathbf{3}}$$

Flavor gauge model

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Flavor representations of fermions:

q_L	u_R	d_R	ℓ_L	e_R
$\mathbf{3}$	$\mathbf{3}$	$\overline{\mathbf{3}}$	$\overline{\mathbf{3}}$	$\mathbf{3}$

Need for additional fermions: **right-handed neutrinos**

non-minimal choice:

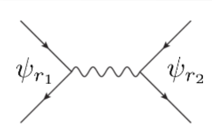
$$\begin{array}{c} \nu_R \\ 1 \times \mathbf{6},\ 4 \times \overline{\mathbf{3}} \end{array}$$

Fermion condensation

By the **MAC method** we can estimate the strength of attraction among fermions.

Charged fermions

$$
\begin{aligned}
&(\text{A.C.})_u = \overline{\mathbf{3}} \times \mathbf{3} \to \mathbf{1} && \Delta C_2 = 8/3 \\
&(\text{A.C.})_d = \mathbf{3} \times \mathbf{3} \to \overline{\mathbf{3}} && \Delta C_2 = 4/3 \\
&(\text{A.C.})_e = \overline{\mathbf{3}} \times \overline{\mathbf{3}} \to \mathbf{3} && \Delta C_2 = 4/3
\end{aligned}
$$



$$
\begin{aligned}
(\text{A.C.}) &= r_1 \times r_2 \to r_{\text{pair}} \\
\Delta C_2 &= C_2(r_1) + C_2(r_2) - C_2(r_{\text{pair}})
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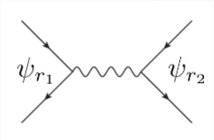

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Neutrinnos in the Nambu—Gorkov formalism

$$(\mathrm{A.C.})_n = \left(\begin{array}{c|c|c} \mathbf{3} \times \mathbf{3} \to \bar{\mathbf{3}} & \bar{\mathbf{3}} \times \mathbf{3} \to \mathbf{1} & \mathbf{6} \times \mathbf{3} \to \mathbf{8} \\ \hline \mathbf{3} \times \bar{\mathbf{3}} \to \mathbf{1} & \bar{\mathbf{3}} \times \bar{\mathbf{3}} \to \mathbf{3} & \mathbf{6} \times \bar{\mathbf{3}} \to \mathbf{3} \\ \hline \mathbf{3} \times \mathbf{6} \to \mathbf{8} & \bar{\mathbf{3}} \times \mathbf{6} \to \mathbf{3} & \mathbf{6} \times \mathbf{6} \to \bar{\mathbf{6}} \end{array}\right) \qquad (\Delta C_2) = \left(\begin{array}{c|c|c} 4/3 & 8/3 & 5/3 \\ \hline 8/3 & 4/3 & 10/3 \\ \hline 5/3 & 10/3 & 10/3 \end{array}\right)$$

$$n_{\mathrm{R}} = \begin{pmatrix} (\nu_{\mathrm{L}\mathbf{3}})^c \\ \nu^1_{\mathrm{R}\bar{\mathbf{3}}} \\ \nu^2_{\mathrm{R}\bar{\mathbf{3}}} \\ \nu^3_{\mathrm{R}\bar{\mathbf{3}}} \\ \nu^4_{\mathrm{R}\bar{\mathbf{3}}} \\ \nu_{\mathrm{R}\mathbf{6}} \end{pmatrix}$$

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right-handed neutrinos in the Majorana channels condense at the highest scale

$$\Lambda_{\mathrm{F}}$$

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right-handed neutrinos in the Majorana channels condense at the highest scale

It drives the spontaneous flavor symmetry breaking.
It suggests the seesaw patern of the neutrino mass matrix.

$$\Lambda_{\mathrm{F}} \sim M_R \sim M_C$$

Is the scenario viable?

A.S., Eur. Phys. J. C 73, (2013) 1

Top-quark condensation model

[MiYa89] [BaHiLi89]

Out of usual Dirac fermions, only the **top-quark** contributes **significantly** to the electroweak scale by its condensate.

single-composite-Higgs-doublet model

Top-quark is too light to saturate the electroweak scale $\quad v_t < 0.68\, v$

The composite Higgs boson is predicted too heavy $\quad M_h > m_t$

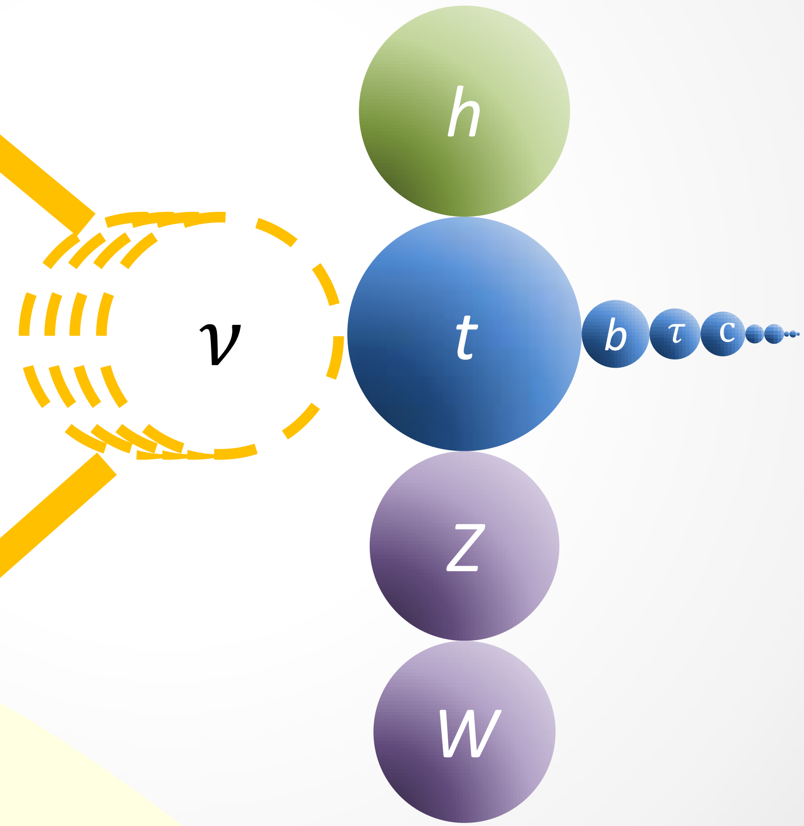
$\Lambda < \Lambda_{\text{Planck}}$

Way out – neutrino condensation

$$-\frac{m_D^2}{M_R} \sim 0.1\,\mathrm{eV}$$

The seesaw mechanism

$$(\bar{\nu}_L \;\; \bar{\nu}_R^c)\begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix}\begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix}$$



$$M_R \sim 10^{14}\,\mathrm{GeV}$$

[Ma'91] [AnKeLiRa'03]

Top-quark and neutrino condensation model

two-composite-Higgs-doublet model

$$\boxed{H_t,\ H_\nu}$$

[AS'13]

to reproduce

- the electroweak scale $v \doteq 246\,\mathrm{GeV}$
- the top-quark mass $m_t \doteq 172\,\mathrm{GeV}$
- the neutrino mass $m_\nu \doteq 0.2\,\mathrm{eV}$
- the Higgs boson mass $M_h \doteq 125\,\mathrm{GeV}$

RESULT:

$$y_t \sim 0.6$$

$$M_{H^+} \sim (200 - 250)\,\mathrm{GeV}$$

$$N_{\nu_R} = \mathcal{O}(10 - 100)$$

$$\Lambda \sim (10^{17} - 10^{18})\,\mathrm{GeV}$$

Lagrangian of the model and composite Higgs doublets

Four-fermion interaction:

$$\mathcal{L} \supset -G_t(\bar{t}_R q_L)(\bar{q}_L t_R) - G_\nu\Big(\sum_{s=1}^{N} \bar{\nu}_R^s \ell_L\Big)\Big(\sum_{s'=1}^{N} \bar{\ell}_L \nu_R^{s'}\Big) - \frac{1}{2} M_R \bar{\nu}_R^s (\nu_R^s)^c + \text{h.c.}$$

The condensation happens at the scale Λ

Specially designed to provide the simplest seesaw pattern.

Two-Higgs-doublet effective description:

$\mu_{t\nu}$

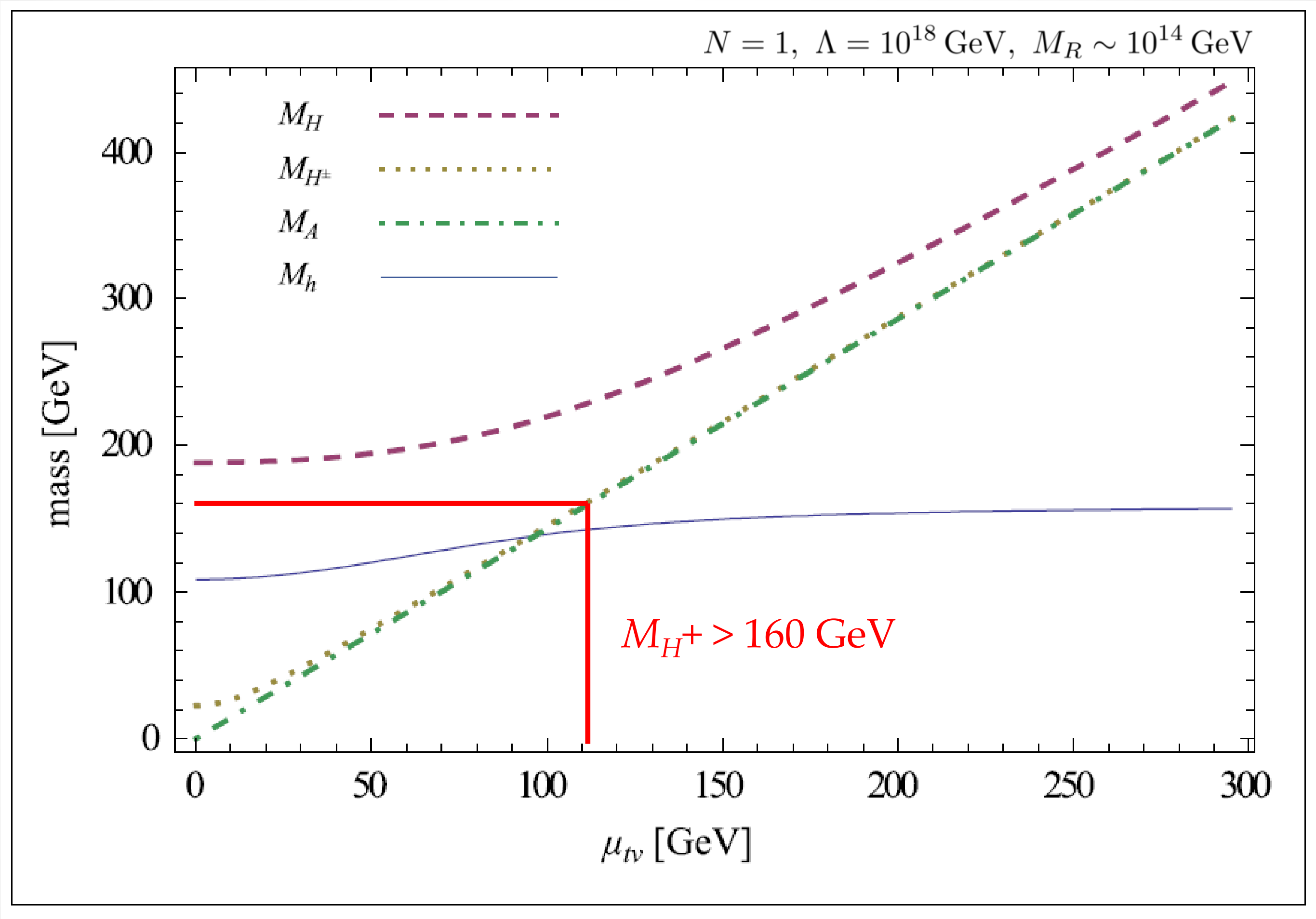
$$\mathcal{L}_{\text{eff}} \supset -y_t(\bar{q}_L t_R)H_t - y_\nu\Big(\sum_s \bar{\ell}_L \nu_R^s\Big)H_\nu - \mathcal{V}(H_t, H_\nu)$$

$$\boxed{\Lambda_{\text{Planck}} > \Lambda > M_R}$$

Mass spectrum of Higgs bosons

$$\boxed{H_t, H_\nu \quad \longrightarrow \quad h, H, A, H^+}$$

Larger values of $\mu_{t\nu}$ are preferred.

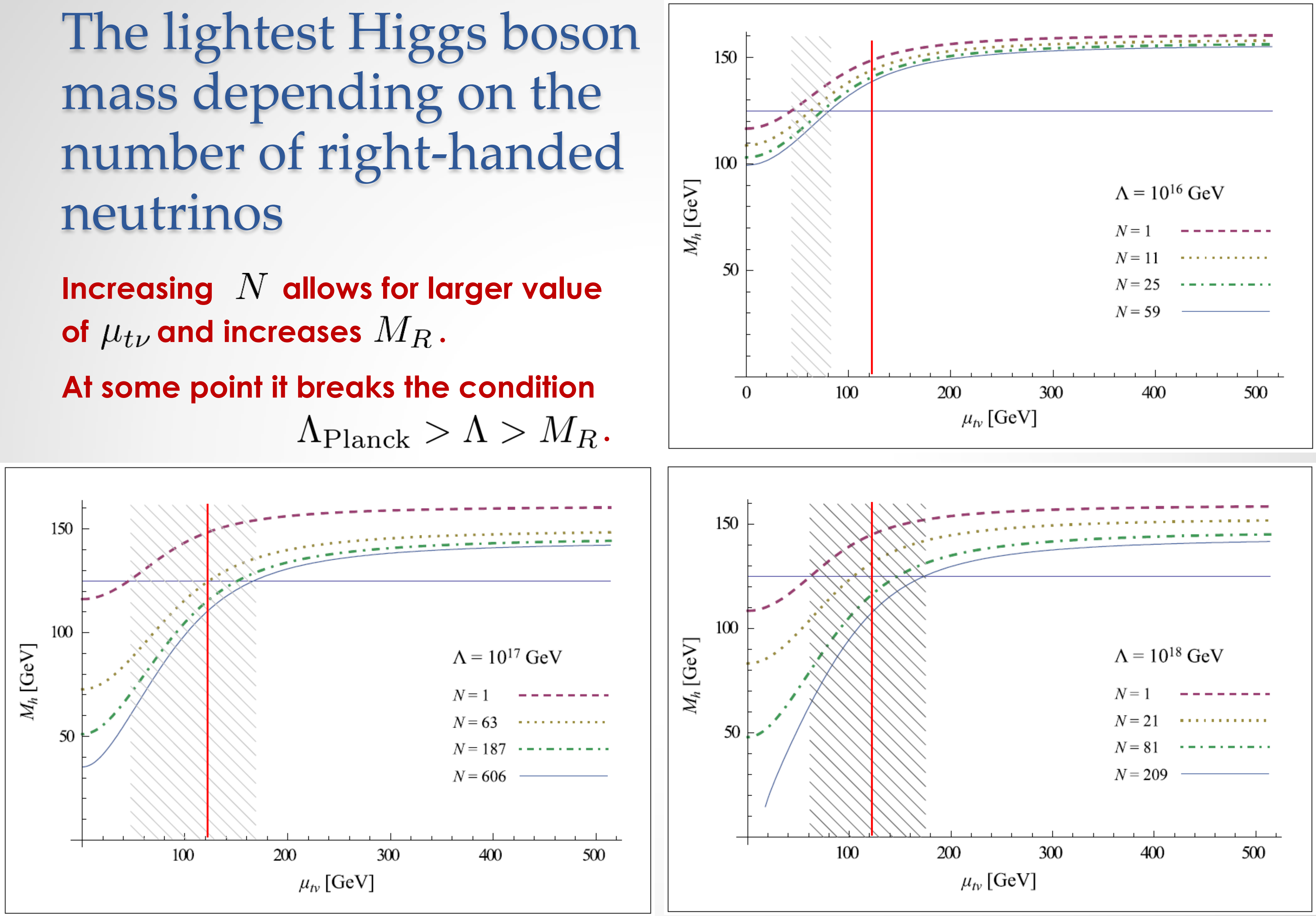


The lightest Higgs boson mass depending on the number of right-handed neutrinos

Increasing N allows for larger value of $\mu_{t\nu}$ and increases M_R.

At some point it breaks the condition

$$\Lambda_{\text{Planck}} > \Lambda > M_R.$$



Number of right-handed neutrinos

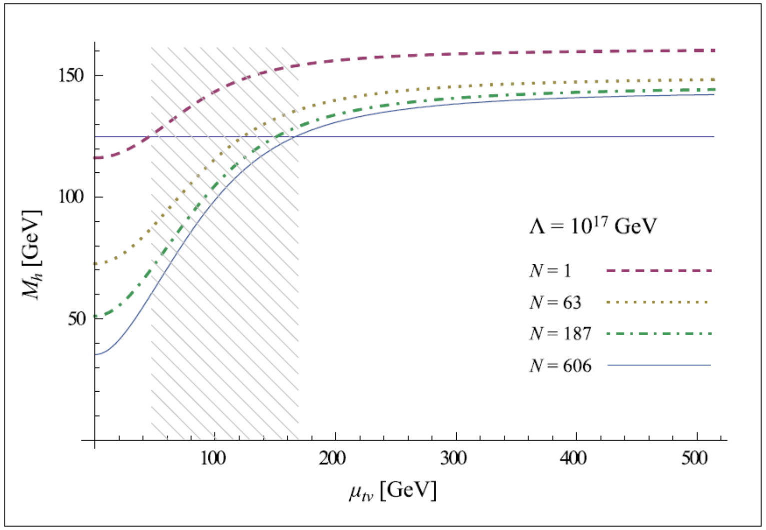
Even though the number being large may be welcome:

- $\mathcal{O}(100)$ is motivated by some string constructions [ElLe'07]
- $\mathcal{O}(10\text{-}100)$ may explain large neutrino mixing [FeKl'12]
- $\mathcal{O}(100)$ improves the standard thermal leptogenesis [Ei'08]

it is better being smaller

- aesthetics (*L*-*R* symmetry wants 3)
- $SU(3)_F$ flavor gauge model requires a moderate number (asymptotic freedom)

Dependence on N



$$
\begin{aligned}
16\pi^2 \tfrac{\mathrm{d}}{\mathrm{d}t} y_\nu &= y_\nu \left[3(N + \tfrac{1}{2})\theta(\mu - M_R) y_\nu^2 - \tfrac{3}{4} g_1^2 - \tfrac{9}{4} g_2^2\right] \\
16\pi^2 \tfrac{\mathrm{d}}{\mathrm{d}t} \lambda_\nu &= 12\lambda_\nu^2 + \lambda_\nu \left[12 N \theta(\mu - M_R) y_\nu^2 - 3 g_1^2 - 9 g_2^2\right] - 12 N \theta(\mu - M_R) y_\nu^4 \\
& \quad + 4\lambda_{t\nu}^2 + 4\lambda_{t\nu}\lambda'_{t\nu} + 2{\lambda'_{t\nu}}^2 + \tfrac{3}{4}(g_1^4 + 2 g_1^2 g_2^2 + 3 g_2^4)
\end{aligned}
$$

$$
\boxed{
\begin{aligned}
3 N y_\nu^2 &= \mathrm{Tr}\,(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu) \\
3 N y_\nu^4 &= \mathrm{Tr}\,(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)^2
\end{aligned}
}
$$

Single right-handed neutrino triplet

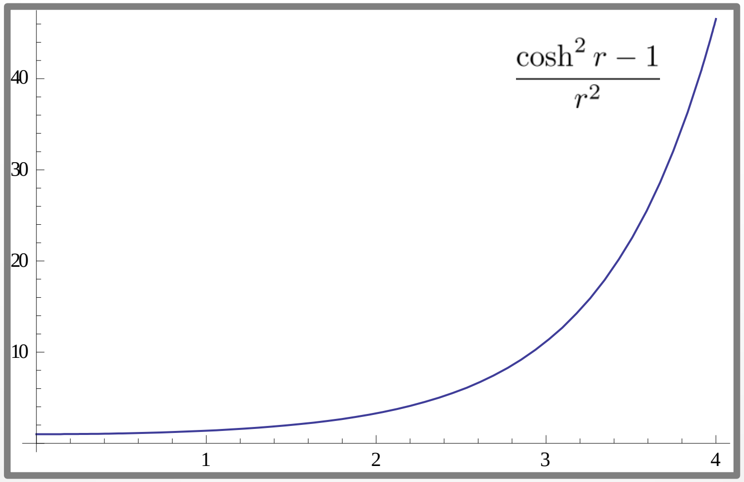
$$\boxed{\mathrm{Tr}\left(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu\right)^n = \left(\frac{M_R}{v_\nu^2}\right)^n \mathrm{Tr}\left(m\, e^{2\mathrm{i}\mathbf{A}}\right)^n \qquad \mathbf{A} = \begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}}$$

$$M_R = M_{R1} = = M_{R2} = M_{R3}$$

$$\mathrm{Tr}\left(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu\right) = \left(\frac{M_R}{v_\nu^2}\right)\left[\sum_i m_i + \frac{\cosh^2 r - 1}{r^2} P_{11}\right]$$

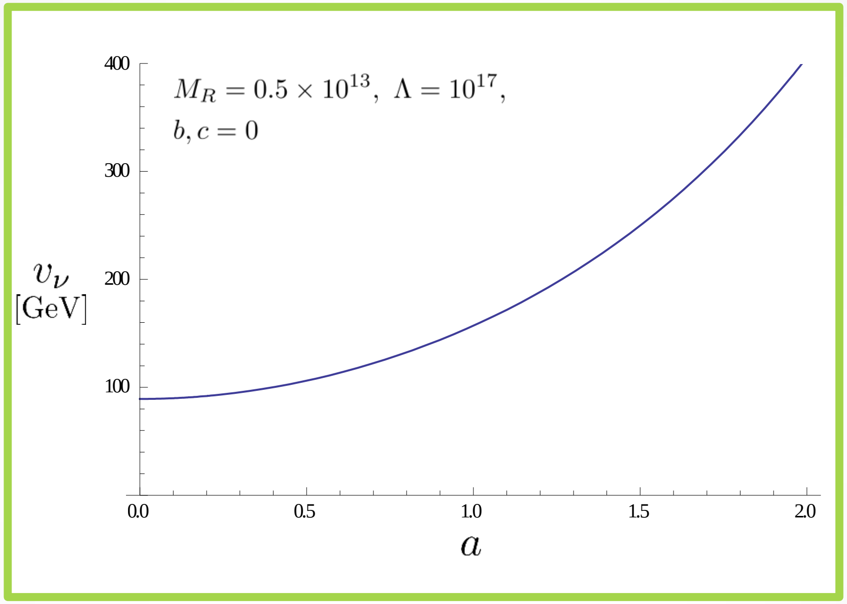
$$\mathrm{Tr}\left(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu\right)^2 = \left(\frac{M_R}{v_\nu^2}\right)^2\left[\sum_i m_i^2 + \frac{\cosh^2 r - 1}{r^2} P_{21} + \left(\frac{\cosh^2 r - 1}{r^2}\right)^2 P_{22}\right] \qquad r^2 = a^2 + b^2 + c^2$$

enhancement of effect of neutrino Yukawa coupling in RGE [RoZh'12]



Single right-handed neutrino triplet

Saturation of the electroweak scale



It allows for Leptogenesis [PaPeYa'03]

$$\mathrm{Im}\left[\,\left\{(Y_\nu^\dagger Y_\nu)_{ij}\right\}^2\,\right] \propto abc\frac{\sinh^3 r \cosh r}{r^3}$$

Conclusions

- **The models** based on the idea of the electroweak symmetry breaking by dynamically generated masses of quarks and leptons represents a framework to **relate various phenomena** (origin of fermion masses and mixings, leptogenesis, dark matter).

- Thanks to neutrino condensation, **they are still viable**.

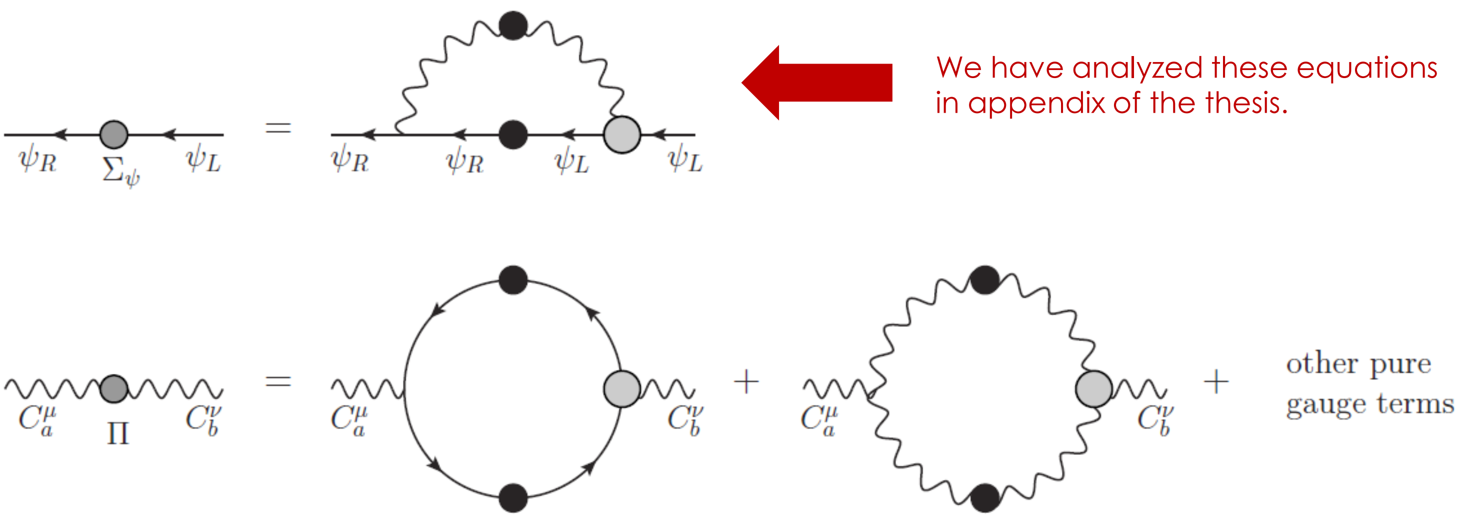
- The scenario seems to be easily falsifiable by measuring coupling properties of the Higgs boson.

Further work consists of

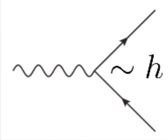
- addressing the realistic mass spectrum and mixing of neutrinos,
- implementing the idea into the multi-Higgs-doublet models.

Dynamical mass generation

Schwinger—Dyson equations



We have analyzed these equations in appendix of the thesis.



non-trivial solution of SDE only if $h \gtrsim 1$

naturally one expects that $\boxed{m_\psi, M_C \sim \Lambda_\mathrm{F}}$