Conformality: SM & Beyond

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CP^3-Origins, Danish IAS, Univ. Southern Denmark

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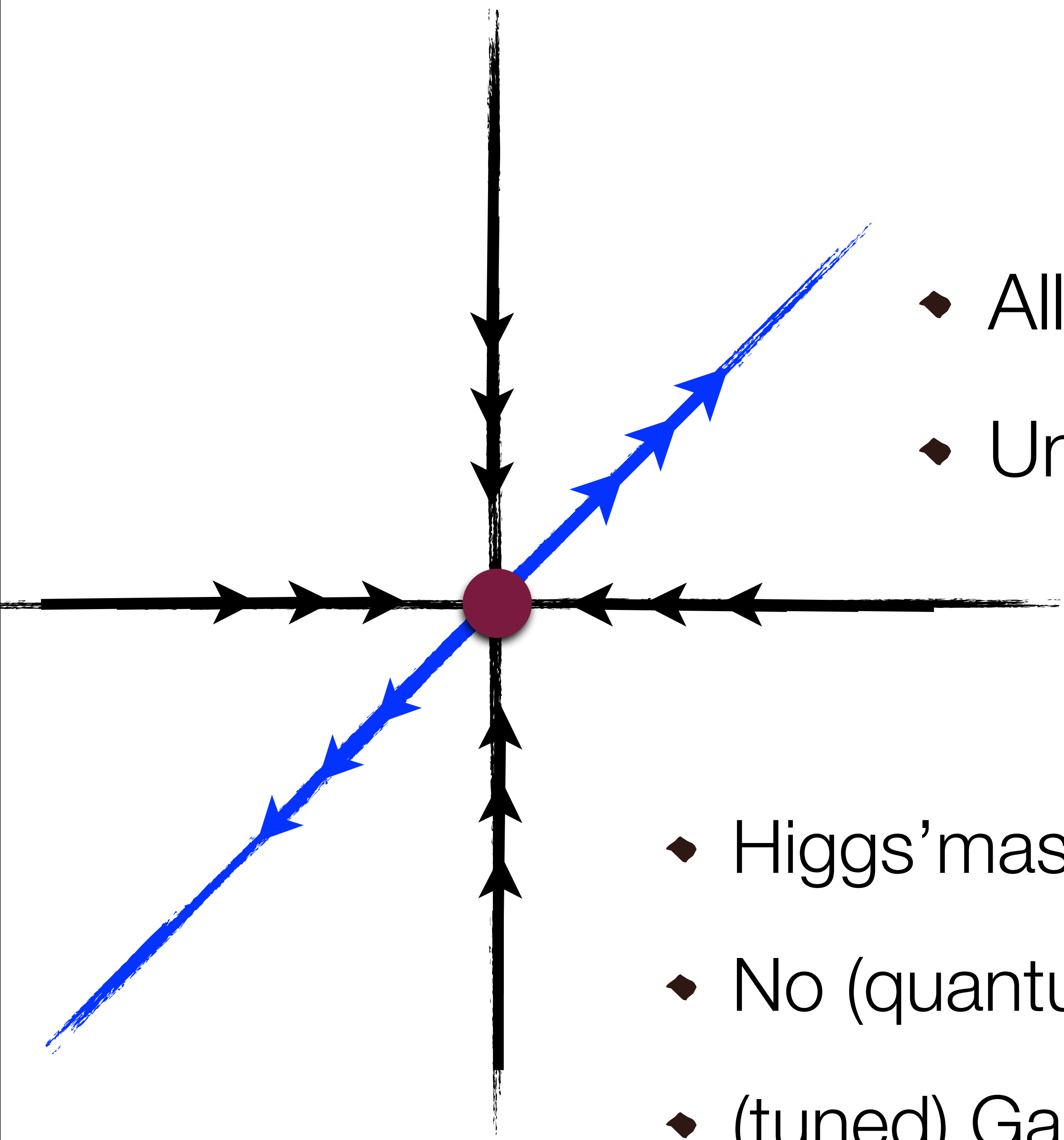
CP^3 Origins
Cosmology & Particle Physics

Points

- Degrees of naturality
- Perturbative conformal naturality
- Conformality organizes PT
- Composite EW is viable

RG (un)naturality



- All stable directions = Fixed point
- Unstable direction = Fine-tuned FP

- Higgs'mass = unstable direction
- No (quantum) symmetry = No protection
- (tuned) Gauge - Yukawa are interesting FTs

In formulæ

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_r)^2 - \frac{1}{2}m^2\phi_r^2 - \frac{\lambda}{4!}\phi_r^4 + \frac{\delta_Z}{2}(\partial_\mu \phi_r)^2 - \frac{\delta_m}{2}\phi_r^2 - \frac{\delta_\lambda}{4!}\phi_r^4$$

$$\phi_B \equiv \sqrt{Z}\phi_r \quad \delta_Z \equiv Z - 1 \quad m^2 \equiv m_0^2 Z - \delta_m \quad \delta_\lambda \equiv \lambda_0 Z^2 - \lambda$$

$$Z = 1 + f_1(\lambda, g_i)\log\frac{\Lambda^2}{m_0^2} + \ldots \qquad \delta_m = f_2(\lambda, g_i)\Lambda^2 + \ldots$$

$$m^2 = m_0^2(1 + f_1(\lambda, g_i)\log\frac{\Lambda^2}{m_0^2}) - f_2(\lambda, g_i)\Lambda^2$$

Degrees of (un)naturality

$$m^2 = m_0^2(1 + f_1(\lambda, g_i)\log\frac{\Lambda^2}{m_0^2}) - f_2(\lambda, g_i)\Lambda^2$$

- Tuning = No prediction for BSM physics*
- Tuning via "classical conformality" $\Lambda = 0, \quad m_0 = 0$
- Delayed naturality $f_2 = 0$ Perturbatively
- Perturbative natural conformality (PNC) $f_2 = 0, \quad m_0 = 0$

*Vacuum stability

Antipin, Mojaza, Sannino 2013

Intriguing PNC model

- Conformal extension of the SM

Antipin, Mojaza, Sannino 2013

$$V_0 = V_0^{SM} + \lambda_{HS} H^\dagger H S^2 + \frac{\lambda_S}{4} S^4 + y_\chi S(\chi\chi + \bar{\chi}\bar{\chi})$$

$$V_0^{SM} = \lambda \left(H^\dagger H\right)^2 - \frac{1}{2}\left(g^2 W_\mu^+ W^{-\mu} + \frac{g^2 + g'^2}{2} Z_\mu Z^\mu\right) H^\dagger H + y_t (\bar{t}_L, 0)\left(i\sigma^2 H^*\right) t_R + \text{h.c.}$$

- 1-loop stability = Veltman conditions

$$f_2 = 0$$

$$\lambda_{HS}(\mu_0) = 6y_t^2(\mu_0) - \frac{9}{4}g^2(\mu_0) - \frac{3}{4}g'^2(\mu_0) \overset{\mu_0 \approx v}{\approx} 4.84\ ,$$

$$\lambda_S(\mu_0) = \frac{8}{3}y_\chi^2(\mu_0) - \frac{4}{3}\lambda_{HS}(\mu_0) \overset{\mu_0 \approx v}{\approx} \frac{8}{3}y_\chi^2(\mu_0) - 6.45$$

$\lambda(\mu_0) \approx 0$ CW flatness

$\mu_0 \simeq 246$ GeV

- Higgs mass and its self-coupling vanish at tree-level

2 predictions

- Coleman E. Weinberg one-loop Higgs mass

$$m_h^2 = \frac{3}{8\pi^2}\Big[\frac{1}{16}\big(3g^4 + 2g^2g'^2 + g'^4\big) - y_t^4 + \frac{\lambda_{HS}^2}{3}\Big]v^2 \qquad \Longrightarrow \qquad m_h \approx 126 \text{ GeV} ,$$

- Tree - level induced S-mass

$$m_S^2 = \lambda_{HS} v^2 \quad \Longrightarrow \quad m_S \approx 541 \text{ GeV}$$

- PNC models are very constrained

- Would be nice to investigate PNC neutrino physics

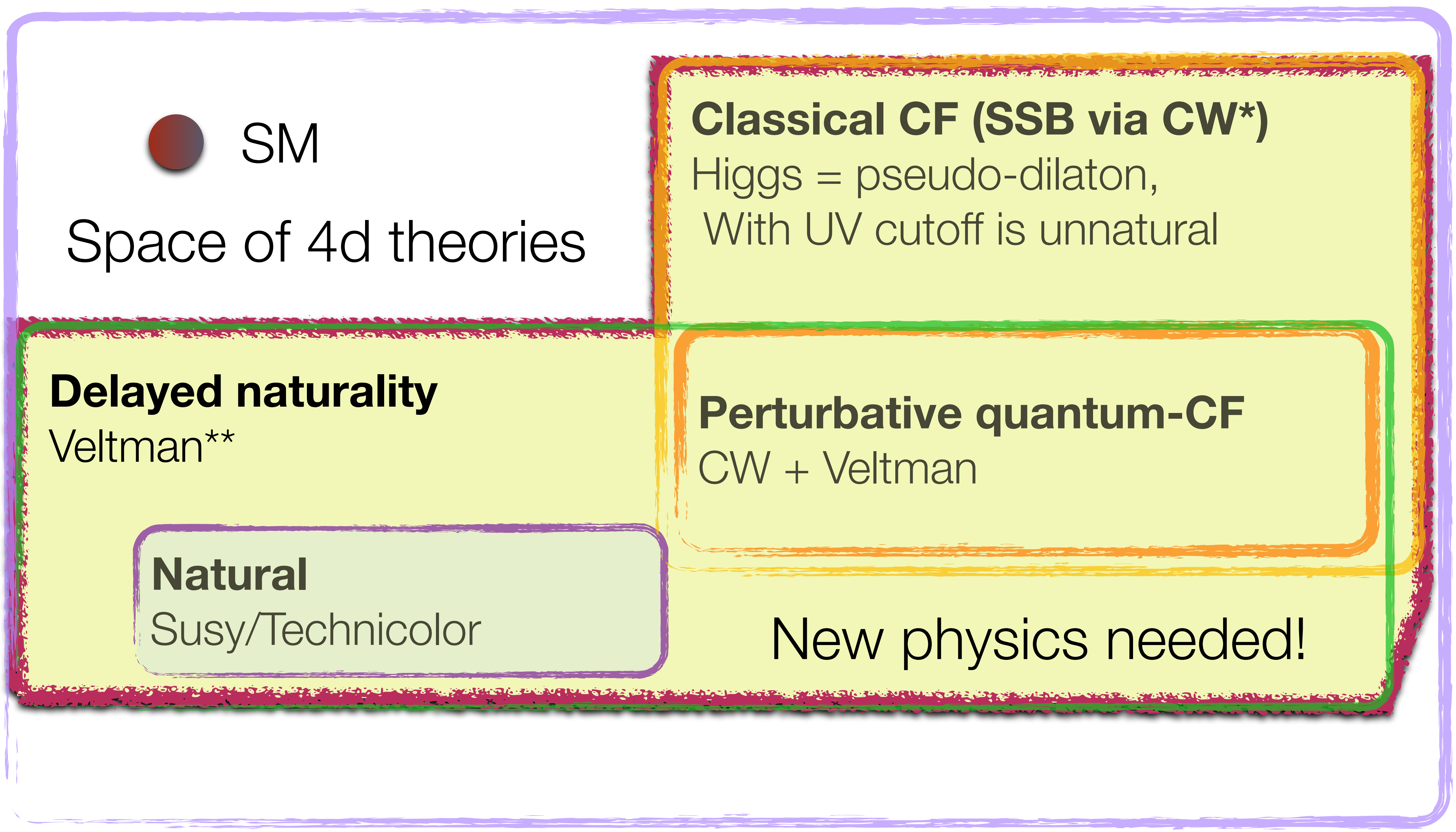
Natural theories

$$m^2 = m_0^2(1 + f_1(\lambda, g_i)\log\frac{\Lambda^2}{m_0^2}) - f_2(\lambda, g_i)\Lambda^2$$

- A symmetry exists protecting $f_2 = 0$

- Cutoff is physical as in composite models

Degrees of naturality



* CW = Coleman-Weinberg

**Perturbative cancellation of quadratic divergences

Gauge - Yukawa theories

CF Organizes PT

Power of conformality

$$\mathcal{L} = \mathcal{L}_{CFT} + \boxed{g_i \mathcal{O}^i}$$ $\longrightarrow$ Quantum correct., marginal oper.

$$g_i = g_i(x)$$ Tool: Curved backgrounds

$$\gamma_{\mu\nu} \to e^{2\sigma(x)} \gamma_{\mu\nu}$$

Conformal transformation

$$g_i(\mu) \to g_i(e^{-\sigma(x)}\mu)$$

$$W = \log\left[\int \mathcal{D}\Phi e^{i\int d^4x \mathcal{L}}\right]$$ Variation of the generating functional

Weyl (anomaly) relations

$$\Delta_\sigma W \equiv \int d^4x\,\sigma(x)\left(2\gamma_{\mu\nu}\frac{\delta W}{\delta\gamma_{\mu\nu}} - \beta_i\frac{\delta W}{\delta g_i}\right) = \sigma\left(aE(\gamma) + \chi^{ij}\partial_\mu g_i\partial_\nu g_j G^{\mu\nu}\right) + \partial_\mu\sigma w^i\,\partial_\nu g_i G^{\mu\nu} + \ldots$$

$$E(\gamma) = R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - 4R^{\mu\nu}R_{\mu\nu} + R^2$$ Euler density

$$G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2}\gamma^{\mu\nu}R$$ Einstein tensor

β_i Beta functions

$a, \quad \chi^{ij}, \quad \omega^i$ Functions of couplings

Weyl relations from abelian nature of Weyl anomaly

$$\Delta_\sigma\Delta_\tau W = \Delta_\tau\Delta_\sigma W$$

Relation to the a-theorem

$$\tilde{a} \equiv a - w^i \beta_i \qquad\qquad \frac{\partial \tilde{a}}{\partial g_i} = \left(-\chi^{ij} + \frac{\partial w^i}{\partial g_j} - \frac{\partial w^j}{\partial g_i} \right) \beta_j$$

$$\frac{d}{d\mu}\tilde{a} = -\chi^{ij}\beta_i\beta_j$$

a-tilde is RG monotonically decreasing if chi is positive definite

Cardy 88, conjecture

True in lowest order PT

Osborn 89 & 91, Jack & Osborn 90

Analyticity: a-tilde bigger in UV

Komargodski & Schwimmer 11, Komargodski 12

Gauge - Yukawa theories

omega is an exact form

Osborn 89 & 91, Jack & Osborn 90

$$\frac{\partial \tilde{a}}{\partial g_i} = \left(-\chi^{ij} + \frac{\partial w^i}{\partial g_j} - \frac{\partial w^j}{\partial g_i}\right)\beta_j \quad \Rightarrow \quad \frac{\partial \tilde{a}}{\partial g_i} = -\beta^i\,, \quad \beta^i \equiv \chi^{ij}\beta_j$$

Gradient flow fundamental relation

$$\frac{\partial \beta^j}{\partial g_i} = \frac{\partial \beta^i}{\partial g_j}\,,$$

Relations among the modified β of different couplings

It is **inconsistent** to expand to the same order in all couplings

Antipin, Gillioz, Mølgaard, Sannino 13

SM & Weyl relations

$$\alpha_1 = \frac{g_1^2}{(4\pi)^2}, \quad \alpha_2 = \frac{g_2^2}{(4\pi)^2}, \quad \alpha_3 = \frac{g_3^2}{(4\pi)^2}, \quad \alpha_t = \frac{y_t^2}{(4\pi)^2}, \quad \alpha_\lambda = \frac{\lambda}{(4\pi)^2}$$

$$\beta_i \equiv \mu^2 \frac{d\alpha_i}{d\mu^2}$$

$$\chi = \mathrm{diag}\left(\frac{1}{\alpha_1^2}\ , \frac{3}{\alpha_2^2}\ , \frac{8}{\alpha_3^2}\ , \frac{2}{\alpha_t}\ , 4\right)$$

- 3 (gauge) - 2(yukawa) - 1(Higgs' coupling) preserves WR
- 3 - 3 - 3 violates WR

Antipin, Gillioz, Krog, Mølgaard, Sannino 13

$$\boxed{2\frac{\partial}{\partial\alpha_t}\beta_\lambda = \frac{\partial}{\partial\alpha_\lambda}\left(\frac{\beta_t}{\alpha_t}\right)} + \mathcal{O}\left(\alpha_i^2\right)$$

$$4\frac{\partial}{\partial\alpha_1}\beta_\lambda = \frac{\partial}{\partial\alpha_\lambda}\left(\frac{\beta_1}{\alpha_1^2}\right) + \mathcal{O}\left(\alpha_i^2\right)$$

$$\frac{4}{3}\frac{\partial}{\partial\alpha_2}\beta_\lambda = \frac{\partial}{\partial\alpha_\lambda}\left(\frac{\beta_2}{\alpha_2^2}\right) + \mathcal{O}\left(\alpha_i^2\right)$$

$$2\frac{\partial}{\partial\alpha_1}\left(\frac{\beta_t}{\alpha_t}\right) = \frac{\partial}{\partial\alpha_t}\left(\frac{\beta_1}{\alpha_1^2}\right) + \mathcal{O}\left(\alpha_i^2\right)$$

$$\frac{2}{3}\frac{\partial}{\partial\alpha_2}\left(\frac{\beta_t}{\alpha_t}\right) = \frac{\partial}{\partial\alpha_t}\left(\frac{\beta_2}{\alpha_2^2}\right) + \mathcal{O}\left(\alpha_i^2\right)$$

$$\frac{1}{4}\frac{\partial}{\partial\alpha_3}\left(\frac{\beta_t}{\alpha_t}\right) = \frac{\partial}{\partial\alpha_t}\left(\frac{\beta_3}{\alpha_3^2}\right) + \mathcal{O}\left(\alpha_i^2\right)$$

$$\frac{1}{3}\frac{\partial}{\partial\alpha_2}\left(\frac{\beta_1}{\alpha_1^2}\right) = \frac{\partial}{\partial\alpha_1}\left(\frac{\beta_2}{\alpha_2^2}\right) + \mathcal{O}\left(\alpha_i^2\right)$$

$$\frac{1}{8}\frac{\partial}{\partial\alpha_3}\left(\frac{\beta_1}{\alpha_1^2}\right) = \frac{\partial}{\partial\alpha_1}\left(\frac{\beta_3}{\alpha_3^2}\right) + \mathcal{O}\left(\alpha_i^2\right)$$

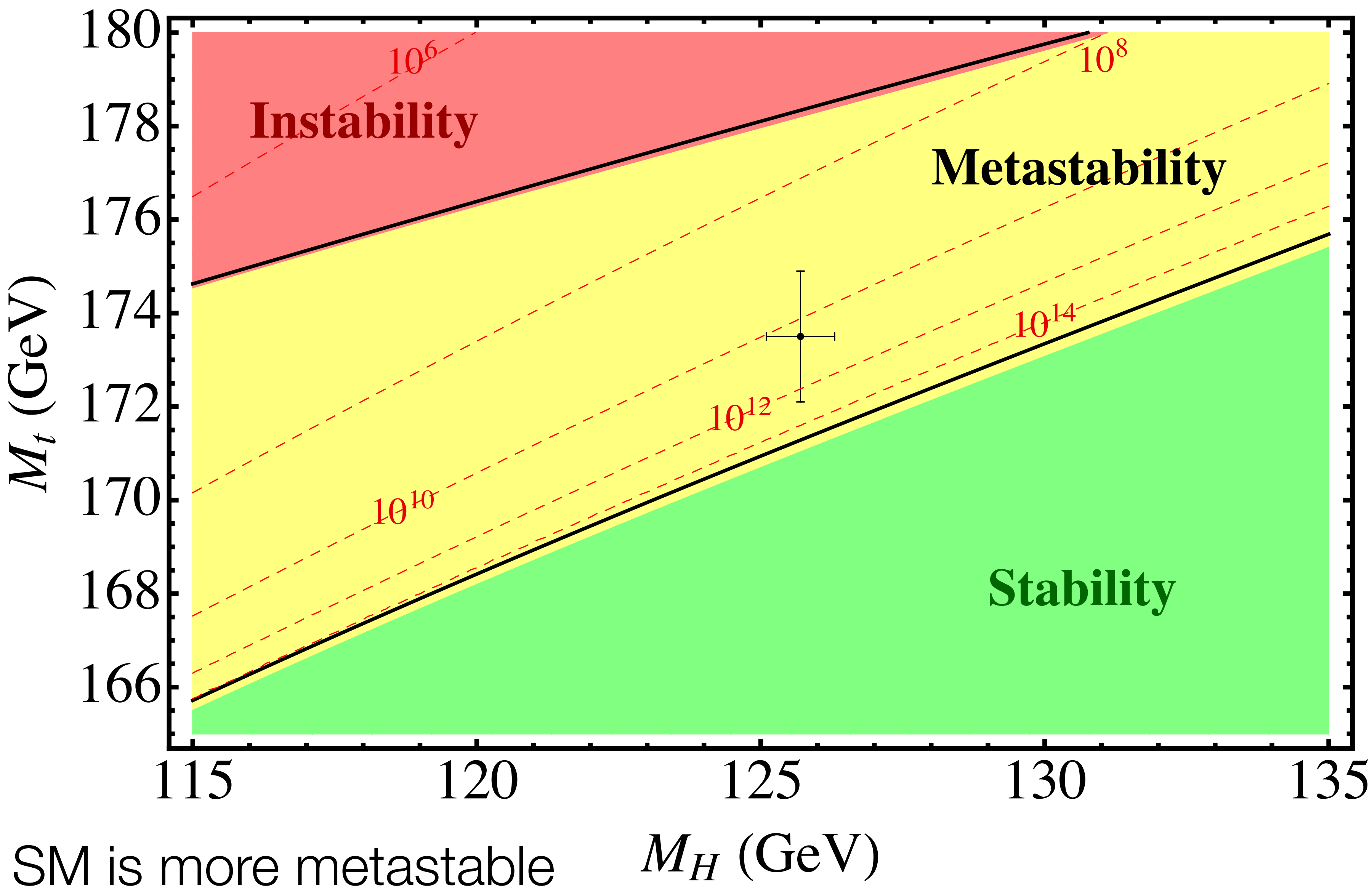
$$\frac{3}{8}\frac{\partial}{\partial\alpha_3}\left(\frac{\beta_2}{\alpha_2^2}\right) = \frac{\partial}{\partial\alpha_2}\left(\frac{\beta_3}{\alpha_3^2}\right) + \mathcal{O}\left(\alpha_i^2\right)$$

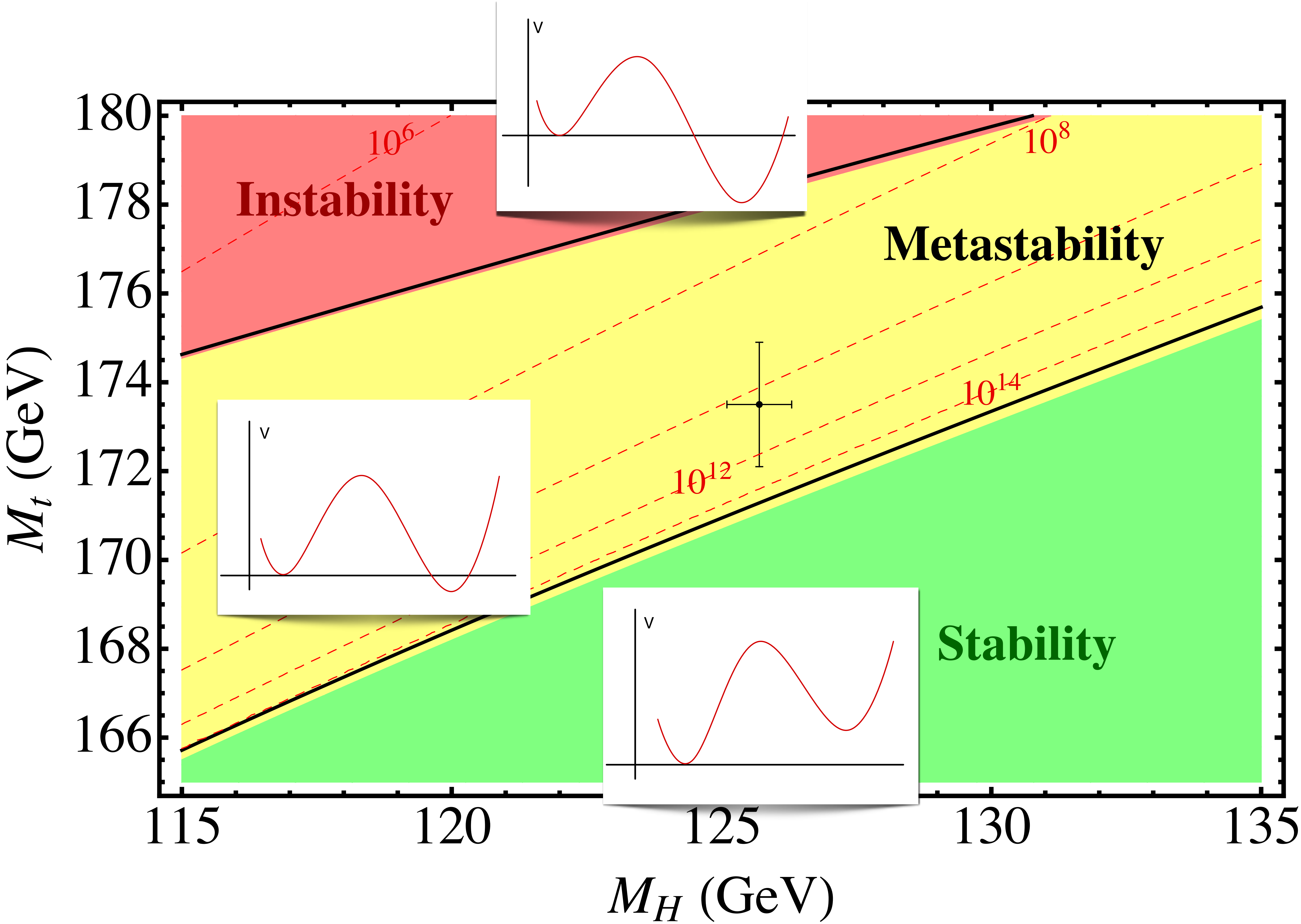
Weyl consistent Vacuum Stability

3-3-3 Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice, Isidori, Strumia 2012 (2-black lines). WR inconsistent

3-2-1 Antipin, Gillioz, Krog, Mølgaard, Sannino 2013. WR consistent

- Smaller top mass
- To do: 4 - 3 - 2
- Only 4 is not known
- Reorganize PT
- At the 3 - 2 - 1 level the SM is more metastable





Instability
Metastability
Stability
10^6
10^8
10^{12}
10^{14}
V
V
V
180
178
176
174
172
170
168
166
115
120
125
130
135
M_t (GeV)
M_H (GeV)

Hold on, fundamental?

- Would be the first time
- Spinors are building blocks
- Scalar theories are fine-tuned

How to get a non-GB light Higgs in composite dynamics?

Composite Higgs dynamics

$$\boxed{DH^{\dagger}DH - V(H)} + \boxed{\overline{\Psi}_L H \psi_R}$$

$$m_W^2 WW \qquad\qquad m_\psi \overline{\psi}_L \psi_R$$

TC $\longleftrightarrow$ Extended TC

TC Higgs

TC - Higgs is the lightest spin-0 scalar made of TC-fermions

$$H \sim c_1 \bar{Q}Q + c_2 \bar{Q}Q\bar{Q}Q + \cdots$$

Will contain also a TC-glue component

QCD lightest scalar is $f_0(500)$ with mass ~ 400-550 MeV

Sannino & Schechter 95 PRD [‘t Hooft 1/N, crossing, chiral, pole mass]
Harada, Sannino & Schechter 95 PRD [$f_0(980)$], 96PRL
Pelaez - Confinement X - lecture

Narrow state in strong dynamics?

Example $f_0(980)$

$$\Gamma = 40 - 100 \text{ MeV} \qquad\qquad m = 990 \pm 20 \text{ MeV}$$

Narrow because near/below 2 kaon threshold

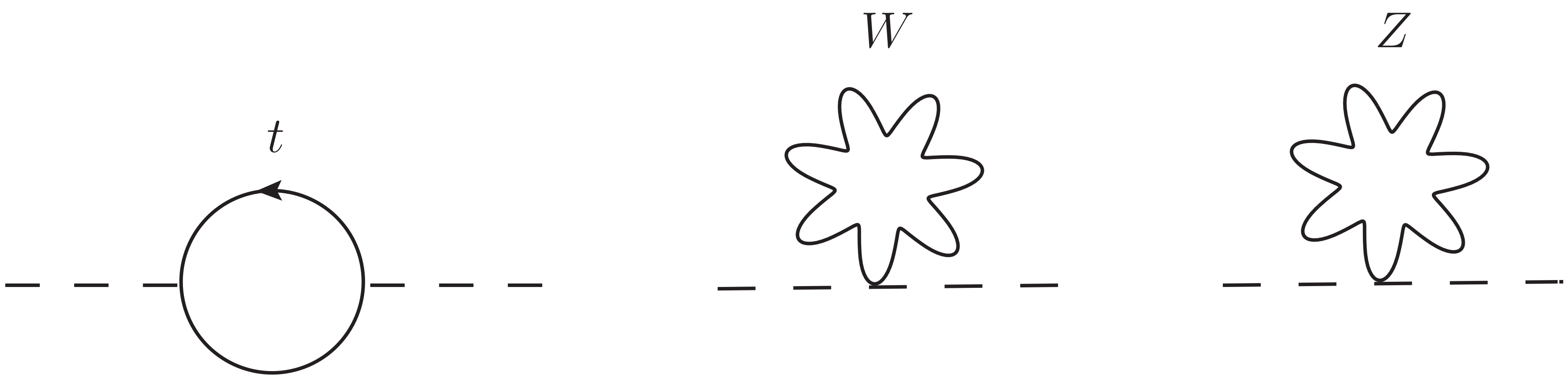
$$m_{2Kaons} \simeq 987.4 \text{ MeV}$$

Harada, Sannino & Schechter 95 PRD [$f_0(980)$], 96PRL [Large N apparent violation]

S. Weinberg 2013

Top - corrections

$$\mathcal{L}_H \supset \frac{2\ m_W^2\ r_\pi}{v}\ H\ W_\mu^+\ W^{-\mu} + \frac{m_Z^2\ r_\pi}{v}\ H\ Z_\mu\ Z^\mu - \frac{m_t\ r_t}{v}\ H\ \bar{t}\ t$$
$$+ \frac{m_W^2\ s_\pi}{v^2}\ H^2\ W_\mu^+\ W^{-\mu} + \frac{m_Z^2\ s_\pi}{2\ v^2}\ H^2\ Z_\mu\ Z^\mu$$



$$M_H^2 = (M_H^{\mathrm{TC}})^2 + \frac{3(4\pi\kappa F_\Pi)^2}{16\pi^2 v^2}\left[-4r_t^2 m_t^2 + 2s_\pi\left(m_W^2 + \frac{m_Z^2}{2}\right)\right] + \Delta_{M_H^2}(4\pi\kappa F_\Pi)$$

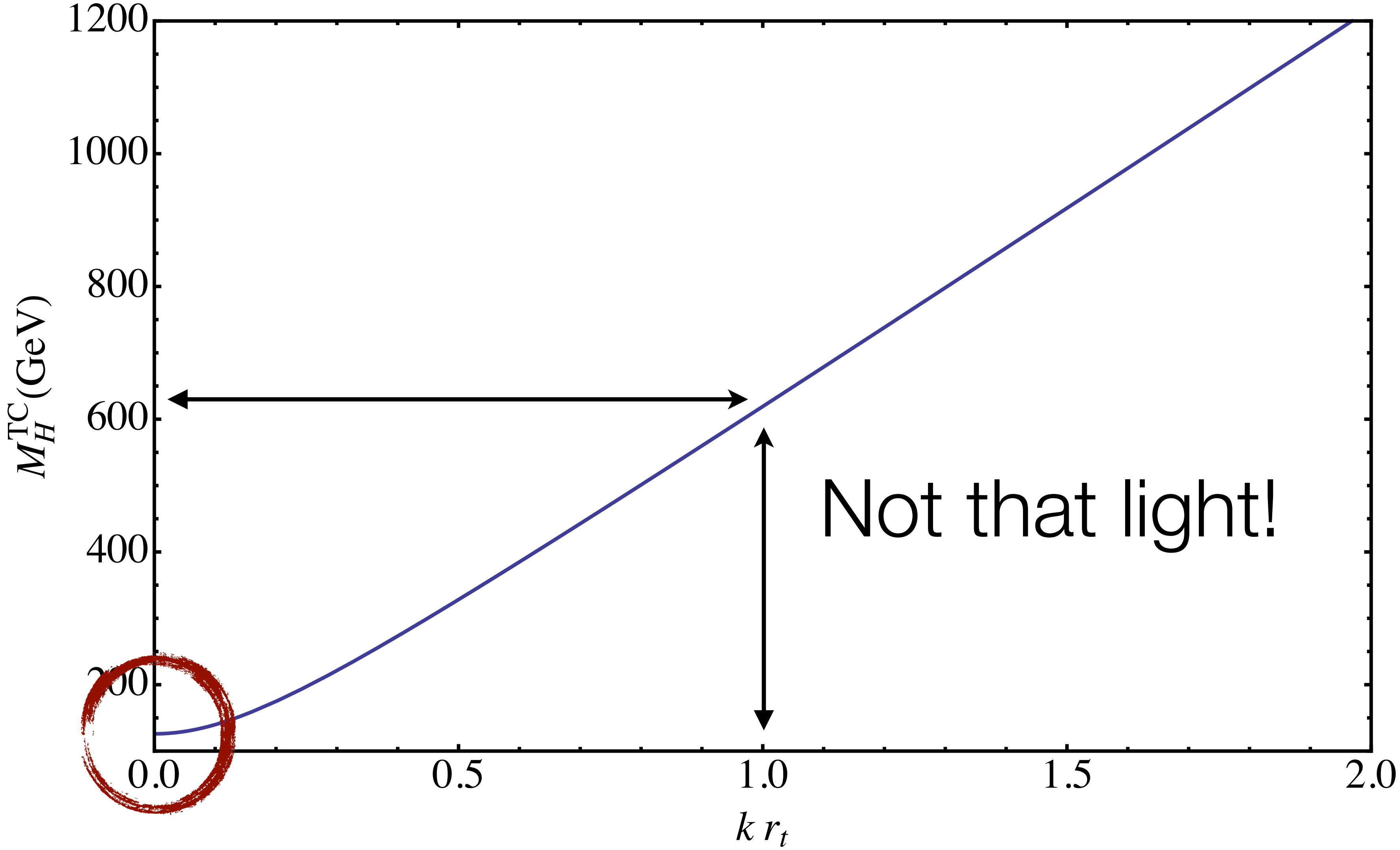
Foadi, Frandsen, Sannino, 1211.1083

How light is the TC-Higgs ?

$$(M_H^{\rm TC})^2 \simeq M_H^2 + 12\ \kappa^2 r_t^2 m_t^2$$

$\kappa\ r_t$ ~ TC x ETC

$$F_\Pi = v$$



Narrow due to kinematics [Similar to $f_0(980)$ in QCD]

Minimal Walking Theories

- SU(2) + 2 Dirac Adjoint — $SU(2)_A$ - MWT
- SU(3) + 2 Dirac Symmetric — $SU(3)_S$ - MWT
- SU(2) +2 Dirac Fund. + .. (U - MWT) — $SU(2)_F$ - MWT
- SO(4) + 2 Dirac Vector — $SO(4)_V$ - MWT
- SU(3) + 2 Dirac Fund. + Ungauged — $SU(3)_F$ - pMWT

Only one N_D gauged: Small S

Realistic SU(3)S MWT

$N_D = 1 \quad d(\text{Symmetric}) = 6$

Sannino & Tuominen hep-ph/0405209

$M_H^{TC} \simeq 735 \text{ GeV}$

Large N scaling

Physical Higgs mass for

$\kappa \; r_t \simeq 1.2$

Lattice: Fodor, Holland, Kuti, Nogradi, Schroeder, Wong, 1209.0391:

$M_\rho \simeq 1754 \pm 104 \text{ GeV}$

$M_{A_1} \simeq 2327 \pm 121 \text{ GeV}$

Early lattice measurements of scalar mass agree with the Large N estimates

Model agrees with LHC data @ 95% CL

Belyaev, Brown, Foadi, Frandsen 2013

Summary

RG (un)naturality

Perturbative natural conformality

Conformality & consistent Gauge - Yukawa PT

A natural avenue: Compositeness

A 125 Higgs via a not-to-light TC Higgs

Promising lattice & pheno studies of Minimal TC

New particles naturally in the (multi)-TeV region