Gegenbauer Goldstones

Ennio Salvioni

1222·2022 800 ANNI

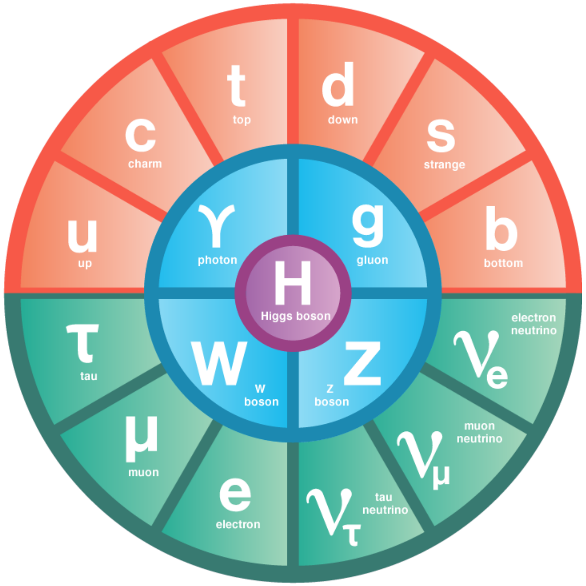
Università degli Studi di Padova

Particle and Astroparticle Theory Seminar
MPIK, May 23, 2022

2110.06941 [JHEP] + 2202.01228 [JHEP] + in progress
with Gauthier Durieux and Matthew McCullough (CERN)

symmetry

The Higgs mystery



What is behind this?

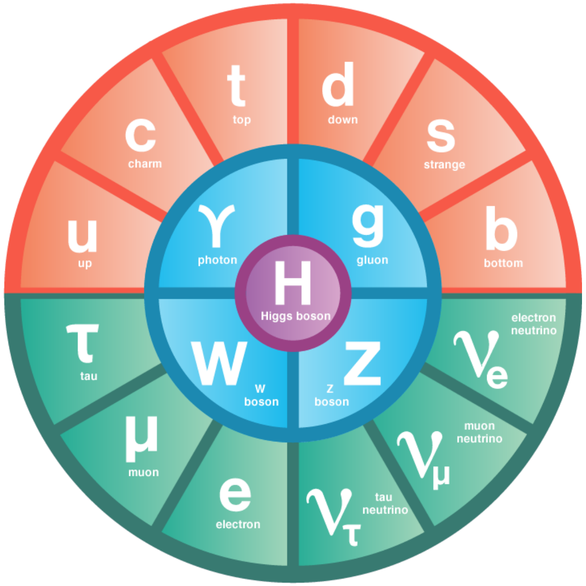
$$V(H) = -m^2|H|^2 + \lambda|H|^4$$

Successful phenomenological model, but lacking deep understanding.

Historical example is superconductivity: H turned out to be Cooper pairs in BCS.

$$H \sim (ee)$$

The Higgs mystery



What is behind this?

$$V(H) = -m^2|H|^2 + \lambda|H|^4$$

Can we calculate it within a more fundamental theory?

Motivation

Other scalars we know: pions, composite pseudo Nambu-Goldstone bosons (pNGBs)

$$\Pi \sim (\bar{q}q) \qquad\qquad \Pi \to \Pi + \theta$$

Old question: could the Higgs field be a pNGB too?

[Kaplan, Georgi 1984]
[Kaplan 1992]
[Agashe, Contino, Pomarol 2004]
and many many others

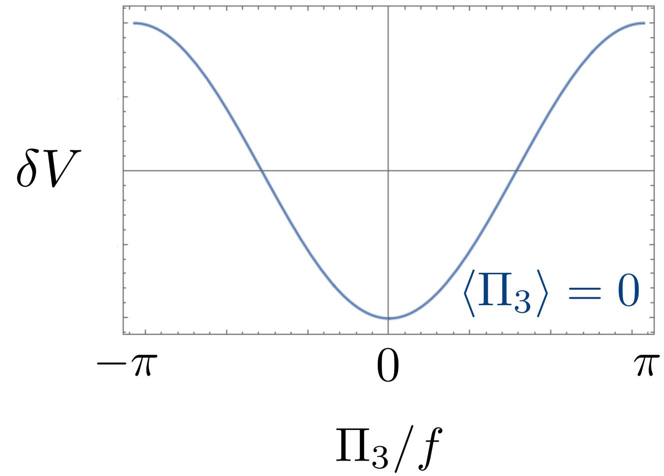
However, important structural difference between pion and Higgs potentials

Motivation

For pions:

$$\delta\mathcal{L}_{\text{ChPT}} \sim Bf^2 \text{Tr}[\Sigma M^\dagger] + \text{h.c.}$$

$$\Sigma = e^{i\Pi^a \sigma^a / f}$$



$$\delta V \sim -Bf^2 m_q \cos\frac{\Pi_3}{f}$$

For Higgs:

$$0 \neq \langle \Pi_h \rangle = v \ll f$$

$v \approx 246 \text{ GeV}$

Higgs couplings to other particles
agree with SM to ~ 20% (LHC Run 2)

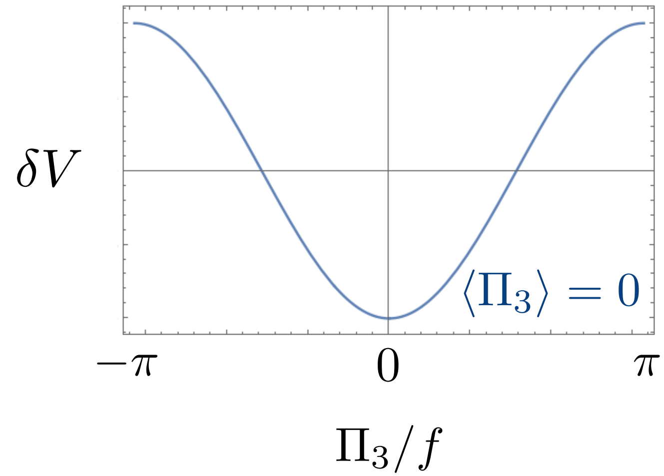
$$\frac{c_{hVV}}{c_{hVV}^{\text{SM}}} = \sqrt{1 - \frac{v^2}{f^2}}$$

Motivation

For pions:

$$\delta\mathcal{L}_{\rm ChPT} \sim Bf^2 \mathrm{Tr}[\Sigma M^\dagger] + \mathrm{h.c.}$$

$$\Sigma = e^{i\Pi^a\sigma^a/f}$$



$$\delta V \sim -Bf^2 m_q \cos\frac{\Pi_3}{f}$$

For Higgs:

$$0 \neq \langle \Pi_h \rangle = v \ll f$$

$v \approx 246$ GeV

Obtaining $v \ll f$ naturally is key question in pNGB Higgs framework

Outline

- Warm-up: Abelian Goldstones
- Non-Abelian: Gegenbauer Goldstones
- Gegenbauer Higgs
- Gegenbauer's Twin

[Durieux, McCullough, Salvioni 2110.06941, JHEP]
[Durieux, McCullough, Salvioni 2202.01228, JHEP]

Abelian Goldstones

Take single Goldstone, arising from spontaneously broken $U(1)$

$$\mathcal{L} = \partial_\mu \Phi^* \partial^\mu \Phi - \lambda \left(\Phi^* \Phi - f^2 \right)^2$$

Make it a pNGB: explicit breaking from operator of charge n

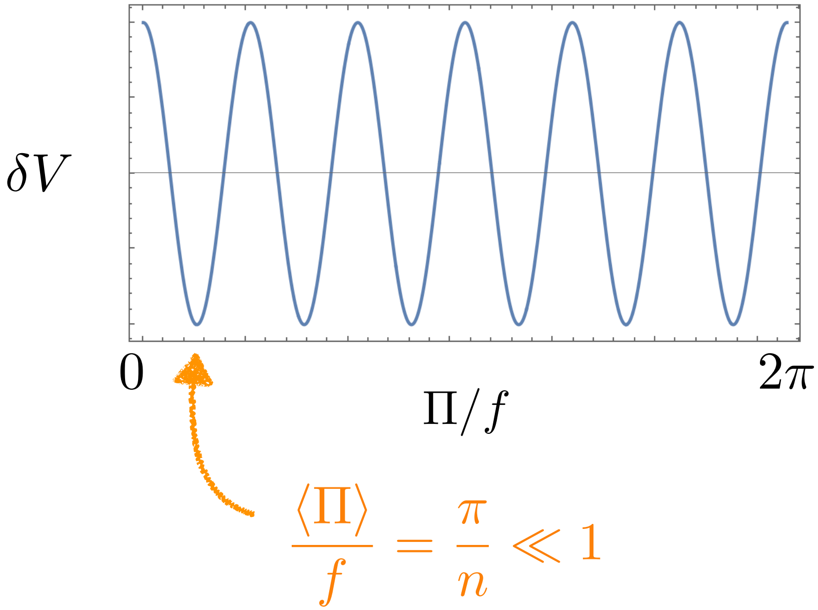
$$\delta V = \frac{\epsilon \, \lambda}{f^{n-4}} \Phi^n + \text{h.c.} \quad \longrightarrow \quad \delta V \sim \epsilon \lambda f^4 \cos\left(\frac{n\Pi}{f} \right)$$

$$\Phi = f e^{i\Pi/f}$$

periodic potential

Abelian Goldstones

example: $n = 6$



discrete symmetry

$$\mathcal{Z}_n : \quad \Pi \to \Pi + \frac{2\pi}{n} f$$

Make it a pNGB: explicit breaking from operator of charge n

$$\delta V = \frac{\epsilon \lambda}{f^{n-4}} \Phi^n + \text{h.c.} \longrightarrow \delta V \sim \epsilon \lambda f^4 \cos\left(\frac{n\Pi}{f}\right)$$

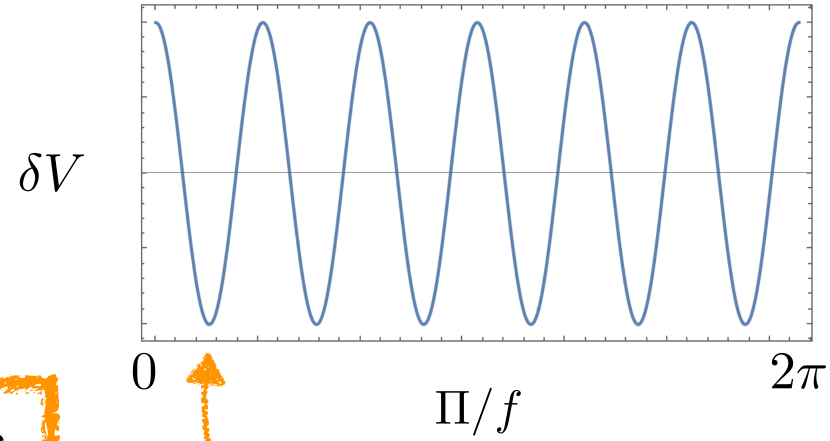
$$\Phi = f e^{i\Pi/f}$$

periodic potential

for interesting work on Abelian setup, see [Hook 2018]

Abelian Goldstones

example: $n = 6$



$$\frac{\langle \Pi \rangle}{f} = \frac{\pi}{n} \ll 1$$

discrete symmetry

$$\mathcal{Z}_n : \quad \Pi \to \Pi + \frac{2\pi}{n} f$$

Standard class of radiatively stable potentials for Abelian Goldstones

Make it a pNGB: explicit breaking from operator of charge n

$$\delta V = \frac{\epsilon \, \lambda}{f^{n-4}} \Phi^n + \text{h.c.} \quad \xrightarrow{\Phi = f e^{i\Pi/f}} \quad \delta V \sim \epsilon \lambda f^4 \cos\left(\frac{n\Pi}{f}\right)$$

periodic potential

for interesting work on Abelian setup, see [Hook 2018]

Outline

✓ Warm-up: Abelian Goldstones

- Non-Abelian: Gegenbauer Goldstones
- Gegenbauer Higgs
- Gegenbauer’s Twin

[Durieux, McCullough, Salvioni 2110.06941, JHEP]
[Durieux, McCullough, Salvioni 2202.01228, JHEP]

Non-Abelian Goldstones

Consider N Goldstone bosons, from SSB of non-Abelian global symmetry

$SO(N+1)/SO(N)$ (best studied pattern for pNGB Higgs)

$$\mathcal{L} = \frac{1}{2}\partial_\mu \Phi^T \partial^\mu \Phi - \lambda \left(\Phi^T \Phi - f^2\right)^2$$

How to get $v \ll f$ naturally?

Non-Abelian Goldstones

Consider N Goldstone bosons, from SSB of non-Abelian global symmetry

$SO(N+1)/SO(N)$ (best studied pattern for pNGB Higgs)

$$\mathcal{L} = \frac{1}{2}\partial_\mu \Phi^T \partial^\mu \Phi - \lambda \left(\Phi^T \Phi - f^2\right)^2$$

Explicit breaking to $SO(N)$ by spurion in

n - index symmetric tensor irrep of $SO(N+1)$

$$\delta V = \frac{\epsilon\,\lambda}{f^{n-4}} K_n^{i_1 \ldots i_n} \Phi_{i_1} \ldots \Phi_{i_n}$$

irrep $\rightarrow$ traceless

Radiatively stable at $O(\epsilon)$ and all loop orders, because only operator allowed.

Corrections at $O(\epsilon^2)$ and higher

[in d = 2: Brézin, Zinn-Justin, Le Guillou 1976]

Enter Gegenbauer

Parametrize

$$\Phi = f\phi \qquad \phi = e^{i\Pi^a T^a/f}\begin{pmatrix}\vec{0}_N \\ 1\end{pmatrix} = \begin{pmatrix}\sin\frac{\Pi}{f}\frac{\vec{\Pi}}{\Pi} \\ \cos\frac{\Pi}{f}\end{pmatrix} \qquad \Pi \equiv \sqrt{\vec{\Pi}^T\vec{\Pi}}$$

$$\delta V = \epsilon\lambda f^4 G_n^{(N-1)/2}\left(\cos\Pi/f\right)$$

potential is a Gegenbauer polynomial

$$\delta V = \frac{\epsilon\,\lambda}{f^{n-4}} K_n^{i_1\ldots i_n}\,\Phi_{i_1}\ldots\Phi_{i_n}$$

irrep $\rightarrow$ traceless

Radiatively stable at $O(\epsilon)$ and all loop orders, because only operator allowed.

Corrections at $O(\epsilon^2)$ and higher

[in d = 2: Brézin, Zinn-Justin, Le Guillou 1976]

Gegenbauers from irreps

$$\tilde{\phi} \equiv \begin{pmatrix} \vec{0}_N \\ 1 \end{pmatrix}$$

Consider scalar function

$$|t\phi - \tilde{\phi}|^{1-N} \quad = \quad \sum_{n=0}^{\infty} t^n K_n^{i_1 \ldots i_n} \phi^{i_1} \ldots \phi^{i_n}$$

//

Taylor expansion $\quad K_n^{i_1 \ldots i_n} = \frac{1}{n!} \frac{\partial^n \phi^{1-N}}{\partial \phi_{i_1} \ldots \partial \phi_{i_n}} \Big|_{\tilde{\phi}}$

traceless, because Laplacian vanishes away from origin

$$\left(1 - 2t \cos \Pi/f + t^2\right)^{(1-N)/2}$$

||

$$\sum_{n=0}^{\infty} t^n G_n^{(N-1)/2}(\cos \Pi/f)$$

generating function for Gegenbauers is

$$\left(1 - 2tx + t^2\right)^{-\alpha} = \sum_{n=0}^{\infty} t^n G_n^{\alpha}(x)$$

Gegenbauers from irreps

$$\tilde{\phi} \equiv \begin{pmatrix} \vec{0}_N \\ 1 \end{pmatrix}$$

Consider scalar function

$$|t\phi - \tilde{\phi}|^{1-N} = \sum_{n=0}^{\infty} t^n K_n^{i_1 \dots i_n} \phi^{i_1} \dots \phi^{i_n}$$

//

Taylor expansion $K_n^{i_1 \dots i_n} = \frac{1}{n!} \frac{\partial^n \phi^{1-N}}{\partial \phi_{i_1} \dots \partial \phi_{i_n}} \Big|_{\tilde{\phi}}$

$$\left(1 - 2t \cos \Pi/f + t^2\right)^{(1-N)/2}$$

traceless, because Laplacian vanishes away from origin

||

generating function for Gegenbauers is

$$\sum_{n=0}^{\infty} t^n G_n^{(N-1)/2}(\cos \Pi/f)$$

$$\left(1 - 2tx + t^2\right)^{-\alpha} = \sum_{n=0}^{\infty} t^n G_n^{\alpha}(x)$$

Gegenbauer polynomials from explicit $SO(N+1) \to SO(N)$ breaking

Gegenbauer?

Leopold Gegenbauer
1849 - 1903

Gegenbauer?

Gegenbauer polynomials can be seen as generalization of Legendre polynomials to $D \neq 3$ spatial dimensions

$$D = 3 \qquad SO(3) \to SO(2)$$

multipole expansion of axi-symmetric function of spacetime coordinates

$$f(\vec{r}) = \sum_{\ell=0}^{\infty} c_\ell P_\ell(\cos\theta)$$

$$(m = 0)$$

They appear in many areas of physics, for example in the expansion of conformal blocks in CFT_d

[Hogervorst, Rychkov 2013]

Here, they arise from explicit breaking of internal symmetry $SO(N+1) \to SO(N)$, variables are pNGB fields

More on radiative stability

Can also see Gegenbauers emerge from Coleman-Weinberg potential:

For general $SO(N)$ invariant potential $V = \epsilon \lambda f^4 G(\cos \Pi / f)$

quadratic piece of one-loop CW is

[Alonso, Jenkins, Manohar 2015]

$' \equiv \frac{\partial}{\partial(\Pi/f)}$

$$V_{\text{quantum}} = \epsilon \lambda f^4 \left[G + \frac{\Lambda^2}{32\pi^2 f^2} \left(G'' + (N-1) \cot \frac{\Pi}{f} G' \right) \right]$$

if $\propto G$, multiplicative renormalization!

More on radiative stability

Can also see Gegenbauers emerge from Coleman-Weinberg potential:

For general $SO(N)$ invariant potential $V = \epsilon\lambda f^4 \, G(\cos \Pi/f)$

quadratic piece of one-loop CW is

[Alonso, Jenkins, Manohar 2015]

$$' \equiv \frac{\partial}{\partial(\Pi/f)}$$

$$V_{\text{quantum}} = \epsilon\lambda f^4 \left[G + \frac{\Lambda^2}{32\pi^2 f^2} \left(G'' + (N-1)\cot\frac{\Pi}{f} G' \right) \right]$$

if $\propto G$, multiplicative renormalization!

Indeed, Gegenbauers satisfy differential equation

$$G_n^{\alpha\,\prime\prime} + 2\alpha \cot\frac{\Pi}{f} G_n^{\alpha\,\prime} + n(n+2\alpha) G_n^\alpha = 0 \quad \longrightarrow \quad \alpha = \frac{N-1}{2}$$

Radiative corrections do not alter functional form

More on radiative stability

Can also see Gegenbauers emerge from Coleman-Weinberg potential:

For general $SO(N)$ invariant potential $V = \epsilon\lambda f^4 G(\cos\Pi/f)$

quadratic piece of one-loop CW is

[Alonso, Jenkins, Manohar 2015]

$$' \equiv \frac{\partial}{\partial(\Pi/f)}$$

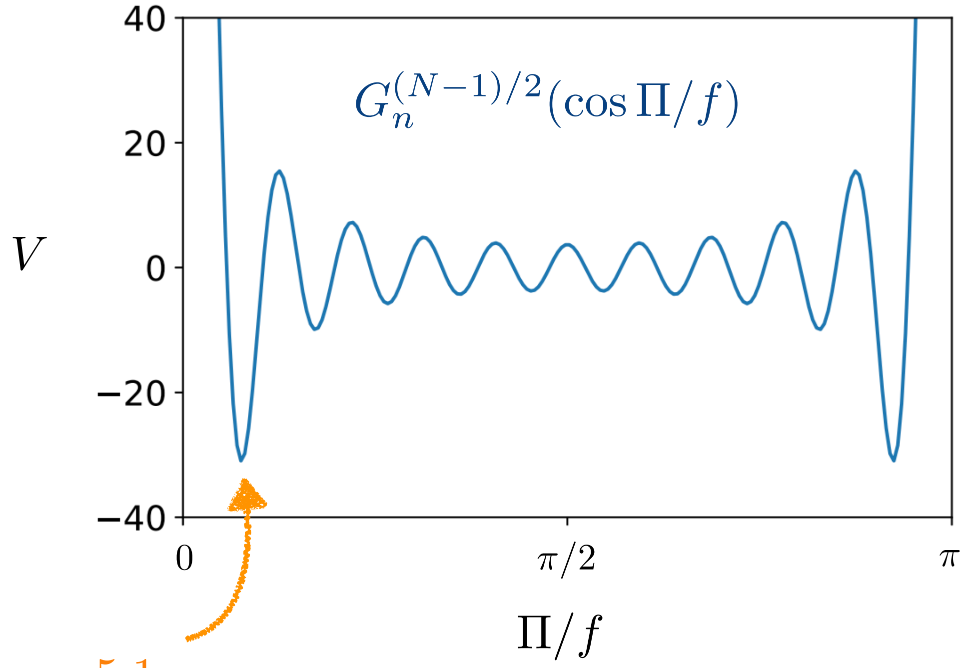
$$V_{\text{quantum}} = \epsilon\lambda f^4 \left[G + \frac{\Lambda^2}{32\pi^2 f^2}\left(G'' + (N-1)\cot\frac{\Pi}{f}G'\right)\right]$$

if $\propto G$, multiplicative renormalization!

Abelian case is recovered for $N = 1$:

$$G'' \propto G \quad \longrightarrow \quad G = \cos\frac{n\Pi}{f}$$

The shape of Gegenbauers



$N=4$

$SO(5)/SO(4)$

Even n

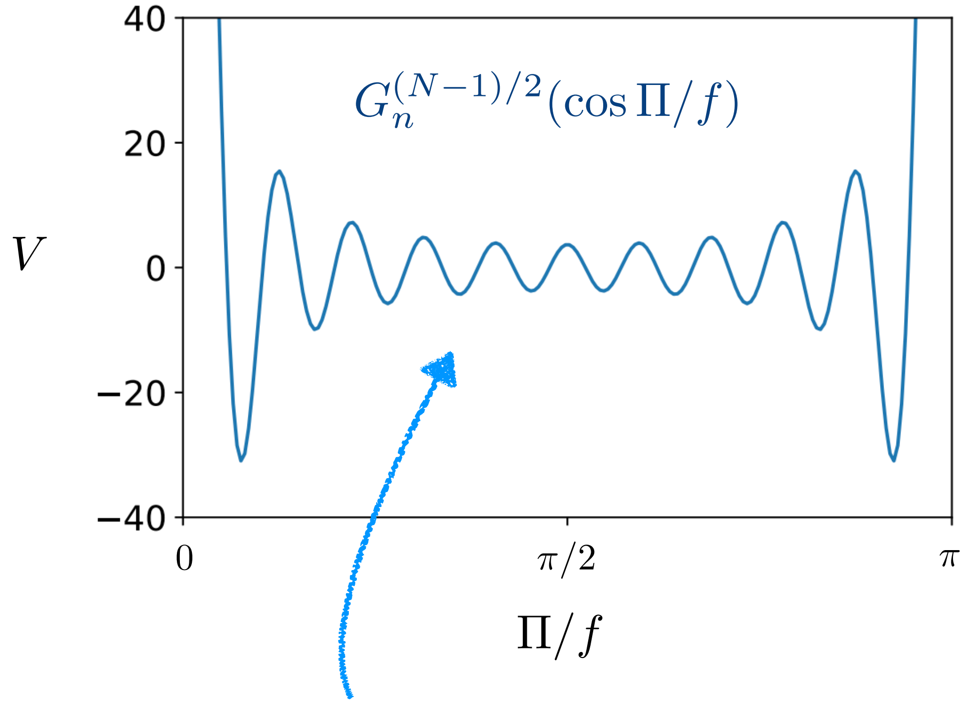
$n=20$

$$\frac{\langle\Pi\rangle}{f} \approx \frac{j_{N/2,1}}{n+\frac{N-1}{2}} \approx \frac{5.1}{n} \ll 1$$

for large n

A radiatively stable way to obtain $\langle\Pi\rangle \ll f$ for non-Abelian Goldstones

The shape of Gegenbauers



$N=4$

$SO(5)/SO(4)$

Even n

$n=20$

Differently from Abelian case, not periodic. Only approximately

$$G_n^\alpha\left(\cos\frac{\Pi}{f}\right) \xrightarrow{n\gg 1} \frac{J_{\alpha-1/2}\left((n+\alpha)\frac{\Pi}{f}\right)}{\Pi^{\alpha-1/2}} \xrightarrow{\frac{\Pi}{f}\gg\frac{1}{n}} \frac{\cos\left((n+\alpha)\frac{\Pi}{f}-\alpha\frac{\pi}{2}\right)}{\Pi^\alpha}$$

The story so far

Gegenbauer polynomials are "building blocks" of $SO(N)$ - invariant potentials for pNGBs of $SO(N+1)/SO(N)$

$$V = \epsilon\lambda f^4 \sum_{n=0}^{\infty} c_n G_n^{(N-1)/2}(\cos\Pi/f)$$

A single Gegenbauer (equivalently, n - index irrep spurion) is radiatively stable

A technically natural way to obtain $\langle\Pi\rangle \ll f$ parametrically, for non-Abelian pNGBs

$$\frac{\langle\Pi\rangle}{f} \approx \frac{5.1}{n}$$

Outline

- ✓ Warm-up: Abelian Goldstones
- ✓ Non-Abelian: Gegenbauer Goldstones
- Gegenbauer Higgs
- Gegenbauer's Twin

[Durieux, McCullough, Salvioni 2110.06941, JHEP]
[Durieux, McCullough, Salvioni 2202.01228, JHEP]

Composite pNGB Higgs

strongly-interacting sector

$m_* \sim \text{few TeV}$

g_*

$$\frac{SO(5)}{SO(4)}$$

$N = 4$ minimal to obtain pNGB Higgs doublet

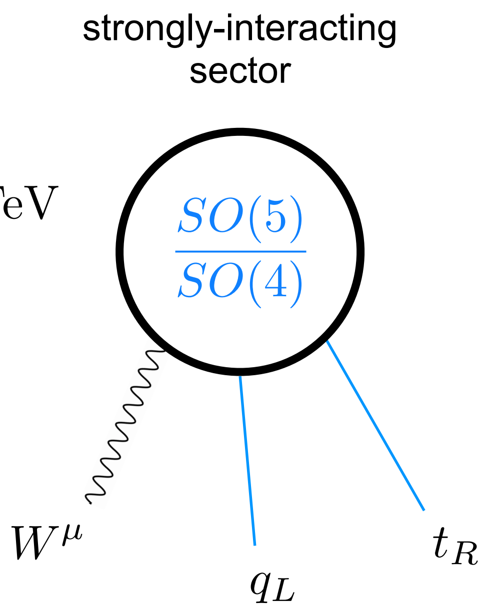
$$f = m_*/g_*$$

[Kaplan, Georgi 1984]
[Kaplan 1992]
[Agashe, Contino, Pomarol 2004]

Composite pNGB Higgs

$m_* \sim$ few TeV

g_*

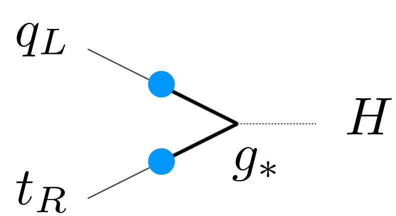


couplings of Standard Model fermions
and gauge fields break $SO(5)$ explicitly

$$V(H) = m_H^2 |H|^2 + \lambda |H|^4$$

$$m_H^2 \sim \left(-\, y_t^2 + g^2 \right) \frac{m_*^2}{16\pi^2} < 0$$

electroweak symmetry breaking
from top quark loops



$$y_t \sim g_* \, \theta_L \theta_R$$

"partial compositeness"

[Kaplan, Georgi 1984]
[Kaplan 1992]
[Agashe, Contino, Pomarol 2004]

The potential

In classic models, top sector loops break EW symmetry:

$$V = \frac{N_c y_t^2}{16\pi^2} M_T^2 f^2 \left(-\sin^2 \Pi/f + \sin^4 \Pi/f \right) + \delta_{\rm gauge} \sin^2 \Pi/f$$

"minimal tuning"
to get $v \ll f$

$$\Delta \sim \frac{v^2}{f^2}$$

(in unitary gauge, $\Pi = h$)

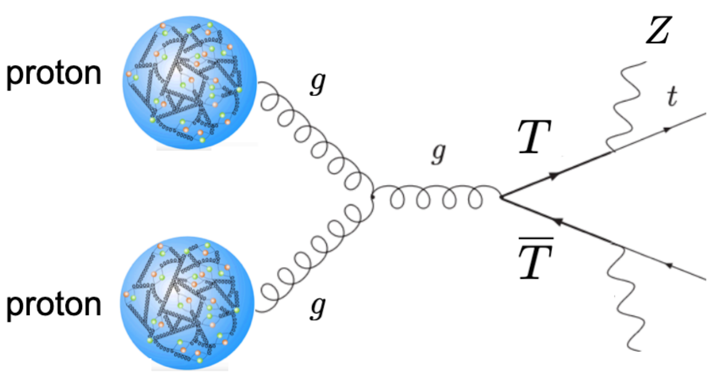
The potential

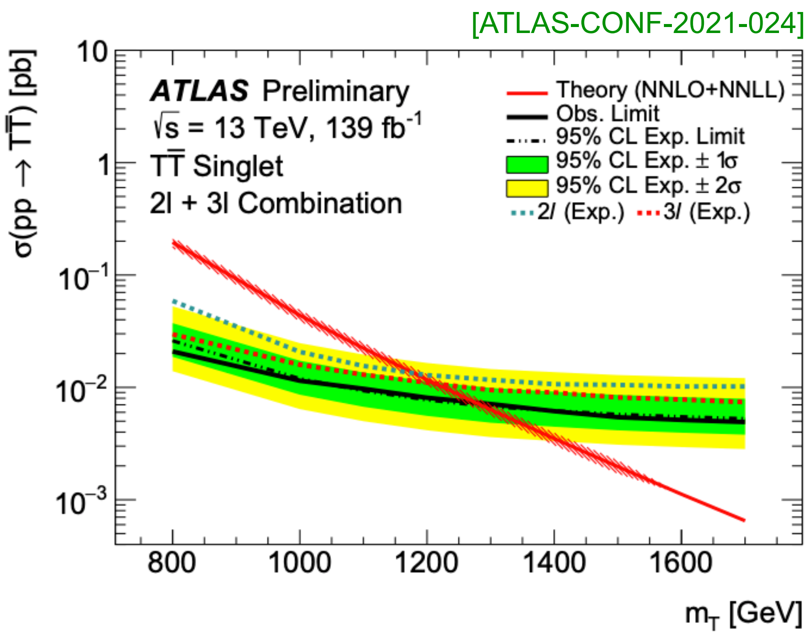
In classic models, top sector loops break EW symmetry:

$$V = \frac{N_c y_t^2}{16\pi^2} M_T^2 f^2 \left(-\sin^2 \Pi/f + \sin^4 \Pi/f \right) + \delta_{\rm gauge} \sin^2 \Pi/f$$

mass of QCD-charged top partners

LHC Run 2: $M_T \gtrsim 1.3\ \text{TeV}$





The potential

Instead, we take EW preserving top contribution + Gegenbauer term:

$$V = \kappa \frac{N_c y_t^2}{16\pi^2} M_T^2 f^2 \left[+ \sin^2 \Pi/f \ + \gamma \, G_n^{(N-1)/2}(\cos \Pi/f) \right]$$

allows to tune overall size

positive sign

size of explicit breaking associated with Gegenbauer

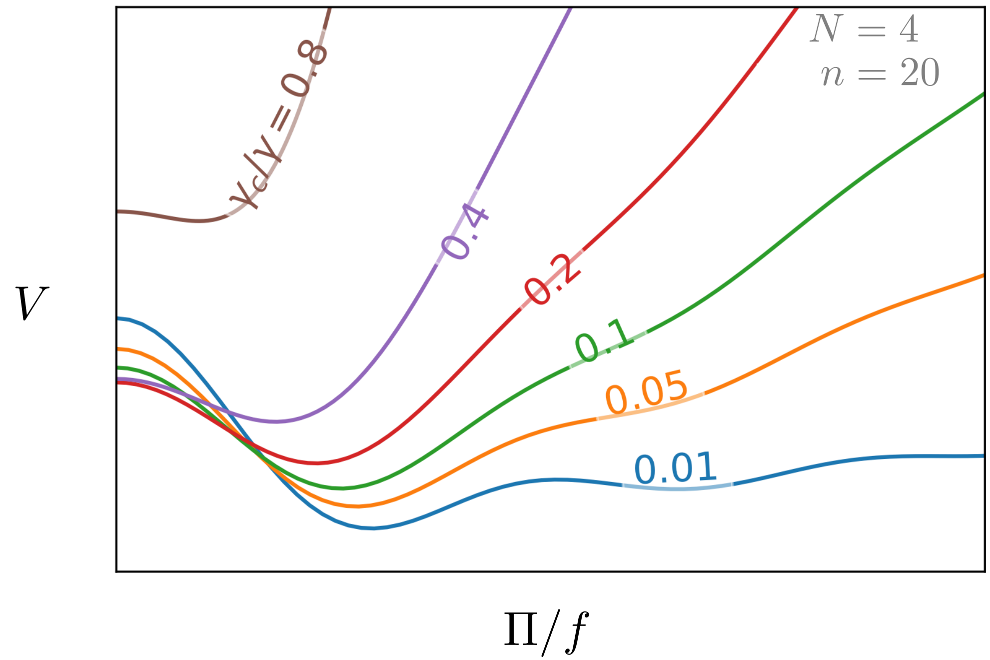
The Gegenbauer is assumed to originate from UV breaking in non-minimal irrep

“Internal” to strong sector - think of quark masses in QCD

The potential

Instead, we take EW preserving top contribution + Gegenbauer term:

$$V = \kappa \frac{N_c y_t^2}{16\pi^2} M_T^2 f^2 \left[+ \sin^2 \Pi/f \;\; + \gamma \, G_n^{(N-1)/2}(\cos \Pi/f) \right]$$



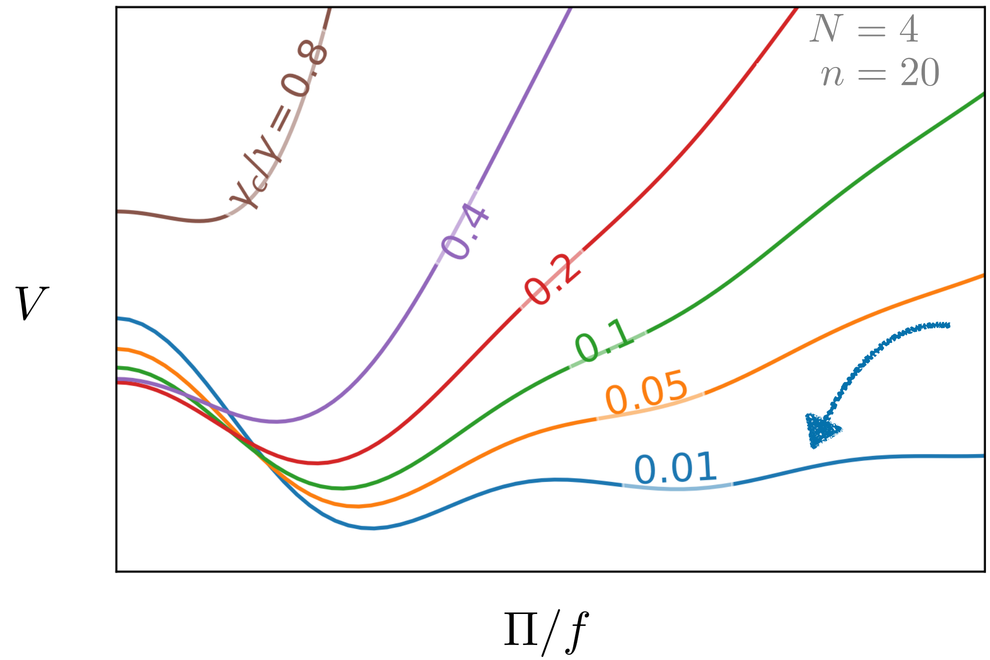
$\gamma_c \approx 8 \times 10^{-4} \, (10/n)^{3.6}$

for $\gamma < \gamma_c$, minimum is at origin

The potential

Instead, we take EW preserving top contribution + Gegenbauer term:

$$V = \kappa \frac{N_c y_t^2}{16\pi^2} M_T^2 f^2 \left[+ \sin^2 \Pi/f \; + \gamma\, G_n^{(N-1)/2}(\cos \Pi/f) \right]$$



Gegenbauer dominates

no tuning to get $v \ll f$

🙂

but need $\kappa \ll 1$ tuning for Higgs mass

😬

for $\gamma < \gamma_c$, minimum is at origin

The potential

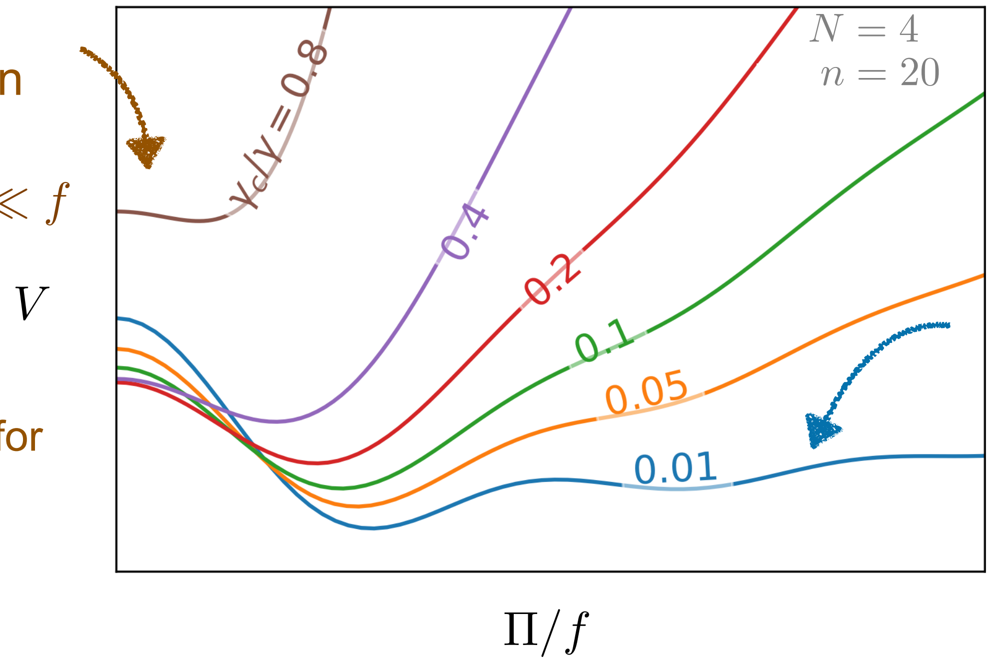
Instead, we take EW preserving top contribution + Gegenbauer term:

$$V = \kappa \frac{N_c y_t^2}{16\pi^2} M_T^2 f^2 \left[+ \sin^2 \Pi/f \ + \gamma \, G_n^{(N-1)/2}(\cos \Pi/f) \right]$$

Gegenbauer is small perturbation

must tune to get $v \ll f$

but $m_h = 125$ GeV for $\kappa \sim O(1)$



Gegenbauer dominates

no tuning to get $v \ll f$

but need $\kappa \ll 1$ tuning for Higgs mass

for $\gamma < \gamma_c$, minimum is at origin

The potential

Instead, we take EW preserving top contribution + Gegenbauer term:

$$V = \kappa \frac{N_c y_t^2}{16\pi^2} M_T^2 f^2 \left[+ \sin^2 \Pi/f \;\; + \gamma\, G_n^{(N-1)/2}(\cos \Pi/f) \right]$$

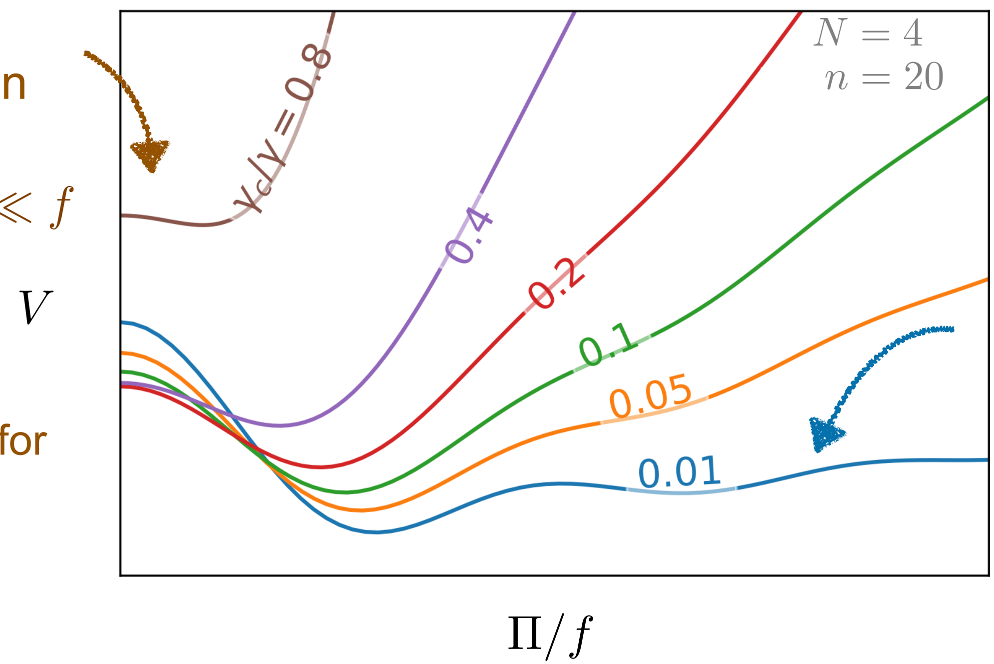
Gegenbauer is small perturbation

must tune to get $v \ll f$

😬

but $m_h = 125$ GeV for $\kappa \sim O(1)$

🙂



Gegenbauer dominates

no tuning to get $v \ll f$

🙂

but need $\kappa \ll 1$ tuning for Higgs mass

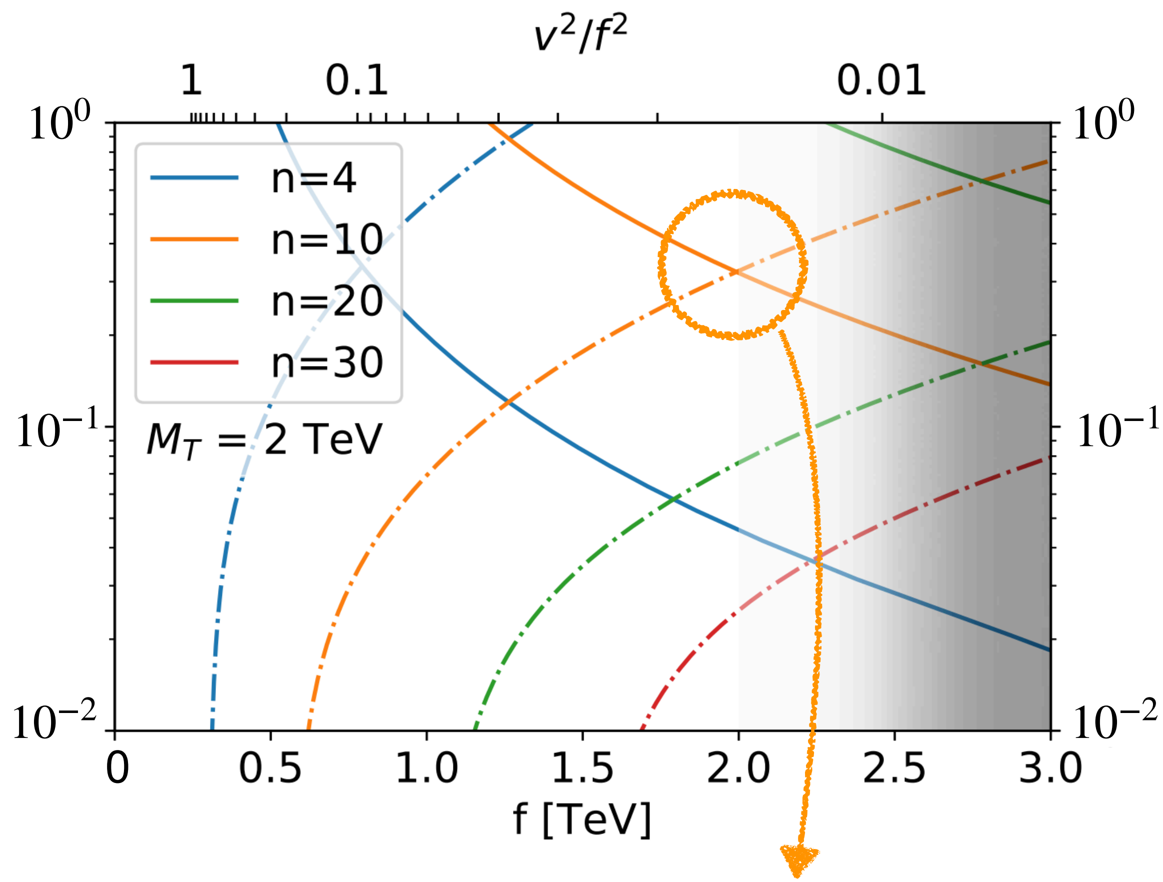
😬

Total tuning minimized in "intermediate" region

Quantifying the fine tuning

$$\Delta = \left(\frac{\partial \log f/v}{\partial \log \gamma}\right)^{-1}$$

vev tuning
(solid)



mass tuning
(dashed)

$n = 10$ $\quad$ $M_T = f = 2\ \text{TeV}$ $\quad$ $\frac{c_{hVV}}{c_{hVV}^{\text{SM}}} \approx 0.8\%$

total tuning better than 10%
and beyond LHC reach

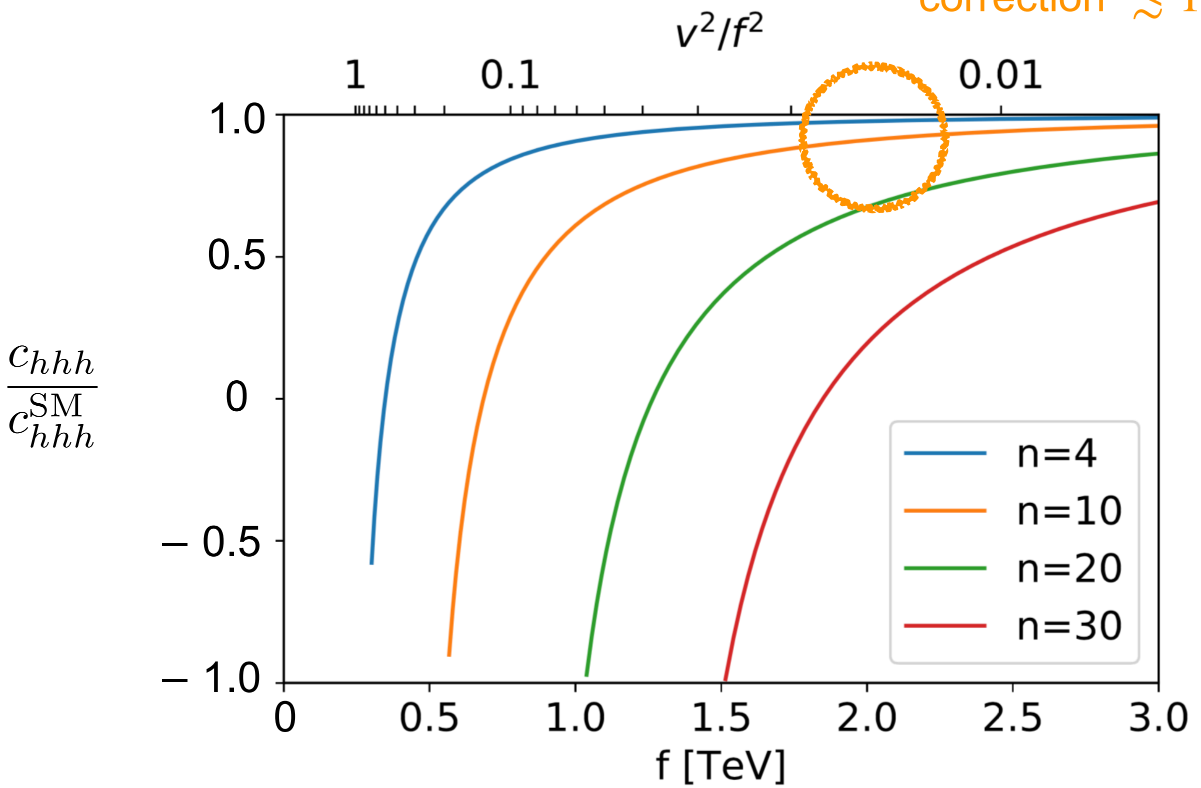
(compare to $2v^2/f^2 \approx 3\%$)

The Higgs cubic coupling

The pure Gegenbauer potential for large *n* has $\frac{c_{hhh}}{c_{hhh}^{\rm SM}} \approx -\frac{N-1}{3} \approx -1$

But top sector contribution overrides this:

correction $\lesssim 10\,\%$



Summary

In principle, Gegenbauer potential could realize $v \ll f$ naturally in composite pNGB models

However, due to LHC lower bounds on QCD-charged top partners, radiative top sector potential is too large and forces some tuning

Higgs cubic coupling is very non-standard for pure Gegenbauer (but top contribution largely overrides this)

Outline

- ✓ Warm-up: Abelian Goldstones
- ✓ Non-Abelian: Gegenbauer Goldstones
- ✓ Gegenbauer Higgs
- ▸ Gegenbauer's Twin

[Durieux, McCullough, Salvioni 2110.06941, JHEP]
[Durieux, McCullough, Salvioni 2202.01228, JHEP]

Gegenbauer's Twin

Gegenbauer potential can realize $v \ll f$ naturally

But for "standard" composite pNGB Higgs, top sector potential still forces some tuning:

QCD-charged top partners must be heavy

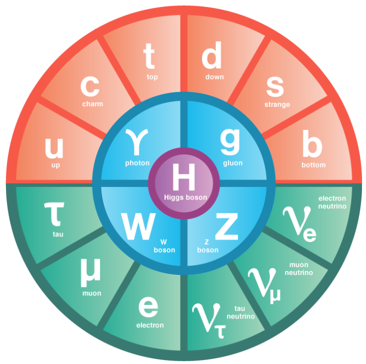
Twin Higgs models can reduce size of top contribution: top partners are QCD-neutral

[Chacko, Goh, Harnik 2005]

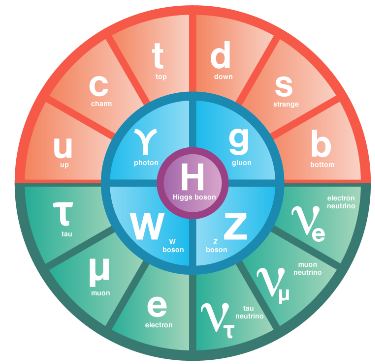
Could Gegenbauer's Twin be fully natural?

Twin Higgs

Standard Model | Twin Standard Model



$$\overset{\mathcal{Z}_2}{\longleftrightarrow}$$



Leading potential for Higgs + Twin Higgs is $SO(8)$ invariant

$$V = \lambda\left(|H_A|^2 + |H_B|^2 - f^2/2\right)^2$$

Spontaneous breaking $SO(8) \rightarrow SO(7)$: 7 Goldstones

3 eaten by gauging $SU(2) \times U(1)$ in Twin sector, 4 form SM Higgs doublet

The Twin protection

Exchange symmetry enforces $SO(8)$ invariance of quadratic corrections to potential

$$\mathcal{L} = y_t Q_A H_A t_A + \hat{y}_t Q_B H_B t_B$$

$$\delta V = -\frac{N_c}{8\pi^2}\left(y_t^2 \Lambda_A^2 |H_A|^2 + \hat{y}_t^2 \Lambda_B^2 |H_B|^2\right)$$

$$A \xleftrightarrow{\mathcal{Z}_2} B \quad \longrightarrow \quad y_t = \hat{y}_t,\ \Lambda_A = \Lambda_B \quad \longrightarrow \quad \delta V \sim (|H_A|^2 + |H_B|^2)$$

$SO(8)$ invariant

Hierarchy problem solved up to scale $\Lambda \lesssim 4\pi f$

Neutral top partners

Exchange symmetry enforces $SO(8)$ invariance of quadratic corrections to potential

$$\mathcal{L} = y_t Q_A H_A t_A + \hat{y}_t Q_B H_B t_B$$

The top partners are neutral under whole SM (& charged under Twin QCD)
They can still be light

$$A \xleftrightarrow{\mathcal{Z}_2} B \longrightarrow y_t = \hat{y}_t,\ \Lambda_A = \Lambda_B \longrightarrow \delta V \sim (|H_A|^2 + |H_B|^2)$$

$SO(8)$ invariant

Hierarchy problem solved up to scale $\Lambda \lesssim 4\pi f$

Twin Higgs potential

Quartic terms do not cancel exactly, but resulting potential with $\mathcal{Z}_2$ is not realistic:

$$v = 0 \qquad \text{or} \qquad v = f$$

$$V_t \approx \frac{3y_t^4 f^4}{64\pi^2}\left[\sin^4\frac{h}{f}\log\frac{a}{\sin^2 h/f} + \cos^4\frac{h}{f}\log\frac{a}{\cos^2 h/f}\right]$$

Need some form of $\mathcal{Z}_2$ breaking, bringing along the usual tuning $\Delta = \frac{2v^2}{f^2}$

[Craig, Katz, Strassler, Sundrum 2015]
[Barbieri, Greco, Rattazzi, Wulzer 2015]

Gegenbauer's Twin

Quartic terms do not cancel exactly, but resulting potential with $\mathcal{Z}_2$ is not realistic:

$v = 0$ or $v = f$

$$V_t \approx \frac{3y_t^4 f^4}{64\pi^2}\left[\sin^4\frac{h}{f}\log\frac{a}{\sin^2 h/f} + \cos^4\frac{h}{f}\log\frac{a}{\cos^2 h/f}\right]$$

Here, we introduce a Gegenbauer contribution:

generalize construction to $SO(8) \to SO(4) \times SO(4)$ explicit breaking

$$V_G^{(n)} = \epsilon f^4 G_n^{3/2}(\cos 2h/f)$$

(in unitary gauge, $\Pi_i = \delta_{i4} h$)

Gegenbauer's Twin

Quartic terms do not cancel exactly, but resulting potential with $\mathcal{Z}_2$ is not realistic:

$$v = 0 \qquad \text{or} \qquad v = f$$

$$V_t \approx \frac{3y_t^4 f^4}{64\pi^2}\left[\sin^4\frac{h}{f}\log\frac{a}{\sin^2 h/f} + \cos^4\frac{h}{f}\log\frac{a}{\cos^2 h/f}\right]$$

Here, we introduce a Gegenbauer contribution:

generalize construction to $SO(8) \to SO(4) \times SO(4)$ explicit breaking

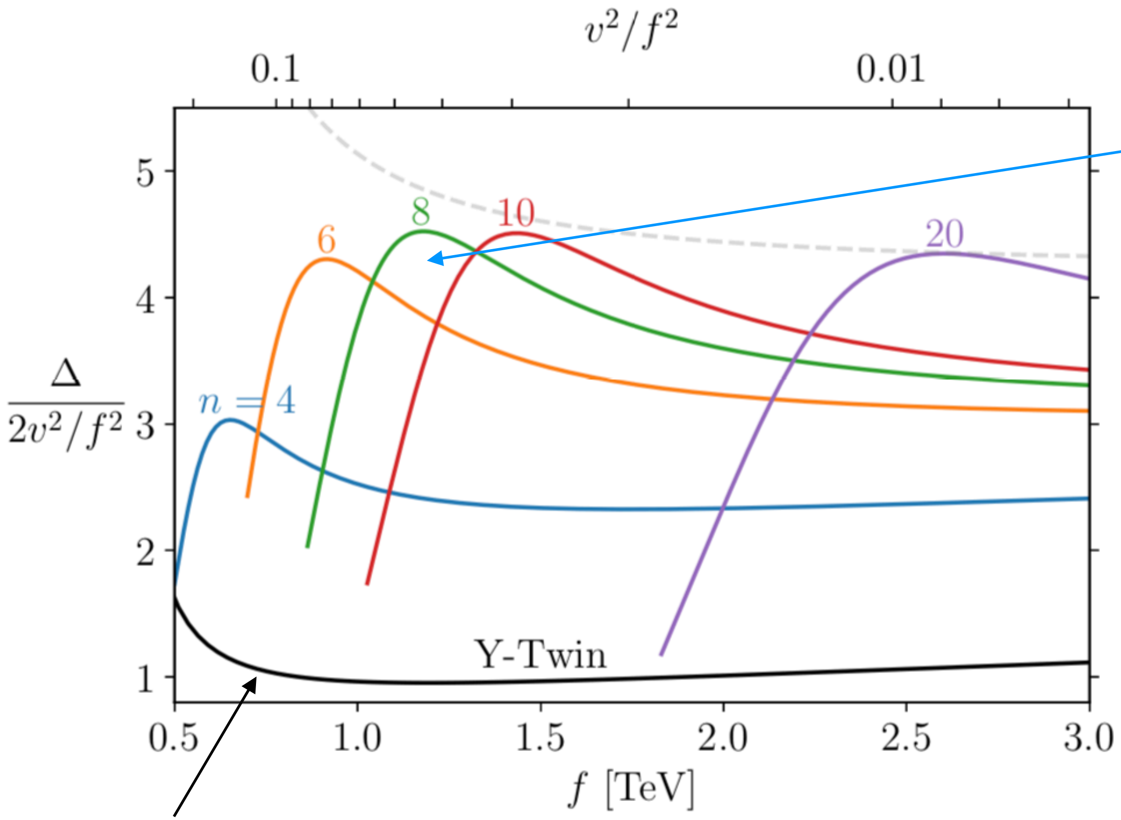
$$V_G^{(n)} = \epsilon f^4 G_n^{3/2}(\cos 2h/f)$$

model parameters
(fixed by Higgs mass and vev)

(in unitary gauge, $\Pi_i = \delta_{i4} h$)

Gegenbauer's Twin

Fine tuning relative to standard Twin model:



fine tuning gain is approximately a factor

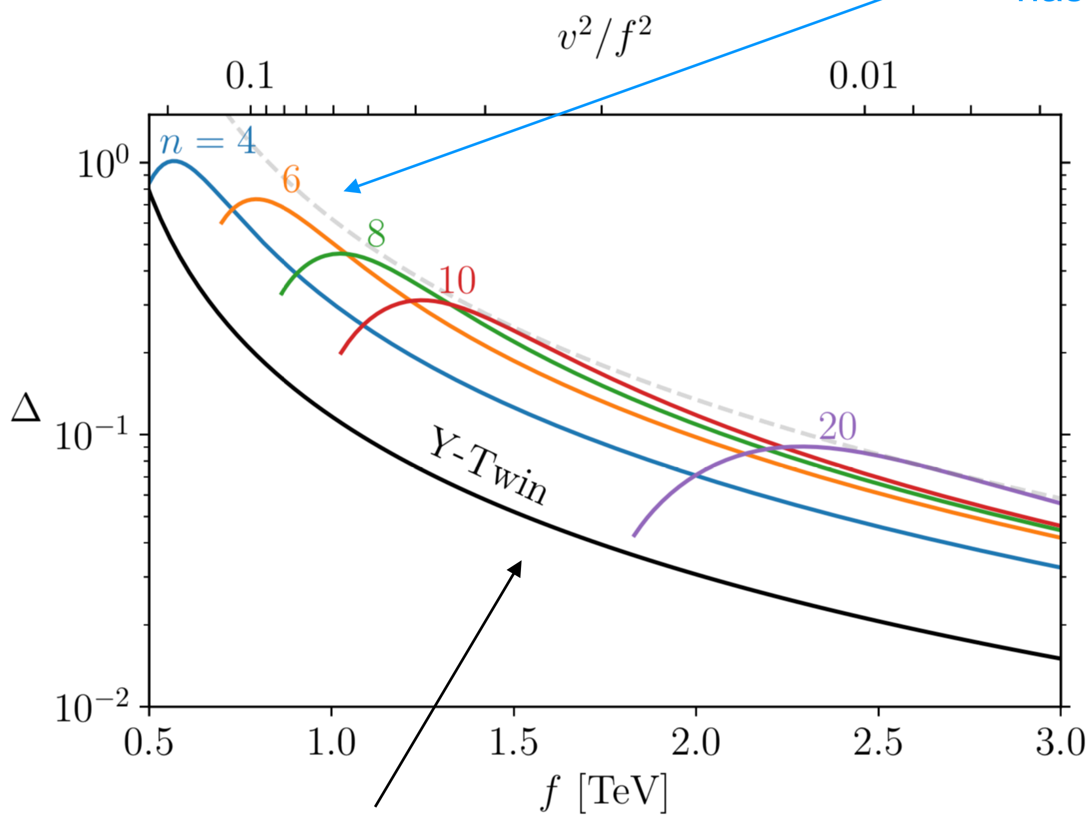
$$\frac{\Delta}{2v^2/f^2} \approx \frac{4\pi^2 m_h^2}{3y_t^4 v^2} \approx 4$$

standard Twin model (Twin hypercharge not gauged)

Gegenbauer's Twin

Absolute fine tuning:

Gegenbauer's Twin with
$n = 6$ or $n = 8$ and $f = 1$ TeV
has essentially no tuning

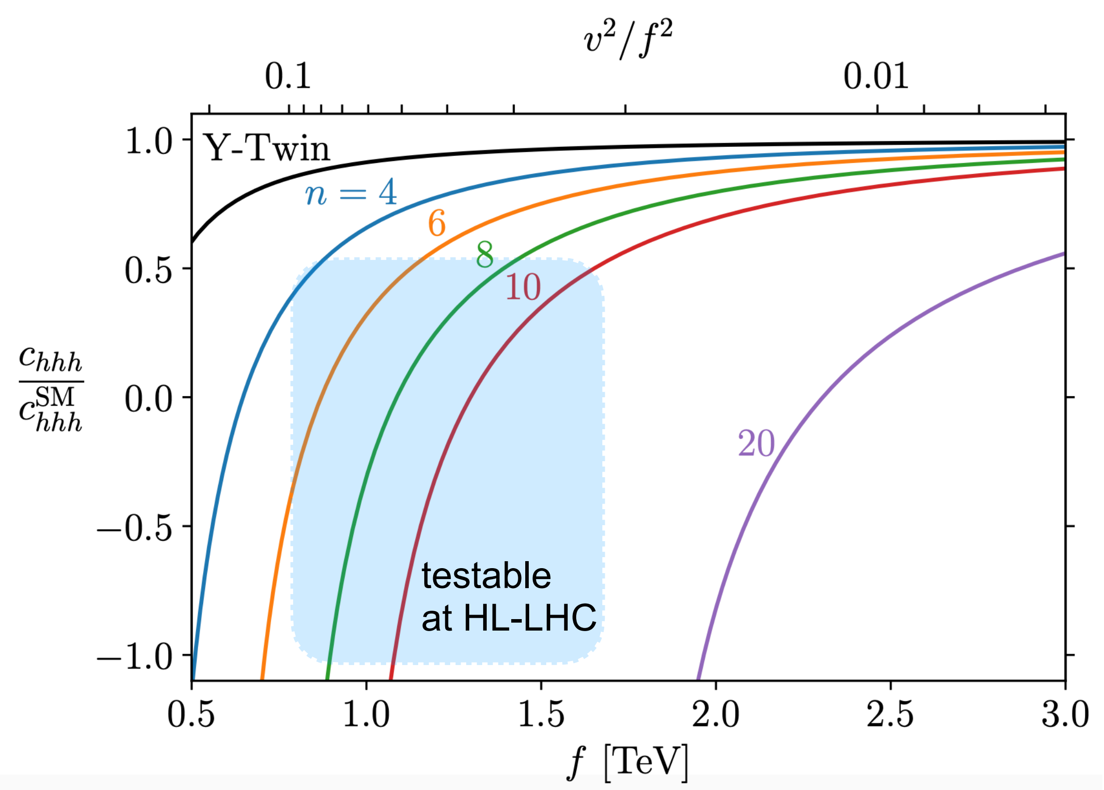


$$\frac{c_{hXX}}{c_{hXX}^{\rm SM}} = \sqrt{1 - \frac{v^2}{f^2}}$$

standard Twin model (Twin hypercharge not gauged)

Higgs cubic coupling

For Gegenbauer's Twin, corrections are parametrically enhanced



"Smoking gun" signal: could even be first deviation observed at LHC

Conclusions

Gegenbauer's Twin shows that fully natural electroweak breaking is still compatible with LHC results

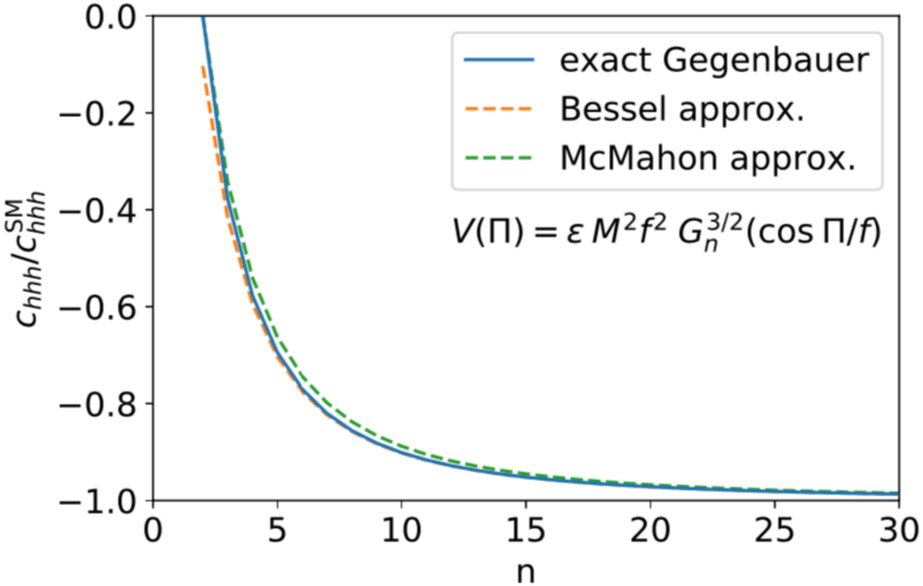
Requires to drop often assumed "minimality criteria" about origin of (explicit) symmetry breaking

General question: can one get such structure accidentally, because lower-dim operators are forbidden? (For example, by gauge symmetries.)

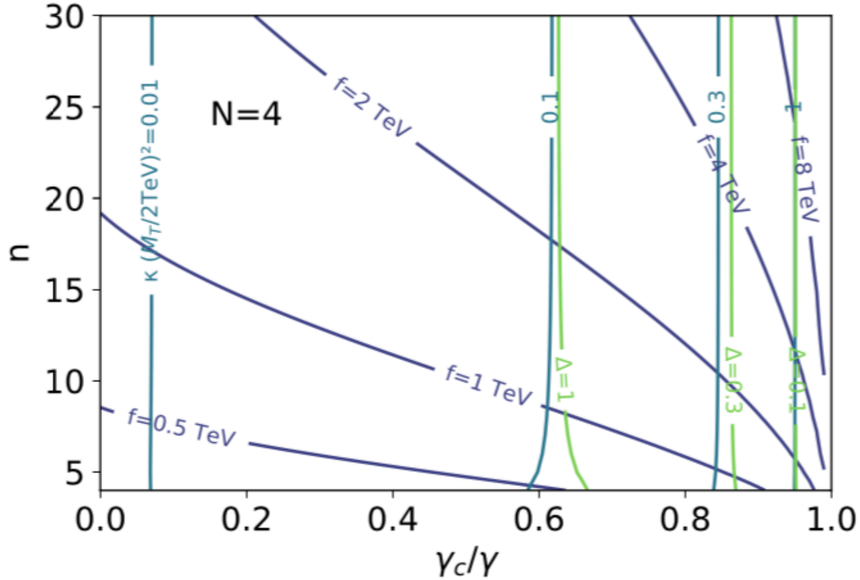
A Gegenbauer top sector? What about other cosets? …

Backup slides

More on Gegenbauer Higgs



$$\frac{c_{hhh}}{c_{hhh}^{\rm SM}} = -\frac{N-1}{3}\;\cos\frac{\langle\Pi\rangle}{f} \approx -\frac{N-1}{3}$$



More on Gegenbauer Higgs

dimension of n - index symmetric traceless irrep of $SO(N+1)$

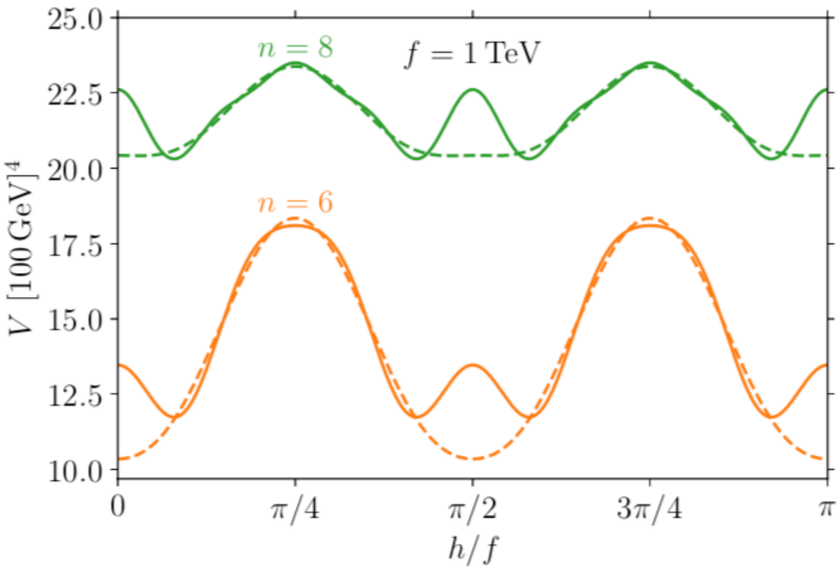
$$\frac{(N+2n-1)(N+n-2)!}{n!(N-1)!} \overset{n \gg N}{\sim} \frac{2n^{N-1}}{(N-1)!}$$

for example $\quad N=4,\ n=10 \quad \rightarrow \quad 506$

$$V_t^{\mathbf{5+5}} = \alpha_t \sin^2 \Pi/f$$

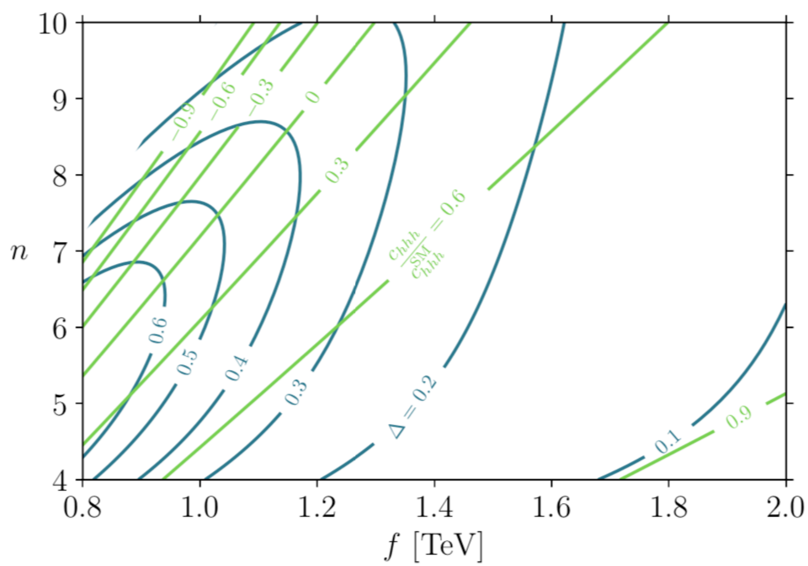
$$\alpha_t \approx \frac{N_c}{8\pi^2}(y_L^2 - 2y_R^2) f^2 (M_1^2 - M_4^2) \int \frac{dp\, p^3}{(p^2 + M_1^2 + y_R^2 f^2)(p^2 + M_4^2 + y_L^2 f^2)}$$

More on Gegenbauer's Twin



$$\delta = \begin{pmatrix} \frac{\partial \log v^2}{\partial \log \epsilon} & \frac{\partial \log v^2}{\partial \log a} \\ \frac{\partial \log m_h^2}{\partial \log \epsilon} & \frac{\partial \log m_h^2}{\partial \log a} \end{pmatrix}$$

$$\Delta = \left(\sum \text{eigenvalues}\,(\delta^T \delta)\right)^{-1/2}$$



$$\left(\frac{\partial \log v^2}{\partial \log a}\right)^{-1} = \frac{8\pi^2 m_h^2}{3y_t^4 f^2\left(1 - \frac{3v^2}{f^2} + \frac{2v^4}{f^4}\right)}$$