Flavour symmetries in the symmetric limit

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Introduction: flavour symmetries

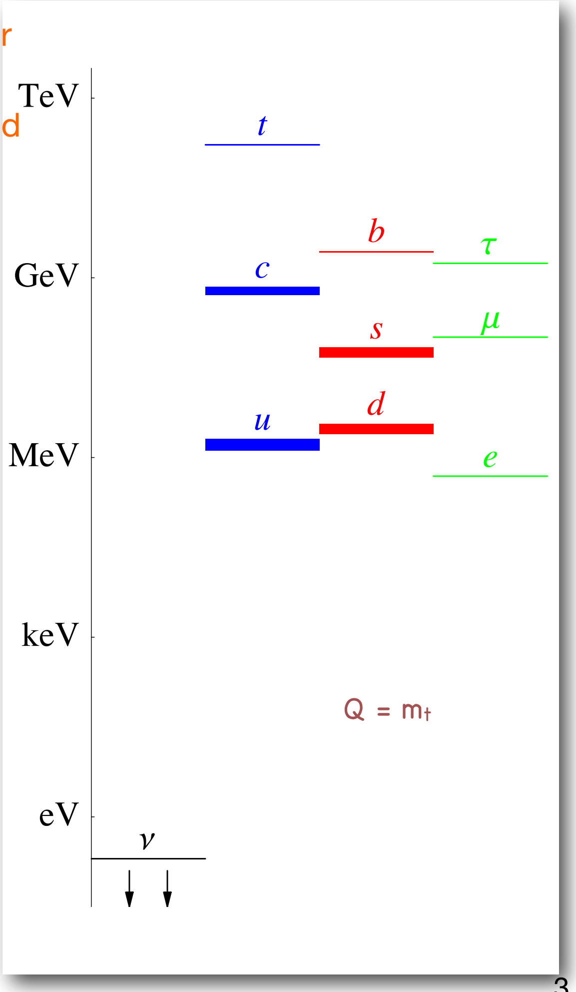
The flavour puzzle in the SM

- 3 families ↔ $U(3)^5$ symmetry of the gauge lagrangian
- Charged fermions: $m_1 \ll m_2 \ll m_3$
  Quarks: $V_{CKM} \sim \mathbf{1}$
- Neutrinos: lighter, milder hierarchy,
  $U_{PMNS} \neq \mathbf{1}$

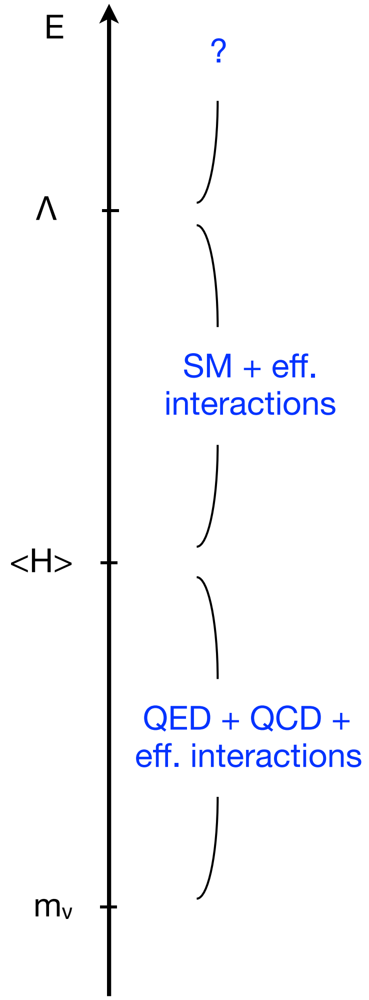
	1	2	3
l	l_1	l_2	l_3
e^c	$(e^c)_1$	$(e^c)_2$	$(e^c)_3$
q	q_1	q_2	q_3
u^c	$(u^c)_1$	$(u^c)_2$	$(u^c)_3$
d^c	$(d^c)_1$	$(d^c)_2$	$(d^c)_3$

family number
(horizontal)
not understood

gauge irreps
(vertical)
understood?



Smallness of neutrino masses and high scales


E

?

Λ

SM + eff. interactions

<H>

QED + QCD + eff. interactions

m_ν

$$\mathcal{L}^{\rm eff}_{E\ll\Lambda} = \mathcal{L}^{\rm ren}_{\rm SM} + \frac{h_{ij}}{2\Lambda}(l_i h)(l_j h) + \text{h.c.}$$

$$m_{u,d,e} = \lambda_{u,d,e}\, v \qquad m_\nu = h\, v \times \frac{v}{\Lambda}$$

$$\Lambda \sim (0.5 \cdot 10^{15}\,\text{GeV})\, h \left(\frac{0.05\,\text{eV}}{m_\nu}\right)$$

This is the framework we consider
The results can be extended to other set-ups

Flavour symmetries

- G_f flavour group acting on “i”, $\mathcal{L}$ invariant
- $$\mathcal{L}_f = \lambda_{ij}\psi_i\psi_j h \quad + \frac{\lambda_{ijk}}{\Lambda_f}\phi_k\psi_i\psi_j h + \ldots$$

ϕ_k scalar “flavon”
(SM invariant)
spontaneously breaks G_f

$$\downarrow \qquad\qquad \downarrow$$

$$m^0_{ij} = \lambda_{ij} v \qquad m^1_{ij} = \lambda_{ijk}\frac{\langle\phi_k\rangle}{\Lambda} v + \ldots$$

$$\boxed{m_{ij} = m^0_{ij}\boxed{+\, m^1_{ij}}}$$

vanishes for $\phi_k = 0$
“symmetric limit”

$$\text{e.g.} \quad m^0_D = \begin{pmatrix} 0 & & \\ & 0 & \\ & & m_b \end{pmatrix} \qquad m^0_U = \begin{pmatrix} 0 & & \\ & 0 & \\ & & m_t \end{pmatrix}$$

$$m^0_E = \begin{pmatrix} 0 & & \\ & 0 & \\ & & m_\tau \end{pmatrix} \qquad V^0_{\mathrm{CKM}} = \mathbf{1}$$

$(\theta_C \text{ undetermined})$

The content of this talk

Q1: can a flavour symmetry acting on the low-scale effective lagrangian (light neutrino Majorana masses) provide an approximate description of lepton flavour in the symmetric limit (neglecting SB effects)?

A1: yes, but only if neutrinos are inverted hierarchical or unconstrained (anarchical). If NH is confirmed, the symmetric limit cannot provide an understanding of lepton mixing (LO role of symmetry breaking effects)

Q2: the Weinberg operator originates from high-scale physics. Is studying the flavour symmetry at low-scale equivalent to studying it at high-scale?

A2: not necessarily. Necessary and sufficient conditions in the case of see-saw I

Q3: can a flavour symmetry constraining a see-saw lagrangian provide an approximate description of lepton flavour in the symmetric limit?

A3: yes, and neutrinos can be normally hierarchical if the high-scale and low-scale actions of the flavour symmetry are not equivalent

Definition of the problem, and **Q1**

Flavour group

- G_f flavour group
  - any: discrete/continuous, abelian/non-abelian, global/gauge, etc
  - includes all “hidden” factors
  - unitary representation, commuting with Poincaré and G_{SM}

Flavour representation

$$g \in G_f : \begin{cases} l_i & \rightarrow & U_l(g)_{ij}\, l_j \\ e^c_i & \rightarrow & U_{e^c}(g)_{ij}\, e^c_j \end{cases} \qquad e^c \leftrightarrow \overline{e_R}$$

Invariant lagrangian, <ϕ> = 0 (low-scale)

$$\mathcal{L}^{(0)}_{\text{low-scale}} = \lambda^E_{ij} e^c_i l_j h^* + \frac{c_{ij}}{2\Lambda} l_i l_j h h$$

$$\rightarrow \boxed{m^{0E}_{ij} e^c_i e_j} + \boxed{\frac{m^{0\nu}_{ij}}{2} \nu_i \nu_j}$$

Symmetric limit

$$\begin{aligned} U_{e^c}(g)^T m_E^0 \; U_l(g) &= m_E^0 \\ U_l(g)^T \; m_\nu^0 \; U_l(g) &= m_\nu^0 \end{aligned}$$

(from the invariance of the lagrangian)

Symmetry breaking

$$\begin{aligned} m_E &= m_E^0 + m_E^1 \\ m_\nu &= m_\nu^0 + m_\nu^1 \end{aligned}$$

invariant under G_f

not invariant under G_f generated by ϕ

The symmetric limit provides an approximate description of lepton flavour

- $m_E \neq 0$ and $m_\nu \neq 0$

$$\begin{aligned} m_E &= \boxed{m_E^0} + \boxed{m_E^1} \\ m_\nu &= \boxed{m_\nu^0} + \boxed{m_\nu^1} \end{aligned}$$

approximate description of lepton observables

moderate correction necessary for an accurate description

$$\text{e.g.} \quad m_D^0 = \begin{pmatrix} 0 & & \\ & 0 & \\ & & m_b \end{pmatrix} \qquad m_U^0 = \begin{pmatrix} 0 & & \\ & 0 & \\ & & m_t \end{pmatrix}$$

$$m_E^0 = \begin{pmatrix} 0 & & \\ & 0 & \\ & & m_\tau \end{pmatrix} \qquad \begin{array}{l} V_{\mathrm{CKM}}^0 = \mathbf{1} \\ (\theta_C \ \text{undetermined}) \end{array}$$

The LO pattern of lepton flavour is determined by symmetry breaking

- $m_E = 0$ or $m_\nu = 0$

$$m_E = \boxed{m_E^0} + \boxed{m_E^1}$$

$$m_\nu = \boxed{m_\nu^0} + \boxed{m_\nu^1}$$

$m_E = 0$ or $m_\nu = 0$
in the symmetric limit

fully determine
the PMNS matrix

$$\text{e.g.} \quad G = A_4 \qquad m_\nu^0 = \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & a \\ 0 & a & 0 \end{pmatrix} \qquad m_E^0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$m_\nu^1\text{: } H_1 \text{ invariant} \qquad m_E^1\text{: } H_2 \text{ invariant}$$

The symmetric limit provides an approximate description of lepton flavour

- $m_E \neq 0$ and $m_\nu \neq 0$

$$
\begin{aligned}
m_E &= \boxed{m_E^0} + m_E^1 \\
m_\nu &= \boxed{m_\nu^0} + m_\nu^1
\end{aligned}
$$

approximate description of lepton observables

Neutrino masses

NH/IH	(a 0 0)	(0 a a)
NH or IH	(a a a)	(a b 0)
	(a b b)	(a b c)

Charged lepton masses

	(A 0 0)
	(A B 0)
	(A B C)

PMNS matrix

$$
\begin{pmatrix} X & X & 0 \\ X & X & X \\ X & X & X \end{pmatrix} \text{ or } \begin{pmatrix} X & X & X \\ X & X & X \\ X & X & X \end{pmatrix}
$$

($X \neq 0$ generic)

G_f U_l U_e leading, in the symmetric limit, to lepton masses and mixings in the above form

- A complete and concise classification is possible, as the predictions in the symmetric limit only depend on the structure of the decomposition of the representations in irreducible components (irreps) and in particular on their
  - **Type** (real, pseudoreal, complex)
  - **Dimension**
  - **Equivalence**
- Notation
  - "**n**": dimension n **complex** or **pseudoreal** irrep
  - "n": dimension n **real** irrep
  - "n, n', n''": dimension n **inequivalent** irreps

G_f U_l U_e leading, in the symmetric limit, to lepton masses and mixings in the above form

U_l, U_{e^c} irreps	masses	ν hierarchy	PMNS zeros
$1 \quad 1 \quad 1$ $1 \quad r \not\supseteq 1$	$(A00)$ (abc)	NH or IH	none
$\mathbf{1} \quad \mathbf{1} \quad \bar{\mathbf{1}}$ $\bar{\mathbf{1}} \quad r \not\supseteq \mathbf{1}, \bar{\mathbf{1}}$	$(A00)$ $(0aa)$	IH	none $(\mathbf{13})$
$1 \quad 1 \quad 1$ $1 \quad 1 \quad r \neq 1$	$(AB0)$ (abc)	NH or IH	none
$\mathbf{1} \quad \mathbf{1} \quad \bar{\mathbf{1}}$ $\bar{\mathbf{1}} \quad \bar{\mathbf{1}} \quad r \neq \mathbf{1}$	$(AB0)$ $(0aa)$	IH	$\mathbf{13}$
$1 \quad 1 \quad 1$ $1 \quad 1 \quad 1$	(ABC) (abc)	NH or IH	none
$\mathbf{1} \quad \mathbf{1} \quad \bar{\mathbf{1}}$ $\bar{\mathbf{1}} \quad \bar{\mathbf{1}} \quad \mathbf{1}$	(ABC) $(0aa)$	IH	$\mathbf{13}, 23, 33$

- Only 6 cases
- Only d = 1 (abelian) irreps
- No pseudoreal irreps (d ≥ 2)
- Neutrinos are either unconstrained (anarchical) or inverted hierarchical
- If NH confirmed, lepton flavour at low-scale can only be accounted for by SB

1+1+1

- "1" = real one-dimensional: f → ± f
- 1+1+1: $U(g)_{ij} = \pm \mathbf{1}_{ij}$
- any m_ν is trivially invariant
- neutrino masses and mixing completely unconstrained
- (anarchy)

SU(5) and SO(10)

- SU(5): assume $U_{\bar{5}} = U_l$ and $U_{10} = U_{ec}$, require $(V_{CKM})_0 = \mathbf{1}$ or V_{12}
  - only unconstrained (anarchical) neutrinos are allowed
- SO(10): assume $U_l = U_{ec} = U_{16}$
  - no solutions

Proof - in 2 steps: masses first, then mixings

Step 1: precise formulation of the problem

Neutrino masses

NH/IH	(a 0 0)	(0 a a)
NH or IH	(a a a)	(a b 0)
	(a b b)	(a b c)

Charged lepton masses

	(A 0 0)
	(A B 0)
	(A B C)

- Given each of the previous 3x6=18 mass patterns, find all G_f, U s.t.
  - $\forall$ invariant m_E, m_ν the mass eigenvalues are in that form
  - $\exists$ invariant m_E, m_ν with mass eigenvalues in that form and generic

Step 1: results charged lepton entries of the same order

lepton masses	decompositions of U_l and U_{e^c}				
$(00A)$ (aaa)	none				
$(00A)$ (aab)	$\mathbf{1}\ \ \overline{\mathbf{1}}\ \ 1$ $\overline{\mathbf{1}}\ \ r \not\supseteq 1, \mathbf{1}$	$1\ \ \mathbf{1}\ \ \overline{\mathbf{1}}$ $1\ \ r \not\supseteq \overline{\mathbf{1}}, \mathbf{1}$	$1\ \ 2$ $1\ \ r \neq 2$		
$(00A)$ $(aa0)$	$\mathbf{1}\ \ \mathbf{1}'\ \ \overline{\mathbf{1}}$ $\overline{\mathbf{1}}\ \ r \not\supseteq \mathbf{1}, \overline{\mathbf{1}'}$	$\mathbf{1}'\ \ \mathbf{1}\ \ \overline{\mathbf{1}}$ $\overline{\mathbf{1}}'\ \ r \not\supseteq \mathbf{1}, \overline{\mathbf{1}}$	$\mathbf{1}\ \ \mathbf{1}\ \ \overline{\mathbf{1}}$ $\overline{\mathbf{1}}\ \ r \not\supseteq \mathbf{1}, \overline{\mathbf{1}}$	$\overline{\mathbf{1}}\ \ \mathbf{1}\ \ \mathbf{1}$ $\mathbf{1}\ \ r \not\supseteq \overline{\mathbf{1}}$	$\mathbf{1}\ \ 2$ $\overline{\mathbf{1}}\ \ r \neq 2$
$(00A)$ $(00a)$	$1\ \ \mathbf{1}\ \ \mathbf{1}'$ $1\ \ r \not\supseteq \overline{\mathbf{1}}, \overline{\mathbf{1}'}$	$\mathbf{1}\ \ \mathbf{1}'\ \ 1$ $\overline{\mathbf{1}}\ \ r \not\supseteq 1, \overline{\mathbf{1}'}$	$1\ \ \mathbf{1}\ \ \mathbf{1}$ $1\ \ r \not\supseteq \overline{\mathbf{1}}$	$\mathbf{1}\ \ \mathbf{1}\ \ 1$ $\overline{\mathbf{1}}\ \ r \not\supseteq 1, \overline{\mathbf{1}}$	$1\ \ \mathbf{2}$ $1\ \ r \not\supseteq \overline{\mathbf{2}}$
$(00A)$ (cba)	$1\ \ 1'\ \ 1''$ $1\ \ r \not\supseteq 1', 1''$	$1\ \ 1\ \ 1'$ $1\ \ r \not\supseteq 1, 1'$	$1'\ \ 1\ \ 1$ $1'\ \ r \not\supseteq 1$	$1\ \ 1\ \ 1$ $1\ \ r \not\supseteq 1$	
$(00A)$ $(0ba)$	$1\ \ 1'\ \ \mathbf{1}$ $1\ \ r \not\supseteq 1', \overline{\mathbf{1}}$	$\mathbf{1}\ \ 1'\ \ 1$ $\overline{\mathbf{1}}\ \ r \not\supseteq 1, 1'$	$\mathbf{1}\ \ 1\ \ 1$ $\overline{\mathbf{1}}\ \ r \not\supseteq 1$	$1\ \ 1\ \ \mathbf{1}$ $1\ \ r \not\supseteq 1, \overline{\mathbf{1}}$	

Step 1: results hierarchies allowed

lepton masses	decompositions of U_l and U_{e^c}
$(0BA)$ (aaa)	none
$(0BA)$ (aab)	$\begin{matrix} \mathbf{1} & \overline{\mathbf{1}} & 1 \\ \overline{\mathbf{1}} & \mathbf{1} & r \neq 1 \end{matrix} \quad \begin{matrix} 1 & \mathbf{1} & \overline{\mathbf{1}} \\ 1 & \overline{\mathbf{1}} & r \neq \mathbf{1} \end{matrix}$
$(0BA)$ $(aa0)$	$\begin{matrix} \overline{\mathbf{1}} & \mathbf{1} & \mathbf{1}' \\ \mathbf{1} & \overline{\mathbf{1}} & r \neq \overline{\mathbf{1}'} \end{matrix} \quad \begin{matrix} \mathbf{1} & \mathbf{1}' & \overline{\mathbf{1}} \\ \overline{\mathbf{1}} & \overline{\mathbf{1}'} & r \neq \mathbf{1} \end{matrix} \quad \begin{matrix} \mathbf{1} & \mathbf{1} & \overline{\mathbf{1}} \\ \overline{\mathbf{1}} & \overline{\mathbf{1}} & r \neq \mathbf{1} \end{matrix} \quad \begin{matrix} \overline{\mathbf{1}} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \overline{\mathbf{1}} & r \neq \overline{\mathbf{1}} \end{matrix}$
$(0BA)$ $(00a)$	$\begin{matrix} 1 & \mathbf{1} & \mathbf{1}' \\ 1 & \overline{\mathbf{1}} & r \neq \overline{\mathbf{1}'} \end{matrix} \quad \begin{matrix} \mathbf{1} & \mathbf{1}' & 1 \\ \overline{\mathbf{1}} & \overline{\mathbf{1}'} & r \neq 1 \end{matrix} \quad \begin{matrix} 1 & \mathbf{1} & \mathbf{1} \\ 1 & \overline{\mathbf{1}} & r \neq \overline{\mathbf{1}} \end{matrix} \quad \begin{matrix} \mathbf{1} & \mathbf{1} & 1 \\ \overline{\mathbf{1}} & \overline{\mathbf{1}} & r \neq 1 \end{matrix}$
$(0BA)$ (cba)	$\begin{matrix} 1 & 1' & 1'' \\ 1 & 1' & r \neq 1'' \end{matrix} \quad \begin{matrix} 1 & 1 & 1' \\ 1 & 1 & r \neq 1' \end{matrix} \quad \begin{matrix} 1' & 1 & 1 \\ 1' & 1 & r \neq 1 \end{matrix} \quad \begin{matrix} 1 & 1 & 1 \\ 1 & 1 & r \neq 1 \end{matrix}$
$(0BA)$ $(0ba)$	$\begin{matrix} 1 & 1' & \mathbf{1} \\ 1 & 1' & r \neq \overline{\mathbf{1}} \end{matrix} \quad \begin{matrix} 1 & \mathbf{1} & 1' \\ 1 & \overline{\mathbf{1}} & r \neq 1' \end{matrix} \quad \begin{matrix} 1 & 1 & \mathbf{1} \\ 1 & 1 & r \neq \overline{\mathbf{1}} \end{matrix} \quad \begin{matrix} \mathbf{1} & 1 & 1 \\ \overline{\mathbf{1}} & 1 & r \neq 1 \end{matrix}$
(CBA) (aaa)	none
(CBA) (aab)	$\begin{matrix} 1 & \mathbf{1} & \overline{\mathbf{1}} \\ 1 & \overline{\mathbf{1}} & \mathbf{1} \end{matrix}$
(CBA) $(aa0)$	$\begin{matrix} \mathbf{1} & \mathbf{1}' & \overline{\mathbf{1}} \\ \overline{\mathbf{1}} & \overline{\mathbf{1}'} & \mathbf{1} \end{matrix} \quad \begin{matrix} \mathbf{1} & \mathbf{1} & \overline{\mathbf{1}} \\ \overline{\mathbf{1}} & \overline{\mathbf{1}} & \mathbf{1} \end{matrix}$
(CBA) $(00a)$	$\begin{matrix} 1 & \mathbf{1} & \mathbf{1}' \\ 1 & \overline{\mathbf{1}} & \overline{\mathbf{1}'} \end{matrix} \quad \begin{matrix} 1 & \mathbf{1} & \mathbf{1} \\ 1 & \overline{\mathbf{1}} & \overline{\mathbf{1}} \end{matrix}$
(CBA) (cba)	$\begin{matrix} 1 & 1' & 1'' \\ 1 & 1' & 1'' \end{matrix} \quad \begin{matrix} 1 & 1 & 1' \\ 1 & 1 & 1' \end{matrix} \quad \begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}$
(CBA) $(0ba)$	$\begin{matrix} 1 & 1' & \mathbf{1} \\ 1 & 1' & \overline{\mathbf{1}} \end{matrix} \quad \begin{matrix} 1 & 1 & \mathbf{1} \\ 1 & 1 & \overline{\mathbf{1}} \end{matrix}$

Comments

- 3 degenerate neutrinos in the symmetric limit cannot be obtained
- no d = 3 irreps
- d = 2 irreps can only appear if $m_e = m_\mu = 0$ in the symmetric limit

Sketch of the proof of step 1

- subspaces in flavour space associated to (zero or non-zero) degenerate m_E masses are invariant under both U_l and U_{ec}

- the U_l and U_{ec} sub-representations corresponding to non-zero degenerate charged lepton masses are conjugated to each other and irreducible.

- the U_l and U_{ec} sub-representations corresponding to zero masses, nor any of their irreducible components, are not conjugated to each other

- each set of degenerate non-zero neutrino masses corresponds to either a real irrep or to a pair of conjugated complex irreps

- none of the remaining irreps (correspond to vanishing neutrino masses) should be real, nor any of them should be conjugated to any other

Step 2: select the cases also leading, in the symmetric limit, to a PMNS matrix close to what observed

- Definition of “close to what observed”: PMNS matrix
  - $|U_{13}| \lesssim 0.16$

$$|U| = \begin{pmatrix} 0.798 \to 0.843 & 0.517 \to 0.584 & 0.137 \to 0.158 \\ 0.232 \to 0.520 & 0.445 \to 0.697 & 0.617 \to 0.789 \\ 0.249 \to 0.529 & 0.462 \to 0.708 & 0.597 \to 0.773 \end{pmatrix}$$

  - $|U_{21}|$, $|U_{31}|$ can be as small as 0.25
  - all other entries larger than 0.45
  - $\begin{pmatrix} X & X & 0 \\ X & X & X \\ X & X & X \end{pmatrix}$ or $\begin{pmatrix} X & X & X \\ X & X & X \\ X & X & X \end{pmatrix}$ with small correction or small accident

Master formula for the PMNS matrix

- $U = H_E P_E V D^{-1} P_\nu^{-1} H_\nu^{-1}$
  - V commutes with U_l (makes m_E diagonal, m_ν diagonal or Dirac blocks)
  - D maximal rotation, if U_l contains conjugated complex irreps (Dirac blocks)
  - P permutations possibly needed to bring mass eigenvalues in standard ordering
  - H rotations up to which U is defined in the symmetric limit
    - (…) $H_\nu = H^*_\nu$(…) for generic neutrino mass pattern (…)
    - (…) $H_E = H_E$ (…) for generic charged lepton mass pattern (…)
- No need to write down any mass matrix, texture explicitly

Example

- $U_l = \mathbf{1} + \mathbf{1} + \mathbf{1}^*$ $U_{ec} = \mathbf{1}^* + r \not\supseteq \mathbf{1}, \mathbf{1}^*$
- Charged lepton masses: (A 0 0)
- Neutrino masses: (0 a a)
- In $U = H_E P_E V D^{-1} P_\nu^{-1} H_\nu^{-1}$
  - $V = V_{23}$
  - $D = D_{12}$
  - P not needed
  - $H_E = (H_E)_{12}$ $H_\nu = (R_\nu)_{12}$ (equivalent to phase redefinition)
- No zeros or an (approximate) zero in 13 if $(H_E)_{12}$ is (approximate) identity

Q2: is studying the flavour symmetry at low-scale equivalent to studying it at high-scale?

[to appear]

Origin of lepton masses (high-scale)

$$\mathcal{L}_{\text{low-scale}} = \lambda^E_{ij} e^c_i l_j h^* + \frac{c_{ij}}{2\Lambda} l_i l_j h h$$ from

Flavour representation

high scale

$$\left\{ \begin{array}{lcl} l_i & \rightarrow & U_l(g)_{ij}\, l_j \\ e^c_i & \rightarrow & U_{e^c}(g)_{ij}\, e^c_j \\ \nu^c_i & \rightarrow & U_{\nu^c}(g)_{ij}\, \nu^c_j \end{array} \right.$$

low scale version

Equivalent, at least in the symmetric limit?

Equivalence of high and low-scale representations (in the symmetric limit)

- By definition, when for each invariant m_ν there exists invariant m_N and M such that $m_\nu = - m_N^T M^{-1} m_N$ (converse is always true)

- Given a low-scale representation does an equivalent high-scale version always exists? **YES**

- Is the low-scale version of a representation always equivalent to the high-scale version? **NO**

- Necessary and sufficient conditions for LS to be equivalent to HS:

1. $U_{\mathbf{vc}}$ vectorlike

   real, or pairs of complex conjugated, or pairs of equivalent pseudoreal

2. The vectorlike part of $U_{\mathbf{l}}$ is contained in $U_{\mathbf{vc}}$

1. U_{ν^c} is not vectorlike

- U_{ν^c} not vectorlike ⇔ M forced to be singular in the symmetric limit: the see-saw formula does not apply

- Example:

Example:	**low-scale**	**high-scale**
$U_l = \mathbf{1}+\mathbf{1}+1$	$m_{ei} = (A\ 0\ 0)$	$m_{ei} = (A\ 0\ 0)$
$U_{e^c} = \bar{\mathbf{1}}+\mathbf{1}+\mathbf{1}$	$m_{\nu i} = (a\ 0\ 0)$	$m_{\nu i} = (a\ 0\ 0)$
$U_{\nu^c} = \bar{\mathbf{1}}+$ real	$U = \begin{pmatrix} ? & ? & X \\ X & X & ? \\ X & X & 0 \end{pmatrix}$	$U = \begin{pmatrix} X & X & ? \\ X & X & X \\ X & X & X \end{pmatrix}$
	$m_E = \begin{pmatrix} & & \\ & & \\ & X & X \end{pmatrix}$	$m_N = \begin{pmatrix} & & \\ & & \\ & X & X \end{pmatrix}$ $m_E = \begin{pmatrix} & & \\ & & \\ & X & X \end{pmatrix}$
	$m_\nu = \begin{pmatrix} X & & \\ & & \\ & & \end{pmatrix}$	$M = \begin{pmatrix} X & X & \\ X & X & \\ & & \end{pmatrix}$ $m_\nu = \begin{pmatrix} & \\ & \boxed{\begin{matrix} X & X \\ X & X \end{matrix}} \end{pmatrix}$
		single RH neutrino dominance det = 0

2. $U_{\nu c}$ is vectorlike but the vectorlike part of U_l is not contained in $U_{\nu c}$

- Example:

	low-scale	**high-scale**
$U_l = \boxed{1+1}+\mathbf{1}$	m_{ei} = (A 0 0)	m_{ei} = (A 0 0)
$U_e^c = 1+\mathbf{1}+\mathbf{1}$	$m_{\nu i}$ = (a b 0)	$m_{\nu i}$ = (a 0 0)
$U_\nu^c = 1+\bar{\mathbf{1}}+\mathbf{1}$	$V = \begin{pmatrix} X & ? & ? \\ ? & X & X \\ 0 & X & X \end{pmatrix}$ or $\begin{pmatrix} ? & ? & X \\ X & X & ? \\ X & X & 0 \end{pmatrix}$	$V = \begin{pmatrix} X & X & ? \\ X & X & X \\ X & X & X \end{pmatrix}$

$$m_E = \begin{pmatrix} & & \\ & & \\ & X & X \end{pmatrix} \qquad m_N = \begin{pmatrix} & & \\ X & & \\ & X & X \end{pmatrix} \qquad m_E = \begin{pmatrix} & & \\ & & \\ & X & X \end{pmatrix}$$

$$m_\nu = \begin{pmatrix} & & \\ & X & X \\ & X & X \end{pmatrix} \qquad M = \begin{pmatrix} & X & \\ X & & \\ & & X \end{pmatrix} \qquad m_\nu = \begin{pmatrix} & \\ & \boxed{\begin{matrix} X & X \\ X & X \end{matrix}} \end{pmatrix}$$

det = 0

Q3: can a flavour symmetry acting on a see-saw lagrangian provide an approximate description of lepton flavour in the symmetric limit?

- If U_{vc} vectorlike and the vectorlike part of U_{l} is contained in U_{vc}: yes, at the same conditions as in the low-scale analysis

- If instead the low- and high-scale analyses are not equivalent, predictive (non-unconstrained) cases corresponding to NH can be found

- The complete list of solutions can be again found based only on the structure of the irrep decompositions

Conclusions

- The **complete** set of lepton flavour **predictions** of any flavour group and representation **in the symmetric limit** has been found, both at low scale (Weinberg operator) and high scale (see-saw). The predictions only depend on the type, dimension, and equivalence of the irrep decompositions.
- In the **low-scale** case: the symmetric limit is close to what observed only if neutrinos are **unconstrained** (anarchical) or **inverted hierarchical**.
- If the present hint for normal hierarchy was confirmed, we would conclude that symmetry breaking plays a leading order role in constraining lepton flavour observables at low scale.
- In the **high-scale** case: the results do not change, except when the low- and high-scale analyses are **not equivalent**. The conditions for equivalence have been found.
- The complete set of additional predictions in the symmetric limit that can obtained at high-scale has been found. A **normal hierarchy** for the neutrinos can be obtained.
- If the present hint for normal hierarchy was confirmed, **a predictive symmetric limit could be close to what observed only because the low- and high-scale actions of the flavour symmetry are not equivalent**. Otherwise, symmetry breaking effects necessary play a leading order role in determining lepton flavour observables.