Running of Radiative Neutrino Masses

(based on arXiv:1502.03098, 1507.06314)

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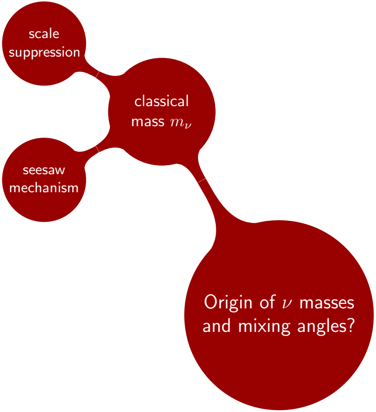
Outline

Introduction

Why Radiative ν Masses?

Origin of ν masses
and mixing angles?

Why Radiative ν Masses?



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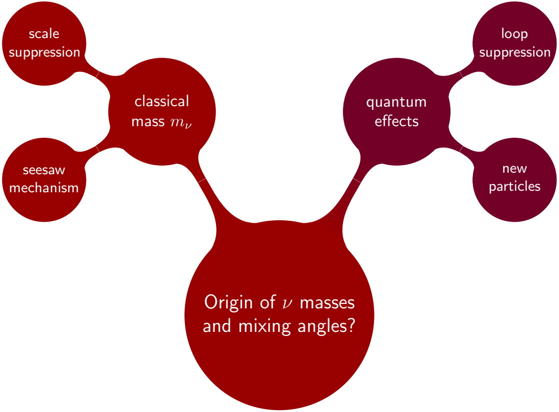

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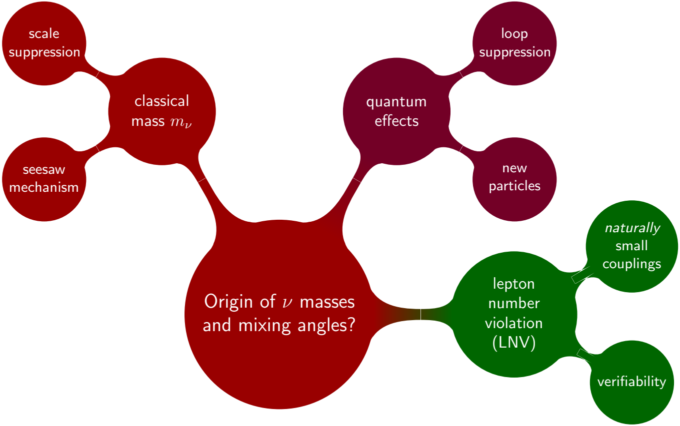

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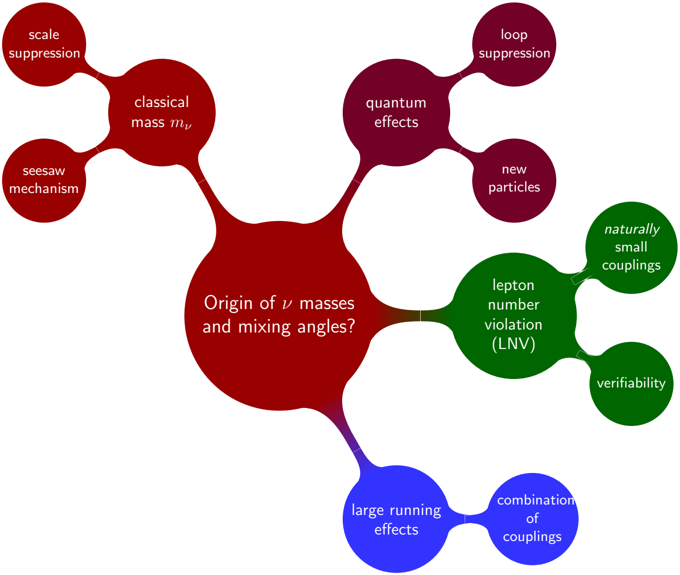

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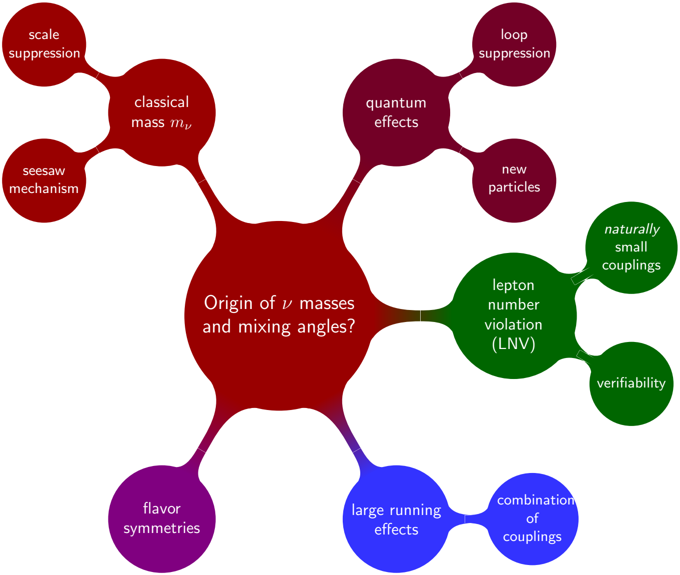

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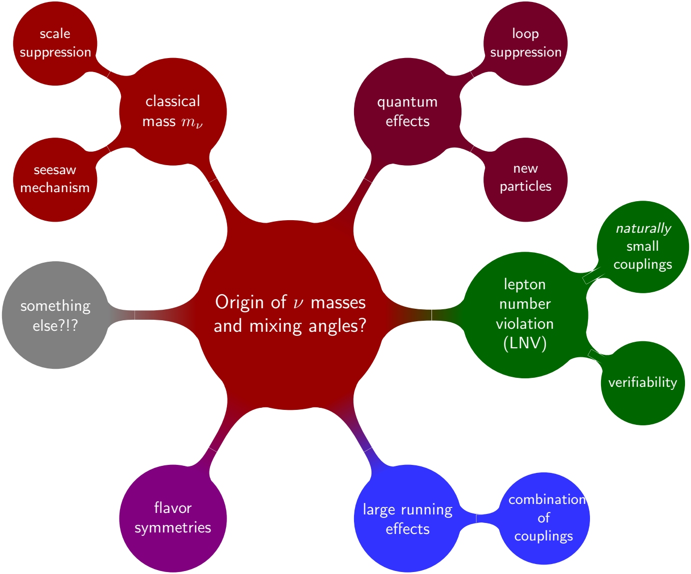
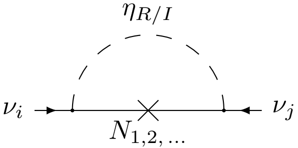
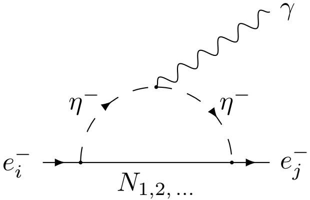


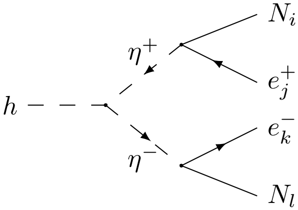
Why Consider Running?



1-loop supressed ν mass ($\lesssim$ eV)



lepton flavor violation (LFV),
e.g. $\mu \to e\gamma$ ($\sim$ MeV)



collider signatures ($\sim$ TeV)

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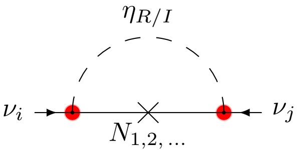

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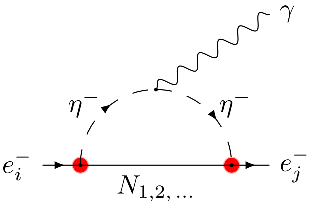

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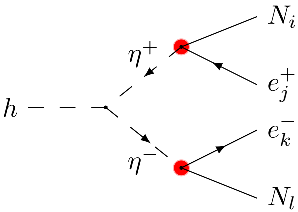

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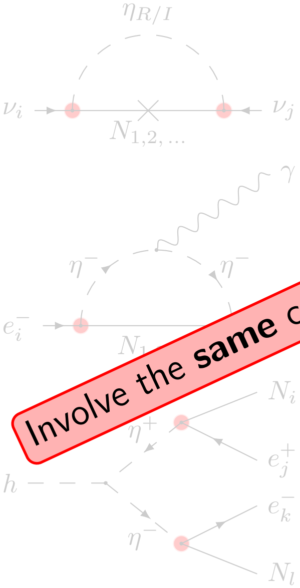

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collider signatures ($\sim$ TeV)

Involve the **same** couplings, but **different** energy scales!

The Scotogenic Model

The Scotogenic Model[Ma, 2006]

(from greek: scotos = darkness)

Particle content

- SM field content
- 2^{nd} scalar doublet η ($Y_\eta = Y_{\text{Higgs}}$)
- ≥ 2 fermion singlets $N_{1,2,\ldots}$

Symmetries

- SM gauge group
- $\mathbb{Z}_2$ parity
  - SM $\mapsto$ SM
  - $\eta \mapsto -\eta$
  - $N_i \mapsto -N_i$
- *inert* or dark sector (stable lightest inert particle)

In short:

New particles with restricted interactions, DM candidate, but **no** neutrino mass at the classical level

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Scalar sector

- SM gauge group broken by Higgs mechanism
  $$SU(3)_c \times SU(2)_L \times U(1)_Y \xrightarrow{\text{EWSB}} SU(3)_c \times U(1)_{em}$$
- physical Higgs: $H = \left(0, v + \frac{1}{\sqrt{2}}h\right)^T$,
  inert scalars: $\eta = \left(\eta^\pm, \frac{1}{\sqrt{2}}(\eta_R + i\eta_I)\right)^T$
- **exact** $\mathbb{Z}_2$ parity $\Leftrightarrow \langle\eta\rangle = 0$

Scalar potential

$$V = m_H^2 H^\dagger H + m_\eta^2 \eta^\dagger \eta + \frac{\lambda_1}{2}\left(H^\dagger H\right)^2 + \frac{\lambda_2}{2}\left(\eta^\dagger \eta\right)^2 + \lambda_3 \left(H^\dagger H\right)\left(\eta^\dagger \eta\right) + \lambda_4 \left(\eta^\dagger H\right)\left(H^\dagger \eta\right) + \frac{\lambda_5}{2}\left[(\eta^\dagger H)^2 + \text{h.c.}\right]$$

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Fermion sector

- Majorana masses for fermion singlets: $M_{1,2,\ldots}$
- only one new Yukawa interaction allowed due to $\mathbb{Z}_2$

$\Rightarrow$ **no** neutrino mass term $(\langle\eta\rangle = 0)$!

Lagrangian

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} - V + \frac{1}{2}\overline{N}_i M_{ij} N_j^{\mathcal{C}} + \mathrm{h.c.} + h_{ij}\overline{N_i}\tilde{\eta}^{\dagger}\ell_{Lj} + \mathrm{h.c.}$$

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Global $U(1)_L$ "lepton number" symmetry

$$\begin{aligned} M = 0 \Rightarrow \quad & L(N) = 1, & L(\eta) = 0, \quad & L(\ell_L) = 1 \\ h = 0 \Rightarrow \quad & L(N) = 0, & L(\eta) = 0, \quad & L(\ell_L) = 1 \\ \lambda_5 = 0 \Rightarrow \quad & L(N) = 0, & L(\eta) = -1, \quad & L(\ell_L) = 1 \end{aligned}$$

't Hooft naturalness

A coupling constant is *naturally* small if the theory's symmetry is enhanced when the coupling vanishes.

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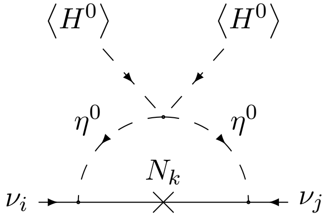
Phenomenology

Scalar masses

$$m_h^2 = 2\lambda_1 v^2 = -2m_H^2 \qquad \text{Higgs}$$

$$\left.\begin{aligned} m_\pm^2 &= m_\eta^2 + v^2\lambda_3 \\ m_R^2 &= m_\pm^2 + v^2\left(\lambda_4 + \lambda_5\right) \\ m_I^2 &= m_\pm^2 + v^2\left(\lambda_4 - \lambda_5\right) \end{aligned}\right\} \text{Inert}$$

Loop-level neutrino masses



Active ν mass:

$$m_{\nu,ij} = \sum_k \frac{M_k \; h_{ki} h_{kj}}{2\,(4\pi)^2} \left\{ \frac{m_R^2}{m_R^2 - M_k^2} \log\left(\frac{m_R^2}{M_k^2}\right) - \frac{m_I^2}{m_I^2 - M_k^2} \log\left(\frac{m_I^2}{M_k^2}\right) \right\}$$

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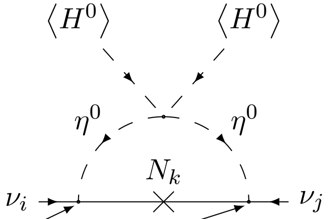

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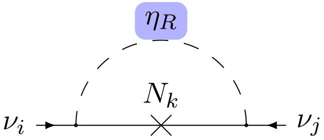

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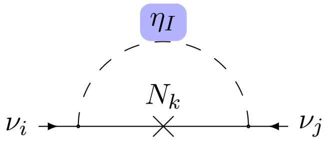

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The Mass Matrix – A Closer Look

Small λ_5 ($m_R \approx m_I$):

$$m_{\nu,\,ij} \approx -\sum_k \lambda_5 \times \frac{1}{(4\pi)^2} \times v\,h_{ki} \times \overbrace{M_k^{-1} f(M_k, m_0)}^{\equiv \Lambda^{-1}} \times v\,h_{kj}$$

$$\left(m_0^2 \equiv \frac{m_R^2 + m_I^2}{2} \right)$$

In general, we find:

$$m_\nu = \text{(LNV coupling)} \times \text{(loop factor)} \times \underbrace{m_D^T \times \text{(loop function)} \times m_D}_{\sim \text{ seesaw formula}}$$

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Running of Neutrino Masses and Mixing Angles

Renormalization Procedure – Technical Steps

$$\mathcal{D}Y_e = Y_e \left\{ \frac{3}{2} Y_e^\dagger Y_e + \frac{1}{2} h^\dagger h + T - \frac{15}{4} g_1^2 - \frac{9}{4} g_2^2 \right\}$$

$$\mathcal{D}h = h \left\{ \frac{3}{2} h^\dagger h + \frac{1}{2} Y_e^\dagger Y_e + T_\nu - \frac{3}{4} g_1^2 - \frac{9}{4} g_2^2 \right\}$$

$$\mathcal{D}M = \left\{ \left(h\, h^\dagger\right) M + M \left(h\, h^\dagger\right)^* \right\}$$

$$\mathcal{D}\lambda_1 = 12\lambda_1^2 + 4\lambda_3^2 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 2\lambda_5^2 + \frac{3}{4}\left(g_1^4 + 2g_1^2 g_2^2 + 3g_2^4\right) - 3\lambda_1\left(g_1^2 + 3g_2^2\right) + 4\lambda_1 T - 4T_4$$

$$\mathcal{D}\lambda_2 = 12\lambda_2^2 + 4\lambda_3^2 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 2\lambda_5^2 + \frac{3}{4}\left(g_1^4 + 2g_1^2 g_2^2 + 3g_2^4\right) - 3\lambda_2\left(g_1^2 + 3g_2^2\right) + 4\lambda_2 T_\nu - 4T_{4\nu}$$

$$\mathcal{D}\lambda_3 = 2\left(\lambda_1 + \lambda_2\right)\left(3\lambda_3 + \lambda_4\right) + 4\lambda_3^2 + 2\lambda_4^2 + 2\lambda_5^2 + \frac{3}{4}\left(g_1^4 - 2g_1^2 g_2^2 + 3g_2^4\right) - 3\lambda_3\left(g_1^2 + 3g_2^2\right) + 2\lambda_3\left(T + T_\nu\right) - 4T_{\nu e}$$

$$\mathcal{D}\lambda_4 = 2\left(\lambda_1 + \lambda_2\right)\lambda_4 + 8\lambda_3\lambda_4 + 4\lambda_4^2 + 8\lambda_5^2 + 3g_1^2 g_2^2 - 3\lambda_4\left(g_1^2 + 3g_2^2\right) + 2\lambda_4\left(T + T_\nu\right) + 4T_{\nu e}$$

$$\mathcal{D}\lambda_5 = \lambda_5\left[2\left(\lambda_1 + \lambda_2\right) + 8\lambda_3 + 12\lambda_4 - 3\left(g_1^2 + 3g_2^2\right) + 2\left(T + T_\nu\right)\right]$$

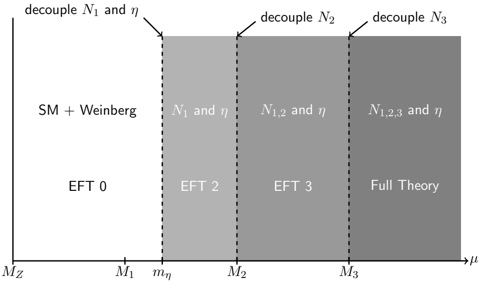
$$\mathcal{D}m_H^2 = 6\lambda_1 m_H^2 + 2\left(2\lambda_3 + \lambda_4\right) m_\eta^2 + m_H^2\left[2T - \frac{3}{2}\left(g_1^2 + 3g_2^2\right)\right]$$

$$\mathcal{D}m_\eta^2 = 6\lambda_2 m_\eta^2 + 2\left(2\lambda_3 + \lambda_4\right) m_H^2 + m_\eta^2\left[2T_\nu - \frac{3}{2}\left(g_1^2 + 3g_2^2\right)\right] - 4\sum_i M_i^2 \left(h\, h^\dagger\right)_{ii}$$

- **calculate RGEs**
- determine matching conditions at mass thresholds
- analytical RGEs for mixing angles and masses ($\theta_{13} \ll 1$)
- compare with numerics

Renormalization Procedure – Technical Steps

Integrate out heavy fields



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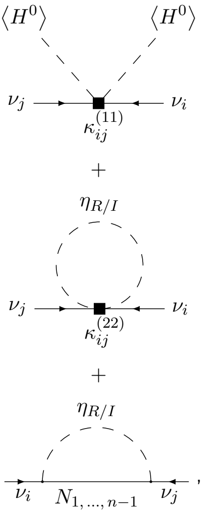
$$-i \overset{(n)}{m}_{\nu,ij} =$$



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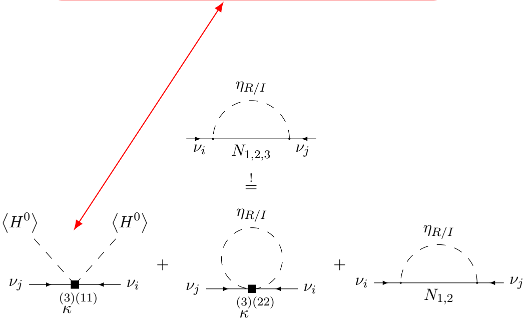
Renormalization Procedure – Technical Steps

$$\kappa^{(11)}_{ij} \left(\overline{\ell^{\mathcal{C}}_{L}}^{a}_{i}\, \epsilon_{ab} H^b\right)\left(\ell_{L j}^{c}\,\epsilon_{cd} H^d\right)$$



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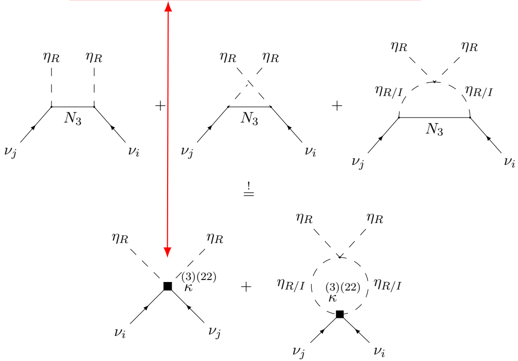
Renormalization Procedure – Technical Steps

$$\kappa^{(22)}_{ij} \left(\overline{\ell^{\mathcal{C}}_{L}}{}^{a}_{i}\, \epsilon_{ab}\eta^{b}\right) \left(\ell_{L}{}^{c}_{j}\, \epsilon_{cd}\eta^{d}\right)$$



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Renormalization Procedure – Technical Steps

$$h^\dagger h = \mathrm{diag}(h_1^2, h_2^2, h_3^2) \quad \& \quad m_\eta \begin{cases} \ll M_{1,2,\ldots} \\ \approx M_{1,2,\ldots} \\ \gg M_{1,2,\ldots} \end{cases}$$

$$(4\pi)^2 m_1' = m_1 \left[C + 2\alpha_h \left[c_{12}^2 c_{13}^2 h_1^2 + s_{23}^2 (c_{12}^2 s_{13}^2 h_2^2 + s_{12}^2 h_3^2) + c_{23}^2 (s_{12}^2 h_2^2 + c_{12}^2 s_{13}^2 h_3^2)\right] + \right. \\ \left. + \alpha_h s_{13}^2 \cos(\delta) \sin(2\theta_{12}) \sin(2\theta_{23}) (h_2^2 - h_3^2)\right]$$

$$(4\pi)^2 m_2' = m_2 \left[C + 2\alpha_h \left[s_{12}^2 c_{13}^2 h_1^2 + s_{23}^2 (s_{12}^2 s_{13}^2 h_2^2 + c_{12}^2 h_3^2) + c_{23}^2 (c_{12}^2 h_2^2 + s_{12}^2 s_{13}^2 h_3^2)\right] + \right. \\ \left. + \alpha_h s_{13}^2 \cos(\delta) \sin(2\theta_{12}) \sin(2\theta_{23}) (h_3^2 - h_2^2)\right]$$

$$(4\pi)^2 m_3' = m_3 \left[C + 2\alpha_h \left[s_{13}^2 h_1^2 + c_{13}^2 (s_{23}^2 h_2^2 + c_{23}^2 h_3^2)\right]\right]$$

$$(4\pi)^2 \theta_{12}' = \frac{\alpha_h}{2} \sin(2\theta_{12}) \frac{\left|m_1 e^{i\phi_1} + m_2 e^{i\phi_2}\right|^2}{\Delta m_{21}^2} (h_1^2 - c_{23}^2 h_2^2 - s_{23}^2 h_3^2) + \mathcal{O}(\theta_{13})$$

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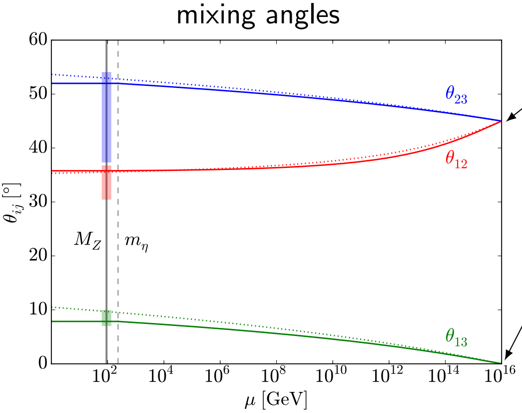
Results – top-down from $\Lambda = 10^{16}$ GeV

For $h_{ij} \sim \mathcal{O}(1)$; $M_{1,2,3} < m_\eta = 350$ GeV; bimaximal mixing:



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- likewise for masses
- $\Delta m^2_{21} \simeq 7.1 \times 10^{-5}\mathrm{eV}^2$ ✓
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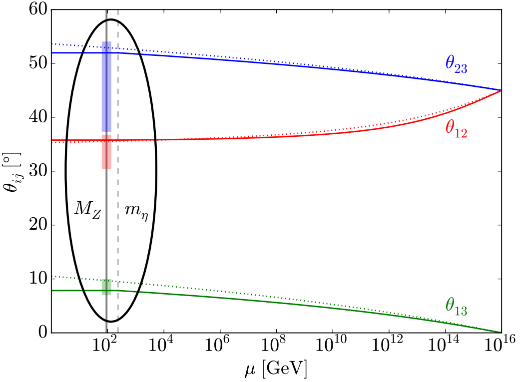

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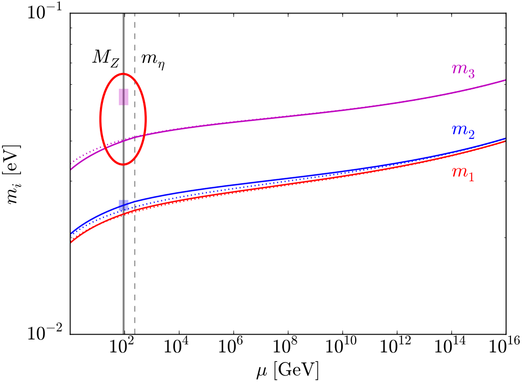

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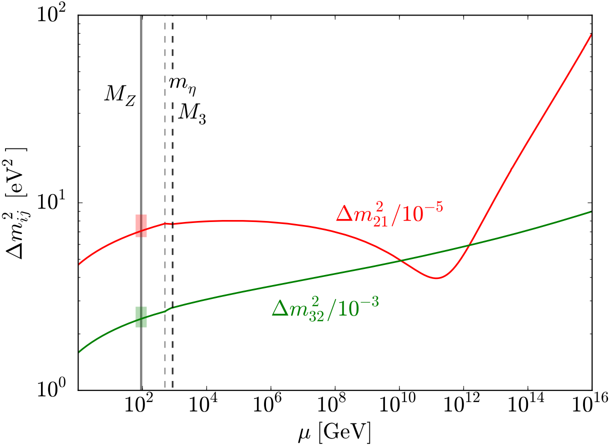
Relaxing the Assumptions on the Yukawas

Now: $m_\eta \approx M_{1,2,3} \approx 600$ GeV and $h^\dagger h$ arbitrary

mass square differences



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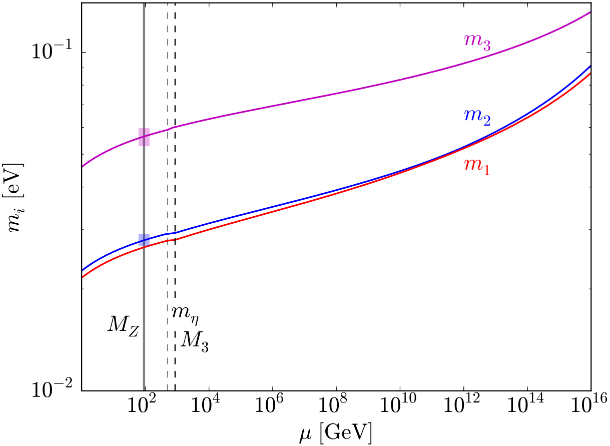

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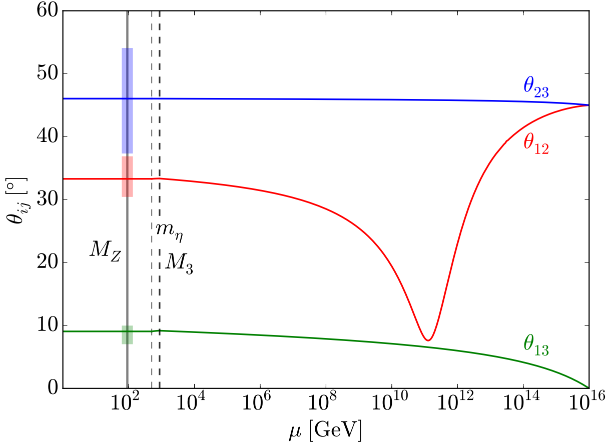
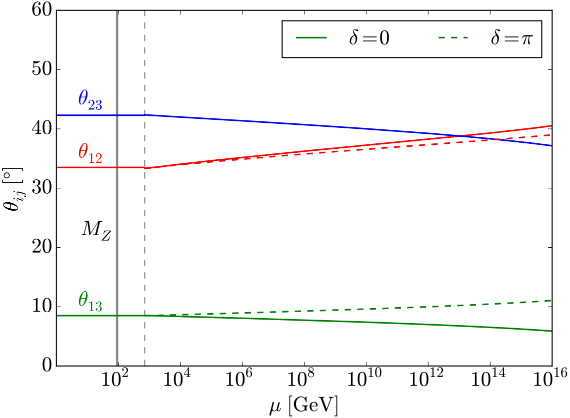


Results – bottom-up

$\phi_{1,2} = 0;\ m_\eta = M_{1,2,3} = 1$ TeV



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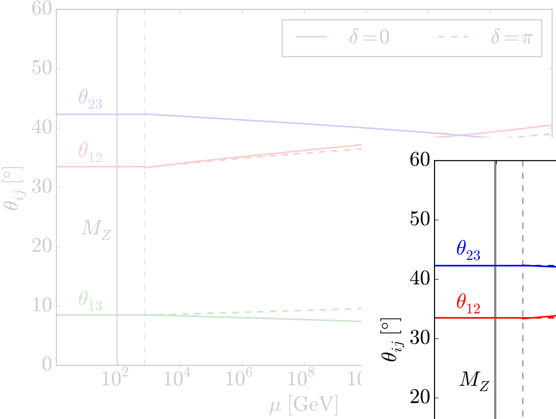
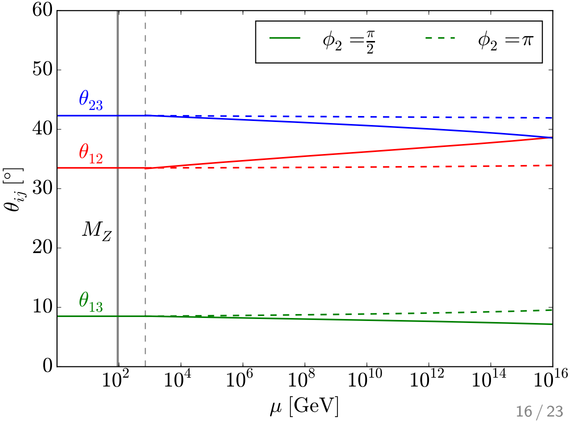


$$\frac{\left|m_2 e^{i\phi_2} + m_3\right|^2}{m_3^2 - m_2^2} \geq \frac{m_3 - m_2}{m_3 + m_2}$$

$\delta = \phi_1 = 0$



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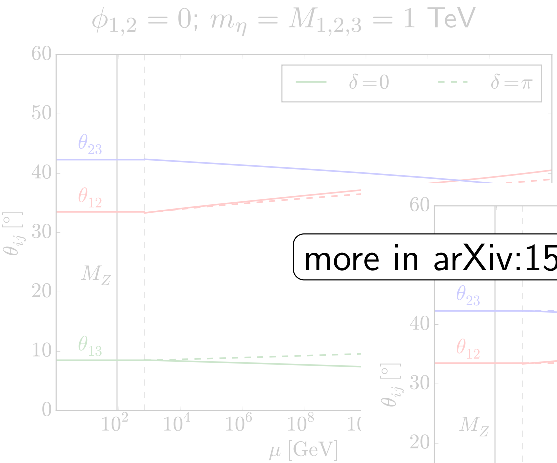
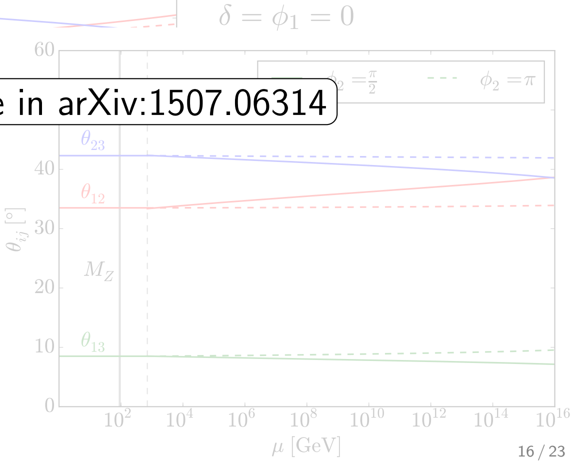




more in arXiv:1507.06314

The Parity Problem

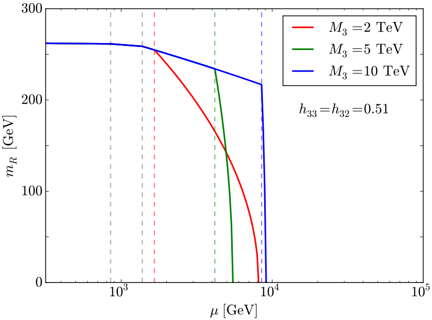
What is the Parity Problem?

- strong mass hierarchy $\left|m_H^2\right| \ll m_\eta^2 \ll M_{1,2,\dots}^2$
  $\Rightarrow$ hierarchy problem
- *fermionic* corrections have opposite sign!
  $\Rightarrow$ $m_\eta^2 < 0$ will cause spontaneous breaking of $\mathbb{Z}_2$ at some point

However:

$\mathbb{Z}_2$ is *crucial* because it guarantees small neutrino masses and the stability of DM!

The Parity Problem – Breaking Scale Λ



- Λ grows with m_η
- $\Lambda \lesssim$ TeV for small m_η

- breaking scale as low as ~ 5 TeV
- DM production?!

more in [A. Merle and MP, arXiv:1502.03098]

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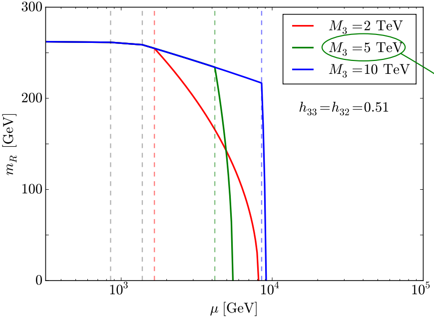

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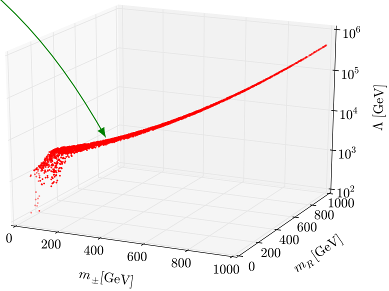

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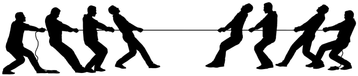
The Parity Problem – Analytical Understanding

EWSB (IR)



$\mathbb{Z}_2$ intact (UV)

$$m_H^2 < 0 \quad \xrightarrow{\text{small enough}} \quad m_\eta^2 \quad \xleftarrow{\text{large enough}} \quad m_\eta^2 \geq \frac{\lambda}{\lambda_1} m_H^2$$

$$\mathcal{D} m_H^2 = 6\lambda_1 m_H^2 + 2\left(2\lambda_3 + \lambda_4\right) m_\eta^2 + m_H^2 \left[2T - \frac{3}{2}\left(g_1^2 + 3g_2^3\right)\right]$$

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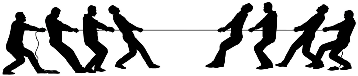

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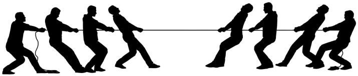
EWSB (IR)  $\mathbb{Z}_2$ intact (UV)

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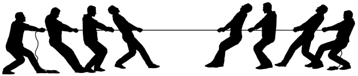
EWSB (IR)  $\mathbb{Z}_2$ intact (UV)

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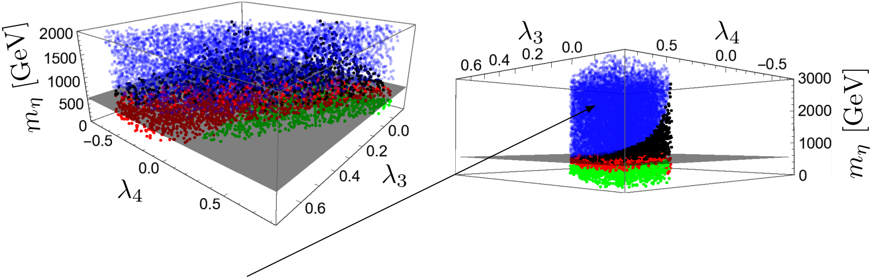
analytically: $$\log \frac{\Lambda}{M_Z} = \frac{4\pi^2 m_\eta^2(M_Z)}{M^2 h^2(M_Z)}$$

Keeping $\mathbb{Z}_2$ intact up to 10^{16} GeV ($M_{1,2,3} \sim 1$ TeV)



Color code: EWSB , valid up to 10^{16} GeV , SSB of $\mathbb{Z}_2$ (+ el. charge)

- for all valid points $m_\eta > 550$ GeV
- physical scalar masses:

$$m_\pm \geq 554 \text{ GeV} \ \& \ m_{R/I} \geq 559 \text{ GeV}$$

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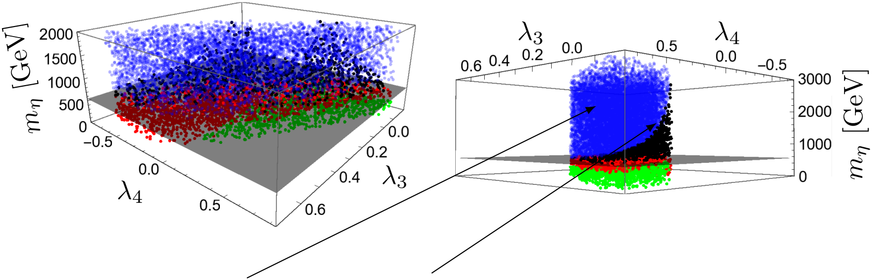

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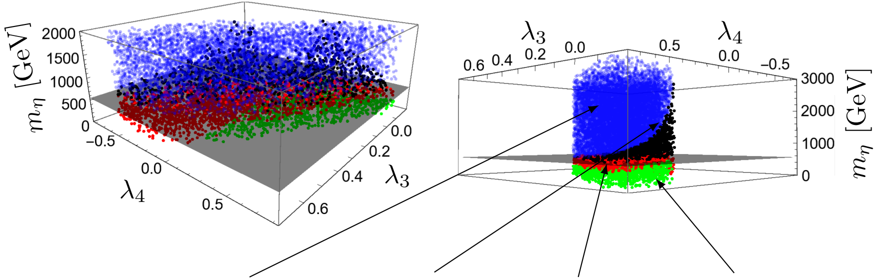

Color code: EWSB , valid up to 10^{16} GeV , SSB of $\mathbb{Z}_2$ (+ el. charge)

- for all valid points $m_\eta > 550$ GeV
- physical scalar masses:

$$m_\pm \geq 554 \text{ GeV } \& \ m_{R/I} \geq 559 \text{ GeV}$$

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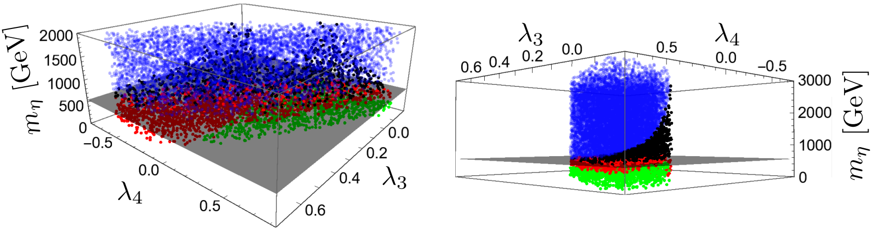

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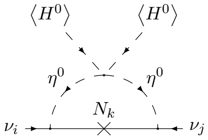
Summary:

- scotogenic model is a paradigm for radiative ν mass
  - $\rightarrow$ LNV, loop suppression and scale suppression
- RG running in such a model can be significant
  - $\rightarrow$ flavor symmetries, collider vs. low-energy data ...
- parity problem is a hierarchy-type problem
  - $\rightarrow$ mass scales cannot be too far apart, light RH neutrinos?!



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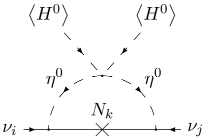

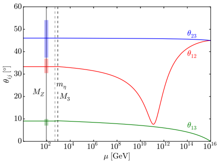

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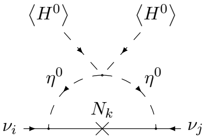

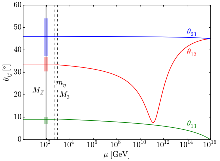

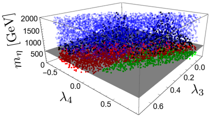

Thank you!