Exploring the phase structure and dynamics of QCD

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Universität Heidelberg & ExtreMe Matter Institute

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GEFÖRDERT VOM
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EMMI

Outline

- Introduction
- Phase structure of QCD
- Hadron spectrum & QCD transport
- Outlook

Outline

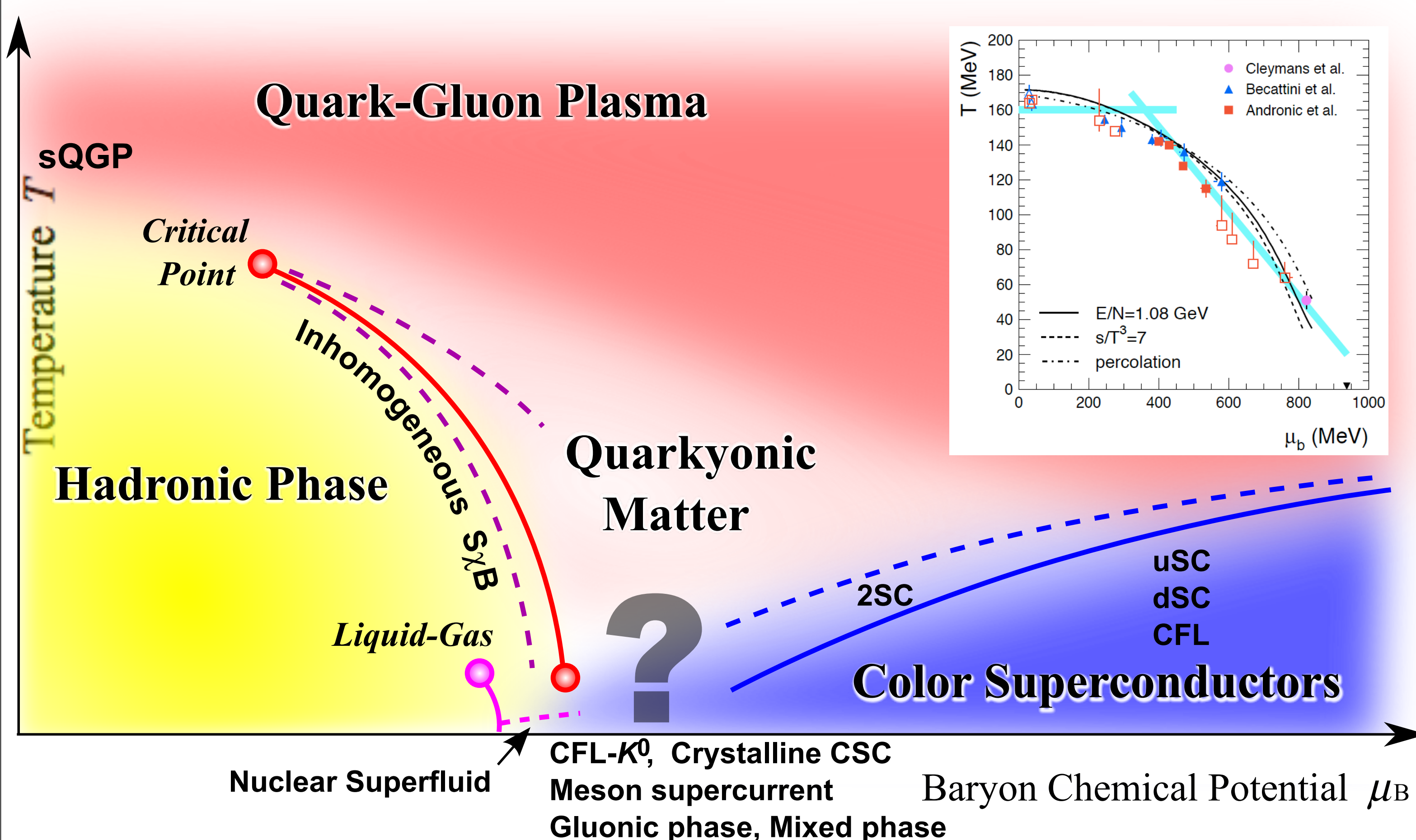
- **Introduction**

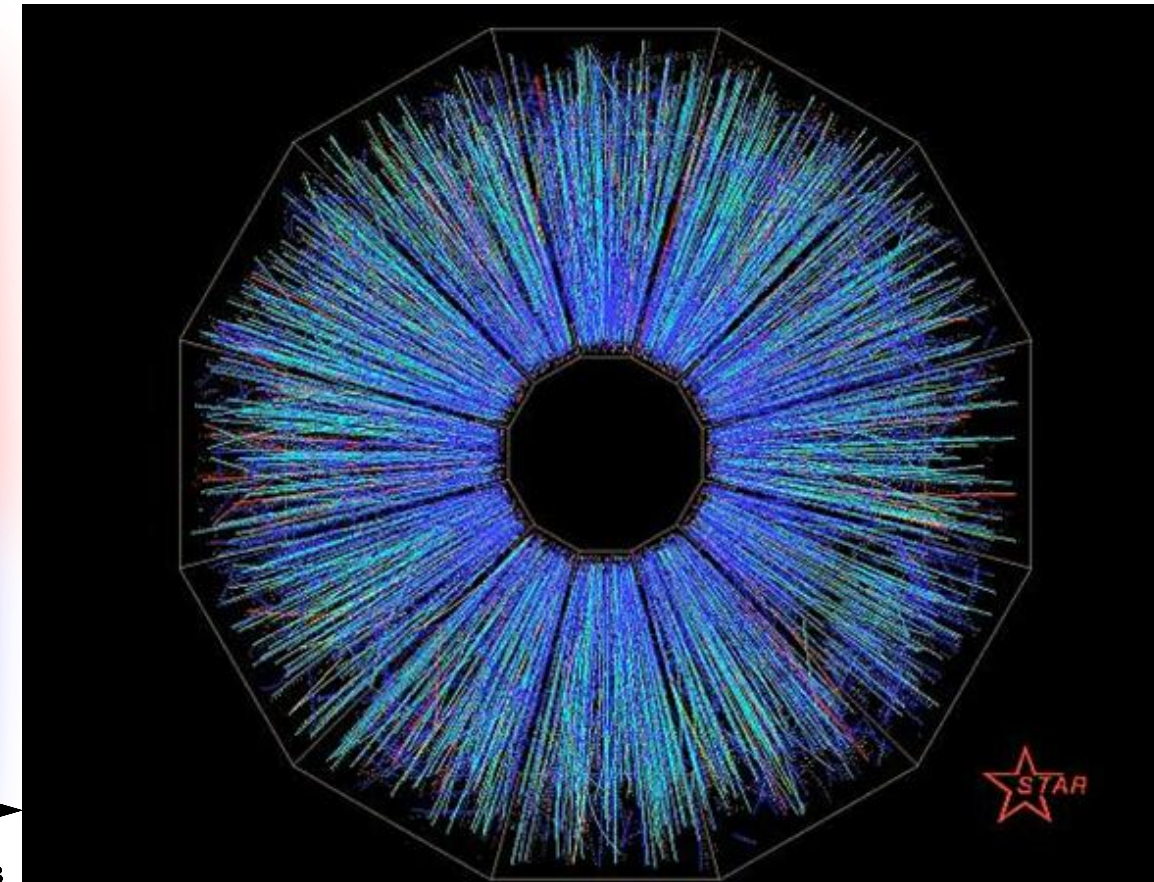
- **Phase structure of QCD**

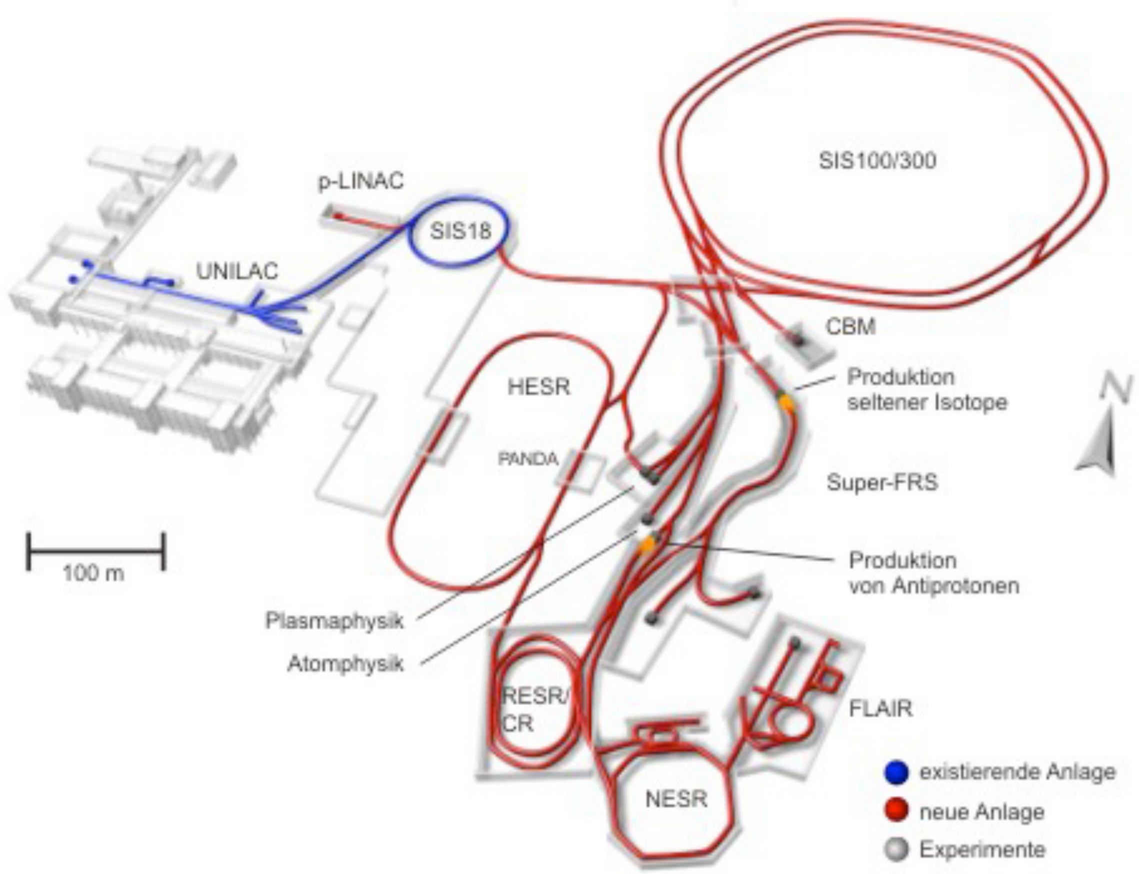
- **Hadron spectrum & QCD transport**

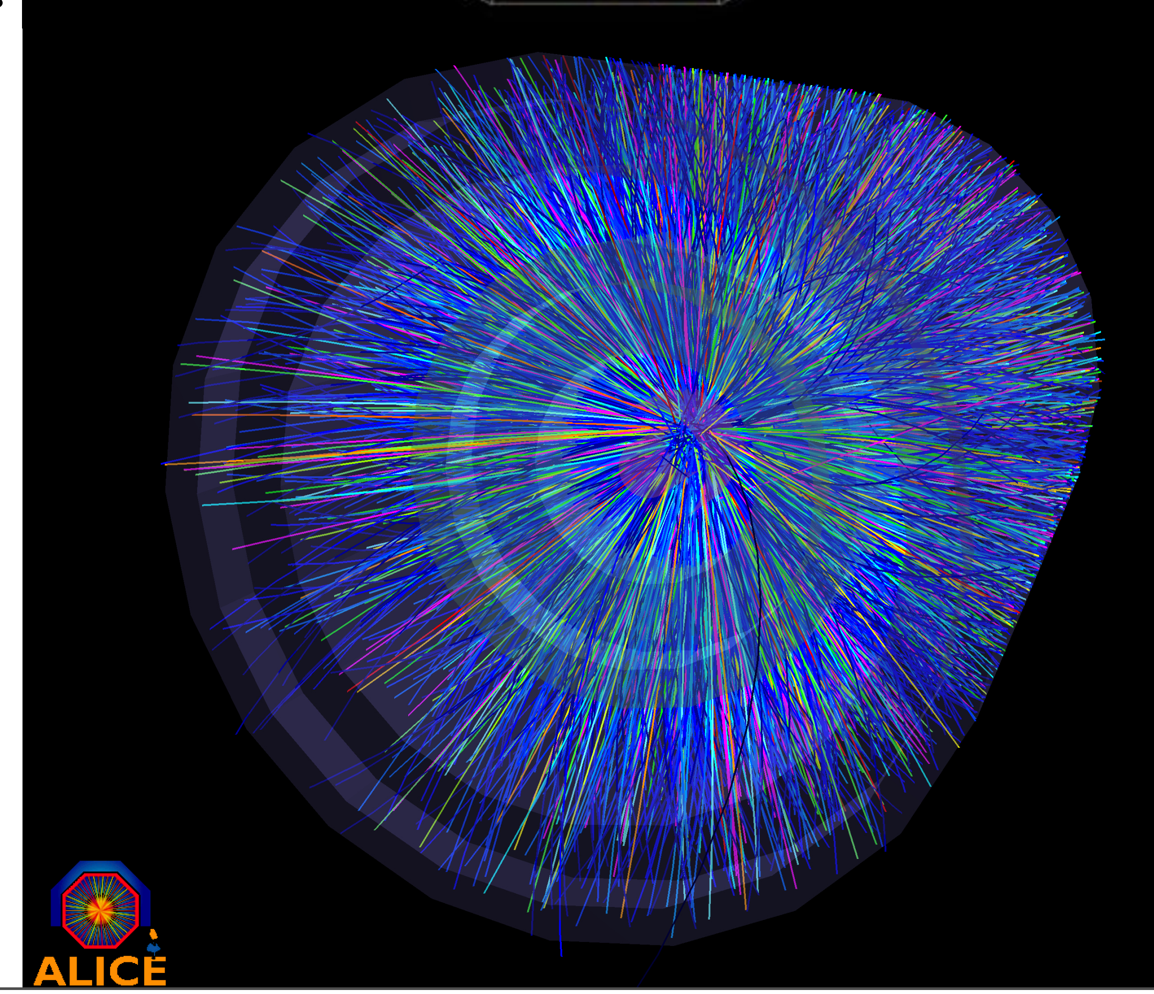
- **Outlook**

Heavy ion collisions

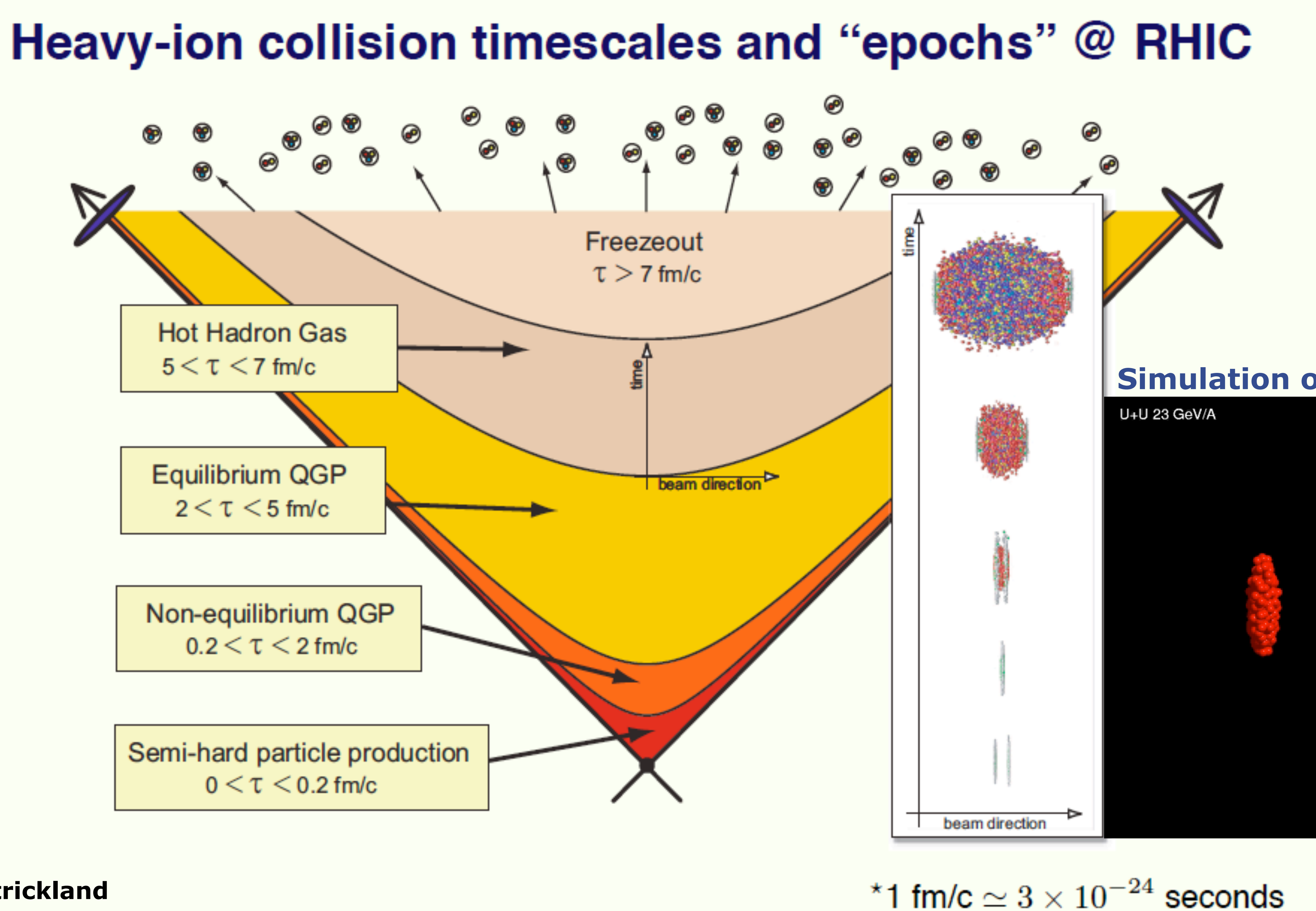






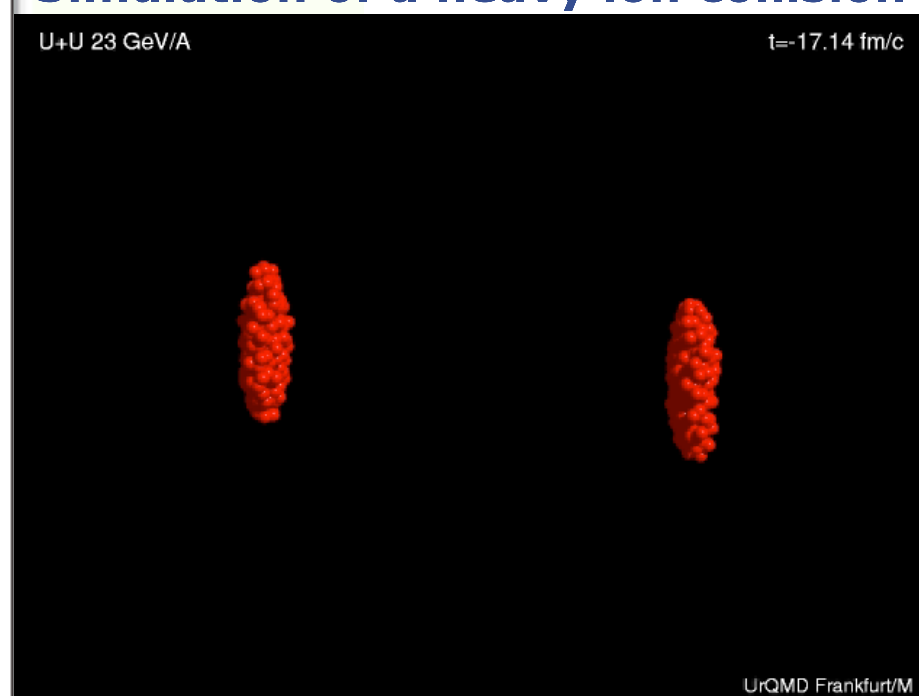


Heavy ion collisions



Strickland

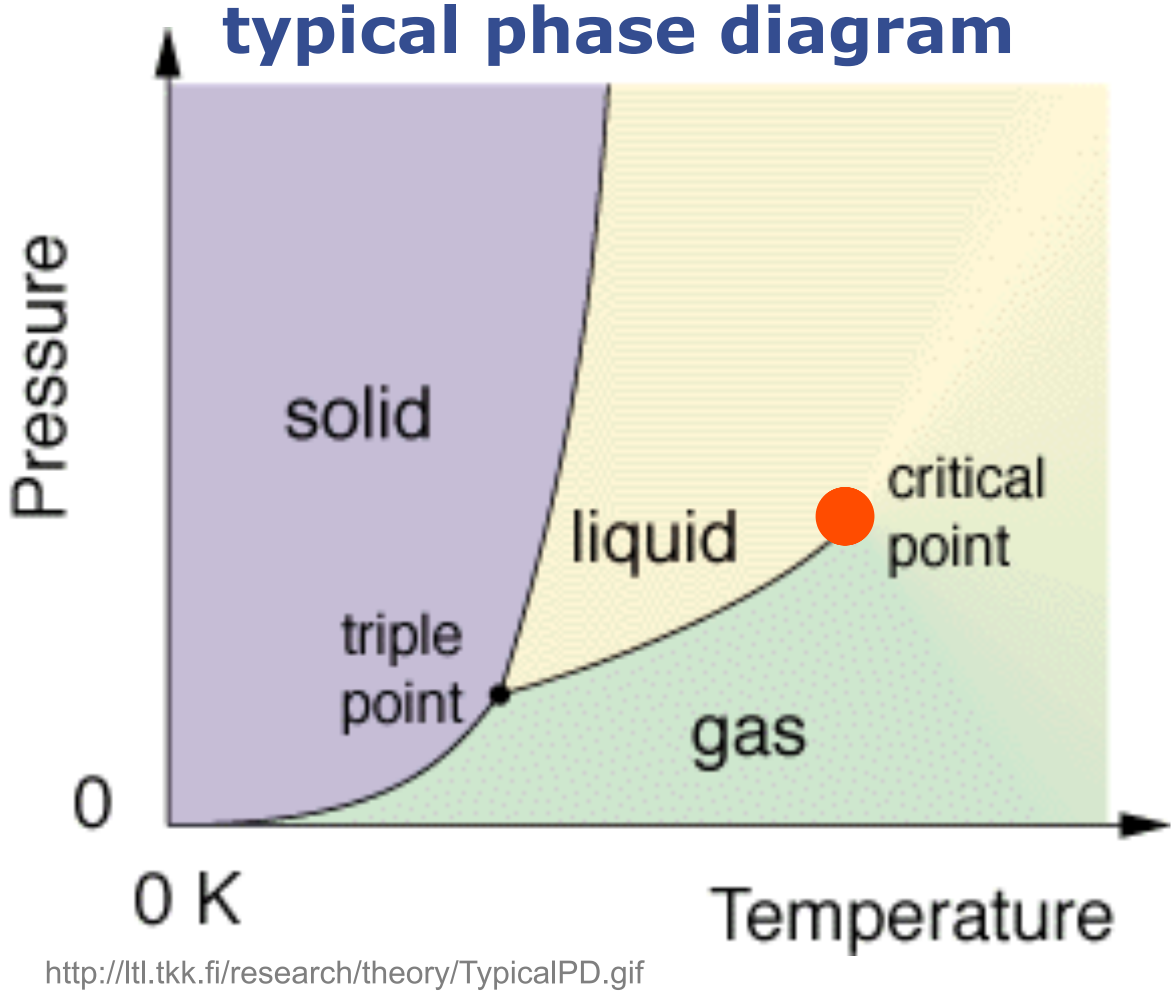
Simulation of a heavy ion collision



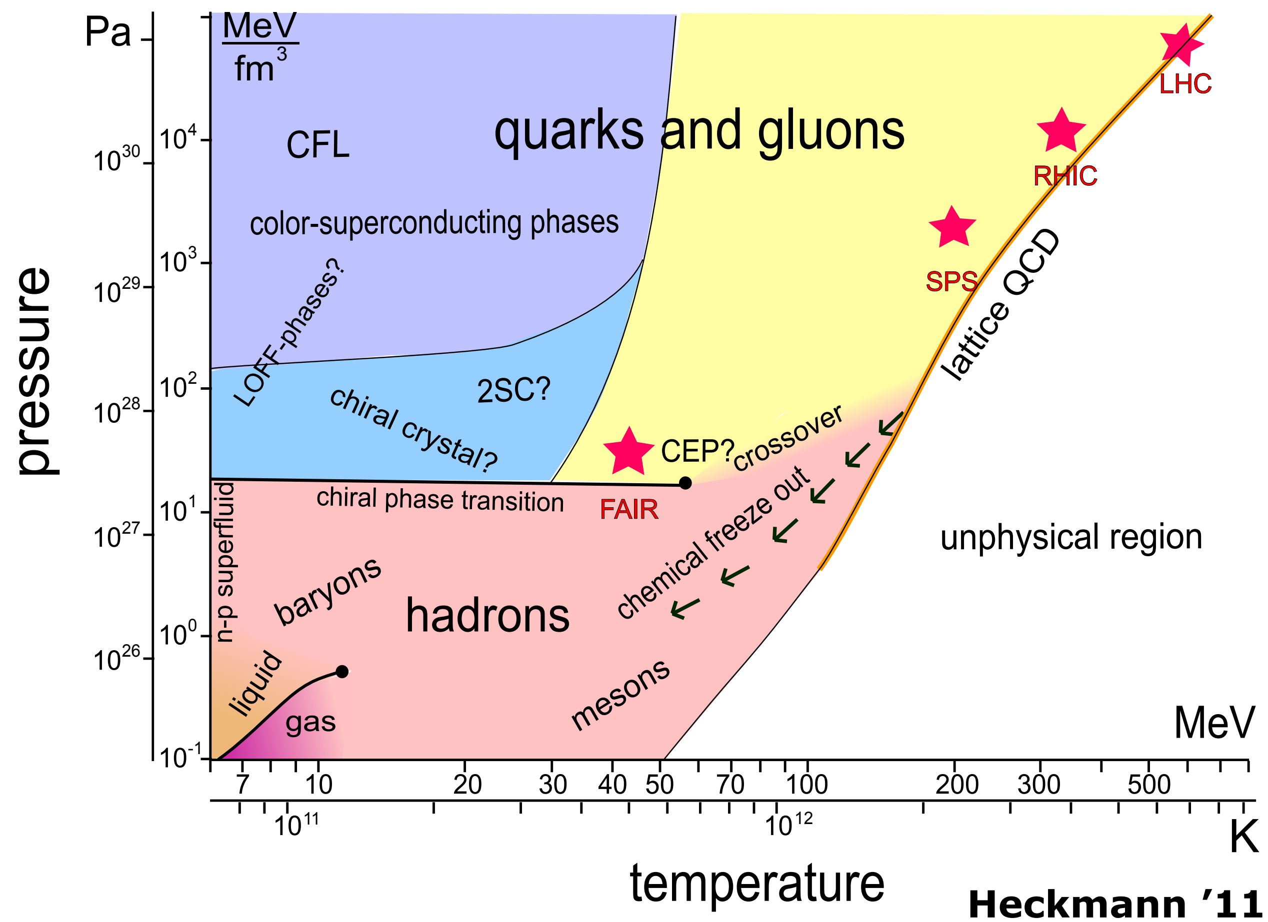
UrQMD Frankfurt/M

Phase diagrams & order parameters

typical phase diagram



phase diagram of QCD



Phases in QCD

quarks massless - massive

chiral condensate $\int_{\vec{x}} < \bar{\mathbf{q}}(\mathbf{x})\mathbf{q}(\mathbf{x}) >$

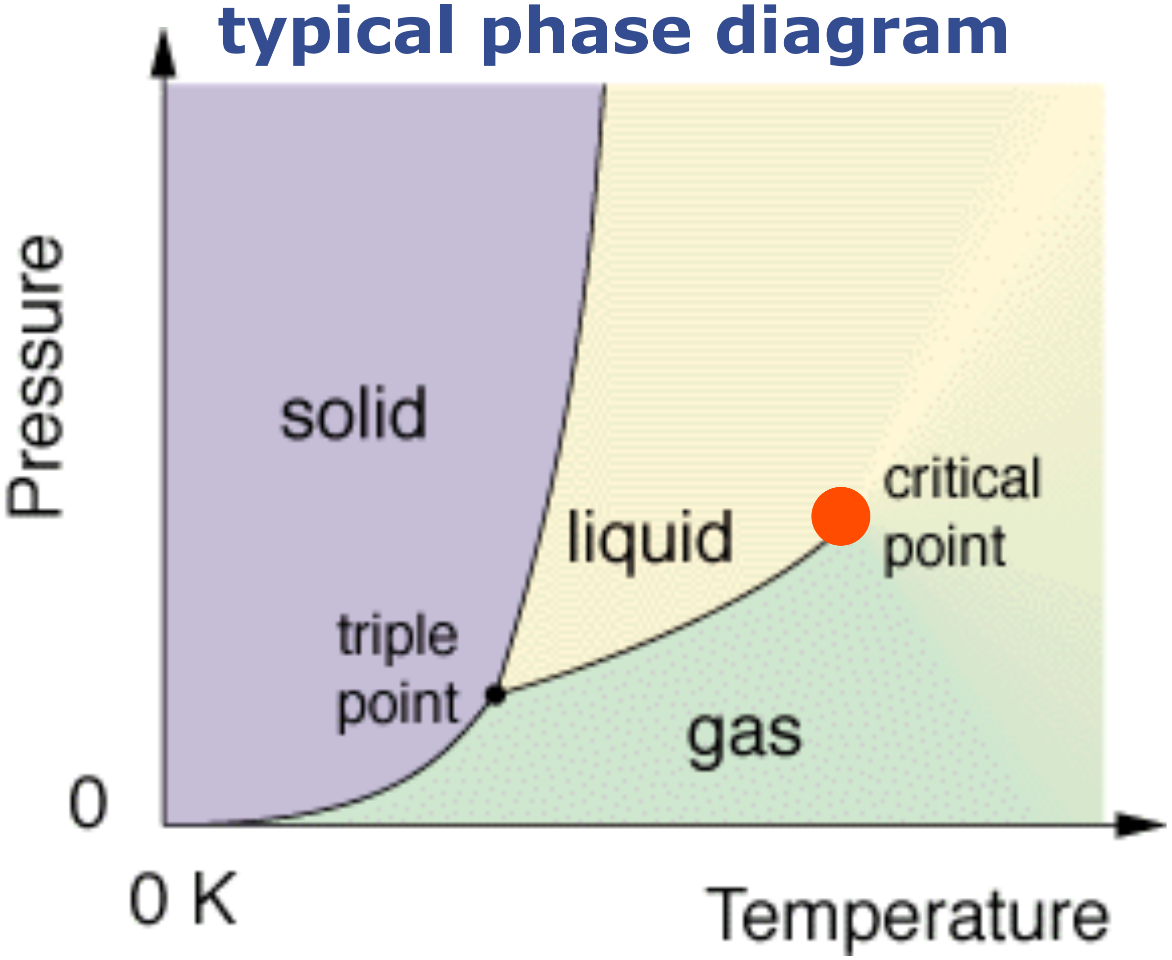
quarks confined - deconfined

Polyakov loop $\mathbf{\Phi} \sim \mathbf{e}^{-\frac{1}{2}\mathbf{F}_{\bar{\mathbf{q}}\mathbf{q}}}$

free energy $\mathbf{F}_{\bar{\mathbf{q}}\mathbf{q}} = \lim_{|\vec{\mathbf{x}}-\vec{\mathbf{y}}|\to\infty} \mathbf{F}_{\bar{\mathbf{q}}(\mathbf{x})\mathbf{q}(\mathbf{y})}$

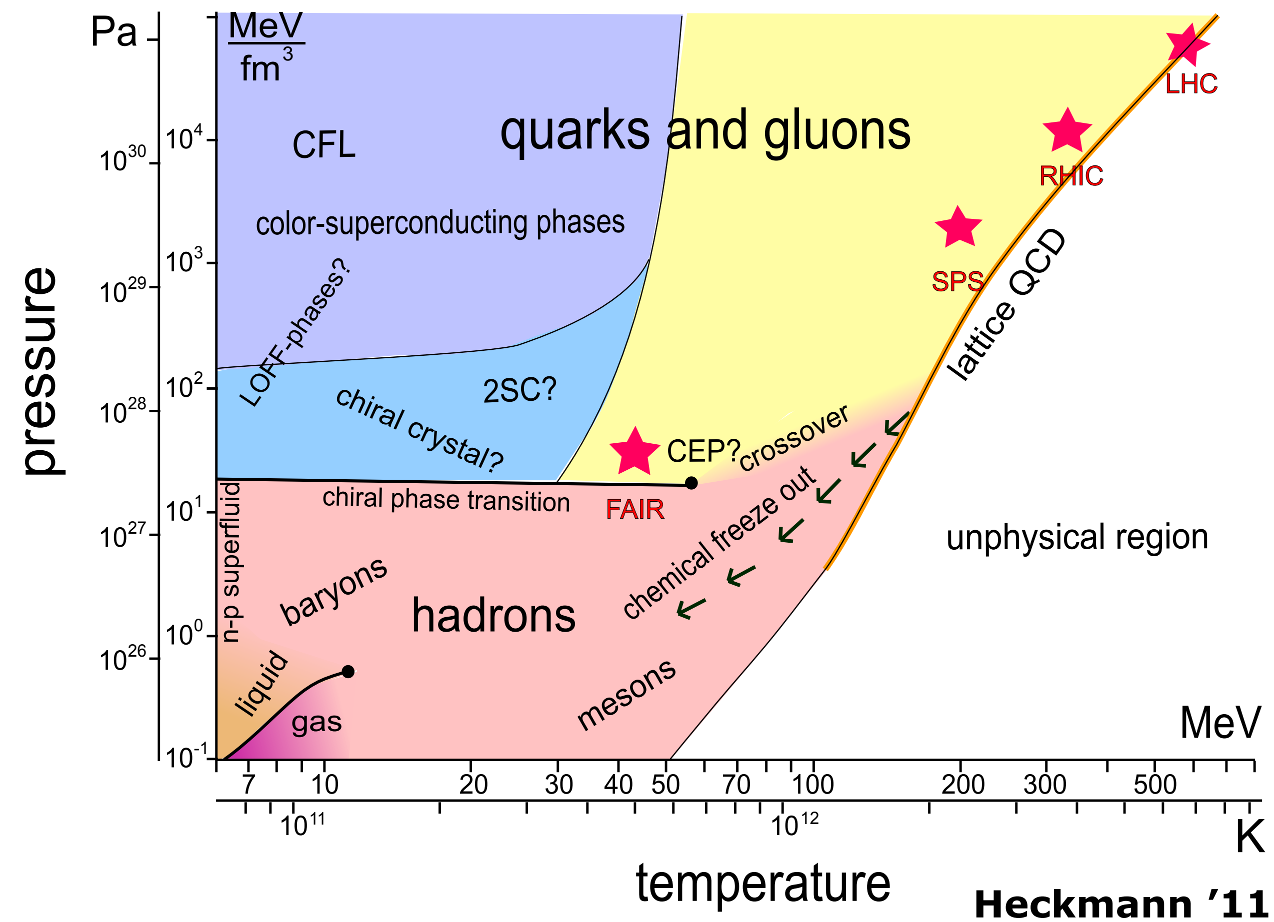
Phase diagrams & order parameters

typical phase diagram



http://ltl.tkk.fi/research/theory/TypicalPD.gif

phase diagram of QCD



Heckmann '11

Phases in QCD

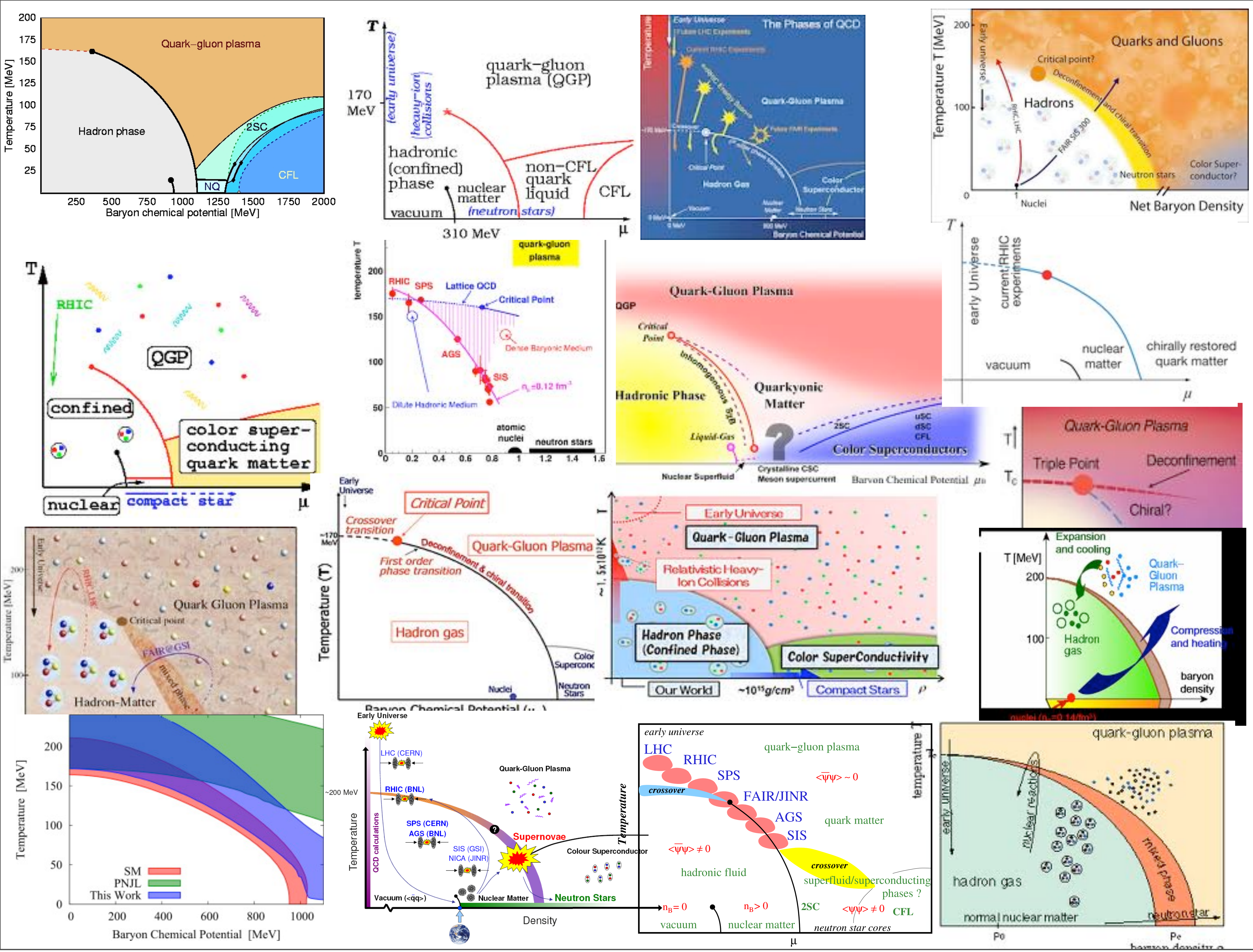
quarks massless - massive

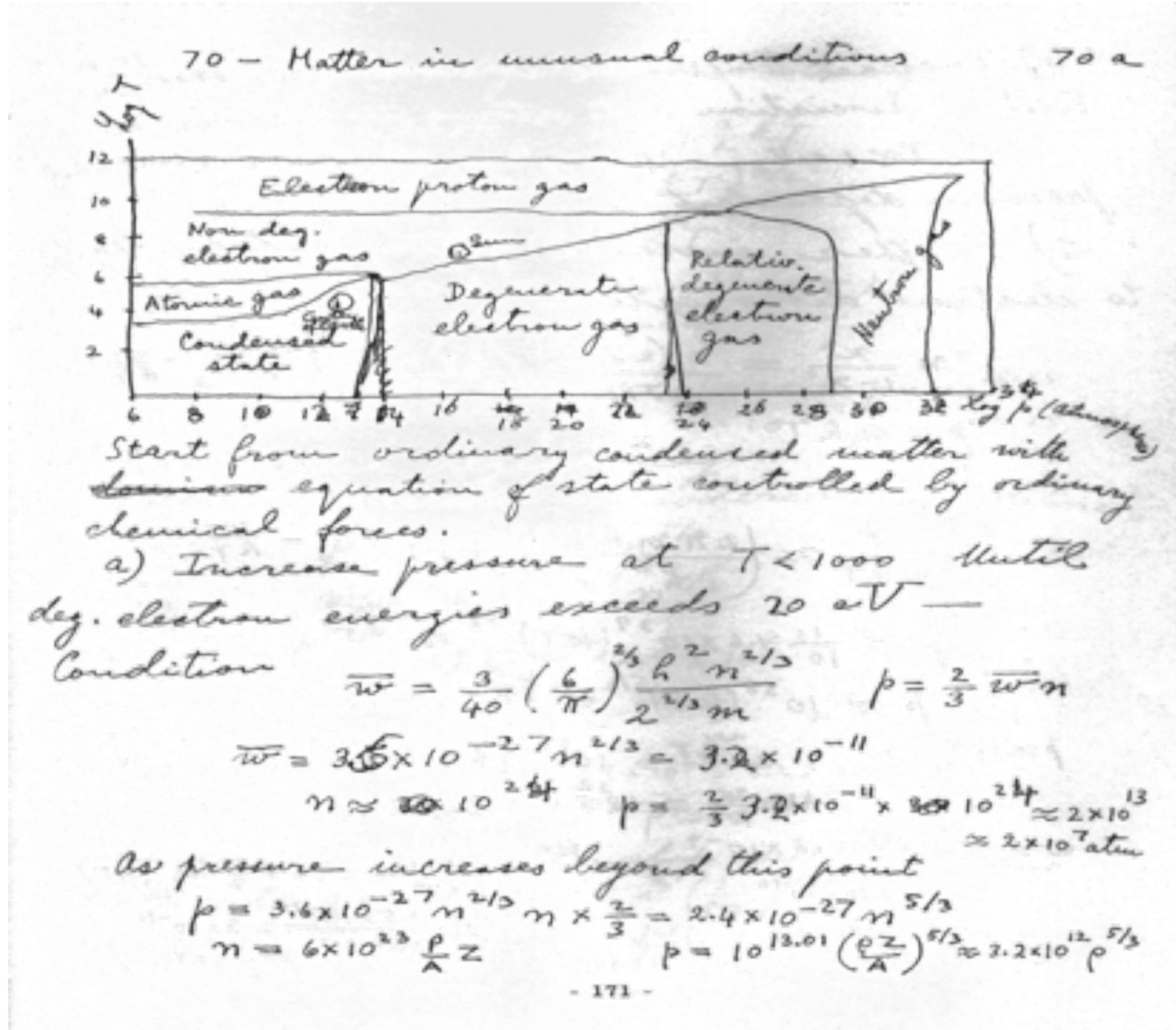
chiral condensate $\int_{\vec{x}} < \bar{q}(x) q(x) >$

quarks confined - deconfined

Polyakov loop $\Phi = \frac{1}{N_c} \langle \mathrm{tr}\, \mathcal{P} e^{i g \int_0^\beta A_0(x)} \rangle$

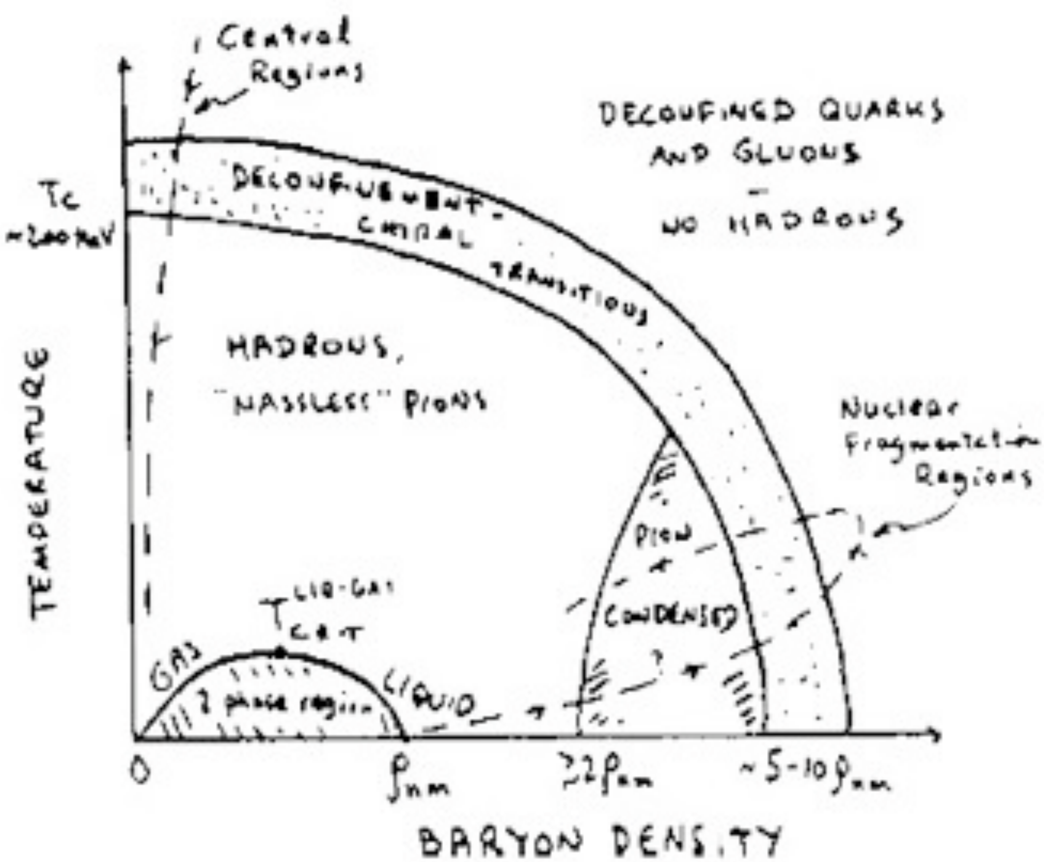
free energy $F_{\bar{q}q} = \lim_{|\vec{x}-\vec{y}| \to \infty} F_{\bar{q}(x) q(y)}$


Quark–gluon plasma
Hadron phase
2SC
NQ
CFL
Temperature [MeV]
Baryon chemical potential [MeV]
quark–gluon plasma (QGP)
170 MeV
early universe
heavy-ion collisions
hadronic (confined) phase
nuclear matter
non-CFL quark liquid
CFL
vacuum
(neutron stars)
310 MeV
The Phases of QCD
Quark-Gluon Plasma
Hadron Gas
Critical Point
Color Superconductor
Baryon Chemical Potential
Quarks and Gluons
Critical point?
Hadrons
Deconfinement and chiral transition
Neutron stars
Color Super-conductor?
Nuclei
Net Baryon Density
Temperature T [MeV]
RHIC
QGP
confined
color super-conducting quark matter
nuclear
compact star
quark-gluon plasma
RHIC
SPS
Lattice QCD
Critical Point
AGS
SIS
Dense Baryonic Medium
Dilute Hadronic Medium
atomic nuclei
neutron stars
Quark-Gluon Plasma
Critical Point
Hadronic Phase
Quarkyonic Matter
Liquid-Gas
2SC
uSC
dSC
CFL
Color Superconductors
Nuclear Superfluid
Crystalline CSC
Meson supercurrent
Baryon Chemical Potential
early Universe
current RHIC experiments
vacuum
nuclear matter
chirally restored quark matter
Quark-Gluon Plasma
Triple Point
Deconfinement
Chiral?
Early Universe
Critical Point
Crossover transition
Quark-Gluon Plasma
First order phase transition
Deconfinement & chiral transition
Hadron gas
Temperature (T)
Color Superconductor
Nuclei
Neutron Stars
Baryon Chemical Potential
Early Universe
Quark-Gluon Plasma
Relativistic Heavy-Ion Collisions
Hadron Phase (Confined Phase)
Color SuperConductivity
Our World
~10¹⁵g/cm³
Compact Stars
Expansion and cooling
T [MeV]
Quark-Gluon Plasma
Hadron gas
Compression and heating
baryon density
Quark Gluon Plasma
Critical point
FAIR@GSI
Hadron-Matter
mixed phase
SM
PNJL
This Work
Temperature [MeV]
Baryon Chemical Potential [MeV]
Early Universe
LHC (CERN)
RHIC (BNL)
SPS (CERN)
AGS (BNL)
SIS (GSI)
NICA (JINR)
Quark-Gluon Plasma
Supernovae
Colour Superconductor
QCD calculations
Vacuum
Nuclear Matter
Neutron Stars
Temperature
Density
early universe
LHC
RHIC
SPS
FAIR/JINR
AGS
SIS
quark–gluon plasma
crossover
quark matter
hadronic fluid
superfluid/superconducting phases ?
2SC
CFL
vacuum
nuclear matter
neutron star cores
quark-gluon plasma
early universe
nuclear reactions
hadron gas
mixed phase
normal nuclear matter
neutron star

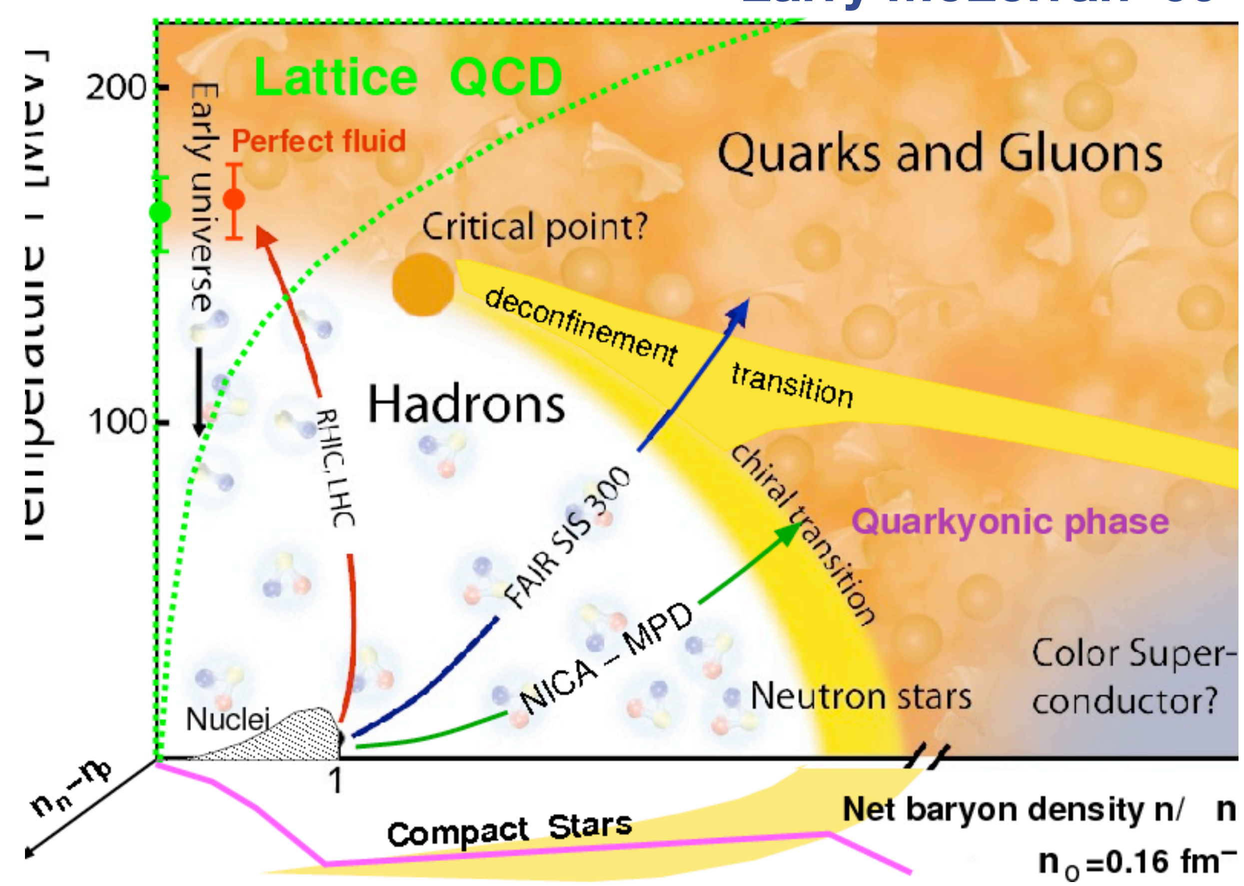


1953 Enrico Fermi

1983 US long range plan, Gordon Baym



Larry McLerran '09



Outline

- Functional Approaches to QCD & the FRG
- Phase structure of QCD
- Hadron spectrum & QCD transport
- Outlook

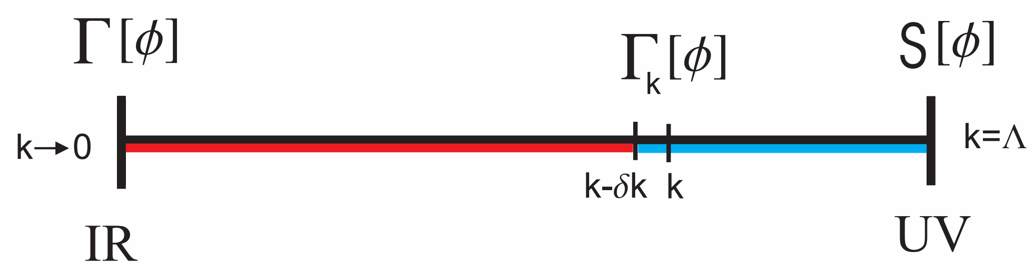
Functional RG for QCD

JMP, AIP Conf.Proc. 1343 (2011)
Nucl.Phys. A931 (2014) 113

Phase diagram survey

JMP, Schladming ’13

free energy at momentum scale k

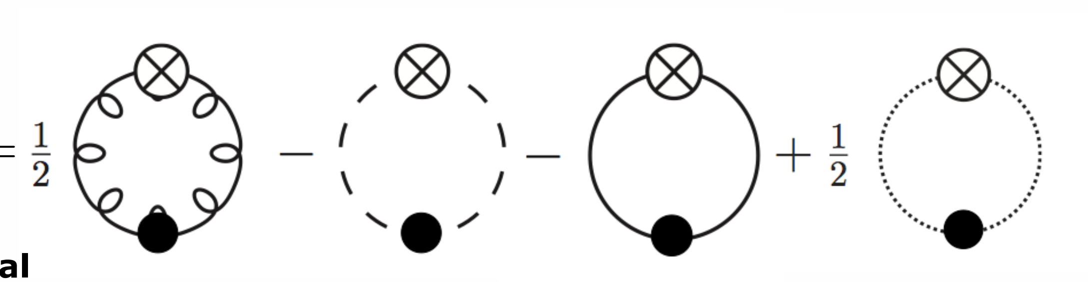


ab initio

glue quantum fluctuations

hadronic quantum fluctuations

$\partial_t \Gamma_k[\phi] = \frac{1}{2}$



free energy/ grand potential

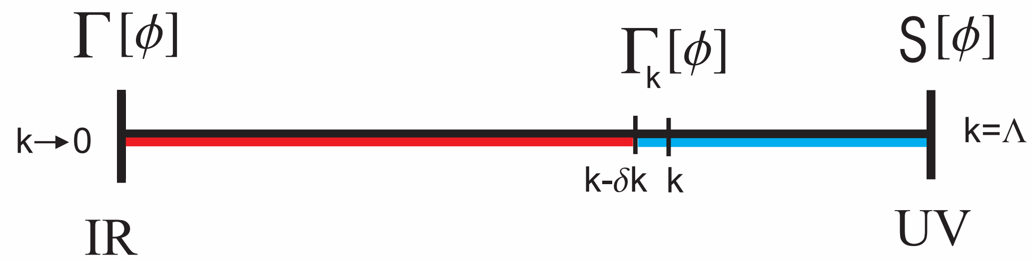
quark quantum fluctuations

RG-scale k: $t = \ln k$

Functional RG for QCD

JMP, AIP Conf.Proc. 1343 (2011)
Nucl.Phys. A931 (2014) 113

free energy at momentum scale k



Phase diagram survey

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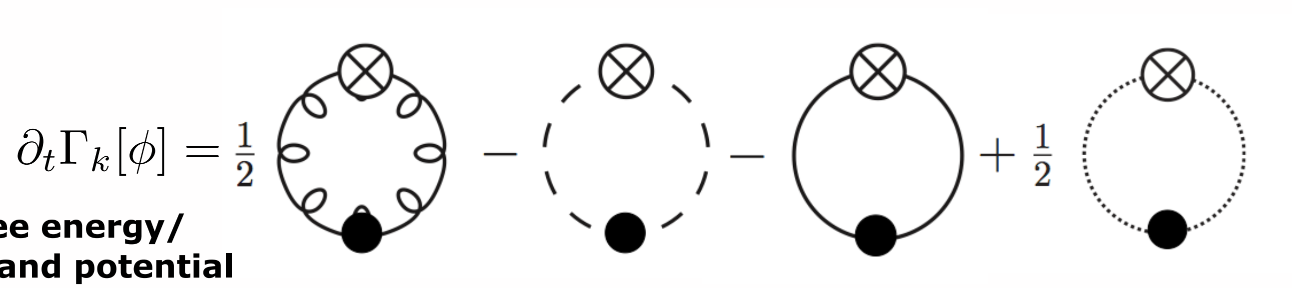

free energy/ grand potential

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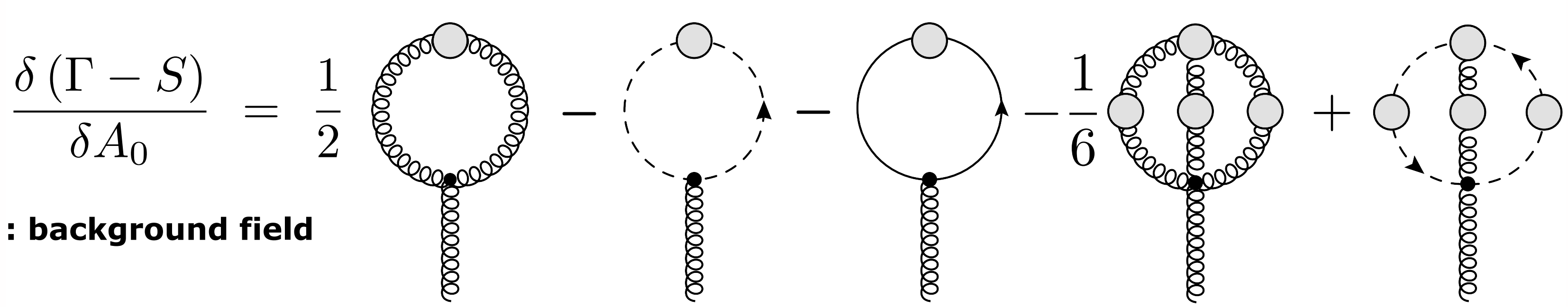
$\text{RG-scale k:}\ t = \ln k$

functional DSE :

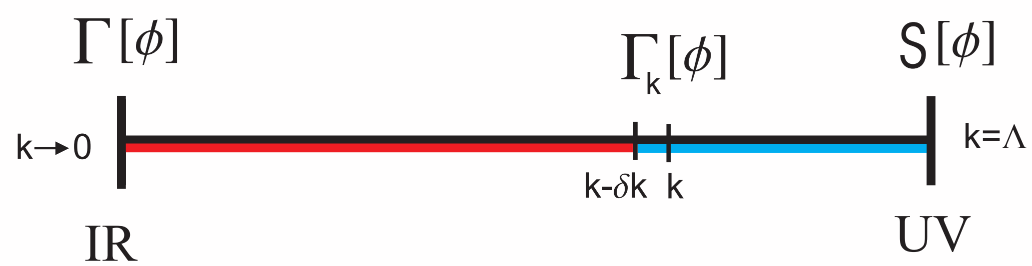


A_0 **: background field**

Functional RG for QCD

JMP, AIP Conf.Proc. 1343 (2011)
Nucl.Phys. A931 (2014) 113

free energy at momentum scale k



Phase diagram survey

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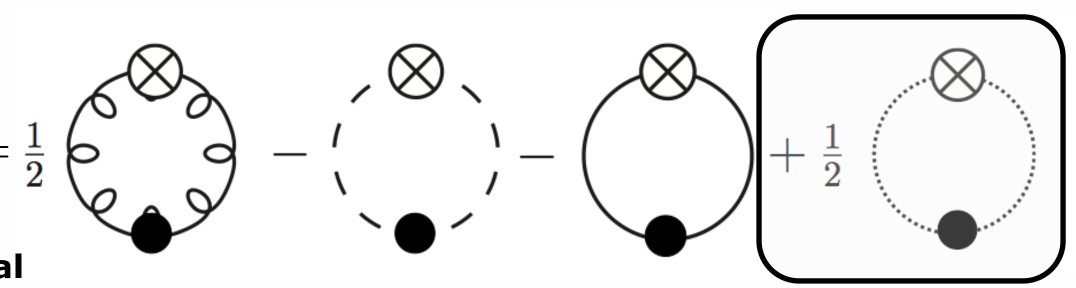

free energy/ grand potential

quark quantum fluctuations

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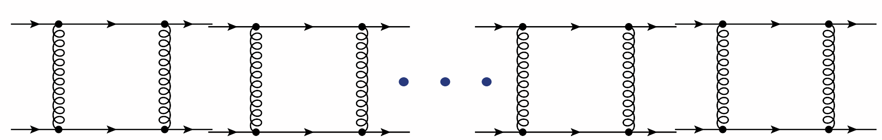
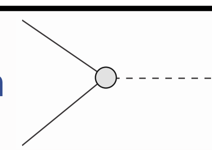


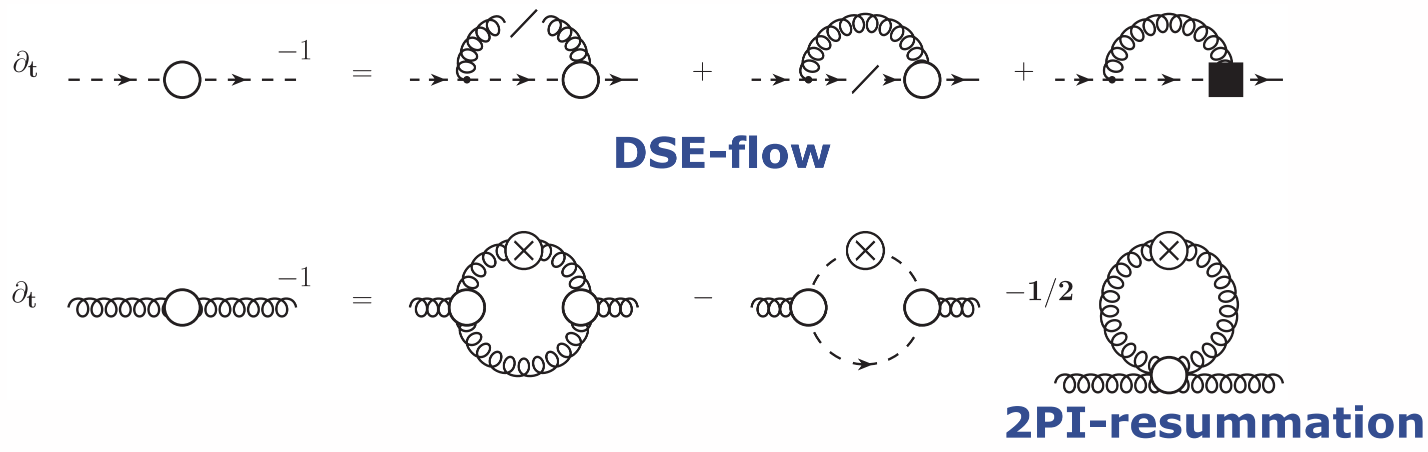
Dynamical hadronisation

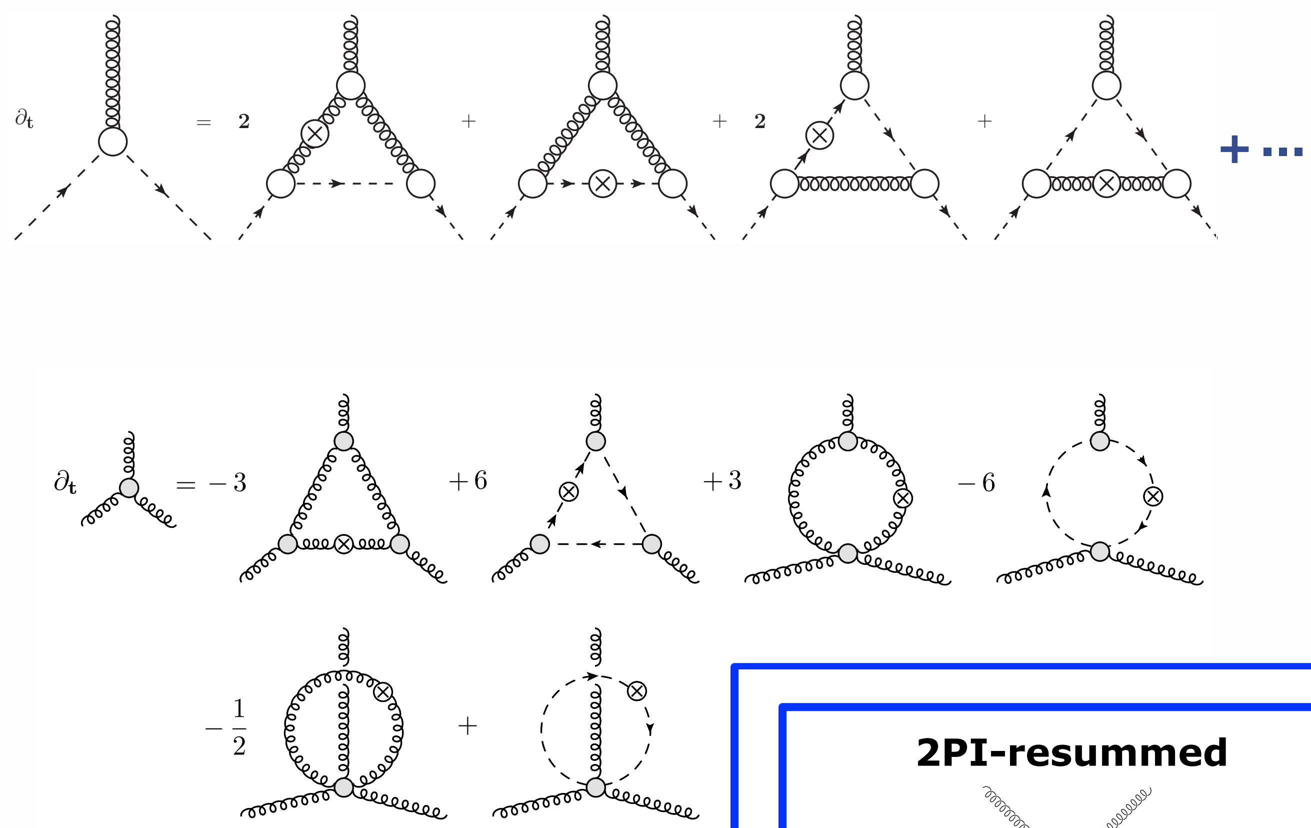


dynamical

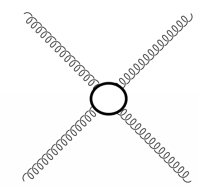
Gies, Wetterich '01
JMP '05
Flörchinger, Wetterich '09

Glue sector



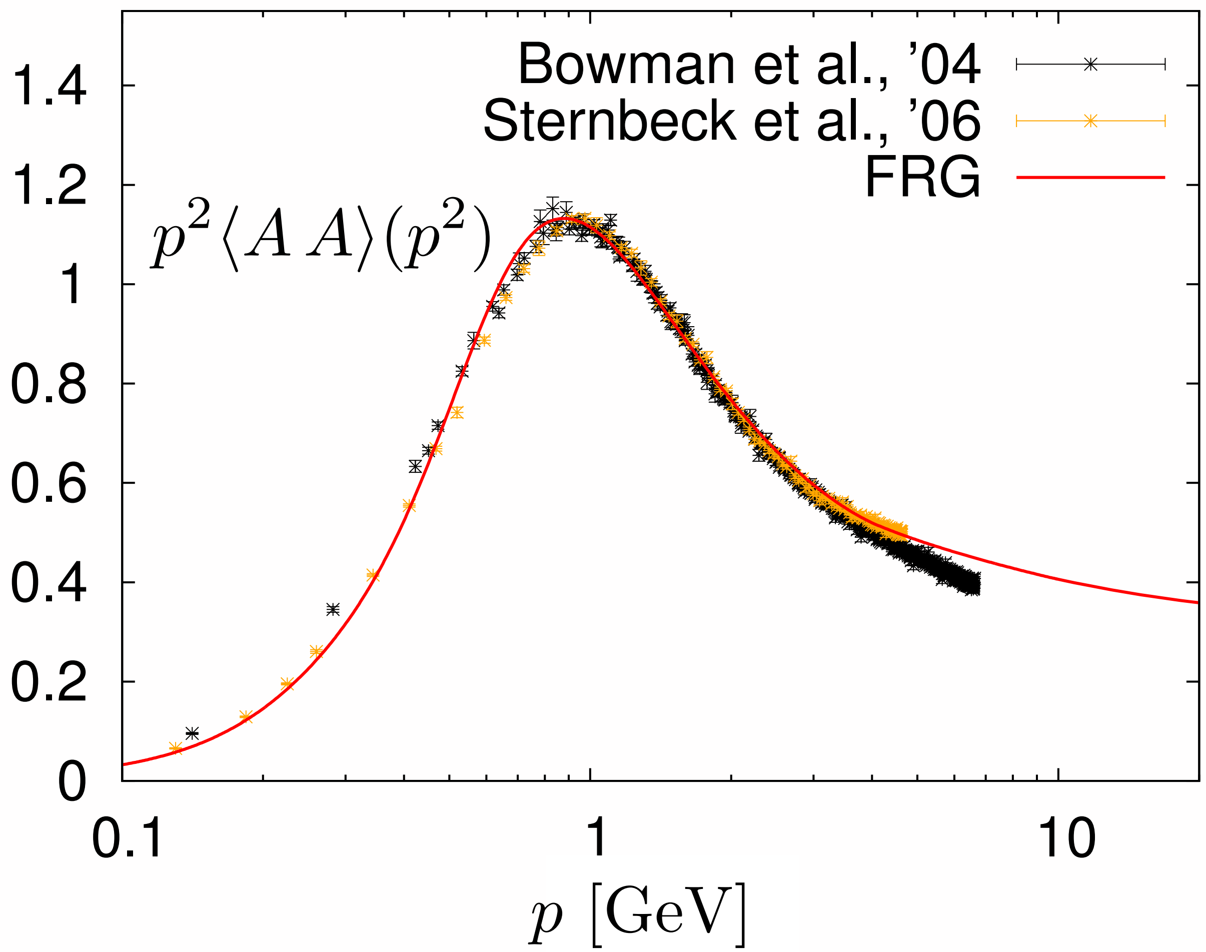


2PI-resummed

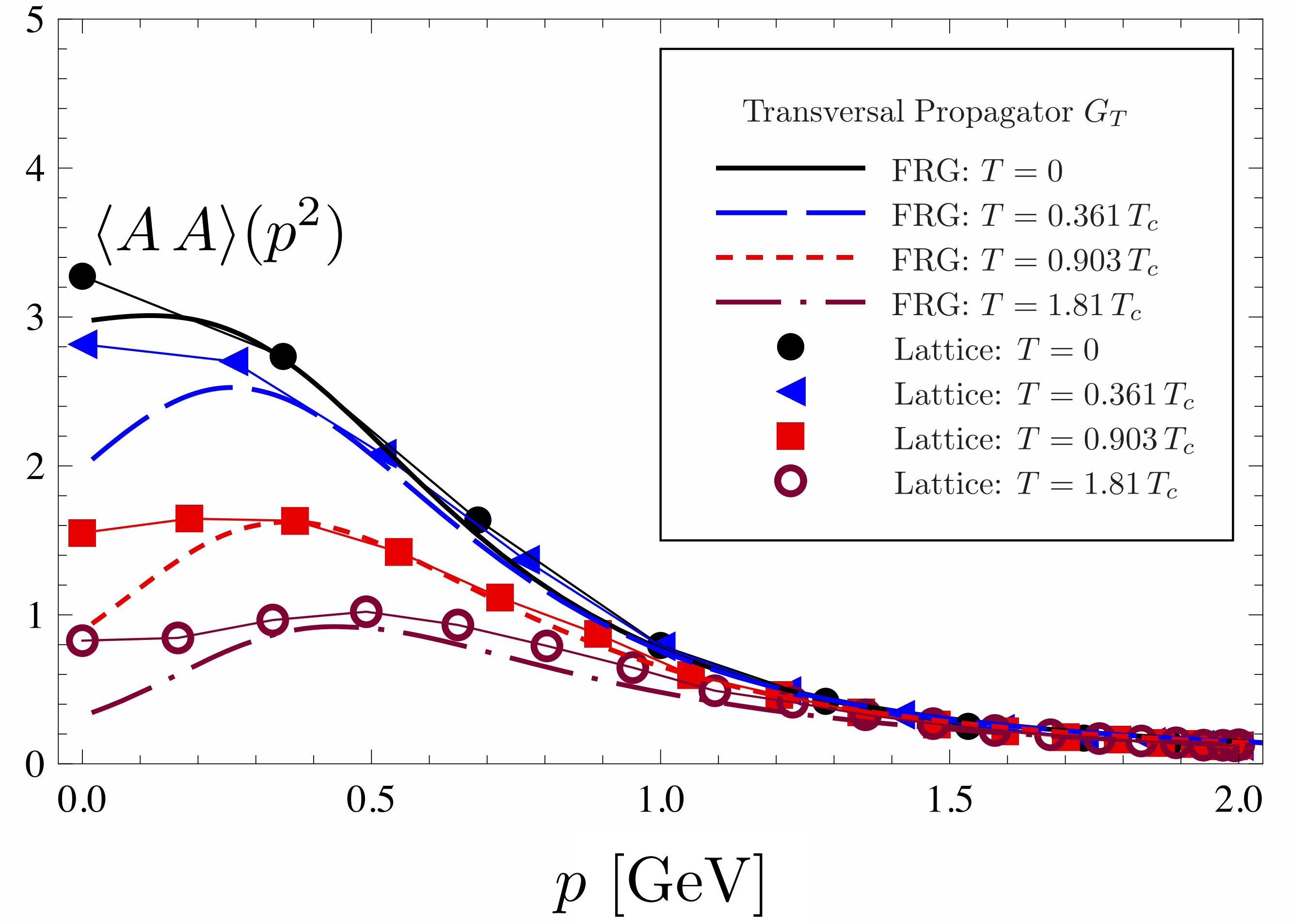


Glue sector

Yang-Mills propagators, T=0



Yang-Mills propagators, finite T



Fischer, Maas, JMP, Annals Phys. 324 (2009) 2408

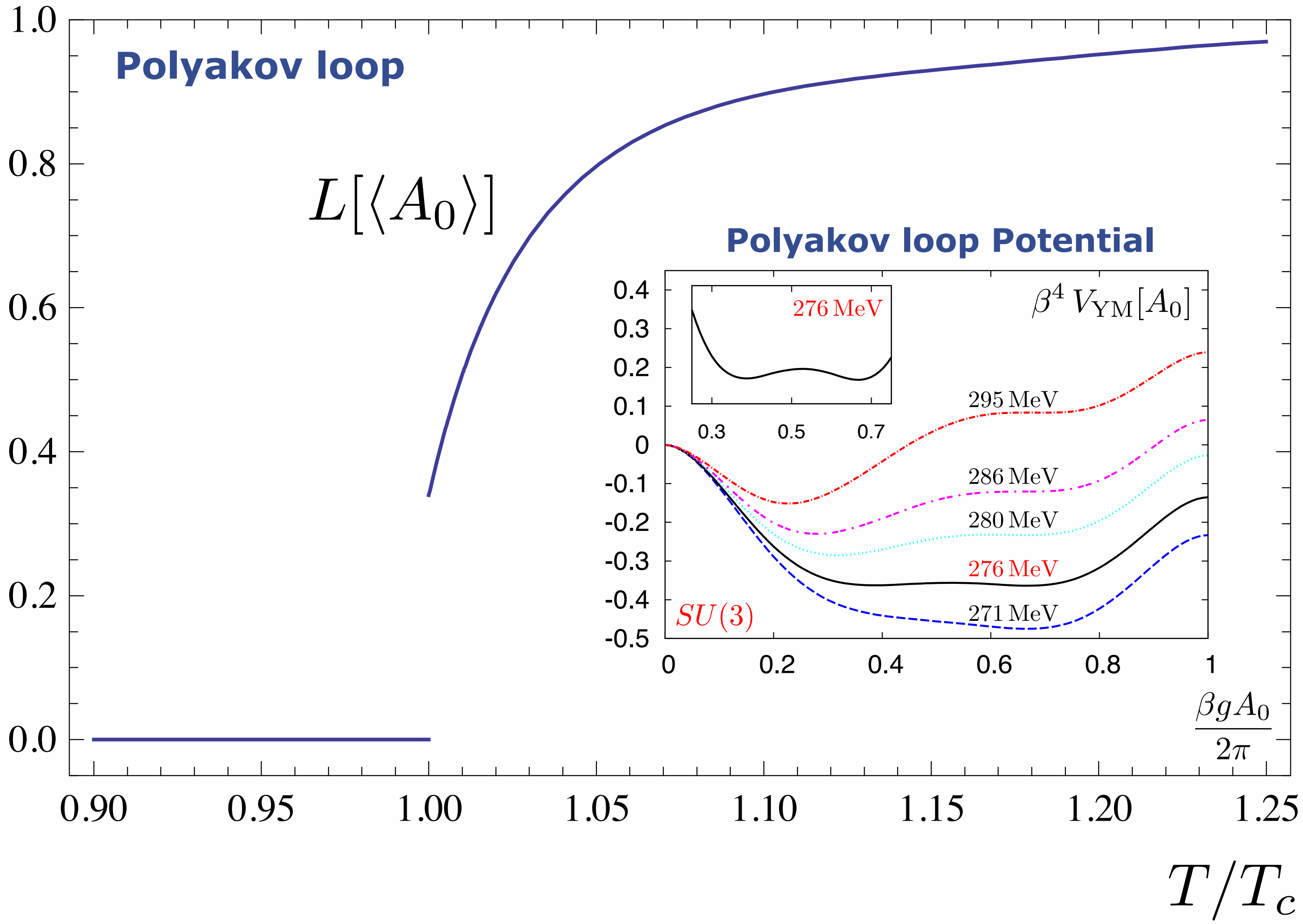
Fister, JMP '14

Fister, JMP, arXiv:1112.5440

Confinement

FRG: Braun, Gies, JMP, PLB 684 (2010) 262

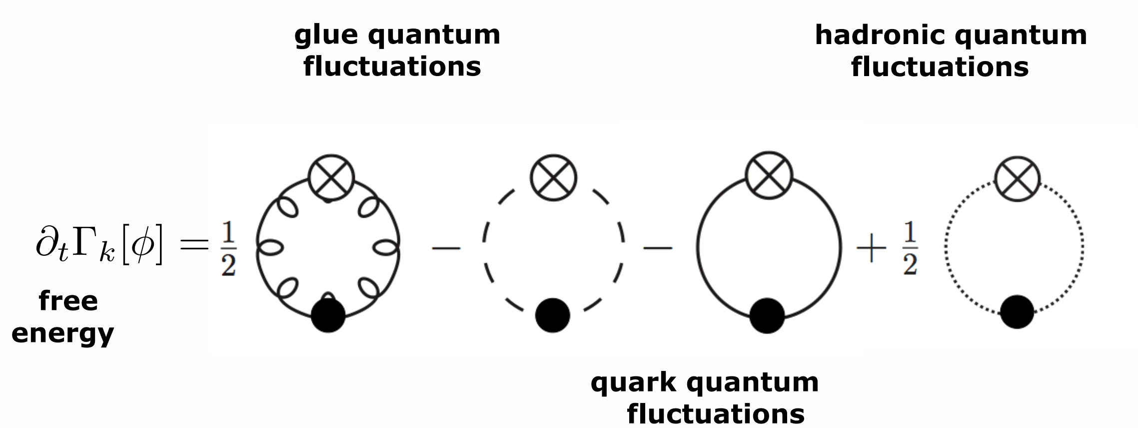
FRG, DSE, 2PI: Fister, JMP, PRD 88 (2013) 045010



$$T_c/\sqrt{\sigma} = 0.658 \pm 0.023$$

$$\text{lattice}: T_c/\sqrt{\sigma} = 0.646$$

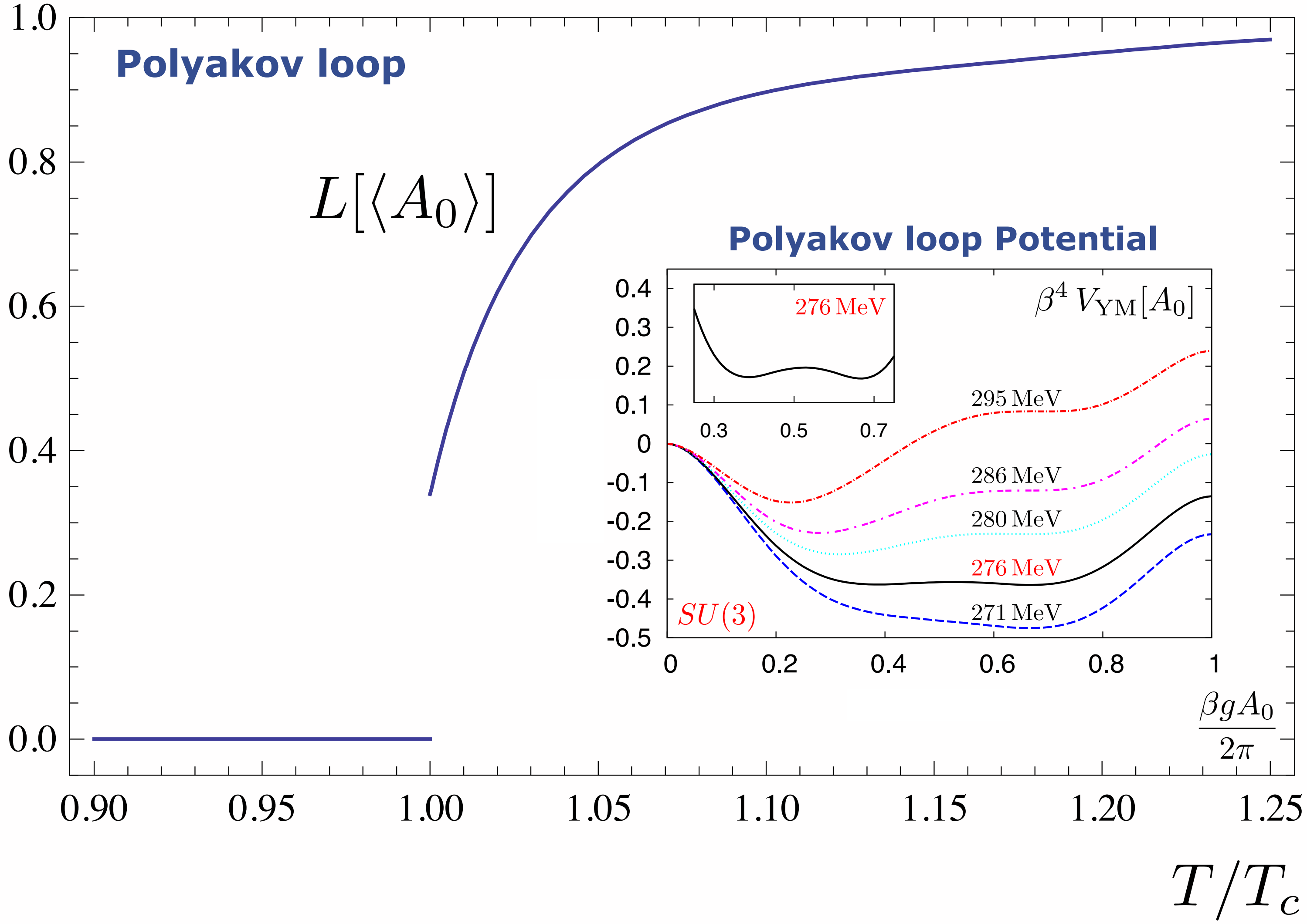
$$L[A_0] = \frac{\mathbf{1}}{\mathbf{N_c}} \mathbf{tr}\, \mathcal{P} \mathbf{e}^{\mathbf{i\, g} \int_0^\beta \mathbf{A_0(x)}}$$



Confinement

FRG: Braun, Gies, JMP, PLB 684 (2010) 262

FRG, DSE, 2PI: Fister, JMP, PRD 88 (2013) 045010



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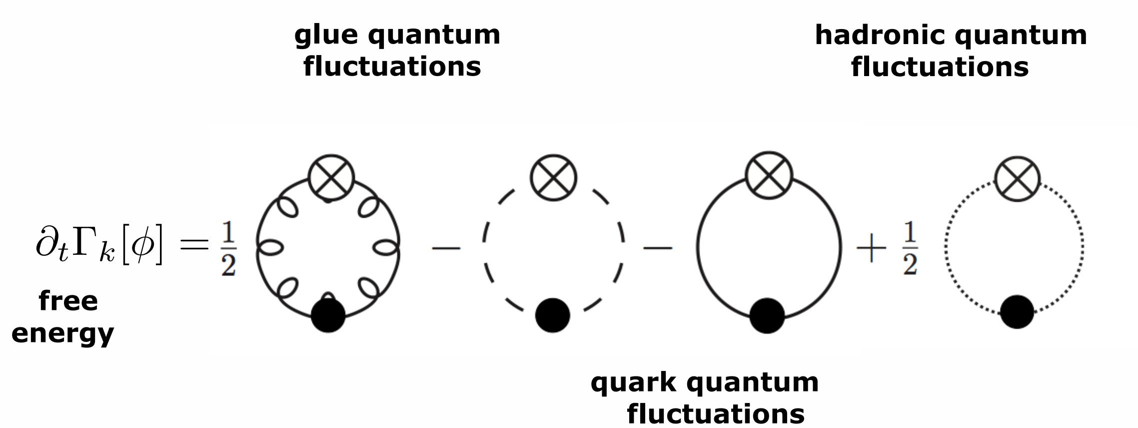


confinement

gluon propagator

gapped relative to

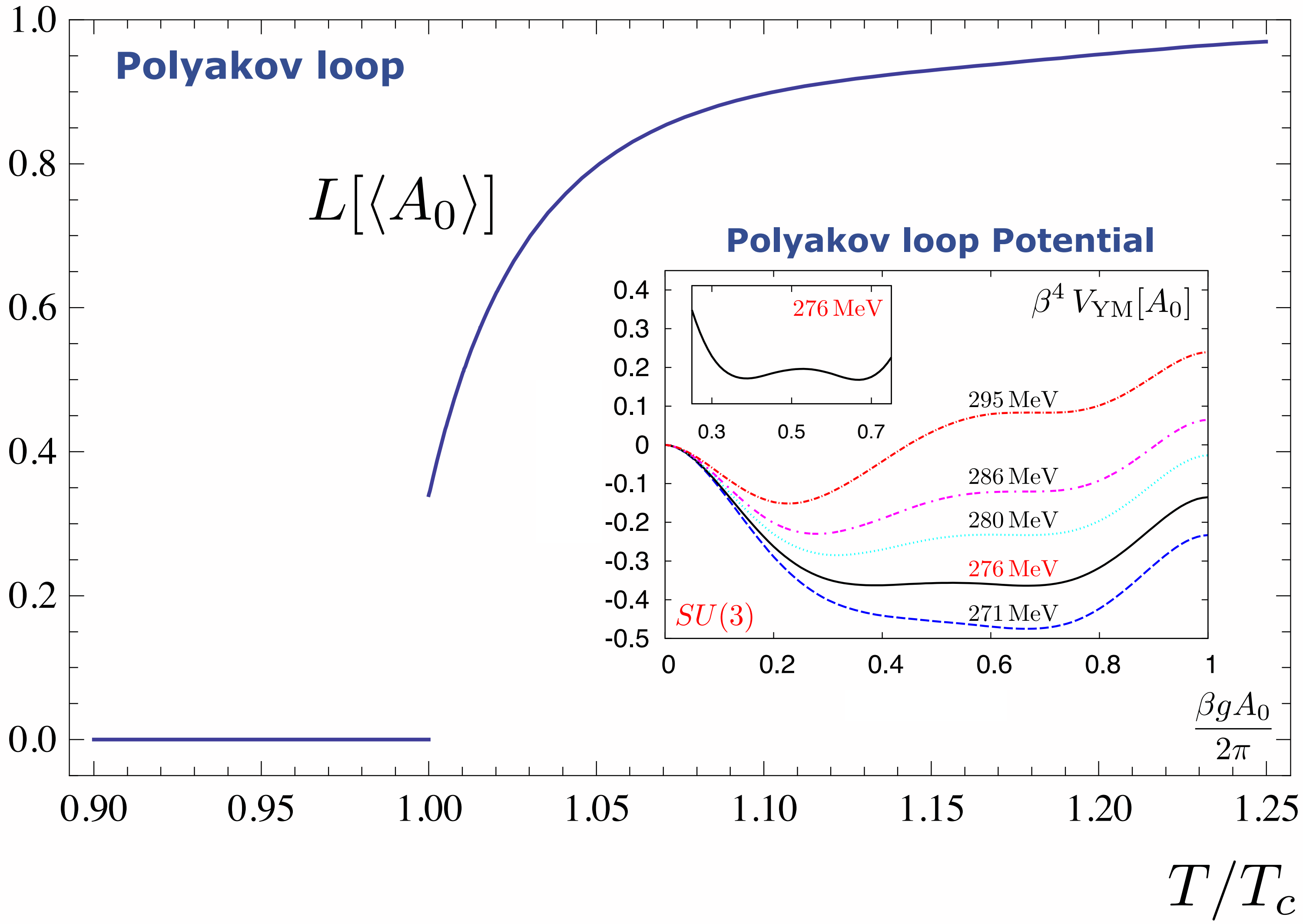
ghost propagator

Braun, Gies, JMP '07
Marhauser, JMP '08
Fister, JMP '13

Confinement

FRG: Braun, Gies, JMP, PLB 684 (2010) 262

FRG, DSE, 2PI: Fister, JMP, PRD 88 (2013) 045010



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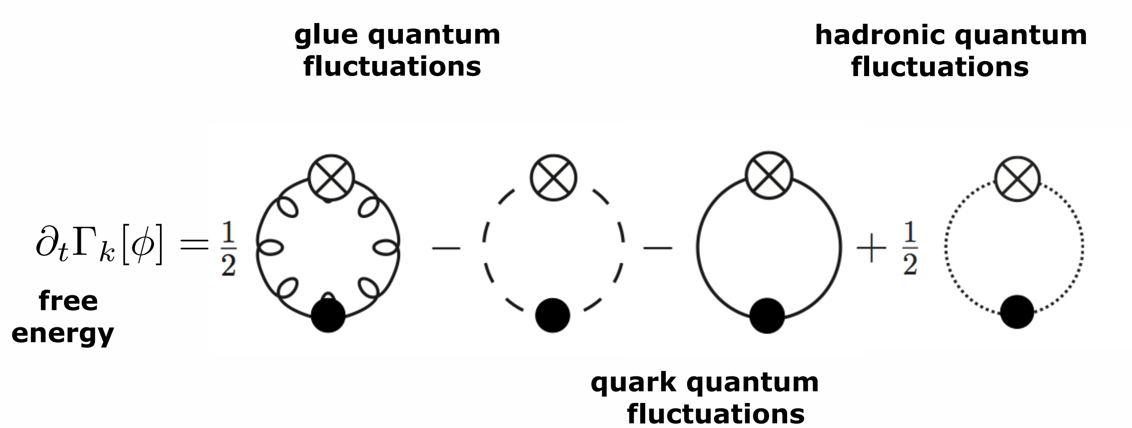
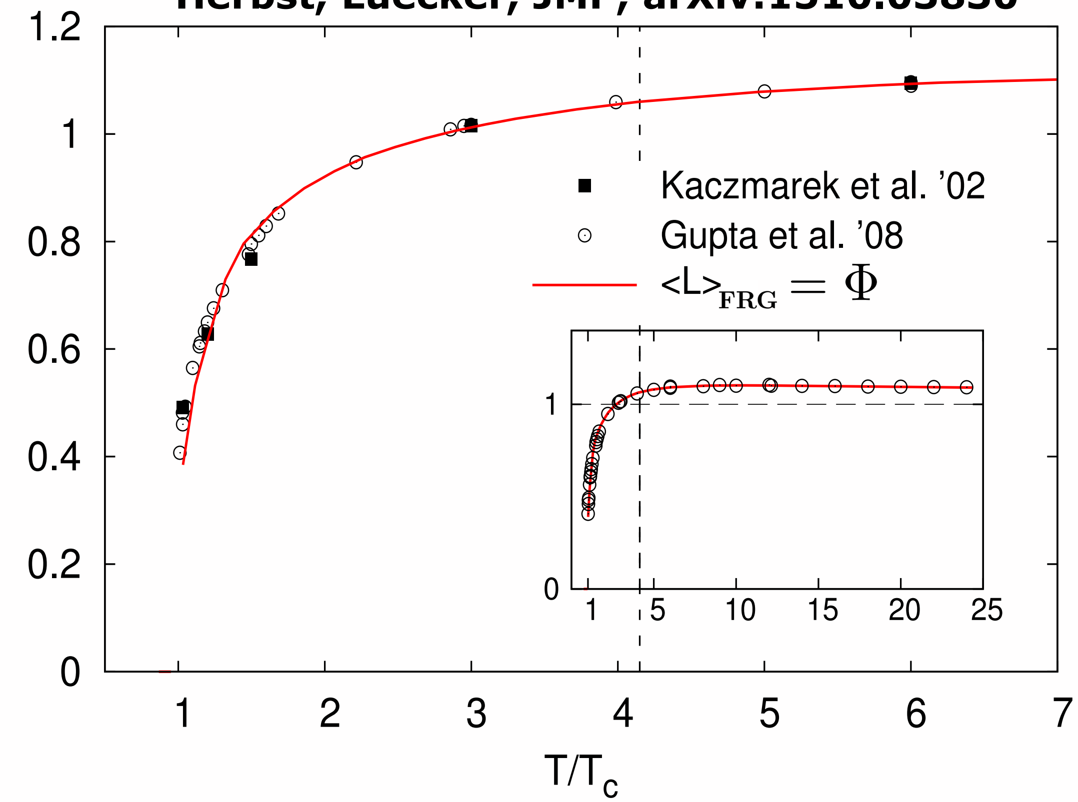


Herbst, Luecker, JMP, arXiv:1510.03830



Functional RG for QCD

fQCD collaboration: **J. Braun, A. Cyrol, L. Fister, W.-j. Fu, M. Mitter, N. Mueller, JMP, F. Rennecke, S. Rechenberger, N. Strodthoff**

Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

Braun, Fister, Haas, JMP, Rennecke, arXiv:1412.1045

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hardQCD: Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

easyQCD: Braun, Fister, Haas, JMP, Rennecke, arXiv:1412.1045

fQCD: workflow

VertEXpand
Mathematica package for the derivation of vertices from a given action using FORM
(Denz,Held,Rodigast; unpub.)

DoFun
Mathematica package for the derivation of functional equations
(Braun,Huber; Comput.Phys. Commun. 183 (2012) 1290-1320)

Action

DoFun
VertEXpand

DoFun
ERGE

Vertices/
Feynman Rules

Symbolic
Flow Equations

FORMTracer high-performance, easy-to-use Mathematica tracing tool using FORM
(Cyrol,Mitter,Pawlowski,Strodthoff; in prep.)

Algebraic
Flow Equations

CreateKernels Mathematica package for the automatic generation of compilable C++ kernels for use in connection with the *frgsolver*
(Cyrol,Mitter,Pawlowski,Strodthoff; unpub.)

Compilable Kernels

frgsolver Flexible, high-performance, parallelized C++ OOP framework for the numerical solution of functional equations
(Cyrol,Mitter,Pawlowski,Strodthoff; unpub.)

Numerical solution

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Bundesministerium für Bildung und Forschung
FWF Der Wissenschaftsfonds.
erc European Research Council
Established by the European Commission

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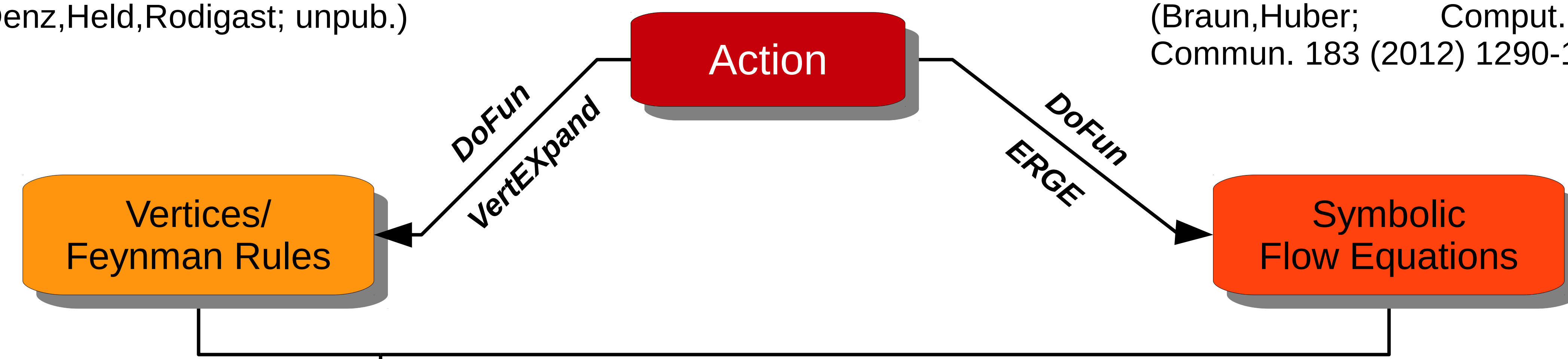

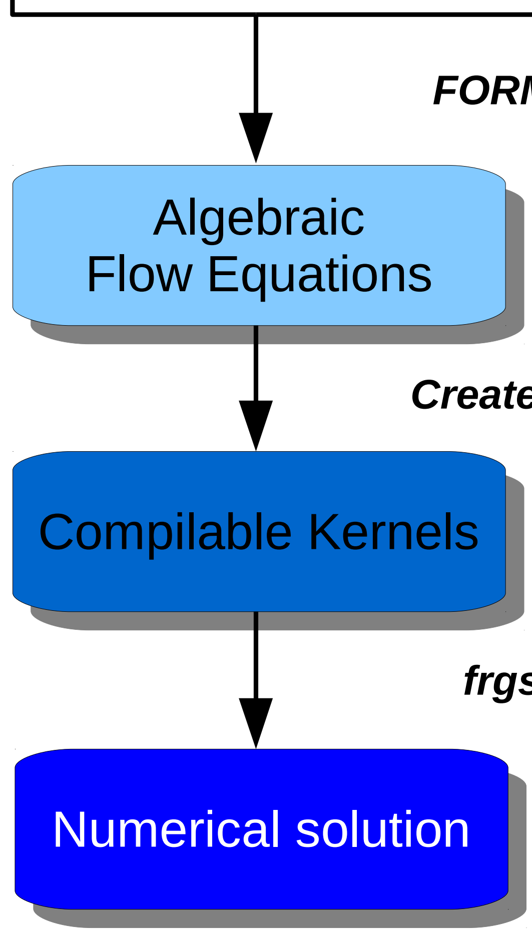

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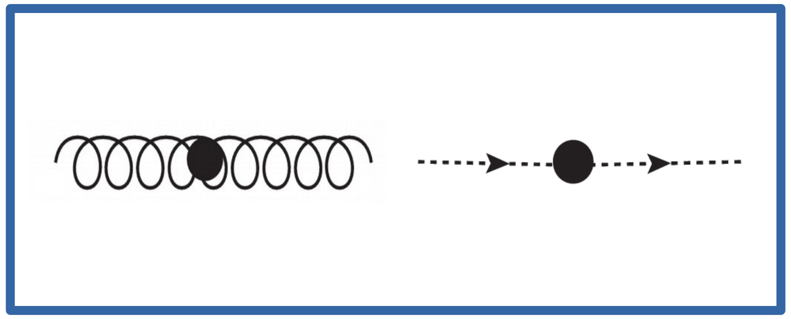
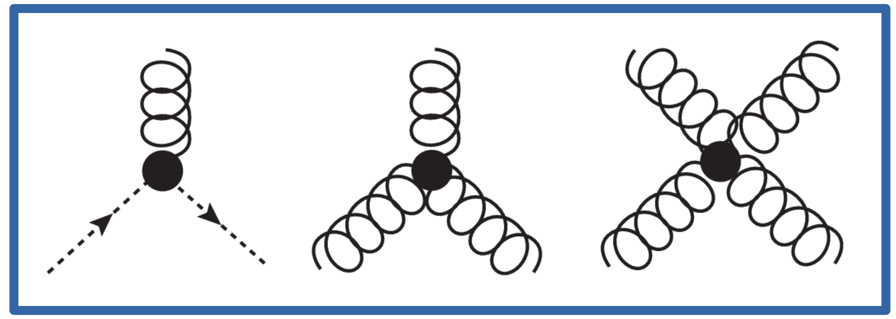
Chiral symmetry breaking

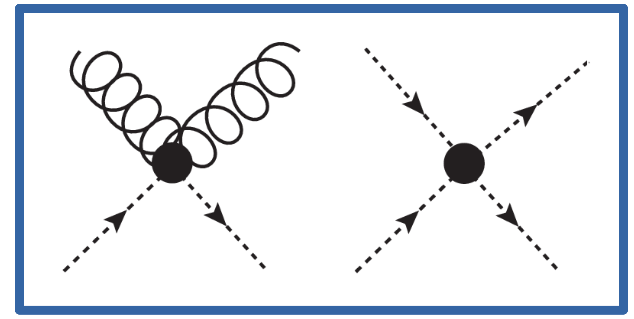
Expansion of effective action in 1PI correlators



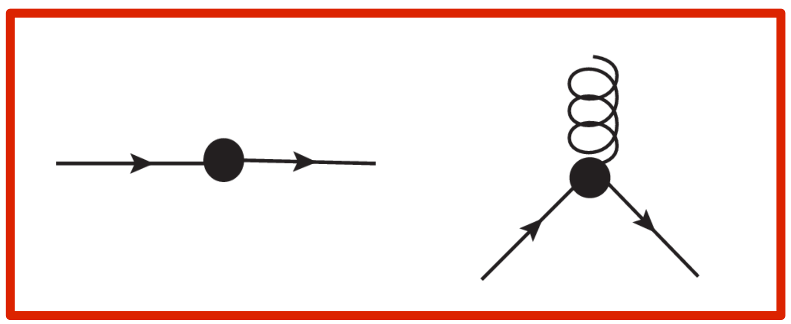
- full mom. dep.



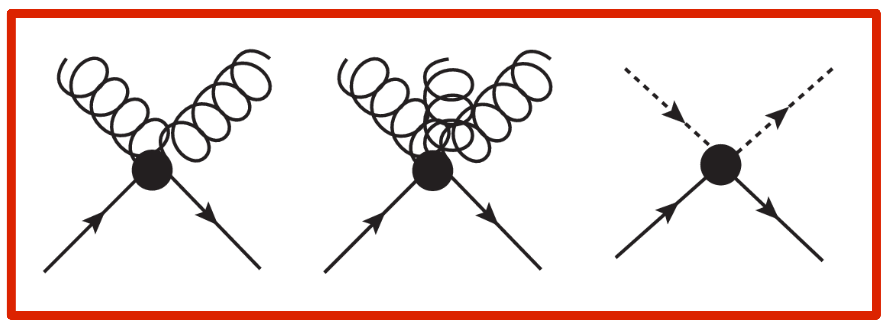
- classical tensor structure
- mom. dep. (sym. channel)



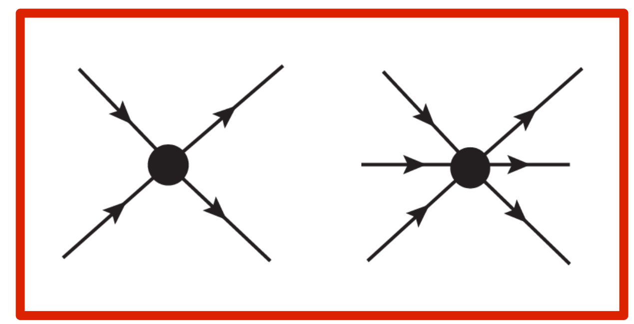
under investigation:
- full tensor structure
- mom. dep. (sym. channel)



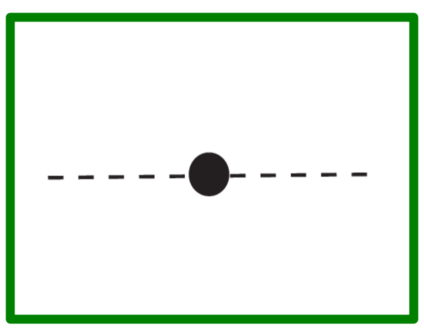
- full tensor structure
- full mom. dep.



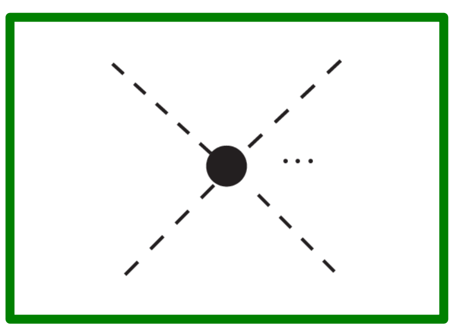
- partial tensor structure
- mom. dep. (sym. channel)



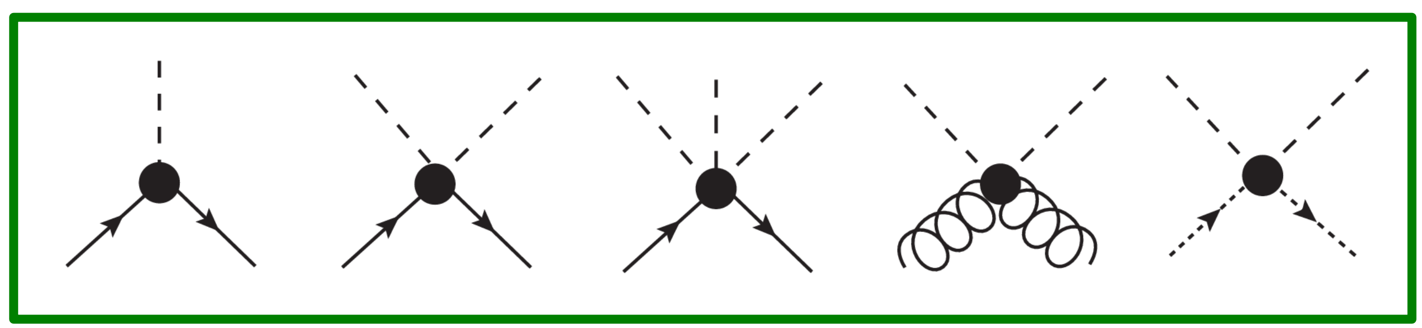
- full tensor structure
- mom. dep. (single channel)



- full mom. dep.



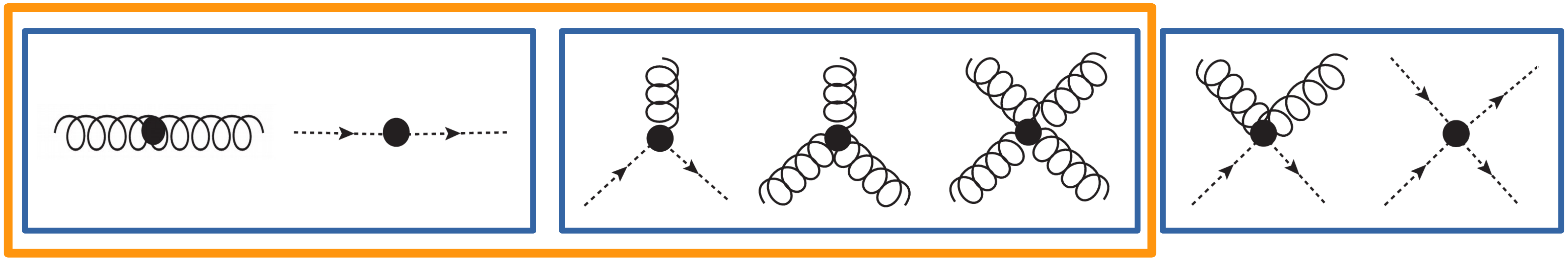
- via effective potential



- full tensor structure
- mom. dep. (sym. channel)

Chiral symmetry breaking

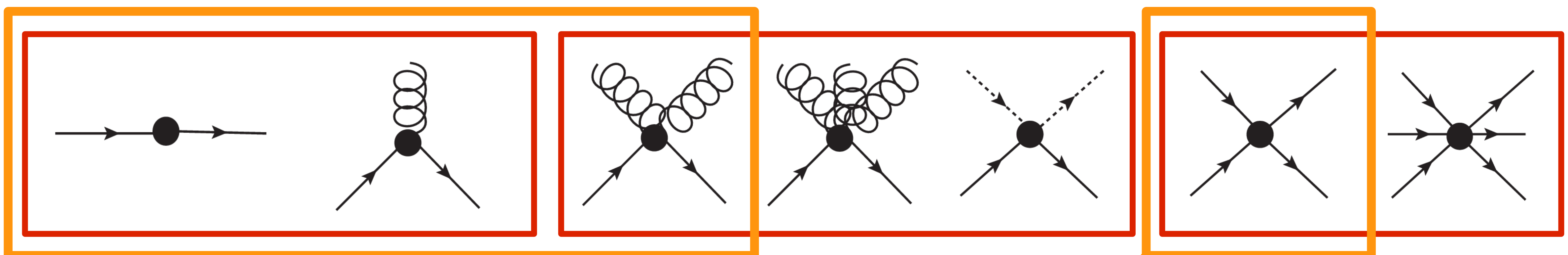
Expansion of effective action in 1PI correlators



- full mom. dep.

- classical tensor structure
- mom. dep. (sym. channel)

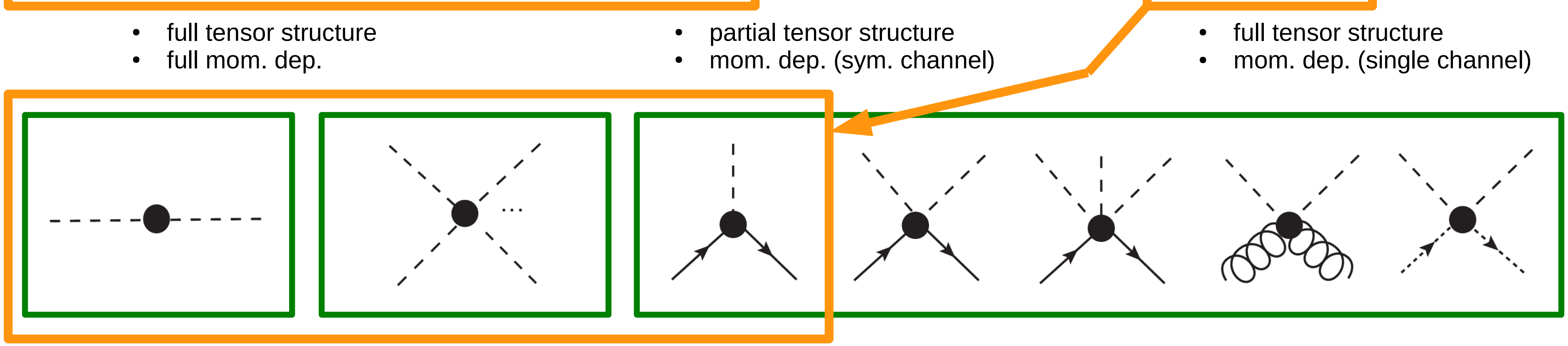
under investigation:
- full tensor structure
- mom. dep. (sym. channel)



- full tensor structure
- full mom. dep.

- partial tensor structure
- mom. dep. (sym. channel)

- full tensor structure
- mom. dep. (single channel)

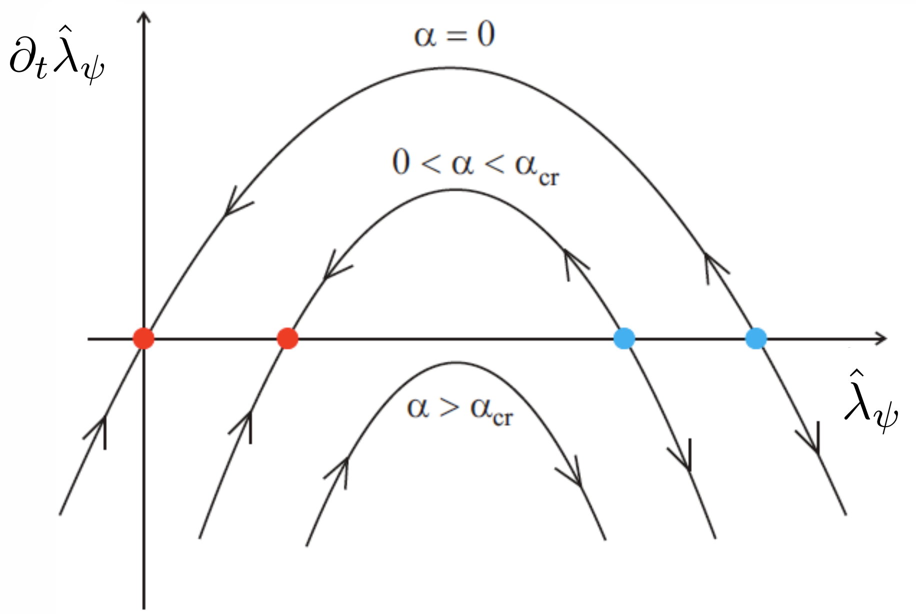


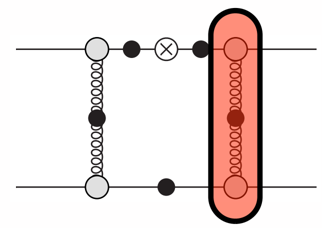
- full mom. dep.

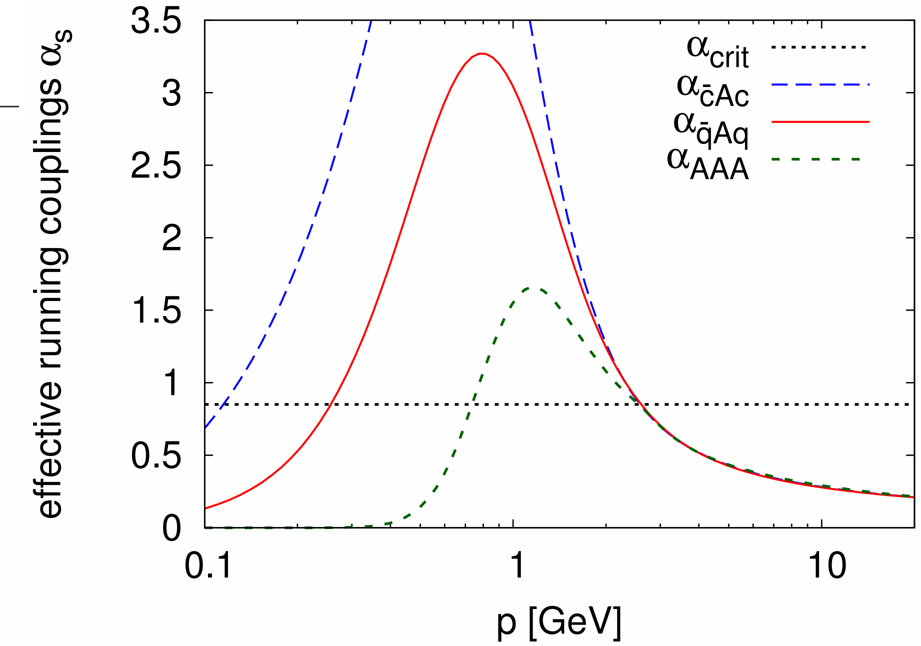
- via effective potential

- full tensor structure
- mom. dep. (sym. channel)

Confinement & symmetry breaking

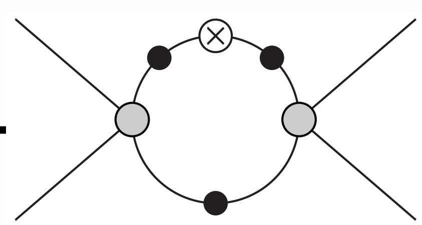
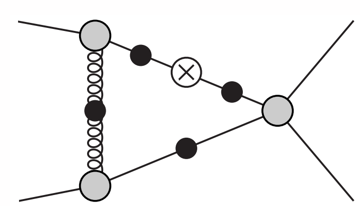
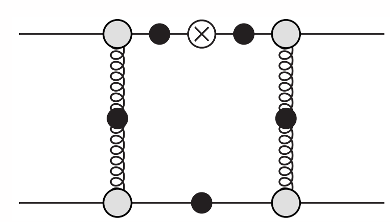


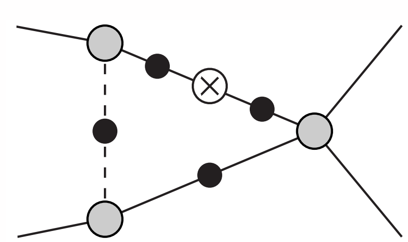
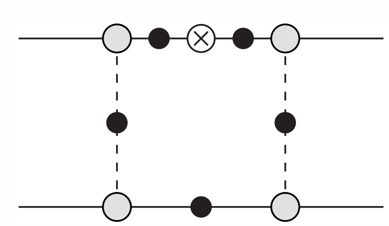




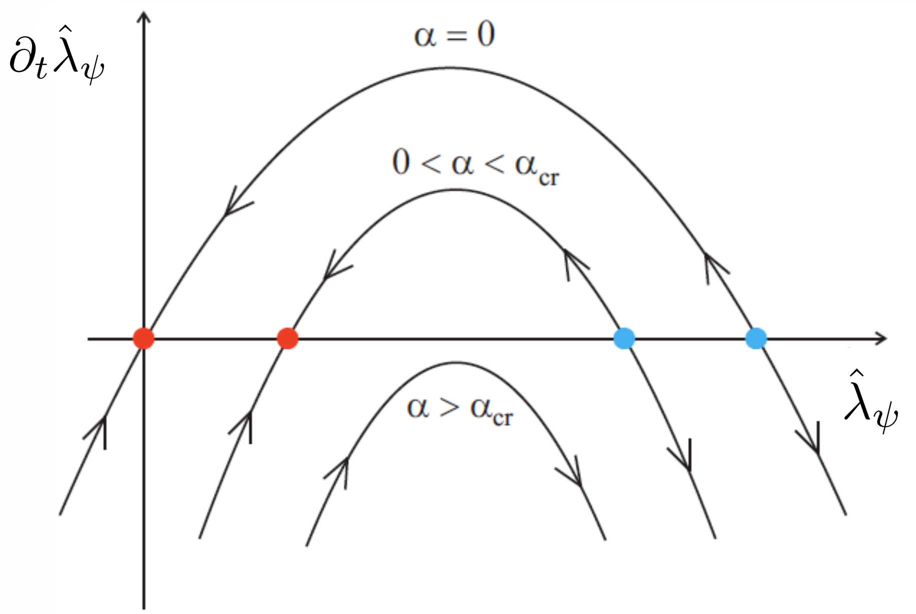
Mitter, JMP, Strodthoff '14

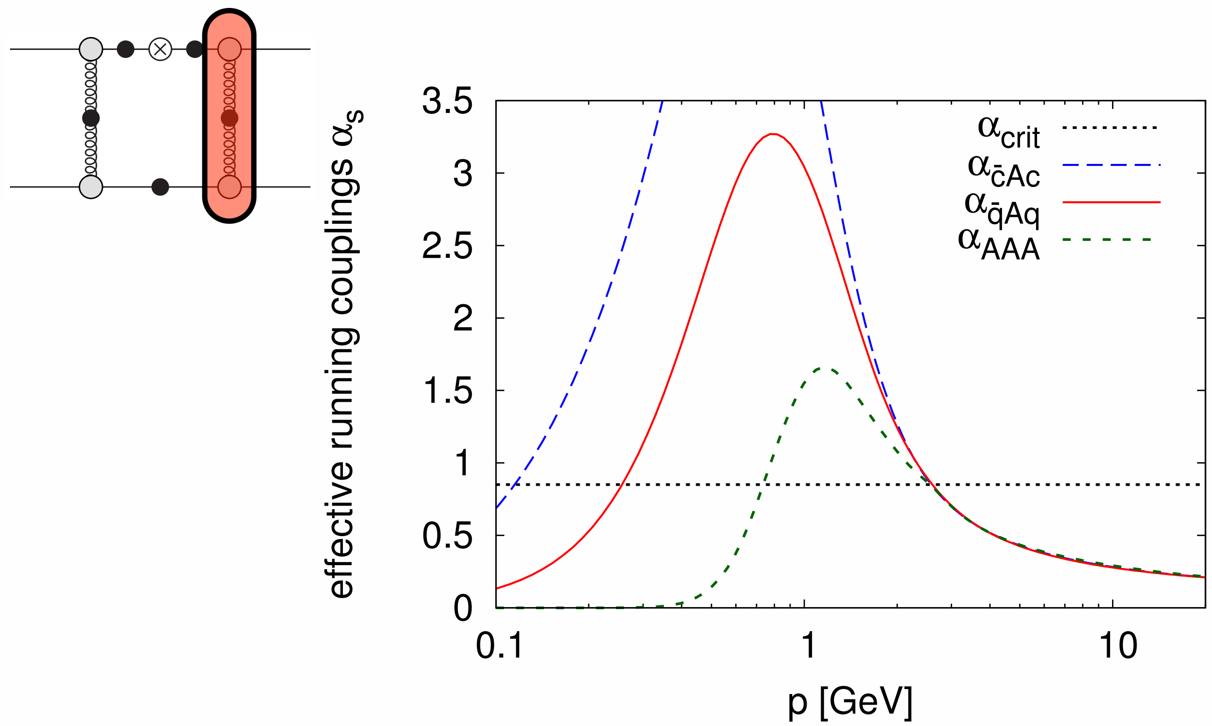
Braun, Fister, Haas, JMP, Rennecke '14

$k\partial_k \hat{\lambda}_\psi = 2\hat{\lambda}_\psi$ +  +  + 

+   + ...

Confinement & symmetry breaking





Mitter, JMP, Strodthoff '14

Braun, Fister, Haas, JMP, Rennecke '14

dynamical correlation of confinement
and
chiral symmetry breaking

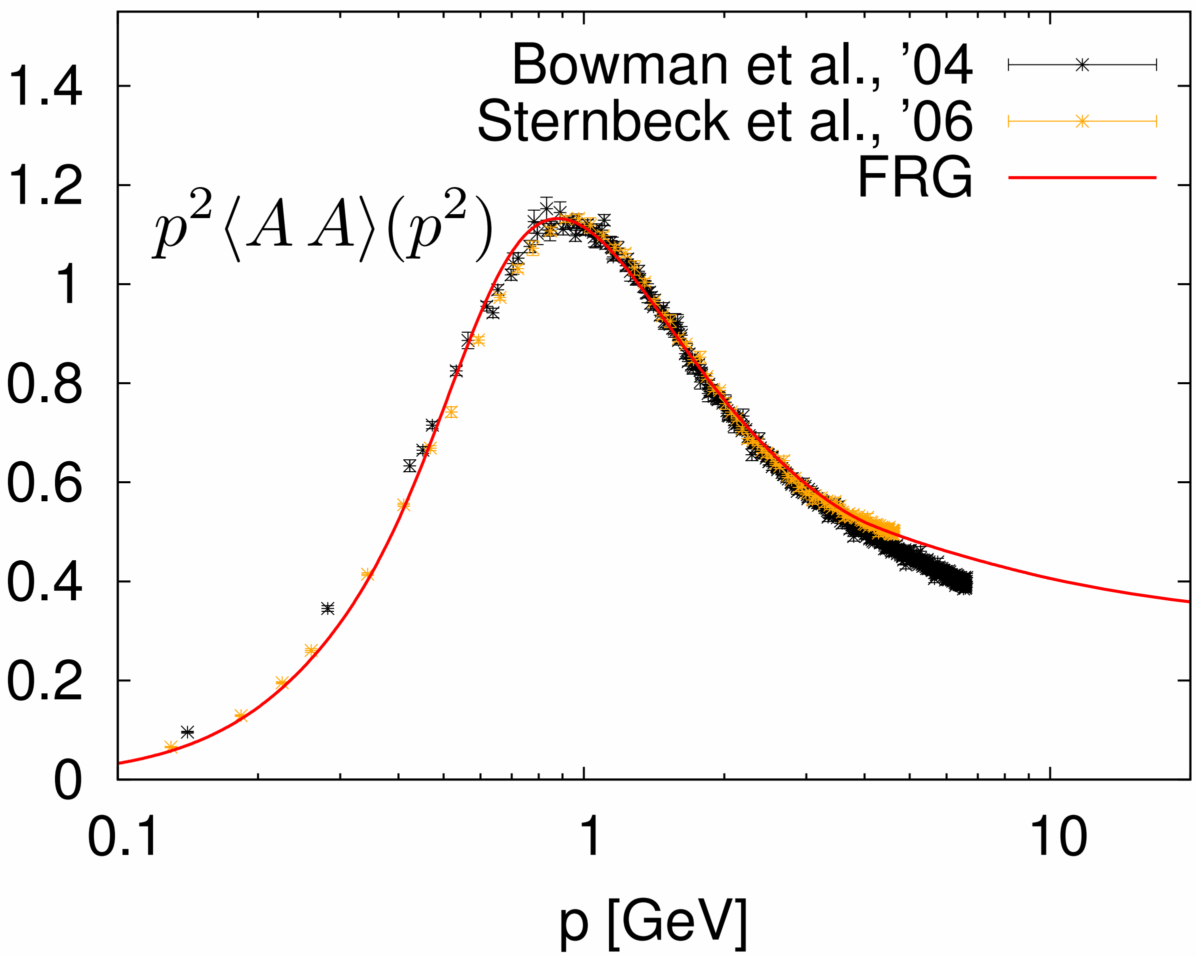
confinement	chiral symmetry breaking
gluon propagator gapped relative to ghost propagator	gluon propagator not gapped too much

Cyrol, Fister, Mitter, JMP, Strodthoff, in prep

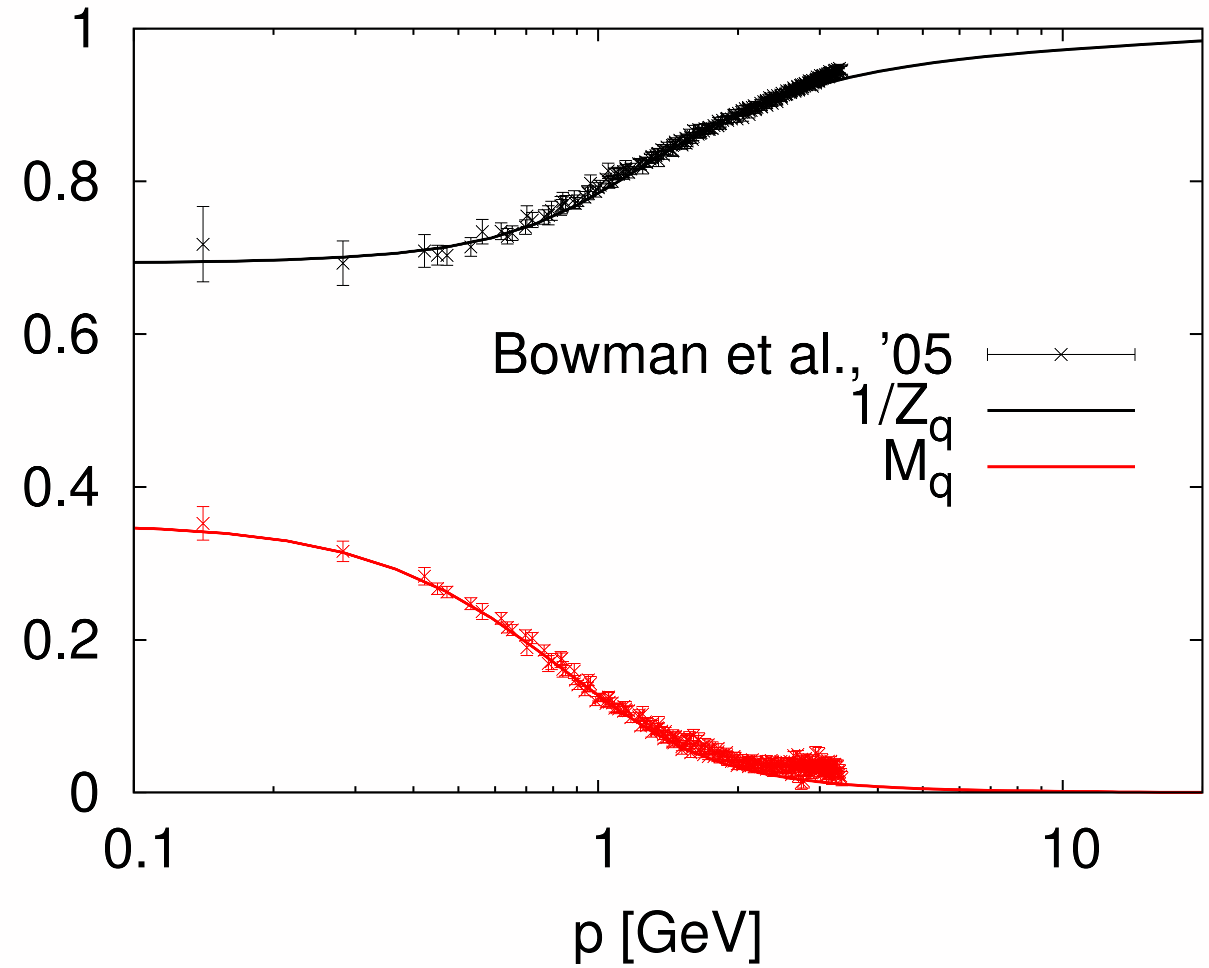
Chiral symmetry breaking

FRG-quenched QCD vs lattice-quenced QCD

quenched gluon dressing



quark propagator



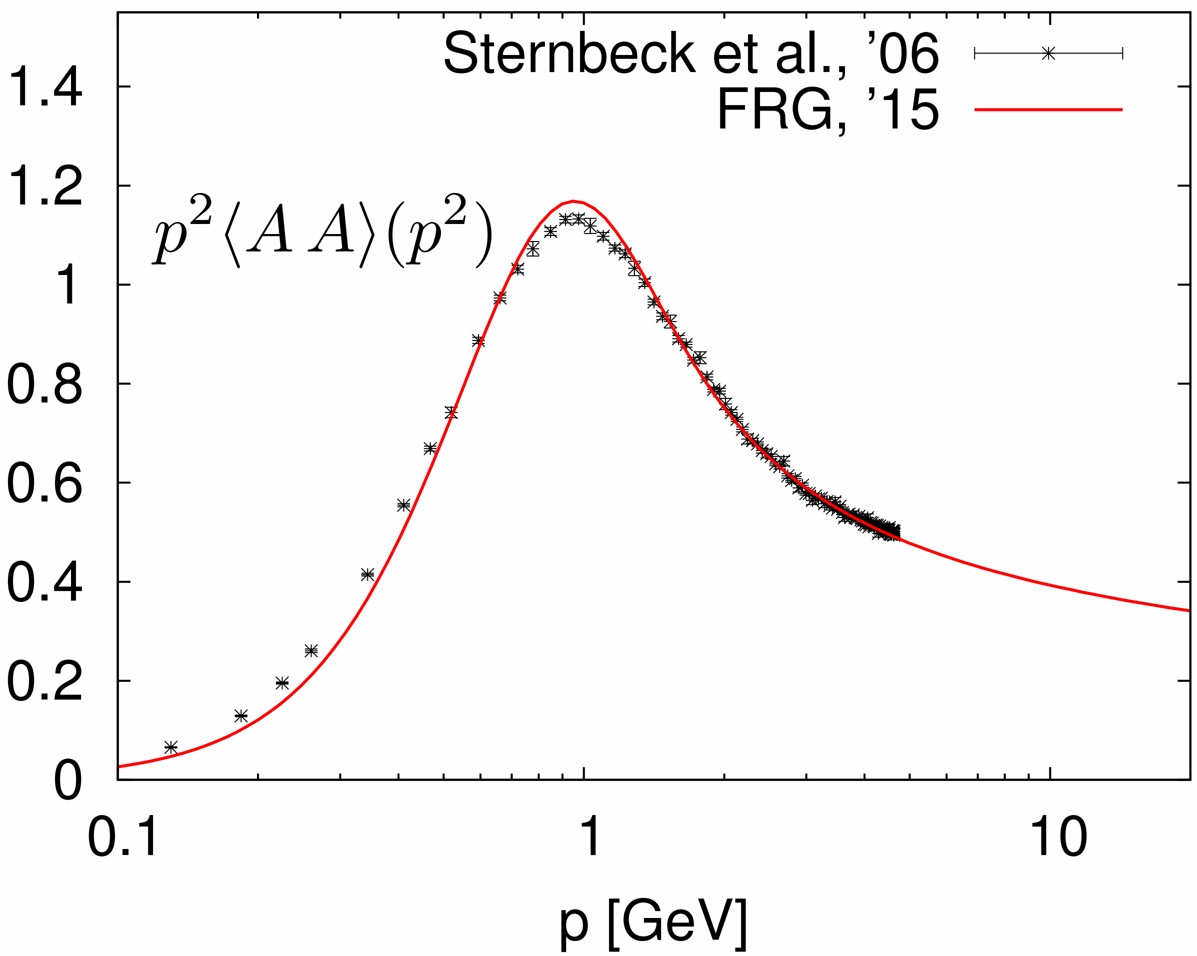
$$\mathbf{N_f = 2}$$

Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

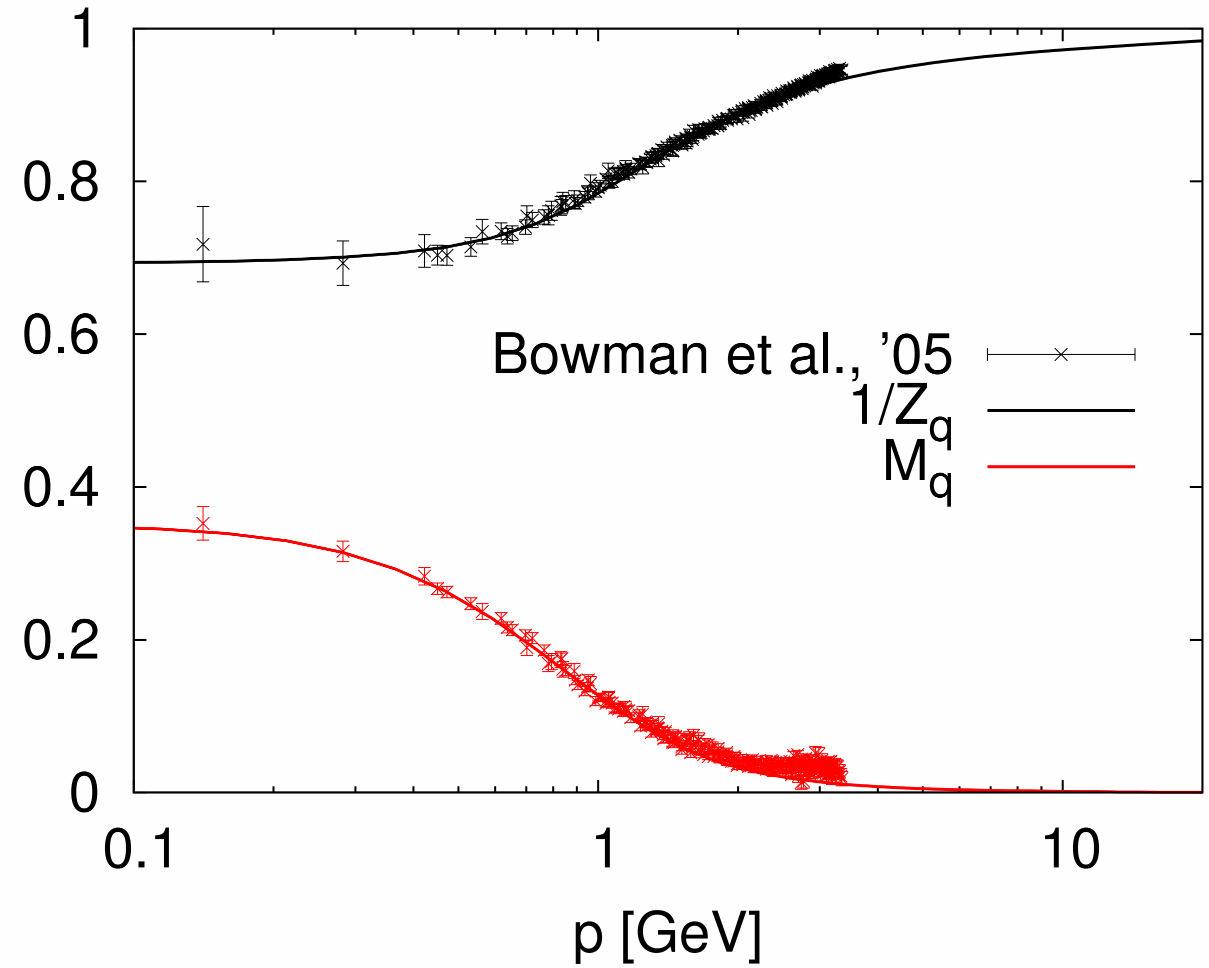
Chiral symmetry breaking

FRG-quenched QCD vs lattice-quenced QCD

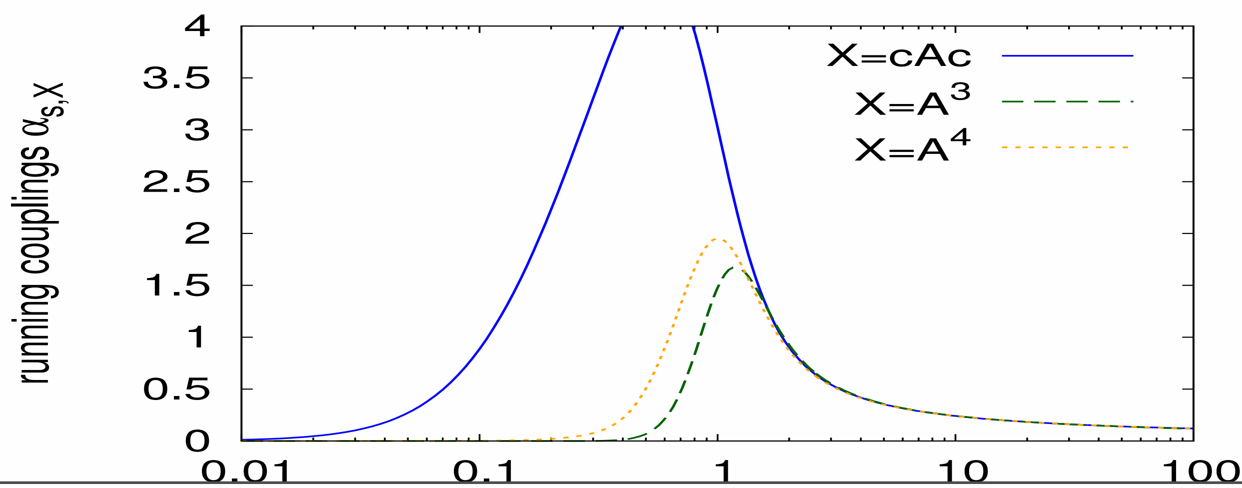
quenched gluon dressing



quark propagator



Cyrol, Mitter, JMP, Strodthoff, in prep



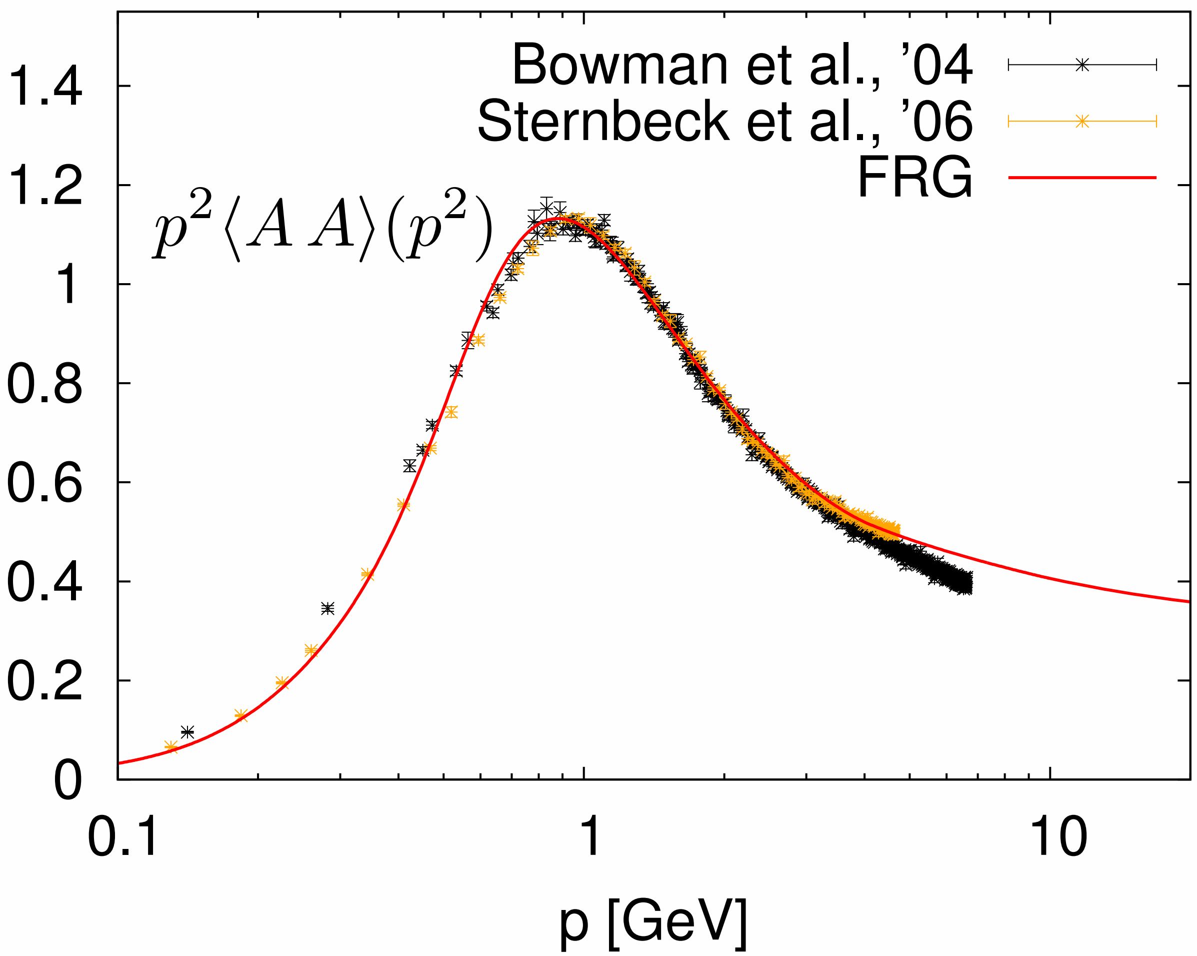
$$N_f = 2$$

Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

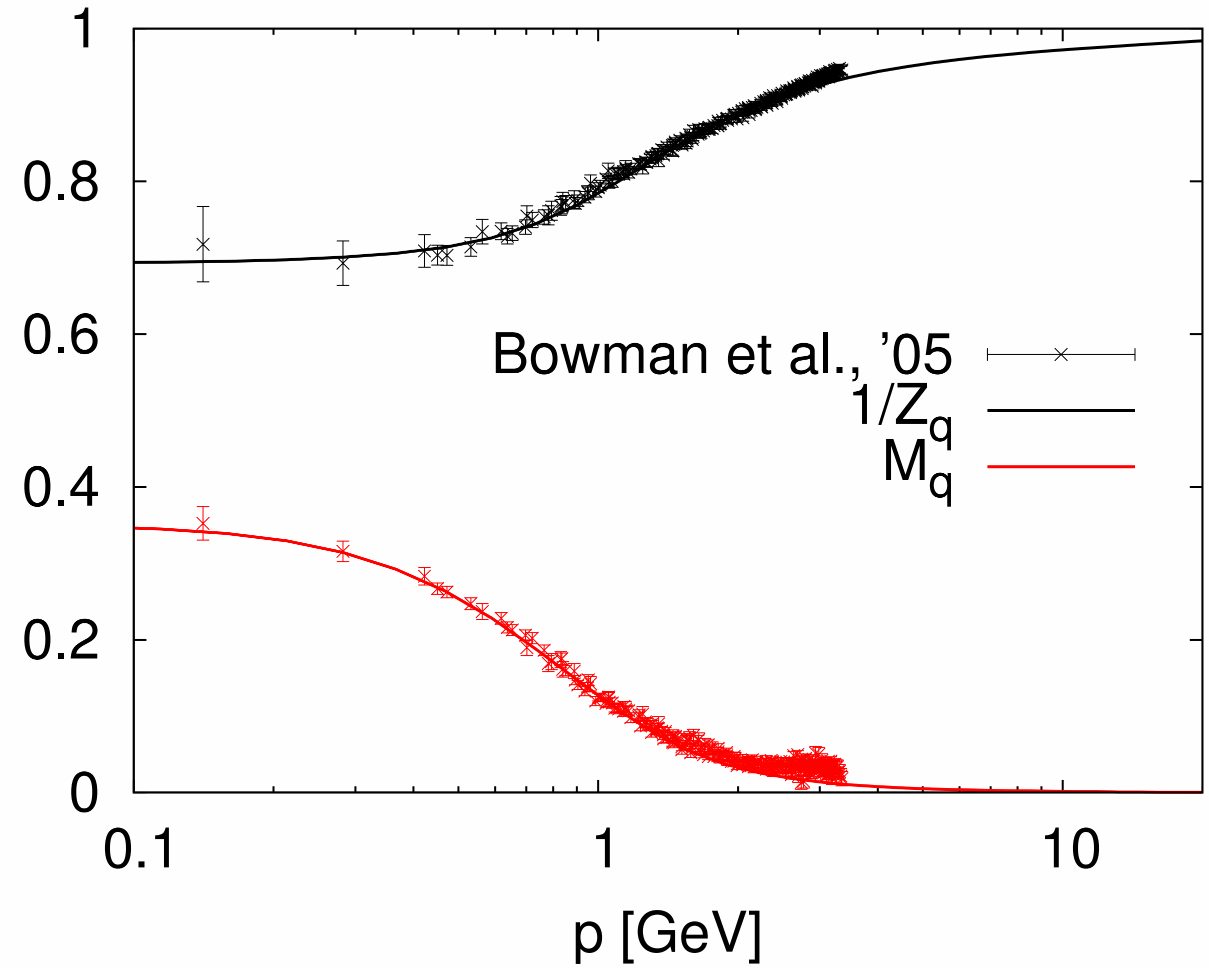
Chiral symmetry breaking

FRG-quenched QCD vs lattice-quenced QCD

quenched gluon dressing



quark propagator



JMP, Rennecke, PRD 90, 076002

Helmboldt, JMP, Strodthoff, PRD 91 (2015) 5, 054010

Braun, Fister, Haas, JMP, Rennecke, arXiv:1412.1045

$\mathbf{N_f = 2}$

systematic error estimate: ~10%

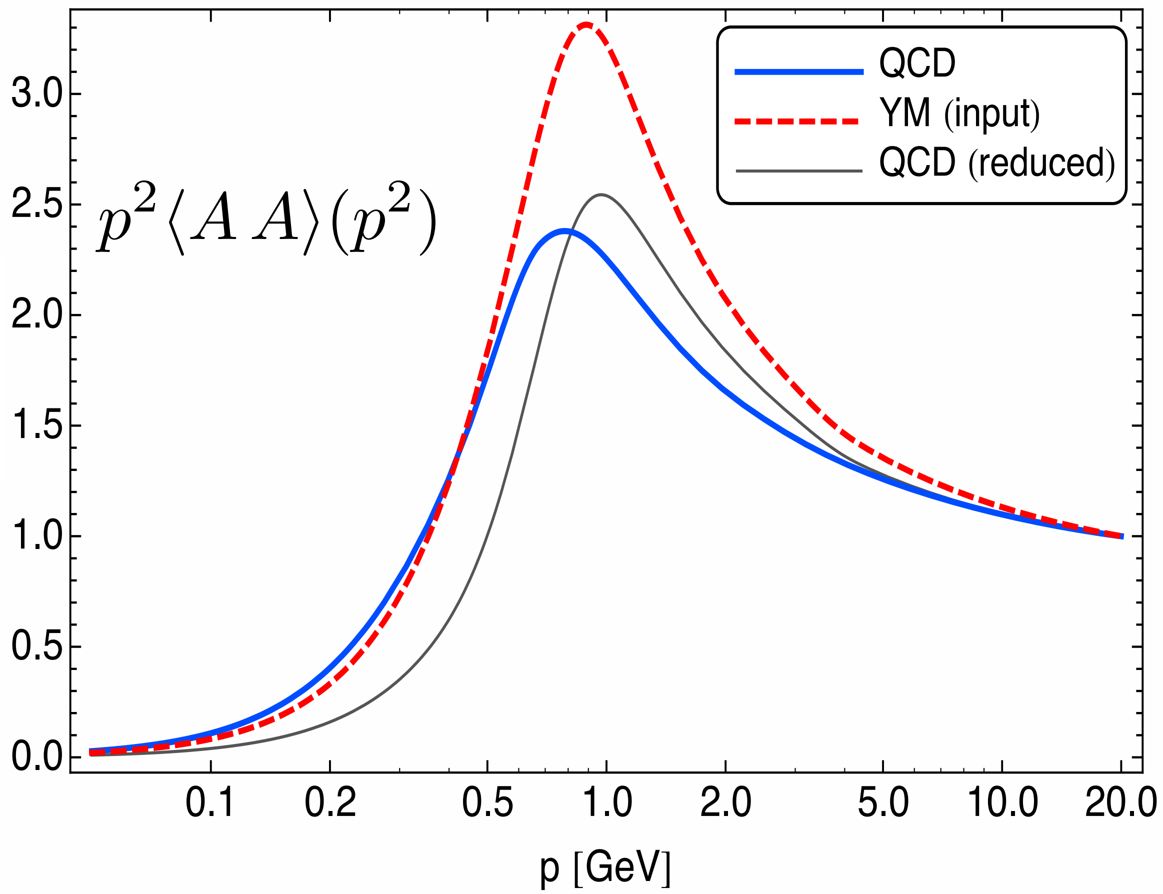
JMP

Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

Chiral symmetry breaking

FRG-quenched QCD vs lattice-quenced QCD

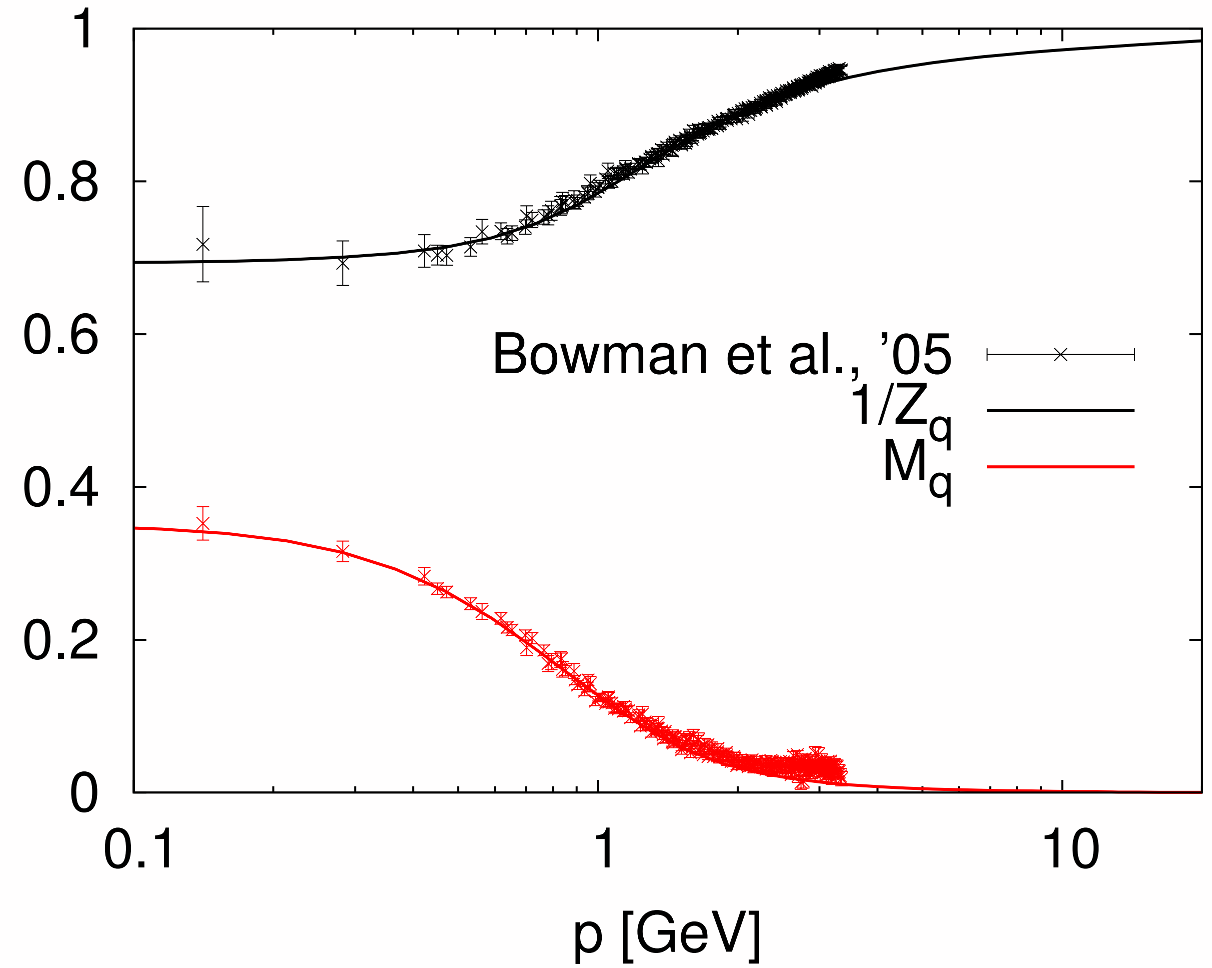
unquenched gluon dressing



Braun, Fister, Haas, JMP, Rennecke, arXiv:1412.1045

Rennecke, arXiv:1504.03585

quark propagator



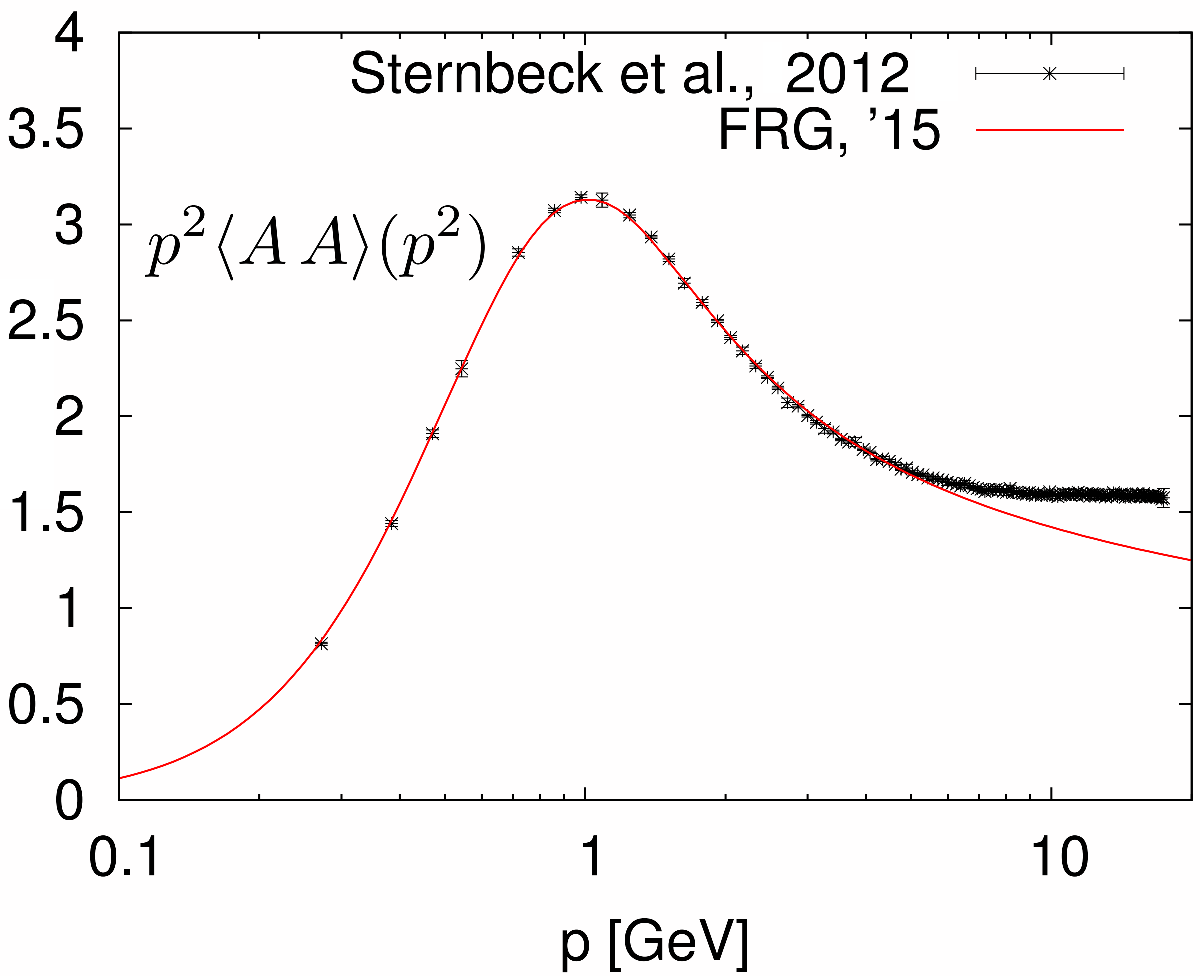
$$\mathbf{N_f = 2}$$

Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

Chiral symmetry breaking

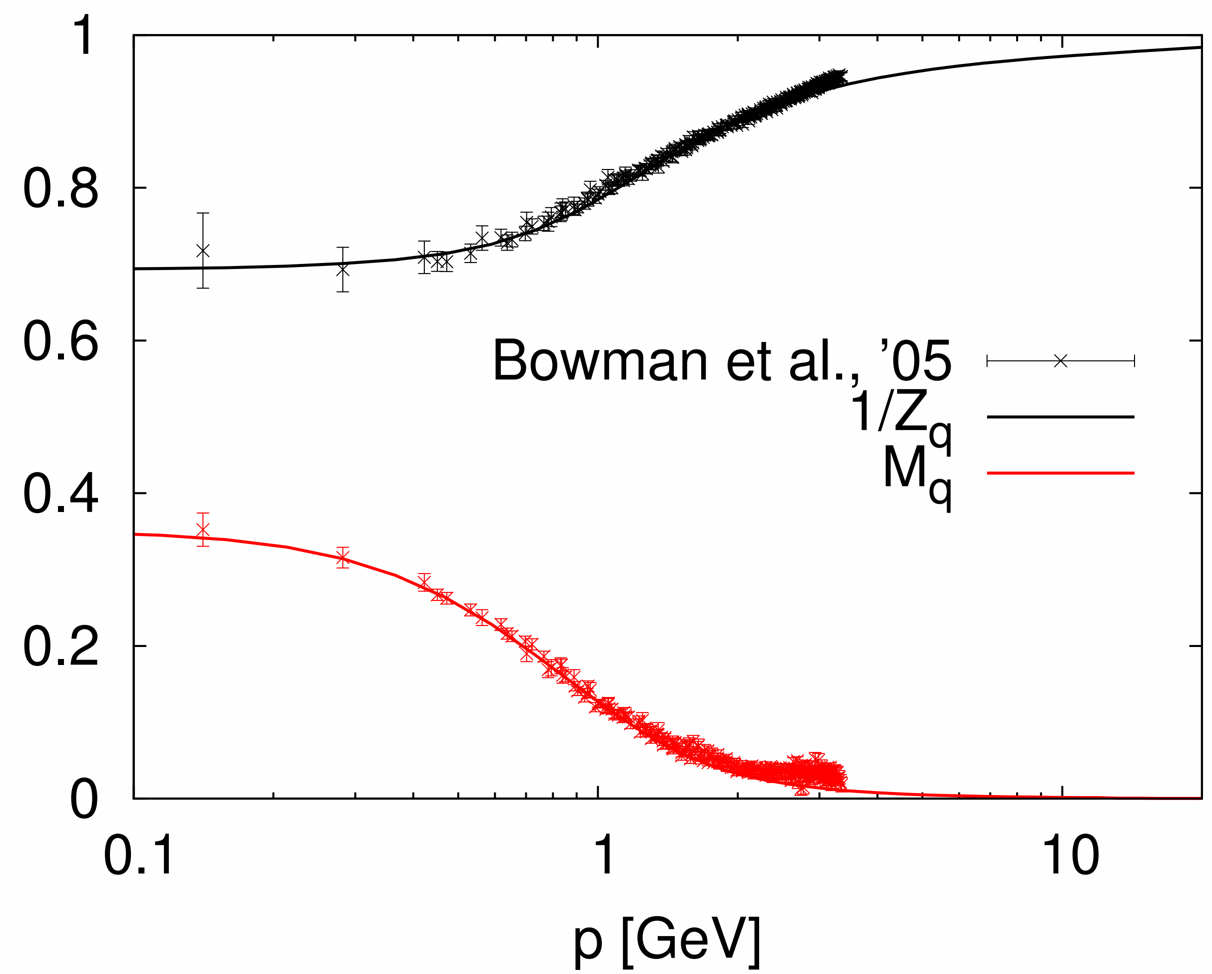
FRG-quenched QCD vs lattice-quenced QCD

unquenched gluon dressing



Cyrol, Mitter, JMP, Strodthoff, in prep.

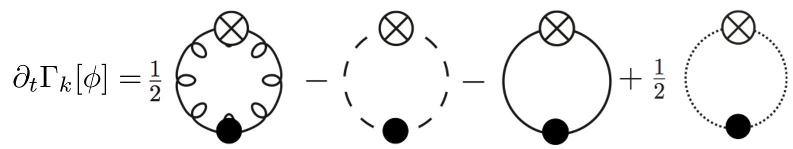
quark propagator



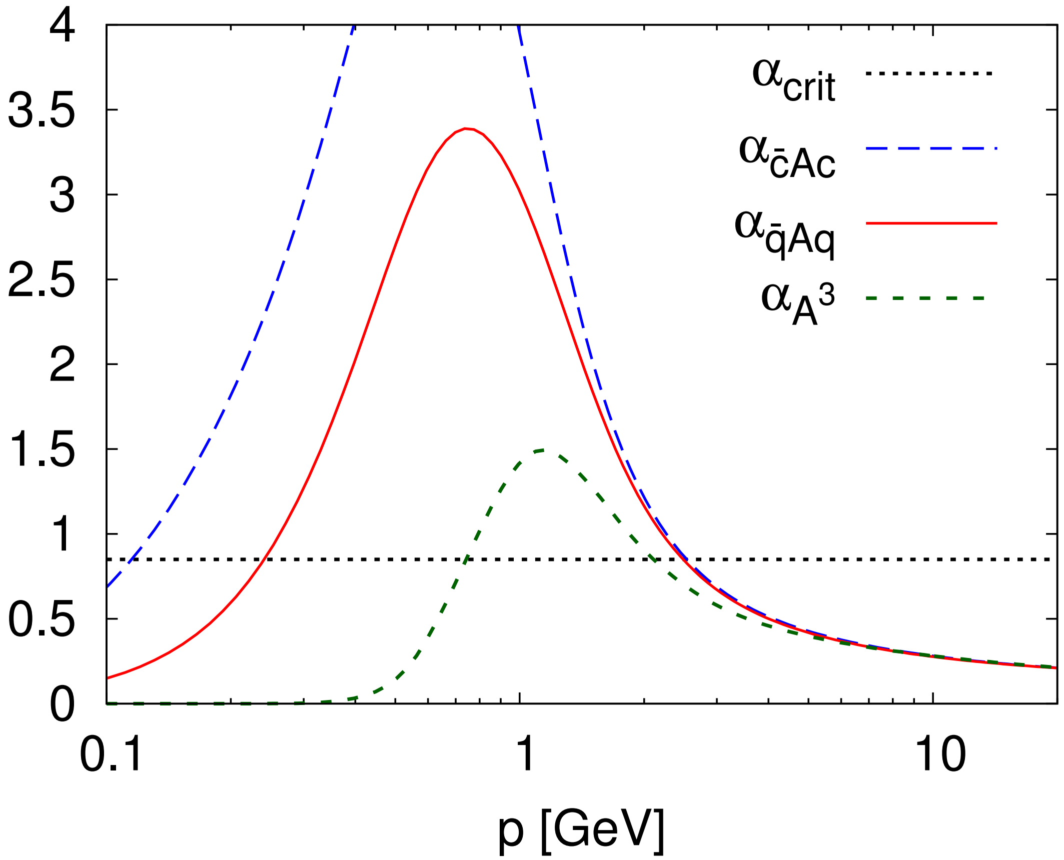
$$\mathbf{N_f = 2}$$

Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

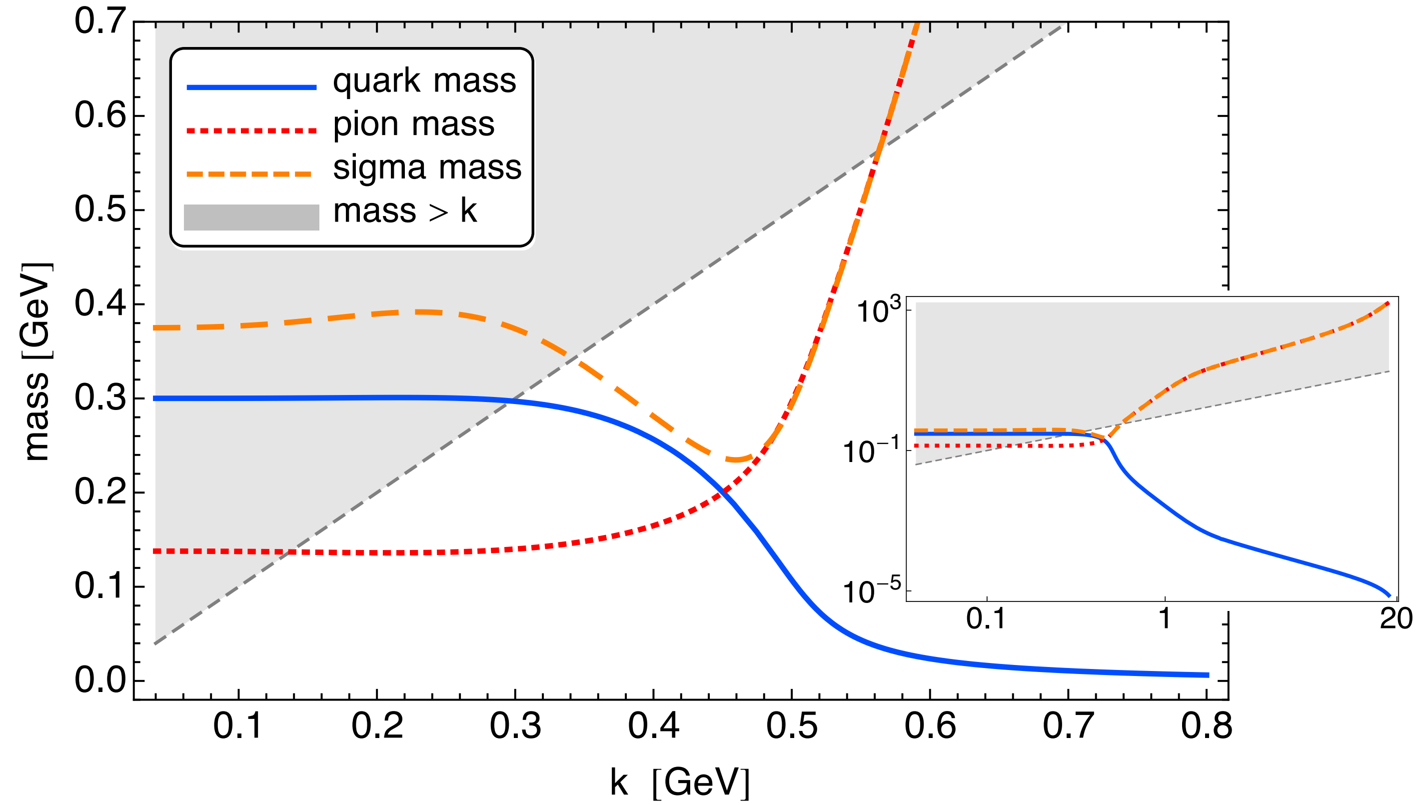
QCD



Sequential decoupling of gluon, quark, sigma, pion fluctuations



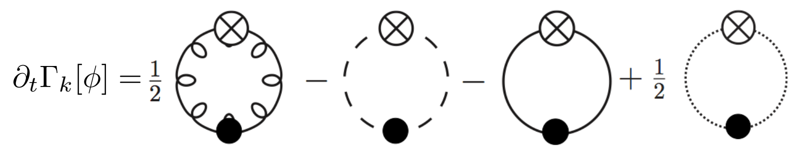
Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

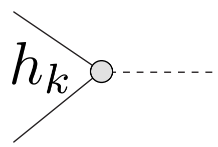


Braun, Fister, Haas, JMP, Rennecke, arXiv:1412.1045

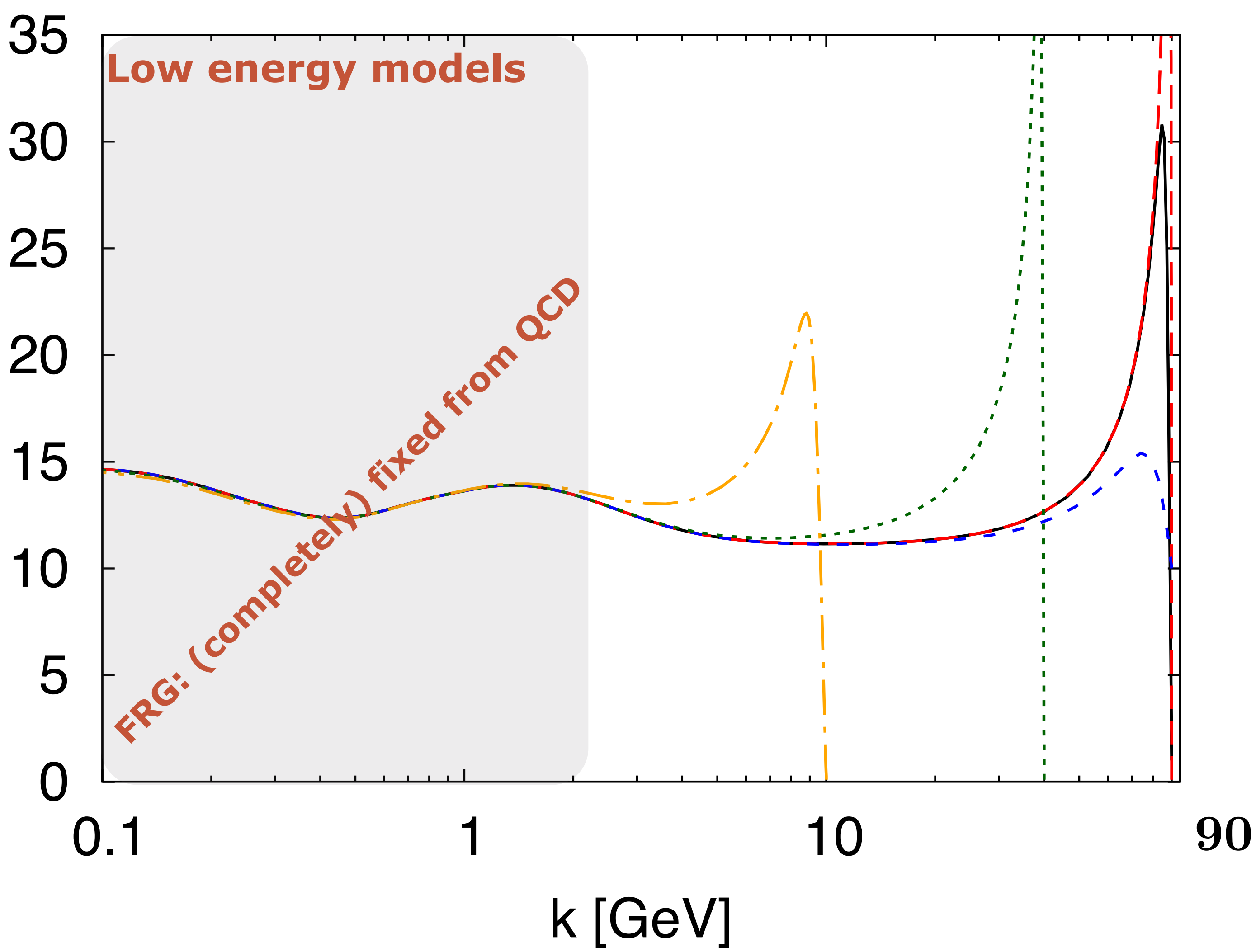
Rennecke, arXiv:1504.03585

QCD



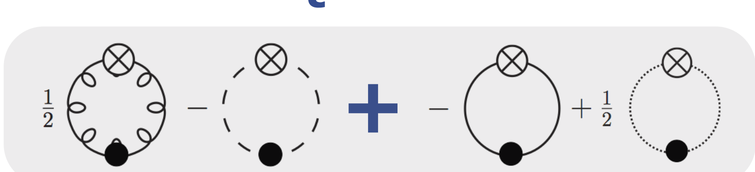


quark-meson coupling



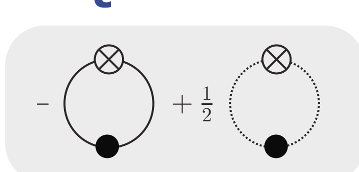
fQCD

PQM-model

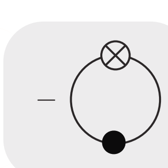


PNJL-model

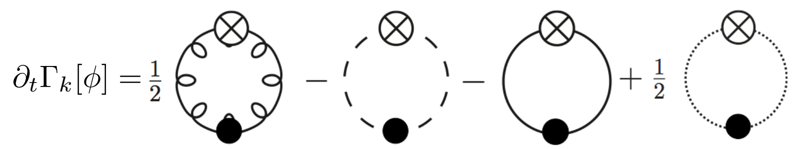
QM-model

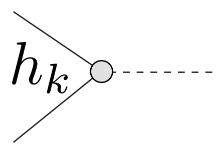


NJL-model

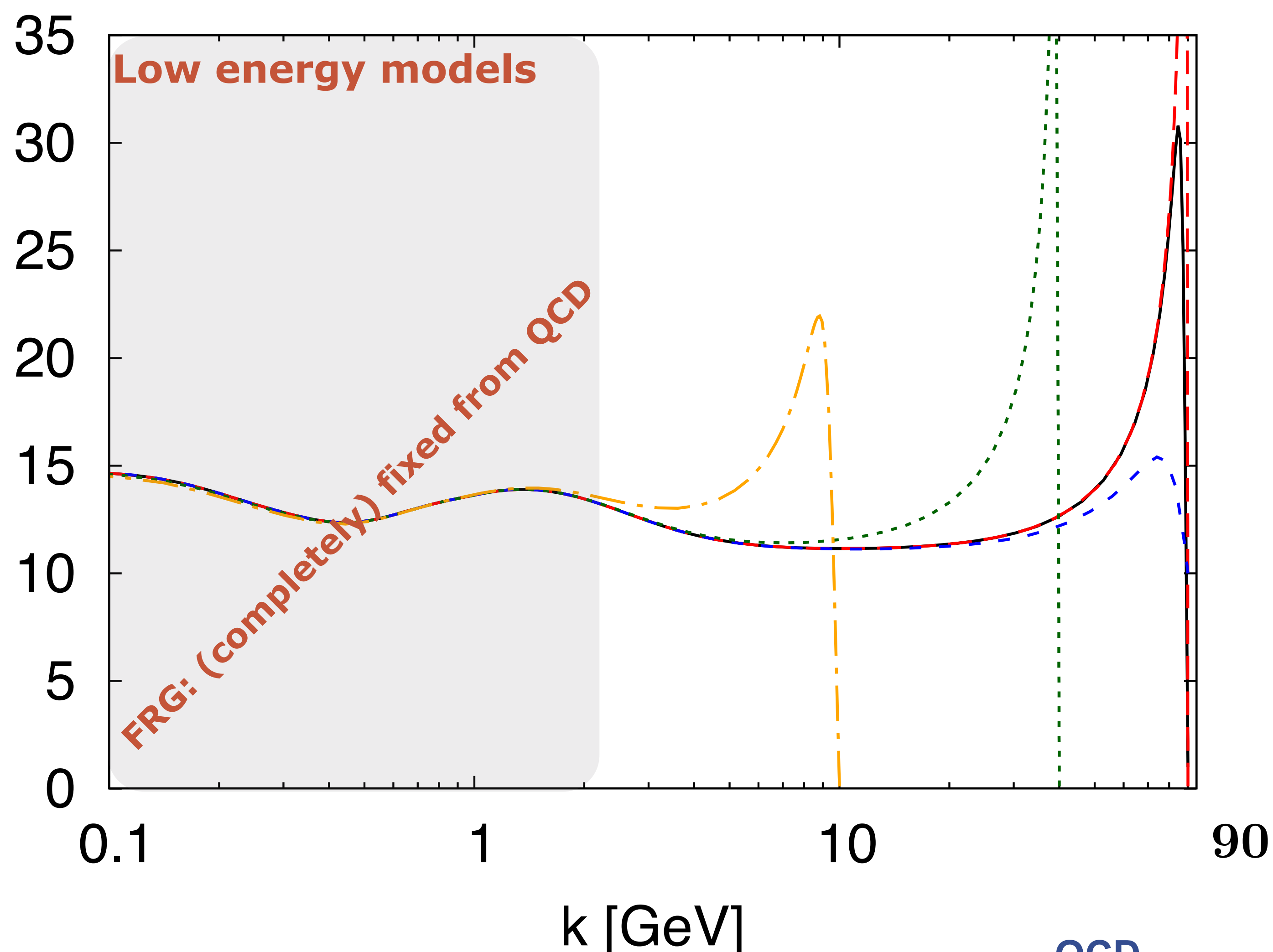


QCD



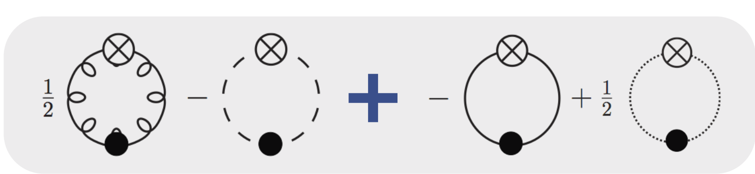


quark-meson coupling

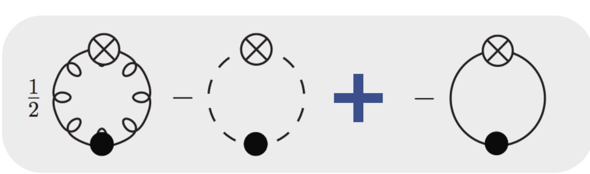


fQCD

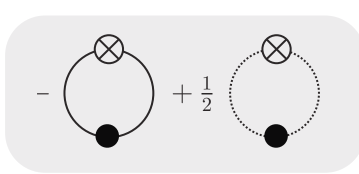
PQM-model



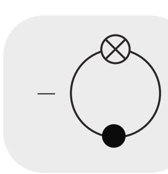
PNJL-model



QM-model



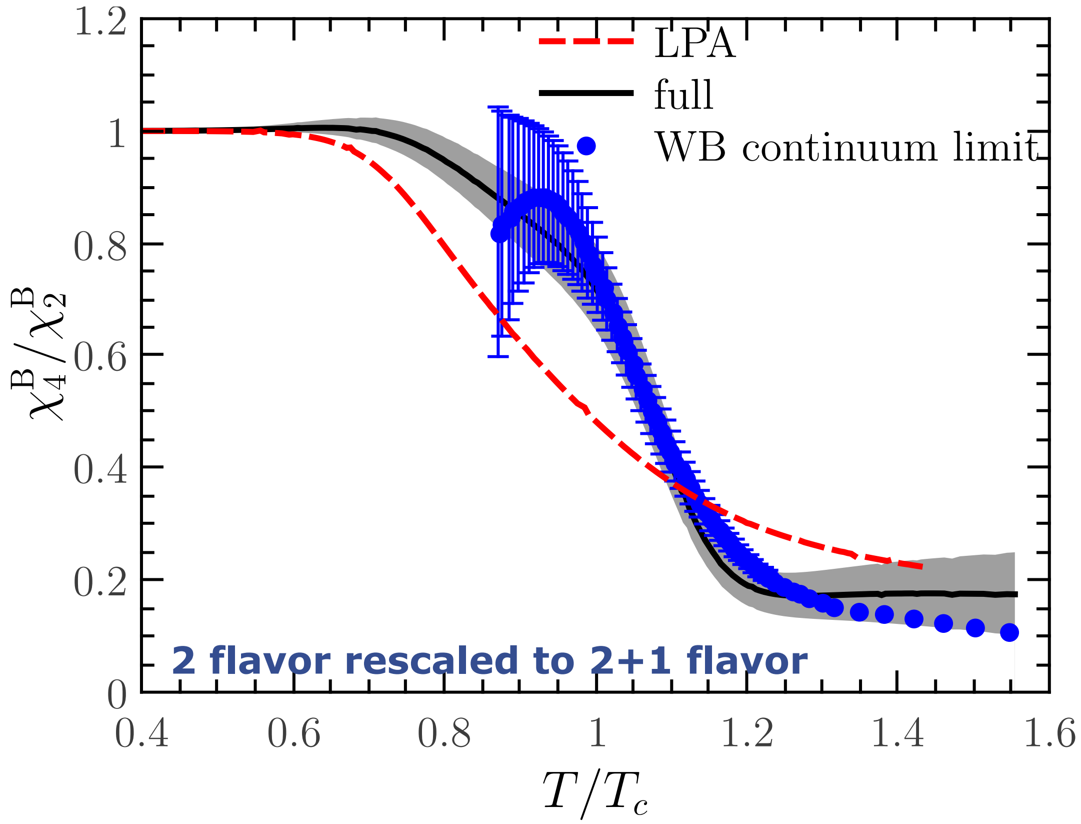
NJL-model



Thermodynamics

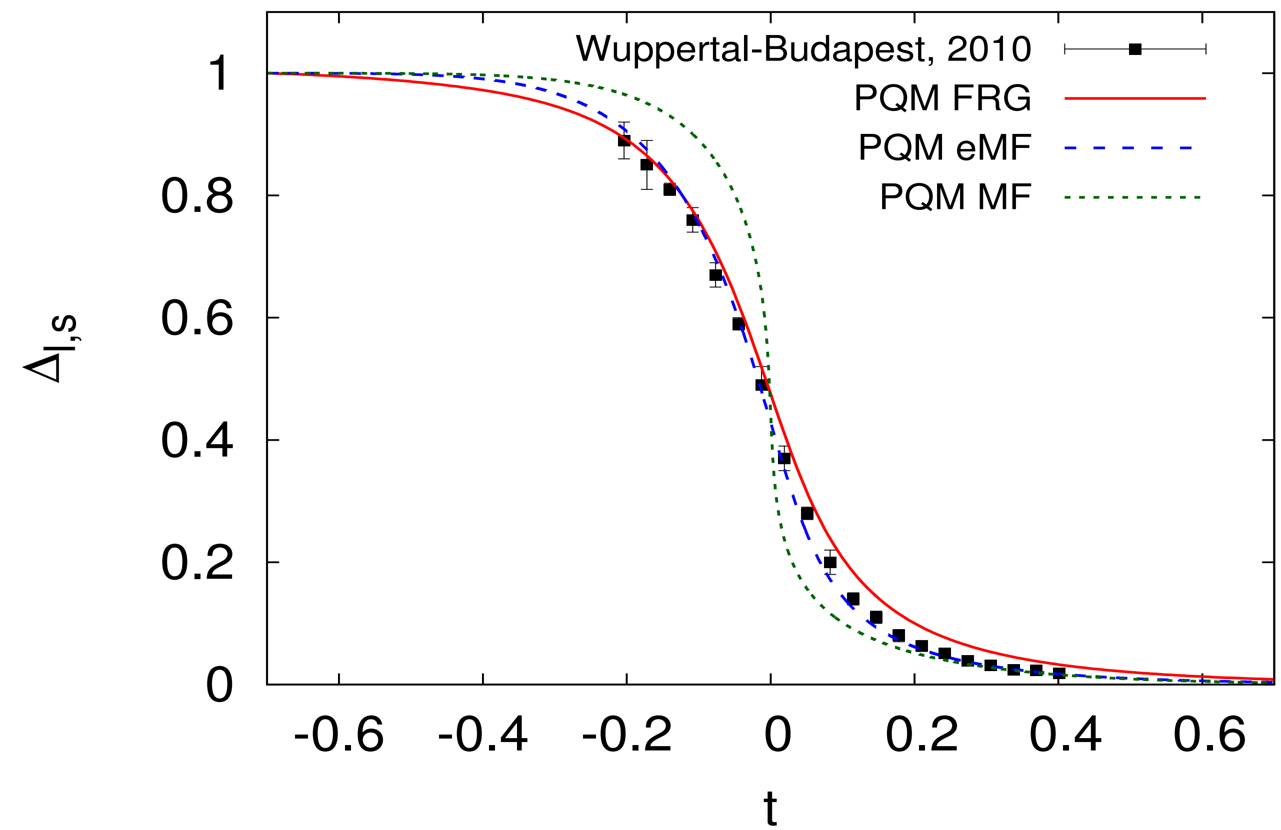
2+1 flavor QCD - enhanced PQM-model

Kurtosis

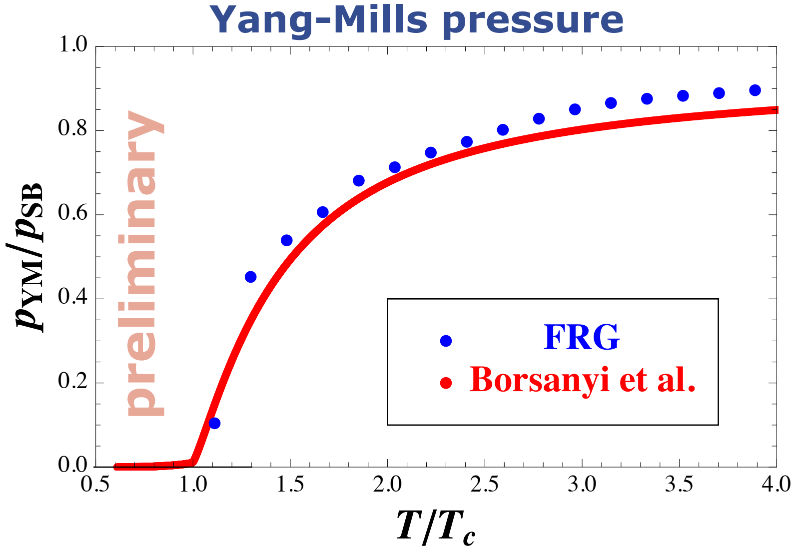


Fu, JMP, arXiv:1508.06504, accepted at PRD

reduced chiral condensate



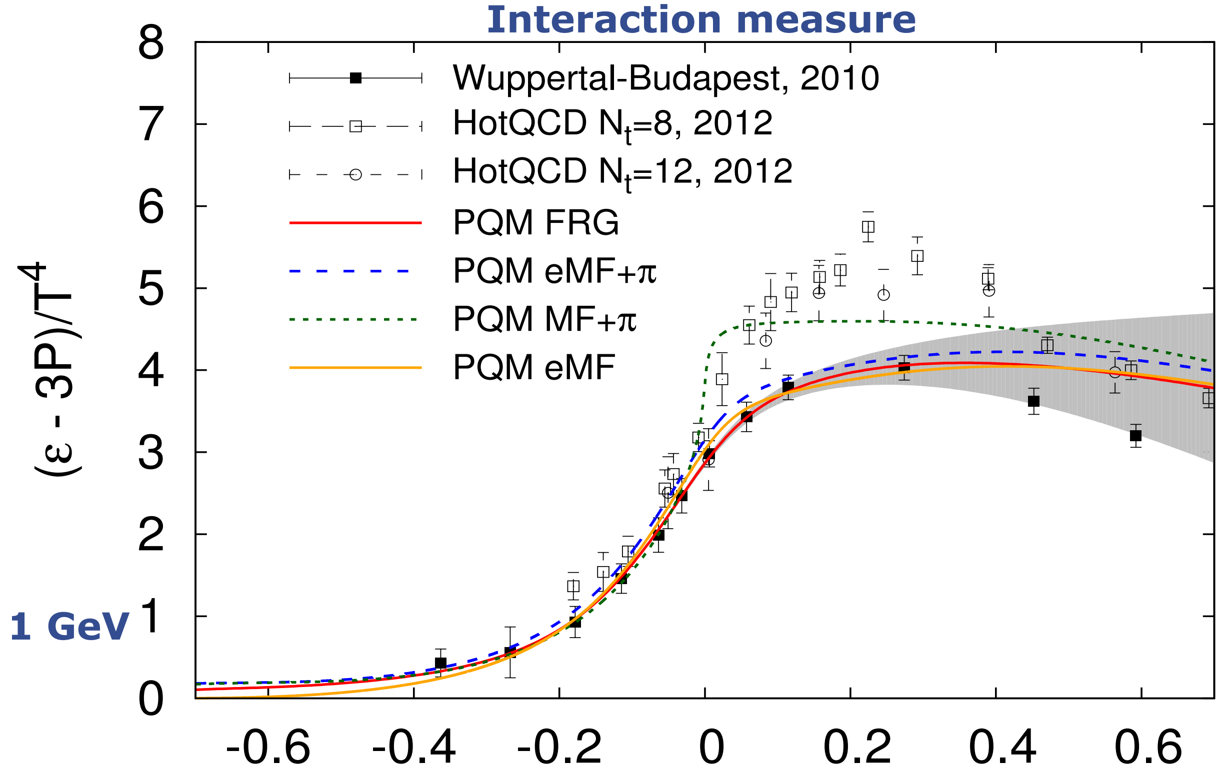
Yang-Mills pressure



Fister, JMP '11

Shaded area:
systematic error estimate
due to low initial UV scale 1 GeV

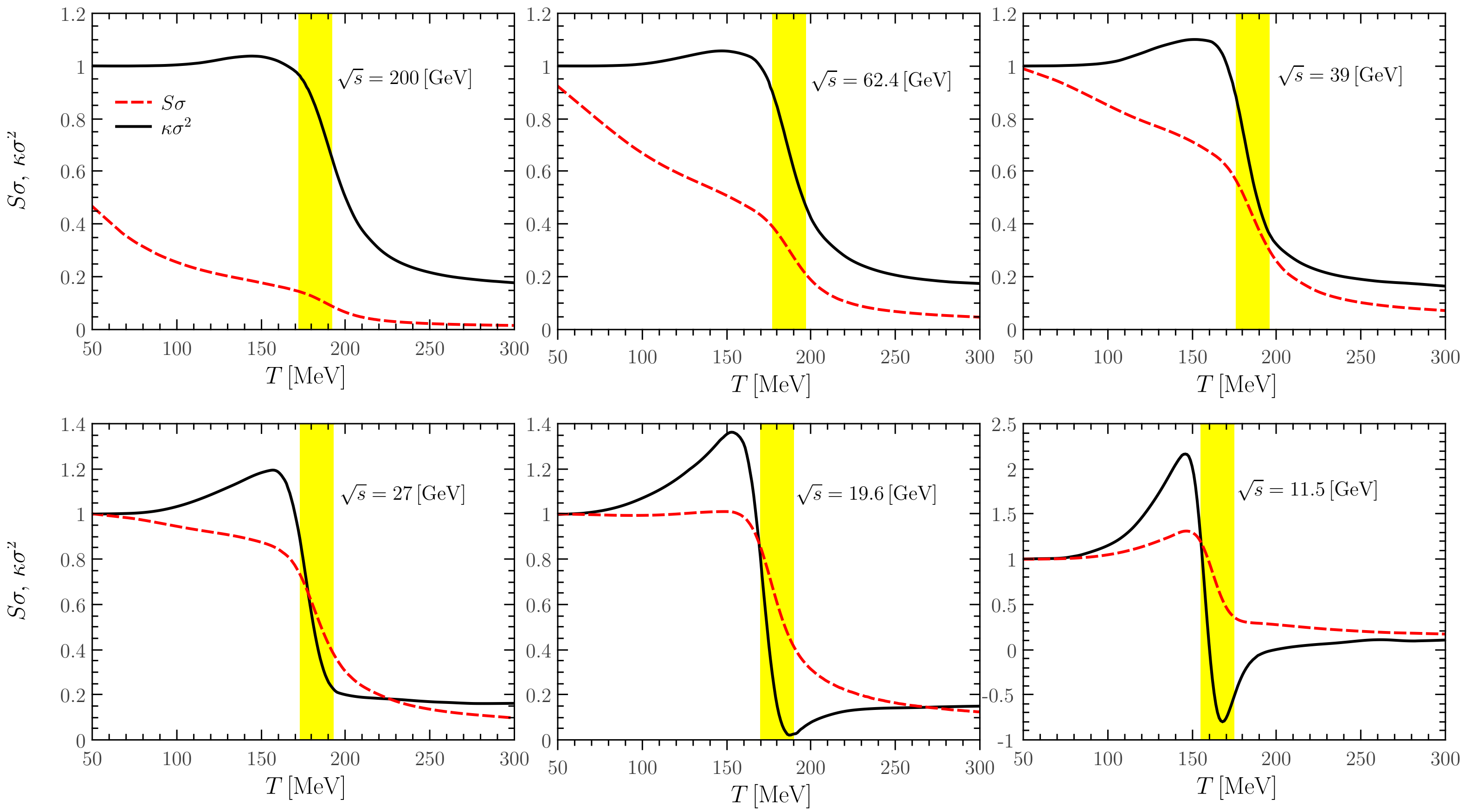
Interaction measure

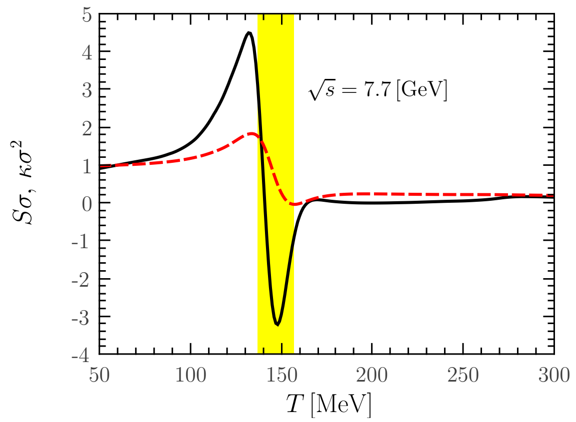


Herbst, Mitter et al, PLB 731 (2014) 248-256

Fluctuations

Skewness, Kurtosis

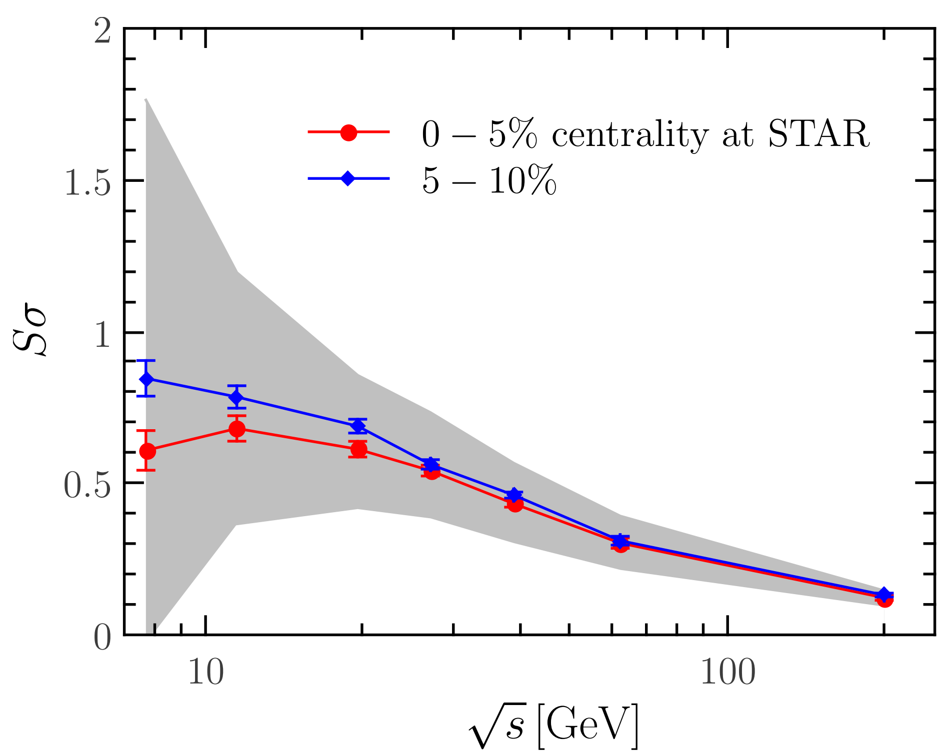


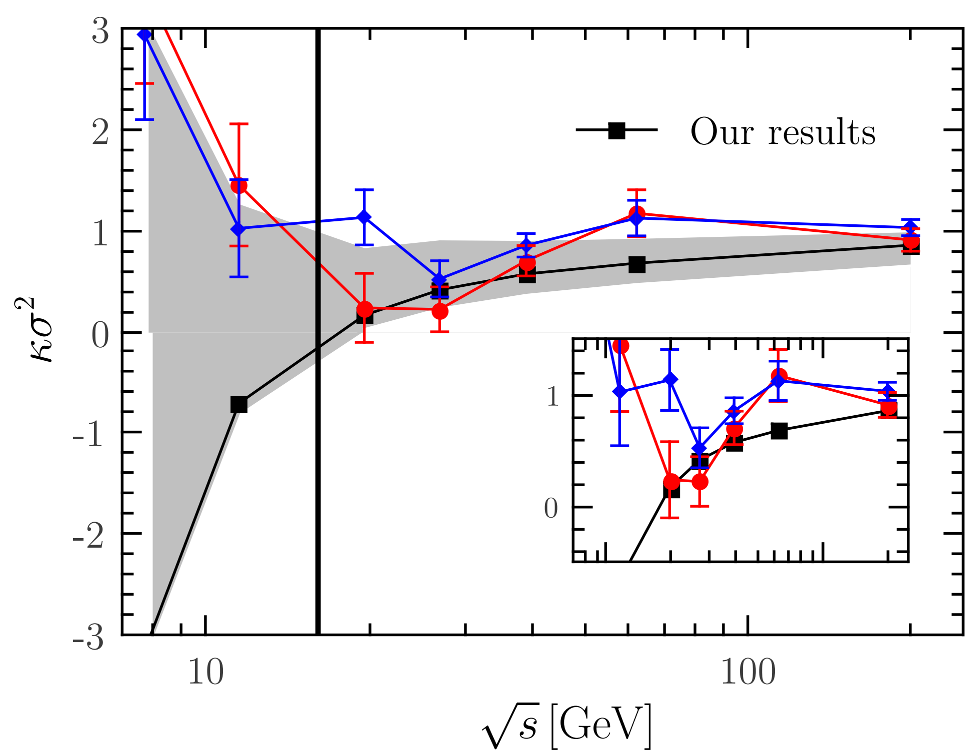


$$\chi_n^{\mathrm{B}} = \frac{\partial^n}{\partial(\mu_{\mathrm{B}}/T)^n}\frac{p}{T^4}$$

$$\sigma^2 = VT^3\chi_2^{\mathrm{B}}$$

$$\kappa = \chi_4^{\mathrm{B}}/(\chi_2^{\mathrm{B}}\sigma^2)$$

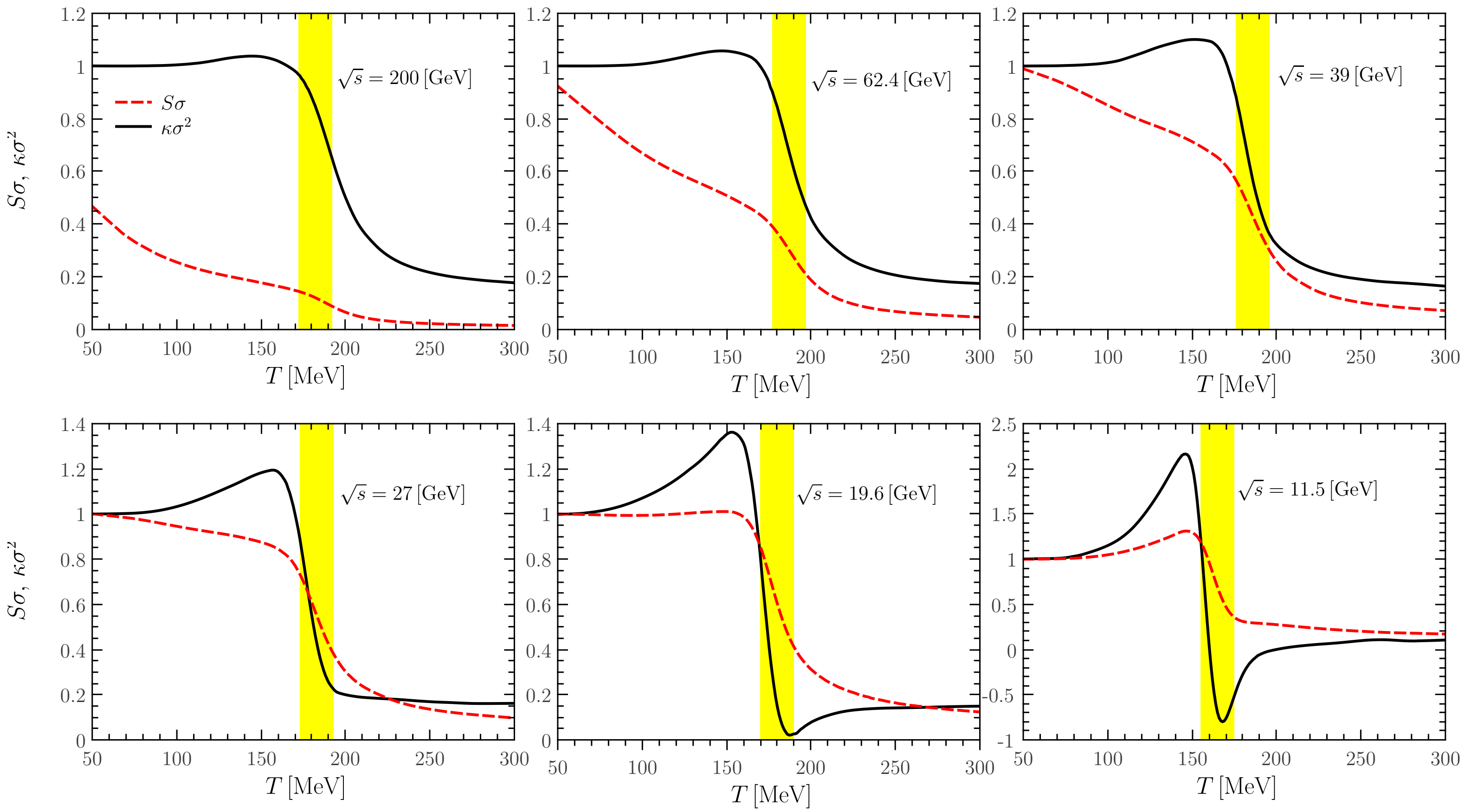


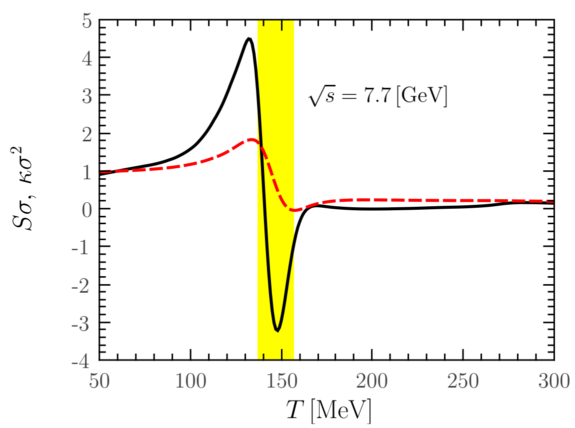


Fu, JMP, in prep.

Fluctuations

Skewness, Kurtosis





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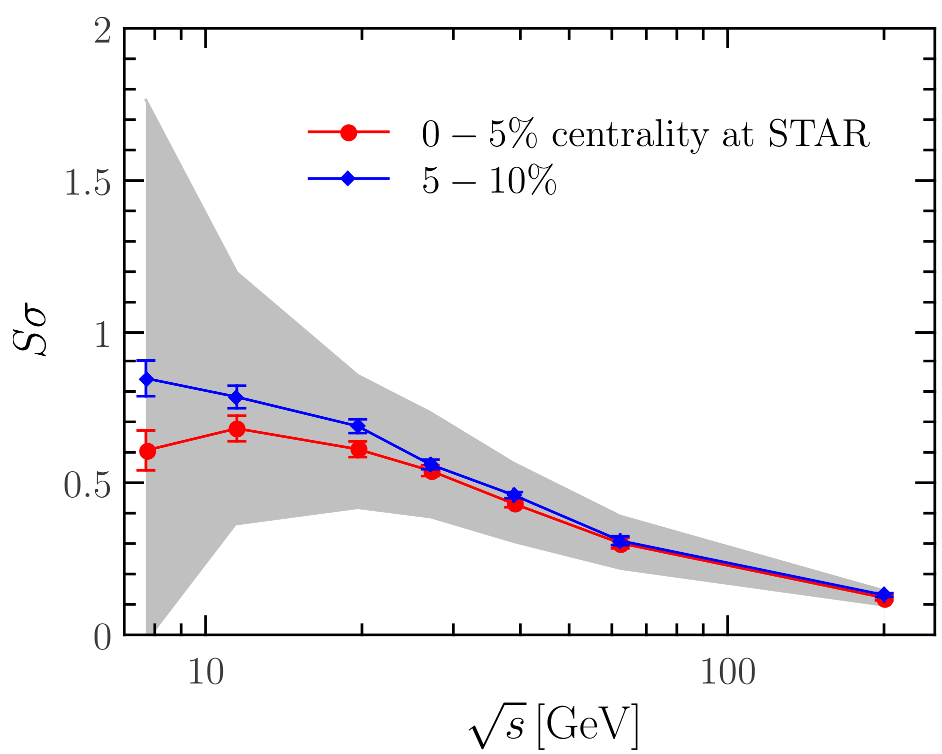
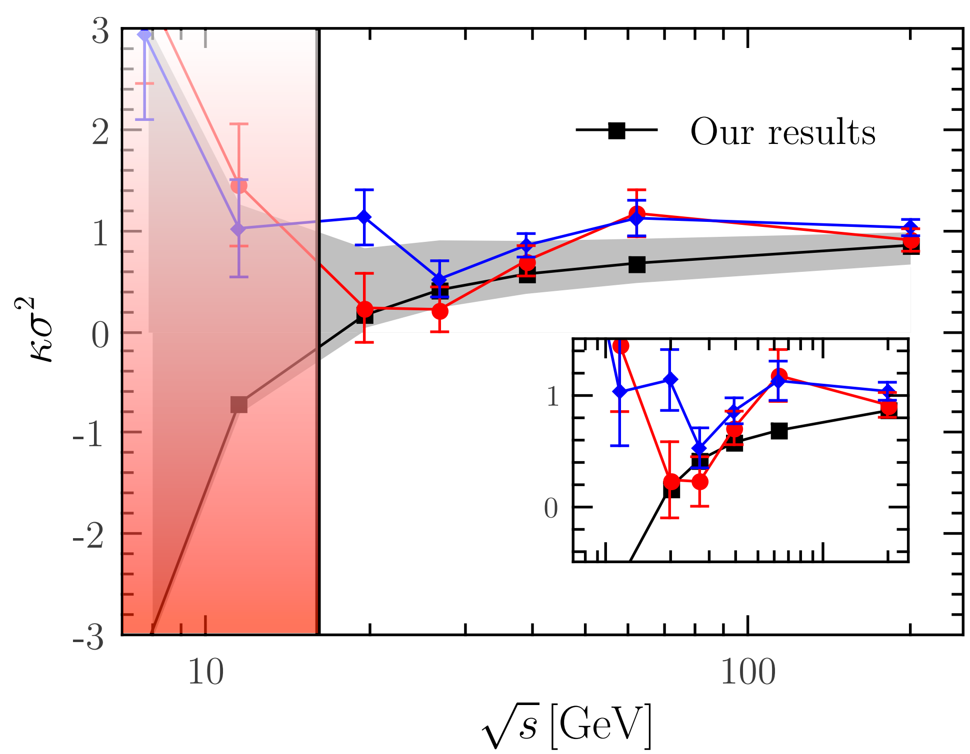
$$\sigma^2 = VT^3\chi_2^{\mathrm{B}}$$

$$\kappa = \chi_4^{\mathrm{B}}/(\chi_2^{\mathrm{B}}\sigma^2)$$

Fu, JMP, Schaefer, Rennecke, work in progress

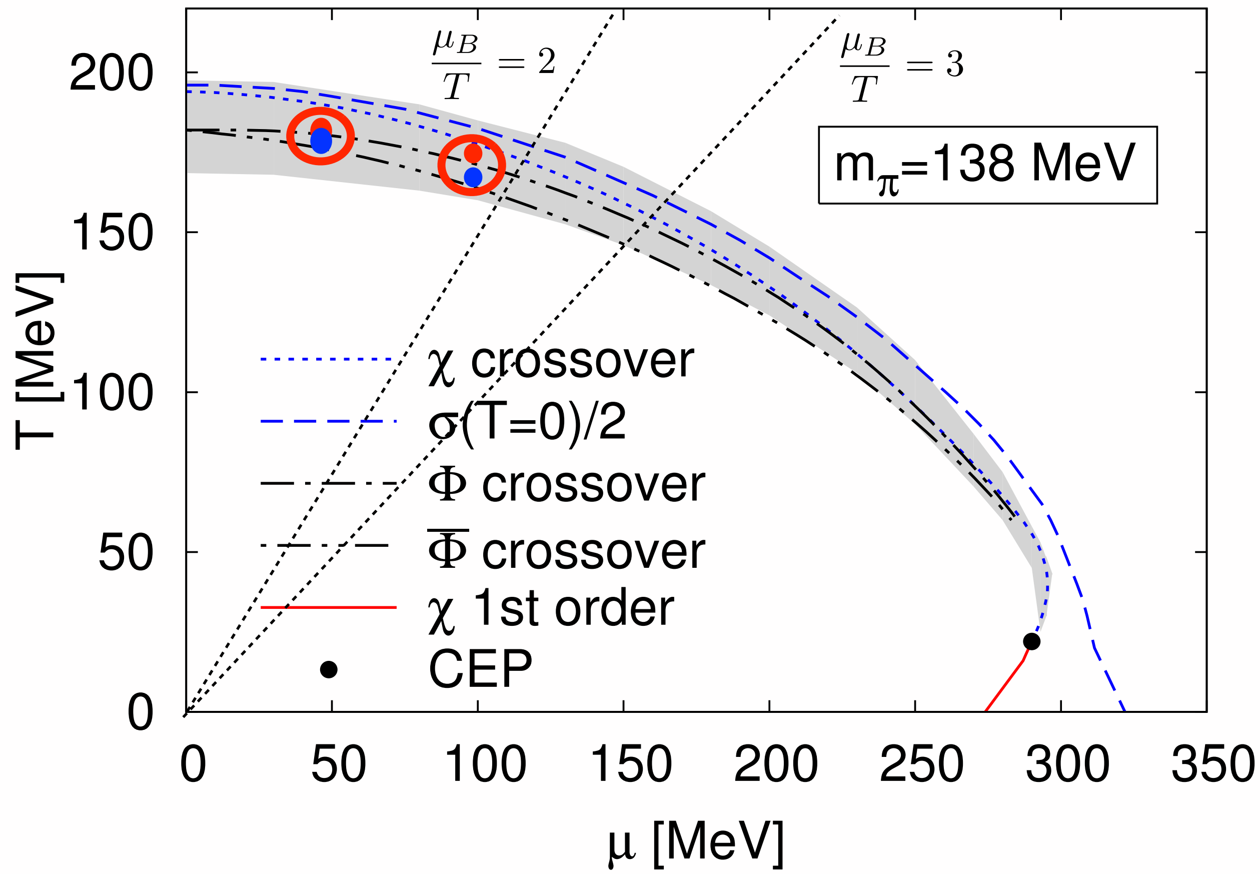




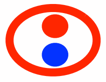
Fu, JMP, in prep.

Phase structure at finite density

Phase diagram of quantised 2-flavor PQM-model



Herbst, JMP, Schaefer, PLB 696 (2011) 58-67
PRD 88 (2013) 1, 014007

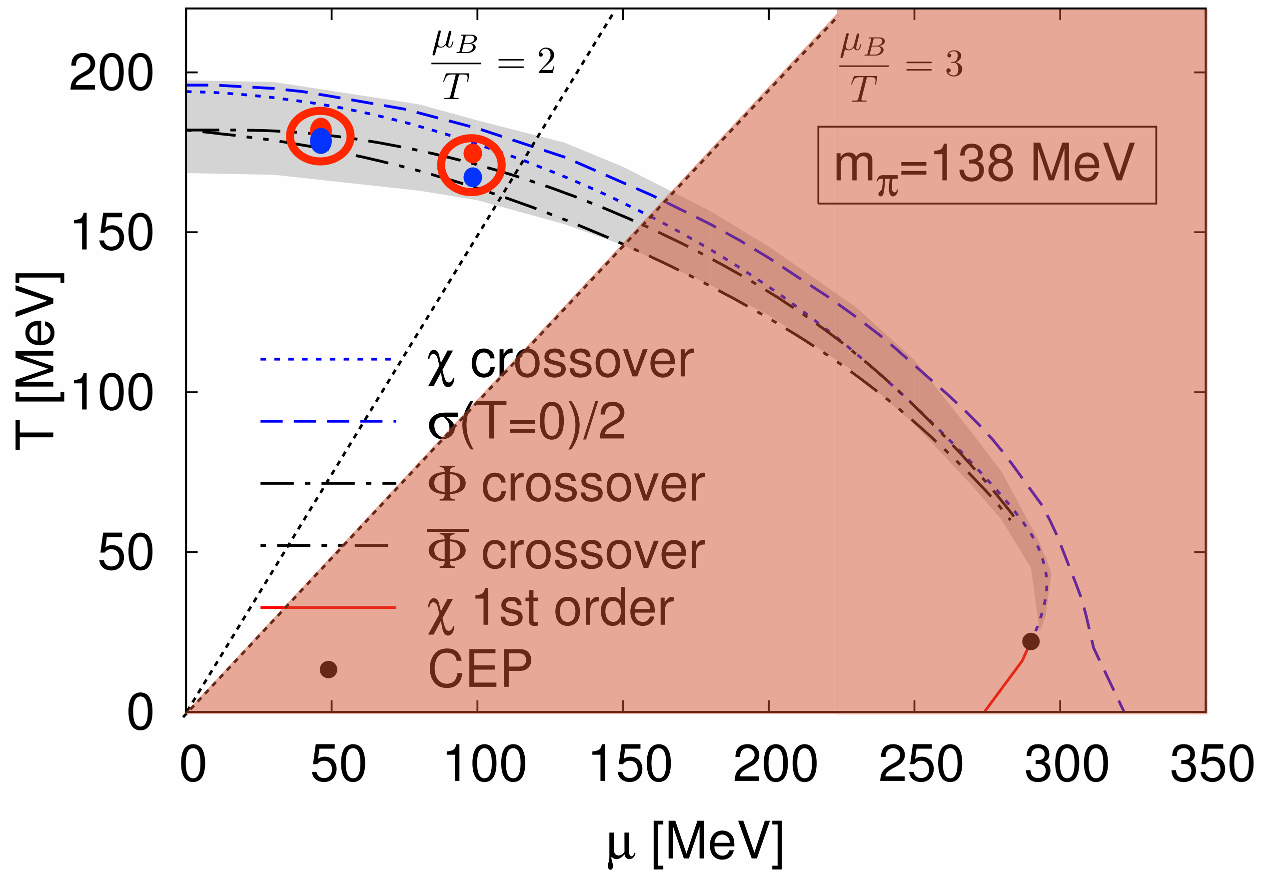


FRG QCD results at finite density

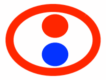
Haas, Braun, JMP '09, unpublished

Phase structure at finite density

Phase diagram of quantised 2-flavor PQM-model



Herbst, JMP, Schaefer, PLB 696 (2011) 58-67
PRD 88 (2013) 1, 014007



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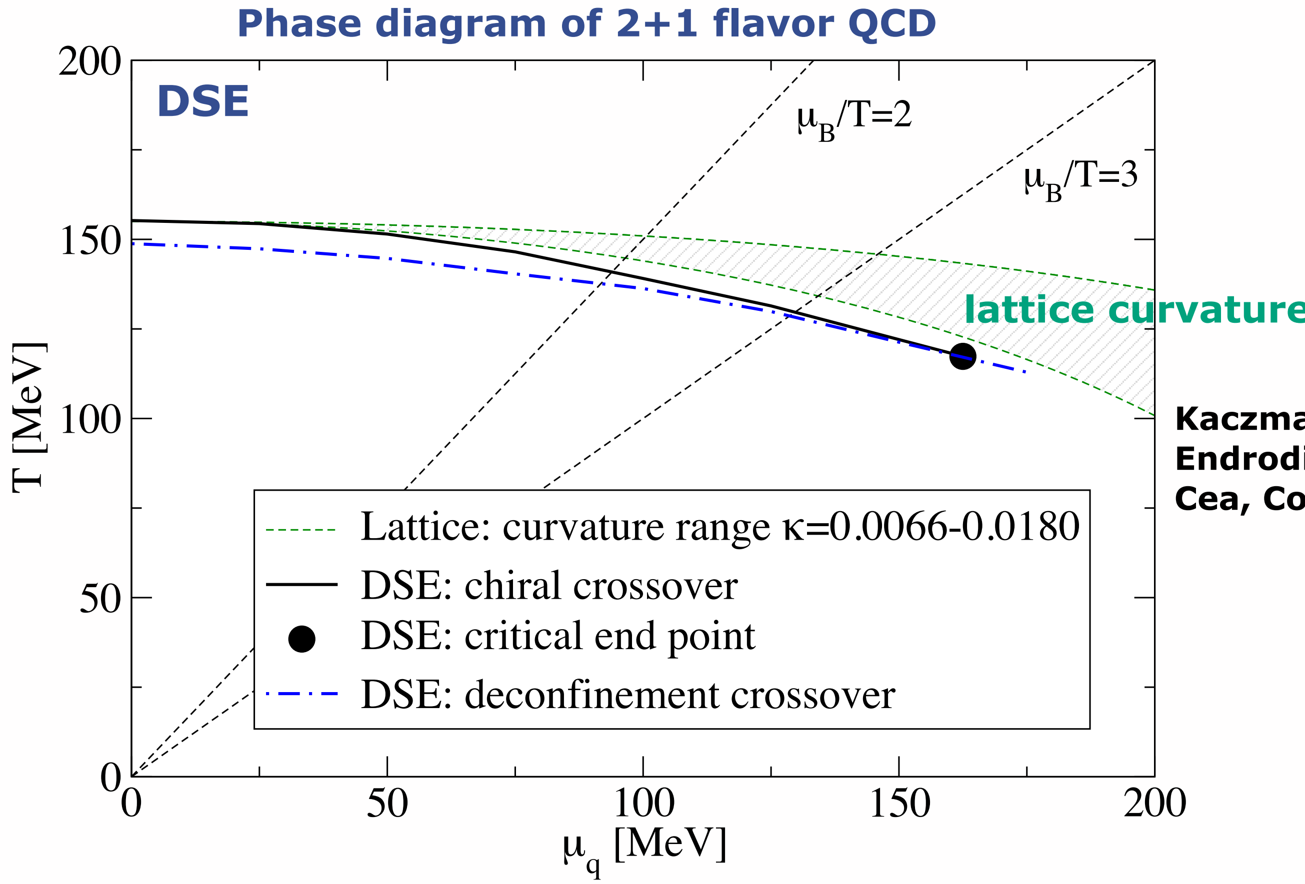
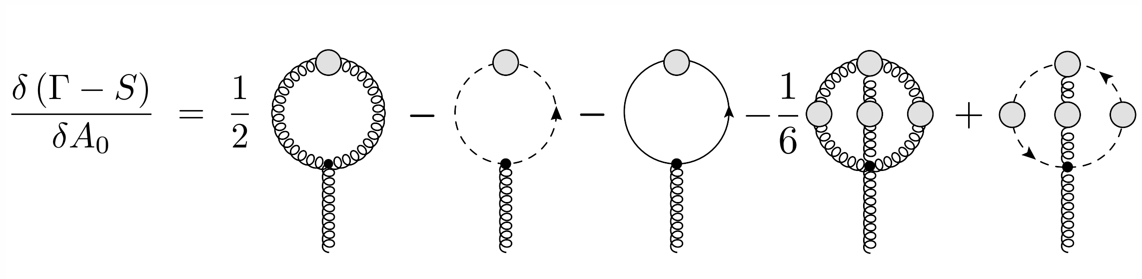
Phase structure at finite density



Kaczmarek at al. '11
Endrodi, Fodor, Katz, Szabo '11
Cea, Cosmai, Papa '14

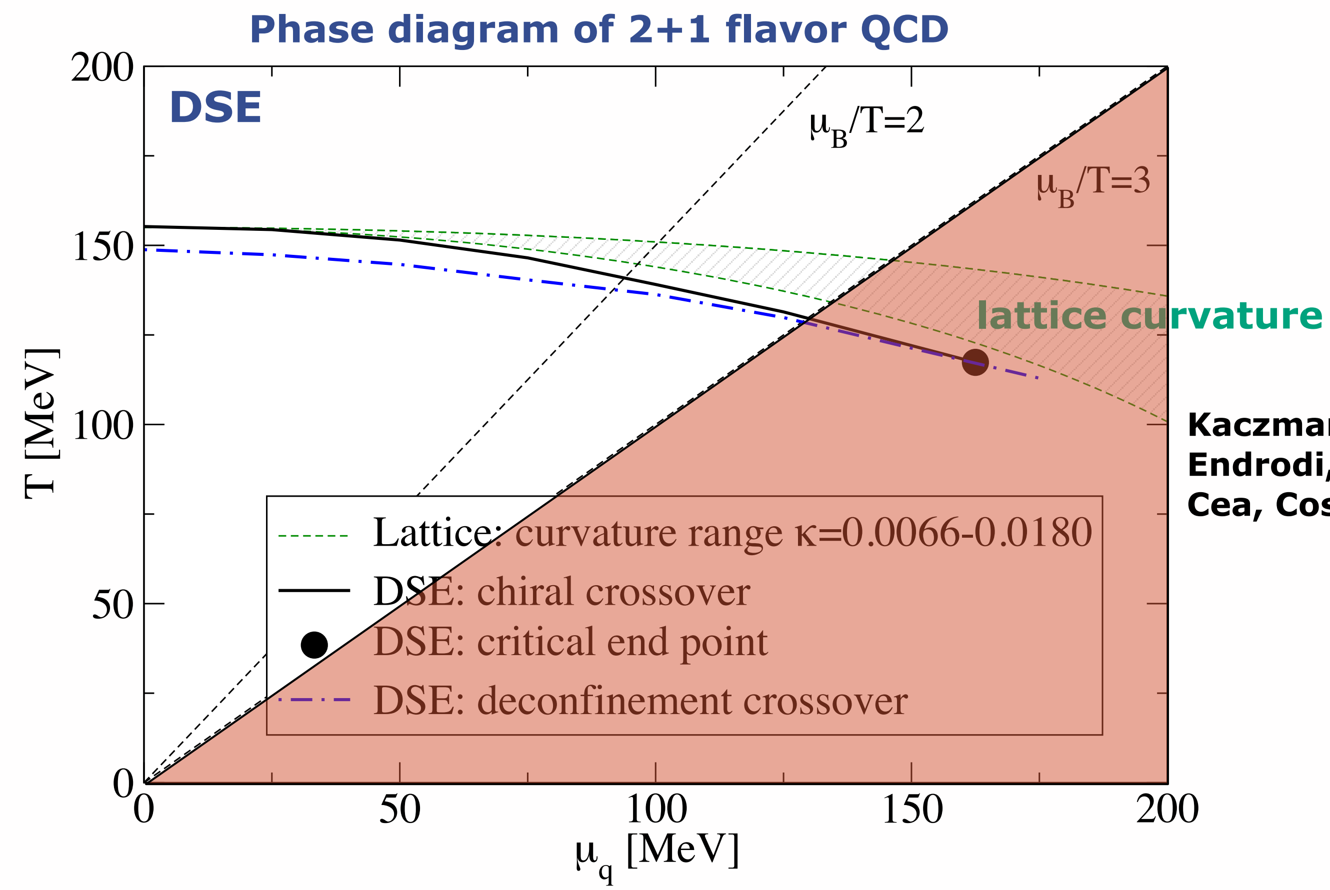
Fischer, Fister, Luecker, JMP, PLB732 (2014) 248

Fischer, Luecker, Welzbacher, PRD 90 (2014) 034022



Fister, JMP, PRD 88 (2013) 045010

Phase structure at finite density



Kaczmarek at al. '11
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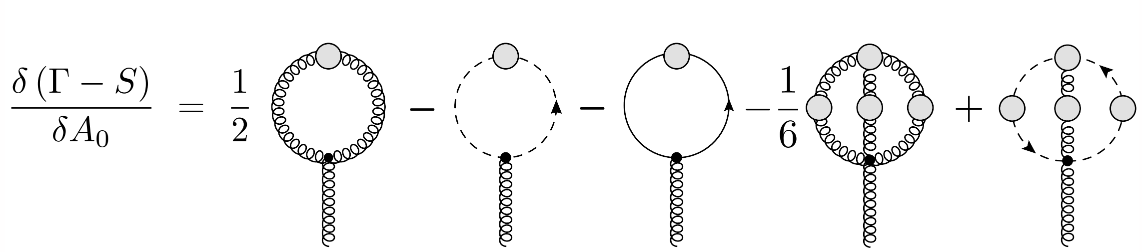
Fischer, Fister, Luecker, JMP, PLB732 (2014) 248

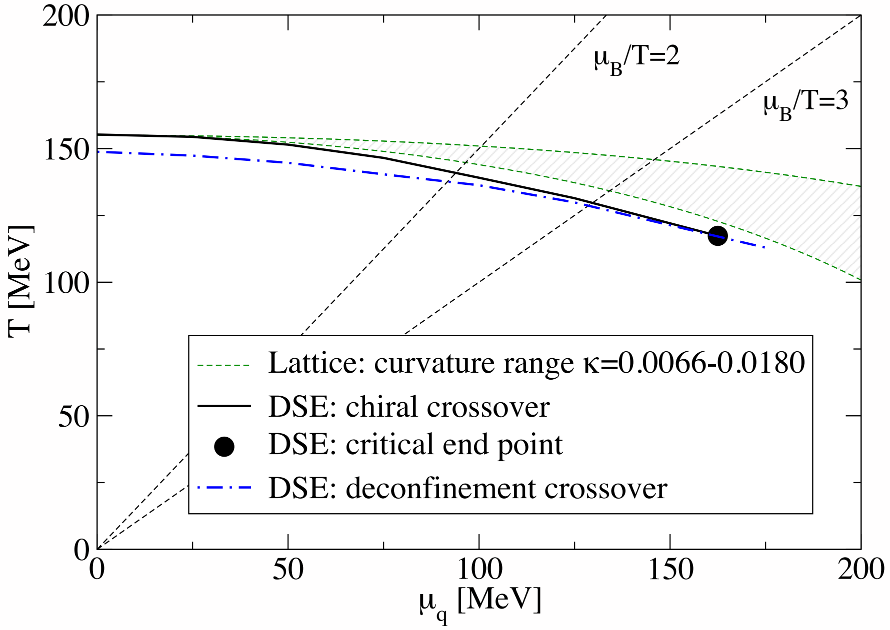
Fischer, Luecker, Welzbacher, PRD 90 (2014) 034022



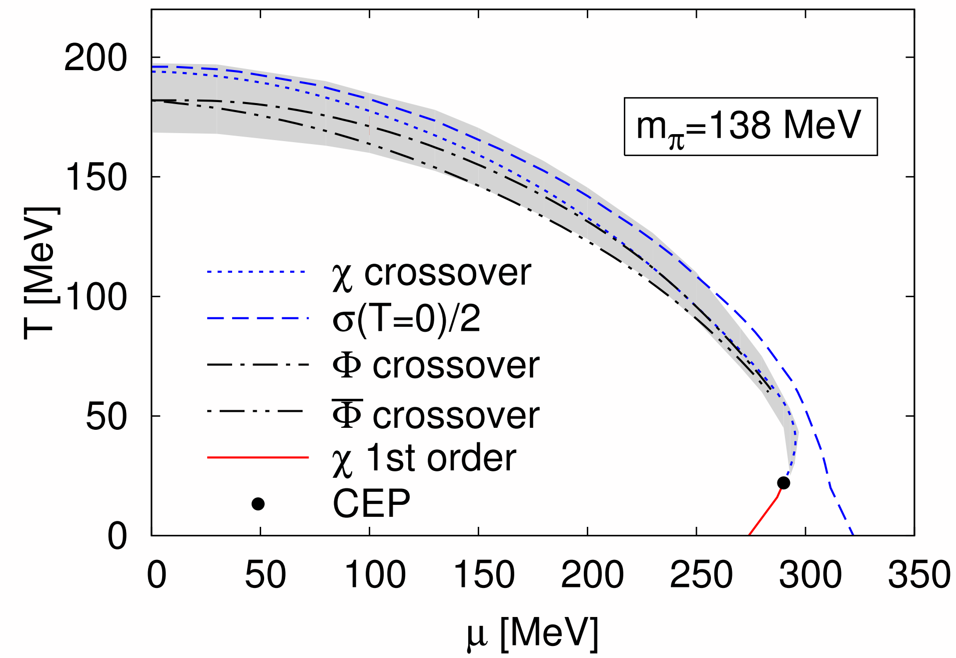
Fister, JMP, PRD 88 (2013) 045010

Phase structure at finite density

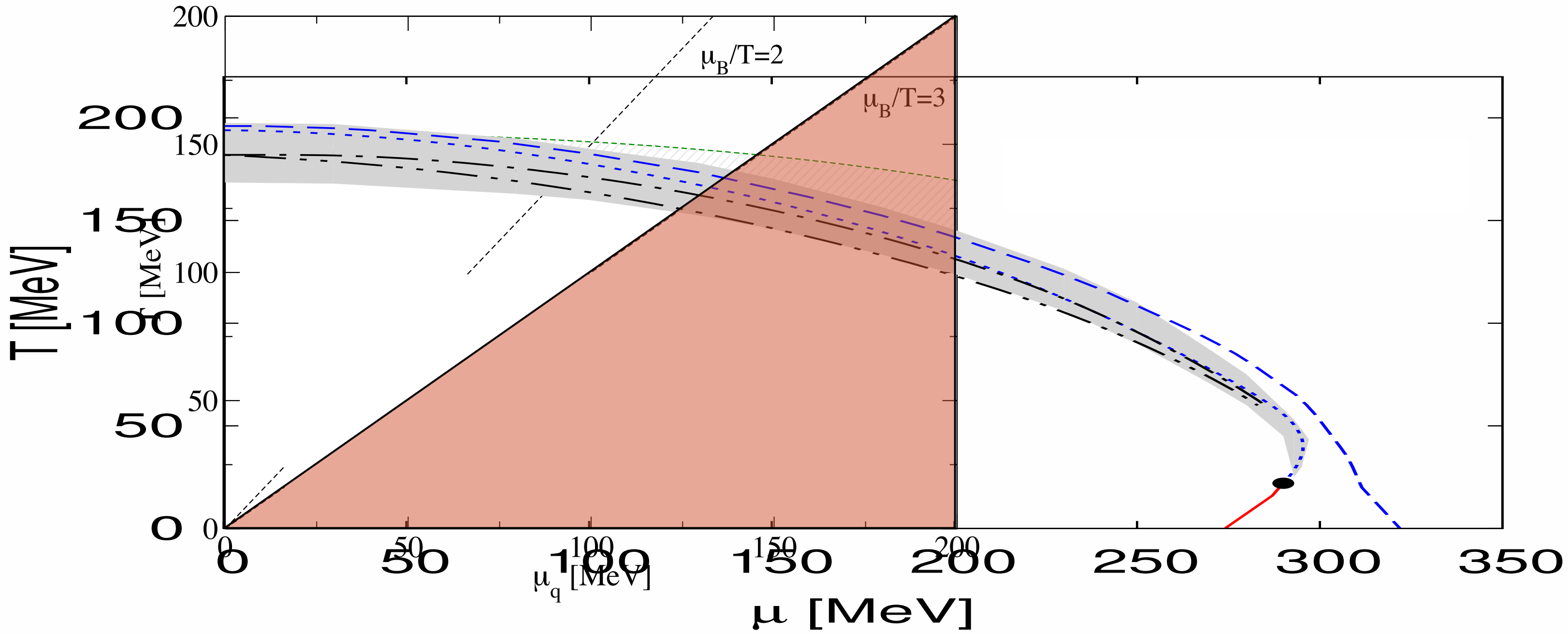
Phase diagram of 2+1 flavor QCD



Phase diagram of quantised 2-flavor PQM-model



Comparison with 2 flavor vs 2+1 flavor scale matching of T_c



Outline

- Functional Approaches to QCD & the FRG

- Phase structure of QCD

- Hadron spectrum & QCD transport

- Outlook

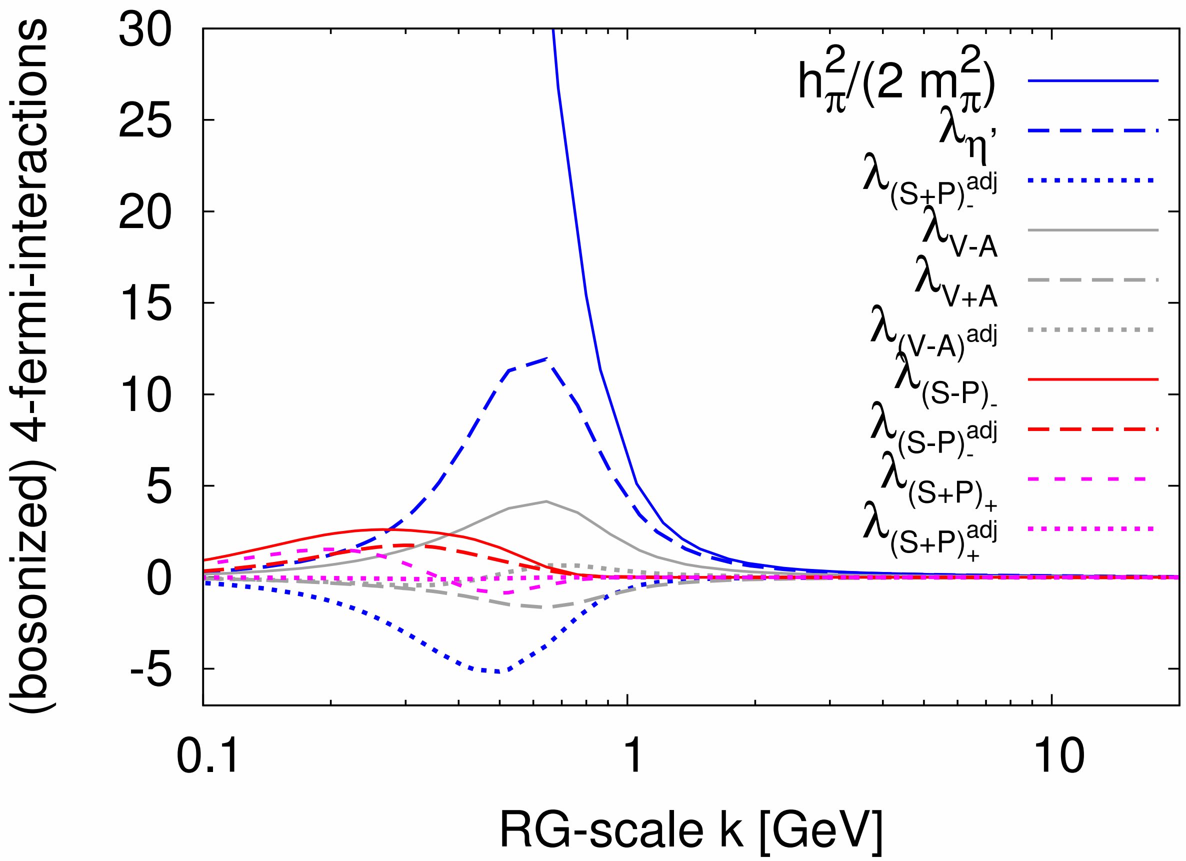
A glimpse at the hadron spectrum

preliminary

four-fermi scattering amplitude at pion pole

$$\langle \bar{q}\vec{\sigma}\gamma_5 q(p) \quad \bar{q}\vec{\sigma}\gamma_5 q(-p)\rangle \quad \to \quad \frac{\chi_{\bar{q}\pi q}\bar{\chi}_{\bar{q}\pi q}}{p^2 - m_\pi^2} \quad + \quad \text{finite terms}$$

$$\Gamma^{(4)}(p_1, p_2, p_3, p_4)$$

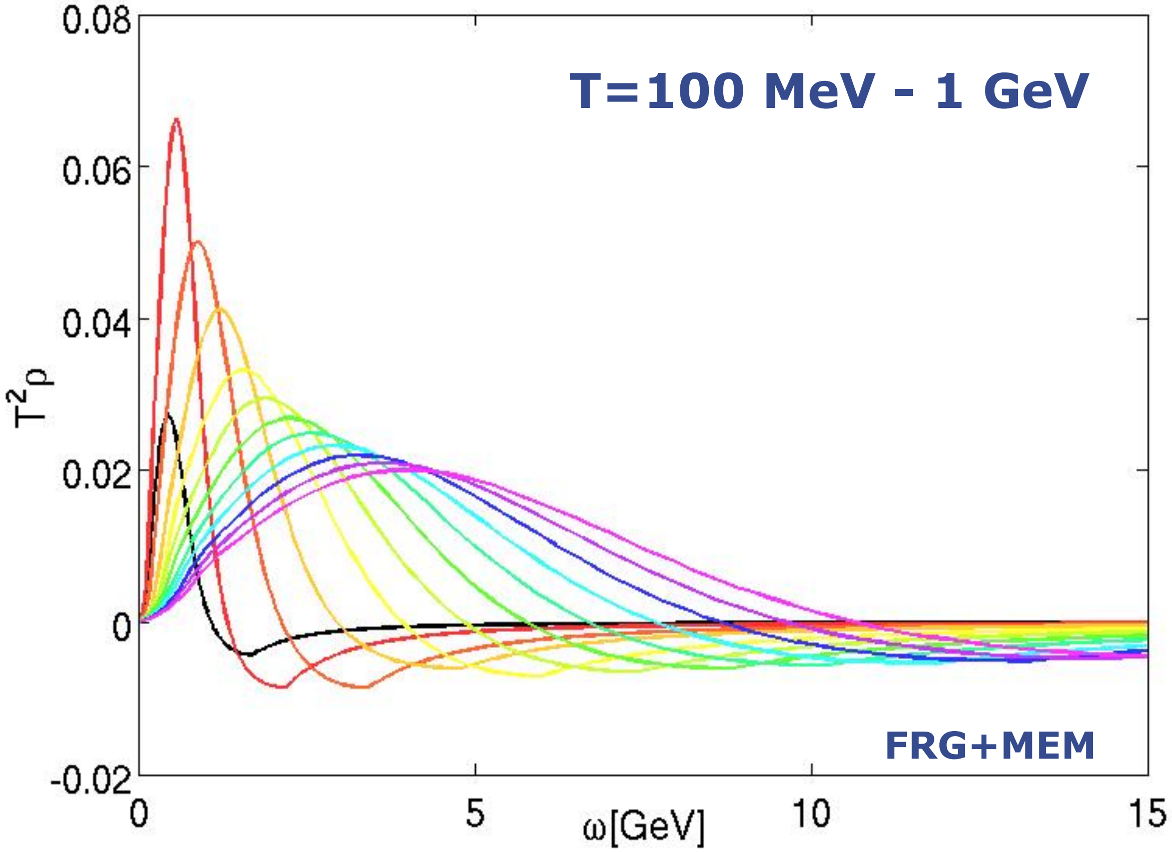


Mitter, JMP, Strodthoff, in preparation

... and now for something completely different ...

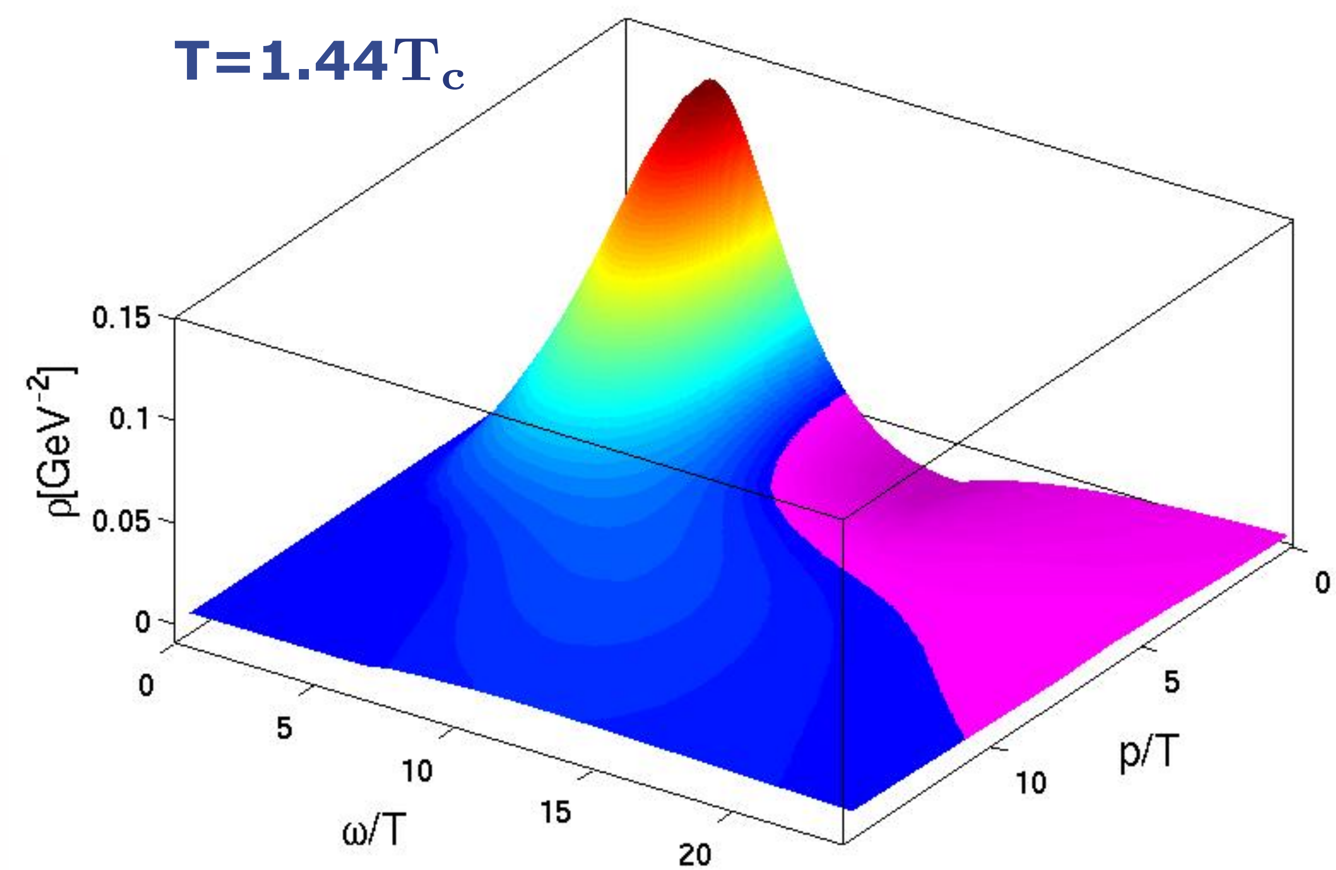
Real time correlation functions & transport

Gluon spectral function at finite T



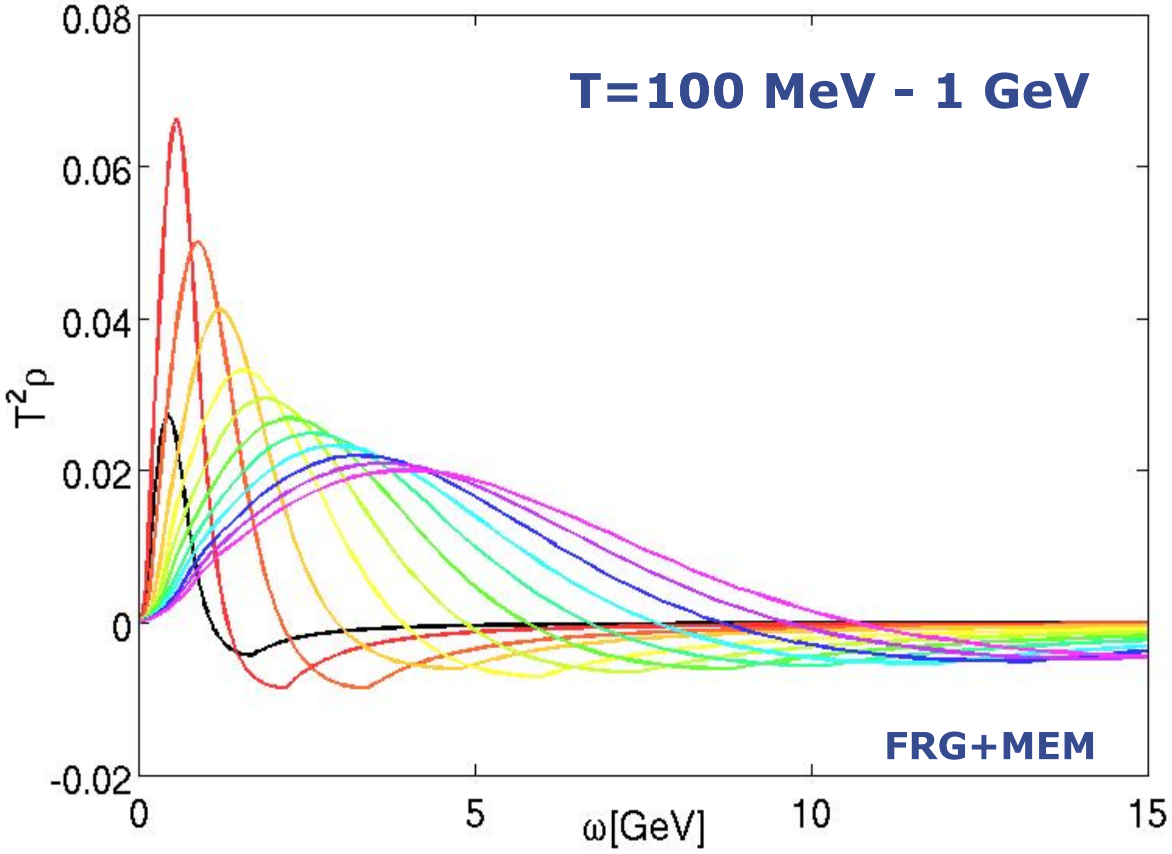
transversal

MEM

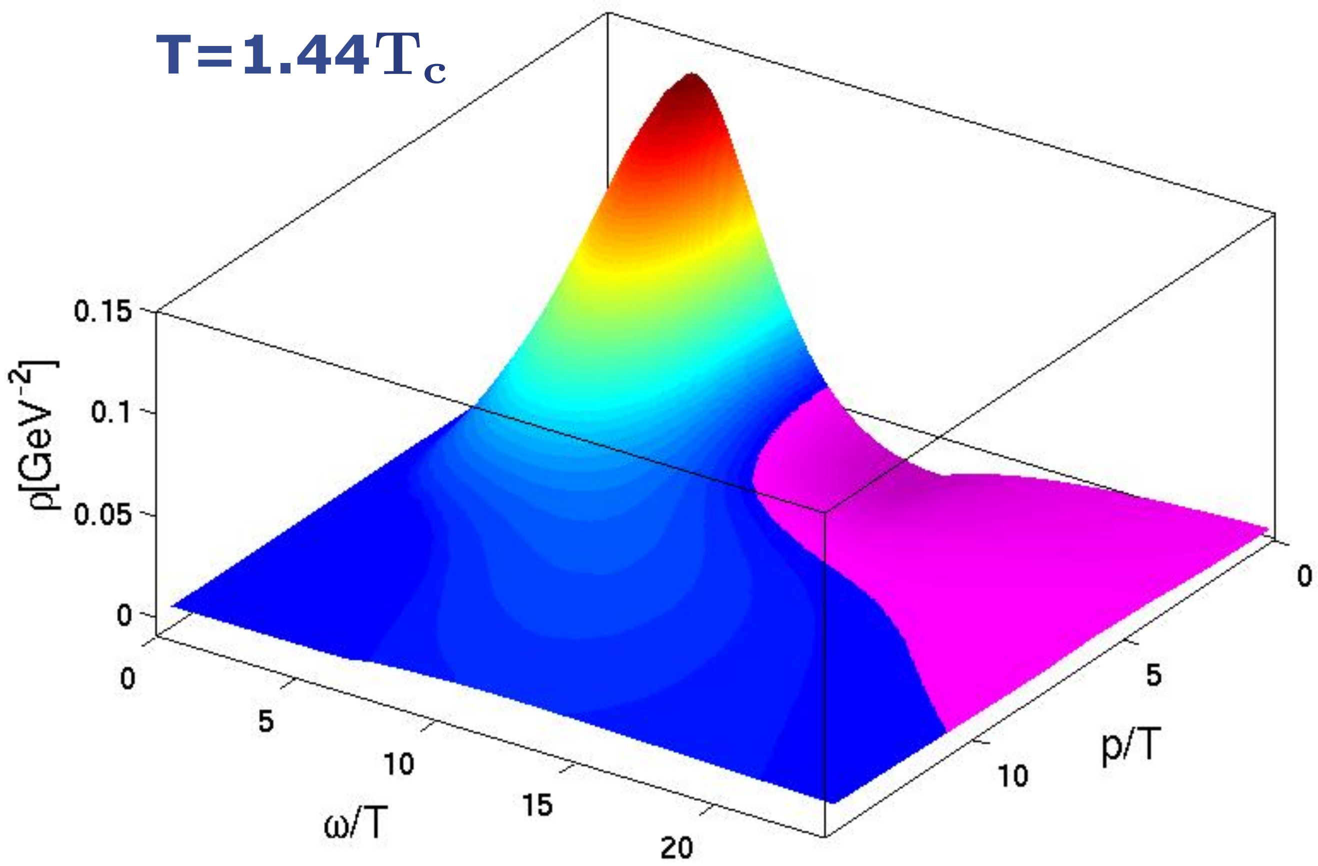


Haas, Fister, JMP, PRD 90 (2014) 9, 091501

Gluon spectral function at finite T



transversal



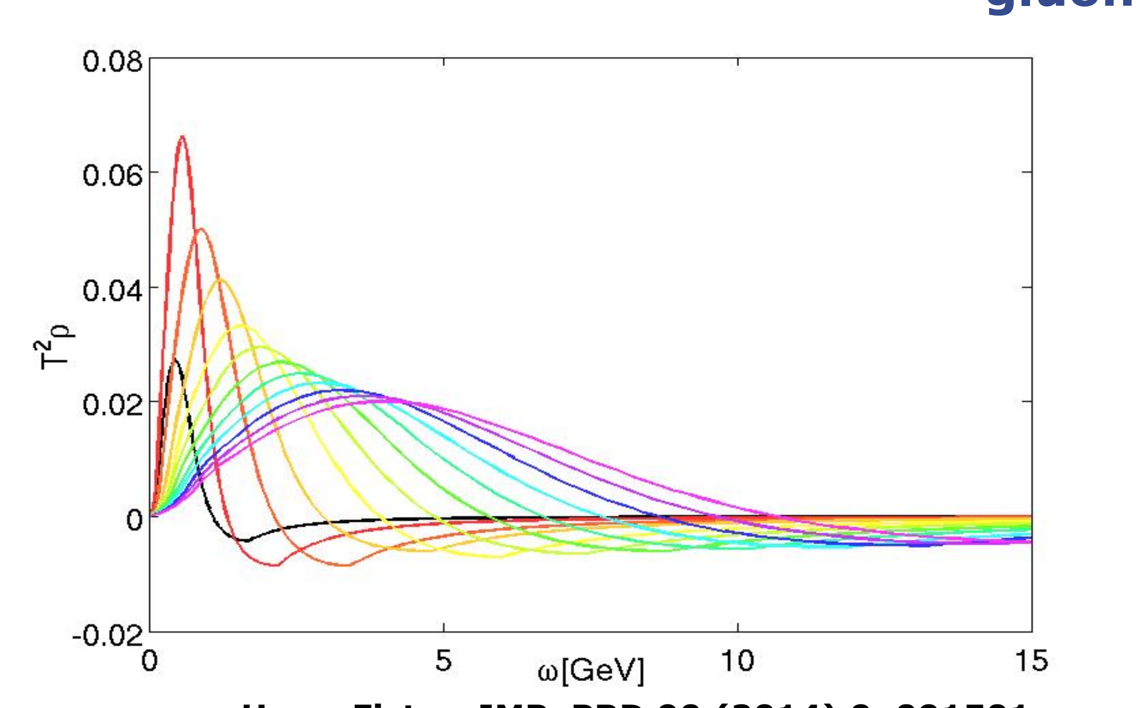
'Those are my methods (principles), and if you don't like them...well, I have others'

direct computation **Groucho Marx**

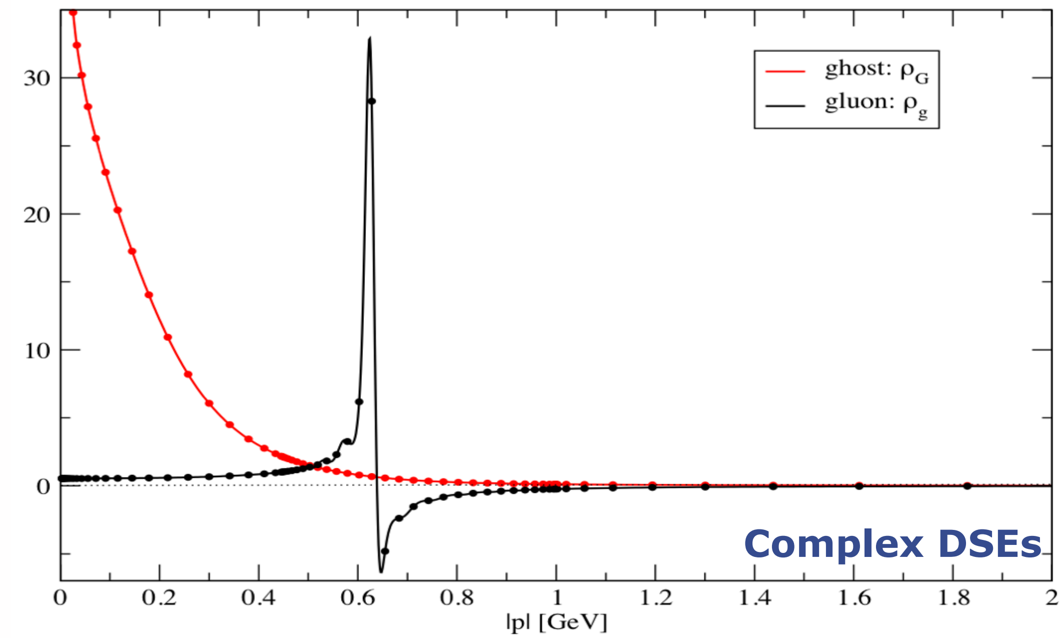
Haas, Fister, JMP, PRD 90 (2014) 9, 091501

Dynamics

gluon spectral functions

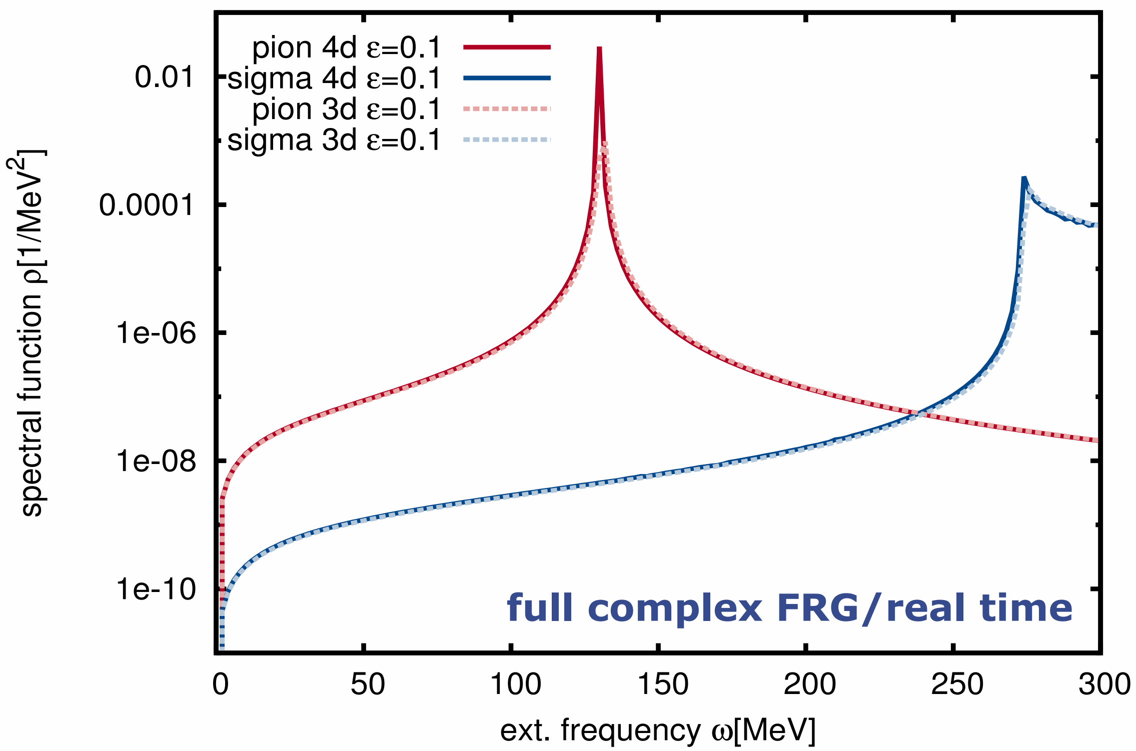


Haas, Fister, JMP, PRD 90 (2014) 9, 091501



Strauss, Fischer, Kellermann, PRL 109 (2012) 252001

pion and sigma spectral functions

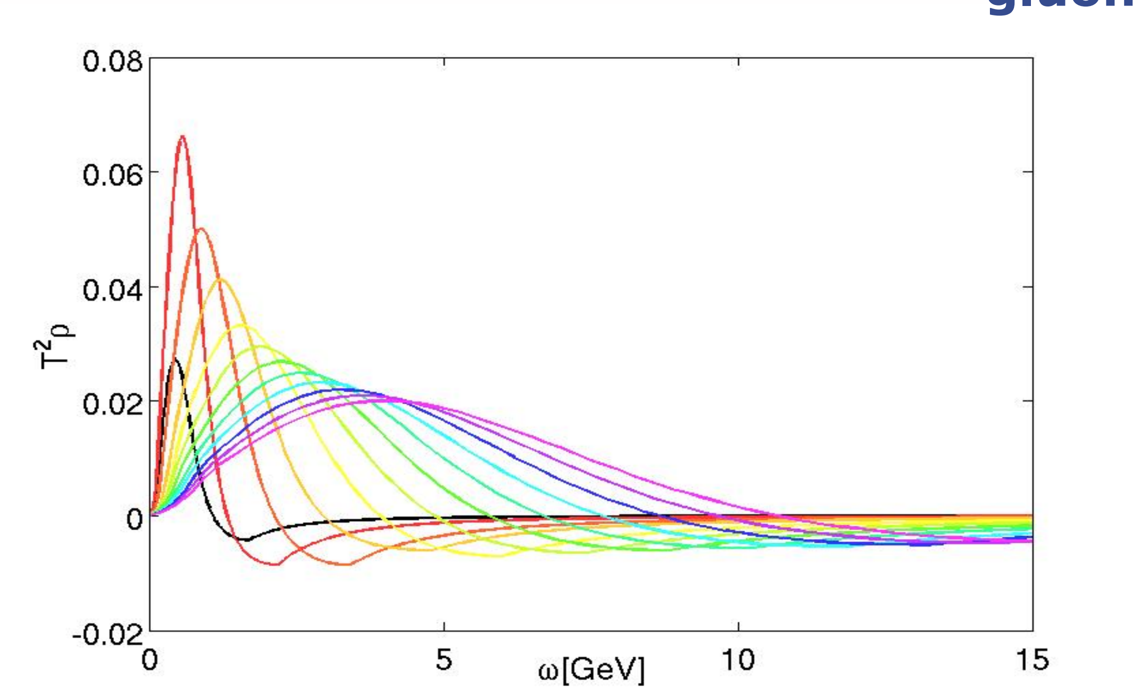


analytic complex FRG

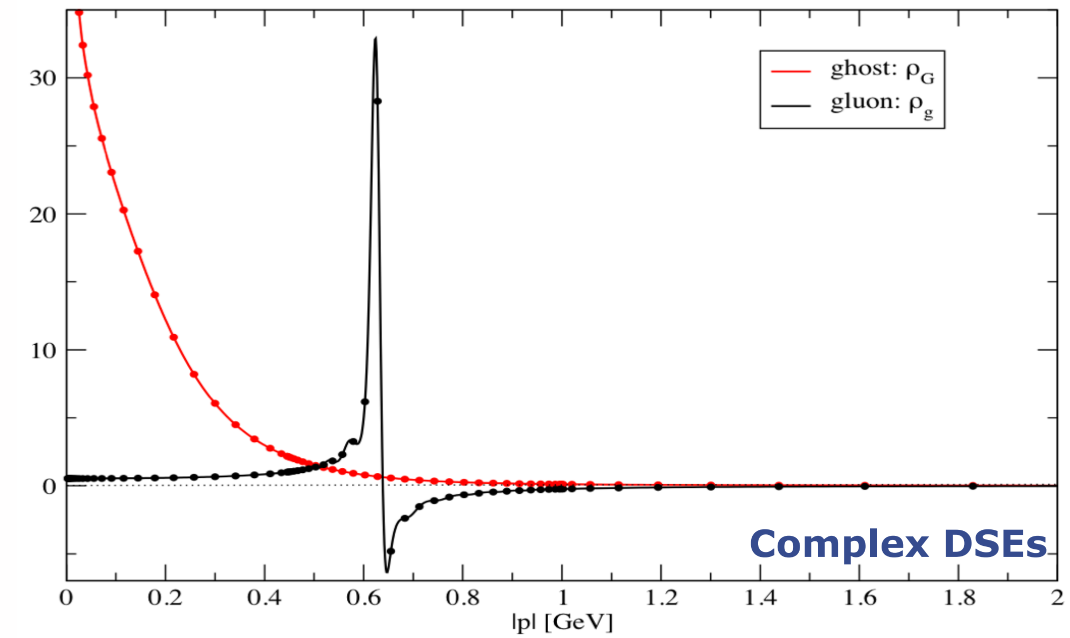
Tripolt, Strodthoff, von Smekal, Wamach, PRD 89 (2014) 034010
Kamikado, Strodthoff, von Smekal, Wambach, EPJ C74 (2014) 2806

Dynamics

gluon spectral functions

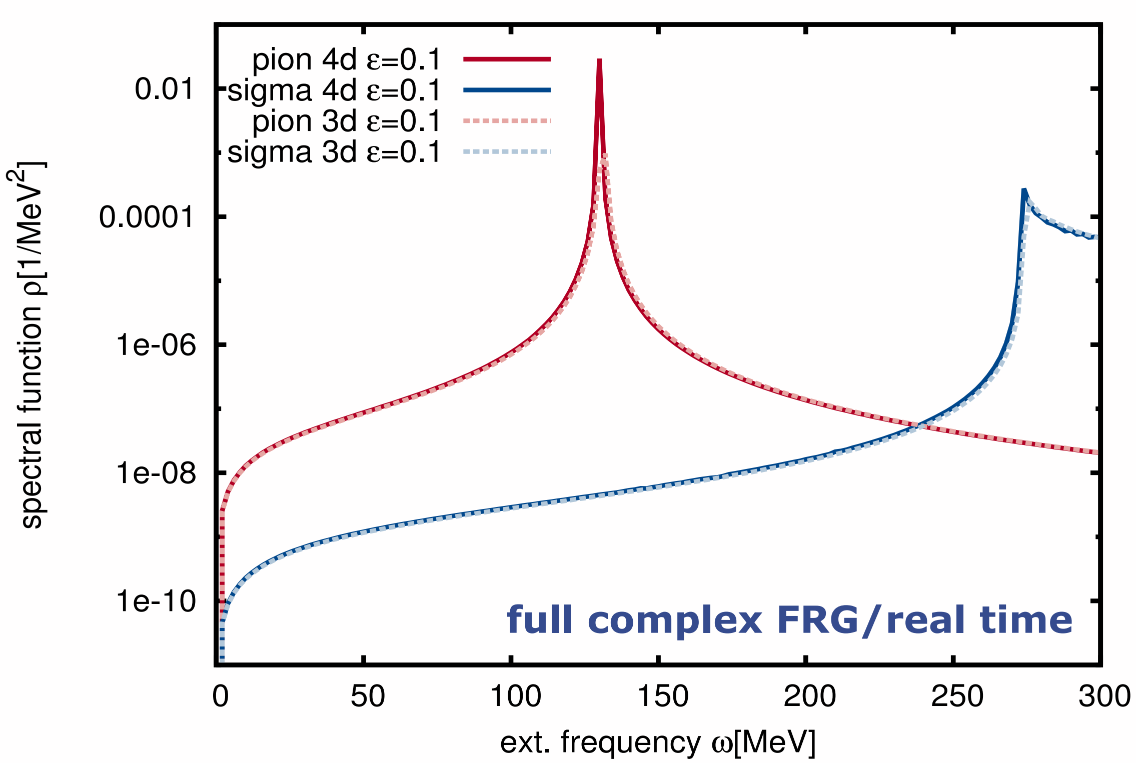


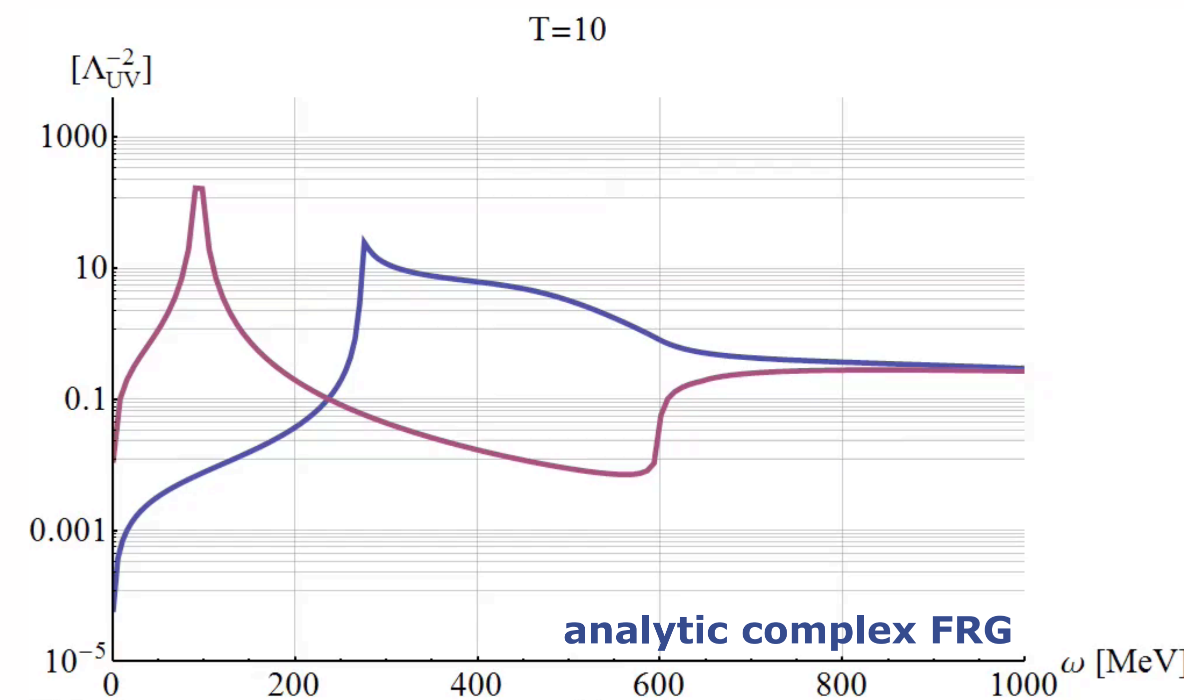
Haas, Fister, JMP, PRD 90 (2014) 9, 091501



Strauss, Fischer, Kellermann, PRL 109 (2012) 252001

pion and sigma spectral functions



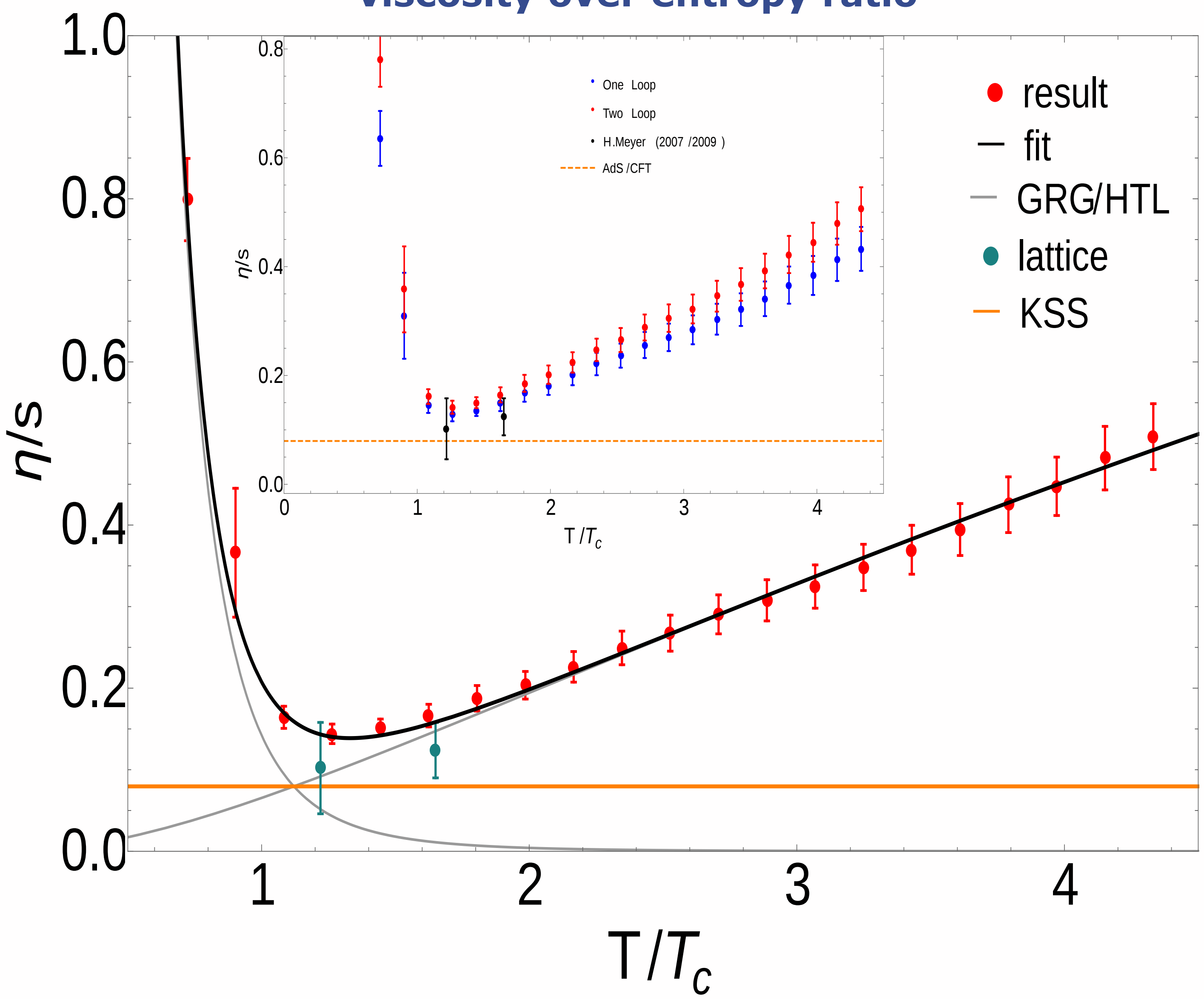


Tripolt, Strodthoff, von Smekal, Wamach, PRD 89 (2014) 034010
Kamikado, Strodthoff, von Smekal, Wambach, EPJ C74 (2014) 2806

Dynamics

transport coefficients

viscosity over entropy ratio

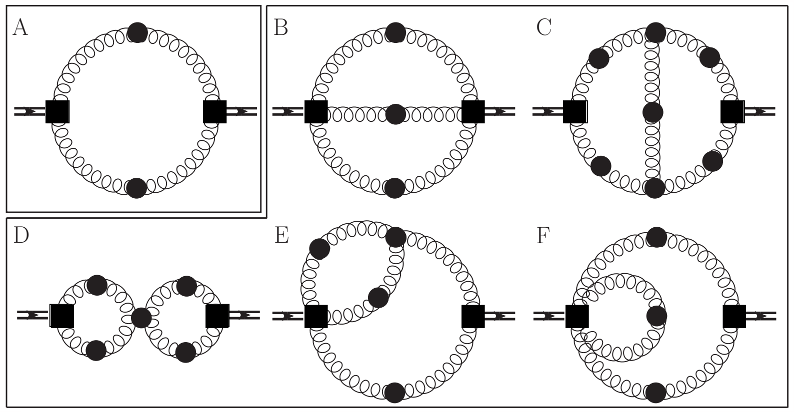


Kubo relation

$$\eta = \frac{1}{20} \frac{d}{d\omega}\bigg|_{\omega=0} \rho_{\pi\pi}(\omega, 0)$$

‘3-loop’ exact functional relation for $\rho_{\pi\pi}$

1 & 2-loop terms



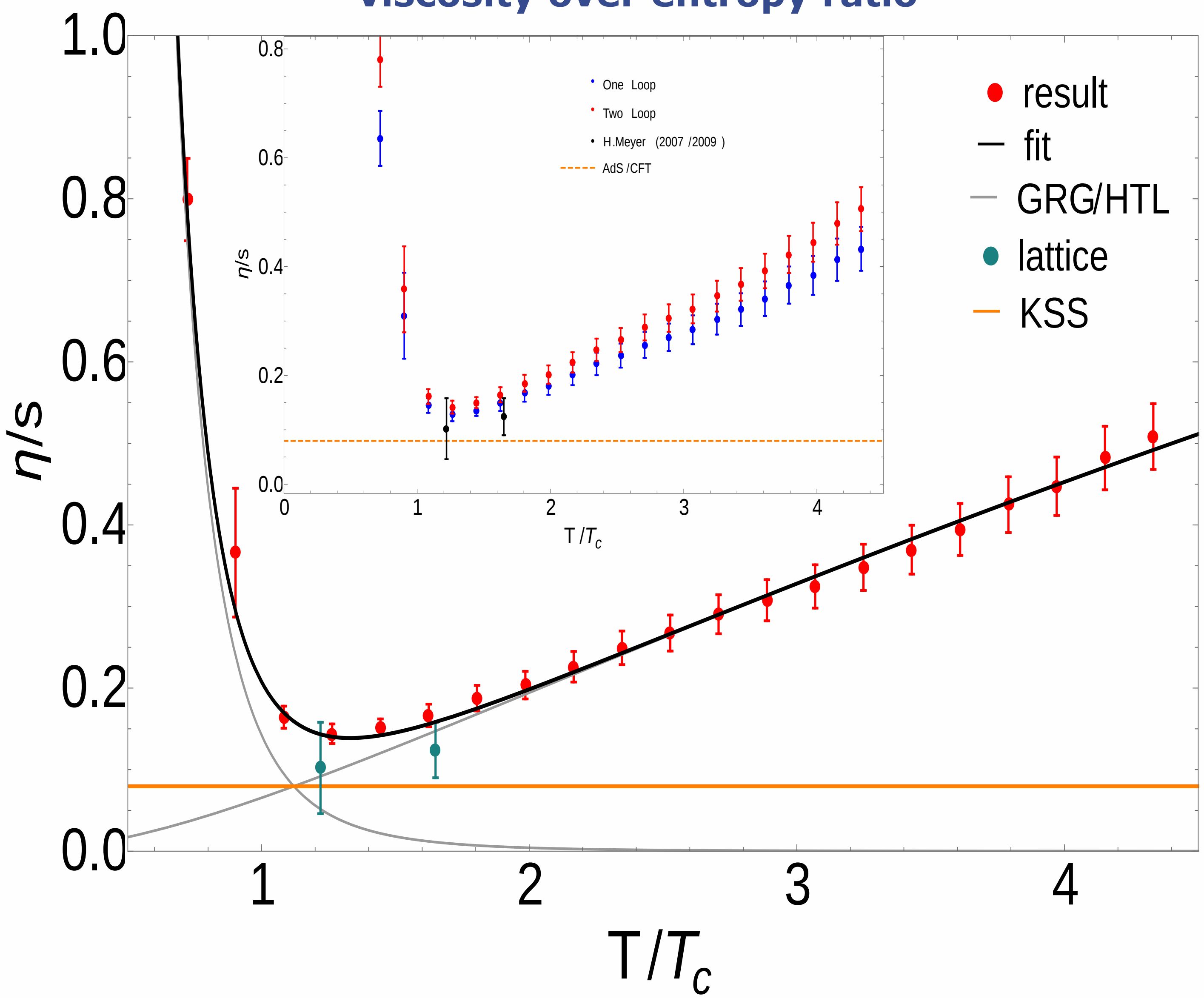
Haas, Fister, JMP, PRD 90 (2014) 9, 091501

Christiansen, Haas, JMP, Strodthoff, PRL 115 (2015) 11, 112002

Dynamics

QCD - estimate for viscosity over entropy ratio

viscosity over entropy ratio



$$\frac{\eta}{s}(T) = \frac{a_{\rm qgp}}{\alpha_s^{\gamma_{\rm qgp}}(cT/T_c)} + \frac{a_{\rm grg}}{(T/T_c)^{\gamma_{\rm grg}}}$$

$$\gamma_{\rm grg} \approx 5$$

$$\gamma_{\rm qgp} \approx 1.6$$

pure glue

$$a_{\rm qgp} \approx 0.15$$

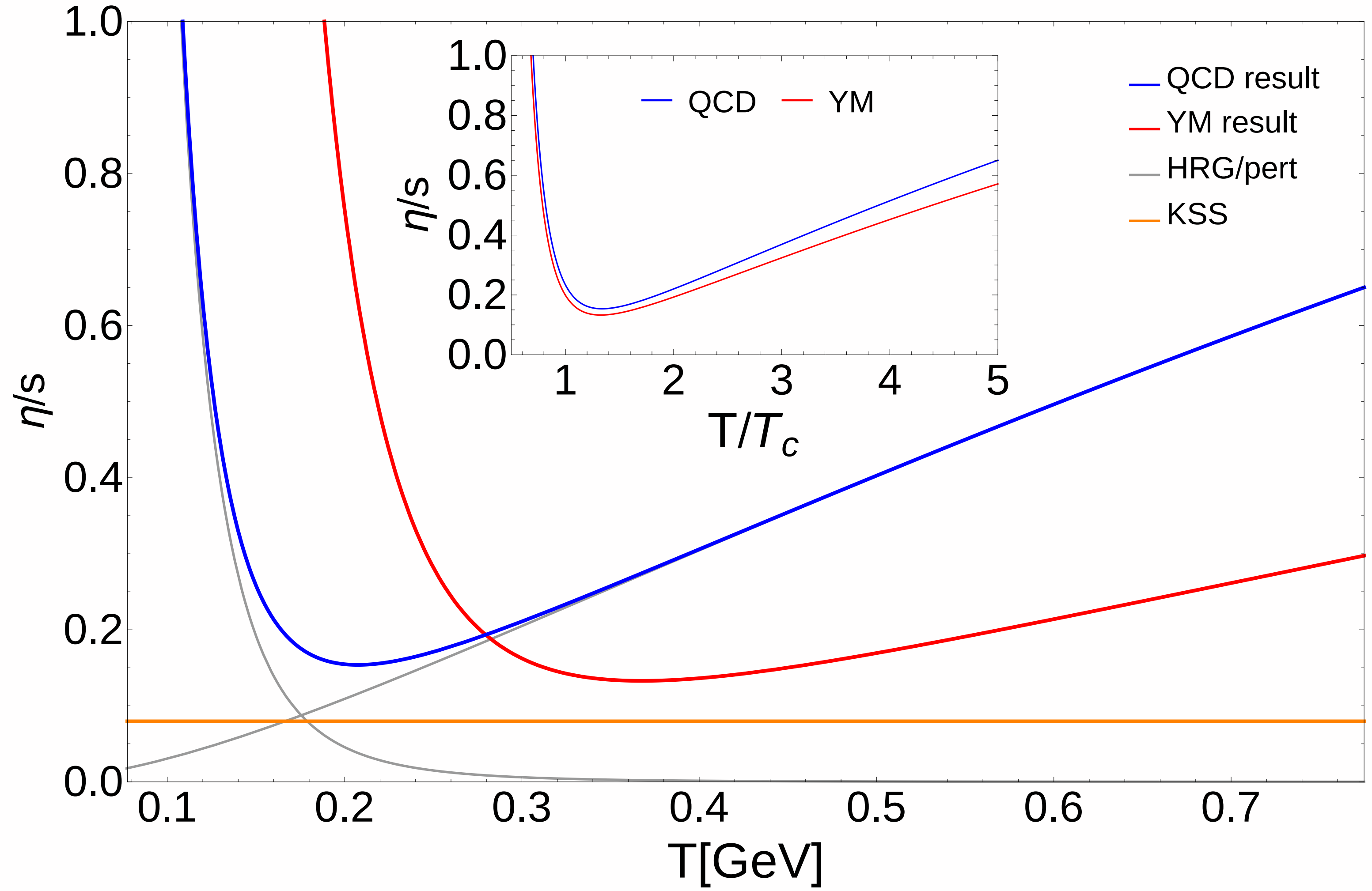
$$a_{\rm hrg} \approx 0.14$$

$$c \approx 0.66$$

Christiansen, Haas, JMP, Strodthoff, PRL 115 (2015) 11, 112002

Dynamics

QCD - estimate for viscosity over entropy ratio



$$\frac{\eta}{s}(T) = \frac{a_{\rm qgp}}{\alpha_s^{\gamma_{\rm qgp}}(cT/T_c)} + \frac{a_{\rm grg}}{(T/T_c)^{\gamma_{\rm grg}}}$$

$$a_{\rm qgp} \approx 0.2$$
$$a_{\rm hrg} \approx 0.16$$
$$c \approx 0.79$$

QCD

$$\gamma_{\rm grg} \approx 5$$
$$\gamma_{\rm qgp} \approx 1.6$$

pure glue

$$a_{\rm qgp} \approx 0.15$$
$$a_{\rm hrg} \approx 0.14$$
$$c \approx 0.66$$

Christiansen, Haas, JMP, Strodthoff, PRL 115 (2015) 11, 112002

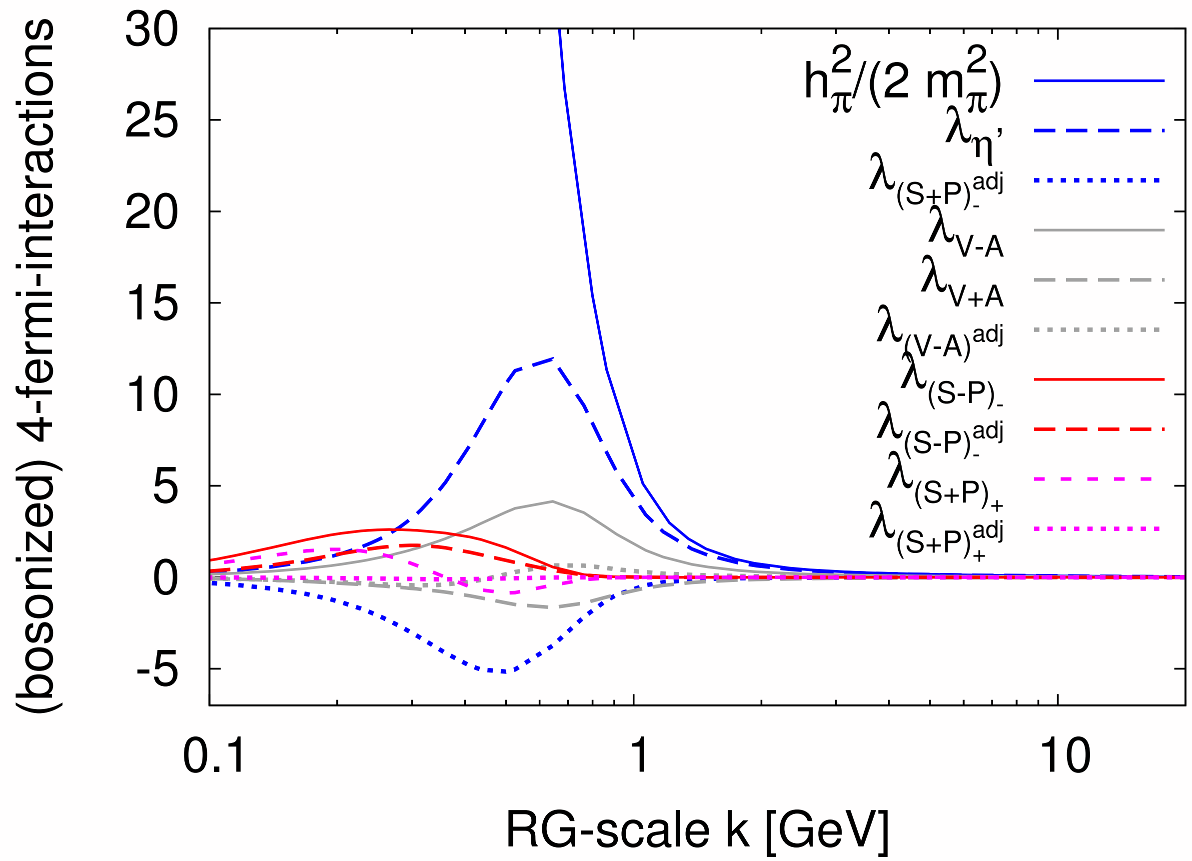
A glimpse at the hadron spectrum

preliminary

four-fermi scattering amplitude at pion pole

$$\langle \bar{q}\vec{\sigma}\gamma_5 q(p) \;\; \bar{q}\vec{\sigma}\gamma_5 q(-p)\rangle \quad \to \quad \frac{\chi_{\bar{q}\pi q}\bar{\chi}_{\bar{q}\pi q}}{p^2 - m_\pi^2} \quad + \quad \text{finite terms}$$

$$\Gamma^{(4)}(p_1, p_2, p_3, p_4)$$



Mitter, JMP, Strodthoff, in preparation

A glimpse at the hadron spectrum

preliminary

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$$\Downarrow \qquad\qquad \Downarrow$$

$$\Gamma^{(4)}_{(\bar{q}\gamma_5\vec{\sigma}q)^2}(p,p,-p,-p) \qquad \frac{\Gamma^{(3)}_{\bar{q}\pi q}\Gamma^{(3)}_{\bar{q}\pi q}}{p^2 - m_\pi^2}$$

Mitter, JMP, Strodthoff, in preparation

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$$\langle \bar{q}\vec{\sigma}\gamma_5 q(p) \quad \bar{q}\vec{\sigma}\gamma_5 q(-p)\rangle \to \frac{\chi_{\bar{q}\pi q}\bar{\chi}_{\bar{q}\pi q}}{p^2 - m_\pi^2} + \quad \text{finite terms}$$

$$\downarrow \qquad\qquad \downarrow$$

$$\Gamma^{(4)}_{(\bar{q}\gamma_5\vec{\sigma}q)^2}(p,p,-p,-p) \qquad \frac{\Gamma^{(3)}_{\bar{q}\pi q}\Gamma^{(3)}_{\bar{q}\pi q}}{p^2 - m_\pi^2}$$

pion decay constant f_π **via normalisation of** $\boldsymbol{\Gamma^{(3)}_{\bar{q}\pi q}}$

aka BSE wave function

recent mini-review on DSE-BSE
Sanchis-Alepuz, Williams, arXiv:1503.05896

Mitter, JMP, Strodthoff, in preparation

A glimpse at the hadron spectrum

preliminary

four-fermi scattering amplitude at pion pole

$$\langle \bar{q}\vec{\sigma}\gamma_5 q(p) \quad \bar{q}\vec{\sigma}\gamma_5 q(-p)\rangle \to \frac{\chi_{\bar{q}\pi q}\bar{\chi}_{\bar{q}\pi q}}{p^2 - m_\pi^2} + \text{finite terms}$$

$$\Gamma^{(4)}_{(\bar{q}\gamma_5\vec{\sigma}q)^2}(p,p,-p,-p) \qquad \frac{\Gamma^{(3)}_{\bar{q}\pi q}\Gamma^{(3)}_{\bar{q}\pi q}}{p^2 - m_\pi^2}$$

pion decay constant $\mathbf{f}_\pi$ **via normalisation of** $\mathbf{\Gamma^{(3)}_{\bar{q}\pi q}}$

$$\mathbf{f}_\pi \simeq \mathbf{99\,MeV}$$

quenched QCD

Mitter, JMP, Strodthoff, in preparation

A glimpse at the hadron spectrum

preliminary

four-fermi scattering amplitude at pion pole

$$\langle \bar{q}\vec{\sigma}\gamma_5 q(p) \quad \bar{q}\vec{\sigma}\gamma_5 q(-p)\rangle \quad \to \quad \frac{\chi_{\bar{q}\pi q}\bar{\chi}_{\bar{q}\pi q}}{p^2 - m_\pi^2} \quad + \quad \text{finite terms}$$

$$\Gamma^{(4)}_{(\bar{q}\gamma_5\vec{\sigma}q)^2}(p,p,-p,-p) \qquad \frac{\Gamma^{(3)}_{\bar{q}\pi q}\Gamma^{(3)}_{\bar{q}\pi q}}{p^2 - m_\pi^2}$$

pion decay constant $\mathbf{f}_\pi$ **via normalisation of** $\boldsymbol{\Gamma^{(3)}_{\bar{q}\pi q}}$

$$\mathbf{f}_\pi \simeq 99\,\mathrm{MeV}$$
quenched QCD

$$\mathbf{f}_\pi \simeq 89\,\mathrm{MeV}$$
unquenched QCD

lattice **Davies et al., PRL 92 (2004) 022001**

$$\frac{\mathbf{f}_\pi^{\text{quenched}}}{\mathbf{f}_\pi^{\text{unquenched}}} \simeq 1.1$$

Mitter, JMP, Strodthoff, in preparation

A glimpse at the hadron spectrum

preliminary

four-fermi scattering amplitude at pion pole

$$\langle \bar{q}\vec{\sigma}\gamma_5 q(p) \quad \bar{q}\vec{\sigma}\gamma_5 q(-p)\rangle \rightarrow \frac{\chi_{\bar{q}\pi q}\bar{\chi}_{\bar{q}\pi q}}{p^2 - m_\pi^2} + \text{finite terms}$$

$$\downarrow \qquad\qquad \downarrow$$

$$\Gamma^{(4)}_{(\bar{q}\gamma_5\vec{\sigma}q)^2}(p,p,-p,-p) \qquad \frac{\Gamma^{(3)}_{\bar{q}\pi q}\Gamma^{(3)}_{\bar{q}\pi q}}{p^2 - m_\pi^2}$$

pion decay constant $\mathbf{f}_\pi$ **via normalisation of** $\mathbf{\Gamma^{(3)}_{\bar{q}\pi q}}$

$\mathbf{f}_\pi \simeq \mathbf{99\,MeV}$ **quenched QCD**

$\mathbf{f}_\pi \simeq \mathbf{89\,MeV}$ **unquenched QCD**

lattice **Davies et al., PRL 92 (2004) 022001**

unquenched **e.g. Horsley et al., PLB 732, 41 (2014)** $\mathbf{f}_\pi^{\text{lattice}} \simeq \mathbf{89}\,\text{MeV}$

Mitter, JMP, Strodthoff, in preparation

Outline

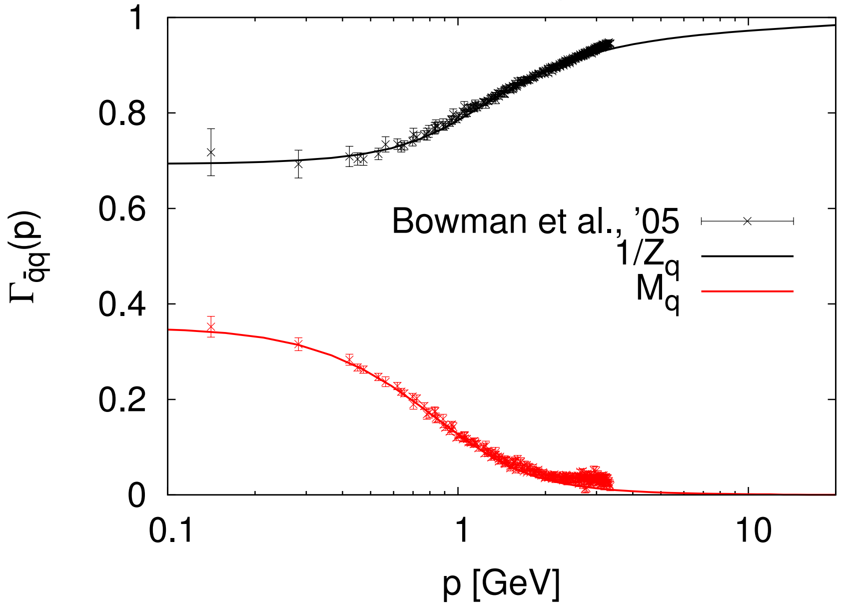
- Functional Approaches to QCD & the FRG
- Vacuum QCD: confinement & chiral symmetry breaking
- Hadron spectrum & QCD transport
- Phase structure of QCD
- Outlook

Summary & Outlook

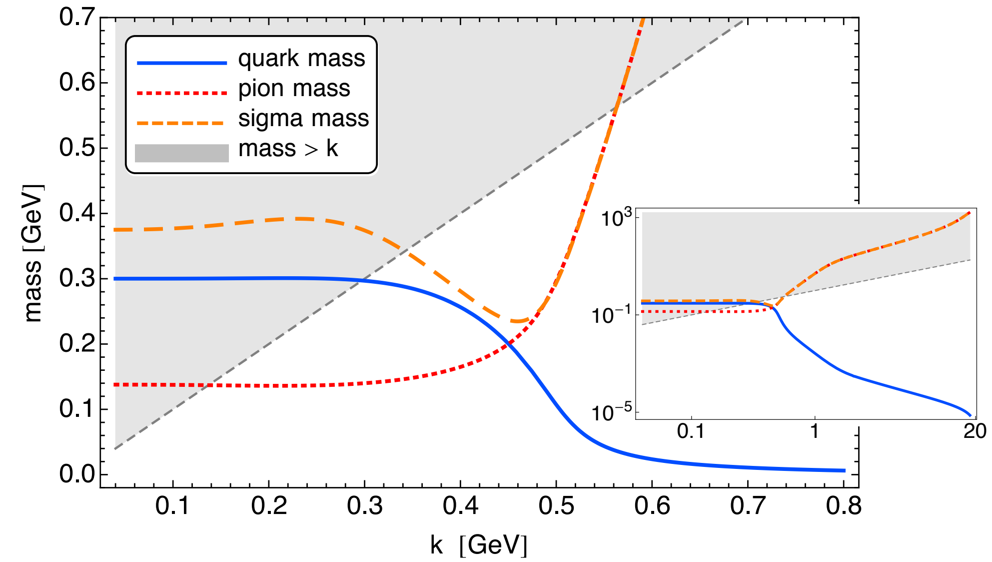
- **Chiral Symmetry Breaking and Confinement**

$$\frac{f_{\pi,\mathrm{FRG}}}{f_{\pi,\mathrm{lattice}}} = 0.99$$

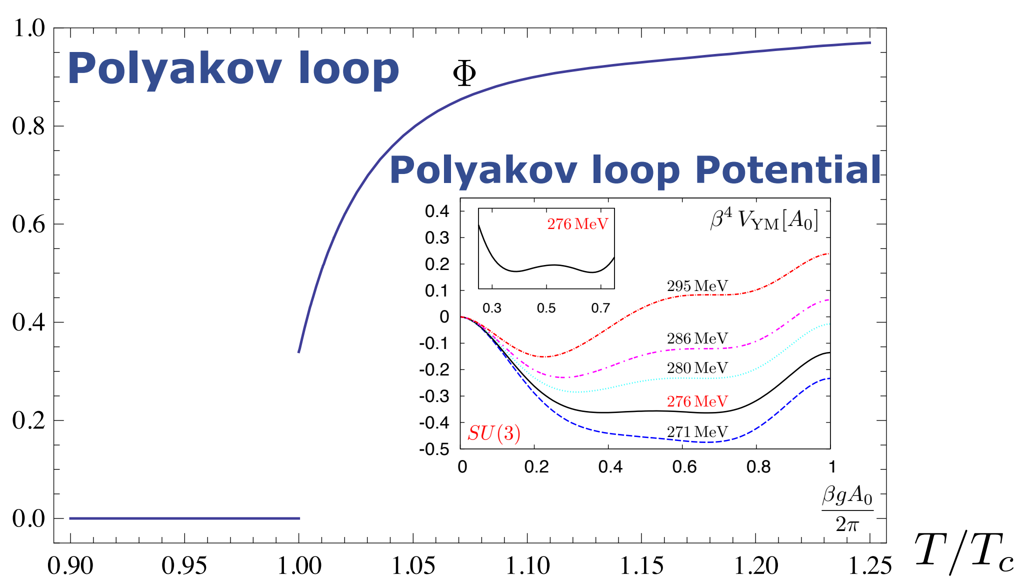
quark propagator



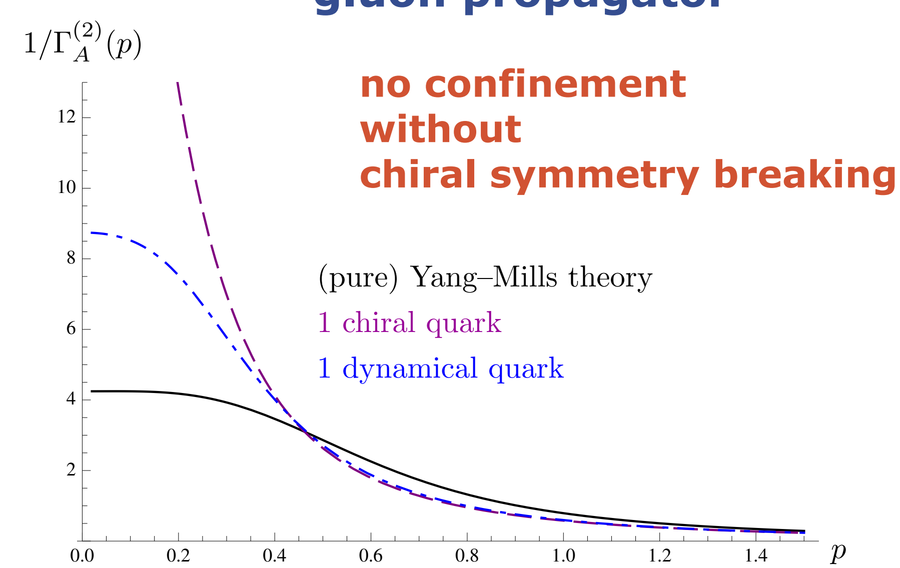
meson masses & decoupling



fQCD



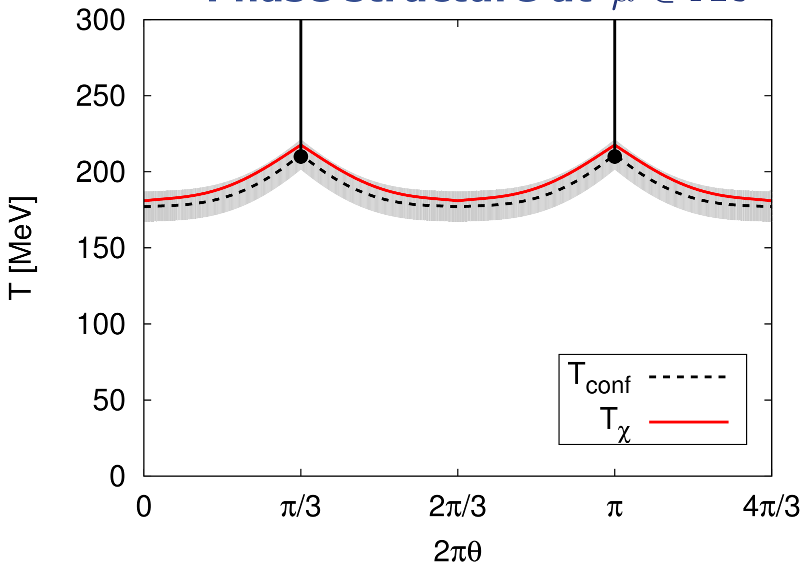
gluon propagator



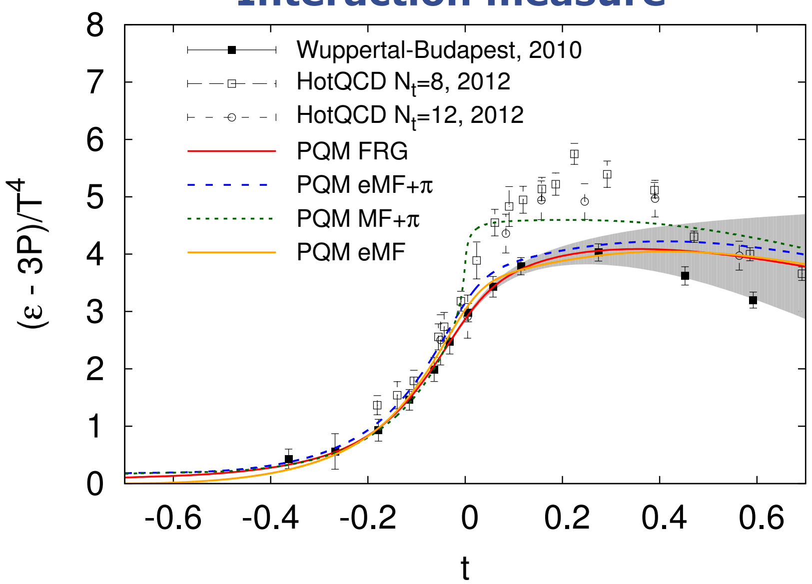
Summary & Outlook

- **Phase structure and Transport**

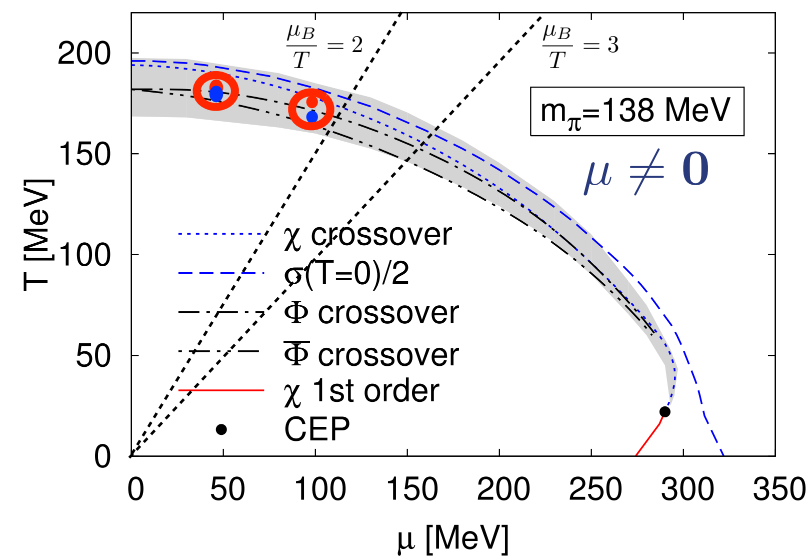
Phase structure at $\mu \in i\mathbb{R}$



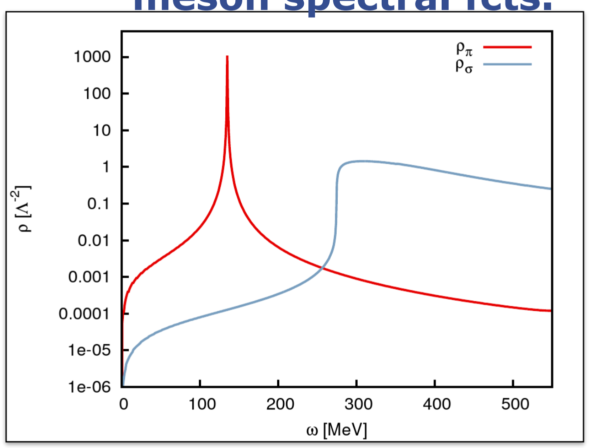
Interaction measure



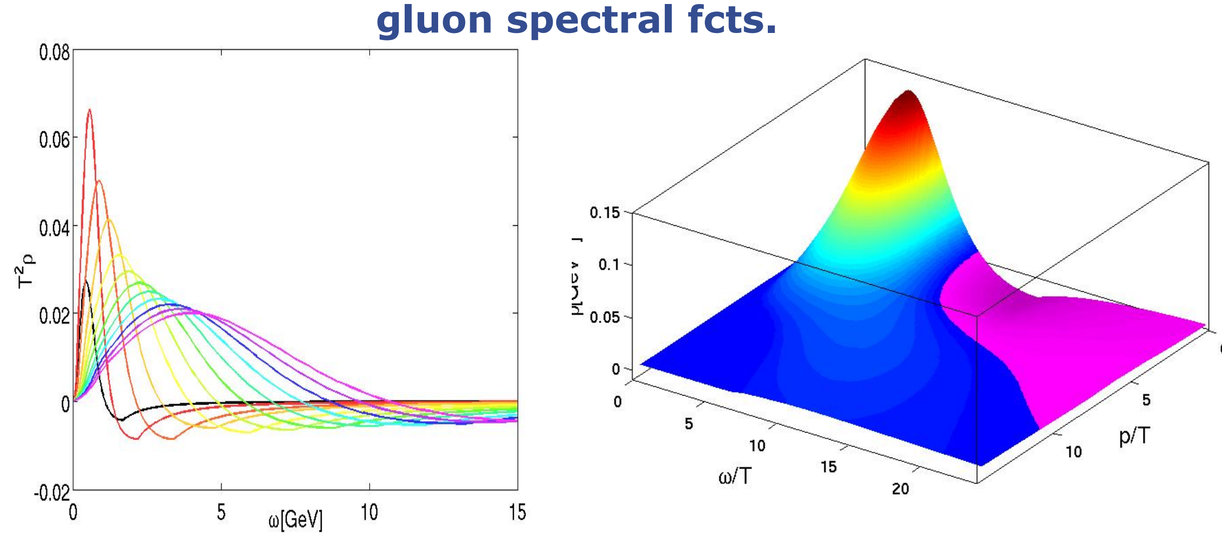
Phase structure at finite density



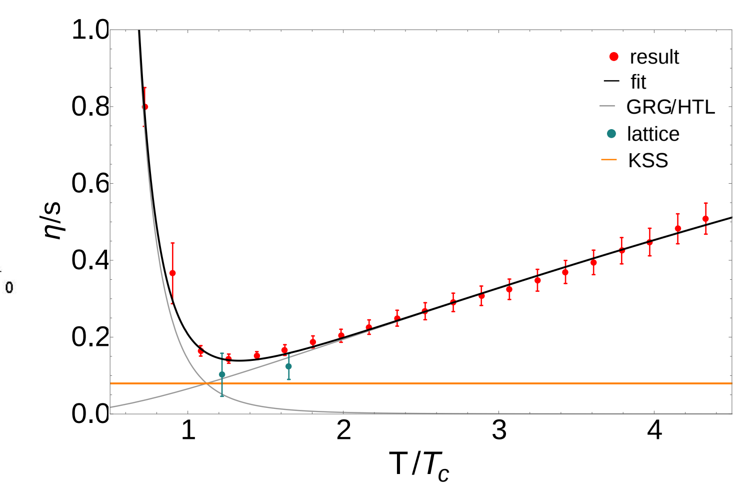
meson spectral fcts.



gluon spectral fcts.



glue viscosity/entropy



Summary & Outlook

- Chiral Symmetry Breaking and Confinement

- Phase Structure and Transport

- Towards quantitative precision

- Baryons, high density regime & CEP, dynamics

- Hadronic properties
  - hadron spectrum & in medium modifications
  - low energy constants