Collective neutrino oscillations, quantum entanglement, and supernova nucleosynthesis

Amol V. Patwardhan

SLAC National Accelerator Laboratory

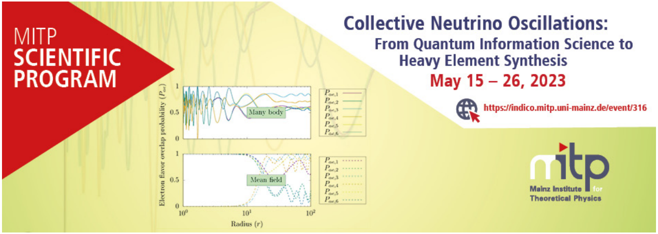
Particle and Astroparticle Theory Seminar, MPIK, Heidelberg

May 15, 2023

References: 1905.00082, 1905.04386, 1908.03511, 2109.08995, 2202.01865, 2205.09384 + a few 23xx.xxxxx (in preparation)

Collaborators: Michael Cervia, Baha Balantekin, Pooja Siwach, Susan Coppersmith, Calvin Johnson, Denis Lacroix, Xilu Wang, Rebecca Surman, Alexander Friedland, Payel Mukhopadhyay, Shuo Xin

MITP workshop happening now!



Collective Neutrino Oscillations: From Quantum Information Science to Heavy Element Synthesis

15.05.2023 to 26.05.2023

Mainz Institute for Theoretical Physics (MITP)

Indico website:

https://indico.mitp.uni-mainz.de/event/316/

Outline

Key motivations and definitions

∗ Neutrinos

- Elementary particles
- No electric charge
- *Not* massless, but **really** small masses
- Don't interact much (only weak interactions)
- Byproduct of some types of nuclear reactions, e.g., decay of a neutron into a proton (but other ways to make them too)

Key motivations and definitions

∗ Supernovae (core-collapse supernovae)

- Explosions of massive stars that are much heavier than our sun
- Final stage in the life cycle of massive stars after multiple stages of nuclear burning
- Triggered by gravitational collapse of stellar core, when it gets so heavy that gravity overcomes pressure support
- Neutrinos have a big part to play

Key motivations and definitions

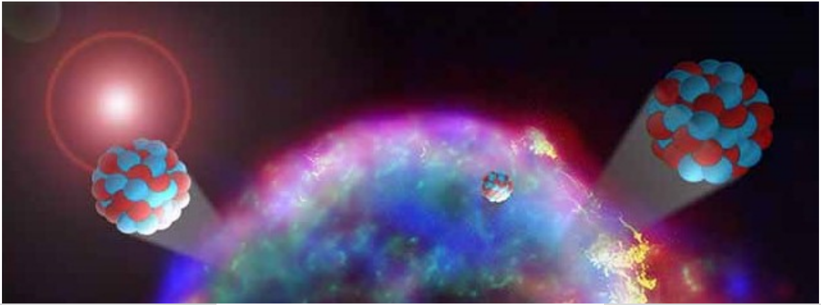
∗ Neutrino flavor oscillations

- Neutrinos come in three ‘flavors’: electron, muon, and tau
- Neutrino produced in a well-defined flavor state evolves into a quantum superposition of *all three flavors* as it propagates, with oscillating amplitudes in each flavor
- In environments with dense neutrino streams, neutrinos can undergo collective flavor oscillations driven by ν-ν interactions
- Subsequent interactions depend on flavor composition — critically important for supernovae and nucleosynthesis

Key motivations and definitions

∗ Nucleosynthesis

- The process of combining protons and neutrons into nuclei
- Takes place in the early universe and in stars
- Early universe ('primordial' or 'big-bang') nucleosynthesis makes Helium — and trace amounts of other light nuclei
- Heavier nuclei are made in stars through a variety of processes
- Neutrinos have a big part to play



Core-collapse supernovae and neutrinos

- Stars with $M_\star \gtrsim 8\, M_\odot$ undergo core collapse & neutronization when core mass exceeds $\sim 1.4\, M_\odot$, i.e., when its gravity surpasses the limit of electron degeneracy pressure support
- Core bounce at nuclear density sends shockwave through infalling material $\rightarrow$ shock eventually loses energy and stalls before it can blow up the star
- Details of the explosion mechanism unknown, but neutrinos expected to play a major role
- CCSNe are neutrino factories: νs are the main carriers of gravitational binding energy ($\sim 99\%$) and lepton number radiated away from the star
  - B.E. $\sim 10^{53}$ ergs $\implies$ $\sim 10^{58}$ νs with $\langle E_\nu \rangle \sim 10$ MeV

Core-collapse supernovae and neutrinos

- Neutrinos depositing $\sim 1\%$ of their energy behind the stalled shock front could revive the shock and explode the star
- ν-induced heating in the aftermath of explosion drives baryonic matter outflows from the surface of the nascent neutron star
- Charged-current weak processes govern the energy deposition and n/p ratio, a crucial input for nucleosynthesis

$$\nu_e + n \longleftrightarrow p + e^-$$
$$\bar{\nu}_e + p \longleftrightarrow n + e^+$$

- Flavor asymmetric processes: thorough understanding of neutrino flavor evolution therefore required

Outline

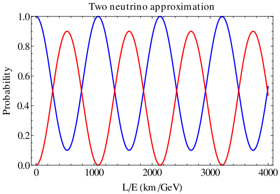
Neutrino oscillations in vacuum

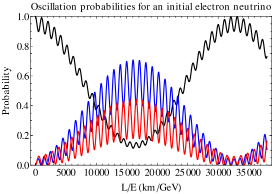
- Neutrino weak-interaction (flavor) eigenstates not aligned with propagation (energy/mass) eigenstates

$$|\nu_e\rangle = \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle$$
$$|\nu_x\rangle = -\sin\theta\,|\nu_1\rangle + \cos\theta\,|\nu_2\rangle$$

- As neutrinos propagate, mass eigenstates gather quantum mechanical phase at different rates, leading to oscillations

$$P_{ex} = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E_\nu}\right)$$





Neutrino oscillations in vacuum

- Vacuum oscillations driven by the free-particle Hamiltonian

$$H_{\rm vac} = \left(\frac{m_1^2}{2p} a_{1\mathbf{p}}^\dagger a_{1\mathbf{p}} + \frac{m_2^2}{2p} a_{2\mathbf{p}}^\dagger a_{2\mathbf{p}} \right) = \omega_{\mathbf{p}}\, \vec{B} \cdot \vec{J}_{\mathbf{p}} \ ,$$

where $\omega_{\mathbf{p}} = \dfrac{\delta m^2}{2p}$, and

$\vec{B} = (0,0,-1)_{\text{mass}} = (\sin 2\theta, 0, -\cos 2\theta)_{\text{flavor}}$.

- Here we have used the mass-basis ‘isospin’ operators

$$J_{\mathbf{p}}^{+} = a_{1\mathbf{p}}^\dagger a_{2\mathbf{p}} \ , \qquad J_{\mathbf{p}}^{-} = a_{2\mathbf{p}}^\dagger a_{1\mathbf{p}} \ ,$$

$$J_{\mathbf{p}}^{z} = \frac{1}{2} \left(a_{1\mathbf{p}}^\dagger a_{1\mathbf{p}} - a_{2\mathbf{p}}^\dagger a_{2\mathbf{p}} \right) \ ,$$

which obey the usual $SU(2)$ commutation relations

$$[J_{\mathbf{p}}^{+}, J_{\mathbf{q}}^{-}] = 2\delta_{\mathbf{pq}} J_{\mathbf{p}}^{z} \ , \qquad [J_{\mathbf{p}}^{z}, J_{\mathbf{q}}^{\pm}] = \pm \delta_{\mathbf{pq}} J_{\mathbf{p}}^{\pm}.$$

Neutrino flavor evolution: matter effects

Matter backgrounds (electrons, nucleons, etc.) modify flavor evolution: "effective mass" through neutrino forward scattering. Mass level crossing $H_{\nu_e\nu_e} = H_{\nu_x\nu_x} \implies$ MSW resonance

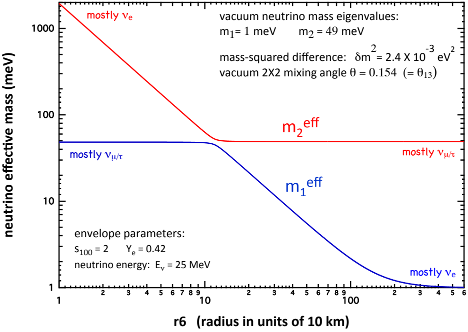


Figure: MSW resonance (figure by George Fuller)

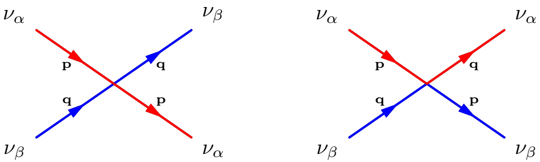
Wolfenstein (1978, '79)
Mikheyev & Smirnov (1985)
Bethe (1986)
Haxton (1986)
Parke (1986)
and so on ...

Neutrino-matter Hamiltonian:

$$H_{\rm mat} = \lambda\, \vec{L} \cdot \vec{J}_{\mathbf{p}}$$

where $\lambda = \sqrt{2}\, G_F n_e$ and $\vec{L} = (\sin 2\theta, 0, \cos 2\theta)_{\rm mass}$

Neutrino-neutrino interactions



- Neutrino-neutrino interaction Hamiltonian

$$H_{\nu\nu} = \frac{\sqrt{2}G_F}{V} \sum_{\mathbf{p},\mathbf{q}} (1 - \cos\vartheta_{\mathbf{pq}})\, \vec{J}_{\mathbf{p}} \cdot \vec{J}_{\mathbf{q}} \ .$$

- Note: here, we only consider interactions which preserve or exchange momenta (or equivalently, flavor), since these can be added coherently

- A many-body coupled quantum system (2^N complex amplitudes) with a complicated geometry on top!

Many-body neutrino Hamiltonian (two-flavor system)

$$H_\nu = \overbrace{\sum_{\mathbf{p}} \omega_{\mathbf{p}} \vec{B} \cdot \vec{J}_{\mathbf{p}}}^{\text{vacuum}} + \overbrace{\lambda \sum_{\mathbf{p}} \vec{L} \cdot \vec{J}_{\mathbf{p}}}^{\text{neutrino-matter}} + \overbrace{\frac{\sqrt{2} G_F}{V} \sum_{\mathbf{p},\mathbf{q}} \left(1 - \cos\vartheta_{\mathbf{pq}}\right) \vec{J}_{\mathbf{p}} \cdot \vec{J}_{\mathbf{q}}}^{\text{neutrino-neutrino}},$$

where $\omega_{\mathbf{p}} = \frac{\delta m^2}{2|\mathbf{p}|}$, $\lambda = \sqrt{2} G_F n_e$, $\vec{B} = (0, 0, -1)$, $\vec{L} = (\sin 2\theta, 0, \cos 2\theta)$,
$\vec{J}_{\mathbf{p}}$: neutrino "isospin" operator ($|\uparrow\rangle = |\nu_1\rangle$, $|\downarrow\rangle = |\nu_2\rangle$)

Many-body neutrino Hamiltonian (two-flavor system)

$$H_\nu = \overbrace{\sum_{\mathbf{p}} \omega_{\mathbf{p}} \vec{B} \cdot \vec{J}_{\mathbf{p}}}^{\text{vacuum}} + \overbrace{\lambda \sum_{\mathbf{p}} \vec{L} \cdot \vec{J}_{\mathbf{p}}}^{\text{neutrino-matter}} + \overbrace{\frac{\sqrt{2}G_F}{V} \sum_{\mathbf{p},\mathbf{q}} (1 - \cos\vartheta_{\mathbf{pq}})\, \vec{J}_{\mathbf{p}} \cdot \vec{J}_{\mathbf{q}}}^{\text{neutrino-neutrino}},$$

where $\omega_{\mathbf{p}} = \frac{\delta m^2}{2|\mathbf{p}|}$, $\lambda = \sqrt{2}G_F n_e$, $\vec{B} = (0,0,-1)$, $\vec{L} = (\sin 2\theta, 0, \cos 2\theta)$,
$\vec{J}_{\mathbf{p}}$: neutrino "isospin" operator ($|\uparrow\rangle = |\nu_1\rangle$, $|\downarrow\rangle = |\nu_2\rangle$)

"single-angle" approximation $\Downarrow$ neglect neutrino-matter term

$$\boxed{H_\nu = \sum_p \omega_p \vec{B} \cdot \vec{J}_p + \mu(r) \vec{J} \cdot \vec{J}}$$

where $\vec{J} = \sum_{\mathbf{p}} \vec{J}_{\mathbf{p}}$ and $\mu(r) = \frac{\sqrt{2}G_F}{V} \langle (1 - \cos\vartheta_{\mathbf{pq}}) \rangle$.

Mean-field (random phase) approximation

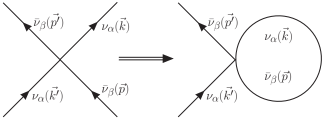


Figure: Volpe *et al.* (2013)

- In an effective one-particle approximation [Sigl and Raffelt (1993), Samuel (1993), Qian and Fuller (1995)], each neutrino considered to interact with an average potential representing all other particles in the medium (including neutrinos)

- Operator product $\mathcal{O}_1\mathcal{O}_2$ approximated as

$$\mathcal{O}_1\mathcal{O}_2 \sim \mathcal{O}_1\langle\mathcal{O}_2\rangle + \langle\mathcal{O}_1\rangle\mathcal{O}_2 - \langle\mathcal{O}_1\rangle\langle\mathcal{O}_2\rangle.$$

  Above expectation values are calculated w.r.t state $|\Psi\rangle$ which satisfies $\langle\mathcal{O}_1\mathcal{O}_2\rangle = \langle\mathcal{O}_1\rangle\langle\mathcal{O}_2\rangle$

Mean-field (random phase) approximation

- This method yields the effective one-particle neutrino interaction Hamiltonian

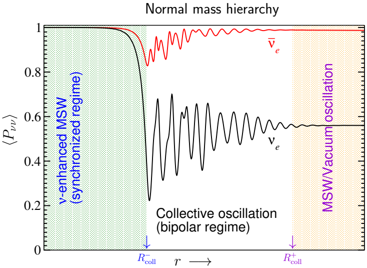
$$H_{\nu\nu}^{\text{RPA}} = \sum_{\mathbf{p},\mathbf{q}} \mu_{\mathbf{pq}} \left[\vec{J}_{\mathbf{p}} \cdot \langle \vec{J}_{\mathbf{q}} \rangle + \langle \vec{J}_{\mathbf{p}} \rangle \cdot \vec{J}_{\mathbf{q}} - \langle \vec{J}_{\mathbf{p}} \rangle \cdot \langle \vec{J}_{\mathbf{q}} \rangle \right]$$

- Together with the one-body terms (H_{vac} and H_{mat}), Ehrenfest's theorem for the evolution of one-body operator expectation values gives:

$$\boxed{\frac{\mathrm{d}\vec{P}_{\mathbf{q}}}{\mathrm{d}t} = \omega_{\mathbf{q}}\, \vec{B} \times \vec{P}_{\mathbf{q}} + \lambda \vec{L} \times \vec{P}_{\mathbf{q}} + 2 \sum_{\mathbf{p}} \mu_{\mathbf{pq}}\, \vec{P}_{\mathbf{p}} \times \vec{P}_{\mathbf{q}},}$$

where $\vec{P}_{\mathbf{q}} = 2\langle \vec{J}_{\mathbf{q}} \rangle$ is called the neutrino "Polarization vector"

Collective flavor oscillations: synchronized and bipolar



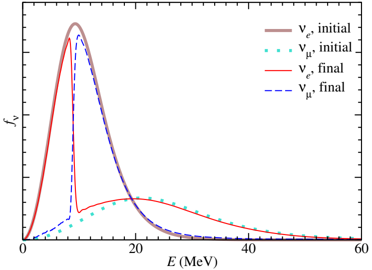
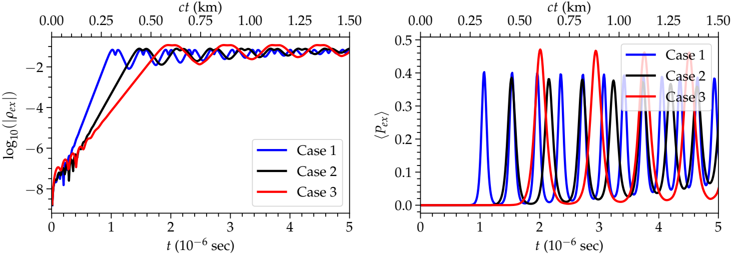


Figure: Taken from Duan et al. (1001.2799). **Left:** regimes for different types of neutrino oscillations in a CCSN environment. **Right:** a neutrino spectral split/swap resulting from collective flavor effects.

‘Fast’ collective flavor transformations

- Fast collective oscillations — driven by flavor-lepton number crossings in the neutrino angular distributions could cause significant flavor conversion on timescales much shorter than bipolar oscillations, i.e., within $\mathcal{O}(1\text{–}10)$ km from the PNS, making them more relevant for shock reheating and nucleosynthesis
- Recent reviews by Chakraborty et al. (1602.02766), Tamborra & Shalgar (2011.01948), and Richers & Sen (2207.03561)



Other cool phenomena in the mean-field limit

- Matter-neutrino resonances
  (Malkus, McLaughlin, Friedland, Wu, Vaananen, Zhu, et al.: 1403.5797, 1507.00946, 1509.08975, 1510.00751, 1607.04671, 1801.07813)
- Collisionally triggered collective flavor instabilities
  (Lucas Johns et al.: 2104.11369, 2206.09225, 2208.11059)
- ‘Halo’ effect from backscattered neutrinos
  (J. F. Cherry et al.: 1203.1607, 1302.1159, 1908.10594, 1912.11489; V. Cirigliano et al.: 1807.07070)
- Decoherence by wave-packet separation
  (Akhmedov, Kopp, Lindner, Kersten, Smirnov, et al.: 1512.09068, 1702.08338)

Outline

1. Core-collapse supernovae and neutrinos
2. Collective neutrino oscillations
3. Beyond the effective one-particle description of collective oscillations
4. Proton-rich nucleosynthesis in supernovae
5. Neutrino flavor mixing and the νp-process

Beyond the mean field, and neutrino entanglement

- The mean-field picture asserts that ν-ν quantum correlations (i.e., entanglement) are strictly prohibited, reducing number of independent amplitudes from 2^N to $2N$ (for two flavors)

- This begs the question as to whether any significant physics is being left out by this approximation

- Early works [Bell et al. (2003), Friedland et al. (2003a, 2003b, 2006), McKellar et al. (2009)] attempted to answer this question using different physical setups. Friedland et al. observed that, in their setup, the build-up of entanglement and the resulting flavor conversion was occurring on timescales suggestive of incoherent effects (and hence unimportant in the large-N limit)

Beyond the mean field, and neutrino entanglement

- Nevertheless McKellar et al. (2009) subsequently pointed out that this does not necessary preclude the multiparticle correlations from being significant.
- In any case, these early analyses had some notable simplifications, such as omitting the one-body term in the Hamiltonian. It is known, even in the mean-field limit that interplay between H_{vac} and $H_{\nu\nu}$ can give rise to interesting phenomena such as spectral splits.
- In light of these omissions, and recent advances in quantum computing/information, a re-examination of this question was (and is) warranted

Recent work

- Patwardhan, Cervia, Balantekin, Siwach, Coppersmith, Johnson, Lacroix et al.: 1905.04386, 1908.03511, 2109.08995, 2202.01865, 2205.09384;

- Rrapaj, Roggero, Xiong, Martin, Duan, Carlson, Illa, Savage, Yeter-Aydeniz et al.: 1905.13335, 2102.10188, 2102.12556, 2103.11497, 2104.03273, 2111.00437, 2112.12686, 2203.02783, 2207.03189, 2210.08656, 2301.07049 — some of these involve simulating on a quantum computer

- More recently (Shalgar & Tamborra: 2304.13050, Johns: 2305.04916) the suitability of the above studies for judging the efficacy of the mean field has been questioned. Suggested possible paths forward include an open quantum system formulation with wavepackets (S&T), or a unified many-body framework with forward and non-forward scatterings (Johns)

Our framework

- Try to ascertain the potential role of many-body neutrino correlations using simple toy systems. Operating with these toy models necessitates being careful about not making sweeping generalizations based on the observed behaviors. Nevetheless, certain patterns or scaling relations can be sought
- Model the neutrino system as N interacting plane waves in a box of volume V, which in general could be time-dependent (to mimic the decreasing density of neutrinos streaming out from a source)
- Examine the evolution of one-body observables (such as expectation values of $\vec{J}_{\mathbf{p}}$) and compare with the mean-field expectation. Additionally, we use entanglement measures (such as bipartite entropy of entanglement) to quantify the degree of multiparticle correlations in the system

Numerical approaches and descriptions

In order to be able to simulate larger and large systems, we have so far explored various numerical methods

- Exploting the integrability of the single-angle Hamiltonian to diagonilize using Bethe Ansatz solutions [Up to $N \simeq 10$] (1905.04386, 1908.03511)
- Brute force numerical integration using 4th order Runge-Kutta with adaptive time step [Up to $N = 16$] (2109.08995)
- Tensor network calculation using a time-dependent variational principle nethod [Up to $N \simeq 50$ or $N \simeq 20$ depending on initial state] (2202.01865)
- Approximate phase-space method to evolve a two-beam neutrino system, wherein the neutrinos in each beam are identical to one another [Up to $N \simeq 100$] (2205.09384)

Single-angle limit: Integrability and Bethe Ansatz

- Eigenvalues and eigenstates obtained using procedure derived from Richardson-Gaudin diagonalization (a.k.a. "Bethe-Ansatz" method) — AVP, Cervia, Balantekin, arXiv:1905.04386

- For a system where each neutrino occupies its own energy mode, the eigenproblem can be mapped onto a system of coupled quadratic equations:

$$\tilde{\Lambda}_q^2 + \tilde{\Lambda}_q = \mu \sum_{\substack{p=1 \\ p \neq q}}^{N} \frac{\tilde{\Lambda}_q - \tilde{\Lambda}_p}{\omega_q - \omega_p}$$

$\tilde{\Lambda}_p$ are related to eigenvalues of the invariants h_p of the single-angle Hamiltonian. Bethe-Ansatz equations shown to be equivalent to polynomial relations between invariants h_p
— Cervia, AVP, Balantekin, arXiv:1905.00082

Single-angle limit: adiabatic many-body evolution

- Eigenvalues and eigenvectors facilitate calculating the adiabatic evolution of the many-body neutrino system, starting from any given initial condition, as μ is varied

- Consider an initial many-body state, $|\Psi_0\rangle \equiv |\Psi(\mu_0)\rangle$
  - Example: in the (two-)flavor-basis, $|\nu_e \nu_x \nu_e \nu_e\rangle$

- May be decomposed into the basis of energy eigenstates: $|\Psi(\mu_0)\rangle = \sum_n c_n \, |e_n(\mu_0)\rangle$

- If μ were to change sufficiently slowly then the system adiabatically evolves into

$$|\Psi(\mu)\rangle \simeq \sum_n c_n e^{-i \int_{\mu_0}^{\mu} \frac{E_n(\mu')}{d\mu'/dt} d\mu'} \, |e_n(\mu)\rangle$$

Summary of entanglement measures

Density Matrix, Polarization Vector, & Entanglement Entropy

Consider a pure, many-body neutrino state $\rho = |\Psi\rangle\langle\Psi|$.
Single-neutrino reduced density matrix: $\rho_q \equiv \mathrm{Tr}_{1,\ldots,\widehat{q},\ldots,N}[\rho]$, given by ($\widehat{\ }$ denotes exclusion)

$$\rho_q = \sum_{i_1,\ldots,\widehat{i_q},\ldots,i_N=1}^{2} \langle \nu_{i_1} \ldots \widehat{\nu_{i_q}} \ldots \nu_{i_N} | \rho | \nu_{i_1} \ldots \widehat{\nu_{i_q}} \ldots \nu_{i_N} \rangle \,,$$

- $S(\omega_q)$, Entropy of entanglement between neutrino q and rest:

$$S(\omega_q) = -\mathrm{Tr}[\rho_q \log \rho_q]$$

- "Polarization vector" of neutrino q, $\vec{P}(\omega_q) = 2\,\langle \vec{J}_q \rangle$, related to the reduced density matrix as:

$$\rho_q = \frac{1}{2}\left(\mathbb{I} + \vec{P}(\omega_q)\cdot\vec{\sigma}\right)$$

Relations between entanglement measures

Entanglement entropy has a one-to-one, inverse relationship with the magnitude of the polarization vector

$$S(P_q) = -\frac{1-P_q}{2}\log\left(\frac{1-P_q}{2}\right) - \frac{1+P_q}{2}\log\left(\frac{1+P_q}{2}\right)$$

with $P_q = |\vec{P}(\omega_q)|$

- $P = 1 \iff S = 0$ (Unentangled)
- $P = 0 \iff S = \log(2)$ (*Maximally* Entangled)

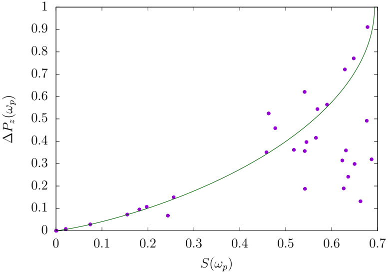
Other studies have used different entanglement measures, including bipartite measures such as Negativity, Renyi entropy, and Left-Right entanglement entropy, as well as multipartitie measures such as n-tangle (e.g., Illa and Savage: 2210.08656)

Correlation of P_z-discrepancies and entanglement entropy

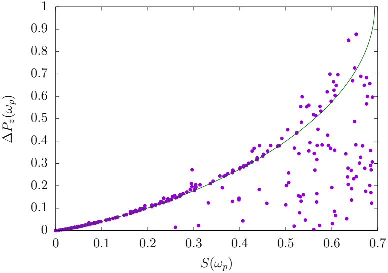
Calculate $\Delta P_z(\omega) \equiv |P_z^{\mathrm{MF}}(\omega) - P_z^{\mathrm{MB}}(\omega)|$ at $r \gg R_\nu$ (i.e., $\mu \approx 0$)

- For $N = 4$: all initial conditions with definite flavor ν_e, ν_x (e.g., $|\nu_e, \nu_x, \nu_x, \nu_x\rangle$)
- For $N = 8$: same ICs as $N = 4$, but with four additional ν_e appended to left or right of spectrum

$(N = 4)$



$(N = 8)$

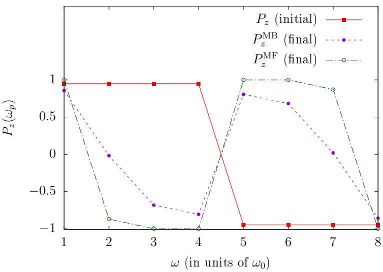


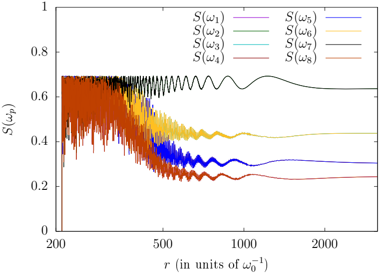
Trendline: $y(S) \equiv P^{\mathrm{MF}}(S) - P^{\mathrm{MB}}(S) = 1 - P(S)$

Example: initial condition with both neutrino flavors

Comparison of final P_z spectra between many-body and mean-field

- Evolve $|\Psi_0\rangle = |\nu_e\nu_e\nu_e\nu_e\nu_x\nu_x\nu_x\nu_x\rangle$ until $r \gg R_\nu$



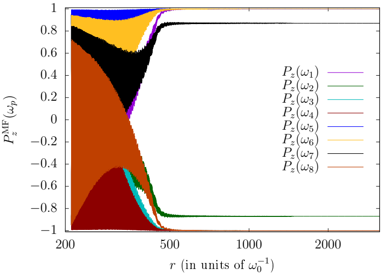


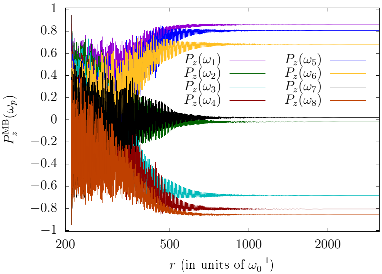
[Cervia, AVP, et al.: 1908.03511]

Spectral split-like features persist in the many-body calculations, but are less sharp relative to mean-field calculations

Comparison of P_z evolution with r

- Same initial conditions, $|\Psi_0\rangle = |\nu_e\nu_e\nu_e\nu_e\nu_x\nu_x\nu_x\nu_x\rangle$





[Cervia, AVP, et al.: 1908.03511]

Spectral splits and entanglement entropy

In [AVP, Cervia, Balantekin: 2109.08995], we extended our calculations to $N = 16$, and noted that the entanglement entropy (and corresponding deviation from the mean-field observables) was seen to be maximum for the neutrinos nearest to the spectral splits. This observation of entanglement localization in certain reigons of the neutrino spectrum motivated the use of tensor networks in a further study (2202.01865).

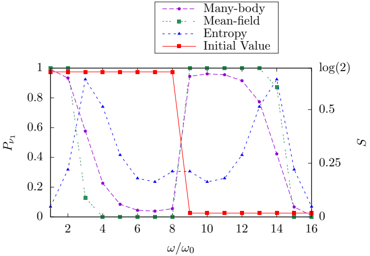


Figure: Initial and final neutrino spectra, along with the respective final state entanglement entropies, for the evolution of initial states $|\nu_e\rangle^{\otimes 8}|\nu_x\rangle^{\otimes 8}$ (left) and $|\nu_e\rangle^{\otimes 12}|\nu_x\rangle^{\otimes 4}$ (right).

Conclusions, Summary and Outlook (Part I)

- Calculations of collective neutrino flavor evolution typically rely on a 'mean-field', i.e., effective one-particle description, the efficacy of which has been (and continues to be) scrutinized
- Using a simple model of interacting neutrino plane waves in a box, certain deviations from the mean-field behaviour are observed in small systems, which can be attributed to multi-particle entanglement in this class of models
- Intriguingly, the magnitude of entanglement appears to be correlated with the locations of the spectral splits
- It has been suggested that these toy models have limited applicability in robustly determining the validity of the mean-field approximation. Nevertheless, it is a useful step that revealed a number of potentially interesting insights that may carry over into more sophisticated future treatments.

Outline

Chart of the nuclides

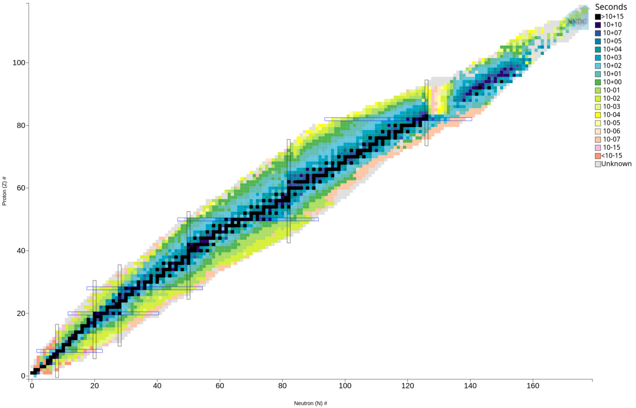


Figure: Chart of Nuclides - National Nuclear Data Center

Chart of the nuclides

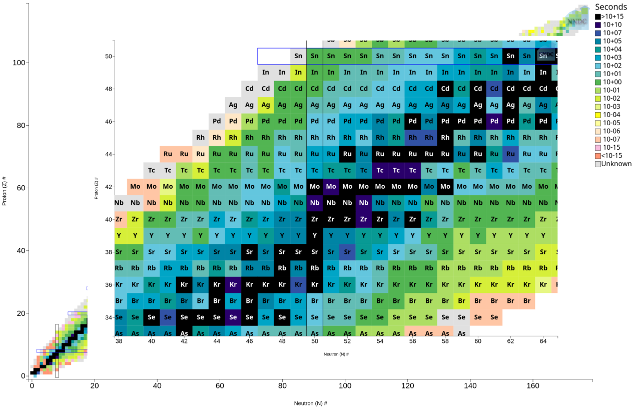


Figure: Chart of Nuclides - National Nuclear Data Center

Proton-rich heavy elements in nature

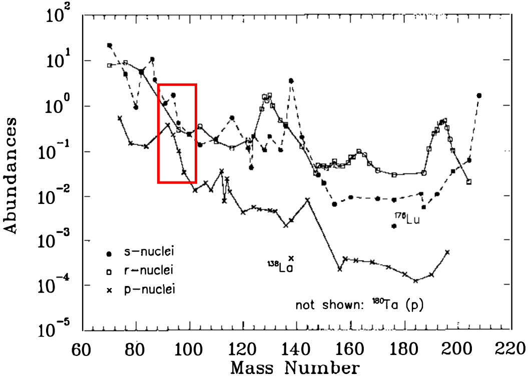


Figure: The solar system abundances of r-nuclei, s-nuclei, and p-nuclei (B. S. Meyer, Annu. Rev. Astron. Astrophys. 1994. 32: 153–190). Most p-nuclides have abundances 1–2 orders of magnitude lower than nearby s- and r-process (neutron-rich) nuclides. Except for ^{92,94}Mo and ^{96,98}Ru.

What about ^{92,94}Mo and ^{96,98}Ru?

- Transmutation of n-rich nuclides likely cannot explain the anomalously high abundances of ^{92,94}Mo and ^{96,98}Ru

- New mechanism proposed in 2005: the νp-process
  Fröhlich *et al.*, PRL 96, 142502 (2006)
  Pruet *et al.*, ApJ 644, 1028 (2006)
  S. Wanajo, ApJ 647, 1323 (2006)
  Wanajo *et al.*, ApJ 729 46 (2011)

PRL **96,** 142502 (2006) PHYSICAL REVIEW LETTERS week ending 14 APRIL 2006

Neutrino-Induced Nucleosynthesis of $A > 64$ Nuclei: The νp Process

C. Fröhlich,[1] G. Martínez-Pinedo,[2,3] M. Liebendörfer,[4,1] F.-K. Thielemann,[1] E. Bravo,[5]
W. R. Hix,[6] K. Langanke,[3,7] and N. T. Zinner[8]
[1]*Departement für Physik und Astronomie, Universität Basel, CH-4056 Basel, Switzerland*
[2]*ICREA and Institut d'Estudis Espacials de Catalunya, Universitat Autònoma de Barcelona, E-08193 Bellaterra, Spain*
[3]*Gesellschaft für Schwerionenforschung, D-64291 Darmstadt, Germany*
[4]*Canadian Institute for Theoretical Astrophysics, Toronto, Ontario M5S 3H8, Canada*
[5]*Departament de Física i Enginyeria Nuclear, Universitat Politècnica de Catalunya, E-08034 Barcelona, Spain*
[6]*Physics Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, USA*
[7]*Institut für Kernphysik, Technische Universität Darmstadt, D-64289 Darmstadt, Germany*
[8]*Institute for Physics and Astronomy, University of Århus, DK-8000 Århus C, Denmark*
(Received 10 November 2005; published 10 April 2006)

The νp-process

- Matter outflows in core-collapse supernovae are accompanied by prodigious ν_e and $\bar{\nu}_e$ fluxes, and these outflows can be proton-rich in certain situations
- Seed nuclei up to ^{56}Ni are formed once the $3\alpha \rightleftharpoons {}^{12}$C reaction falls out of equilibrium, and these remain in quasi-equilibrium with $A > 56$ nuclei till the outflow cools to $T \sim 3\,$GK
- $\bar{\nu}_e$ capture on free protons (in a p-rich wind) converts a small fraction ($\sim$ few %) of protons into neutrons, triggering (n,p) and (n,γ) reactions to bypass the β^+ decay waiting points. These, combined with (p,γ), keep the flow moving along the p-rich side for $3\,$GK $> T > 1.5\,$GK
- At $T \lesssim 1.5\,$GK, Coulomb barriers inhibit further (p,γ) reactions, and subsequent (n,p) and (n,γ) reactions drive the nuclear flow back towards stability

However . . .

Several questions raised in the intervening years regarding the νp-process efficacy

- Rauscher et. al (2013) have argued that νp cannot account for the bulk of the ^{92}Mo in the solar system due to its inability to co-produce ^{92}Nb.
- Difficulties have been reported in producing the correct isotopic ratios, as well as required absolute yields of ^{92,94}Mo and ^{96,98}Ru [e.g., Fisker *et al.* (2009), Bliss *et al.* (2018)]
- Recent calculations [Jin et al., Nature vol. 588, pg. 57–60 (2020)] reported heavy suppression of νp-process yields as a result of an in-medium enhancement of the triple-α reaction rate†. A nail in the coffin of the νp-process?

† **Note:** an enhancement in the $3\alpha \rightarrow ^{12}$C rate leads to increased seed-nuclei formation and lowers the proton-to-seed ratio, decreasing νp-process potency

High-entropy subsonic outflows to the rescue

[A. Friedland, P. Mukhopadhyay, AVP, *in preparation*]

- We discovered that subsonic outflows are much more conducive to optimal νp-process yields
- Outflow spends more time in the $3\,\text{GK} > T > 1.5\,\text{GK}$ band where the νp-process operates optimally
- Also, the material remains closer to NS compared to supersonic outflows, allowing for greater exposure to $\bar{\nu}_e$ fluxes which make neutrons needed for (n, p) and (n, γ) reactions
- Triple-α enhancement still hurts the νp-process, but may not kill it completely!
- In addition, a high entropy $S \gtrsim 80$ is required to obtain good yields — corresponds to $M_{\text{PNS}} \sim 1.8\,M_{\odot}$ for $R_{\text{PNS}} = 19\,\text{km}$

A comparison: subsonic vs supersonic outflows

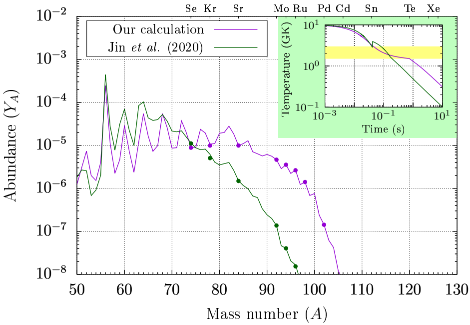


Figure: Nucleosynthesis yields in a νp-process simulation with a subsonic outflow profile (purple) obtained by solving the outflow equations [using a $13\,M_\odot$ progenitor model, with $M_{\text{PNS}} = 1.8\,M_\odot$ and $R_{\text{PNS}} = 19\,\text{km}$], and with a supersonic outflow profile (green) described in a parametric form with entropy $S = 80$ by Jin *et al.* (2020). The subsonic outflow shows $\sim$ 2 orders of magnitude higher yields of Mo and Ru.

Outline

Neutrino mixing and electron (proton) fraction

- The nuclear composition of a p-rich outflow at 3 GK consists of mainly protons, αs, and seed nuclei, with their abundances depending on the proton fraction prior to freeze-out from nuclear statistical equilibrium (NSE) at $T \simeq 6$ GK. νp-process efficacy depends on proton-to-seed ratio at 3 GK

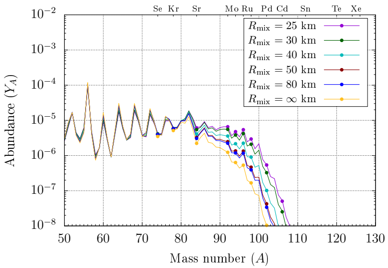
- The electron (or proton) fraction (Y_e) prior to NSE freeze-out set by ν_e and $\bar{\nu}_e$ capture rate competition. Since $\bar{\nu}_e$ have higher average energies, a luminosity hierarchy $L_{\nu_e} > L_{\bar{\nu}_e}$ is required for p-richness ($Y_e > 0.5$). Moreover, any mechanism that enhances the ν_e average energies, such as mixing between ν_e and the more energetic $\nu_{\mu,\tau}$ flavors, could make the outflow more proton-rich, improving the νp-process efficacy.

Neutrino flavor mixing implementation

- Flavor mixing implemented as complete and sharp flavor equilibration among ν_e, ν_μ, and ν_τ (and among $\bar{\nu}_e$, $\bar{\nu}_\mu$, $\bar{\nu}_\tau$) at radius R_{mix}, so that the energy distributions of each flavor at $r > R_{\text{mix}}$ are given by $(f_{\nu_e} + f_{\nu_\mu} + f_{\nu_\tau})/3$, where f_{ν_α} are the initial distributions (see also: Xiong *et al.*, arXiv:2006.11414)

- Effect of neutrino mixing examined over three regimes:
  (a) before NSE freeze-out ($T \gtrsim 6$ GK),
  (b) between NSE and QSE freeze-out (6 GK $\gtrsim T \gtrsim$ 3 GK),
  (c) after QSE freeze-out (3 GK $\gtrsim T \gtrsim$ 1.5 GK).
  Increasing ν_e and $\bar{\nu}_e$ average energies by flavor mixing has varying effects across these regimes. Typical hierarchy between ν_e and $\nu_{\mu,\tau}$ average energies is more pronounced than that between $\bar{\nu}_e$ and $\bar{\nu}_{\mu,\tau}$ $\Longrightarrow$ flavor equilibration increases ν_e average energy much more than it does for $\bar{\nu}_e$.

Neutrino flavor equilibration and the νp-process



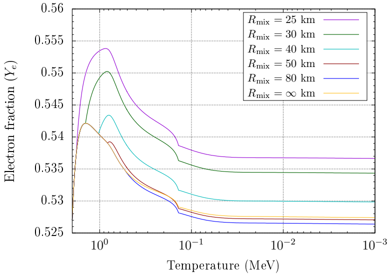


Figure: Nucleosynthesis calculations with different flavor equilibration radii $R_{\rm mix}$. **Left:** Abundance vs Mass number. **Right:** Electron fraction vs Temperature.

[AVP, A. Friedland, P. Mukhopadhyay, and S. Xin, *in preparation*]
In our model, we study these different regimes by varying the radius $R_{\rm mix}$. Flavor equilibration is found to universally improve the νp-process efficacy, more so if it occurs closer to PNS.

Conclusions, Summary and Outlook (Part II)

- νp-process nucleosynthesis continues to remain a plausible explanation for the origin of Mo and Ru p-rich isotopes
- Subsonic profiles with self-consistently modeled outflow physics can give robust νp-process yields, by increasing the neutrino exposure
- Neutrino flavor mixing close to the PNS surface can improve p-nuclide yields considerably, primarily through enhancement of the early proton-to-seed ratio, but also through increased neutron prodcution during later stages
- The effect of neutrino mixing demonstrated using the simple flavor equilibration model motivates future studies which couple a nucleosynthesis network with increasingly sophisticated treatments of collective neutrino oscillations and transport (including fast-flavor transformations and any potential beyond-the-mean-field effects).