Conformal Standard Model

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Based on K. Meissner and HN, PLB648(2007)090001

and joint work with

P. Chankowski, A. Lewandowski and K. Meissner,

Mod. Phys. Lett. A30(2015)1550006, arXiv:1404.0548

A. Latosinski, A. Lewandowski and K. Meissner,

JHEP1510 (2015) 170, arXiv:1507.01755

Related/previous work (not complete...)

- **Survival of SM up to** M_{PL} [Froggat,Nielsen(1996)]
- **Conformal symmetry and electroweak hierarchy**
  [Bardeen (1995);Meissner,HN(2007)]
- **Coleman-Weinberg symmetry breaking**
  [Elias et al.(2003); Hempfling(1996);Foot et al.(2010);...]
- **Conformal symmetry and phenomenology**
  [Holthausen, Kubo, Lindner, Smirnov(2013;...)]
- **Conformal models with** $(B-L)$ **gauging**
  [Iso,Okada,Orikasa(2009); + Takahashi(2015)]
- **Asymptotic safety and conformal fixed point at** M_{PL}
  [Wetterich,Shaposhnikov(2010)]
- νMSM **model** [Asaka,Blanchet,Shaposhnikov(2005);...]

The electroweak hierarchy problem

A main focus of BSM model building over many years:

$$m_R^2 = m_B^2 + \delta m_B^2 \ , \quad \delta m_B^2 \ \propto \ \Lambda^2 \quad \textbf{and } m_R^2 \ll \Lambda^2$$

But is this really a problem?

- **Not in renormalized perturbation theory because $\Lambda \to \infty$ and because renormalisation "does not care" whether an infinity is quadratic or logarithmic!** *(as exemplified by dimensional regularisation which does not even "see" quadratic divergences for $d = 4 + \varepsilon$)*.
- **Yes, if SM is embedded into Planck scale theory and Λ is a *physical* scale (cutoff) $\Rightarrow$ quadratic dependencies on cutoff imply extreme sensitivity of low energy physics to Planck scale physics.**

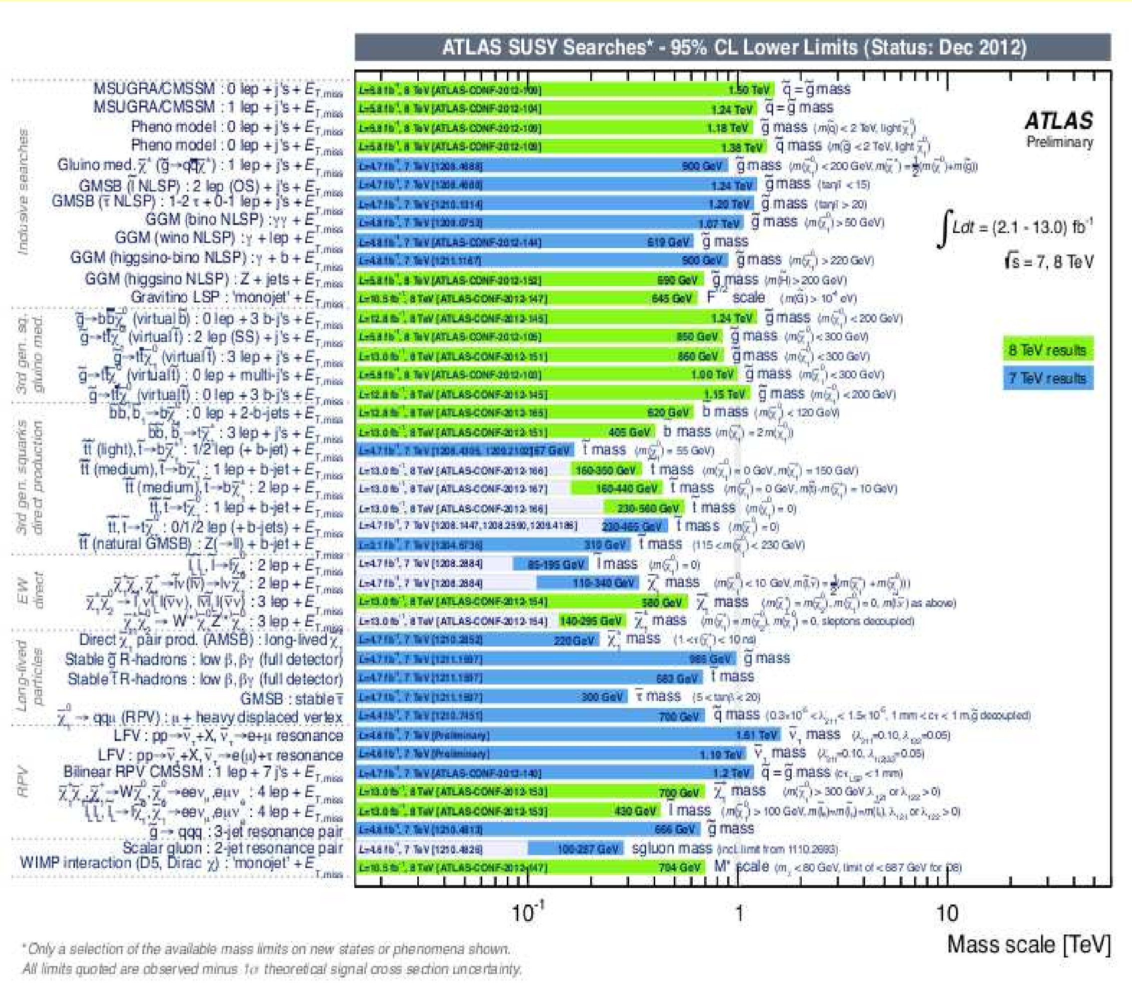
Two popular proposed solutions

- **Low energy supersymmetry:** exact cancellation of quadratic divergences by (softly broken) supersymmetry $\Rightarrow$ choice of cutoff Λ does not matter, can formally send $\Lambda \to \infty$ and adopt any convenient renormalisation scheme.
- **Technicolor** (motivated by QCD): no fundamental scalars $\Rightarrow$ no quadratic divergences $\Rightarrow$ H boson would have to be composite (could still be true...)

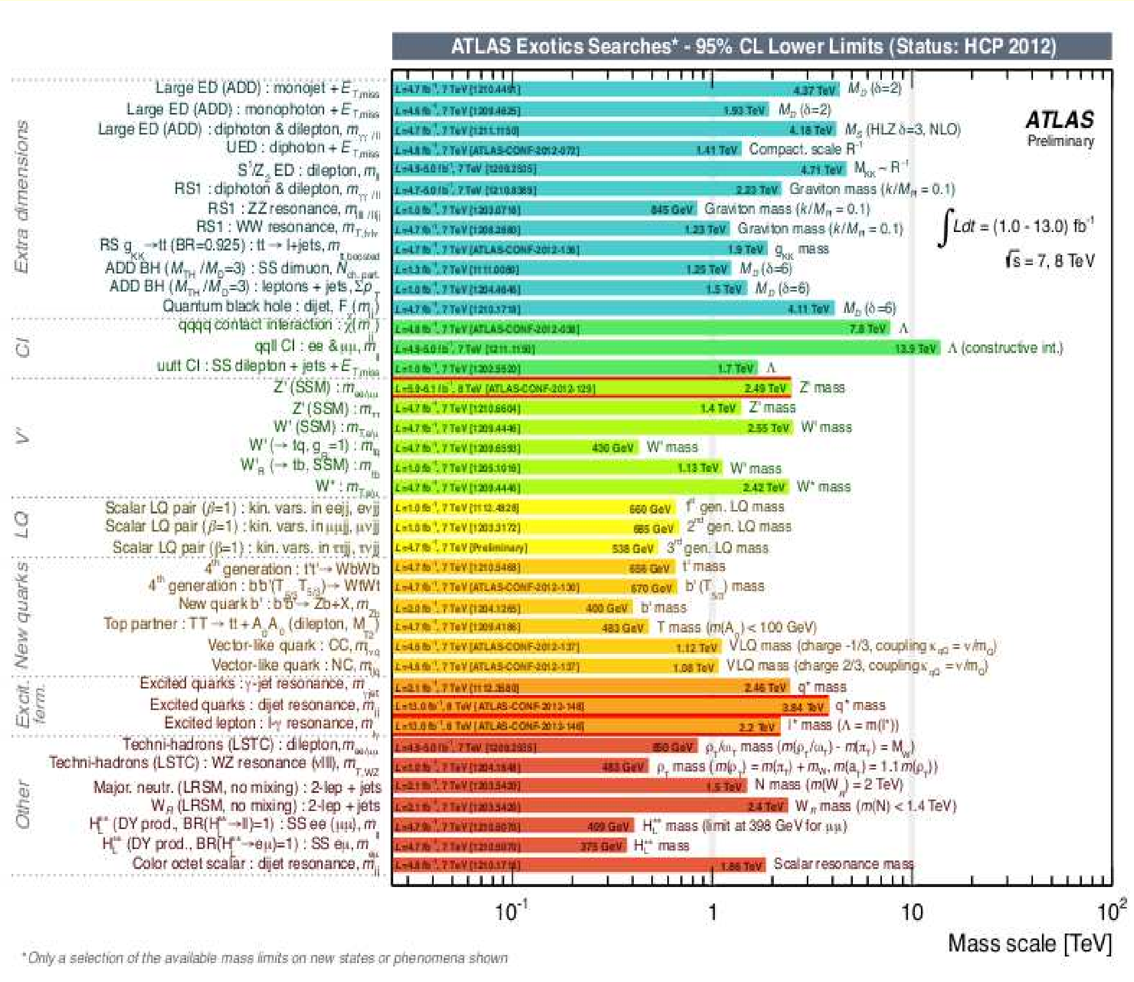
... as well as a number of other ideas

NB: these proposals would only solve the *technical part* of the hierarchy problem (= stabilising small numbers against large perturbative corrections), but would *not* explain the observed hierarchy of scales!

Low energy supersymmetry?



Low energy exotics?



Absence of any evidence (so far) from LHC for either of these options $\Rightarrow$ explore alternative options $\Rightarrow$

Can the SM survive all the way to Planck scale $M_{\rm PL}$?

In this talk: explore (softly broken) conformal symmetry for a *minimal extension of usual SM* as an alternative option.

NB: this proposal does without *low* energy supersymmetry, but supersymmetry is probably still essential for a finite and consistent theory of quantum gravity.

Realisation of such a scenario would move the SUSY breaking scale back up to the Planck scale, but make no explicit assumptions about Planck scale theory other than its UV finiteness ($\equiv$ UV completeness).

Reminder: the conformal group $SO(2,4)$

This is an old subject! [see e.g. H.Kastrup, arXiv:0808.2730]

Conformal group = extension of Poincaré group (with generators $M_{\mu\nu}, P_\mu$) by five more generators D and K_μ:

- **Dilatations (D) :** $\qquad x^\mu \to e^\alpha x^\mu$
- **Special conformal transformations (K^μ):**

$$x'^\mu = \frac{x^\mu - x^2 \cdot c^\mu}{1 - 2c \cdot x + c^2 x^2}$$

$e^{i\alpha D} P^\mu P_\mu e^{-i\alpha D} = e^{2\alpha} P^\mu P_\mu \;\Rightarrow$ ***exact*** **conformal invariance implies that one-particle spectrum is either continuous ($= \mathbb{R}_+$) or consists only of the single point $\{0\}$.**

Consequently, conformal group cannot be realized as an exact symmetry in nature.

Conformal Invariance and the Standard Model

Fact: Standard Model of elementary particle physics is conformally invariant at tree level except for explicit mass term $m^2\Phi^\dagger\Phi$ in potential $\Rightarrow$

Masses for vector bosons, quarks and leptons $\rightarrow$

Can 'softly broken conformal symmetry' ($\equiv$ 'SBCS') stabilize the electroweak scale w.r.t. the Planck scale?

Concrete implementation of this idea requires

- **Consistency conditions:**
  - **absence of Landau poles up to M_{Pl}**
  - **absence of instabilities of effective potential up to M_{Pl}**
- **Absence of any intermediate mass scales between $M_{\rm EW}$ and $M_{\rm PL}$ ('grand desert scenario').**

Evidence for large scales other than M_{Pl}?

- **(SUSY?) Grand Unification:** $m_X \geq \mathcal{O}(10^{16}\,\mathrm{GeV})$**?**
  - But: proton refuses to decay (so far, at least!)
  - SUSY GUTs: unification of gauge couplings at $\geq \mathcal{O}(10^{16}\,\mathrm{GeV})$
- **Light neutrinos** $(m_\nu \leq \mathcal{O}(1\,\mathrm{eV}))$ **and heavy neutrinos** $\rightarrow$ **most popular (and most plausible) explanation of observed mass patterns via seesaw mechanism:**

  [Gell-Mann,Ramond,Slansky; Minkowski; Yanagida]

$$m_\nu^{(1)} \sim \frac{m_D^2}{M}\,,\; m_D = \mathcal{O}(m_W) \;\Rightarrow m_\nu^{(2)} \sim M \geq \mathcal{O}(10^{12}\,\mathrm{GeV})?$$

- **Strong CP problem** $\Rightarrow$ **need *axion*** $a(x)$**?** **Limits e.g. from axion cooling in stars** $\Rightarrow$

$$\mathcal{L} = \frac{1}{4f_a} a F^{\mu\nu}\tilde{F}_{\mu\nu} \qquad \textbf{with } f_a \geq \mathcal{O}(10^{10}\,\mathrm{GeV})$$

NB: axion is (still) an attractive CDM candidate.

Conformal Invariance and Quantum Theory

Important Fact: classical conformal invariance is *generically* broken by quantum effects (unlike SUSY!) $\Rightarrow$

- Impose anomalous Ward identity
$$\Theta^{\mu}{}_{\mu} = \sum_{n} \beta^{(n)}(g) \mathcal{O}^{(n)}(x)$$
[W. Bardeen, FERMILAB-CONF-95-391-T, FERMILAB-CONF-95-377-T]
and try radiative symmetry breaking *à la* Coleman-Weinberg. But: quadratic divergences?

- Admit soft breaking (=explicit mass terms) as is commonly done for MSSM like models, but insist on cancellation of quadratic divergenes

NB: it is known that option (1) does not work for usual SM with one physical Higgs, but with one extra complex scalar (as in our model) there is more freedom.

Coleman-Weinberg Mechanism (1973)

- **Idea: spontaneous symmetry breaking by radiative corrections $\Longrightarrow$ can small mass scales be explained via *conformal anomaly* and *effective potential*?**

$$V(\varphi)_{\rm eff} = \frac{\lambda}{4}\varphi^4 \;\rightarrow\; V_{\rm eff}(\varphi) = \frac{\lambda}{4}\varphi^4 + \frac{9\lambda^2\varphi^4}{64\pi^2}\left[\ln\left(\frac{\varphi^2}{\mu^2}\right) + C_0\right]$$

- **But: radiative breaking spurious for pure φ^4 theory as is easily seen in terms of RG improved potential**

$$V_{\rm eff}^{RG} = \frac{1}{4}\lambda(L)\varphi^4 = \frac{\lambda}{4}\cdot\frac{\varphi^4}{1-(9\lambda/16\pi^2)L} \qquad L \equiv \ln\left(\frac{\varphi^2}{\mu^2}\right)$$

- **With more scalar fields finding minima and ascertaining their stability is much more difficult, as there is no similarly explicit formula for $V_{\rm eff}^{RG}(\varphi_1,\varphi_2,\ldots)$.**

Softly broken conformal symmetry (SBCS)

Assume existence of a UV complete and finite fundamental theory, such that Λ is a physical cutoff to be kept finite, and impose vanishing of quadratic divergences at particular distinguished scale Λ ($= M_{\rm PL}$?) :

- **Bare mass parameters should obey $m_B(\Lambda) \ll M_{\rm PL}$;**
- **there should be neither Landau poles nor instabilities for $M_{\rm EW} < \mu < \Lambda$ (manifesting themselves as the unboundedness from below of the effective potential depending on the running scalar self-couplings);**
- **all couplings $\lambda_R(\mu)$ should remain small (for the perturbative approach to be applicable and stability of the effective potential electroweak minimum).**

Furthermore use known SM values of couplings and masses as input parameters at $\mu = M_{\rm EW}$.

Bare vs. renormalized couplings

With cutoff Λ and normalization scale μ we have

$$\lambda_B(\mu, \lambda_R, \Lambda) = \lambda_R + \sum_{L=1}^{\infty} \sum_{\ell=1}^{L} a_{L\ell}\, \lambda_R^{L+1} \left(\ln \frac{\Lambda^2}{\mu^2} \right)^{\ell},$$

so that $\lambda_B = \lambda_R$ for $\mu = \Lambda$, and

$$m_B^2(\mu, \lambda_R, m_R, \Lambda) \;=\; m_R^2 \;-\; \hat{f}^{\text{quad}}(\mu, \lambda_R, \Lambda)\, \Lambda^2 + \; m_R^2 \sum_{L=1}^{\infty} \sum_{\ell=1}^{L} c_{L\ell}\, \lambda_R^{L} \left(\ln \frac{\Lambda^2}{\mu^2} \right)^{\ell}$$

Crucial fact: coefficient of Λ^2 can be written as a function of the bare coupling(s) only, *i.e.* $\hat{f}^{\text{quad}}(\mu, \lambda_R, \Lambda) \equiv f^{\text{quad}}(\lambda_B(\mu, \lambda_R, \Lambda))$.

Thus, keeping the physical cutoff Λ finite we can set

$$f^{\text{quad}}(\lambda_B) = 0$$

NB: this condition would not make sense if $\Lambda \to \infty$ where bare couplings are expected to become singular!

Quadratic divergences in Standard Model

[M. Veltman(1982);Y.Hamada,H.Kawai,K.Oda, PRD87(2013)5; D.R.T.Jones,PRD88(2013)098301]

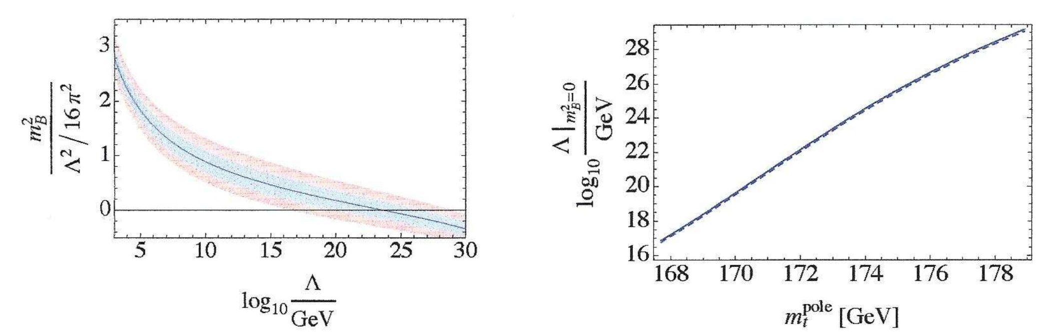


Figure 3: Left: The bare Higgs mass m_B^2 in units of $\Lambda^2/16\pi^2$ vs the UV cutoff scale Λ. The blue (narrower) and pink (wider) bands represent the one and two sigma deviations of m_t^{pole}, respectively. Right: The UV cutoff scale at which the bare mass m_B^2 becomes zero as a function of m_t^{pole}. The solid (dashed) line corresponds to the scale where m_B^2 ($m^2_{B,\,\text{1-loop}}$) becomes zero. In both panels, we have taken the central values $\alpha_s(m_Z) = 0.1184$ and $m_H = 125.7\,\mathrm{GeV}$.

Only one scalar: $f^{\mathrm{quad}}(\lambda_R(\mu)) = 0$ **for** $\mu \approx 10^{24}\mathrm{GeV} \gg M_{\mathrm{PL}}$ **!**

Is the Standard Model doomed?

[Y.Hamada,H.Kawai,K.Oda, PRD87(2013)5]

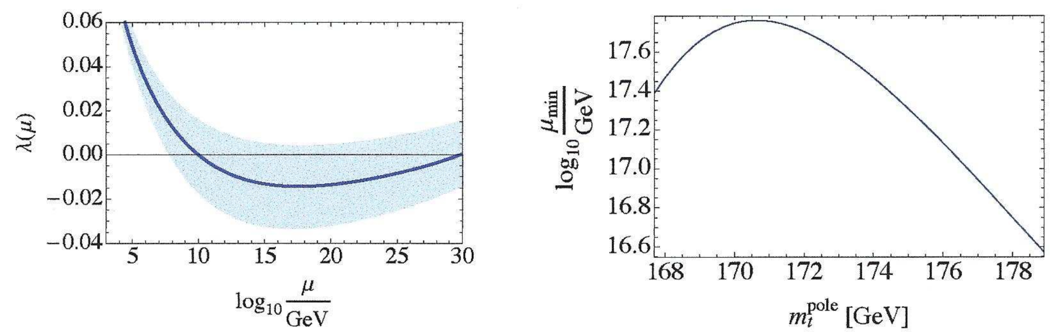


Figure 2: Left: $\overline{\mathrm{MS}}$ running of the quartic coupling λ. The band corresponds to the 1σ deviation $m_t^{\mathrm{pole}} = 173.3 \pm 2.8\,\mathrm{GeV}$. Right: The scale μ_{min} at which $\lambda(\mu)$ takes its minimum value, as a function of m_t^{pole}. In both panels, low energy inputs are given by the central values $\alpha_s(m_Z) = 0.1184$ and $m_H = 125.7\,\mathrm{GeV}$.

$\lambda_R(\mu)$ **becomes negative for** $\mu > 10^{10}\,\mathrm{GeV}$ $\Rightarrow$ **instability?**

$\rightarrow$ **might also be relevant to cosmology!**

Minimal extension of SM = CSM

[K. Meissner,HN, PLB648(2007)312; Eur.Phys.J. C57(2008)493]

- **Start from conformally invariant (and therefore renormalizable) fermionic Lagrangian** $\mathcal{L} = \mathcal{L}_{kin} + \mathcal{L}'$

$$\begin{aligned}\mathcal{L}' \ := \ & \Big(\bar{L}^i \Phi Y^E_{ij} E^j + \bar{Q}^i \epsilon \Phi^* Y^D_{ij} D^j + \bar{Q}^i \epsilon \Phi^* Y^U_{ij} U^j + \\ & + \bar{L}^i \epsilon \Phi^* Y^\nu_{ij} \nu^j_R + \phi \nu^{iT}_R \mathcal{C} Y^M_{ij} \nu^j_R + \text{h.c.} \Big) - V(\Phi, \phi)\end{aligned}$$

- **Besides usual $SU(2)$ doublet Φ: new scalar field $\phi(x)$**

$$\phi(x) = \varphi(x) \exp\left(\frac{ia(x)}{\sqrt{2}\mu} \right)$$

- **No fermion mass terms, all couplings dimensionless**
- Y^U_{ij}, Y^E_{ij}, Y^M_{ij} **real and diagonal:** $Y^M_{ij} = y_{N_i} \delta_{ij}$
  Y^D_{ij}, Y^ν_{ij} **complex** $\rightarrow$ **parametrize family mixing (CKM)**
- **Neutrino masses from usual seesaw mechanism (but with** $\langle \phi \rangle < \mathcal{O}(1\,\text{TeV})$ **and** $Y^\nu \sim 10^{-6}$ $\Rightarrow$ **no new large scales needed!)**

Scalar Sector of CSM

Right-chiral neutrinos and one complex scalar $\Rightarrow$

$$V(\Phi,\phi) = m_H \Phi^\dagger\Phi + m_\phi^2|\phi|^2 + \lambda_1(\Phi^\dagger\Phi)^2 + 2\lambda_3(\Phi^\dagger\Phi)|\phi|^2 + \lambda_2|\phi|^4$$

where $\Phi = (\Phi_1, \Phi_2)$ is the $SU(2)_{\rm EW}$ doublet and ϕ is the complex extra gauge singlet. At the minimum

$$\sqrt{2}\langle\Phi_i\rangle = v_H\delta_{i2} \ , \quad \sqrt{2}\langle\phi\rangle = v_\phi$$

with mass eigenstates h^0 and φ^0

$$\begin{pmatrix} h^0 \\ \varphi^0 \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \sqrt{2}\,\mathrm{Re}(\Phi_2 - \langle\Phi_2\rangle) \\ \sqrt{2}\,\mathrm{Re}(\phi - \langle\phi\rangle) \end{pmatrix}, \qquad (1)$$

with masses $M_h < M_\varphi$ and $|\tan\beta| < 0.3$ (from existing experimental bounds if $h^0 =$ SM Higgs-Boson).

Scalar sector can be further enlarged $\Rightarrow$ more 'sterile scalars', possibly to also explain axion as a pseudo-Goldstone boson. [cf. arXiv:1507.01755]

Quadratic divergences in CSM

Two physical scalars $\Rightarrow$ two conditions (at one loop)

$$16\pi^2 f_1^{\text{quad}}(\lambda, g, y) \;=\; 6\lambda_1 + 2\lambda_3 + \frac{9}{4}g_w^2 + \frac{3}{4}g_y^2 - 6y_t^2$$

$$16\pi^2 f_2^{\text{quad}}(\lambda, g, y) \;=\; 4\lambda_2 + 4\lambda_3 - \sum_{i=1}^{3} y_{N_i}^2$$

- **Start from known values of electroweak couplings g_y, g_w, y_t at $\mu = M_{\text{EW}}$ and evolve them to $\mu = M_{\text{PL}}$.**
- **Choose λ_1, y_N and determine λ_2 and λ_3 from $f_k^{\text{quad}} = 0$**
- **Evolve all couplings back to $\mu = M_{\text{EW}}$ and check whether all consistency requirements are satisfied.**

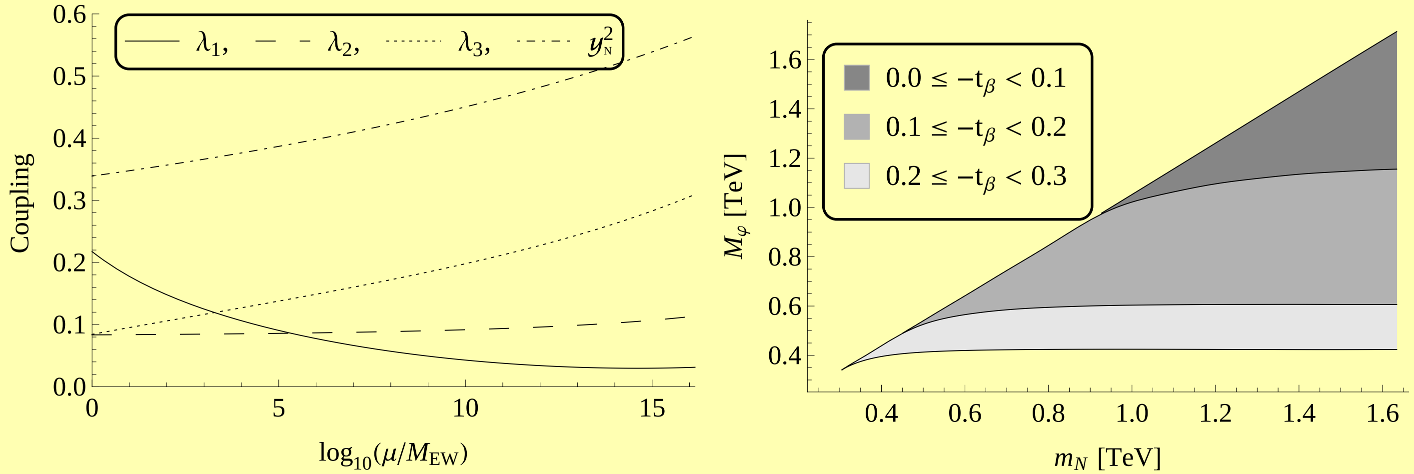
$\Rightarrow$ leads to a range of possible values for new heavy scalar φ^0 and heavy neutrinos (with $m_N < 1\,\text{TeV}$).

β-functions at one loop

$$
\begin{aligned}
\tilde{\beta}^{(1)}_{\lambda_1} &= 24\lambda_1^2 + 4\lambda_3^2 - 3\lambda_1\left(3g_w^2 + g_y^2 - 4y_t^2\right) + \\
&\quad + \frac{9}{8}g_w^4 + \frac{3}{4}g_w^2 g_y^2 + \frac{3}{8}g_y^4 - 6y_t^4 \\
\tilde{\beta}^{(1)}_{\lambda_2} &= 20\lambda_2^2 + 8\lambda_3^2 + 2\lambda_2 \sum_{i=1}^{3} y_{N_i}^2 - \sum_{i=1}^{3} y_{N_i}^4 \\
\tilde{\beta}^{(1)}_{\lambda_3} &= \frac{1}{2}\lambda_3 \left\{ 24\lambda_1 + 16\lambda_2 + 16\lambda_3 - \left(9g_w^2 + 3g_y^2\right) + 2\sum_{i=1}^{3} y_{N_i}^2 + 12y_t^2 \right\} \\
\tilde{\beta}^{(1)}_{g_w} &= -\frac{19}{6}g_w^3\,, \quad \tilde{\beta}^{(1)}_{g_y} = \frac{41}{6}g_y^3, \quad \tilde{\beta}^{(1)}_{g_s} = -7g_s^3, \\
\tilde{\beta}^{(1)}_{y_t} &= y_t \left\{ \frac{9}{2}y_t^2 - 8g_s^2 - \frac{9}{4}g_w^2 - \frac{17}{12}g_y^2 \right\}, \\
\tilde{\beta}^{(1)}_{y_{N_j}} &= \frac{1}{2}y_{N_j} \left\{ 2y_{N_j}^2 + \sum_{i=1}^{3} y_{N_i}^2 \right\}
\end{aligned}
$$

where $\tilde{\beta} \equiv 16\pi^2\beta$**.**

Admissible parameter ranges



- **Couplings remain small (also at two loops)**
- M_φ **grows with decreasing mixing angle** β
- **Usual seesaw mechanism applies, with small light neutrino masses (for** $Y_\nu \sim 10^{-6}$**) and** $m_N < 1\,\mathrm{TeV}$**?**
- **Stability of electroweak vacuum can be arranged**
- **Caveats: scheme dependencies, threshold effects?**

What might be observed

- h^0 **decay width is decreased:** $\Gamma_{h^0} = \cos^2\beta\,\Gamma_{SM} < \Gamma_{SM}$
- φ^0 **decay width:** $\Gamma_{\varphi^0} = \sin^2\beta\,\Gamma_{SM} + \cdots$
  - **First term: narrow resonance ('shadow Higgs boson')**
  - **Other terms: decay of heavy scalar into two or three h^0's, and two heavy neutrinos (if kinematically allowed).**
- $\rightarrow$ **new scalar boson is main prediction!**

  ... but not compatible with diphoton excess at 750 TeV!
- **Decay via two h^0 bosons might produce spectacular signatures with 5,...,8 leptons coming out of a single vertex. But: rates *vs.* background?**
- **Proposal can be easily discriminated against other extensions of SM that might produce similar signatures, but that would come with a lot of extra baggage (accompanying signatures).**

Outlook

- **Usual SM probably cannot survive to Planck scale $\Rightarrow$ requires *some* extension, if only to accommodate right-chiral neutrinos.**
- **A conformally motivated extension of the SM can in principle satisfy all consistency requirements, if properly embedded into a UV complete theory.**
- **Model accommodates axions naturally: $f_a \propto m_W^2/m_\nu$.**
- **Low energy supersymmetry may after all *not* be required for stability of electroweak scale...**
- **... but is probably needed for a UV complete theory of quantum gravity and quantum space-time.**

Conclusion: Nature is probably still a bit smarter than us, and may have a few more tricks up her sleeve!

Conformal invariance from gravity?

Here *not* from scale (Weyl) invariant gravity, but:

$N = 4$ **supergravity** $\qquad 1[2] \oplus 4[\frac{3}{2}] \oplus 6[1] \oplus 4[\frac{1}{2}] \oplus 2[0]$

coupled to n vector multiplets $\qquad n \times \{1[1] \oplus 4[\frac{1}{2}] \oplus 6[0]\}$

Gauged $N = 4$ SUGRA: [Bergshoeff,Koh,Sezgin; de Roo,Wagemans (1985)]

- **Scalars** $\phi(x) = \exp(L_I{}^A T^I{}_A) \in SO(6,n)/SO(6) \times SO(n)$
- **YM gauge group** $G_{\rm YM} \subset SO(6,n)$ **with dim** $G_{\rm YM} = n + 6$

[Example inspired by 'Groningen derivation' of conformal M2 brane ('BGL', 'ABJM') theories from gauged $D = 3$ SUGRAs]

Although this theory is *not* conformally invariant, the conformally invariant $N = 4$ SUSY YM theory nevertheless emerges as a $\kappa \to 0$ limit, which 'flattens' spacetime (with $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$) and coset space

$$SO(6,n)/((SO(6) \times SO(n)) \longrightarrow \mathbb{R}^{6n} \ni \phi^{[ij]\,a}(x)$$

However: conformality of limit requires extra restrictions, in particular *compact* gauge group:

$$G_{\rm YM} \subset SO(n) \subset SO(6,n)$$

Exemplify this claim for scalar potential: with

$$C_{ai}{}^{j} = \kappa^2 f_{abc} \phi_{[ik]}{}^{b} \phi^{[jk]\,c} + \mathcal{O}(\kappa^3) \quad , \quad C_{ij} = \kappa^3 f_{abc} \phi_{[ik]}{}^{a} \phi^{b\,[kl]} \phi_{[lj]}{}^{c} + \mathcal{O}(\kappa^4)$$

potential of gauged theory is $(m,n = 1,\ldots,6;\ \kappa|z|<1)$

$$V(\phi) = \frac{1}{\kappa^4} \frac{(1-\kappa z)(1-\kappa z^*)}{1-\kappa^2 z z^*} \left(C_{ai}{}^{j} C^{ai}{}_{j} - \frac{4}{9} C^{ij} C_{ij} \right) = \operatorname{Tr} \left[X_m , X_n \right]^2 + \mathcal{O}(\kappa)$$

Idem for all other terms in Lagrangian! Unfortunately

- $N=4$ **SYM is quantum mechanically conformal theory** $\rightarrow$ **no conformal anomaly** $\rightarrow$ **no symmetry breaking!**
- **Thus need non-supersymmetric vacuum with** $\Lambda = 0$

$\Rightarrow$ **must look for a better theory with above features!**

Metamorphosis of CW mechanism?

But: if we embed SM in a *UV finite theory of quantum gravity* what is the origin of (conformal) anomalies?

Conjecture: If this (unknown) theory admits a classically conformal flat space limit, CW-like contributions could arise from *finite logarithmic* (in κ) *quantum gravitational corrections* $\Rightarrow$ identify $v \sim M_{Pl}$!

- **CW-like corrections would not be due to UV divergences, but rather to the fact that gravity is *not conformal* – this would be the only 'footprint' that quantum gravity leaves in low energy physics.**
- **Observed mass spectrum and couplings in the SM could have their origin in quantum gravity.**