Neutrino Propagation with Non-unitarity

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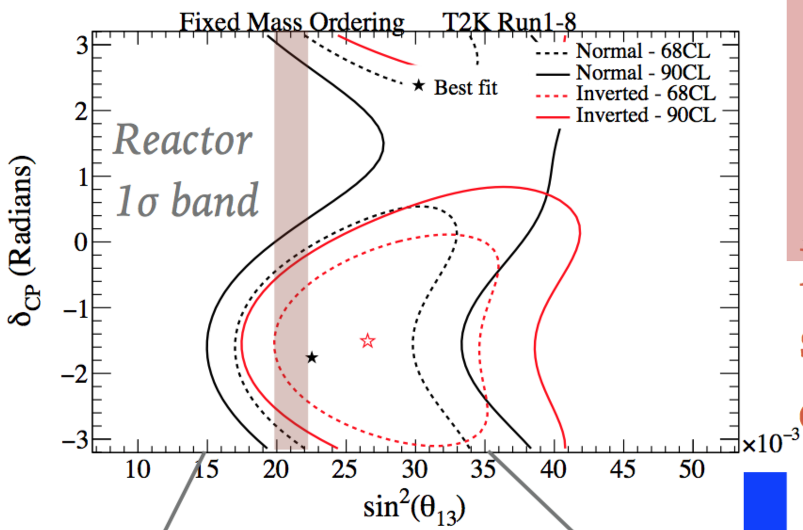
Chee Sheng Fong, HM, Hiroshi Nunokawa, arXiv:1609.08623,

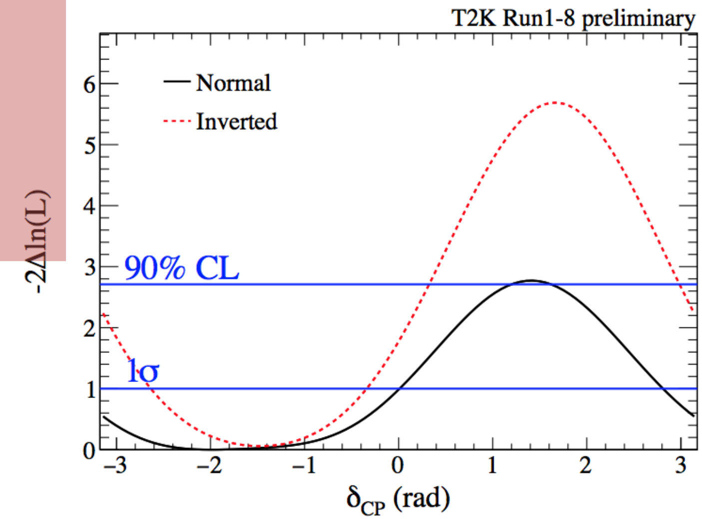
OSCILLATION PARAMETER SENSITIVITIES (2017)

Without the reactor experiment constraint on $\sin^2 2\theta_{13}$

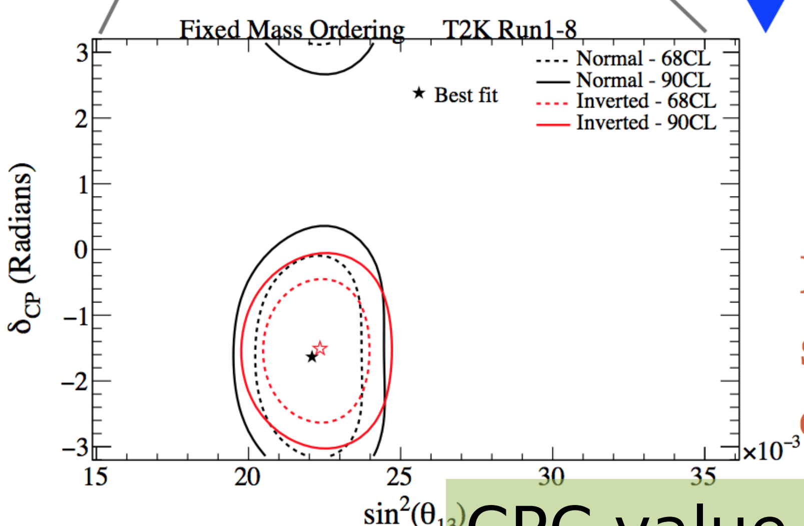
Mark Hartz, KEK Colloquium Aug 2017



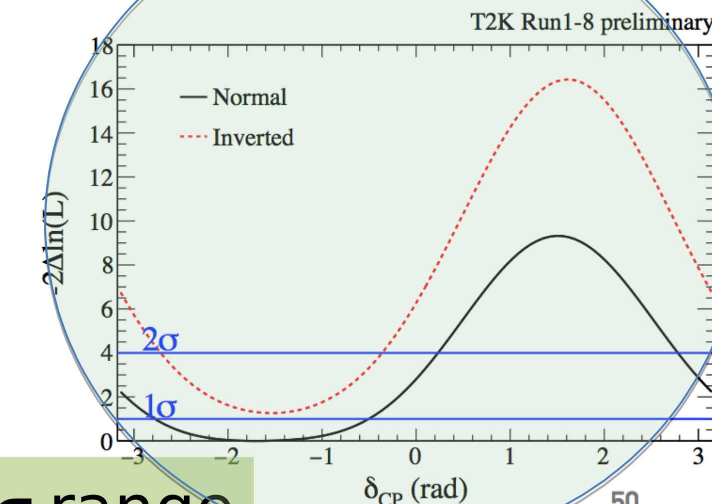
Integrate out $\sin^2\theta_{13}$ dependence



Reactor constraint on $\sin^2(2\theta)_{13}$ (PDG2016)



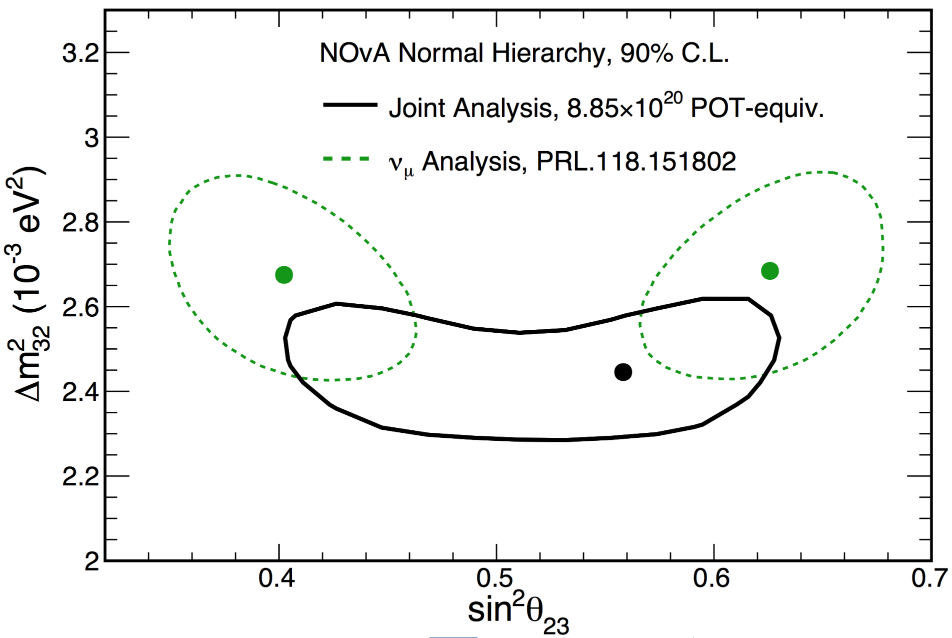
Integrate out $\sin^2\theta_{13}$ dependence



CPC value outside 2σ range

ns shared.

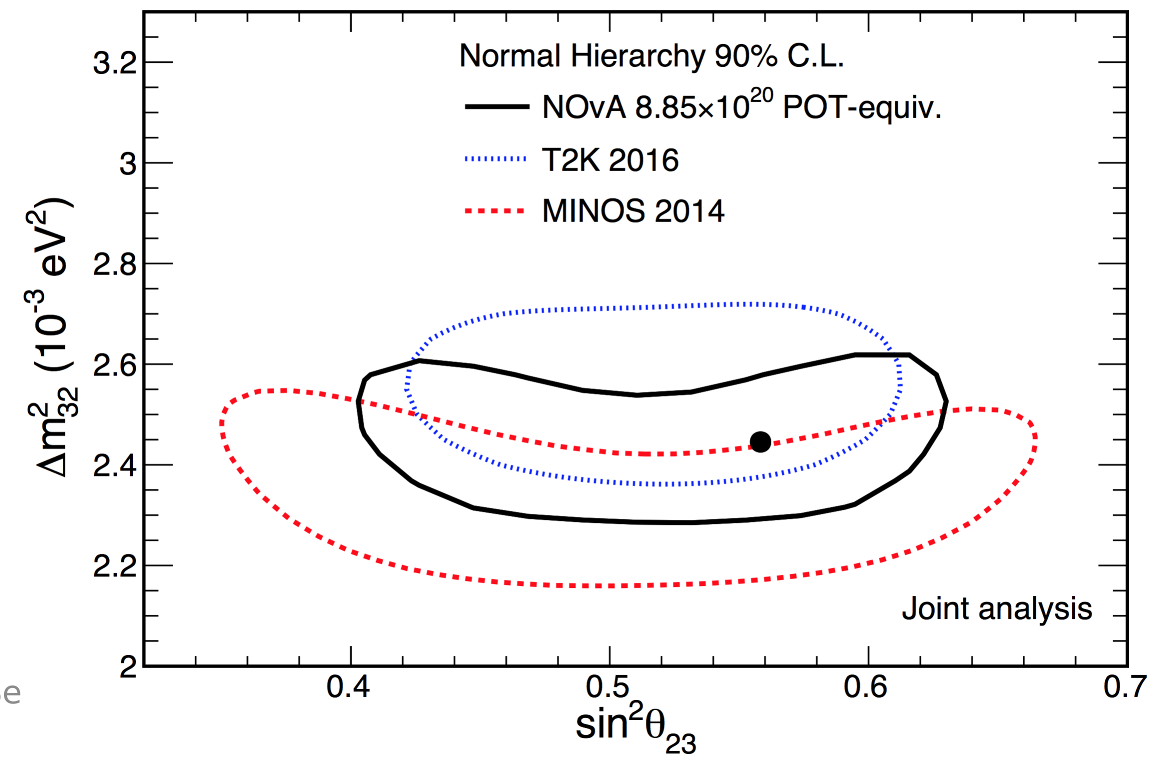
NOvA Preliminary



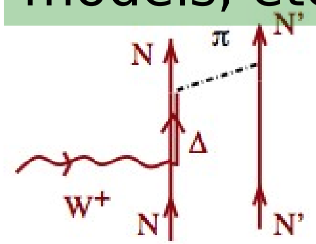
Agree on near maximal θ_{23}

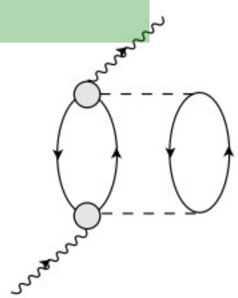
Alexander Radovic
Wine&Cheese@ Fermilab
Jan 2018

NOvA Preliminary



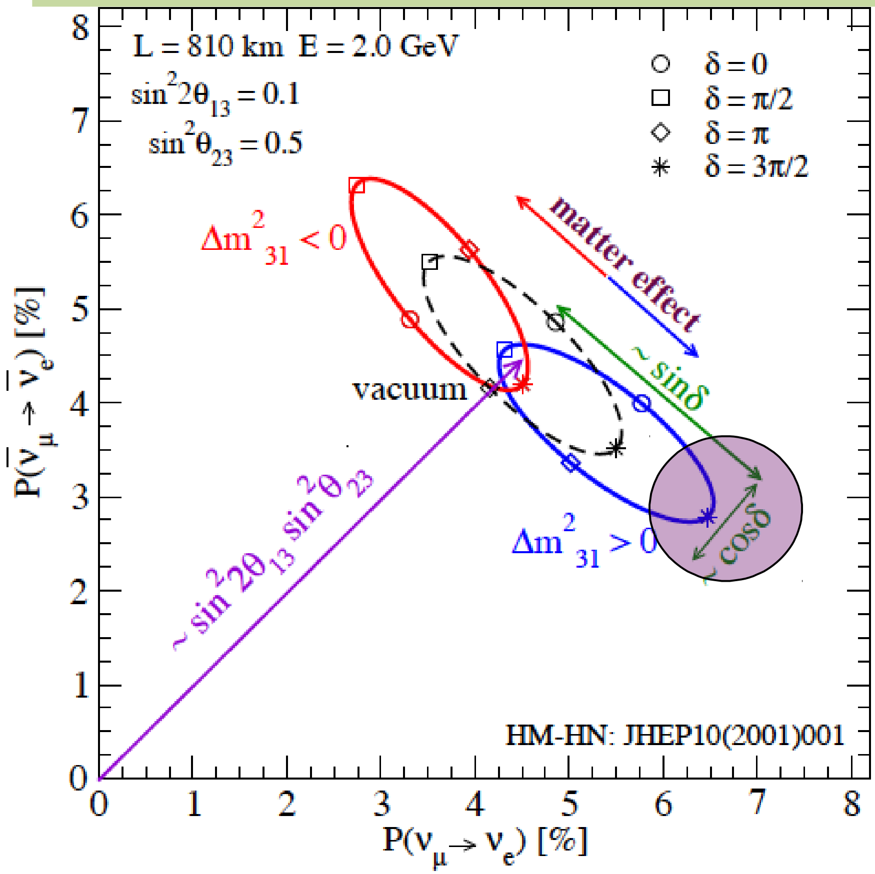
Improving on nuclear physics models, etc

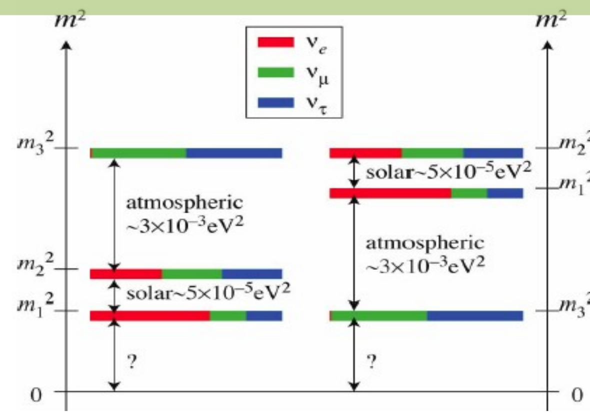




δ=3π/2 (or –π/2) implies that we are at the tip of the ellipse ⮕ the best case for NOvA

P-\bar{P} bi-probability diagram, proposed by HM-H.Nunokawa, JHEP 2001





Sign of Δm^2_{31} distinguishes normal vs inverted mass ordering

δ and sign Δm^2_{31} couple because (Δm^2_{31} ✉ $-\Delta m^2_{31}$, δ ✉π-δ) symmetry in vacuum (JHEP 2001)

Assuming all these go through well, the key question is "What is left?" and "what is most important among them?"

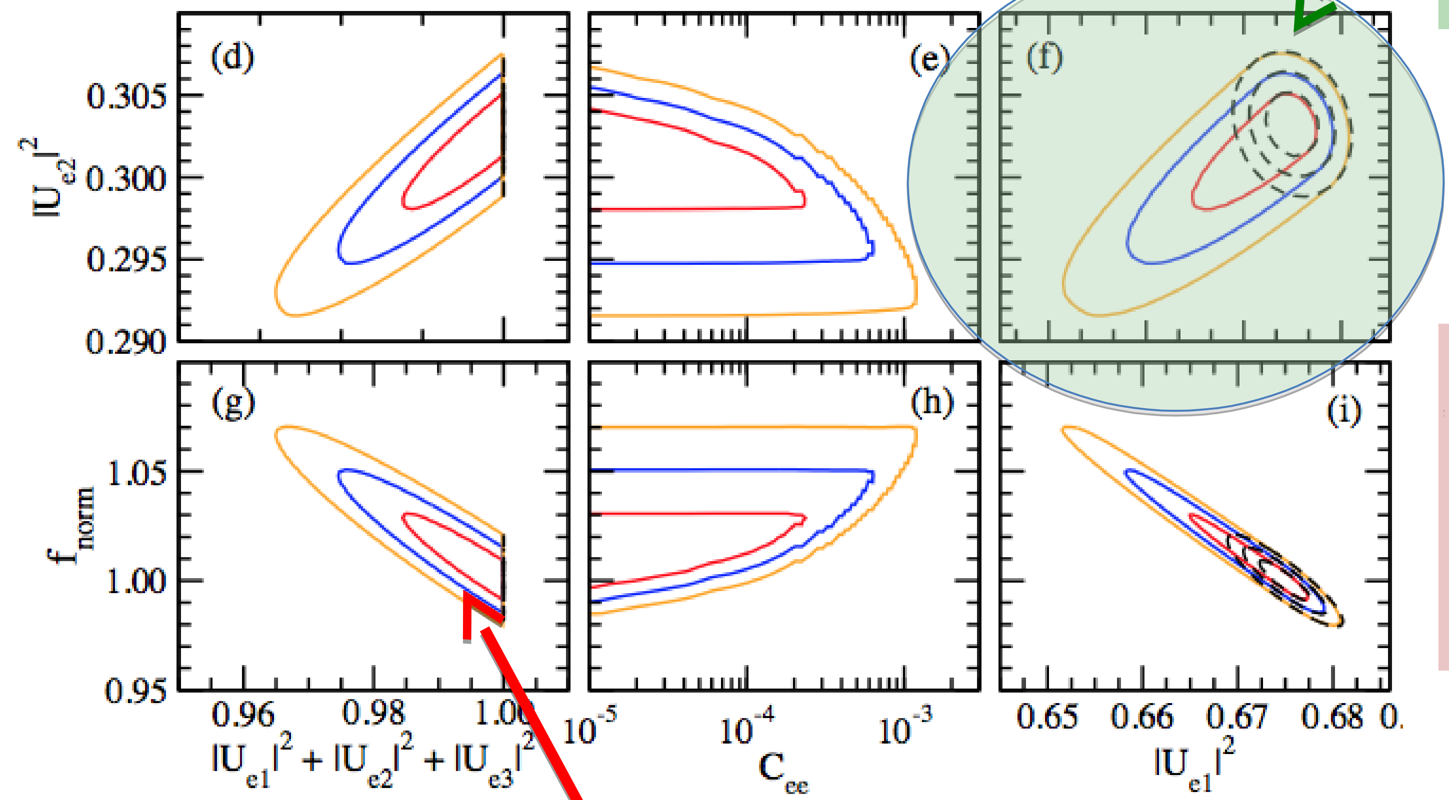
My answer is:

- Paradigm Test !

JUNO can m
easure
$|U_{e1}|^2 + |U_{e2}|^2$
$+ |U_{e3}|^2$

Separate determination $|U_{e1}|^2$ and $|U_{e2}|^2$: JUNO 5 years

Separate measurement of Ue1 and Ue2: 3% flux error assumed



JUNO Resolving Δm^2_{31} vs. Δm^2_{32} waves

$1 - (|U_{e1}|^2 + |U_{e2}|^2 + |U_{e3}|^2)$ can be constrained to ~1% level by JUNO !

Chee Sheng Fong, HM, Hiroshi Nunokawa, IHEP 2017

Method 1: Unitarity triangle

Yes, model-independent!

$$U_{e1}U^*_{\mu 1}+U_{e2}U^*_{\mu 2}+U_{e3}U^*_{\mu 3}=0,$$

- Determine $|U_{e1}|$, $|U_{e2}|$, $|U_{e3}|$, separately
  JUNO (+Daya Bay etc)
- Can we do "ν_μ-JUNO"? Determine $|U_{\mu 1}|$, $|U_{\mu 2}|$, $|U_{\mu 3}|$, separately
  2^{nd} OM
- For JPARC beam, L=300x30=9000 km, pretty hard….
- 6×10^{20} ν_e-bar/s ($1GW_e$) 6×10^{20}

Method 2: Models with unitarity violation

- Prepare generic model of unitarity violatio n
- Constrain these models by confronting th em with experiments

- Question: Can I do it in a completely mod el-independent way?
- Unitarity test is a passive way

High vs lo w scale u nitarity vi olation

New Physics at high energies:

- This is Orthodox way, well studied ..
- high scale UV ✉ pioneering work by Antusch, Biggio, Fernandez-Martinez, Gavela, and Lopez-Pavon, JHEP2006

Note: (3+N) model invented by Schechter-Valle in 1980

- But, neutrino experiments will not be the best player for unitarity test
- the reason is: high-energy ✉ SU(2) x U(1) prevails ✉ charge lepton gives stronger constraints

My suggestion today is low-E UV

New Physics at low energies:
relatively new option

- "low scale": heavy leptons/neutrinos do commu nicate with light ν system, i.e., participate to nu os cillation
- Various scenarios are proposed which involve "ne w physics" at low energies: motivated by LSND-Mi niBoone, DAMA, etc.

[21] A. E. Nelson and J. Walsh, "Short Baseline Neutrino Oscillations and a New Light Gauge Boson," Phys. Rev. D **77** (2008) 033001 doi:10.1103/PhysRevD.77.033001 [arXiv:0711.1363 [hep-ph]].

[22] M. Pospelov and J. Pradler, "Elastic scattering signals of solar neutrinos with enhanced baryonic currents," Phys. Rev. D **85** (2012) 113016 Erratum: [Phys. Rev. D **88** (2013) no.3, 039904] doi:10.1103/PhysRevD.85.113016, 10.1103/PhysRevD.88.039904 [arXiv:1203.0545 [hep-ph]].

[23] R. Harnik, J. Kopp and P. A. N. Machado, "Exploring nu Signals in Dark Matter Detectors," JCAP **1207** (2012) 026 doi:10.1088/1475-7516/2012/07/026 [arXiv:1202.6073 [hep-ph]].

[24] K. S. Babu, A. Friedland, P. A. N. Machado and I. Mocioiu, "Flavor Gauge Models Below the Fermi Scale," JHEP **1712** (2017) 096 doi:10.1007/JHEP12(2017)096 [arXiv:1705.01822 [hep-ph]].

Plus many more !!

- Orthodoxy seems challenged, e.g., WIMP dark mat ter, low-E SUSY, day one NP, ..

High- vs low-energy unitarity violation

Low-energy UV

- lepton flavor universality: YES
- zero distance neutrino flavor transition: NO
- Kinematical effect of sterile nu emission: YES

High-energy UV

- lepton flavor universality: NO
- zero distance neutrino flavor transition: YES
- Kinematical effect of sterile nu emission: YES (if kinematically allowed)

High-energy unitarity violation

Antusch et al JHEP 2006

- When high mass sector integrated out we have effective Lagrangian of light neutrinos and leptons but with unitarity violation

Aiming at model-independent formulation !!

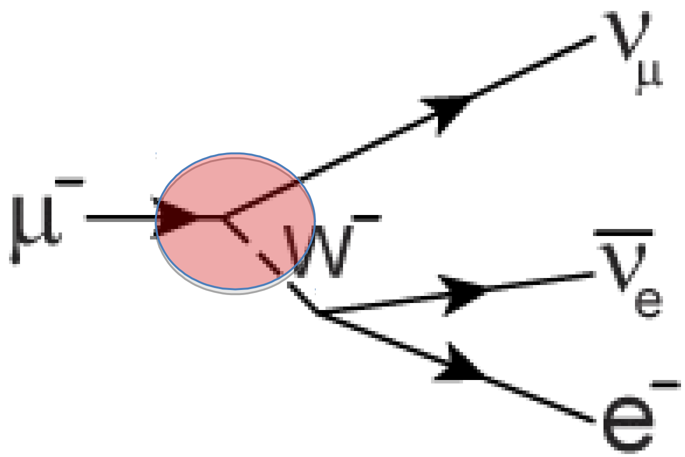
$$\mathcal{L}^{eff} = \frac{1}{2}\left(\bar{\nu}_i i \partial\!\!\!/ \nu_i - \overline{\nu^c}_i m_i \nu_i + h.c.\right) - \frac{g}{2\sqrt{2}}\left(W^+_\mu \,\bar{l}_\alpha \gamma_\mu (1-\gamma_5) N_{\alpha i} \nu_i + h.c.\right)$$
$$- \frac{g}{2\cos\theta_W}\left(Z_\mu \,\bar{\nu}_i \gamma^\mu (1-\gamma_5) (N^\dagger N)_{ij} \nu_j + h.c.\right) + \dots$$

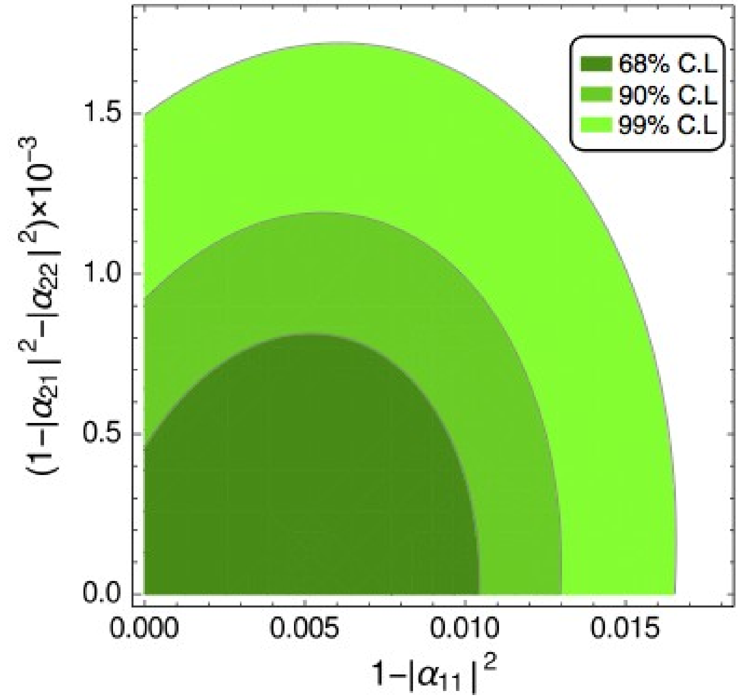
$$\nu_\alpha = N_{\alpha i}\,\nu_i \,. \qquad \langle \nu_i | \nu_j \rangle = \delta_{ij}\,, \qquad G_F = \frac{G_F^M}{\sqrt{(NN^\dagger)_{ee}(NN^\dagger)_{\mu\mu}}}\,.$$

$$|\nu_\alpha\rangle = \frac{1}{\sqrt{(NN^\dagger)_{\alpha\alpha}}} \sum_i N^*_{\alpha i}\,|\nu_i\rangle \equiv \sum_i \tilde{N}^*_{\alpha i} |\nu_i\rangle\,,$$

Flavor nu states NOT orthogonal with each

Flavor non-universality





- μ ✉ ν_i: $N_{\mu i}{}^*$
- ν_i✉ ν_μ: $N_{\mu i} / \sqrt{\ }(NN^+)_{\mu\mu}$ because

Escrihuela etal PRD2015

- $<\nu_i \mid \nu_\mu> = N_{\mu i} / \sqrt{\ }(NN^+)_{\mu\mu}$
- Then, $= \Sigma_i N_{\mu i} N_{\mu i}{}^* / \sqrt{\ }(NN^+)_{\mu\mu} = \sqrt{\ }(NN^+)_{\mu\mu}$
- $G_\mu = G_F \sqrt{\ }(NN^+)_{\mu\mu} \sqrt{\ }(NN^+)_{ee}$

Monologu
e.. by
myself

I am not sure if I understand theory of high scale unitarity violation, …

- Antusch et al JHEP 2006 treatment probab
ly fine
- But, treatment of truncated active 3 nu spa
ce looks ad hoc
- 3 active nu spans complete space of neutr
al leptons ✉ ¥sum |n><n| = 1, then there is
no room for non-unitary evolution
- Heavy sterile limit in (3+N) model? Looks g
ood idea, but it seems nontrivial

3 active+N-sterile ν model for Low-E unitarity violation

Other models of Low-E UV?

My goal today

- Introduce a model for Low-energy scale UV
- (3+N) model
- Show to what extent it can be formulated independently of details of the sterile sector models (mass spectrum, active-sterile mixing, etc.)
- Reveal how to discriminate between low-scale vs high-scale UV

3 active +N sterile unitary model in vacuum

3x3

Schechter-Valle, PRD1980

$$\nu_\alpha = \mathbf{U}_{\alpha i}\tilde{\nu}_i, \qquad \mathbf{U} = \begin{bmatrix} U & W \\ Z & V \end{bmatrix}$$

NxN

$$i\frac{d}{dx}\nu = H\nu.$$

$$H = \mathbf{U}\begin{bmatrix} \Delta_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \Delta_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \Delta_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Delta_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & \Delta_{3+N} \end{bmatrix}\mathbf{U}^\dagger$$

$$\Delta_i \equiv \frac{m_i^2}{2E} \quad (i = 1, 2, 3), \qquad \Delta_J \equiv \frac{m_J^2}{2E} \quad (J = 4, \cdots, 3+N).$$

3+N model for low-E UV and modest requests on it

- By 3+N model I mean (3+N) space is unitary, but not in 3 active nu space

Unique? Probably not. General Low-E UV model hard to construct. My strategy is….

- Requirement: The prediction of the 3+N model must be independent of details of N sterile sector
- After fulfilling this criterion we will show what is the *difference* between High-E vs Low-E UV

Probability in vacuum

$$
\begin{aligned}
P(\nu_\beta \to \nu_\alpha) &= \left|\sum_{k=1}^{3} U_{\alpha k} U^*_{\beta k}\right|^2 \qquad (4.5)\\
&-2\sum_{j\neq k} \mathrm{Re}\left(U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k}\right) \sin^2 \frac{(\Delta_k - \Delta_j)x}{2} + \sum_{j\neq k} \mathrm{Im}\left(U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k}\right) \sin(\Delta_k - \Delta_j)x\\
&+\sum_{J} |W_{\alpha J}|^2 |W_{\beta J}|^2\\
&+\sum_{J\neq K}\left[\mathrm{Re}\left(W^*_{\alpha J} W_{\beta J} W_{\alpha K} W^*_{\beta K}\right)\cos(\Delta_K - \Delta_J)x + \mathrm{Im}\left(W^*_{\alpha J} W_{\beta J} W_{\alpha K} W^*_{\beta K}\right)\sin(\Delta_K - \Delta_J)x\right]\\
&+2\sum_{j=1}^{3}\sum_{K=4}^{3+N}\left[\mathrm{Re}\left(U^*_{\alpha j} U_{\beta j} W_{\alpha K} W^*_{\beta K}\right)\cos(\Delta_K - \Delta_j)x + \mathrm{Im}\left(U^*_{\alpha j} U_{\beta j} W_{\alpha K} W^*_{\beta K}\right)\sin(\Delta_K - \Delta_j)x\right].
\end{aligned}
$$

What is this?

- Active-active, active-sterile, sterile-sterile oscillations
- If Δm^2_{as} (Δm^2_{ss}) > 0.1 eV^2, "fast oscillation" due to active-sterile and sterile-sterile Δm^2 are averaged out

$$\left\langle \sin\left(\frac{\Delta m^2_{Ji} x}{2E}\right)\right\rangle \approx \left\langle \sin\left(\frac{\Delta m^2_{JK} x}{2E}\right)\right\rangle \approx 0,$$

Fast oscillation averaged out by decoherence

$$\left|\delta\left(\frac{\Delta m^2_{ab}x}{2E}\right)\right| = \left|\frac{\Delta m^2_{ab}}{2E}\delta x - \frac{\Delta m^2_{ab}x}{2E^2}\delta E\right| \gtrsim 2\pi.$$

i. *Spatial resolution.* In this case, decoherence happens if

$$\delta x \gtrsim \frac{4\pi E}{|\Delta m^2_{ab}|}.$$

ii. *Energy resolution.* In this case, decoherence happens if

$$\delta E \gtrsim \frac{4\pi E^2}{|\Delta m^2_{ab}|x}.$$

$$\frac{\Delta m^2_{21}x}{4E} \sim 1 \qquad \text{or} \qquad \frac{|\Delta m^2_{31}|x}{4E} \sim 1.$$

LBL reactor (JUNO) | accelerator

$\Delta m^2 > 0.1\ \mathrm{eV}^2$

$$|\Delta m^2_{Ja}| \gtrsim \pi\left(\frac{\Delta m^2_{21}}{\delta E/E}\right) \approx 7.9\times 10^{-3}\mathrm{eV}^2\left(\frac{\delta E/E}{0.03}\right)^{-1} \quad \text{(for LBL reactor)},$$

$$|\Delta m^2_{Ja}| \gtrsim \pi\left(\frac{|\Delta m^2_{31}|}{\delta E/E}\right) \approx 7.5\times 10^{-2}\mathrm{eV}^2\left(\frac{\delta E/E}{0.1}\right)^{-1} \quad \text{(for accelerator)}.$$

Decoherence in ν production from pion decay

Hernandez-Smirnov PLB2012

- Decoherence parameter $\xi = (\Delta m^2/2E) \ / \ \Gamma_\pi$
- 2 states resolved if $\xi \ \gg 1$ decoherence
- Estimate ξ assuming pion decay $\Gamma=(m_\pi/E_\pi)\Gamma_0$ (Γ_0=pion width at rest)
- Approximation $E=\alpha E_\pi$
- $\xi = \Delta m^2 \ / \ 2\alpha \ m_\pi \Gamma_0$
- $1/\alpha = 2.35$
- $\xi = 0.34 \ (\Delta m^2 \ / \ 1 \text{ eV}^2)$ decoheren
  t if $\Delta m^2 > 1\text{-}\ 10 \text{ eV}^2$

P looks almost standard one, but the re is a new term

Probability leakage !

Appearanc e

$$P(\nu_\beta \to \nu_\alpha) = \mathcal{C}_{\alpha\beta} + \left| \sum_{j=1}^{3} U_{\alpha j} U^*_{\beta j} \right|^2 - 2 \sum_{j \neq k} \mathrm{Re}\left(U_{\alpha j} U^*_{\beta j} U^*_{\alpha k} U_{\beta k} \right) \sin^2 \frac{(\Delta_k - \Delta_j)x}{2}$$

$$- \sum_{j \neq k} \mathrm{Im}\left(U_{\alpha j} U^*_{\beta j} U^*_{\alpha k} U_{\beta k} \right) \sin(\Delta_k - \Delta_j)x,$$

U = non-unitary "MNS"

Disappearan ce

$$P(\nu_\alpha \to \nu_\alpha) = \mathcal{C}_{\alpha\alpha} + \left(\sum_{j}^{3} |U_{\alpha j}|^2 \right)^2 - 4 \sum_{k>j}^{3} |U_{\alpha j}|^2 |U_{\alpha k}|^2 \sin^2 \frac{(\Delta_k - \Delta_j)x}{2},$$

$$\mathcal{C}_{\alpha\beta} \equiv \sum_{J=1}^{N} |W_{\alpha J}|^2 |W_{\beta J}|^2, \qquad \mathcal{C}_{\alpha\alpha} \equiv \sum_{J=1}^{N} |W_{\alpha J}|^4$$

Order ~ W^4, small!!

P-leaking term:
It must be obvious to exist, right?

- There is a N sterile sector which can communicate with active nu sector
- So the probability leaks to sterile sector
- Yet, not emphasized before…
- ~ W^4, Too small?

$$\delta_{\alpha\beta} = \sum_{j=1}^{3} U_{\alpha j} U_{\beta j}^* + \sum_{J=4}^{N+3} W_{\alpha J} W_{\beta J}^*.$$

Then, $\left|\sum_{j=1}^{3} U_{\alpha j} U_{\beta j}^*\right|^2 = \left|\sum_{J=4}^{N+3} W_{\alpha J} W_{\beta J}^*\right|^2$ in the appearance channel ($\alpha \neq \beta$), $\left(\sum_{j=1}^{3} |U_{\alpha j}|^2\right)^2 = \left(1 - \sum_{J=4}^{N+3} |W_{\alpha J}|^2\right)^2 = 1 - \mathcal{O}(W^2)$ in the disappearance channel

Term kept by S. Parke and M. Ross Lonergan, PRD 2017 is also 4th order in W

Summary: There exists sterile-sector model independent P formula if $\Delta m^2_{as} > 0.1$ eV^2

Disappearance

$$P(\nu_\alpha \to \nu_\alpha) = \mathcal{C}_{\alpha\alpha} + \left(\sum_j^3 |U_{\alpha j}|^2 \right)^2 - 4 \sum_{k>j}^3 |U_{\alpha j}|^2 |U_{\alpha k}|^2 \sin^2 \frac{(\Delta_k - \Delta_j) x}{2},$$

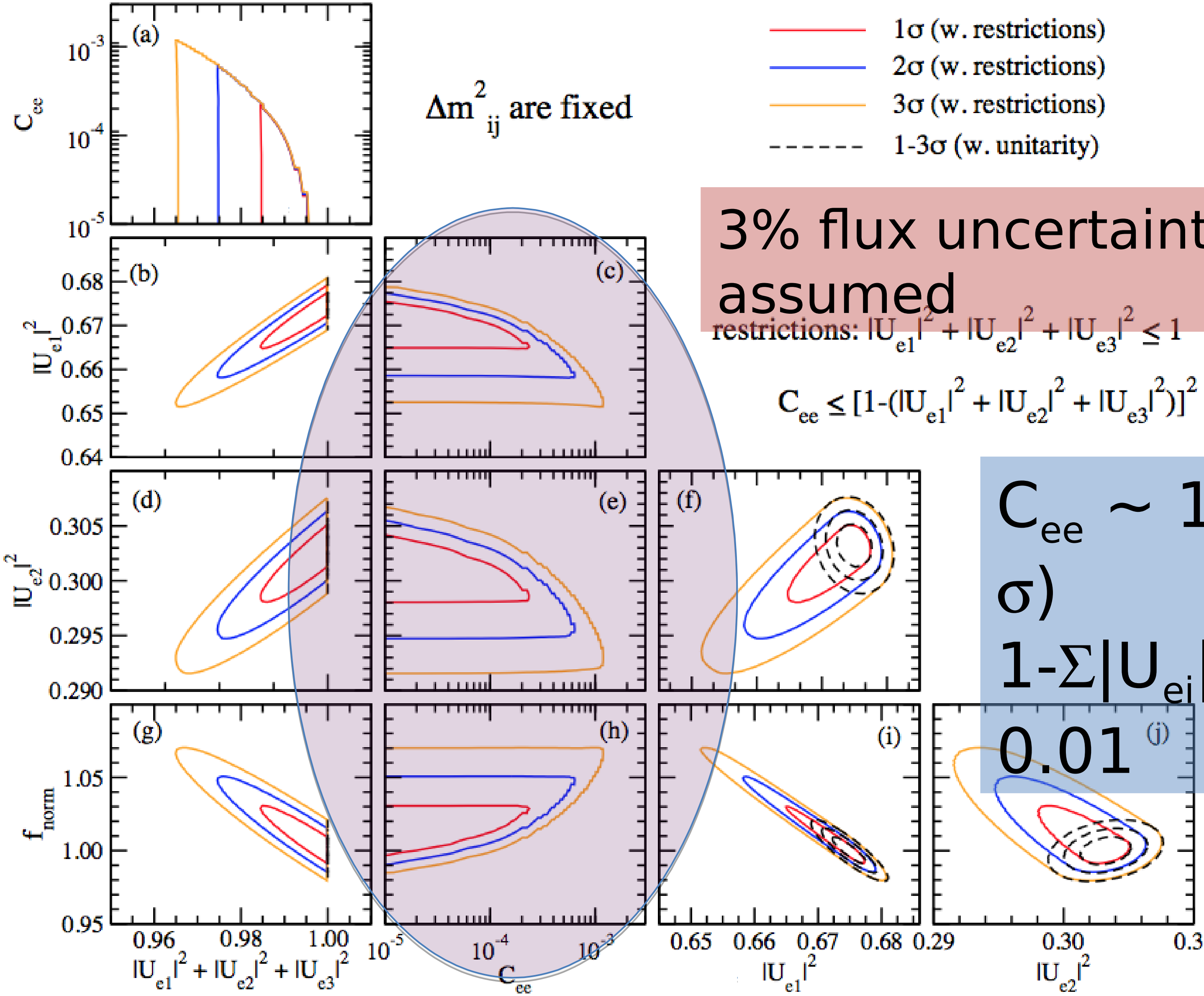
$$\mathcal{C}_{\alpha\beta} \equiv \sum_{J=1}^N |W_{\alpha J}|^2 |W_{\beta J}|^2, \qquad \mathcal{C}_{\alpha\alpha} \equiv \sum_{J=1}^N |W_{\alpha J}|^4$$

- A constant leaking term $C_{\alpha\beta}$ (= distinguishes between low-E vs high-E unitarity violation !!)
- Unitary MNS ✉ non-unitary "U"

UV effect is in: (1) explicit W correction term, (2) non-unitary U matrix

Invitation to non-unitary world..

Sensitivity to C_{ee} and $1-\Sigma|U_{ei}|^2$ from JUNO 5 years



3% flux uncertainty assumed

$C_{ee} \sim 10^{-4}$ (1 σ)

$1-\Sigma|U_{ei}|^2 \sim 0.01$

Parke-Ross-Lonergan PRD2016

Without P leaking term!

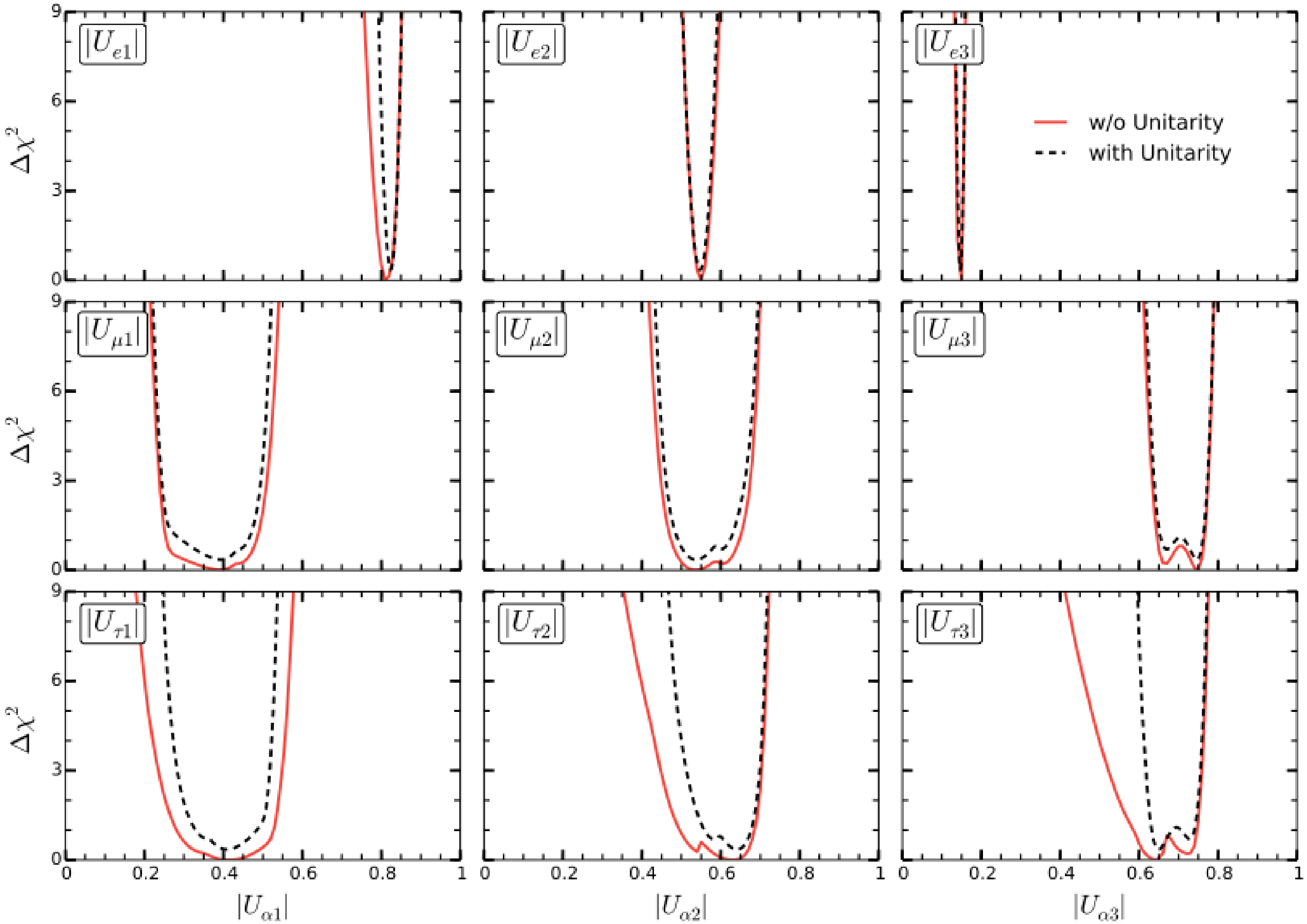


FIG. 1. Marginalized 1-D $\Delta\chi^2$ for each of the magnitudes of the 3×3 neutrino mixing matrix elements, without (red solid) and with (black dashed) the assumption of unitarity. The x-axis is the magnitude of each individual matrix element, and the y-axis is the associated $\Delta\chi^2$ after marginalization over all parameters other than the one in question. This analysis was performed for the normal hierarchy, the inverse hierarchy providing the same qualitative result.

Constraints on unitarity violation (Parke-Ross -Lonergan)

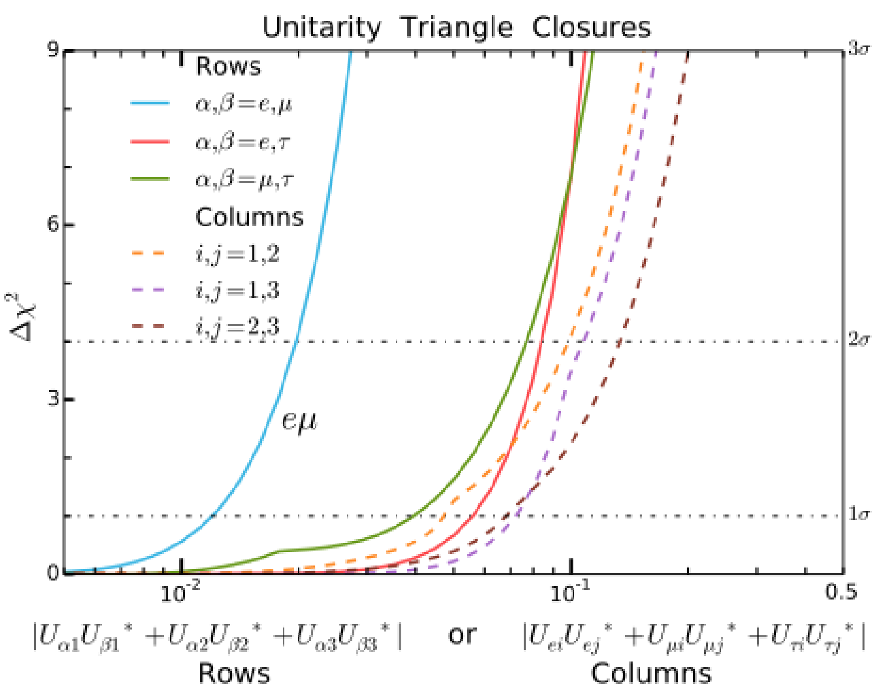


FIG. 2. 1-D $\Delta\chi^2$ for the absolute value of the closure of the three row (solid) and three column (dashed) unitarity triangles when considering new physics that enters above $|\Delta m^2| \geq 10^{-2}$ eV^2. There is one unique unitarity triangle, the $\nu_e\nu_\mu$ row unitarity triangle, in that it does not contain any ν_τ elements and hence is constrained to be unitary at a level half an order of magnitude better than the others. By comparison to Fig. 3 one can clearly see that the Cauchy-Schwartz constraints are satisfied.

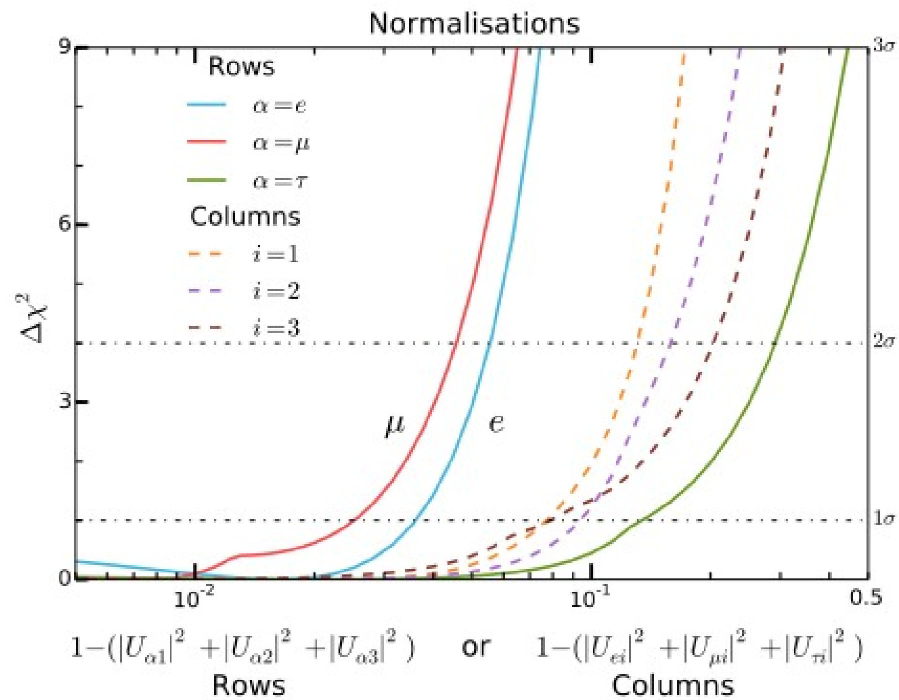


FIG. 3. 1-D $\Delta\chi^2$ for deviation of both U_{PMNS} row (solid) and column (dashed) normalizations, when considering new physics that enters above $|\Delta m^2| \geq 10^{-2}$ eV^2.

Sterile model-independent P: Prevail to “in matter” ?

Small-UV perturbation theory

Chee Sheng Fong, HM, Hiroshi Nunokawa, arXiv:1712.02798

$$H = \mathbf{U}\begin{bmatrix} \mathbf{\Delta_a} & 0 \\ 0 & \mathbf{\Delta_s} \end{bmatrix}\mathbf{U}^\dagger + \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} \equiv H_{\rm vac} + H_{\rm matt}$$

where $\mathbf{\Delta_a} = \text{diag}(\Delta_1, \Delta_2, \Delta_3)$ and $\mathbf{\Delta_s} = \text{diag}(\Delta_4, \Delta_5, \cdots, \Delta_{N+3})$.

$$A = \begin{bmatrix} \Delta_A - \Delta_B & 0 & 0 \\ 0 & -\Delta_B & 0 \\ 0 & 0 & -\Delta_B \end{bmatrix}$$

$$\mathbf{U} = \begin{bmatrix} U & W \\ Z & V \end{bmatrix}$$

$$\Delta_A \equiv \frac{a}{2E}, \qquad \Delta_B \equiv \frac{b}{2E},$$

$$a = 2\sqrt{2}G_F N_e E \approx 1.52 \times 10^{-4}\left(\frac{Y_e \rho}{\text{g cm}^{-3}}\right)\left(\frac{E}{\text{GeV}}\right)\text{eV}^2,$$

$$b = \sqrt{2}G_F N_n E = \frac{1}{2}\left(\frac{N_n}{N_e}\right)a.$$

Small-UV perturbation theory: change to vacuum mass basis

$$\tilde{H} = \tilde{H}_{\text{vac}} + \tilde{H}_{\text{matt}} = \begin{bmatrix} \mathbf{\Delta_a} & 0 \\ 0 & \mathbf{\Delta_s} \end{bmatrix} + \mathbf{U}^\dagger \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} \mathbf{U}.$$

$$\tilde{H}_0 = \begin{bmatrix} \mathbf{\Delta_a} + U^\dagger A U & 0 \\ 0 & \mathbf{\Delta_s} \end{bmatrix}, \qquad \tilde{H}_1 = \begin{bmatrix} 0 & U^\dagger A W \\ W^\dagger A U & W^\dagger A W \end{bmatrix}.$$

X=unitary matrix, diagonalize H_0(3x3)

$$\mathbf{X}^\dagger \tilde{H}_0 \mathbf{X} = \begin{bmatrix} X^\dagger \left(\mathbf{\Delta_a} + U^\dagger A U \right) X & 0 \\ 0 & \mathbf{\Delta_s} \end{bmatrix} = \begin{bmatrix} \mathbf{h} & 0 \\ 0 & \mathbf{\Delta_s} \end{bmatrix} \equiv \hat{H}_0,$$

$$\hat{H}_1 = \mathbf{X}^\dagger \tilde{H}_1 \mathbf{X} = \begin{bmatrix} 0 & (UX)^\dagger A W \\ W^\dagger A (UX) & W^\dagger A W \end{bmatrix}.$$

Do perturbation theory in hat basis

$$S_{aa} = (UX)\hat{S}_{aa}(UX)^\dagger + (UX)\hat{S}_{aS}W^\dagger + W\hat{S}_{Sa}(UX)^\dagger + W\hat{S}_{SS}W^\dagger,$$

Do W perturbation to 4th order to keep P leaking term

- Did we find ~W^4 P leaking term? Yes!
- How about what is the role of the rest?

Typical W^4 term

$$
\begin{aligned}
& P(\nu_\beta \to \nu_\alpha)^{(4)}_{2nd} \equiv 2\mathrm{Re}\left[\left(S^{(0)}_{\alpha\beta} \right)^* S^{(4)}_{\alpha\beta}[4]_{\text{ diag}} \right] \\
&= 2\mathrm{Re}\Bigg\{ \sum_n \sum_k \sum_K \Bigg[-\frac{x^2}{2} \frac{1}{(\Delta_K - h_k)^2} e^{-i(h_k - h_n)x} - \frac{2(ix)}{(\Delta_K - h_k)^3} e^{-i(h_k - h_n)x} \\
&- \frac{(ix)}{(\Delta_K - h_k)^3} e^{-i(\Delta_K - h_n)x} - \frac{3}{(\Delta_K - h_k)^4} \left(e^{-i(\Delta_K - h_n)x} - e^{-i(h_k - h_n)x} \right) \Bigg] \\
&\times (UX)_{\alpha k} (UX)^*_{\beta k} (UX)^*_{\alpha n} (UX)_{\beta n} \\
&\times \left\{ (UX)^\dagger A W \right\}_{kK} \left\{ W^\dagger A (UX) \right\}_{Kk} \left\{ (UX)^\dagger A W \right\}_{kK} \left\{ W^\dagger A (UX) \right\}_{Kk} \\
&+ \sum_n \sum_k \sum_K \sum_{m \neq k} \Bigg[\frac{(ix)}{(\Delta_K - h_k)^2 (h_m - h_k)} e^{-i(h_k - h_n)x} \\
&- \frac{(ix)}{(\Delta_K - h_k)^2 (\Delta_K - h_m)} e^{-i(\Delta_K - h_n)x} + \frac{(h_k + 2h_m - 3\Delta_K)}{(\Delta_K - h_k)^3 (\Delta_K - h_m)^2} e^{-i(\Delta_K - h_n)x} \\
&+ \frac{1}{(\Delta_K - h_m)^2 (h_m - h_k)^2} e^{-i(h_m - h_n)x} - \frac{(\Delta_K + 2h_m - 3h_k)}{(\Delta_K - h_k)^3 (h_m - h_k)^2} e^{-i(h_k - h_n)x} \Bigg] \\
&\times (UX)_{\alpha k} (UX)^*_{\beta k} (UX)^*_{\alpha n} (UX)_{\beta n} \\
&\times \left\{ (UX)^\dagger A W \right\}_{kK} \left\{ W^\dagger A (UX) \right\}_{Km} \left\{ (UX)^\dagger A W \right\}_{mK} \left\{ W^\dagger A (UX) \right\}_{Kk} \\
&+ \sum_n \sum_k \sum_{K \neq L} \Bigg[-\frac{x^2}{2} \frac{1}{(\Delta_K - h_k)(\Delta_L - h_k)} e^{-i(h_k - h_n)x} - (ix) \frac{(\Delta_K + \Delta_L - 2h_k)}{(\Delta_K - h_k)^2 (\Delta_L - h_k)^2} e^{-i(h_k - h_n)x} \\
&- \frac{1}{(\Delta_K - h_k)^3 (\Delta_L - \Delta_K)} e^{-i(\Delta_K - h_n)x} + \frac{1}{(\Delta_L - h_k)^3 (\Delta_L - \Delta_K)} e^{-i(\Delta_L - h_n)x} \\
&+ \frac{1}{(\Delta_K - h_k)^3 (\Delta_L - h_k)^3} \left\{ \Delta_L^2 + \Delta_L \Delta_K + \Delta_K^2 - 3h_k (\Delta_L + \Delta_K) + 3h_k^2 \right\} e^{-i(h_k - h_n)x} \Bigg] \\
&\times (UX)_{\alpha k} (UX)^*_{\beta k} (UX)^*_{\alpha n} (UX)_{\beta n} \\
&\times \left\{ (UX)^\dagger A W \right\}_{kK} \left\{ W^\dagger A (UX) \right\}_{Kk} \left\{ (UX)^\dagger A W \right\}_{kL} \left\{ W^\dagger A (UX) \right\}_{Lk} \\
&+ \sum_n \sum_k \sum_{K \neq L} \sum_{m \neq k} \Bigg[\frac{(ix)}{(\Delta_K - h_k)(\Delta_L - h_k)(h_m - h_k)} e^{-i(h_k - h_n)x} \\
&- \frac{1}{(\Delta_K - h_k)^2 (\Delta_L - h_k)^2 (h_m - h_k)^2} \left\{ \Delta_K \Delta_L + (h_m - 2h_k)(\Delta_K + \Delta_L) + 3h_k^2 - 2h_m h_k \right\} e^{-i(h_k - h_n)x} \\
&+ \frac{1}{(\Delta_K - h_m)(\Delta_L - h_m)(h_m - h_k)^2} e^{-i(h_m - h_n)x} \\
&+ \frac{1}{(\Delta_K - \Delta_L)(\Delta_K - h_k)^2 (\Delta_K - h_m)} e^{-i(\Delta_K - h_n)x} - \frac{1}{(\Delta_K - \Delta_L)(\Delta_L - h_k)^2 (\Delta_L - h_m)} e^{-i(\Delta_L - h_n)x} \Bigg] \\
&\times (UX)_{\alpha k} (UX)^*_{\beta k} (UX)^*_{\alpha n} (UX)_{\beta n} \\
&\times \left\{ (UX)^\dagger A W \right\}_{kK} \left\{ W^\dagger A (UX) \right\}_{Km} \left\{ (UX)^\dagger A W \right\}_{mL} \left\{ W^\dagger A (UX) \right\}_{Lk} \Bigg\}. \qquad \text{(C.5)}
\end{aligned}
$$

$$
\begin{aligned}
\left|S^{(2)}_{\alpha\beta}\right|^2_{\mathrm{1st}} &= \sum_{k,K}\sum_{l,L} \frac{1}{(\Delta_K - h_k)(\Delta_L - h_l)} \\
&\times \Bigg[x^2 e^{-i(h_k-h_l)x} - (ix)\frac{e^{-i(\Delta_K-h_l)x} - e^{-i(h_k-h_l)x}}{(\Delta_K-h_k)} + (ix)\frac{e^{-i(h_k-\Delta_L)x}-e^{-i(h_k-h_l)x}}{(\Delta_L-h_l)} \\
&+ \frac{1}{(\Delta_K-h_k)(\Delta_L-h_l)}\left\{e^{-i(\Delta_K-\Delta_L)x} + e^{-i(h_k-h_l)x} - e^{-i(\Delta_K-h_l)x} - e^{-i(h_k-\Delta_L)x}\right\}\Bigg] \\
&\times (UX)_{\alpha k}(UX)^*_{\beta k}\left\{(UX)^\dagger AW\right\}_{kK}\left\{W^\dagger A(UX)\right\}_{Kk} \\
&\times (UX)^*_{\alpha l}(UX)_{\beta l}\left\{(UX)^\dagger AW\right\}_{lL}\left\{W^\dagger A(UX)\right\}_{Ll} \\
&+ \sum_{k\neq m}\sum_{K}\sum_{l\neq n}\sum_{L} \frac{1}{(h_m-h_k)(\Delta_K-h_k)(\Delta_K-h_m)}\frac{1}{(h_n-h_l)(\Delta_L-h_l)(\Delta_L-h_n)} \\
&\times \left[(\Delta_K-h_k)\,e^{-ih_m x} - (\Delta_K-h_m)\,e^{-ih_k x} - (h_m-h_k)e^{-i\Delta_K x}\right] \\
&\times \left[(\Delta_L-h_l)\,e^{+ih_n x} - (\Delta_L-h_n)\,e^{+ih_l x} - (h_n-h_l)e^{+i\Delta_L x}\right] \\
&\times (UX)_{\alpha k}(UX)^*_{\beta m}\left\{(UX)^\dagger AW\right\}_{kK}\left\{W^\dagger A(UX)\right\}_{Km} \\
&\times (UX)^*_{\alpha l}(UX)_{\beta n}\left\{(UX)^\dagger AW\right\}_{nL}\left\{W^\dagger A(UX)\right\}_{Ll} \\
&+ \sum_{k,K}\sum_{l,L}\frac{1}{(\Delta_K-h_k)(\Delta_L-h_l)}\left(e^{-i\Delta_K x}-e^{-ih_k x}\right)\left(e^{+i\Delta_L x}-e^{+ih_l x}\right) \\
&\times \left[(UX)_{\alpha k}W^*_{\beta K}\left\{(UX)^\dagger AW\right\}_{kK} + W_{\alpha K}(UX)^*_{\beta k}\left\{W^\dagger A(UX)\right\}_{Kk}\right] \\
&\times \left[(UX)^*_{\alpha l}W_{\beta L}\left\{W^\dagger A(UX)\right\}_{Ll} + W^*_{\alpha L}(UX)_{\beta l}\left\{(UX)^\dagger AW\right\}_{lL}\right] \\
&+ \sum_{K}|W_{\alpha K}|^2|W_{\beta K}|^2 + \sum_{K\neq L} e^{-i(\Delta_K-\Delta_L)x}W_{\alpha K}W^*_{\beta K}W^*_{\alpha L}W_{\beta L}. \qquad \text{(C.1)}
\end{aligned}
$$

P leaking term

Do W perturbation to 4th order to keep P leaking term

- Did we find ~W^4 P leaking term? Yes!
- How about what is the role of the rest?
- To answer the question let us first exami ne W^2 terms

After averaging out fast oscillations..

$$
\begin{aligned}
&P(\nu_\beta \to \nu_\alpha)^{(0+2)} \\
&= \left| \sum_{j=1}^{3} U_{\alpha j} U^*_{\beta j} \right|^2 - 2 \sum_{j \neq k} \mathrm{Re}\left[(UX)_{\alpha j} (UX)^*_{\beta j} (UX)^*_{\alpha k} (UX)_{\beta k} \right] \sin^2 \frac{(h_k - h_j)x}{2} \\
&- \sum_{j \neq k} \mathrm{Im}\left[(UX)_{\alpha j} (UX)^*_{\beta j} (UX)^*_{\alpha k} (UX)_{\beta k} \right] \sin(h_k - h_j)x \\
&+ 2\mathrm{Re}\Bigg\{ \sum_{m} \sum_{k,K} \frac{1}{\Delta_K - h_k} \left[(ix) e^{-i(h_k - h_m)x} - \frac{e^{-i(h_k - h_m)x}}{(\Delta_K - h_k)} \right] \\
&\times (UX)_{\alpha k} (UX)^*_{\beta k} (UX)^*_{\alpha m} (UX)_{\beta m} \left\{ (UX)^\dagger A W \right\}_{kK} \left\{ W^\dagger A (UX) \right\}_{Kk} \\
&- \sum_{m} \sum_{k \neq l} \sum_{K} \frac{1}{(h_l - h_k)(\Delta_K - h_k)(\Delta_K - h_l)} \\
&\times \left[(\Delta_K - h_k)\, e^{-i(h_l - h_m)x} - (\Delta_K - h_l)\, e^{-i(h_k - h_m)x} \right] \\
&\times (UX)_{\alpha k} (UX)^*_{\beta l} (UX)^*_{\alpha m} (UX)_{\beta m} \left\{ (UX)^\dagger A W \right\}_{kK} \left\{ W^\dagger A (UX) \right\}_{Kl} \\
&- \sum_{m} \sum_{k,K} \frac{e^{-i(h_k - h_m)x}}{(\Delta_K - h_k)} \Bigg[(UX)_{\alpha k} W^*_{\beta K} (UX)^*_{\alpha m} (UX)_{\beta m} \left\{ (UX)^\dagger A W \right\}_{kK} \\
&+ W_{\alpha K} (UX)^*_{\beta k} (UX)^*_{\alpha m} (UX)_{\beta m} \left\{ W^\dagger A (UX) \right\}_{Kk} \Bigg] \Bigg\},
\end{aligned}
$$

Zeroth order in W

Large denominator suppression ✉ Always comes with matter potential

Do it: W perturbation to 4th order to keep P leaking term

- Did we find $\tilde{W}^4$ P leaking term? ⇒ Yes!
- How about what is the role of the rest?
- To answer the question let us first examine W^2 terms
- If we impose $\Delta m^2_{jK} > 0.1$ eV^2, then all the $\tilde{W}^2$ correction terms are small ⇒ negligible

Always comes with matter potential

- Then, all the $\tilde{W}^4$ terms can be ignored except for P leaking term

$$\frac{|A|}{\Delta m^2_{Jk}} = 2.13 \times 10^{-3} \left(\frac{\Delta m^2_{Jk}}{0.1\text{eV}^2}\right)^{-1} \left(\frac{\rho}{2.8\,\text{g/cm}^3}\right) \left(\frac{E}{1\text{ GeV}}\right),$$

A simple formula for oscillation probability in matter w/o unitarity: leading order in W pertur bation

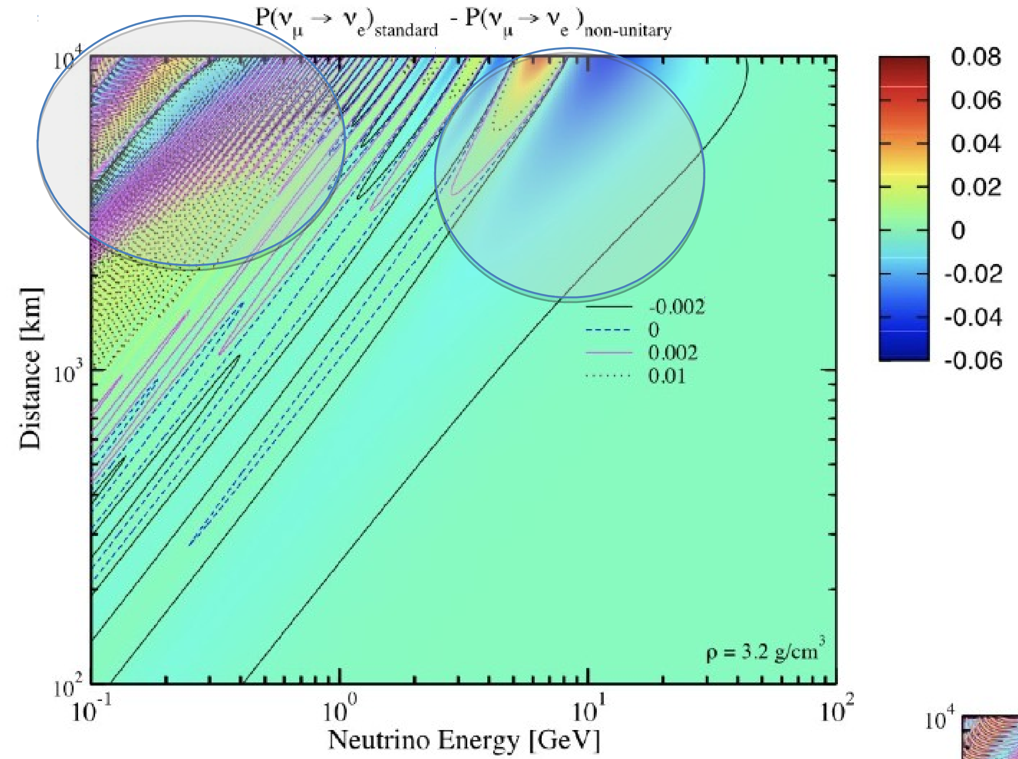
$$
\begin{aligned}
P(\nu_\beta \to \nu_\alpha) &= \mathcal{C}_{\alpha\beta} + \left| \sum_{j=1}^{3} U_{\alpha j} U^*_{\beta j} \right|^2 \\
&- 2 \sum_{j \neq k} \mathrm{Re} \left[(UX)_{\alpha j} (UX)^*_{\beta j} (UX)^*_{\alpha k} (UX)_{\beta k} \right] \sin^2 \frac{(h_k - h_j)x}{2} \\
&- \sum_{j \neq k} \mathrm{Im} \left[(UX)_{\alpha j} (UX)^*_{\beta j} (UX)^*_{\alpha k} (UX)_{\beta k} \right] \sin(h_k - h_j)x,
\end{aligned}
$$

X=...

$$
\mathbf{U} = \begin{bmatrix} U & W \\ Z & V \end{bmatrix}
$$

- All W^2 & W^4 terms avaraged out or suppressed if $\Delta m^2 >$ 0.1 eV^2 except for P leaking term!!
- UV effect is in: (1) explicit W correction term, (2) non-un itary U matrix

Where is the region of large UV?

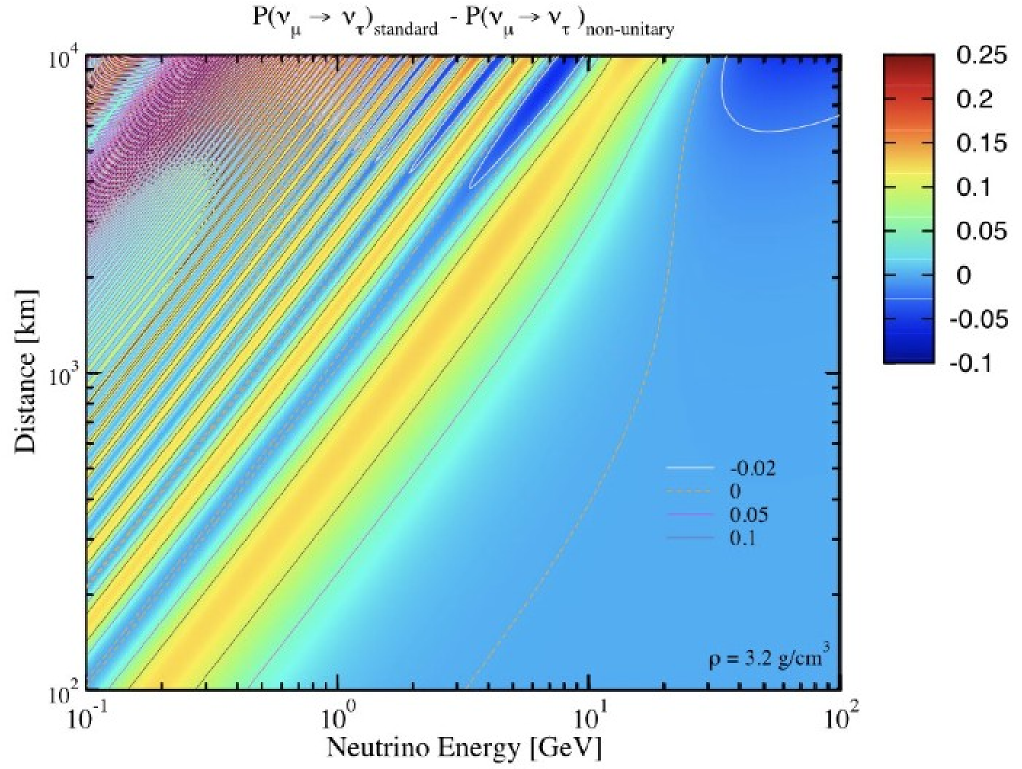


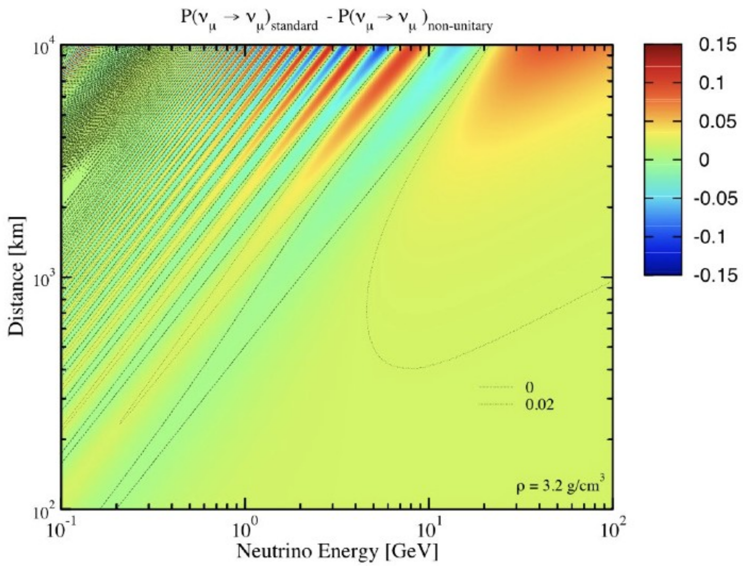
- Leading order terms = Zeroth order in W

ΔP large at solar- and atm- MSW enhanced regions

$\alpha_{11} = 0.990,\ \alpha_{21} = -0.0141,\ \alpha_{22} = 0.995,\ \alpha_{31} = -0.0445,$

$\alpha_{32} = -0.0316,\ \alpha_{33} = 0.949.$

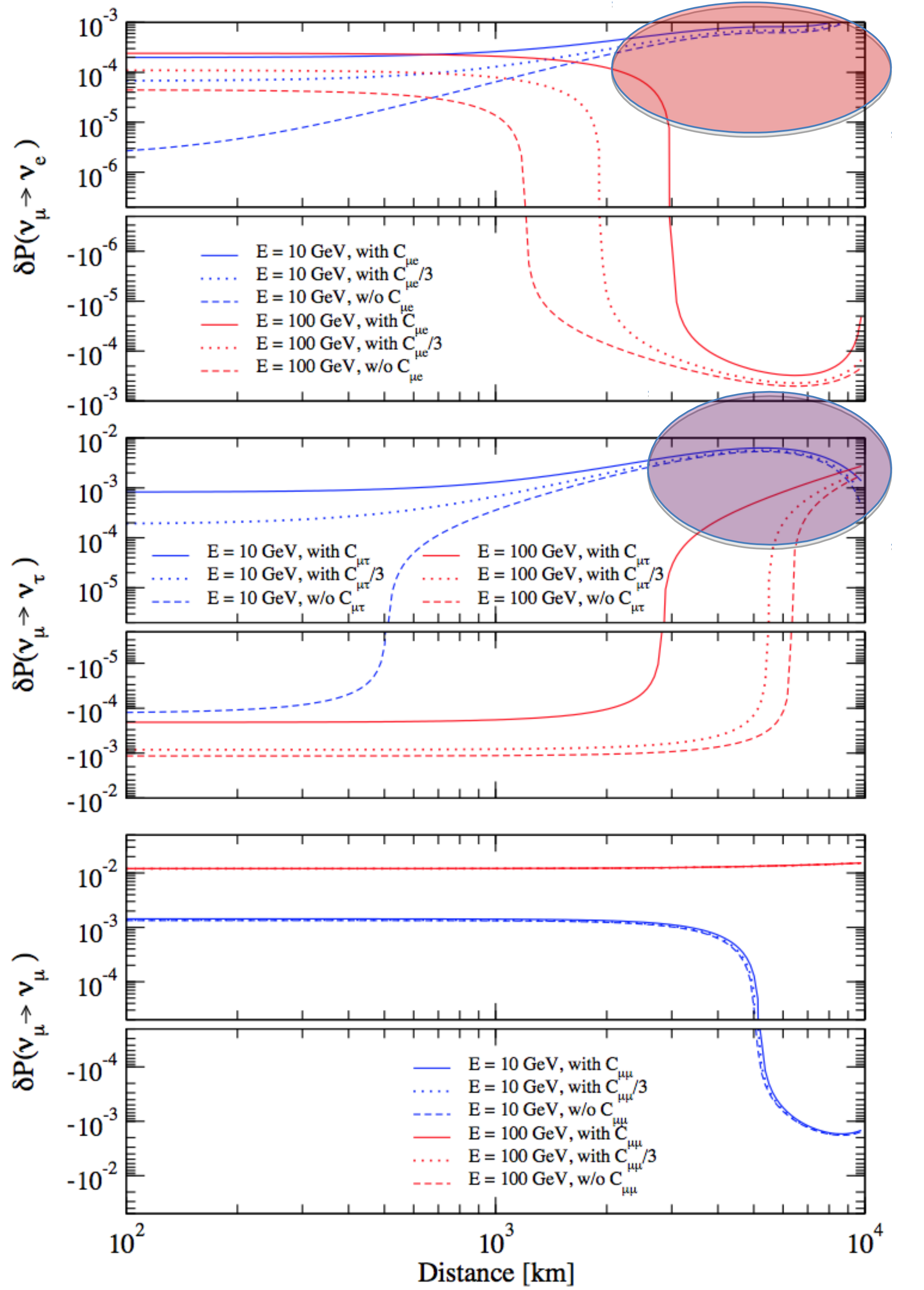




nar@M

Large $\sim W^2$ corrections?

- Order W2 correcti on terms
- small in most of the regions of L-E, but sizeable in li mited places
- High energy, long baseline IceCub e, PINGU, Hyper-K



How to pr
oceed?

How to proceed?

- Find hint for non-unitarity → can be done with leading order P, e.g., (implicit) order W^2 correction in disappearance channels

$$\left(\sum_{j=1}^{3} |U_{\alpha j}|^2\right)^2 = \left(1 - \sum_{J=4}^{N+3} |W_{\alpha J}|^2\right)^2 = 1 - \mathcal{O}(W^2)$$

- This step is being done by many people: common to high-E and low-E UV

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Plus many more !!

How to proceed? 2

- Then, if we see UV, the next step would be:
- Detect P leaking term $C_{\alpha\beta}$
- Detect explicit W^2 corrections

To distinguish low-E UV from high-E UV

- So far, we only did "JUNO" with known flux
- Detecting $C_{\alpha\beta}$ (in accelerator) requires near detecto r measurement
- T2K/T2HK ✉ ND at 300m ✉ OM for $\Delta m^2=3$ eV^2 ✉ ND before averaged out, if we limit to $\Delta m^2 <\tilde{}\, 0.3$ eV^2
- W^2 terms ✉ IceCube, Hyper-K atmospheric nu?

Conclusion (1st part)

- Mixing parameter measurement in progress ✉ looks converging
- Accumulating hints for lepton CP violation
  δ ~ $3\pi/2$ δ could me measured much earlier than we thought?
- δ ~ $3\pi/2$ implies NOvA could determine Nu mass ordering
- ~3 σ evidence for both CP and mass ordering before Hyper-K and DUNE?

Conclusion (non-unitarity)

- General structure of nu oscillation in active nu sector of (3+N) unitary system is analyzed in vacuum and in matter in the context of low-E unitarity violation
- A new term, the "probability leaking term" found (leaking to sterile sector)
- Distinguishes between Low-E vs High-E unitarity violation
- Conditions for sterile sector model-independent P in vacuum and in matter are elucidated

  $m_J^2 > 0.1\ \mathrm{eV}^2$
- Likely to be insensitive to sterile interactions

Conclusion (non-unitarity2)

- JUNO analysis shows one can constrain UV in ν_e row at a high level
- $C_{ee} \sim 10^{-4}$, $1-\Sigma|U_{ei}|^2 \sim 0.01$ (both 1 σ)
- Non-unitarity effect in the leading order (W^0) seems sizeable in solar- and atm MSW regions (Probability level)
- generally requires $L \sim 3000\text{-}10^4$ km
- W^2 correction sizeable in limited L-E regions
- distinguishes between low-E from high-E UV

$L \sim 3000\text{-}10^4$ km