Status of flavour symmetries

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MPIK, April 13^{th}, 2015

MAX-PLANCK-INSTITUT
FÜR KERNPHYSIK

Outline of the talk

I. Experimental status and motivations for flavour symmetries.
II. Theoretical status and recent developments (selection).
III. Summary and conclusions.

I. Experimental status and motivations for flavour symmetries

Lepton masses and mixing parameters

m_e, m_μ, m_τ known very precisely.

parameter	best fit	1σ-range	3σ-range
Δm^2_{21} $[10^{-5}\,\mathrm{eV}^2]$	7.50	7.33 – 7.69	7.02 – 8.09
Δm^2_{31} $[10^{-3}\,\mathrm{eV}^2]$	2.457	2.410 – 2.504	2.317 – 2.607
$\lvert\Delta m^2_{32}\rvert$ $[10^{-3}\,\mathrm{eV}^2]$	2.449	2.401 – 2.496	2.307 – 2.590
$\sin^2\theta^\ell_{12}$	0.304	0.292 – 0.317	0.270 – 0.344
$\sin^2\theta^\ell_{23}$	0.452	0.424 – 0.504	0.382 – 0.643
$\sin^2\theta^\ell_{23}$	0.579	0.542 – 0.604	0.389 – 0.644
$\sin^2\theta^\ell_{13}$	0.0218	0.0208 – 0.0228	0.0186 – 0.0250
$\sin^2\theta^\ell_{13}$	0.0219	0.0209 – 0.0230	0.0188 – 0.0251
δ^ℓ	1.7π	$1.3\pi - 1.9\pi$	$0 - 2\pi$
δ^ℓ	1.4π	$1.1\pi - 1.8\pi$	$0 - 2\pi$

[Gonzalez-Garcia et al., JHEP **1411** (2014) 052]
Upper line: normal spectrum, lower line: inverted spectrum.

m_0 unknown, Majorana phases unknown, δ^ℓ unconstrained at 3σ.

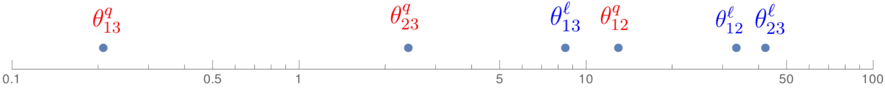
Quark masses and mixing parameters

$m_u/(10^{-6} v)$	7.4	${}^{+1.5}_{-3.0}$
$m_d/(10^{-5} v)$	1.58	${}^{+0.23}_{-0.10}$
$m_s/(10^{-4} v)$	3.12	${}^{+0.17}_{-0.16}$
$m_c/(10^{-3} v)$	3.60	± 0.11
$m_b/(10^{-2} v)$	1.639	± 0.015
$m_t/(10^{-1} v)$	9.861	${}^{+0.086}_{-0.087}$
$\sin^2\theta_{12}^q/10^{-2}$	5.080	± 0.032
$\sin^2\theta_{23}^q/10^{-3}$	1.770	± 0.054
$\sin^2\theta_{13}^q/10^{-5}$	1.325	± 0.038
δ^q/π	0.385	± 0.017

The quark masses and the parameters of the CKM matrix in the $\overline{\mathrm{MS}}$ scheme at $\mu = M_Z$ computed within the Standard Model in [Antusch, Maurer, JHEP **1311** (2013) 115]. $v = 174.104\,\mathrm{GeV}$.

All observables known with good precision.

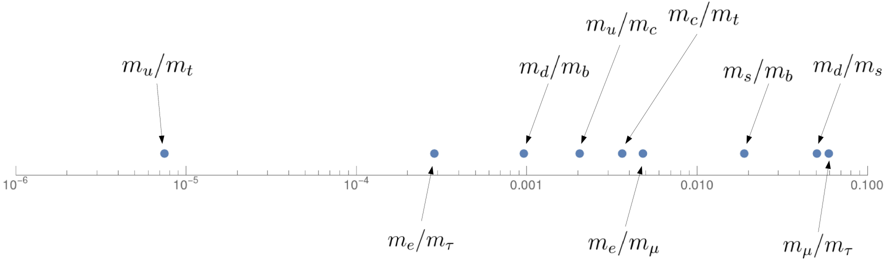
Mixing angles



Quark sector: Two mixing angles very small. Cabibbo angle $\theta^q_{12} \simeq 13^\circ$.

Lepton sector: All mixing angles larger than 8°.

Mass hierarchies



From **very strong** to **moderate**:

$$\frac{m_u}{m_t} < 10^{-5}, \quad \frac{m_\mu}{m_\tau} \simeq 0.06.$$

Neutrino sector: Masses of the three light neutrinos may show only **mild hierarchy** or may even be close to degenerate. However, also $m_0 \ll 0.05$ eV possible. ($\rightarrow$ strong hierarchy).

Motivation for flavour symmetries in these data?

Lepton sector:

✗ $U_{\text{PMNS}} = U_{\text{TBM}}$ $(\theta^\ell_{12} \approx 35.2^\circ,\ \theta^\ell_{23} = 45^\circ,\ \theta^\ell_{13} = 0^\circ)$?

- $\theta^\ell_{13} \approx 8 - 9^\circ \rightarrow$ quite far from zero.
- θ^ℓ_{23} between 38° and $53^\circ \rightarrow$ significant deviation from 45° possible.

? Models for $U_{\text{PMNS}} \approx U_{\text{TBM}}$? Other structures following from flavour symmetries only?

? Large mixing angles $\leftrightarrow$ Mild neutrino mass hierarchy?

Quark sector:

✗ $U_{\text{CKM}} = \mathbb{1}$? $\theta^q_{12} \approx 13^\circ \rightarrow$ nonzero.

? Models for $U_{\text{CKM}} \approx \mathbb{1}$? Difference to U_{PMNS} predictable by flavour symmetries?

? Small mixing angles $\leftrightarrow$ Strong quark mass hierarchy?

In order to have more clues on flavour symmetries, we would like to know the mass matrices M_u, M_d, M_ℓ and M_ν.

What do we know about the elements of the mass matrices themselves?

. . . unfortunately, not much.

However, some statements can be made.

Quark sector:

- Freedom of weak-basis transformations: Even in the basis where M_1 diagonal: Rotation of *right-handed* quark fields

$$M_2 \to M_2 V \quad (V \text{ unitary})$$

  does not change physical observables. $\Rightarrow$ Elements of mass matrices not observable.

$$\to \text{Only } M_d M_d^\dagger \text{ and } M_u M_u^\dagger \text{ observable.}$$

What do we know about the elements of the mass matrices themselves?

Lepton sector:

- In case of Dirac neutrinos: Same argument as for quarks.
- Majorana neutrinos: Situation different. In basis where **charged-lepton mass matrix M_ℓ diagonal**:

$$M_\nu = U^*_{\mathsf{PMNS}} \mathrm{diag}(m_1, m_2, m_3) U^\dagger_{\mathsf{PMNS}} \rightarrow \text{ function of phys. obs.}$$

$\rightarrow$ in principle observable!

Direct measurement of neutrino mass matrix elements?

Direct measurement in lepton-number violating processes:

- $|(M_\nu)_{ee}| = m_{\beta\beta}$ in neutrinoless double beta decay (if other contributions to the $(\beta\beta)_{0\nu}$-rate negligible). Currently only upper bounds: $m_{\beta\beta} \lesssim 0.4\,\mathrm{eV}$ [EXO-200].
- Other elements from extremely rare lepton-number violating processes[1]: E.g. $\Gamma_{K^+\to\pi^-\mu^+\mu^+} \propto |(M_\nu)_{\mu\mu}|^2 \to$ from experimental side hopeless.

$\Rightarrow$ Try to reconstruct M_ν from data on neutrino masses and mixing.

[1] W. Rodejohann, Phys. Rev. D 62, 013011 (2000); J. Phys. G 28, 1477 (2002).

Reconstruct M_ν from data

$$|(M_\nu)_{\alpha\beta}| = \left|\sum_{k=1}^{3} m_k e^{2i\sigma_k} V_{\alpha k} V_{\beta k}\right|$$

depends on the nine parameters:

$$m_0,\ \Delta m^2_{21},\ \Delta m^2_{31}, \theta^\ell_{12},\ \theta^\ell_{23},\ \theta^\ell_{13}, \delta^\ell,\ \rho,\ \sigma.$$

- m_0: bounds from cosmology, $(\beta\beta)_{0\nu}$, ^{3}H-decay: $m_0 \lesssim 0.3\,\mathrm{eV}$
- $\Delta m^2_{21},\ \Delta m^2_{31}, \theta^\ell_{12},\ \theta^\ell_{23},\ \theta^\ell_{13}, \delta^\ell$: global fits of oscillation data,
- $\rho,\ \sigma$: totally unconstrained: Have to be treated as free parameters in $[0, 2\pi)$.

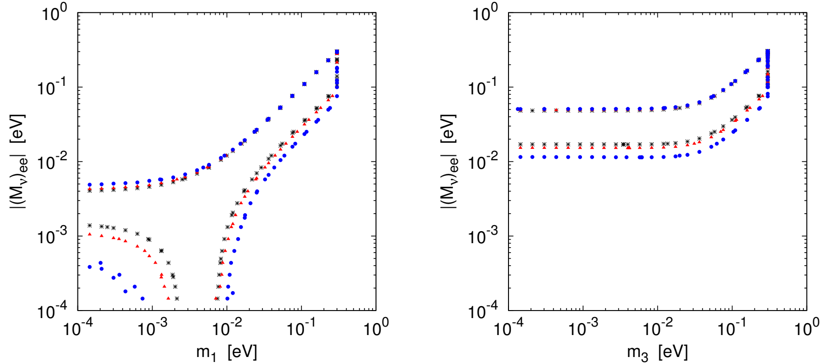
$\rightarrow$ Only upper bound for m_0, Majorana phases $\rho,\ \sigma$ unconstrained, δ^ℓ unconstrained at 3σ.

Allowed ranges for the $|(M_\nu)_{\alpha\beta}|$

A. Merle and W. Rodejohann (2006):[2] Plots of the allowed ranges of $|(M_\nu)_{\alpha\beta}|$ versus the smallest neutrino mass m_0.

2012: W. Grimus, POL repeated analysis with new data:

$\rightarrow$ At 3σ plots still in agreement with plots of Merle and Rodejohann.



[2] A. Merle and W. Rodejohann, Phys. Rev. D **73** (2006) 073012.

Correlations of the elements of the neutrino mass matrix

Idea: Due to improved data $\to$ correlation plots of $|(M_\nu)_{\alpha\beta}|$

Majorana neutrinos $\Rightarrow$ 6 independent matrix elements

$\Rightarrow$ 15 pairings, 2 spectra $\Rightarrow$ 30 correlation plots.

Among these 30 correlations: Five manifest at 3σ:

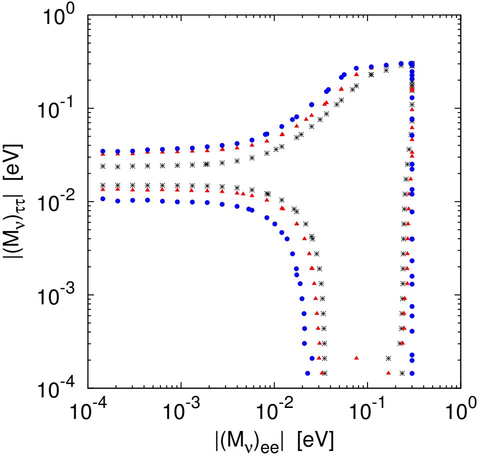
$$\begin{array}{lcll}
|(M_\nu)_{ee}| & \text{vs.} & |(M_\nu)_{\mu\mu}| & \text{(normal spectrum)} \\
|(M_\nu)_{ee}| & \text{vs.} & |(M_\nu)_{\mu\tau}| & \text{(normal spectrum)} \\
|(M_\nu)_{ee}| & \text{vs.} & |(M_\nu)_{\tau\tau}| & \text{(normal spectrum)} \\
|(M_\nu)_{\mu\mu}| & \text{vs.} & |(M_\nu)_{\mu\tau}| & \text{(normal spectrum)} \\
|(M_\nu)_{\mu\tau}| & \text{vs.} & |(M_\nu)_{\tau\tau}| & \text{(normal spectrum)}
\end{array}$$

May be summarized as:

"If one matrix element is small, the other one must be large."

Correlations of the elements of the neutrino mass matrix

Forero et al.:[3] best fit: $*$, 1σ : ▲, 3σ : $\bullet$; normal spectrum



[3] Forero et al., Phys. Rev. D 86 (2012) 073012.

Status

In total: With improved precision in lepton data: No obvious hints for symmetries.

Remaining playground: Predictions for δ^ℓ (unconstrained at 3σ).

Biggest open questions in experiment:

- Search for neutrinoless double beta decay. $\rightarrow$ Possibility to decide question of neutrino nature.
- Extensions of the electroweak scale scalar sector?

Biggest open questions in theory:

- Why is mixing in the lepton sector so different from the quark sector? Related to different mass hierarchies?

II. Theoretical status and recent developments

Flavour symmetries

$\Rightarrow$ Two possibilities:

- **Abelian symmetries** $\rightarrow$ Zeros in the mass matrices: "texture zeros". No relations between matrix elements.
- **Non-Abelian symmetries.** Relations between matrix elements.

For both cases: huge amount of papers: looking up in INSPIRE-HEP:

- find t texture $\rightarrow$ 477 citeable papers.
- find t flavor or flavour and symmetry $\rightarrow$ 767 citeable papers.

Nevertheless, no compelling model has been found!

$\rightarrow$ Main road to proceed: **Concentrate on systematic studies** rather than model building.

Systematic studies of flavour symmetry groups

Need classification of finite groups:

- **Abelian groups classified**: Direct products of cyclic groups $\mathcal{A} \simeq \mathbb{Z}_m \times \mathbb{Z}_n \times \dots$
- **non-Abelian groups not classified** in general.

→ Can use two helpful tools:

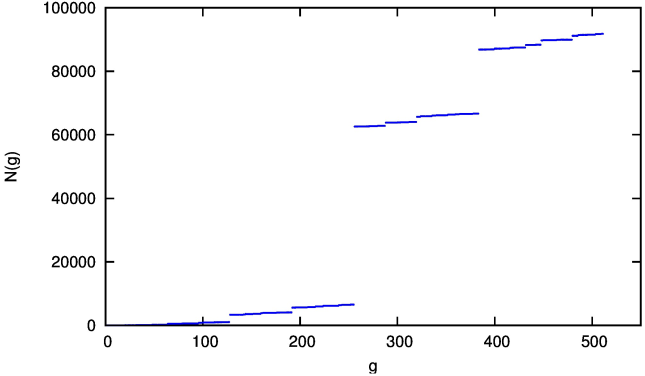
- the *SmallGroups Library*,
- the computer algebra system **GAP** (Groups, Algorithms and Programming)[4]

- SmallGroups library contains information on all finite groups up to order 2000 (except 1024).
- GAP: read information from the library and calculate character tables, irreps,...

[4] www.gap-system.org

How many small finite groups are there?

$N(g)$ ... number of non-Abelian groups of order $\leq g$.



Numbers soon get large. Particularly many groups of orders $2^m 3^n$, *e.g.* $\approx 5 \times 10^{10}$ groups of order $2^{10} = 1024$.

Systematic studies of flavour symmetry groups

With additional restrictions like 3-dim. irreps, ...

$\rightarrow$ *Group scans* up to order $\sim 10^3$ become feasible.
$\rightarrow$ Frequently used in the literature.

Procedure:

- Choose desired properties of groups,
- Use GAP and the SmallGroups library to extract all groups up to a given order (< 2000) fulfilling the conditions,
- investigate the found groups and construct models, derive predictions *etc.*

Powerful tool, but only groups up to some maximal order can be investigated.

SU(3)-subgroups

Often used: groups with 3-dimensional faithful irreps (U(3)-subgroups).

$\rightarrow$ not classified.

But: SU(3)-subgroups classified.
$\rightarrow$ Classification into five classes:

(A) Abelian groups: Of the form $\mathbb{Z}_m \times \mathbb{Z}_p$.

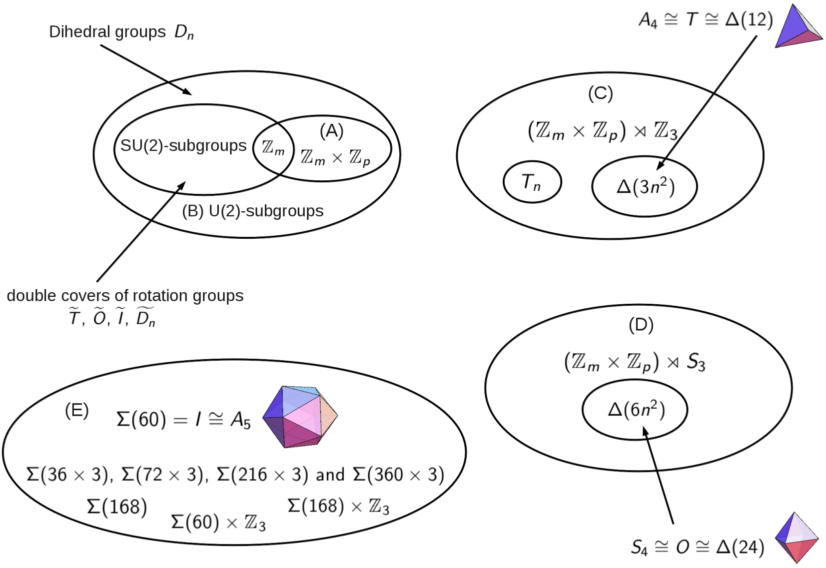
(B) Finite subgroups of $SU(3)$ with faithful 2-dimensional representations [U(2)-subgroups].

(C) Groups of the form $(\mathbb{Z}_m \times \mathbb{Z}_p) \rtimes \mathbb{Z}_3$.

(D) Groups of the form $(\mathbb{Z}_m \times \mathbb{Z}_p) \rtimes S_3$.

(E) Exceptional finite subgroups of SU(3).

Classification of the finite subgroups of SU(3)



General statements possible?

Only SU(3)-subgroups with faithful 3-dim. irreps classified completely.

$\rightarrow$ For all other group-based studies: SmallGroups library can be used.

Can general statements be made without the use of group scans?

$\rightarrow$ One amazing example using number theory instead of group theory.

$\rightarrow$ Uses the framework of residual symmetries in the mass matrices.

Residual symmetries in the mass matrices

Approach intensively studied recently:

$\rightarrow$ Lam; Hernandez, Smirnov; Toorop, Feruglio, Hagedorn; Holthausen, Lim, Lindner; Hagedorn, Meroni, Vitale; King, Neder, Stuart; Lavoura, POL; ...

Transformations leaving mass matrices invariant (lepton sector with Majorana neutrinos):

$$T^\dagger M_\ell M_\ell^\dagger T = M_\ell M_\ell^\dagger, \quad S^T M_\nu S = M_\nu.$$

with T and S unitary. Such transformations *always exist*, because $M_\ell M_\ell^\dagger$ and M_ν are diagonalizeable via

$$U_\ell^\dagger M_\ell M_\ell^\dagger U_\ell = \mathrm{diag}(m_e^2,\, m_\mu^2,\, m_\tau^2), \quad U_\nu^T M_\nu U_\nu = \mathrm{diag}(m_1,\, m_2,\, m_3).$$

Residual symmetries in the mass matrices

$$\Rightarrow T = U_\ell \,\mathrm{diag}\left(e^{i\alpha_1},\, e^{i\alpha_2},\, e^{i\alpha_3}\right) U_\ell^\dagger, \quad S = U_\nu \,\mathrm{diag}\left(s_1,\, s_2,\, s_3\right) U_\nu^\dagger$$

with $s_i \in \{-1, +1\}$.

$$\Rightarrow T \in \mathrm{U}(3), \quad S \in \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2.$$

Idea of residual symmetries: Some of the possible T and S are elements of a discrete flavour symmetry group G_F, *i.e.* they are *residual symmetries*.

$$\text{SSB: } G_F \to G_\ell, G_\nu.$$

$$T_i \text{ generate } G_\ell \subset U(3),\ S_j \text{ generate } G_\nu \subset \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2.$$

Thus, if (strong enough) *residual symmetries* T_i, S_j imposed: U_ℓ and U_ν *determined*!

$\Rightarrow$ Mixing matrix $U = U_\ell^\dagger U_\nu$ completely determined by residual symmetries G_ℓ and G_ν.

Residual symmetries in the mass matrices

- If U completely determined by symmetry $\rightarrow$ mixing matrix independent of masses.
- If U only partially determined, one row or column of U is fixed.
- The elements of U are only determined up to permutation of rows and columns (mass orderings not fixed by symmetry).
- U is only determined up to rephasing. $\Rightarrow$ Majorana phases cannot be predicted.
- Procedure with group scans: Scan for finite groups G_F with U(3)-subgroups (for G_ℓ) and $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$-subgroups (for G_ν). Compute U and compare to experiment.

General statement possible

Using results from number theory (theorem of Conway and Jones about vanishing sums of roots of unity):

R.M. Fonseca and W. Grimus,
Classification of lepton mixing matrices from residual symmetries
JHEP **1409** (2014) 033 [arXiv:1405.3678 [hep-ph]].

Result of this paper:

- Assuming Majorana neutrinos $\rightarrow$ Classification of **all** possible mixing matrices fully determined by residual symmetries.
- 16 sporadic mixing matrices, 1 infinite series of mixing matrices (all found in literature).
- Only members of the infinite series of mixing matrices are compatible with the data at 3σ.

Mixing matrices completely determined by flavour symmetry

Infinite series[5] predicts: $\delta^\ell = 0$

$$|U|^2 = \frac{1}{3}\begin{pmatrix} 1 & 1+\mathrm{Re}\,\sigma & 1-\mathrm{Re}\,\sigma \\ 1 & 1+\mathrm{Re}(\omega\sigma) & 1-\mathrm{Re}(\omega\sigma) \\ 1 & 1+\mathrm{Re}(\omega^2\sigma) & 1-\mathrm{Re}(\omega^2\sigma) \end{pmatrix}; \quad \omega = e^{2\pi i/3}; \quad \sigma = e^{2\pi i p/n}.$$

"trimaximal mixing"

Special case: $\mathrm{Re}(\sigma^6) = 1 \Rightarrow$ tribimaximal mixing, *e.g.* $\sigma = e^{2\pi i/6}$:

$$|U|^2 = \begin{pmatrix} 1/3 & 1/2 & 1/6 \\ 1/3 & 0 & 2/3 \\ 1/3 & 1/2 & 1/6 \end{pmatrix} \xrightarrow{\text{perm.}} \begin{pmatrix} 2/3 & 1/3 & 0 \\ 1/6 & 1/3 & 1/2 \\ 1/6 & 1/3 & 1/2 \end{pmatrix}$$

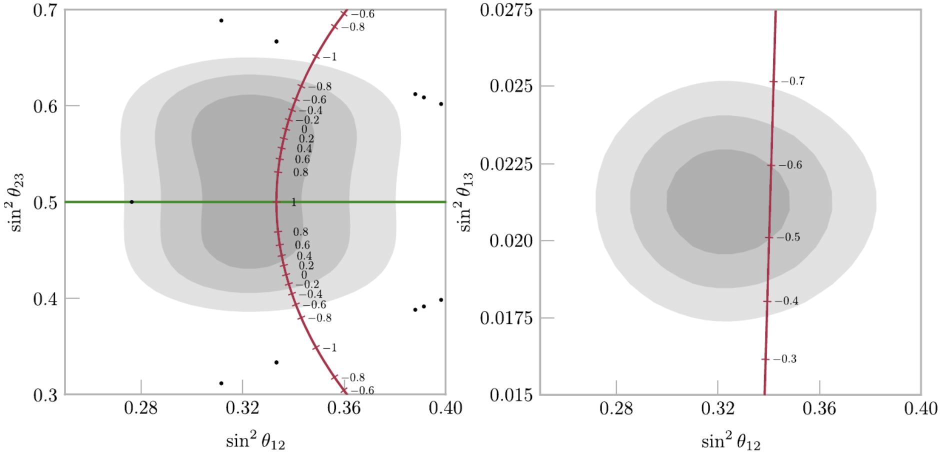
Physical predictions of infinite series only depend on $\mathrm{Re}(\sigma^6)$.

Allowed range at three sigma: $-0.69 \lesssim \mathrm{Re}(\sigma^6) \lesssim -0.37$

[5] Series studied earlier in: R. de Adelhart Toorop, F. Feruglio and C. Hagedorn, Nucl. Phys. B **858** (2012) 437; C. Hagedorn, A. Meroni and L. Vitale, J. Phys. A **47** (2014) 055201.

Infinite series → trimaximal mixing

Plots by courtesy of W. Grimus and R. Fonseca (taken from “Classification of lepton mixing matrices from residual symmetries” [arXiv:1405.3678 [hep-ph]]).



Infinite series $\rightarrow$ trimaximal mixing

$\rightarrow$ Smallest symmetry group G_F which can give a mixing matrix in agreement with experiment at 3σ is a group of type (D):

$$(\mathbb{Z}_{18} \times \mathbb{Z}_6) \rtimes S_3 \quad \text{of order 648.}$$

$$\Rightarrow \sigma = \exp(2\pi i/9) \Rightarrow |U|^2 \approx \begin{pmatrix} 0.65 & 0.33 & 0.02 \\ 0.28 & 0.33 & 0.39 \\ 0.08 & 0.33 & 0.59 \end{pmatrix}$$

$$\Rightarrow s_{12}^2 \approx 0.33,\ s_{23}^2 \approx 0.40,\ s_{13}^2 \approx 0.020.$$

$\rightarrow$ Alternative to TBM, however needs much larger group (TBM needs S_4 of order 24).

Résumé: Residual symmetries in the mass matrices

Résumé:

- All possible mixing matrices determined through residual symmetries known.
- All these mixing matrices have been studied in the literature. No convincing model has emerged.
- Very large flavour symmetry groups needed to reproduce experimental results.

Abelian flavour symmetries and texture zeros

Want to study flavour symmetries where masses and mixing not decoupled.

Simplest possibility: Abelian flavour symmetries $\leftrightarrow$ texture zeros.

$\rightarrow$ **Relations between fermion mass ratios and mixing angles!**

$\rightarrow$ Goal: Systematic studies of texture zeros in the lepton and quark sector.

Systematic analysis of texture zeros in the lepton sector

Setting:

	fields	mass term	mass matrix
charged leptons	$3\ \ell_{iL}$, $3\ \ell_{iR}$	$-\overline{\ell_L} M_\ell \ell_R + \text{H.c.}$	M_ℓ complex 3×3
Dirac νs	$3\ \nu_{iL}$, $3\ \nu_{iR}$	$-\overline{\nu_R} M_D \nu_L + \text{H.c.}$	M_D complex 3×3
Majorana νs	$3\ \nu_{iL}$	$\frac{1}{2}\nu_L^T C^{-1} M_L \nu_L + \text{H.c.}$	M_L c. symm. 3×3

Mostly studied situations:

- Diagonal M_ℓ + TZ in the neutrino mass matrix[6]

  Example: $M_\ell \sim \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & 0 \\ 0 & 0 & \times \end{pmatrix}$, $M_D \sim \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & 0 \\ \times & \times & \times \end{pmatrix}$.

- TZ in M_ℓ and M_ν, both non-diagonal + additional restrictions (e.g. Hermitian mass matrices, Fritzsch textures).

[6] *Dirac νs:* studies by Hagedorn, Rodejohann,. . . ; *Majorana νs:* Frampton, Glashow, Marfatia; Xing,. . .

Systematic studies of texture zeros in the lepton mass matrices

$\rightarrow$ Our aim: Study **all types of texture zeros (without other imposed symmetries)** in M_ℓ and M_ν.

POL and Walter Grimus:
A complete survey of texture zeros in the lepton mass matrices.
JHEP **07** (2014) 090 [arXiv:1406.3546].

Three parts:

1. Classification of texture zeros in the lepton mass matrices.
2. Which sets of texture zeros are compatible with the experimental observations?
3. What is the predictive power of texture zeros?

Classification of texture zeros in the lepton mass matrices

How many different patterns of texture zeros (PTZ) in the lepton mass matrices?

$\rightarrow$ number is **huge**:

- Dirac neutrinos: (M_ℓ, M_D): $2^9 \times 2^9 = 262144$ different PTZ.
- Majorana neutrinos: (M_ℓ, M_L): $2^9 \times 2^6 = 32768$ different PTZ.

Freedom of **weak-basis transformations** $\rightarrow$ some PTZ are **physically equivalent**.

$\Rightarrow$ Can divide PTZ into equivalence classes.

- Dirac neutrinos 2^{18} PTZ $\rightarrow$ **570 classes.**
- Majorana neutrinos 2^{15} PTZ $\rightarrow$ **298 classes.**

χ^2-analysis

Tested which PTZ are compatible with the experimental observations **at tree-level** by means of a χ^2-analysis.

About **75%** of all PTZ are **compatible** with the exp. observations!

For further investigation: Went through list of compatible textures and kept only maximally restrictive PTZ.

Maximally restrictive patterns of texture zeros

A set of texture zeros compatible with experiment is **maximally restrictive**, if imposing an additional texture zero it becomes incompatible with the physical observations.

$\rightarrow$ Out of all possibilities: Only about **30 maximally restrictive viable classes of texture zeros** for each Dirac and Majorana neutrinos and normal and inverted neutrino mass spectrum.

Predictivity measures for models with texture zeros

Predictive power of texture zeros?

Idea behind our predictivity measures

Given a viable set of texture zeros and **fixing all but one** of the (already measured) observables to their experimentally observed values, how much can the **remaining observable at most deviate** from its experimental or best-fit value?

$\rightarrow$ Defined such measures w.r.t. all **lepton physics observables**.

Results of the predictivity analysis

Predictive power of the **maximally restrictive PTZ** compatible with the data:

	Dirac	Majorana
charged-lepton masses	✗ (none)	✗ (none)
ν-oscillation parameters	✗ (none)	✗ (none)
smallest ν-mass m_0	✓ (mostly $m_0 = 0$)	✓ (mostly $m_0 = 0$)
Dirac phase δ	✓ (only 1 model)	✓ (mostly $\delta \sim \pi$)
Majorana phases	—	✗
$m_{\beta\beta}$	—	✓

Comment on $m_{\beta\beta}$:

Normal spectrum: $m_{\beta\beta}$ can be large ($> 0.1\,\mathrm{eV}$) or small.

Inverted spectrum: $m_{\beta\beta} < 0.1\,\mathrm{eV}$.

$\rightarrow$ *Only weak predictive power!*

Systematic analysis of texture zeros in the quark sector

Did the same analysis for the quark mass matrices: All possible texture zeros in M_d and M_u, without other imposed symmetries.

POL and Walter Grimus:
A complete survey of texture zeros in general and symmetric quark mass matrices, arXiv:1501.04942 [hep-ph].

Choice of scale at which the zeros should be realized: $\mu = M_Z$.

Reason: At higher scales running of quark masses and mixing angles becomes model-dependent. $\rightarrow$ Statements not possible without implementation of the texture zeros in a specific model!

Main difference to study of Dirac neutrinos: In quark sector **all 10 parameters known** (six masses, three mixing angles, one phase).

Texture zeros in general quark mass matrices

Result for general quark mass matrices:

27 maximally restrictive textures. None of them can predict any observable.

Similar to Dirac neutrinos: There only the not yet measured observables could be predicted.

$\rightarrow$ Add further constraints. *E.g.* assume symmetric quark mass matrices (can be motivated in SO(10)-GUTs.)

Result: 15 maximally restrictive textures. Several of them are predictive w.r.t. some of the light-quark masses m_u, m_d, m_s.

Most predictive texture in symmetric quark mass matrices

$$M_d \sim \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix}, \quad M_u \sim \begin{pmatrix} 0 & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & \times \end{pmatrix}$$

$$\Rightarrow \sin\theta_{12}^q \simeq \sqrt{\frac{m_d}{m_s}}, \quad \sin\theta_{13}^q \simeq \sqrt{\frac{m_u}{m_t}}.$$

$\rightarrow$ Example for a framework with

Strong mass hierarchy $\leftrightarrow$ Small mixing angles.

Mild neutrino mass hierarchy and large lepton mixing?

Have seen that in quark sector texture zeros can imply:

Strong mass hierarchy $\leftrightarrow$ Small mixing angles.

Idea in lepton sector: Texture zeros leading to

Mild neutrino mass hierarchy $\leftrightarrow$ Large mixing angles?

Maximal atmospheric neutrino mixing from texture zeros

Frampton, Glashow, Marfatia (2002)[7]:

$$M_\ell = \mathrm{diag}(m_e,\, m_\mu,\, m_\tau) \Rightarrow \text{no effect on } U_{\mathrm{PMNS}}$$

In this framework: Seven experimentally viable cases of two (independent) texture zeros: $M_\nu \sim$

$$A_1:\begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix},\quad A_2:\begin{pmatrix} 0 & \times & 0 \\ \times & \times & \times \\ 0 & \times & \times \end{pmatrix},\quad B_1:\begin{pmatrix} \times & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix},$$

$$B_2:\begin{pmatrix} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & 0 \end{pmatrix},\quad B_3:\begin{pmatrix} \times & 0 & \times \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix},\quad B_4:\begin{pmatrix} \times & \times & 0 \\ \times & \times & \times \\ 0 & \times & 0 \end{pmatrix},$$

$$C:\begin{pmatrix} \times & \times & \times \\ \times & 0 & \times \\ \times & \times & 0 \end{pmatrix}.$$

[7] P.H. Frampton, S.L. Glashow and D. Marfatia, Phys. Lett. B 536 (2002) 79.

Maximal atmospheric neutrino mixing from texture zeros

For B_3 and B_4 one finds that in the limit of a quasi-degenerate neutrino mass spectrum (*i.e.* very mild hierarchy):

$$|U_{\mu j}| \approx |U_{\tau j}|,$$

which implies

$$\sin^2\theta_{23} \approx \frac{1}{2} \quad (\Leftrightarrow \theta_{23} \approx 45^\circ)$$

and

$$\sin\theta_{13}\cos\delta \approx 0.$$

- Close to maximal atmospheric neutrino mixing.
- Since θ_{13} not very small: close to maximal CP violation ($\delta = \pi/2,\, 3\pi/2$).

Maximal atmospheric neutrino mixing from texture zeros

Approximate expression for $\sin^2\theta_{23}$ for cases B_3 and B_4 in the limit of quasi-degeneracy of the neutrino masses:

$$\sin^2\theta_{23} \simeq \frac{1}{2} \mp \frac{1}{8}\frac{\Delta m_{31}^2}{m_1^2}(1+\sin^2\theta_{13}) + \ldots$$

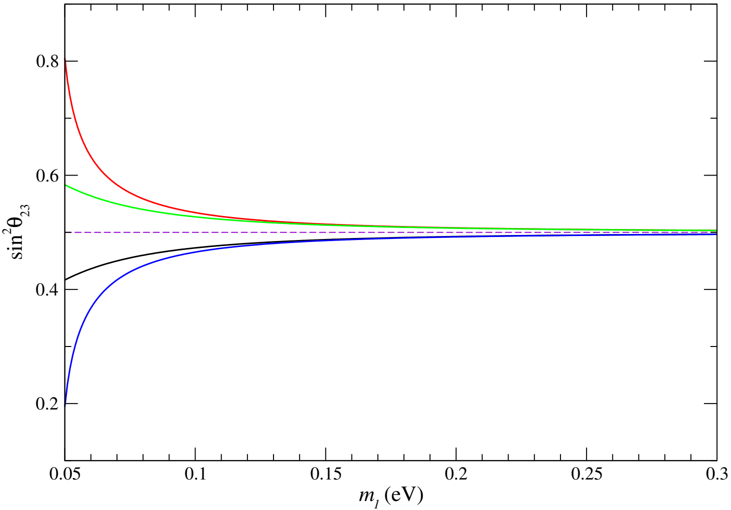
In the limit of quasi-degeneracy:

$\theta_{23} \to 45^\circ$ **irrespective** of the values of θ_{12} and θ_{13}!

Maximal atmospheric neutrino mixing from texture zeros

(Δm^2_{21}, Δm^2_{31}, $\sin^2\theta_{12}$ and $\sin^2\theta_{13}$ fixed to best fit values.)

B_3 (inverted) B_4 (normal) B_3 (normal) B_4 (inverted)



III. Summary and conclusions

Summary

- No compelling experimental hints for flavour symmetries.
- Main theoretical motivations:
  - Difference between quark and lepton mixing.
  - Relation between mass hierarchies and mixing angles?

Theoretical developments:

- Group scans using the SmallGroups library.
- Residual symmetries in the lepton mass matrices.
- Classification of all mixing matrices fully determined by residual symmetries (independent of group scans; general argument using number theory).
- Systematic investigation of texture zeros:
  - "Pure" texture zeros show no predictive power.
  - Need additional assumptions like symmetric mass matrices, quasi-degenerate neutrino mass spectrum.

Topics not covered in this talk: Generalized CP-symmetries, model building, explanations of fermion mass hierarchy.

Conclusions

- Experimental hints for flavour symmetries are weak.
- $\rightarrow$ Transition from model building to systematic studies.
- Wish to "understand" flavour sector still keeps the field active.

Thank you for your attention!